

Elastic Foam Characterization for Rate-Dependent Nonlinear Material Model Development

by

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Abstract of “Elastic Foam Characterization for Rate-Dependent Nonlinear Material Model Development”

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Elastic polymer foams are ubiquitous in cushioning, impact protection, and similar applications. Consisting of solid elastic polymer matrix and gaseous phases, elastomeric foams are classified into two major categories: open-cell and closed-cell. These structures elicit a behavior typified by tension-compression nonlinearity and asymmetry, shear-coupling, and a nonlinear Poisson’s function. The compressive response characteristically displays a linear-plateau-densification stress-strain profile. Many foams are anisotropic, locally non-affine, and rate- and temperature-sensitive due to microstructural and material effects.

A suite of experiments and tools were designed to evaluate foam behavior and inform model development. This was accomplished in three phases: first, setup and instrumentation; second, quasistatic experiments with a non-localizing, isotropic foam; and third, multi-rate experiments for viscoelasticity. The experimental platform consisted of custom load frames instrumented for digital image correlation (DIC). A new DIC algorithm, using the iterative deformation method and quality factors for cross-correlation, overcame challenges of measuring full-field strain on foam. Stress-strain and axial-transverse strain from tension and compression experiments were used to calibrate a novel continuum model co-developed with collaborators. This model was validated using shear without and with pre-compression, indentation, and inhomogeneous tension. Higher strain-rate experiments were used to calibrate time-dependency in an extended model.

The total corpus of work includes new experimental and analysis tools and results to investigate the mechanics of elastic polymer foams, namely: a new DIC algorithm; an experimental and analytical platform for model development; and, experiments and tools for strain rates ranging from quasistatic to impact-like. These advances yield an improved workflow for characterizing, and thus utilizing, elastomeric foam in engineering applications.

Curriculum Vitæ

For undergraduate work, Alex attended Clarkson University (Potsdam, NY), where he received a Bachelor of Science with Honors in Mechanical Engineering with a Minor in Mathematics in 2014. In 2014 he began graduate work at Brown University with Dr. K.-S. Kim and Dr. Christian Franck in the Mechanics of Solids group in the School of Engineering. He was granted a Master of Science in Engineering from Brown University in 2016. He served as a supplemental teaching assistant for high-school summer courses, undergraduate-, and graduate-level courses. The final year of his graduate program was spent co-appointed as a visiting scholar at the University of Wisconsin-Madison in the Mechanical Engineering department.

Alex was awarded the James R Rice Graduate Fellowship in Solid Mechanics in 2014-15 and was a National Science Foundation Graduate Research Fellowship Honorable Mention in 2014. In 2015 he was awarded a National Science Foundation Graduate Research Fellowship, and subsequently was a Society of Experimental Mechanics International Student Paper Competition finalist in 2018. He was granted a National Institute of Standards and Technology National Research Council Postdoctoral Research Associateship in 2019.

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Chapter 1

Introduction

1.1 Motivation

Protective equipment has been a field of remarkable importance for millennia. For example, in ca. 600 BCE the Servian constitution likely reorganized Etruscan armies in early Rome into five classes on the basis of the level of protective equipment citizen-soldiers could procure. Later the advent (ca. 50 BCE) and adoption (ca. 120 CE, during the reign of Hadrian) of the heavily armored cataphracterii [1] were pivotal to the evolution of Roman military forces. Throughout the Middle Ages and Early Modern periods in Europe, hard protective equipment, e.g. the stereotypical suit armor of the chivalric period, was a definitive feature of warfare and power projection until the widespread advent of gunpowder weapons in the 17th century radically altered the purpose and scope of protective equipment on the battlefield [2]. Monolithic armor was generally confined to Europe – in China, leather, lamellar, and scale armors were developed from at least the 4th century BCE [3] until the advent of modern weapons. These armor concepts have typically relied on hard external shell-like layers with some amount of cushioning provided leather and/or cloth backing or under-layer. Soft armors, such as boiled leather or mail provided complaint solutions, but offered limited protection from bludgeoning and/or piercing – this soft-armor limitation is one that modern advanced polymer equipment may be able to eliminate.

In the modern paradigm, concepts of armoring have been translated to numerous fields of endeavor beyond the battlefield, although battlefield protection remains a key issue. In civilian pursuits, such as gridiron and association football, hockey, or other sports, hard plastic armoring or helmets backed by a layer of cushioning foam is ubiquitous. In warfare, metal-plate armor has largely been relegated to the province of vehicles, and “lightweight” ceramics [4] and woven composite, e.g. Kevlar [5], have become the norm for personnel armor and helmets of armed forces worldwide. To be effective protective equipment must first be worn, and thus design for low weight, comfort, and ease-of-use is an important consideration – hence soft-armor concepts. Furthermore, to encourage adoption and make protective equipment universally available and saleable, inexpensive solutions are important. Meeting these criteria involves sophisticated design engineering and advanced materials.

A staple material for cushioning and impact protection, two key aspects of protective equipment, is the randomly porous elastomeric polymer (i.e. elastomeric foam), of which many types exist and are under active development [6–8]. Elastomeric foams consist of a heterogeneous distribution of an elastic polymer matrix material and gas-filled free space, as shown in the example images in Fig. 1.1 (see also Sec. 1.2.1). Depending on matrix polymer and geometrical properties these foams can exhibit high specific energy absorption due to dissipative micromechanical and matrix material effects, (i.e., viscoelasticity) and offer the capability to withstand repeatable deformations, and hence use [9] (see also Sec. 1.2.2). Due to the porous nature foams are lightweight – relative densities (i.e. $\frac{\rho^*}{\rho_0}$, where ρ^* is the bulk density of the foam and ρ_0 is the bulk density of the matrix material) are typically lower than 30% and as low as 0.3% or lower – and due to the random nature can be inexpensively fabricated at commercial process scales [10]. These aspects make elastic polymer foams attractive for modern protective equipment. However, to effectively design and deploy solutions using materials exhibiting complicated constitutive responses, such as advanced composites [11] or foams [10], material models must be sufficiently stable and predictive, which generally requires thorough understanding of material. For models to be adopted in practice, they must be straightforward to calibrate and/or contain a minimum number of parameters.

To this end, this work focuses on the characterization of low relative density, isotropic, non-localizing, open cell polyurethane-based foam (the importance of each of these features is discussed presently). This provides as a starting point within the realm of elastic polymer foams, since this class of material is widely used for protective equipment. The experimental aspects of a combined experimental-computational effort to understand and develop a tractable and predictive hyperelastic continuum model will be emphasized.

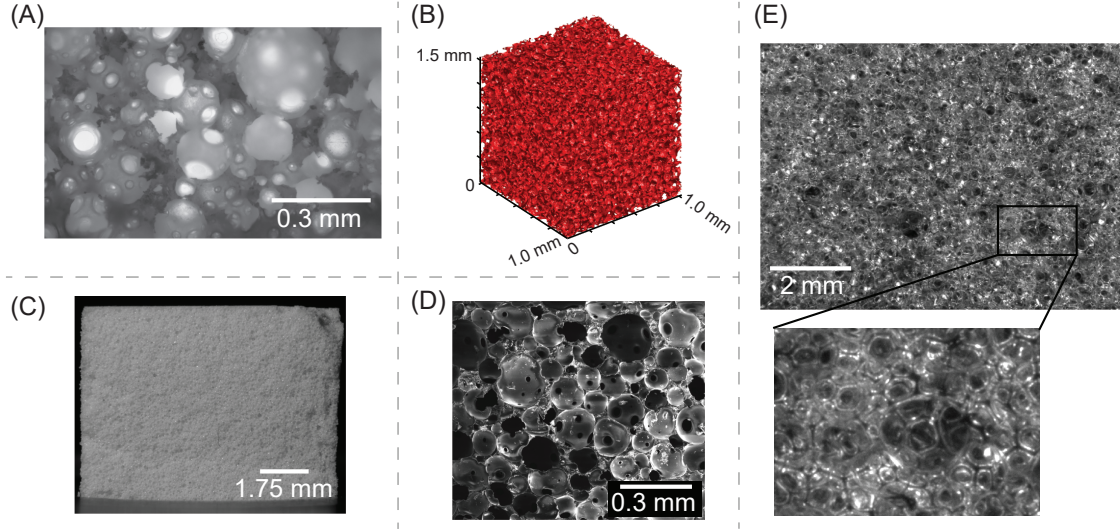


Figure 1.1: Images of open-cell polymer foams at various scales and using assorted imaging modalities. (A) Optical microscopy image of a moderate density polyurethane (PU) foam. (B) Micro-computed tomography image of the same PU foam. (C) Standard SLR camera image of the same PU foam. (D) Scanning electron microscopy image of the same PU foam. (E) Standard scientific camera with tele-microscopy lens image of a low-density helmet liner foam; *inset* Digital zoom-in of a single large pore. In all images, note the random cellular structure with moderate to low relative density of the matrix polymer material.

1.1.1 Challenges of elastomeric foams

General and predictive models for elastomeric foams remain elusive, despite concerted effort to generate representations based on, e.g., hyperelastic theory [12–17], micromechanical principals [18–21], or direct finite element modeling [22–26] (see Sec. 1.2.4 for a more complete summary of modeling aspects). Due to the elastomeric matrix material and porous structure, these foams are compliant and capable of undergoing large, reversible deformations involving substantial tension-compression asymmetry and changes in volume. The porous structure

typically gives rise to strong isochoric-volumetric coupling and a varying Poisson’s function, see for example [27–29]. In addition, many elastomeric foams display a highly dissipative, time-dependent mechanical response, manifesting in strain rate-dependence, hysteresis under reversed loading, and changes in strain localization characteristics [30–33].

Of interest for the hyperelastic response are both the high degree of non-linearity including substantial asymmetry between tension and compression, and the non-linear asymmetric Poisson’s function (i.e. instantaneous tangent Poisson’s ratio; see, e.g., [29,34]). These features are due to the differing responses of the (typically) gas-filled pore space in compression and tension, buckling of cell walls in compression, and other microstructural characteristics. In compression the initial response is approximately linear elastic – broadly mimicking the base polymer – but as compressive strain increases the cell walls of the pores undergo elastic buckling, leading to a plateau in uniaxial stress and transverse strain [7, 10, 35–37]. As the free void space is reduced toward zero, i.e. the apparent density approaches that of the bulk polymer, the stress begins to increase rapidly. As diagrammed in Fig. 1.2 these three domains of the response are referred to as the “linear elastic”, “plateau”, and “densification” regions. For similar reasons, analogous phenomena occur in the axial versus transverse strain function. In tension, the response is weakly non-linear and the Poisson’s function is idealized to take constant value of $\nu = 0.25$ [15] (in experiments, see Chapter 3 or [28], I observe a value of approximately $\nu = 0.27$).

By conducting several example experiments, one can easily observe several additional challenging responses of a candidate polymer foam that will be address in this corpus of work. The specific foam in this test case is Poron XRD (provided to me in sheet form by Rogers Corp., Rogers, CT) – an elastic polyurethane-based foam that is used in sports equipment and cushioning applications, and which will be the major focus of the characterization efforts of Chapters 3 and 4. The observed characteristics of Fig. 1.3 are typical for this class of foamed elastomers [10]. Three key aspects of the response in roughly increasing generality are: increased shear compliance and dissipation under applied pre-compression (i.e., multiaxial loading - see also Sec. 1.2.3 and [38]) as shown in Fig. 1.3A, strain-rate sensitivity (Fig. 1.3B), and changes in both compliance and response “regime” (i.e. earlier onset of densification)

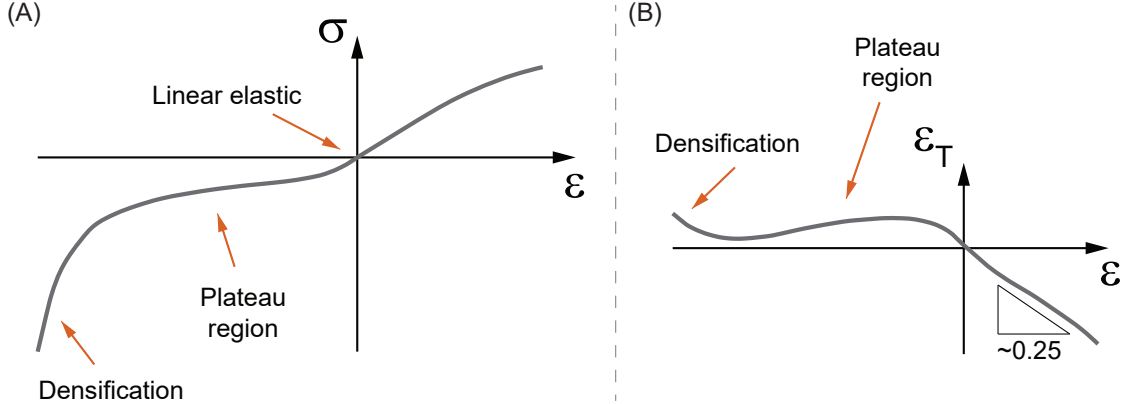


Figure 1.2: Schematic diagrams of the typical stress-strain and axial-lateral strain responses to applied uniaxial stress for an open-cell elastic foam. (A) Axial stress-strain, note the asymmetry with a highly nonlinear compressive response exhibiting the linear-plateau-densification behavior typical of large deformation in foam. These elastic foams typically do not have stress peaks or softening behavior in the linear-plateau regime. While nonlinear, the tension response is typically a much simpler form than compression. (B) The typical instantaneous (i.e., secant) Poisson's function. Note the asymmetry of similar form to stress-strain and the approximately $\nu \approx 0.25$ Poisson's ratio in tension.

with relative void space within the foam – quantified via the void volume fraction or porosity, $\phi_0 = 1 - \rho^*$, see Fig. 1.3C. Each of these aspects is important to capture the total response of the foam, but require extensive experimental and modeling effort to systematically observe and faithfully represent in a tractable form.

The most straightforward and universal of the three parameters is the void volume fraction (ϕ_0 , equivalently, the relative density ρ^*), which is often measured via bulk density for foams with well-known matrix materials, heuristic mathematical estimates based on pore geometry, or micro-computed tomography, and is a direct experimental parameter used nearly universally in modeling efforts [10]. Rate-sensitivity is a common feature of many foams, particularly polymer foams, and has been attributed to several sources [7] including micro-inertia of ligaments (e.g. carried by compaction shock fronts, e.g. [39,40]), viscosity of void space fluid [17,41], and simple rate-sensitivity of the matrix material [34,42,43]. See Chapter 4 for more details regarding rate-sensitivity. Finally, the most elusive and poorly understood of these three effects in the isochoric-volumetric (also referred to as volumetric/distortional) coupling under multi-axial loading. Although this behavior has been observed in experiments, notably [38,44,45], it has seldom been considered during modeling

efforts.

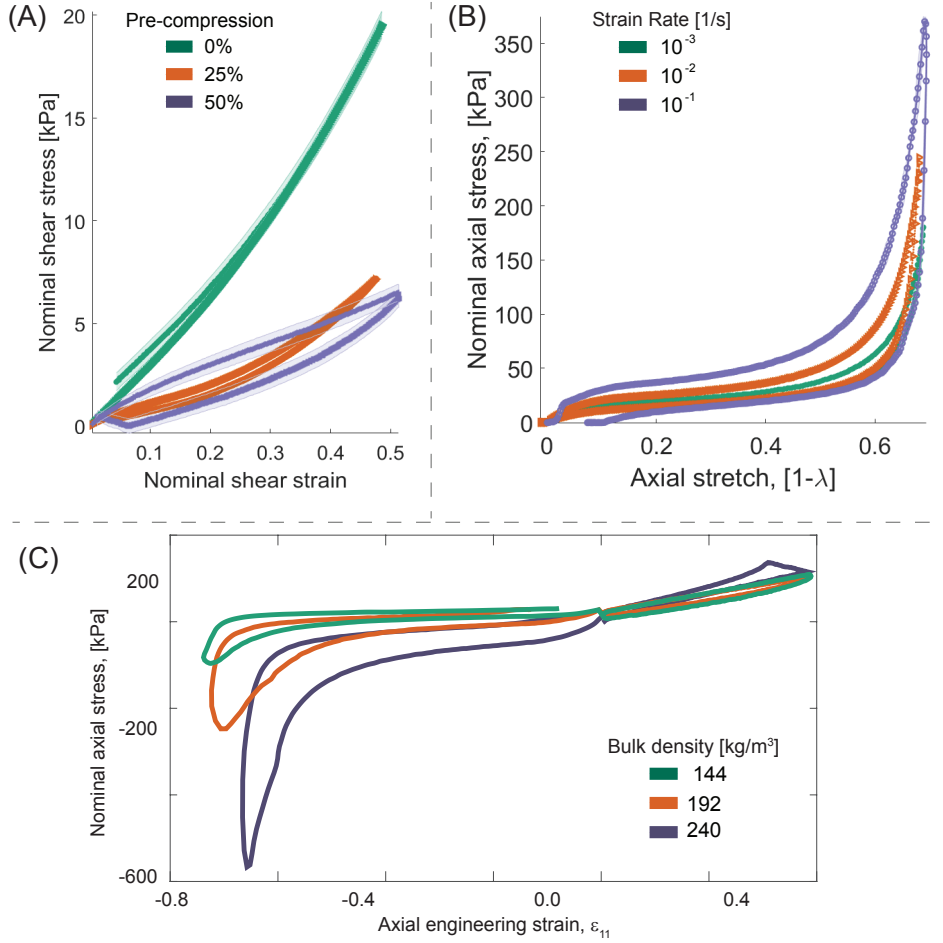


Figure 1.3: Various experimental results showing assorted dependencies of an elastic polymer foam (Poron XRD) to applied deformation. (A) Increased shear compliance with various levels of compressive deformation applied prior to shear (volumetric-deviatoric coupling). (B) Strain rate sensitivity showing that elevated rates lead to stiffer response with increase dissipation. This is generally a matrix-material dominated effect. (C) Uniaxial stress-strain response for various bulk densities of the same matrix polymer, showing overall behavior sensitivity to structural parameters (i.e., the void volume fraction ϕ_0).

Finally, there remain several important aspects of the deformation behavior of many (elastomeric) foams that are typical, but less general and can often be avoided through careful selection of material and/or experimental preparation, these include: strain localization, anisotropy, and temperature dependency. These are outside the scope of the present work to account for directly, but either do not occur in the candidate XRD material in typical testing (viz. localization) or at in the complete material response (viz. anisotropy) or have

been explicitly accounted for in the experimental programs (viz. temperature). For example, a high-porosity, very-low-density foam manufactured by Team Wendy for US Army helmet liners shows the occurrence of both localization (1.4A; see also [34, 46–48]) and anisotropy (Fig. 1.4B; see also [49–51]) in baseline experiments. In Chapter 5 I describe an experiment to observe this directly, but suffice to say, these are important aspects of the foam response that require more advanced models (e.g., [52]) and experiments to adequately capture. Additionally, since elastomeric foam are typically formed from polymers in the rubbery temperature regime, the ambient temperature effects the overall stress-strain response of the foam (again, preliminary experiments demonstrating this phenomenon are discussed in Chapter 5; see also [37, 53–57]). For polyurethane foams, e.g. the XRD candidate material, the operational temperature of protective equipment will be relatively near to the glass transition temperature of the base polymer, and thus marked mechanistic changes are likely with changes in operational environment.

1.1.2 Approach

In this work, I describe a combined experimental and computation program to develop material models for elastic polymer foams that are predictive over a wide range of strain magnitudes, deformation modes, and strain rates. Throughout, we employed experimental and computational tools tractable to implement (or have provided open-source implementations) and calibrate for researchers and practicing engineers. By coupling the experimental and computational efforts, we attempted to tailor both the experimental program and modeling techniques to complement one another. In this way, we approach this challenging material in a systematic way, and obtain as complete a model as possible given the inevitable constraints of time, equipment, and funding. To break down the general problem of capturing the behavior of elastomeric foam, we isolate several key aspects of the foam response by selection of material and experimental parameters to obtain baseline features, before increasing the complexity. As a starting point, we begin our characterization effort where current models struggle – continuum-level large reversible deformations and multi-mode deformation. To solve the hyperelastic continuum formulations the ubiquitous finite element (FE) method has

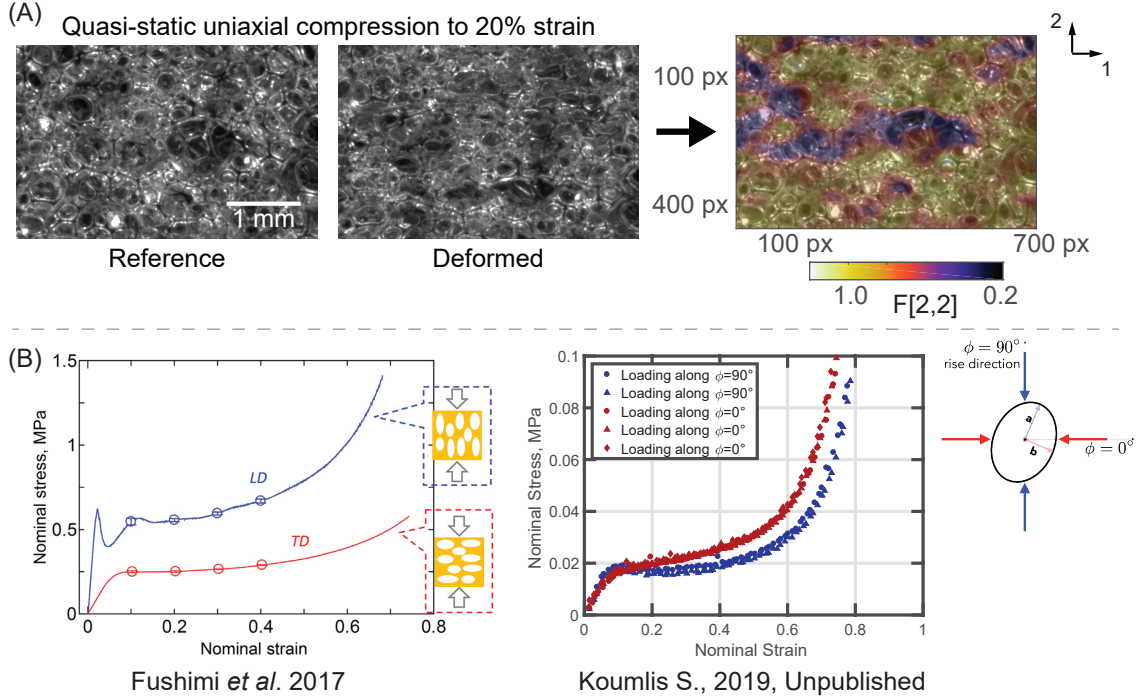


Figure 1.4: Illustrations of the development of strain localization and anisotropic response in a low-density elastomer foam. (A) Images of the reference and deformed (20% applied global compressive strain) and the resultant map of the axial component of the deformation gradient tensor, $F[2,2]$. This shows the highly inhomogeneous distribution of strain. (B) (Left) Example of anisotropy in the very-low-density polymer foam from Fushimi *et al.* [51] showing a dramatic difference between the longitudinal direction (LD) and transverse direction (TD) in both stress magnitude and response characteristics. (Right) Example stress-strain plots from specimens excised at 0deg and 90deg from the rise direction of the foam shown in (A), demonstrating a difference in stress magnitude, but not mechanism (from unpublished experiments conducted at Drexel University by Stylianos Koumlis).

been used with a non-linear large-deformation material model informed by, and calibrated to, experimental measurements.

The breakdown of key aspects to reconstruct to complete foam response was conducted as follows. First, preliminary experimental work to establish a suitable material and basic deformation characteristics was conducted by Ms. Jennifer Cardona and Mr. Matthew Petersen under my advisement at slow, non-quasistatic strain rates (typically $\dot{\epsilon}_{\text{applied}} \approx 10^{-3} \text{ 1/s}$). Second, informed by this and other initial data, which included isotropy testing and stress relaxation, and subsequent initial modeling efforts, I began with uniaxial stress experiments (compression first, then tension). By employing a non-localizing foam at a

single density and quasistatic strain rate ($\dot{\epsilon}_{\text{applied}} = 2 \times 10^{-4} \text{ 1/s}$) the constitutive modeling requirements are simplified to fitting a stress-strain and an axial-lateral strain function. Third, I then incorporate multiple densities of foam into the experimental program (see Chapter 3). Fourth, with a fully developed model and several validation experimental configurations, I scale across strain rates for a fixed density, and then expand to multiple densities (see Chapter 4). In a similar manner, the effects of temperature, anisotropy, and/or localization can be incrementally probed experimentally and added to the modeling effort. Although a cursory discussion of these aspects is given in Chapter 5, a complete study of these aspects remains as future work.

Collecting complete and accurate experimental measurements is its own challenge. I have chosen to use full-field 2D kinematic measurements (i.e., in-plane surface displacements and strains) in conjunction with engineering stresses. Full-field 2D data presents a balance between completeness of data and obtainability of testing equipment for model calibration. Due to the inherently rough and changeable surface topology of foams undergoing large deformations a new, bespoke digital image correlation algorithm (qDIC, see Chapter 2 and [27]) was developed to handle the low signal-to-noise environment. The fully 2D nature of the displacement data collected was used for internal experimental validation (e.g., checking for unwanted shear in uniaxial experiments or evidence of localization in compression) or for validation of the computational model in inhomogeneous loading configurations – model calibration used only axial and transverse strains and axial engineering stresses.

To generate appropriate data for model development and validation I designed and deployed several new experimental testing modes that were matched to simulations (e.g., see Chapter 3 and [28]). To begin, I adapted custom load frame from a micro-indentation developed by Dr. Eyal Bar-Kochba for my elastomeric foam specimens of approximately $1 \times 1 \times 1 \text{ cm}^3$ volume. Experiment designs included new strategies for uniaxial compression and tension specialized for the characteristics of low-density polymer foams, i.e., soft, large deformation, large volumetric strains, and challenging lubrication. Additionally, a double-lap shear fixture to impose simple-shear-like deformation was designed and used as a validation case. Both spherical and conical indentation experiments were implemented. Finally, an

inhomogeneous tension mode for validation was implemented by modification to the standard tension specimen.

To address rate dependence and dissipation effects in the foams, experiments were conducted over a range of strain rates (see Chapter 4). Three primary means of elevated strain rate testing were designed and implemented, and two were used in practice. First, the system used for quasistatic experiments was reconfigured to operate at strain rate ranges up to approximately 10^{-1} 1/s. Second, we built a new linear-induction-based linear actuator-driven load frame with faster actuation and velocity reversal, coupled with a high-speed camera, extending the strain rate regime up to about $\dot{\epsilon}_{applied} = 10^2$ 1/s in compression and $\dot{\epsilon}_{applied} = 40$ 1/s in tension. Third, a drop-weight-based system was designed and built for strain rates ranges between $\dot{\epsilon}_{applied} = 10^2$ 1/s and $\dot{\epsilon}_{applied} = 10^3$ 1/s in compression (this system was ultimately unused for this work due to limitations of testing capabilities, i.e. compressive loading with non-constant strain rate, and time). Design and implementation of the second and third systems was led, under my advisement, by Mr. Jialiang Tao with initial development of the drop-weight system by Ms. Lauren Chanen. With these experimental platforms in place, I am able to conduct experiments and generate data for soft polymer foams over strain rate range from quasistatic $\dot{\epsilon}_{applied} = 2 \times 10^{-4}$ 1/s to impact-like $\dot{\epsilon}_{applied} = 10$ 1/s rates – more than five decades of strain rate in total – and probe the entire range of use-typical deformation rates.

1.2 Background

Many excellent summaries exist for both individual classes or testing regimes of foam material and the field of cellular materials in general (e.g. [8], [7], or [58]). In particular, see the canonical textbook of Gibson and Ashby [10] and, for a more theoretical treatment of micromechanics aspects, Nemat-Nasser and Hori [59] or the overview of porous materials edited by Ehlers and Bluhm [60]. A recent summary of elevated rate behaviors in foams by Sun and Li ([7]) presents a through outlook on the state of our understanding of dynamic-rate events in foam. In this section I briefly summarize the classes of materials and discuss

approaches for both experiments and modeling, focusing on those most relevant to elastic open-cell polymer foams. To begin, differences in foam structure are described, i.e. as relating to geometrical considerations necessary for experiments and modeling. This is followed by a discussion of the classes and characteristics of polymer foams. Various experimental techniques and efforts to understand foams are introduced, and finally, modeling techniques are briefly discussed.

1.2.1 Foam structure

For randomly dispersed 3D cellular solids (i.e., “foams”) the two broadest categories are closed-cell and open-cell. Here, I define the solid phase the matrix material and the remaining open volume (whether truly evacuated void, or gas- or fluid-filled) as “void” space, and the geometry of the matrix-and-void configuration as the “microstructure” of the foam. In closed-cell foam, each void is separated from its neighbors (see the schematic in Fig. 1.5A), typically by membrane-like webs, shells, or walls in moderate to low density foams, and the matrix material is contained in edges, vertices, and faces. In contrast, in open-cell foam the void spaces are interconnected and the solid material is contained in edges and vertices [10, 61] (see the schematic in Fig. 1.5B), which are often termed ligaments or struts. The void volumes are often called pores (in particular for high relative density foam) or cells. Open-cell foams are generally gas- or liquid-permeable, whereas closed-cell foams are impermeable [6, 61]. Often, a mixture of cell types exist (i.e., closed-cell foams typically have 1%-10% open-cell content [6]) due to the random nature of the foaming process [62]. Details of geometrical features including edges, vertices, faces, cell size and shape are a key factors in deformation response [57, 58, 63–66] and have been extensively studied – as a representative sample, reports have been observational [36, 67–72], as part of modeling efforts [24, 35, 73–78], and for computational mesh generation [7, 23, 26, 61, 79–84]. Since we focus predominately on continuum-level response, an in-depth discussion or analysis of these aspects is beyond the scope of this summary.

Important characteristics of the overall foam response arising due to microstructural features, as with bulk materials, are the degree of anisotropy and the effective Poisson’s

ratio. As expected, for highly random microstructures the foam response is typically isotropic (see, e.g., [28]). For many foams, the cell shape, orientation, and/or distribution is anisotropic [50, 74, 85] (e.g., elliptical cells with directional bias) leading to anisotropic overall behavior [7, 51, 86]. Often, these structural features, e.g. axisymmetric elongation in the rise or spray direction, yield approximately transversely isotropic overall properties, although orthotropy is also possible, as is matrix material anisotropy [10, 77, 87]. As discussed in Sec. 1.1.1, the effective, i.e. “secant”, Poisson’s ratio ($\nu_{sec} = -\frac{\epsilon_{22}}{\epsilon_{11}}$, where the 1-direction is the loading direction and the 2-direction is transverse to the loading direction) is typically variable as a function of deformation. Hence a Poisson’s function or tangent Poisson’s ratio ($\nu_{sec} = -\frac{d\epsilon_{22}}{d\epsilon_{11}}$) is often used [29, 34, 67, 88, 89]. Many low density open-cell foams exhibit negative tangent Poisson’s ratio, likely due to buckling processes in struts, and so called “re-entrant” foams [90] are designed to have negative overall Poisson’s ratio (i.e., auxeticity) and a considerable body work discusses the design, fabrication, and characteristics of auxetic foams (including increased indentation resistance and energy adsorption [91, 92]), for examples see [16, 68, 93–95].

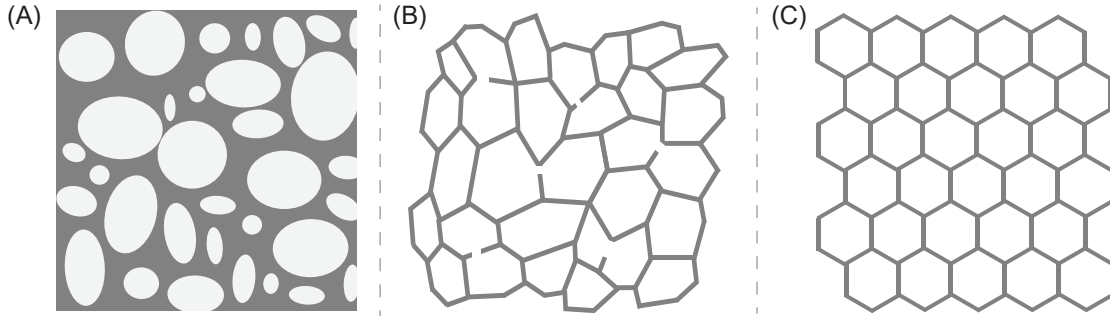


Figure 1.5: Illustrative 2D “cut-plane”-type schematics showing the general character of three archetypal classes of cellular materials. (A) Closed-cell foam where cells or pores are isolated by webs connected at edges and vertices. At higher void volume fractions (larger ϕ_0) the open cavities are separated by thin flat or shell-like webs. (B) Open-cell foam composed of struts of matrix material joined at vertices. Although shown as typically six-sided here in general the structures are random with variable connectivity. (C) The most widely studied and classical regularly structured (i.e., “architected”) cellular solid, a honeycomb, with regular hexagonal cells.

Beyond auxetic foams, many cellular materials have designed or “architected” microstructures. Of these the “honeycomb” structures consisting of regular hexagonal cells (see the

schematic of Fig. 1.5) has received a great deal of attention [10,96]. Concomitant with the advent of widespread and high resolution additive manufacturing, porous and cellular materials with tailor-able properties arising from engineered (i.e., topologically optimized) micro-architecture have become a popular research field, see [97–100]. Designed characteristics can include: aforementioned Poisson’s ratio (i.e. auxetics), stiffness [101] or stiffness-to-weight ratio [102], energy dissipation or phononic bandgap metamaterials (e.g. [103]), thermodynamics, or microfluidics [97]. Architected materials have remarkable properties, but custom-designed microstructures usually result in high fabrication and unit costs, and while reducing cost is an active research field, these materials are generally beyond the scope of this work. A noteworthy cross-over between fully designed microstructures and random foams is syntactic foams, in which pores are formed via hollow-shell spheres in a binder material [104], or for example see [105] or [81] as more recent examples (syntactic are often triphasic, as opposed to the typical biphasic polymer foam).

1.2.2 Polymer foams

In general, porous and foamed materials exist across virtually all major classes of solids, and many biological and natural material are foam-like [10]. Among the typically fabricated options are metallic foams, e.g. closed- or open-cell aluminum (e.g., [106,107]), graphene/graphite foams (e.g. [108]), multiphasic (e.g. glass/epoxy matrices [105]) foams (typically from syntactic processes [104]), ceramic foams (e.g., [109]), closed- or open-cell rigid polymer foams [6,82,110], or elastic polymer foam either closed- (e.g. [111]), or, my focus in this work, open-cell elastomeric foams [57,112,113]. Each of these classes of matrix material offer a diverse literature of exciting technology and applications, but for human-scale protective equipment and cushioning, and in particular inexpensive multi-use equipment, elastomeric foams dominate [114], despite the lack of accessible, predictive models available (see Sec. 1.1.1).

Both rigid, crushable polymer foams and softer elastic polymer foams are used for impact protection, although in different contexts – rigid foams are more common in packaging or one-time-use gear. Rigid foams are often made from base polymers such as polystyrene

(e.g., [115]), or highly crosslinked polyurethane e.g., [40], or various other types of thermosets and thermoplastics [6, 10]. Rubbery polymers, i.e. elastomers, are matrix materials for softer or elastomeric foams, specifically, lightly crosslinked polyurethanes [69], polyethylene [36], ethylene-vinyl acetate [62], or polypropylene (e.g., [116]) are common choices. Many techniques exist for fabricating both rigid and soft foams [6, 10, 117], but are outside the scope of this discussion. However, in brief, three typical methods include: homogeneous nucleation (spontaneous void growth due to local thermodynamics), heterogeneous nucleation (void growth about a filler particle), or via a blowing agent (gas entrapment) [6, 117]. Importantly, regarding difference in mechanical response, rigid foams characteristically exhibit a stress peak during the linear-to-plateau regime and a very flat plateau associated with plastic yield of the struts, whereas elastomeric foam is characteristically monotonic in stress-strain with a less-flat plateau associated with elastic buckling of struts [10].

1.2.3 Experimental approaches for elastic foams

Material testing and experimental data collection for foams has been an ongoing process for decades. The American Society for Testing and Materials (ASTM) and International Organization for Standardization (ISO) both have standards for testing cellular materials in various modes. In particular, standard ASTM D3574-17 defines a testing methods flexible cellular materials, which includes compression force-deflection and tension using a dogbone geometry (see Fig. 1.6A) [118]. The ISO standards catalogue ISC 83.100 defines testing procedures for cellular material [119], and within this ISO 1798:2008 defines standards for tension or ISO 1922:2018 for shear properties. These standards, however, are designed to establish baseline performance measures for validation of manufactured materials rather than produce data for material modeling. The majority of experiments for material modeling have focused on uniaxial compression and/or tension with compression receiving the majority of attention. For example [15, 34, 49, 67, 68, 73–75, 82, 84, 95, 111, 113, 120–128] all rely on uniaxial compression and/or tension, typically on load frame apparatus at “quasistatic” rates. These low-rates are on the order of $\dot{\epsilon}_{\text{applied}} \approx 10^{-3} \text{ 1/s}$ although no established criteria exist to define a suitable “quasistatic” rate. Shear has also occasionally been employed [28, 32, 87, 129]. It

has become clear that the transverse response to uniaxial stress is also important to capture and several techniques have been employed, including direct image-based measurements of the edges (for example, see Fig. 1.6B from Widdle *et al.* [125] or [29, 68, 127]), markings (such as in [15]) or cell morphology (see [75]) on the foam surface, or full-field measurements such as digital image correlation [27, 34, 67].

To extract more complete information, validate models, and explore use-case deformation modes, multi-axial and inhomogeneous testing has also been used, although less extensively. An early example for rigid foam is the axial stress plus pressurization experiments of Zaslowsky [130] which are similar to the higher-rate experiments of Viot [131] or purely hydrostatic experiments (in triaxial compression or via a 3D “Maltese-cross”-type tension) such as in [87, 132–134]. Indentation (e.g., as outlined in [118]) has more often been used for validation (see for example Fig. 1.6B Briody *et al.* [129], or [28]) and material comparisons (such as for foam grading) [91, 92, 114] than for model fitting. Confined compression (see Fig. 1.6B Liu *et al.* [135]) is a relatively straightforward multi-axial test that lends itself to micro-computed tomography (CT) based measures due to the high degree of axisymmetry possible (for example, [136]). Biaxial testing in either compression [137], such as via the compression method of [138] (see Kossa *et al.* of Fig. 1.6B) or biaxial tension (see Seibert *et al.* [53], Fig. 1.6B or [87, 139]). A final class of multi-axial experiments include fully combined loading situations such as those of Ling *et al.* in Fig. 1.6B with adjustable contributions of shear and compression [44], or the hexapod-based experiments of [45] which allow a large degree of control of the complete deformation state.

For elevated rate experiments, i.e. intermediate strain rate ($\dot{\epsilon}_{\text{applied}} \approx 10^1 - 10^3$ 1/s) and high ($\dot{\epsilon}_{\text{applied}} \approx 10^3 - 10^5$ 1/s) strain rate, a variety of systems have been used [7]. At intermediate rates, pneumatic impacts (e.g. [17]), drop-weight or drop-tower systems [42, 71, 140, 141], a flywheel impactor [131], small-strain rheological testing [9, 142], or high-speed load frames [57, 87, 143] have all been used. The Split-Hopkinson Pressure Bar (SHPB, i.e. Kolsky Bar) is extensively used for high-rate testing (e.g., [29, 34, 37, 111, 123, 128, 144]), although shock tube or gas-gun-based experiments have also been conducted [40]. A selection of studies have used more than one of these techniques to assess foams over a wide range of

rates (e.g., [41, 56]). For an overview of dynamic rate experimental implementation specifics see Chapter 4.

Finally, a tool for experiments that has proven extremely useful to investigate geometry, kinematics, and deformation mechanisms has been volumetric X-ray imaging, i.e. synchrotron (e.g., [84, 140, 145]) or micro-computed tomography (μ CT, including [69–71, 76, 81, 139, 146]) imaging. As identified in Sec. 1.2.1, the shape and configuration of cells or pores in the foam are critical to determine overall response. Observations, e.g. in Elliott *et al.* of Fig. 1.6C, of strut or cell wall buckling have been instrumental in revealing deformation mechanisms during collapse (such as in [51, 69, 121, 147, 148]) or enabling direct FE meshing (for examples see [24, 26, 149, 150]). Volumetric imaging has also been used for displacement measurement at the whole-material and cell-scales using digital volume correlation (DVC, see the example of Hu *et al.* in Fig. 1.6C) [81, 136, 151]. DVC has been used, for example, to discover and visualize localization [71, 136, 152] and inhomogeneities [153, 154] in displacement or measure Poisson’s ratio [154].

1.2.4 Modeling approaches for elastic foams

Note: portions of this subsection have been adapted from my previously published work [28].

Several general classes of hyperelastic model have been proposed to capture the features of elastomeric foams. A selection of representative or commonly used models fit for a moderate density XRD foam are shown in Fig. 1.7 (adapted from [28]). The first, most well-known class of models utilizes a free-energy density function that depends on the principal invariants of the Cauchy-Green deformation tensors, including the Blatz-Ko hyperelasticity model [15] and its extensions, e.g., [155, 156]. Although the Blatz-Ko-based models can capture the compression/tension asymmetry up to moderate stretches the functional form is incapable of describing the highly compressive response. Other invariant-based hyperelastic approaches include polynomial-type free-energy density functions [128, 139, 157]. Micromechanics-derived hyperelastic approaches, which explicitly incorporate the effect of porosity or other micro-scale features, also capture the moderate stretch regime well, but break down beyond this range

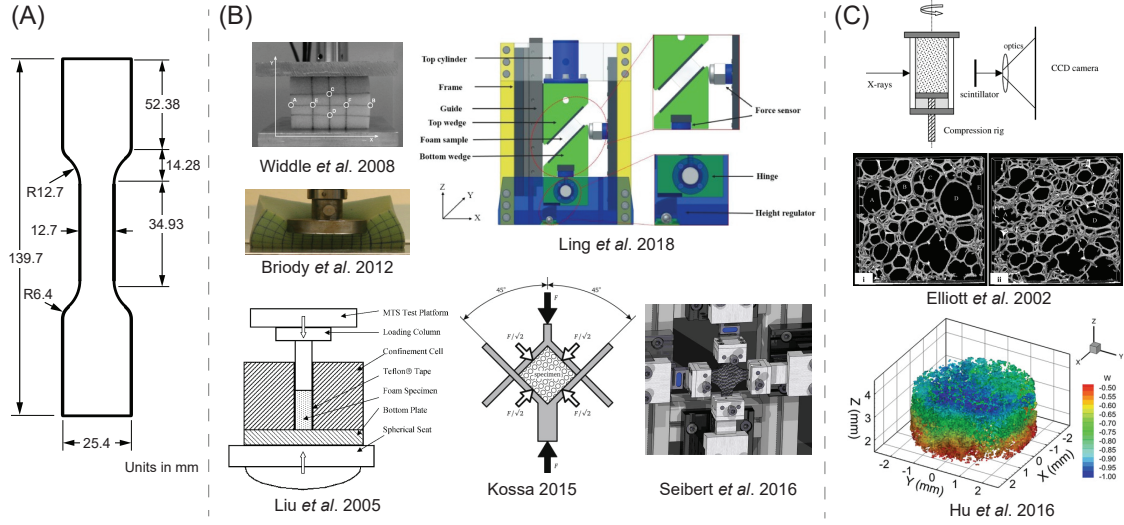


Figure 1.6: Selected experimental techniques used to investigate polymeric, typically elastic, foams. (A) The ASTM D3574 standard dumbbell tension specimen geometry, one of two dumbbell geometries designated in standard D3574-17 for testing of flexible cellular materials. (B) Testing geometries sampled from the elastic foam measurement literature. From top left in counterclockwise order, these consist of: uniaxial compression [125], indentation force deflection-style inhomogeneous compression [129], confined compression [135], biaxial compression [138], biaxial cruciform-type tension [53], and variable mixed-mode shear and compression [44]. (C) Micro-computed tomography (μ CT) to assess internal deformations of micro- and meso-scale structures. Top: a typical μ CT scanner arrangement and pre- to post-deformation images of a low-density elastic polyurethane foam [69]. Bottom: an example of digital volume correlation (DVC) displacement mapping to reconstruct full-field volumetric behavior [136].

(see Chapter 3) since structural non-linearity and instability begin to dominate [19–21, 158]. Alternatively, phenomenological Ogden-type, stretch-based hyperelastic models [12, 13] are often used to describe data beyond the moderate stretch regime, e.g., [14, 124, 129, 159–161], but generalization beyond the fitted region typically yields poor results.

While phenomenological classes of hyperelastic modeling approaches (e.g., Ogden-Storakers-type so called “hyperfoam”) are capable of capturing the features of the uniaxial, equilibrium, hyperelastic response of elastomeric foams, resultant complete models can be unwieldy to work with, challenging to fit, and/or difficult or expensive to conduct the correct experimental programs to generate relevant data e.g., [125, 162]. For example, the biphasic Ogden-type continuum model due to Markert [17] captures an extensive set of the large deformation responses, but contains 27 fitting parameters and has no clear calibration pathway to fully

describe the model. Material parameter fitting to experimental data is often a challenge, and a given model typically has minimal extrapolative potential. This yields poor predictions of deformation in loading scenarios not used in fitting or instability beyond the fitted range.

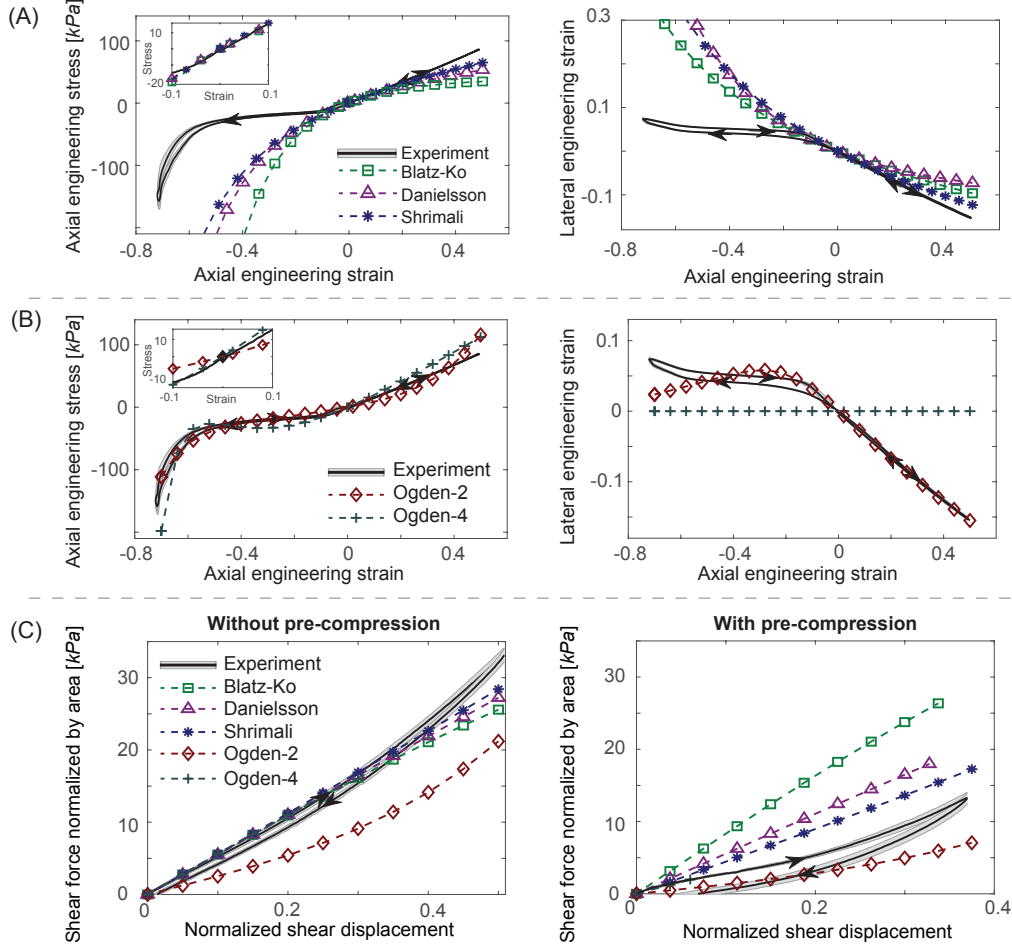


Figure 1.7: Comparison of representative models fit to moderate density (192 kg/m^3) XRD foam results (adapted from [28]). Blatz-Ko [15] and Ogden-Storakers-based models [12, 13] are phenomenological hyperelastic formulations. Danielsson [19] is a micromechanics-based model, and Shrimali [21] is a homogenization-based closed-cell hyperelastic model. (A) Axial stress-strain and lateral axial strain response for small-to-moderate deformation models (see inset for small-deformation regime). (B) Ogden-Storakers model for 2-term and 4-term forms. The 2-term model captures the small-deformation response (see inset) whereas the 4-term model captures large-deformation response. (C) Simple-shear-like deformation without and with a compressive 25% engineering pre-strain.

As an alternative to direct hyperelastic models, neural network and fully micromechanical models have been attempted. While providing remarkable fitting results compared to an Ogden-type model, the neural network based models [163–165] rely on extensive training data.

Due to their nature as sophisticated parameter identification and fitting techniques it is unclear what physical intuition is gained, or if they will remain predictive outside the training cases. Fully-micromechanical modeling based on strut-type unit cells, e.g., [22,23,73,78,166], or unit cells involving cylindrical or spherical voids, e.g., [21,64,167], or statistical representations, e.g., [76,95,168], has been used to capture the precise behavior of low- to moderate-density, typically closed-cell, elastomeric foams under certain conditions; however, while very insightful, such modeling approaches can be difficult to deploy in practice – often requiring less immediately obtainable experimental data, such as micromechanical characterizations or base elastomer rheology, and extensive computational resources.

1.3 Summary of Accomplishments

The major contributions encompassed in this work are presented in the following three chapters, followed by a discussion of preliminary efforts to understand secondary effects in elastomeric foams and, finally, conclusions. Chapter 2 discusses the development of a new digital image correlation (DIC) algorithm that uses cross-correlation quality factors and the iterative deformation method with Fast Fourier-space cross-correlation to reconstruct full-field 2D displacements. This combination of features allows the algorithm to accurately measure displacements despite the unique challenges presented by elastomeric foams, i.e., large axial and small transverse displacement, poor speckle quality, and loss of speckles. Furthermore, the use of quality factors enables automatic reference image updating, a hybrid incremental and cumulative mode, when degeneracy of matching is detected. Chapter 3 describes the experimental design, execution, and results for continuum model development. Experimental systems for compression, tension, shear, and indentation were developed. Testing focused on three densities of a polyurethane foam designed for impact protection applications, and results are used to calibrate a novel material model developed with collaborators. Chapter 4 examines the extension of the work in chapter three to higher “intermediate” strain rates up to $10^4/s$, to extend the model to physically relevant impact-like rates. The final chapter, Chapter 5, discusses initial efforts to measure temperature effects, localization, and matrix material properties via nanoindentation. It then provides an overall summary of the work.

Chapter 2

A q-Factor-Based Digital Image Correlation Algorithm (qDIC) for Resolving Finite Deformations with Degenerate Speckle Patterns

Note: A version of this chapter is published in the journal *Experimental Mechanics*. Any published data or figures included have been done so with co-authors' consent.

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2.1 Introduction to Digital Image Correlation

Digital image correlation (DIC) is a longstanding image based non-contact, full-field displacement measurement technique [169–171]. Extensive effort has been expended in developing and improving DIC algorithms, quantifying errors, and extending the technique to 3D surface (3D-DIC) [172, 173] and full volumetric (digital volume correlation) measurement [174–178]

since its inception. Substantial prior work has focused on accurately resolving small (i.e. sub-pixel), spatially varying displacement fields of either unknown (local DIC) or known (global DIC) character and/or boundary conditions [171,179]. Several contemporary algorithms have implemented DIC-based tracking for large finite deformations and have discussed the ongoing challenges distinct to resolving finite deformations within the DIC framework [180,181].

In the DIC method, intensity variations in an image of the specimen surface encode a unique descriptor for a particular sub-region of the image. The intensity variations can be due to natural specimen texture, or, more typically, a pattern of random “speckles” that are applied to the specimen prior to testing. Full-field displacements are determined by tracking the motion of image subsets between an image of the reference (i.e., undeformed) configuration and a current (i.e., deformed) configuration by cross-correlation of image subset pairs [171]. Usually, a transform, or shape, function of order n is adopted to map the reference configuration to the deformed configuration, and the transformation parameters are iteratively adjusted to minimize a cross-correlation matching error metric. Subpixel accuracy is achieved by interpolation of the pixel-level cross-correlation.

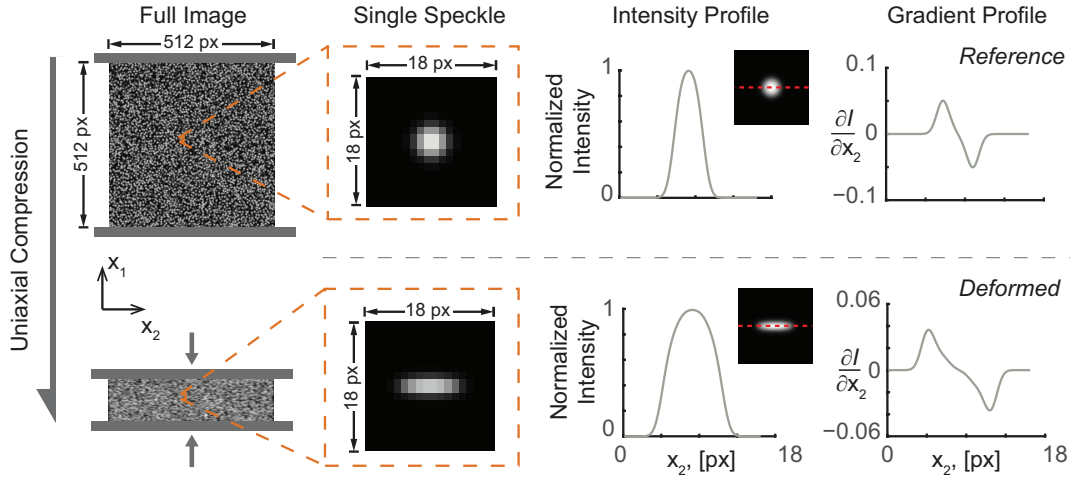


Figure 2.1: Schematic and synthetically generated example images undergoing finite deformation uniaxial compression. At large deformations, the speckles and pattern become highly anisotropic. In the transverse direction, the information-carrying content of the speckle, the grayscale gradient, becomes degenerate in both its magnitude and sharpness (slope), decreasing the overall quality of the motion reconstruction process

For finite deformations, particularly in compression dominated modes, the matching between reference and deformed configurations is more challenging for three primary reasons:

(1) the speckle pattern becomes highly distorted, (2) the portion of the image field of view subtended by the specimen is reduced, and (3) large 0^{th} -order (rigid body) motion exists. Each of these effects are apparent in the diagram of Fig. 2.1. Thus, for tracking large finite deformations, DIC algorithms have used “reliability-guided” non-linear optimization schemes to obtain the parameters of a higher-order shape function and utilized incremental (image n is compared to image $n + 1$) or hybridized incremental-cumulative image switching, rather than a purely cumulative (image 0 is compared to image n) based comparison [180, 181]. Cumulatively switching offers lower error than incremental switching until image decorrelation begins, and it is therefore important to carefully choose reference update points in a hybrid scheme.

A copious portion of the literature concerning DIC has discussed error sources, propagation, mitigation, and error evaluation strategies at various measurement stages throughout the system, from the speckling pattern to the final processing steps [171, 179, 182, 183]. The accuracy of the displacement reconstruction depends upon the quality and characteristics (e.g., the gradients of intensity captured in a subset) of the intensity pattern from which image subsets are sampled, in addition to the algorithm details [184–187]. For finite deformations, speckle pattern quality can become degenerate, particularly in the transverse directions to the principal deformation axis. The plots of spatial variation in intensity and intensity gradient in Fig. 2.1 illustrate this in a synthetically generated image pair. Speckle pattern degeneracy as a result of deformation-based image distortion or from regions that contain insufficient gradient information has been quantified by a number of techniques. These were recently reviewed in detail by Dong and Pan [186] and by Crammond and co-authors [187]. The speckle pattern assessment techniques provide a quality parameter either for an entire image or an image subset using either image intensity distributions, e.g., by measuring intensity gradients [185, 188], fluctuations [189], speckle size/morphology [187, 190, 191], or entropy [190, 192], or correlation-based heuristics, such as autocorrelation peak sharpness [193] or autocorrelation tallest peak to secondary peak ratio [194]. These metrics provide a deterministic way for quantifying the information carrying potential and signal-to-noise ratio within a DIC image. The autocorrelation-based measures are related to the idea of

cross-correlation quality factors used within particle image velocimetry algorithms that shall be discussed in detail later in this work.

To approach the issue of poor regions for correlation or decorrelation due to image distortion within the DIC algorithm, several authors [180, 181] have developed "reliability-guided" DIC (RG-DIC) implementations. In the RG-DIC technique, peak magnitudes for zero-normalized cross-correlation (ZNCC) below a predefined threshold are used as indicators for points to be skipped and re-evaluated when a better initial guess for certain shape function parameters is available from nearby interrogation points. While the peak-value of the ZNCC offers a basic indication of decorrelation, it is an incomplete metric of the signal quality of matching. More complete evaluation of the loss-of-signal in cross-correlation is possible with the use of a cross-correlation quality-factor (q-factor) [195, 196]. There are four common q-factor metrics. First, the simplest and least informative is the peak-to-peak ratio, in which the highest peak in the cross-correlation space is compared to the second highest peak [197]. Second, the peak to root-mean-square ratio compares the overall signal noise magnitude to the magnitude of the peak [198]. Third, the peak to correlation energy quantifies the distinctness of the primary peak energy from the correlation energy [195]. The fourth, peak to information entropy, compares the random correlation noise to the height of the primary correlation peak [196]. The final three q-factors are based on analytical derivation from signal processing theory and the definition of signal-to-noise. When incorporated into a DIC algorithm, q-factors provide a reliable and mathematically robust method for evaluating each measurement point, and I show that this improves measurement accuracy.

The algorithm presented herein utilizes a non-traditional technique originally introduced for particle velocimetry applications of image correlation, termed the iterative deformation method (IDM) [199] in place of non-linear optimization of the deformation shape functions. With the IDM a simple matching function with no optimization parameter is employed to estimate the motion field, and this motion field estimate is used to iteratively warp the reference and deformed configuration images. The total deformation field motion tracking problem is thus linearized into a series of 0^{th} -order deformation steps, such that large, arbitrary deformations are reconstructed accurately and few switching points are required in

the hybridized reference updating scheme.

The organizational structure adopted for this chapter is as follows. In Sect. 2.2 the basic 2D adaptation of Fast Iterative Digital Volume Correlation (FIDIVC) and its features are described, the addition of q-factors for both point-wise quality assurance and global reference switching is introduced, and the incremental-cumulative finite deformation summation scheme is discussed. Section 2.3 presents the validation of the new q-factor based 2D-FIDVC (qDIC) algorithm. Finally, my new algorithm is used to experimentally measure the full-field deformation of an elastomeric foam in Sect. 2.4, which features poor speckle characteristics due to the high porosity of the material.

2.2 q-Factor Based Fast Iterative Digital Image Correlation (qDIC)

The new algorithm that forms the basis for this chapter introduces q-factors to a computationally efficient implementation for large deformation DIC. Considered in addition to the underlying IDM framework, this improves the overall accuracy for finite deformation measurements in environments with poor speckle patterns. The q-factor is used as a loss-of-signal metric in both local evaluation of individual measurement points and global evaluation of image-pair decorrelation. For local assessment, the individual displacement measurement point quality is estimated and used as a threshold criterion. To reconstruct large deformations with global quality information, the algorithm employs a hybrid incremental-cumulative image pairing scheme with switching informed by the global q-factor assessment. A version of the algorithm is also available for 3D volumetric images.

2.2.1 Fast Iterative Digital Image Correlation

A 2D version of FIDVC, which is extensively described by Bar-Kochba *et al.* [200], formed the basis for this work. The basic 3D to 2D conversion of FIDVC is termed FIDIC; both FIDVC and FIDIC are made freely available¹ to the community. The FIDVC algorithm is a

¹See GitHub, <https://github.com/FranckLab> or <https://github.com/ALandauer>

Fast Fourier Transform-based implementation of the IDM [199, 201] designed to reconstruct large deformations in 3D volumes, the salient features of which are summarized below and outlined in the *Modified 2D-FIDVC framework* box of Fig. 2.2. For either technique an image pair containing an image of the reference configuration of the specimen (the reference image) and a second image of the deformed configuration (the deformed image) are analyzed to reconstruct the encoded differential motion field.

During DIC processing, the images are divided into overlapping subsets with a given initial square subset size, w_i , and spacing, d_i , and the motion of each subset center is tracked via a matching function. The final resolution of any subset-based algorithm is determined by the final combined subset size and spacing. Whereas the subset size determines the range of recoverable spatial frequencies of the motion field within the image, the subset spacing sets the spatial resolution between two independent measurement points. In the IDM scheme, the subset size and spacing are refined to w and d respectively during the iterative deformation process, allowing low spatial frequency components of the displacement to be captured in initial iterations, and higher frequency signals to be reconstructed as the process continues. In general, the motion field estimate $d\mathbf{u}$ between subsets of image n and $n + m$ for iteration k is computed via

$$I_n^{k-1}(\mathbf{x}) \otimes \hat{I}_{n+m}^{k-1}(\mathbf{x}) \rightarrow d\mathbf{u} \quad (2.2.1)$$

where $\otimes$ is the cross-correlation operator, and k is an integer counter starting at one. See the diagram in Fig. 2.3(a) for a schematic description of this process. The image subset windows are treated independently such that computation is highly parallelizable. In the cross-correlation space returned for each subset, a 3^{rd} -order Gaussian polynomial is fit to the region surrounding the peak value to determine the peak location, and hence the displacement estimate, with subpixel resolution. To improve accuracy and compensate for the moving-average nature of the cross-correlation function, a modular transfer function with

weighing support for frequency fluctuation suppression as given by Nogueira et al. [202], i.e.,

$$\begin{aligned}\omega(\mathbf{x}) = & \left(\prod_{i=1}^3 12 \left[\left| \frac{x_i}{w} \right|^2 - 12 \left| \frac{x_i}{w} \right| + 3 \right. \right. \\ & + 0.15 \cos(4\pi x_i/w) + 0.2 \cos(6\pi x_i/w) \\ & \left. \left. + 0.1 \cos(8\pi x_i/w) + 0.05 \cos(10\pi x_i/w) \right] \right)^{1/2}\end{aligned}\quad (2.2.2)$$

is incorporated into the cross-correlation operation, such that the complete formulation becomes

$$C(d\mathbf{u}) = \sum_{\mathbf{x}=-w/2}^{w/2} \omega(\mathbf{x}) f(\mathbf{x}) \cdot \omega(\mathbf{x} + d\mathbf{u}) \hat{f}(\mathbf{x} + d\mathbf{u}) \quad (2.2.3)$$

where f and $\hat{f}$ are the reference and deformed image subsets. As in Bar-Kochba *et al.* [200], the cross-correlation C may be computed in Fourier space from

$$C(d\mathbf{u}) = \mathcal{F}^{-1} \{ \mathcal{F} \{ \omega(\mathbf{x}) f(\mathbf{x}) \} \times \mathcal{F} \{ \omega(\mathbf{x} + d\mathbf{u}) \hat{f}(\mathbf{x} + d\mathbf{u}) \} \}, \quad (2.2.4)$$

where $\mathcal{F}$ indicates a Fourier transform. A normalized cross correlation formulation, which is used throughout this work, is optionally available although at considerable time-cost, in which case the Matlab formulation (based on [203]) is directly utilized. For volumetric 3D images the non-normalized FFT is highly recommended to reduce computational cost.

From the incremental motion field $d\mathbf{u}$, the total motion field must be built up by a series of linearized iterative steps. This is accomplished through the summation of the incremental fields,

$$\mathbf{u}^k = \sum_k \mathbf{u}^{k-1} + d\mathbf{u} \quad (2.2.5)$$

and symmetric warping of the original reference image (I^k) and deformed image ($\hat{I}^k$) by the new total displacement field estimate, i.e.,

$$\begin{aligned}I^k(\mathbf{x}) &= I^0(\mathbf{x} - \mathbf{u}^k/2) \\ \hat{I}^k(\mathbf{x}) &= \hat{I}^0(\mathbf{x} + \mathbf{u}^k/2).\end{aligned}\quad (2.2.6)$$

The warping function utilizes bicubic spline interpolation since the increased computation

cost of the higher-order interpolant for 2D images is negligible. For the 3D case, linear interpolation is used to substantially reduce cost.

In general, the IDM has no guarantee of convergence and can become unstable, so a specialized low-pass convolution filter $p(\xi)$ [204] is applied during displacement summation, such that Eq. 2.2.5 becomes

$$\mathbf{u}^k = p * \left(\sum_k \mathbf{u}^{k-1} + d\mathbf{u} \right). \quad (2.2.7)$$

Additional spurious displacement measurement points (due to, e.g., mismatching or poorly defined cross-correlation peaks) are removed with a universal median test [205] and replaced with an energy-minimizing plate metaphor-based interpolant using the displacement values from the adjacent measurement points. Following the procedure described above, the algorithm continues iterating until a stopping criterion for convergence is met. Since the IDM attempts to warp the images toward each other, the relevant measure for ending iterations is image similarity. For this implementation, an error-rate based formulation is adopted where the change in normalized sum-of-squared-difference between the images in the image pair is tested against the stopping criterion. A stopping criterion is defined for each spatial resolution and as the criteria are met, the algorithm refines subset size and spacing until the final resolution, which is set by the user, e.g., $w = 16$ pixels (px) and $d = 8$ px, is achieved. After final convergence, the displacement field is output. For an image series, qDIC can be run in either incremental or cumulative mode. Total displacements are either given by summation of the displacement increments (incremental) or computed directly with reference to the initial stress-free configuration (cumulative).

2.2.2 Cross-Correlation Assessment via q-factors

As shown in Figs. 2.2 and 2.3(a), the incremental displacement measure, $d\mathbf{u}$, at each iteration step k is obtained by cross-correlating the subset image intensities in the reference and deformed images. A key improvement of the q-factor based qDIC method over the basic FIDIC is to add an additional level of signal quality assessment during cross-correlation

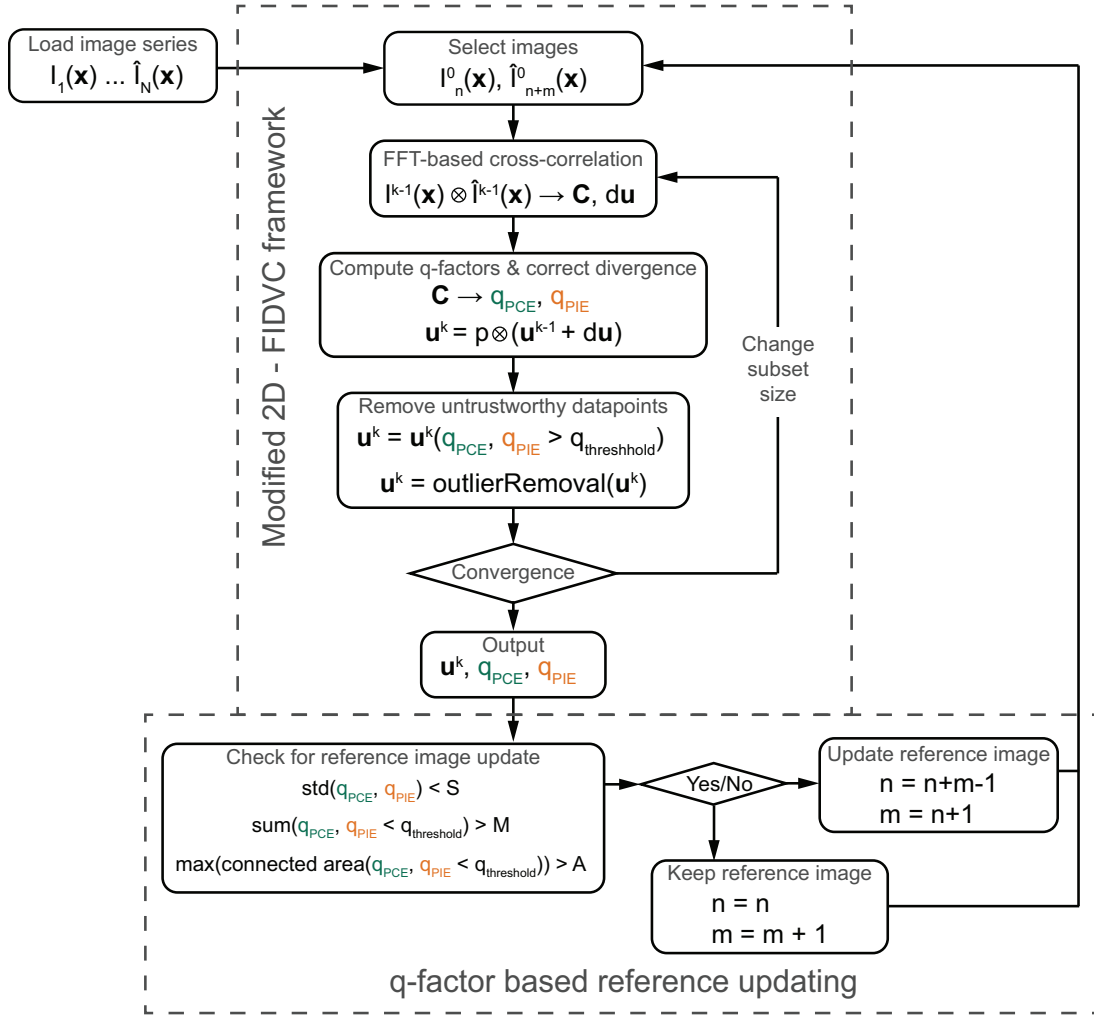


Figure 2.2: Algorithm outline of the major steps to measure displacements from an image series containing N images. In the *Modified 2D - FIDVC framework* block, the displacement between an image pair consisting of a reference image (I_n , the n 'th image in the series) and a deformed image ($\hat{I}_{n+m}$, the $(n+m)$ 'th image in the series) is measured and q-factors for each measurement point are calculated. The displacement measurement is conducted in an iterative deformation method framework where at each iteration, k , the images are warped with the displacement estimate from the previous iteration until they converge to identical images. The process is repeated with decreasing subset size and spacing until the final desired resolution is reached. In *q-factor based reference updating*, several q-factor based metrics are checked for decorrelation between the reference and deformed images. The algorithm checks whether the standard deviation of q-factor values, the total number of low q-factors, and the maximum connected area of low q-factors is small. If any check fails, a reference update is performed wherein the first image in the image pair is updated to the $(n+m-1)$ 'th image. Otherwise, the second image in the image pair is updated with the next $(n+m+1)$ image in the series

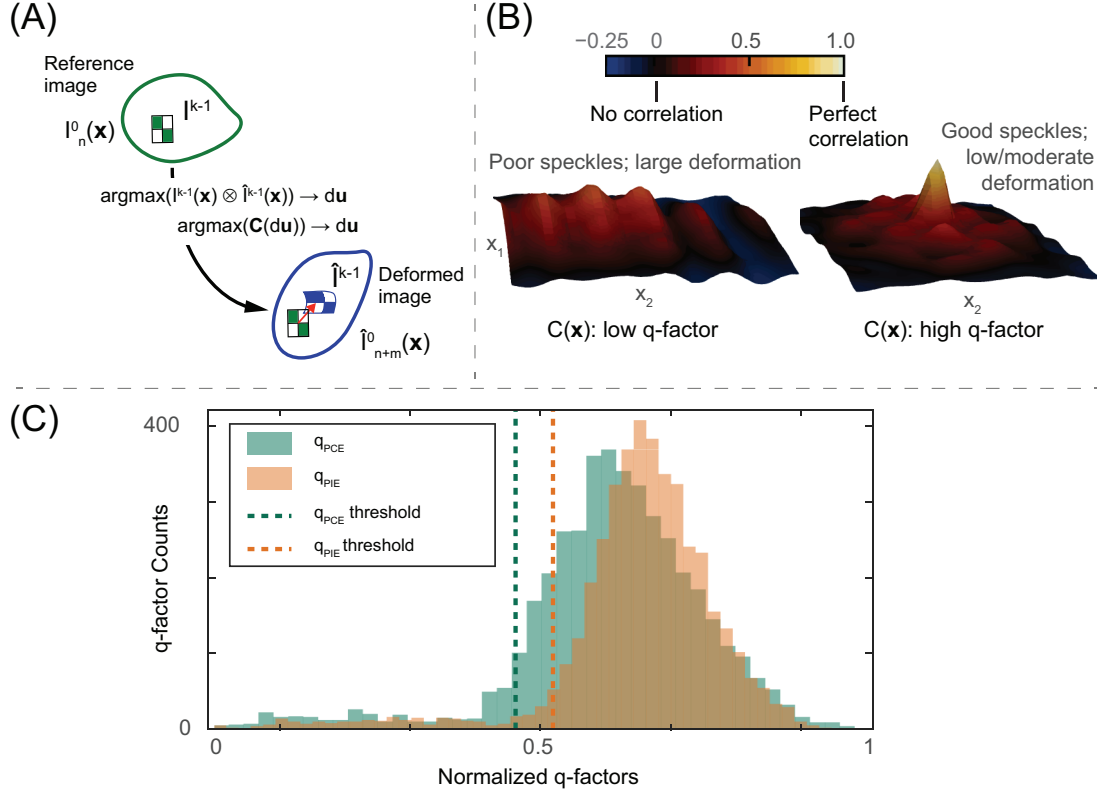


Figure 2.3: Quality factor (q-factor) based assessment in DIC. (a) For a given iteration, k , of mapping a reference image I_n^0 (the n 'th image in an image series) to a deformed image $\hat{I}_{n+m}^0$ (the $(n+m)$ 'th image) the displacement vector at a point in the image plane, $\mathbf{x}$, is given by the offset of the peak of the cross-correlation function between the images at that point. (b) Whether the peak of the cross-correlation function is sharp, strong, and uniquely defined depends upon the local image conditions. A cross-correlation quality factor (q-factor) is needed to assess if the cross-correlation function topology contains information sufficient to accurately and reliably measure the corresponding subset displacement. (c) In this work, two q-factors have been used as reliable metrics to judge cross-correlation quality. These are the peak-to-correlation-energy ratio (q_{PCE}) and the peak-to-information-entropy ratio (q_{PIE}). All q-factors are normalized, and a quality threshold is established as one standard deviation below the mean of the best fit Gaussian to each q-factor histogram

via a performance metric. Various metrics to estimate the signal-to-noise-ratio (SNR) of cross-correlation between two signals are well established [195]. These techniques have been discussed extensively in the context of particle image velocimetry (PIV) for estimating motion fields in fluid flows [206]. By quantifying the sharpness and distinctness of the cross-correlation coefficient peak and comparing it to the underlying signal, these metrics offer a robust technique to establish the quality of the cross-correlation itself, hence the term quality-factors of cross-correlation. The q-factors directly evaluate the ability of the cross-correlation coefficient, $C(\mathbf{x})$, to give accurate and reliable displacement measurement results, and assess how trustworthy a particular motion estimate is. The distinction between a high and low q-factor is illustrated in Fig. 2.3(b).

Two q-factors are of particular note, the peak-to-correlation-energy ratio (q_{PCE}) and the peak-to-information-entropy ratio (q_{PIE}). The physical origin of both these methods is rooted in information theory, and they have been shown to offer reliable performance in providing accurate displacement measures over a variety of image sets [195, 196]. The q_{PCE} compares the magnitude of the strongest peak to the normalized correlation signal energy by computing

$$q_{PCE} = \frac{|C(\mathbf{x})_{max}|^2}{\frac{1}{L} (\sum_L |C(\mathbf{x})|^2)} \quad (2.2.8)$$

where $C(\mathbf{x})$ is the cross-correlation coefficient and L is the size of the cross-correlation space. Based on the Shannon entropy [207] of the image cross-correlation coefficient, the q_{PIE} is computed from a 30-bin histogram of the cross-correlation space,

$$\frac{1}{q_{PIE}} = - \sum_{i=1}^{30} p_i \log p_i \quad (2.2.9)$$

where p_i is the probability of finding a given point in the i^{th} bin. The remaining two q-factors, first-to-second peak and peak to root-mean-squared signal, capture fewer features of the correlation space ($C(\mathbf{x})$) in a single metric and were found to offer less robust performance. Both the q-factors can be normalized in the range $[0, 1]$ for a given population, e.g., the set of q-factors drawn from correlating all subset points within an image pair.

Once q-factors have been computed, a threshold level must be defined to distinguish high

and low quality correlation points, which in turn define a cutoff for reliable versus unreliable displacement estimates. For a given iteration in the IDM, the number of cross-correlated measurement points and q-factors is set by the image size and subset spacing. The population of q-factors for each q-factor type generally features a normal, Gaussian-like distribution sampled about an unknown mean. As one or more image regions become decorrelated, e.g., due to image degradation, the distribution becomes bi-modal. To establish an appropriate q-factor threshold, a maximum likelihood estimator (MLE) was used to fit a bi-modal Gaussian density function to the q-factor histogram. If the mean $\pm$ standard deviation of the two estimated peaks overlapped, no second peak was recorded, and the MLE was recomputed for a uni-modal Gaussian density function. For either case, the quality threshold was the largest-q-factor-value peak minus one standard deviation (see Fig. 2.3(c)). Measurement points failing the quality threshold test for either or both q-factors were discarded from the displacement field and replaced with interpolated values via the plate-metaphor interpolation scheme as described in Sect. 2.2.1, supported by points with high q-factor scores. Edge points and discarded points within a prescribed distance from an edge typically have low, poor-quality cross-correlation coefficients due to limited data along the image periphery. Thus, in the final iteration these points are discarded.

2.2.3 q-factor Based Reference Configuration Updating

A balance of error sources exists between incremental and cumulative mode steps. Incremental steps contain less decorrelation error than cumulative image comparisons beyond image $n + 1$, but random errors are summed and may quickly grow. To minimize this effect, a hybridized incremental-cumulative technique was implemented for updating the reference image during a time-lapse experiment, as outlined in Fig. 2.2 in the *q-factor based reference updating* box. In this scheme the occurrence of extensive decorrelation was evaluated via the q-factor map of the final iteration of a given image pair. Three features of the q-factor map are important when considering overall decorrelation between an image pair. First, the total number of thresholded q-factors must not exceed a given percentage of the total displacement measurement points, typically set to 15%. Second, the largest simply-connected

area multiplied by its eccentricity plus a small factor to prevent degeneracy must not exceed a specified area, typically 121 px. Third, and most importantly, the standard deviation of normalized q-factors, Gaussian distribution described in Sect. 2.2.2, must not exceed a predefined level, typically set to $0.13 \text{ px} \pm 0.01 \text{ px}$. These threshold numbers were empirically established by analyzing datasets of known displacement. The variability in width is to account for changes in sample size. If any of these conditions are exceeded, decorrelation is indicated. The reference image is updated to the image immediately prior to the current deformed configuration image, the location of the reference update is flagged, and the IDM is rerun with the new reference image. If adjacent images fail the q-factor check the pair is flagged again and the algorithm continues to the next image.

2.2.4 Incremental and Hybrid Displacement Cumulation

DIC algorithms are usually run in either incremental or cumulative mode. However, while measuring large deformation in cumulative mode, significant decorrelation can occur between images in a pair (image 0 compared to image n) for large n , which leads to large displacement errors. To circumvent this issue, qDIC is executed in a hybrid incremental-cumulative mode and I explicitly utilize the q-factor metrics to inform reference image updates when significant decorrelation between an image pair occurs. Subsequently, the displacements from this hybrid incremental-cumulative mode are converted to cumulative displacement measures.

This displacement conversion is conducted as follows, and the Matlab implementation was developed in collaboration with Dr. Mohak Patel. Consider a time-lapse image series analyzed from image 0 to image N , where significant decorrelation is detected using q-factors when reconstructing the cumulative displacement between image 0 and image $m + 1$. Consequently, image m is updated as the reference image, and the cumulative displacements are then computed between image m to image N . Up until image m the cumulative displacement (u_p^0 where $0 < p \leq m$) between image 0 and image p is computed at the grid points $\mathbf{x}_{grid_0}$ defined by the subset spacing in image 0. While between image m and image h , the cumulative displacement ($\mathbf{u}_q^m$ where $m < q \leq h$), between image m and image q , is computed at the grid points $\mathbf{x}_{grid_m}$ defined by subset spacing in image m . The original grid points $\mathbf{x}_{grid_0}$ in image

0 move to positions $\mathbf{x}_{grid_0} + \mathbf{u}_m^0$ in image m . The cumulative displacements $\mathbf{u}_q^m$ computed at $\mathbf{x}_{grid_m}$ in image m are interpolated using bi-cubic splines onto the original grid points in image m and summed with u_m^0 to compute the total cumulative displacement u_q^0 . This step is similarly repeated for multiple reference image updates to compute the total cumulative displacement.

2.3 Validation of the qDIC algorithm

To assess the improvement in displacement measurement accuracy by employing both the $qPCE$ and $qPIE$ q-factor metrics, I generated synthetic image stacks with analytically imposed displacement fields. A four stage validation process was adopted where the performance, namely accuracy and precision, was first assessed for zero-displacement images with varying image distortion applied to both images in a pair. Second, the performance on a homogeneous displacement field was characterized. Third, I applied a known inhomogeneous displacement field, and fourth, the algorithm was executed on the SEM Challenge #14 dataset. Three minimum subset sizes were employed throughout the validation process.

2.3.1 Synthetic Image Generation

Synthetic images were constructed by seeding higher-intensity regions (i.e. speckles) into a dark background. Seeding locations were semi-randomly generated using the Poisson Disk sampling algorithm with a cut-off radius 0.5 px greater than the seeded speckle characteristic radius, typically 3 px. An integer number of seed locations were used such that the final fill factor of speckles was approximately 50%, typically 50,000 - 65,000 particles for a 2 MPx initial image.

Speckles were generated at high resolution, convolved with a point spread function (PSF), downsampled, and interpolated into the complete image. The original speckle intensity was defined by a top-hat function of radius r_0 , given by

$$S(x_1, x_2) = \frac{1}{1 + \exp(-2\gamma [1 - x_1^2/r_0^2 - x_2^2/r_0^2])} \quad (2.3.1)$$

which is parameterized by $\gamma = 1000$, the decay rate of the intensity. Image formation optics were idealized by a symmetric Gaussian approximation to the Bessel function of the PSF for a 40x microscope objective with green ($\lambda_0 = 532nm$) speckles,

$$PSF(x_1, x_2) = \exp\left(-\frac{x_1^2}{2s^2} - \frac{x_2^2}{2s^2}\right) \quad (2.3.2)$$

where $s = \frac{0.21L}{NA}$, with $L = \frac{\lambda_0 r}{0.5 \times 10^{-6}}$ and $NA = 0.7$ is the numerical aperture of the lens, mimicking the configuration used for volumetric imaging. Here r is the integer-valued radius of the speckle on the final pixel grid.

The speckle image is the convolution of the separable PSF and the intensity distribution of the speckle, thus,

$$B(x_1, x_2) = PSF(x_1, x_2) * S(x_1, x_2)$$

$$B(x_1, x_2) = \sum_{\xi_1=-2r}^{2r} \exp\left(-\frac{\xi_1^2}{2s^2}\right) \times \left[\sum_{\xi_2=-2r}^{2r} \exp\left(-\frac{\xi_2^2}{2s^2}\right) I(x_1 - \xi_1, x_2 - \xi_2) \right] \quad (2.3.3)$$

gives the image of a single speckle on a grid of $4r + 1$ by $4r + 1$ pixels with the speckle at the center. After convolution, speckle images were downsampled to the final pixel grid via binning of intensities to mimic the light collection of an image sensor. For each seed point location in the final image the downsampled speckles were added using bicubic spline interpolation for the subpixel level intensity distribution.

After all speckles were placed into the final image, noise was added. Two types of random noise were used to mimic noise present in typical experimental images. First, white Gaussian noise was added at a signal-to-noise ratio of 25 and a background black-level of approximately 2% to simulate random noise. Second, each pixel was resampled from a Poisson distribution to simulate shot noise during image acquisition. Any pixels above the saturation limit were then set to the limit, and the whole image was renormalized to 255 grey-levels (i.e. an 8-bit greyscale range). A 512×512 px^2 region in the center of the image was output as the final synthetic image. An analogous process was developed for synthetic volumetric image generation.

2.3.2 Analytical Displacement Fields

To prescribe deformation fields in the synthetic images, the same image generation process was conducted with two additional steps. The locations of the seed points were changed according to the known, analytical deformation map, and the individual speckle images for each location were deformed according to the local, affine deformation. By simulating the motion field in this way large-deformation interpolation was avoided, reducing potential artifacts introduced due to the image warping process.

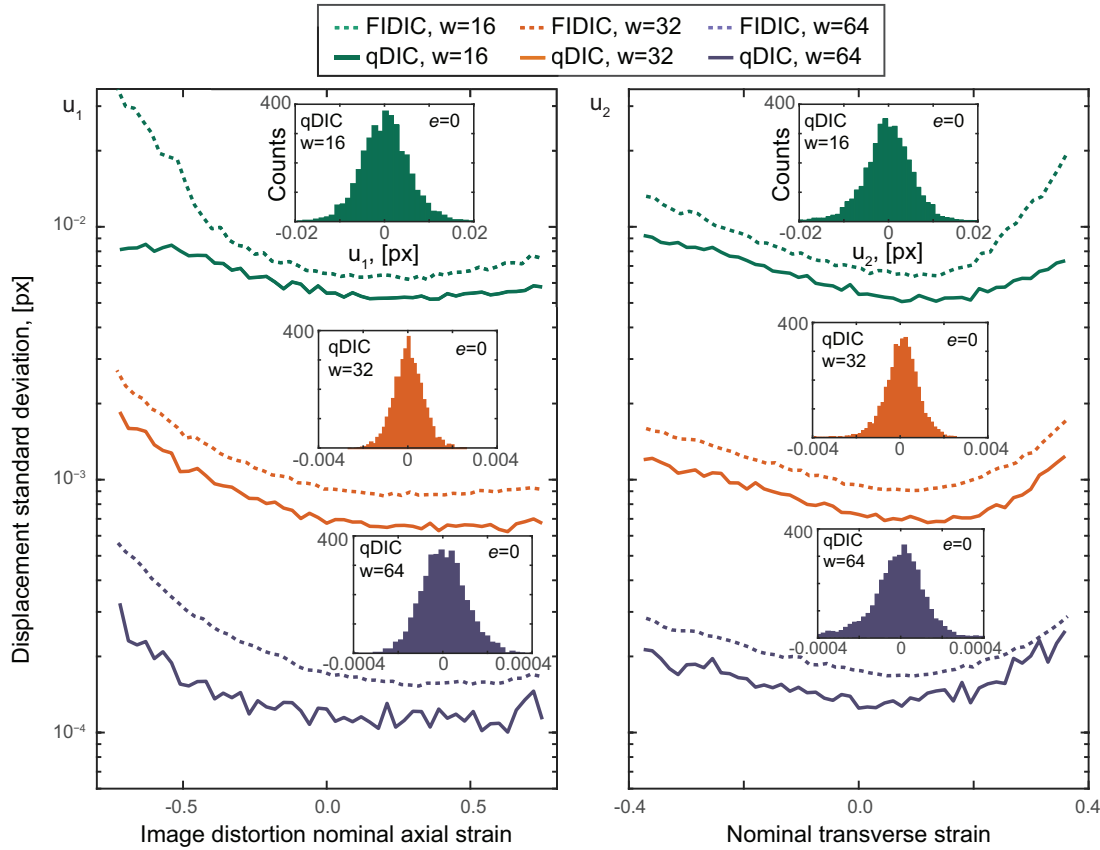


Figure 2.4: Standard deviation error from a rigid-body motion zero displacement field, plotted as a function of image distortion. Images in each pair contain unique synthetic noise and were analytically distorted via incompressible uniaxial compression or tension fields centered in the image in x_2 and the bottom edge of the image in x_1 to the given strain levels. The u_1 error is plotted as a function of the nominal axial image distortional strain, and u_2 error is plotted as a function of the transverse distortional strain for minimum subset sizes $w = 16, 32, 64$ px. The qDIC algorithm has lower error and is a flatter function of image distortion, indicating improved robustness to image noise and degenerating speckle quality. (*Insets*) Histograms of the recovered displacement for the distortion-free, zero-displacement cases for each qDIC minimum subset size (baseline)

To alter bead locations, the deformation mapping function for the image is applied to each seed point. At seed point n the reference position X_i^n is updated to the deformed position x_i^n by adding the displacement $u_i^n(X_i^n)$ evaluated from the deformation mapping function, i.e.,

$$x_i^n = X_i^n + u_i^n(X_i^n), \quad (2.3.4)$$

which are the locations now to be seeded with speckle images. Speckles in the deformed configuration that moved farther than $2r$ pixels outside of the original image boundary were discarded. Speckles within this margin region were seeded, but the image was cropped to the original dimensions after seeding such that the image boundaries suffered no loss of speckle information.

Each speckle was deformed according to an affine approximation to the underlying displacement field before seeding into the deformed image. The analytical displacement field in a $2r$ px square centered at the seed point is densely sampled at ten times the pixel sampling frequency, and the local deformation gradient is determined via a least-squares plane fitting procedure (see Sect. 2.4.2). This deformation gradient is used to warp the high-resolution speckle image before convolution with the PSF and downsampling to the nominal post-binning size. The appropriately deformed speckle image for each seed point n is then added to the final image, as discussed in Sect. 2.3.1. Non-rigid speckles were thus simulated for large deformation without introducing interpolation artifacts.

For all experiments, initial image sizes were chosen such that the final 512×512 px² region used as input to the qDIC algorithm was fully seeded with speckles throughout the deformation process.

2.3.3 Zero-Displacement Noise Floor Assessment with Image Distortion

To determine the noise floor as a function of speckle warping, and thus signal degradation, a series of zero-displacement image pairs at various levels of axial strain (schematically illustrated in Fig. 2.1) was generated. In this synthetic dataset, each image contained unique Gaussian and Poisson noise but seed points were identical. Distorted images for

pairs undergoing incompressible uniaxial compression and tension with prescribed nominal strains up to 75% in 1% increments were created to simulate the increasing degeneracy of the speckle pattern with increasing distortion (Fig. 2.4). This provides an assessment of the qDIC displacement measurement result between each image pair at given applied nominal axial strains. The mean and the standard deviation of the normally distributed displacement histogram yields a visual representation of the algorithm’s accuracy and precision, respectively, in recovering the applied rigid body motion.

The noise level and matching error depend upon the amount of spatial averaging that the algorithm utilizes. Therefore, the minimum subset size, w , is a key factor in setting the noise floor, since smaller minimum subset sizes yield less spatial averaging, usually resulting in an increased noise floor, in the final displacement field. Three minimum subset sizes, $w = 16, 32, 64$ px, were utilized for the FIDIC and qDIC implementations presented here. Both algorithms include an error-minimizing iterative process and outlier removal. Since the qDIC algorithm extends outlier removal from the standard post-measurement fluctuation-based paradigm [200] to include the newly introduced q-factors, the noise level is reduced. Both of these effects are present in the plot of standard deviation (1σ) of displacement versus the image distortional strains in Fig. 2.4. Representative displacement distributions for 0% distortion (baseline) are shown in histogram form for each subset minimum size in the three insets in Fig. 2.4. The characteristic distribution remains Gaussian-like, but the width dramatically decreases for larger subset sizes, due to the intrinsic smoothing operation of the larger correlation subset windows. Representative displacement distributions for 0% distortion (baseline) are shown in histogram form for each subset minimum size in the three insets in Fig. 2.4. The characteristic distribution remains Gaussian-like, but the width dramatically decreases for larger subset sizes, due to the intrinsic smoothing operation of the larger correlation subset windows. Mean and standard deviation of displacement and Lagrangian strain components are provided in Table 2.1 for the zero-strain case. Strain estimates are computed via an optimal-11 spatial differentiation kernel (discussed further in [200]). Although two techniques for computing strain are demonstrated in this work, strain computation technique best practice strongly depends upon application particulars

and this topic has been widely discussed elsewhere [171, 180, 200, 208]. The focus of qDIC is primarily on the reconstruction of the encoded motion, i.e. displacement field, rather than its strain field.

Table 2.1: Mean and standard deviation of displacement and Lagrangian strain ($\mathbf{E}$) for the zero-displacement, non-distorted image pair. Since the differentiation kernel is symmetric $E_{12} \equiv E_{21}$ everywhere

	$w = 16$		$w = 32$		$w = 64$	
	Mean	St Dev	Mean	St Dev	Mean	St Dev
u_1 [px]	1.1×10^{-4}	5.5×10^{-3}	2.2×10^{-5}	6.7×10^{-4}	3.2×10^{-6}	1.2×10^{-4}
u_2 [px]	-1.1×10^{-4}	5.4×10^{-3}	-2.5×10^{-5}	7.3×10^{-4}	-1.3×10^{-5}	1.3×10^{-4}
E_{11} [%]	-1.2×10^{-4}	1.1×10^{-2}	2.5×10^{-5}	1.6×10^{-3}	6.6×10^{-6}	2.6×10^{-4}
E_{22} [%]	-5.4×10^{-5}	1.1×10^{-2}	-8.8×10^{-7}	1.7×10^{-3}	5.2×10^{-6}	2.6×10^{-4}
E_{12} [%]	-4.3×10^{-5}	7.0×10^{-3}	-1.1×10^{-5}	9.9×10^{-4}	-2.0×10^{-6}	1.3×10^{-4}

2.3.4 Homogeneous Deformation Reconstruction Assessment

To assess the algorithm's performance in a more realistic deformation, an image dataset consisting of single images with a homogeneous deformation field was evaluated. The analytical field for this case was uniaxial compression to 75% nominal compressive strain. The center of the displacement field was located at the bottom center of the image, i.e., $x_1 = 0$, $x_2 = 255.5$ px. Transverse displacements were symmetric about the centerline of the image and u_1 increased linearly with x_1 . Contours of these displacement fields are visualized in Fig. 2.5(b).

The mean displacement error is assessed by computing the mean squared pointwise displacement difference from the analytical field. This error measurement formulation is expressed as,

$$u_{err} = \frac{\sqrt{\sum_I (\mathbf{u}_{DIC} - \mathbf{u}_{analytical})^2}}{N} \quad (2.3.5)$$

where $\sum_I$ indicates a sum across all measurement points in the image and N is the total number of measurement points. The plot of mean displacement error versus nominally applied axial strain is given in Fig. 2.5, comparing the performance for both the FIDIC and qDIC algorithms at minimum subset sizes of $w = 16, 32, 64$ px.

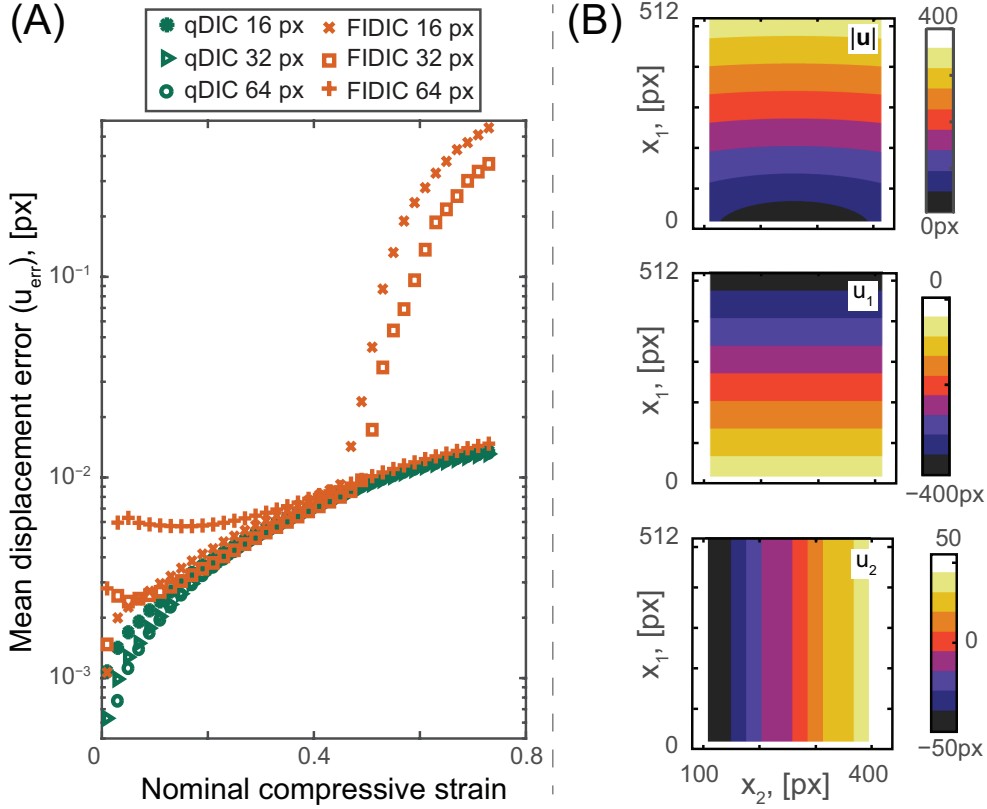


Figure 2.5: Mean displacement error determined from an analytically imposed homogeneous deformation field. (a) Mean displacement error as a function of applied strain. (b) Contour plots of the displacement magnitude, $|u|$, u_1 , and u_2 of the recovered deformation field via qDIC

The plot in Fig. 2.5(a) shows that the error levels in both algorithms are comparable for displacement fields that are not highly distorted. The increased spatial averaging of larger minimum subset sizes tends to reduce error levels, which is expected for a homogeneous deformation field. At a critical strain (between 45% and 50% depending on the experimental details) the FIDIC algorithm accuracy, for smaller subset sizes, begins to decrease dramatically. Increases in error rate for both u_1 and u_2 occur at the same deformation step for FIDIC, but not qDIC, which benefits from the q-factor based hybrid incremental-cumulative switching scheme (Fig. 2.2).

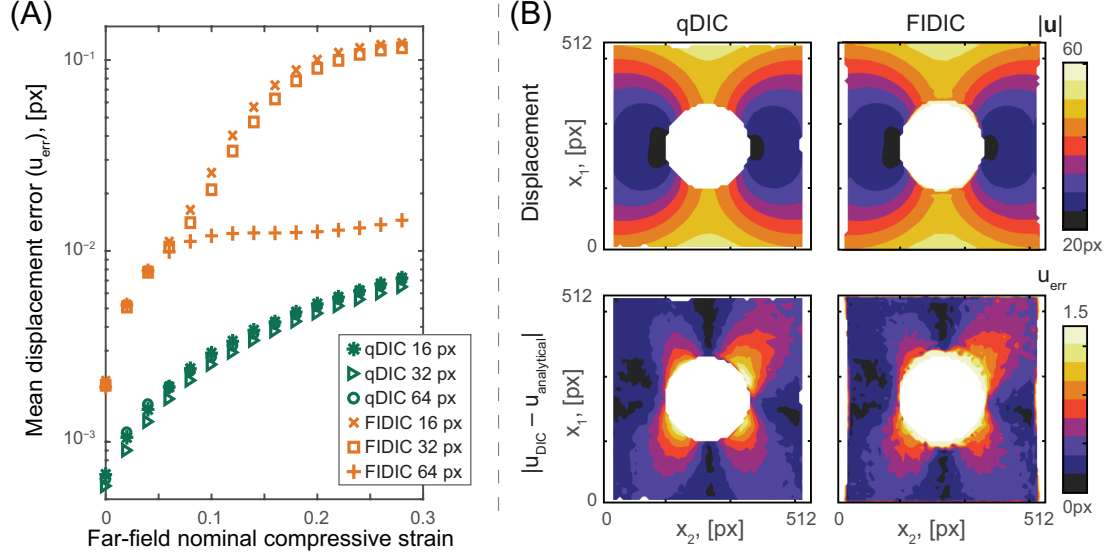


Figure 2.6: Assessment of qDIC performance under inhomogeneous loading. Note that the maximum far-field strain attainable is limited to approximately 30%, primarily to prevent full hole closure. (a) Comparison of the displacement error magnitude (see Eq. 2.3.5) as a function of applied far-field strain at three minimum subset sizes for each algorithm. (b) Contour plots of the final displacement magnitude fields from both qDIC and FIDIC, and the absolute error from the analytical displacements. A minimum subset size $w = 32$ px was used for both algorithms in these plots

2.3.5 Inhomogeneous Deformation Reconstruction Accuracy

Since the most useful applications for DIC techniques involve the resolution of highly localized deformation fields, I evaluated the performance of the qDIC algorithm for an image set undergoing an inhomogeneous loading scenario. Here, I chose the well-characterized hole in an infinitely-thick plate problem featuring a hole of radius a under applied far-field compression. For this set of tests the hole was centered within each image. The linear elastic solution for the displacement fields is given by the Michell solution [209] under plane strain boundary conditions,

$$u_1(x_1, x_2) = \frac{a\sigma_\infty}{8\mu} \left\{ \frac{r}{a} (\kappa + 1) \cos \theta + \frac{2a}{r} [(\kappa + 1) \cos \theta + \cos 3\theta] - \frac{2a^3}{r^3} \cos 3\theta \right\} \quad (2.3.6)$$

$$u_2(x_1, x_2) = \frac{a\sigma_\infty}{8\mu} \left\{ \frac{r}{a} (\kappa - 3) \sin \theta + \frac{2a}{r} [(1 - \kappa) \sin \theta + \sin 3\theta] - \frac{2a^3}{r^3} \sin 3\theta \right\} \quad (2.3.7)$$

where $\mu = \frac{E}{2(1+\nu)}$, $\kappa = \frac{3-\nu}{1+\nu}$, $r = \sqrt{x_1^2 + x_2^2}$, and $\theta = \arctan(x_2/x_1)$. The control variable to impose displacement is the far-field stress σ_∞ , which is modulated such that strain steps are produced in approximately 1% increments. The two material properties, Young's modulus and Poisson's ratio, were taken as $E = 1$ MPa and $\nu = 0.3$ respectively. The hole radius was set to $a = 120$ px. The maximum axial compressive strain was restricted by the onset of the central hole closing. Although the linear elastic assumption no longer holds at these strain levels, however it suffices to provide well-defined analytical motion field for validation the reconstruction capability of the DIC algorithms.

The mean displacement error as a function of the applied far-field strain is given in Fig. 2.6(a), while Fig. 2.6(b) shows contours of the recovered displacement magnitude and displacement error between the analytically imposed and reconstructed displacement fields. As can be seen from Fig. 2.6(a) the error level as a function of minimum subset size behaves differently between the FIDIC and qDIC algorithms. For the qDIC algorithm the $w = 32$ px subset size balances spatial averaging and noise suppression, resulting in the lowest overall error. The higher inherent noise with FIDIC requires the additional smoothing of $w = 64$ px to reduce mean error, but is unable to fully capture the higher spatial frequency displacement signal content of the u_1 field at that subset size.

The difference in reconstruction noise in the displacement signal is evident in the contour plots of Fig. 2.6(b), both of which are for a subset of $w = 32$ px. When examining both contour plots, it is visually apparent that qDIC outperforms FIDIC near boundaries where signal quality is lost due to incomplete information content in some subsets and in high-gradient regions where speckle distortions are greater. This is borne out by contours of displacement error magnitude, where large-gradient regions (i.e. tight banding in the displacement magnitude contours) tend to have comparatively large errors. Furthermore, the displacement errors in FIDIC are generally noisier.

2.3.6 SEM Challenge #14

The Society for Experimental Mechanics provides several benchmark cases for evaluating DIC algorithm performance. The qDIC algorithm targets large displacement and high spatial

frequency fields. Although not designed for large-deformations, the Sample #14 L5 test is a

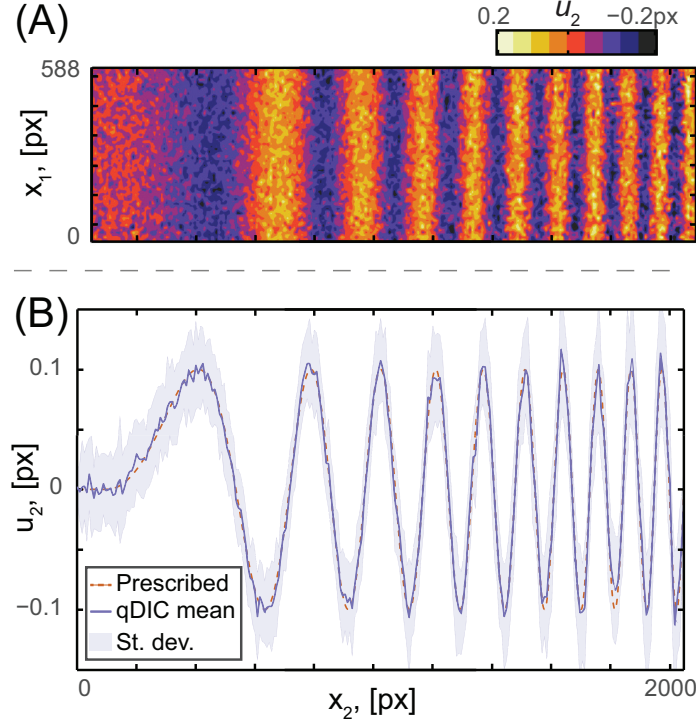


Figure 2.7: Contour and averaged displacement plots measured from the Society for Experimental Mechanics DIC Challenge Sample #14 L5 dataset. (a) Contour plot of measured u_2 displacement over the entire image area, using a minimum subset size $w = 16$ px and spacing $d = 8$ px. (b) Mean and one standard deviation (shaded region) of u_2 displacement averaged across the x_1 direction. The prescribed analytical displacement (dashed line) is shown for comparison

standard benchmark evaluating the reconstruction capability of DIC algorithms for varying spatial frequency signals. The image pair of Sample #14 L5 was analyzed via the qDIC algorithm, and the contour plot of the resultant displacement field is shown in Fig. 2.7(a). The mean and standard deviation of the resultant displacement field were taken in the x_1 direction for each pixel in x_2 , and compared to the provided imposed displacement. Although optimized for large deformations this algorithm reconstructed the high-spatial frequency subpixel displacement field with overall pointwise absolute error of $5.4 \pm 4.4\%$ (0.0054 ± 0.0044 px) and root mean squared displacement deviation ($\langle (u_{qDIC} - u_{analytical})^2 \rangle^{1/2} = 0.007$ px). At the highest spatial frequency in this sample, the qDIC algorithm continues to accurately track the displacement with minimal aliasing or amplitude attenuation due to the high spatial frequency reconstruction capacity of the IDM.

2.4 Experimental Application of qDIC

As a real-world application of the new qDIC algorithm, an experimental test case consisting of uniaxial compression of an elastomeric foam to 65% nominal compressive strain was considered. In this experiment, a scientific camera and optical system was employed to image a face of the foam specimen that had been speckled. An image series was taken through the loading-unloading process, such that the entire deformation process was recorded. This basic experiment modeled those performed for foam characterization work described in Chapters 3 and 4.

2.4.1 Experimental Setup

Load Frame Configuration. The load frame consisted of a custom benchtop system attached to a vibration isolated optical table. The loading apparatus was comprised, from top to bottom, as in Fig. 2.8(a), of a linear actuator (Ultra Motion, Cutchogue, NY; $0.6\mu m$ step resolution), load cell (LCFD-50, Omega Engineering, Stamford, CT), load alignment pivot, top and bottom platens, and two 90° opposed linear translation stages in an aluminum frame. Control and measurement was achieved via a data acquisition system (National Instruments, Austin, TX) operated through LabView (National Instruments, Austin, TX). A schematic of the arrangement is shown in Fig. 2.8(a). Shear and bending loads on the specimen were minimized by the load alignment pivot consisting of a spherical indenter tip in contact with the flat top surface of the top platen. Platen thickness was chosen to minimize bending and ensure uniform contact pressure along the top surface of the sample to impose uniaxial compression. Top and bottom platens were machined from lubricious graphitic carbon steel and carefully lubricated to minimize frictional confinement (i.e., barreling). To this end, contact areas of the platens were dusted with $35\mu m$ spherical glass beads, such that rolling friction dominated the initial stages of compression. A layer of powdered graphite lubricant minimized friction during subsequent compression.

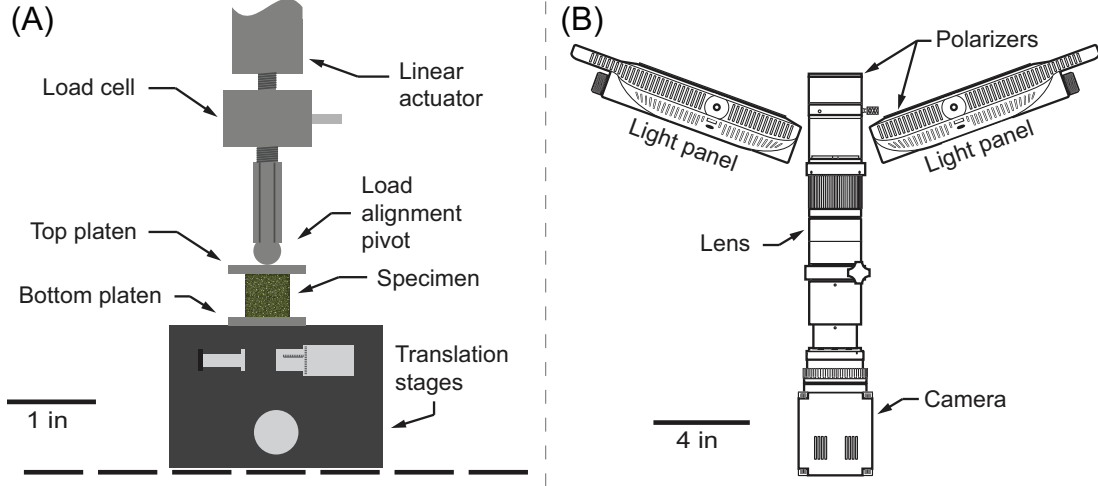


Figure 2.8: Schematic of the experimental uniaxial compression setup. (a) Front view of the compression device, showing the important components needed to deform the specimen, while minimizing shear and frictional forces and measuring the applied force. (b) Top view of the imaging platform used to collect experimental data for the qDIC algorithm. Light panels with linear polarization illuminated the specimen, while a long-distance microscope lens with perpendicular polarization formed images for the full-format scientific camera

Optical System Configuration. An optical system for single camera imaging of the specimen surface consisted of a camera with control computer, lens, polarizers, and lights was affixed to the optical table facing the load frame, see Fig. 2.8(b). The camera is a full-format 5.5 MPx greyscale sCMOS unit with $6.5\mu\text{m}$ pixel pitch (edge5.5, PCO-Tech Inc., Romulus, MI). A linearly polarized long-distance microscopy lens (K2/S, Infinity Photo-Optical Company, Boulder, CO) with $660\text{ mm} - \infty$ focal distance and 1.8x magnification was used for image formation. The specimen surface was illuminated via a pair of high-intensity LED light panels (Lykos Daylight, Manfrotto, South Upper Saddle River, NJ) having a linear polarizing film (American Polarizers Inc., Reading, PA) attached at a 90° offset from the lens polarizer. The cross-polarization reduces glare and specular reflections from the load frame and specimen, improving image quality for DIC [210].

Specimen Preparation. The material used for this test case is a polyurethane-based $9\frac{lb}{ft^3}$ (void volume fraction approximately 90%) open-cell elastomeric foam that has been designed for impact protection (XRD, Rogers Corp., Rogers, CT). Specimens were excised from sheets of raw material via a vertical bandsaw and are approximately 13mm x 10mm x

10mm in size. One 13mm x 10mm face of the specimen is designated as the DIC face and prepared appropriately. A black background paint (Acrylic Ink!, Liquitex Artist Materials, Piscataway, NJ) with white speckling paint (High Flow, Golden Artist Colors Inc, New Berlin, NY) was used. Both paints are low-viscosity acrylic compounds and are applied using a gravity-feed dual-action airbrush (HP-C Plus, Iwata Medea Inc, Portland, OR) to create small, regularly sized, and high-contrast speckles. After speckling, the specimen cross-sectional area was measured, and the specimen was placed on the bottom platen, moved into the load frame such that the speckled face was at the focal plane of the lens, and the top platen set in place. Alignment under the compression device was an incremental process of applying small-to-moderate deformations while minimizing angular discrepancy between top and bottom platens. After alignment, the specimens were allowed to relax for approximately 1 hr prior to testing.

2.4.2 Experimental Procedure

Once aligned, a displacement controlled uniaxial compression experiment was conducted. A symmetric triangular displacement function peaking at 65% nominal compressive strain at a nominal strain rate of 2×10^{-4} 1/s was utilized. Throughout the duration of the experiment, images were taken at 0.1 Hz and load data were acquired at 50 Hz. The image series was downsampled by a factor of three for computational tractability and input into the qDIC. A region of interest as shown in Fig. 2.9(a) was selected and the algorithm was run on 12 cores with 64Gb of memory on the Brown University Center for Computation and Visualization cluster. The qDIC settings were as given in Table 2.1.

Homogenized spatial derivatives of displacement were taken by utilizing a global plane fitting procedure to the displacement field of a given displacement component, $i = 1, 2$. An example of such a displacement field pair is shown in Fig. 2.9(b). For a given displacement step, n , the coefficients of the best-fit plane p_{kl} determined by a robust bisquare linear least-squares fitting, are computed such that,

$$u_i^n(x_j)_{plane} = p_{i0}^n + p_{ij}^n x_j \quad (2.4.1)$$

Table 2.2: Digital image correlation specification details for the experimental test case

Specification	Details
Technique	Single Camera q-factor Based Digital Image Correlation
Camera noise	0.3 %
Pre-filtering	Gaussian, [3 px, 0.2]
Min. Subset Size	32 px
Min. Step Size	8 px
Correlation criterion	FFT-based ZNCC
Interpolation	Bicubic splines
Image size	513 x 1281 px ²
Measurement points	10465
Computation time per image	40.2 s $\pm$ 17.9 s
Number of images	231 (0.1 Hz)
Pixel-to- μm conversion	1 px = 6.2 μm
Resolution: noise	0.05 px = 0.31 μm

where the summation convention is implied for $i, j = 1, 2$, and terms with zero subscripts are typically negligibly small rigid offsets. The deformation gradient is simply,

$$F_{ij} = p_{ij} + \delta_{ij}. \quad (2.4.2)$$

The polar decomposition of F_{ij} is $F_{ij} = R_{ik}U_{kj}$ where R_{ik} is the rotation tensor and U_{kj} is the right stretch tensor. The principal stretches are determined from the spectral decomposition of the right stretch tensor U_{kj} . The rotation tensor R_{ij} was determined experimentally to be close to identity, since rotations are $O(10^{-2})$ smaller than stretches, thus $F_{ij} \approx U_{ij}$. In direct notation the spectral decomposition of $\mathbf{U}$ is then

$$\mathbf{U} = \lambda \mathbf{e}_1 \otimes \mathbf{e}_1 + \lambda_T \mathbf{e}_2 \otimes \mathbf{e}_2, \quad (2.4.3)$$

where λ is the stretch in the loading direction, λ_T is the stretch in the transverse direction, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are the basis vectors of the measurement space (x_1, x_2) , as given in Fig. 2.9. Under the conditions outlined above the nominal strain, as used in the validation cases, is simply $e = \lambda - 1$. The nominal uniaxial stress was computed by normalizing the measured force by the undeformed specimen cross-sectional area in the typical fashion. Image-based data and force data were temporally aligned by manually matching the onset of loading with

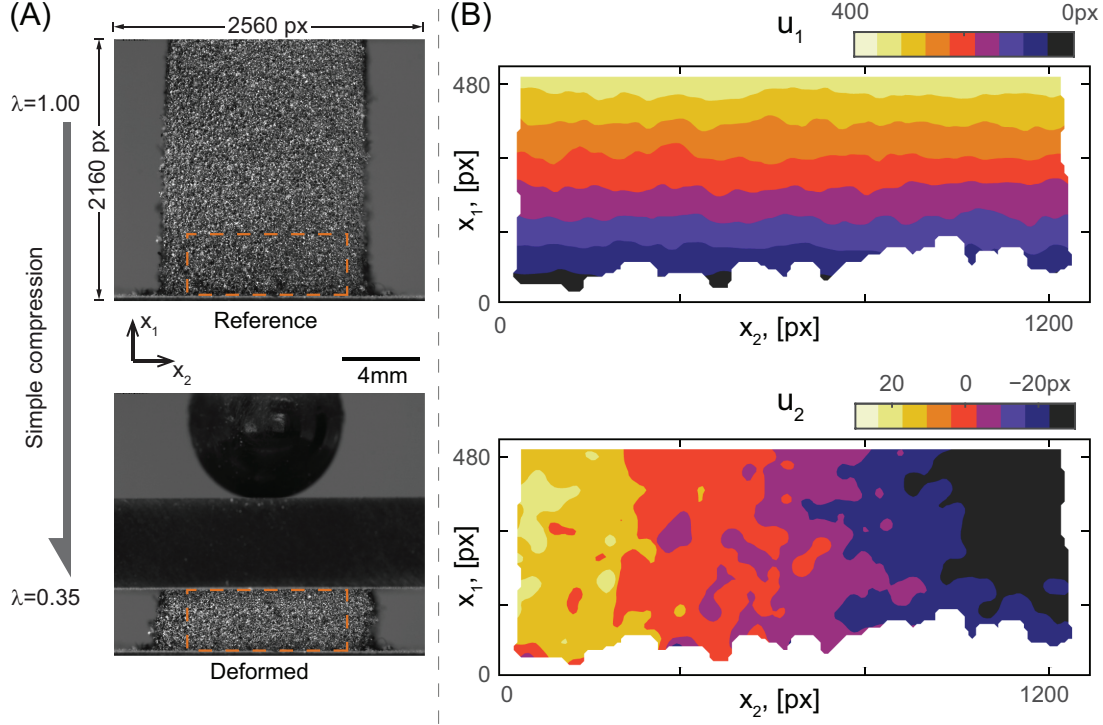


Figure 2.9: Experimental uniaxial compression images and contours of measured displacement fields. In this test case the foam undergoes large deformations and severe speckle distortions occur. The qDIC algorithm reconstructs up to about 400 pixels of displacement motion during the deformation process. (a) Example images from the experimental dataset showing the initial reference and fully deformed configuration. Note the relatively small Poisson effect and spherical load alignment pivot in the deformed image. The region of interest used to extract DIC images is denoted by the dashed box. (b) Contours of the qDIC measured displacement field for the fully-deformed configuration show flat uniform banding in u_1 indicating that little shear or displacement localization occurs. Banding is less distinct for u_2 since the signal level of the small lateral expansion is closer to the measurement noise floor

the first observable deformation, and both engineering stress and stretch were resampled to 500 datapoints in the time domain using bi-cubic spline interpolation.

2.4.3 Experimental Results

Initial diagnostic results from the qDIC on a zero-displacement noise assessment image pair are given in Table 2.1. The data listed show that the imaging system has relatively little shot noise. The spatial resolution is set by the choice of minimum subset size during the iterative refinement process; the minimum size of $w = 32$ is suitable to achieve the measurement goal while balancing spatial resolution and noise content in the images. Given this configuration,

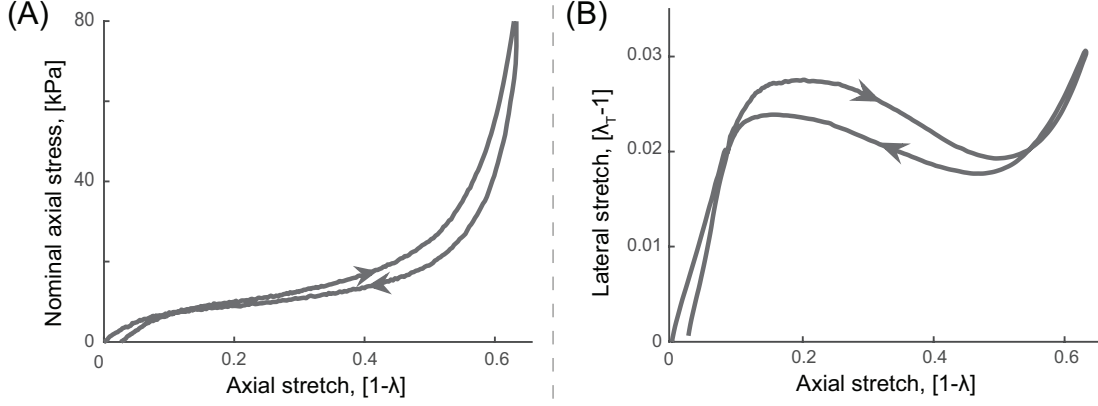


Figure 2.10: Experimentally measured nominal axial stress-stretch and transverse behavior of a polyurethane-based $9 \frac{lb}{ft^3}$ open-cell elastomeric foam undergoing large deformation uniaxial compression. (a). The resultant engineering stress-stretch curve reconstructed from qDIC and force measurements at quasi-static $2 \times 10^{-4} \frac{1}{s}$ strain rates. (b) The transverse engineering stretch $(\lambda_T - 1)$ computed from the qDIC displacement fields as a function of applied axial stretch $(1 - \lambda)$

the noise-based displacement error indicates that the speckle pattern before deformation is sufficient for subpixel accuracy at a noise floor $O(10^{-2})$ px.

Displacement contours measured with qDIC from imaging results (e.g. Fig. 2.9(a)) for the fully-deformed configuration are shown in Fig. 2.9(b). For the axial u_1 displacement component case, the regular spacing of bands indicates a homogeneous displacement field with no compaction band formation or other localization observable on the surface of the specimen. The parallel and horizontal nature demonstrates that shear, rotation, and bending are minimal. Despite the large deformation and associated measurement window size loss, qDIC tracks up to approximately 400 px of displacement. Resolving such large displacement, which has been extremely challenging with many previous DIC formulations, is a unique feature of the q-factor based hybrid incremental-cumulative qDIC computation scheme (Fig. 2.2). For transverse u_2 displacement, the banding is less well defined due to the high degree of compressibility, minimal transverse stretch, and significant speckle distortion (Fig. 2.9(a) resulting in a much lower signal-to-noise ratio in the u_2 displacements). The slight skewedness, curvature, and asymmetry visible particularly between $x_2 = 1000$ to 1200 px is due in large part to asymmetrical frictional confinement as a result of imperfections in the lubrication - an issue endemic to large deformation compression experiments.

Final stress-stretch and transverse-axial stretch curves are plotted in Fig. 2.10 for the complete load-unload cycle. In the stress-stretch plot of Fig. 2.10(a) the initial linear, plateau, and densification behavior typical of open cell foams is readily apparent. A small amount of hysteresis occurs, particularly at large levels of compression, which may be due to rate-dependant (i.e. polymer matrix viscoelasticity) or rate independent (e.g. frictional) effects. The apparent permanent set is a due to viscoelastic transients that do not completely decay during the imaging time-series. In the transverse versus axial stretch plot shown in Fig. 2.10(b) a similar characteristic pattern of increase-plateau-increase occurs, but at a much smaller amplitude. Note that although the plateau region appears to exhibit substantial auxeticity, this is in part a visual artifact. The decrease in transverse strain is less than $\sim 0.75\%$ over about 30% axial strain, i.e. a linearized Poisson-like ratio $\nu \approx -0.025$, during the plateau. The low noise floor of qDIC is needed in this challenging environment so that these small transverse strains remain measurable despite large axial deformation.

2.5 Conclusion

This chapter introduced and described a new 2D DIC technique, based on the finite deformation IDM FIDVC technique, which incorporates the concept of a cross-correlation quality factor. A volumetric implementation is also available, but not thoroughly tested to date. Two q-factors were utilized in this new qDIC technique, the peak-to-correlation-energy ratio and the peak-to-information-entropy. The q-factors improved the robustness and accuracy of the DIC for distorted speckle patterns arising from large finite deformations. By including the q-factors as a metric for image decorrelation an intelligent hybrid incremental-cumulative switching scheme was implemented.

The new algorithm showed improved performance over FIDIC across all validation and benchmarking cases, i.e., rigid-body, homogeneous and inhomogenous modes of deformation displacement fields. However, although notably improved performance is observed in this implementation, more sophisticated q-factor based DIC schemes are possible in future

development. Future directions to investigate include several avenues: implementing high-order shape function optimization with q-factors as a part of the matching routine to initialize the IDM; or, point-cloud based subset selection and/or adaptive remeshing could significantly streamline and improve generality of the technique. Lower-level detail improvements, such as other ways identify and account for poor quality factors (for example, through machine-learning based threshold determination) or improved interpolation in poor regions (such as weighing factors for displacement guesses based on q-factors rather than binary masking and in-painting) can make the algorithm more sophisticated than the initial naive, but nonetheless effective, implementation described in this chapter.

Chapter 3

Experimental Characterization of Open-Cell Elastomeric Foams at Quasistatic Rates for Hyperelastic Constitutive Modeling

Note: A version of this chapter is has previously been published in the *Journal of the Mechanics and Physics of Solids*. Any published data or figures included have been done so with co-authors' consent. See [28]:

AK Landauer, X Li, C Franck, DL Henann, (2019) "Experimental characterization and hyperelastic constitutive modeling of open-cell elastomeric foams" *Journal of the Mechanics and Physics of Solids*. p. 103701, 2019. doi: 10.1016/j.jmps.2019.103701.

3.1 Introduction and Background

As described in Chapter 1, and to summarize the information relevant to this chapter presented therein, open-cell elastomeric foams are materials consisting of two continuous phases: a polymeric elastomer matrix and a connected gas-filled pore space. Due to the

elastomeric matrix and porous structure, these materials are highly compliant and capable of undergoing large, reversible deformations involving substantial volume change. Consequently, the mechanical behavior of elastomeric foams is quite complex, involving large deformations, substantial compressibility, and strong tension/compression asymmetry. In particular, in compression, the response is initially linear, but as the axial strain magnitude increases, the response gradually transitions into a “plateau” in which both the axial stress and the lateral strain vary minimally as the axial strain magnitude increases (for a representative plot see Fig. 1.2). The plateau regime is generally associated with buckling in the elastomeric-matrix microstructure and concomitant closure of the pore space [10]. Then, as the material is compressed beyond the plateau regime, rapid stiffening is observed in the axial stress response as the bulk response of the base polymer begins to dominate. However, in tension neither a plateau regime nor a rapidly stiffening response is observed. This chapter focuses on the mechanics of elastic foams that exhibit neither deformation localization nor permanent set.

For this chapter a commercially-available impact protection foam (Poron XRD, Rogers Corp, Rogers, CT) is a candidate example material. This polyurethane-based open-cell elastomeric foam (a single example of which was used as a example case in Chapter 2) is investigated at three relative densities. As expected, the response in all three cases is highly nonlinear and displays substantial asymmetry between tension and compression. Importantly, the plateau region for XRD is not completely flat and increases slightly with applied strain. I observe homogeneous deformation for all three densities with no evidence of localization in my compression experiments, in contrast to many low-density elastomeric foams (e.g., [34]). Upon unloading in both compression and tension, some hysteresis but no permanent set is observed. This is in contrast to the inelastic behavior of crushable foams, e.g., [87, 110, 211, 212], which yield and display permanent set upon unloading.

Due to the complexity of the mechanical behavior of open-cell elastomeric foams, developing predictive constitutive models remains challenging despite the ubiquity of these materials. There is a substantial history of both experimental characterization and constitutive modeling of porous and foam materials, see the canonical textbook by Gibson and Ashby ([10]), or reviews of the subject e.g., [74, 150, 213]. This chapter focuses on developing experimental

tools and calibration for a hyperelastic modeling approach used to capture the equilibrium, elastic response of the XRD foam discussed in the preceding paragraph. The XRD foam is a highly random semi-open (i.e. composed of open-sided shells rather than the more classical strut-like members of many fully-open cell polymer foams) and thus is isotropic (see Fig. 3.2), reducing the number of experiments needed to capture the full behavior.

Numerous experimental efforts have been conducted to measure the behavior of elastic foams. Since the response is most strikingly nonlinear in compression, and many applications are compression dominated, the majority have focused on compression behavior, e.g. [34,51,62,74,95,121,125,135,137,151,160,214–216]. Tension and shear modes have also been addressed, although less extensively e.g., [14,15,116,126,162,217,218]. Multi-modal loading with some combination of shear, compression, and tension is least common, but quite important since coupling between volumetric and deviatoric deformations is a feature of open-cell elastomeric foam response [25,44,45,129]. The majority of these efforts have reported only the loading, and not unloading, path for a given loading mode, which calls into question the quasistatic nature of loading and balance of viscous and elastic forces present. It furthermore obviates the usefulness of the data for understanding dissipative effects, which are a key aspect of design for impact protection (Chapter 4 deals with experiments to capture these rate-based effects).

Due to the complicated and multifaceted nature of the response many modeling approaches have been attempted, typically these are highly parameterized hyperelastic representations. These have been described in more detail elsewhere (viz. Chapter 1); for this chapter it is useful to note that a number of isotropic, logarithmic-strain-based hyperelasticity models have been proposed in the literature. Specifically, it was shown historically that a simple quadratic free-energy function employing the logarithmic strain is capable of capturing experimental data for a wide class of materials in the moderate-stretch regime [219,220]. Since then, various quadratic free energy-based logarithmic strain schemes have been successfully applied to incompressible and nearly-incompressible materials (i.e., rubbery materials) in the large-stretch regime, e.g., [221–224]. This chapter discussed experiments and, briefly, the attendant a constitutive modeling approach developed by collaborators Li and Henann

to extend this to highly-compressible materials, such as porous elastomers (i.e., elastic polymers foam). We decompose the free-energy function into terms that represent the coupled volumetric/distortional, purely distortional, and purely volumetric contributions to the response and select phenomenological fitting functions for each term. In this way accurate and straightforward data collection widely available in the experimental community can be captured for open-cell polyurethane foam and used to directly and systematically calibrate our model.

The purpose of this chapter is to describe a comprehensive methodology for experimental characterization and overview the constitutive modeling of non-localizing, isotropic elastomeric foam materials under quasi-static, equilibrium loading. The accurate full-field kinematic measurements needed for model calibration from in these experiments are obtained using the quality-factor-based digital image correlation (qDIC) of Chapter 2 (see also [27]), which is crucial to measure both axial and lateral strains under large deformations. Our isotropic hyperelasticity model adopts principal invariants based on the logarithmic strain tensor [225,226] that were proposed [227], rather than the classical invariants the Cauchy-Green deformation tensors. The reasoning for this choice is two-fold: the logarithmic-strain invariants may be associated with specific aspects of deformation, enabling a more straightforward path to choosing and calibrating phenomenological fitting functions that capture experimental data, and this invariant space was shown to be more accurate in representing deformation in the moderate-stretch regime that is a common operational domain for these foams.

Initial experiments used for model development and material parameter estimation were homogeneous simple compression and tension, where the full-field data was reduced to two scalar measurements (axial and transverse strain) and used in conjunction with uniaxial engineering stress for calibration. After calibration of the model, a suite of validation tests were used to probe the predictive capability of the model in a variety of deformation modes. These modes typified compression, shear, and tension-dominated situations. For compression-dominated loading I used two finite-indentation configurations, for shear loading I employed a double-lap shear geometry, and for tension I used a standard dogbone with holes

removed. The intent is to provide an accessible characterization and modeling framework for highly-compressible elastic materials via the initial experiments. In future, the model may be extended to consider other specific, perhaps micromechanically-based, fitting functions in concert with image and/or nanoindentation-based experimental information, and the range of fitted and acceptable (e.g., closed-cell) materials may be extended.

The remainder of Chapter 3 is organized as follows. In Section 3.2, I describe my experimental program for characterizing the mechanical response of the isotropic, polyurethane-based, elastomeric foam using homogeneous simple compression/tension experiments, describe the general approach for validation testing. The complete experimental setup is discussed in Section 3.3. Then, in Section 3.4, I present a brief summary of the hyperelastic constitutive model for the elastic behavior of isotropic, elastomeric foams as applied to the XRD foam. In Section 3.5, this model is validated in the three configurations mentioned above, namely: spherical and conical indentation, shear both without and with fixed-displacement pre-compression, and finally, tension of a specimen with circular holes. In Section 3.6, I compare the proposed model with other important models from the literature, which were initially shown in Fig. 1.7. Finally, the chapter closes with a summary and a discussion of the limits of the experiments and modeling approach, and mentions possible future research directions in Section 3.7.

3.2 Experimental Program

In this section, I describe my process for experimentally testing the mechanical response of isotropic, elastomeric foams. I conducted two classes of primary experiments: (1) homogeneous characterization experiments that are utilized to develop constitutive equations and estimate material parameters and (2) inhomogeneous validation experiments that are utilized to test material model predictions. First, for homogeneous characterization testing, I performed simple compression and simple tension experiments. Kinematic data in these experiments are obtained using full-field digital image correlation via the qDIC algorithm of

Chapter 2, which enables both axial and lateral strains to be obtained despite low signal-to-noise and large deformations. Moreover, since full-field displacement data are available, I ensure that no unintended inhomogeneous deformation (e.g., specimen barreling or buckling, compaction banding, surface wrinkling, etc.) occurs during nominally homogeneous characterization experiments. Second, I performed a variety of inhomogeneous validation tests, namely, spherical and conical indentation, shear both without and with pre-compression, and tension of a specimen with circular holes, each of which are discussed further in Section 3.5. These validation experiments probe the constitutive model in compression, shear, and tension and verify its predictive capability.

3.2.1 Foam material

The experimental testing procedures described and the constitutive theory briefly outlined here are intended to be applicable to a large class of isotropic, open-cell elastomeric foams, provided that the material is capable of deforming homogeneously and does not exhibit localized deformation, such as compaction banding. In this chapter, I consider three different relative densities of a single type of foam material. As mentioned above, the foam selected is a polyurethane-based open-cell elastomeric foam, “XRD”, developed by Rogers Corporation for impact mitigation applications. XRD has been the subject of previous studies in the literature [9, 27, 128], which have typically been observational. The three relative densities considered, as specified by the manufacturer, are 144 kg/m^3 , 192 kg/m^3 , and 240 kg/m^3 , which are hereafter referred to as the low, moderate, and high density foams, respectively.

To get a sense of the foam microstructure, Fig. 3.1 shows (A) a macro-scale image, (B) a micro-scale image, and (C) a 3D volume render obtained via micro-computed tomography (μCT) all for a representative sample of the moderate density foam. The pore structure is highly random, and based on optical microscopy measurements, the mean pore diameter is approximately $70 \mu\text{m}$ with a maximum pore diameter of up to about $500 \mu\text{m}$ for the moderate density foam. In addition to the random microstructure, preliminary characterization of the foam in each of the three orthogonal directions of the as-received material displayed no discernible physical difference in the stress-strain behavior, see Fig. 3.2. Thus, we choose to

model the foam material as isotropic.

3.2.2 Void Volume Fraction

Prior to testing each of the foam densities, the void volume fraction (i.e. porosity) of the foam was assessed via a water permeation experiment, wherein the void space of the foams was flooded with water by immersed compression and vacuum degassing and the absorbed mass of water was measured. The experiment was conducted by first excising a foam specimen of approximately $1.2 \times 1.2 \times 1.2 \text{ cm}^3$ from the original sheet material. The initial volume (V_{total}) was measured via water displacement and calipers. The initial (dry) mass was taken. The foam was submersed in water and firmly squeezed and released while submerged. The beaker containing foam and water was placed in a vacuum chamber for circa 20 min. Upon removal the foam was squeezed underwater again. The fully saturated specimen was then removed from the beaker (with no dripping), reweighed and measured with calipers. The foam was desiccated via uniaxial compression, and effluent water volume measured in a graduated tube. The void volume ratio was computed by

$$V_p = \frac{mass_{wet} - mass_{dry}}{\rho_{H_2O}}, \quad (3.2.1)$$

and

$$\phi_0 = \frac{V_p}{V_{total}} \quad (3.2.2)$$

where V_p is the pore volume, V_{total} is the volume of the foam specimen, and ρ_{H_2O} is the mass density of water at room temperature. As a secondary check, the volume of water removed during compression was measured and used as V_p in the equation 3.2.2. Using this method, the undeformed void volume fractions (ϕ_0) of the three foam densities are estimated at between $\phi_0 = 0.68\% - 0.94\%$, $\phi_0 = 63\% - 83\%$, and $\phi_0 = 68\% - 0.94$ for low, moderate, and high densities respectively, based on a series of three repeat measurements on each foam.

Based on a tomographic reconstruction, see Fig. 3.1(C), an example moderate density specimen was measured to have $\phi_0 = 71\%$. However, in this preliminary experiment no measurement phantom was available to calibrate for this material and the reconstruction

is relatively noisy due to the high x-ray permeability of the base polymer. Thus, although not quantified directly, error arising due to binarization uncertainty is relatively high due to the large surface-to-volume ratio and high aspect ratio members in the XRD foam, and this quantification is similarly vague. Due to the overall uncertainty in these measurements and challenges associated with collecting accurate data, the continuum framework was designed with the relative volume fraction (i.e. locking strain, $J_{min} = 1 - \phi_0$) as a fitted parameter.

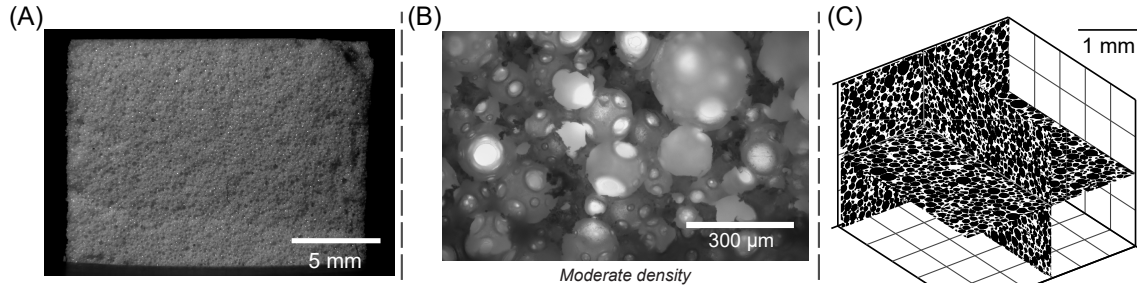


Figure 3.1: Three images of a representative moderate density foam sample at various scales. (A) The cut face of a foam cuboid. (B) Optical microscopy image of the microstructure of the surface of a cut face. (C) Micro-computed tomography 3D volumetric render of the foam microstructure (watershed-based binarization, in which black is void).

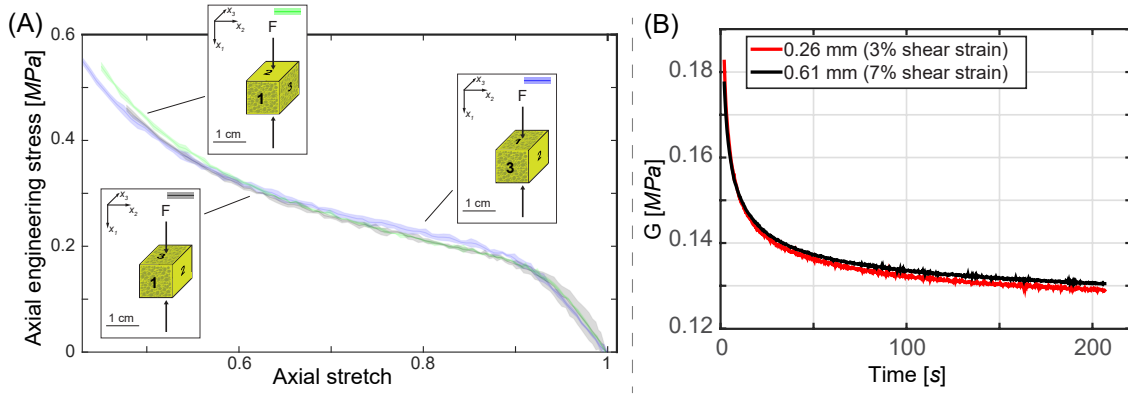


Figure 3.2: Exploratory experimental results for isotropy and time decay. (A) Initial results for a low density foam tested quasistatically (i.e., similarly to the procedure of Sec. 3.3.1) along three orthogonal axes (see insets), demonstrating the minimal anisotropy in this foam. The shaded region shows one standard deviation for three experiments on a representative low density specimen. No preparation difference, foaming mechanism changes, or images from the experiments indicated that the degree of isotropy would change for moderate or high density cases. (B) Initial stress relaxation experiments conducted in a simple-shear-like (single-lap, ASTM D4255 Procedure A) configuration to two strain levels, showing that dominant time dependent processes operate at a $O(10s)$.

3.2.3 Specimen preparation

The details of specimen preparation depend on the specific type of test, but in general, specimens are collected from as-received ca. 12.9 mm-thick sheets of bulk material, speckled for digital image correlation, and then installed in appropriate fixtures. For compression testing, foam cuboids of approximately $10 \times 10 \times 12.9 \text{ mm}^3$ are excised with a bandsaw. For tension testing, dogbone specimens are cut using a 75 W CO₂ laser cutter (Universal Laser Systems Inc., Scottsdale, AZ) to the ASTM D3574 standard geometry for tensile testing of foam polymers. For indentation experiments (Section 3.5.1), $25.4 \times 25.4 \times 12.9 \text{ mm}^3$ samples are laser cut and bonded to an aluminum backer on one of the large faces using a two-part epoxy (ITW Polymers Adhesives, Danvers, MA). For each shear experiment (Section 3.5.2), two $30 \times 30 \times 12.9 \text{ mm}^3$ specimens are laser cut, and each large, square face is epoxied to an aluminum backer. Finally, circular holes are cut out of tension specimens to be used for the third type of validation testing (Section 3.5.3). DIC measurements are used to obtain kinematic data in all experiments except for indentation testing, and to prepare specimens for DIC analysis, a face orthogonal to the testing axis is speckle-painted for each specimen. To speckle the specimens, a black base coat of acrylic ink (Liquitex Artist Materials, Piscataway, NJ) is airbrushed (HP-C Plus, Iwata Medea Inc, Portland, OR) on the surface, followed by a white speckle layer (Golden Artist Colors Inc., New Berlin, NY). For each configuration, the optical train employed is configured such that this technique yields speckles of approximately 3 to 7 px in size. Due to the inherent DIC challenges posed by mechanical testing of elastomeric foams – namely, that large speckle distortions and speckle loss due to cell collapse or opening arise when the material undergoes finite deformations – a custom DIC procedure is utilized [27], which incorporates the concept of cross-correlation quality factors to improve the accuracy of DIC measurements involving distorted speckle patterns undergoing large deformations.

Finally, for all experiments, both characterization and validation, three specimens are tested, and each specimen is tested three times (i.e., nine total experiments per condition). For repeated experiments on the same specimen, I ensured that the wait-time between experiments was sufficiently long to allow complete relaxation of any time-dependent material

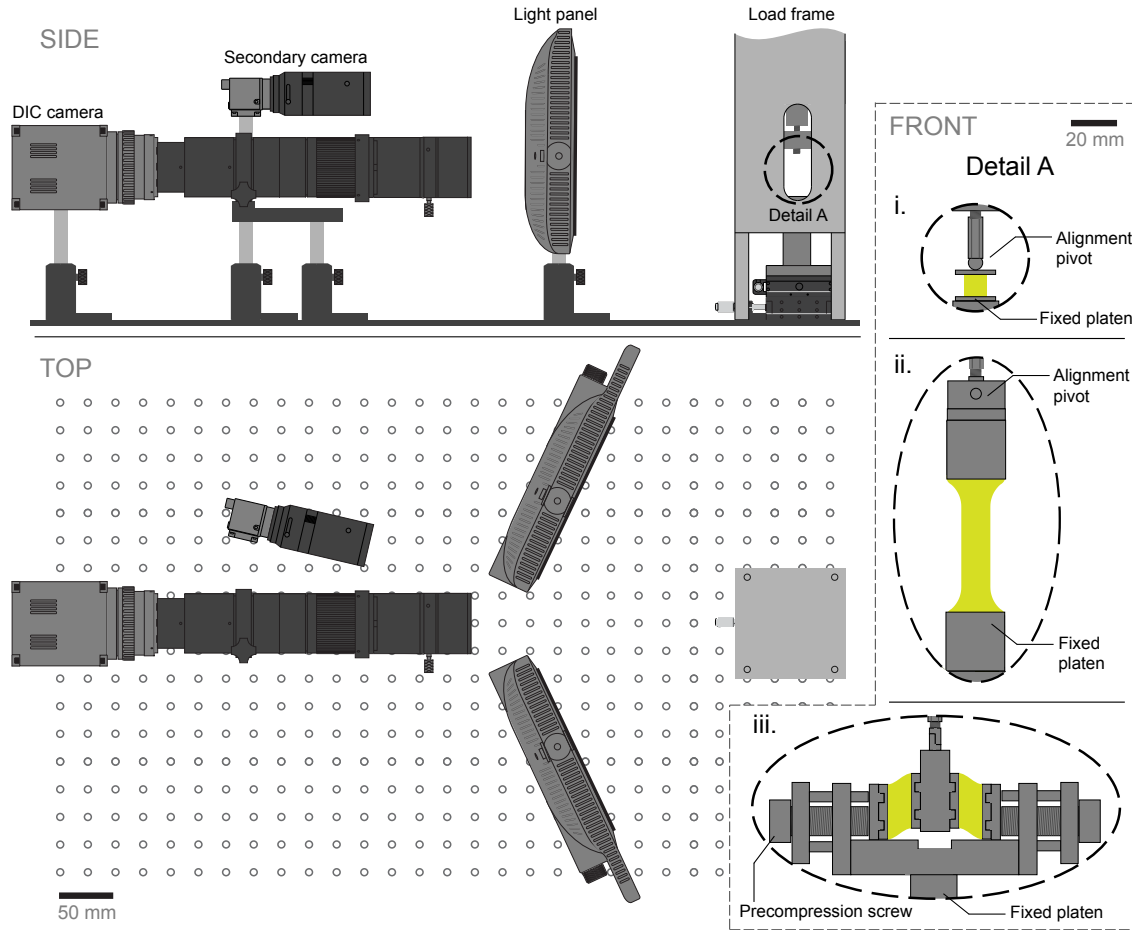


Figure 3.3: Schematic of the top and side views of the experimental setup as deployed on a vibration-isolated optical breadboard. The arrangement of the two cameras with lenses, light panels with linearly polarizing filters, and load frame is shown. Detail A gives front-view diagrams (i.e., as seen by the cameras) of the fixtures used for (i) simple compression, (ii) simple tension, and (iii) shear.

physics, typically 3 hours or more in total test time. Preliminary experiments investigating shear modulus, see Fig. 3.2(B), demonstrate that the dominate rates in the foam are $O(10\text{ s})$; secondarily, observations from experiments conducted with inter-test times varying from ca. 45 min to 3 hr show that beyond a total test time of approximately 2.5 hr minimal differences are observed in the stress-strain response.

3.3 Experimental setup

A schematic layout of the optical system and loading apparatus used in my experiments is shown in Fig. 3.3. For all experiments, the loading apparatus and control software are the same as that described in Chapter 2 and [27]. Specifically, a screw-driven linear actuator (Ultra Motion, Cutchogue, NY) and a load cell (LCFD-50, Omega Engineering, Stamford, CT) with a custom LabView (National Instruments, Austin, TX) interface that allows synchronized actuator displacement control, actuator displacement measurement, imaging, and force data collection. The optical system utilized to collect data for two-dimensional DIC is comprised of a custom control computer, a scientific camera (edge5.5, PCO-Tech Inc., Romulus, MI), a long-distance microscopy lens (K2/S, Infinity Photo-Optical Company, Boulder, CO), polarizing filters (American Polarizers Inc., Reading, PA and Edmund Optics, Barrington, NJ), and LED light panels (Lykos Daylight, Manfrotto, South Upper Saddle River, NJ). These parts are assembled on the same vibration-isolated table as the loading apparatus. Additionally, for all experiments, a secondary imaging camera with a field-of-view encompassing the complete specimen and load frame is used to detect any potential anomalous loading conditions (e.g., onset of specimen failure at the grips). A temperature and humidity gauge was also placed in the field of view of the secondary camera for the majority of experiments and monitored such that experiments were conducted between 22°C and 25°C and 10% and 40% relative humidity.

For each experiment type, a custom load-frame insert is used to grip specimens during loading. The three primary fixtures – simple compression, simple tension, and shear – are diagrammed in the insets of Fig. 3.3. By using modular inserts, the control procedures and DIC parameters (see Table 3.1) remain nearly consistent across experiment types. For simple compression, the insert consists of graphitic steel top and bottom platens compressed via a spherical indenter tip attached to the load cell. The spherical indenter tip acts as a pivot point, allowing any misalignment to be identified and eliminated during alignment prior to compression experiments. The contact areas between the platens and the specimen are lubricated with graphite powder and spherical glass microbeads (diameter 20-30 μm). For

simple tension, screw-actuated wedge-and-pin-type grips, with the gripping-force screw acting as the pin, are attached to the base of the load frame and load cell. The attachment to the load cell is via a tie-rod-end-type spherical pivot with a sliding joint, allowing for 4 degrees of freedom (three rotations, one translation) during loading. For the shear validation experiments, a symmetric “double-lap-shear” insert, involving two specimens with aluminum backers epoxied to both sides, is utilized, as shown in inset (iii) of Fig. 3.3. Akin to the shear testing of ASTM D4255 Procedure B, the center platen undergoes a vertical displacement prescribed by the linear actuator to shear the specimens. Additionally, horizontal screws on each side of the fixture may be adjusted to subject the specimens to a uniaxial pre-compression, which I set to be equal for both specimens in order to maintain symmetry, and which is held constant during the subsequent shear deformation.

3.3.1 Details of simple compression and tension tests

The characterization experiments, i.e., simple compression and tension, are conducted in displacement control to engineering strain limits of -0.75 (axial stretch of $\lambda = 0.25$) in compression and 0.5 (axial stretch of $\lambda = 1.5$) in tension and are loaded and unloaded at a constant, quasi-static engineering strain-rate magnitude of 2.0×10^{-4} 1/s where the choice of quasistatic rate was informed by preliminary shear stress relaxation experiments (see Fig. 3.2) and minimization of hysteresis in the load-unload curves. Illustrative example images of reference and deformed specimens for the high density foam are shown in Figs. 3.4(A) and (B) for compression and tension, respectively, where the direction of compression/tension is identified as the x_1 -direction, and the selected regions-of-interest for DIC analysis are outlined with dashed lines. Contours of the resultant two-dimensional displacement fields, i.e., $u_1(x_1, x_2)$ and $u_2(x_1, x_2)$ calculated using the custom qDIC procedure are also shown in Fig. 3.4 at axial stretches of $\lambda = 0.35$ for compression and $\lambda = 1.50$ for tension, In both examples, the contours reveal an axial displacement field u_1 that depends linearly on the axial coordinate x_1 and a lateral displacement field u_2 that depends linearly on the lateral coordinate x_2 , which indicates that the intended homogeneous deformation without shearing and with minimal barreling has been achieved.

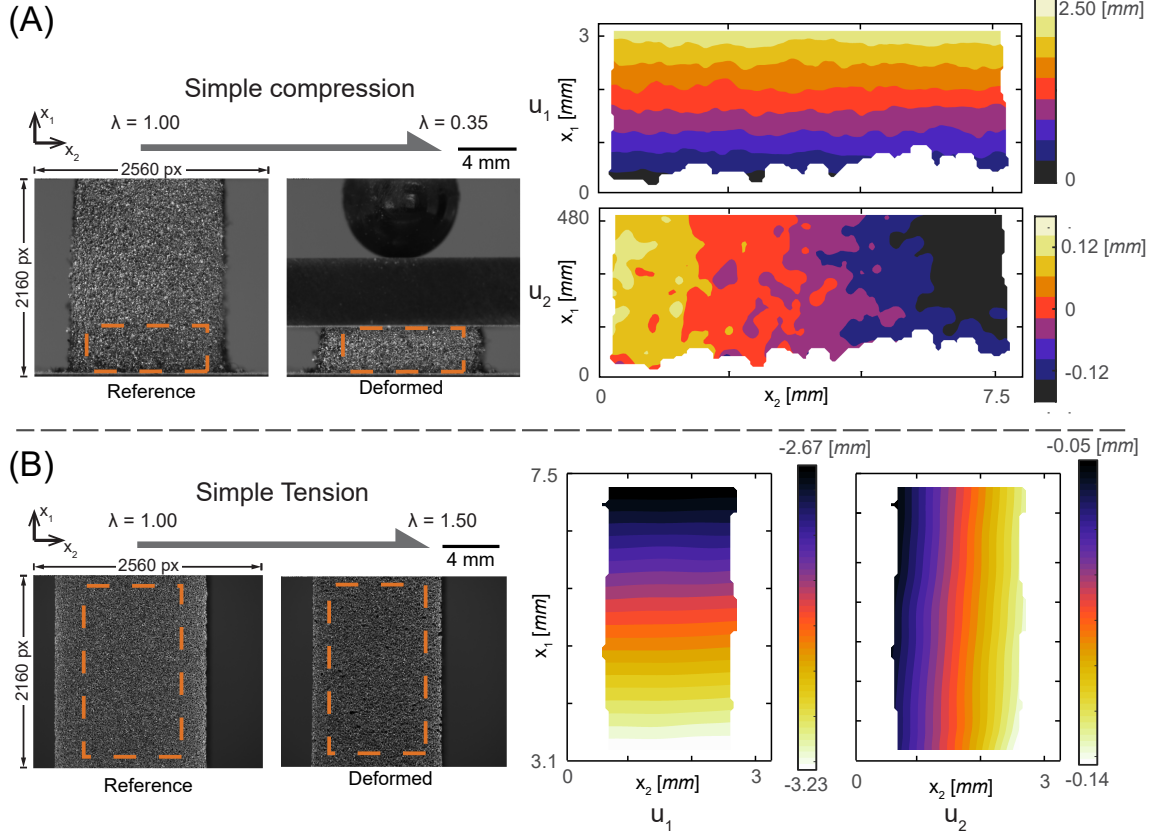


Figure 3.4: Example kinematic data collected from (A) simple compression (adapted from [27]) and (B) simple tension experiments on the high density foam. (*Left*) Images of the respective specimens in the reference and deformed states with the regions-of-interest for DIC analysis outlined with dashed lines. DIC regions are typically approximately $1200 \times 350 \text{ px}^2$ for compression and approximately $800 \times 1900 \text{ px}^2$ for tension. (*Right*) Contours of the DIC-calculated, two-dimensional displacement fields for the axial (u_1) and lateral (u_2) displacement components.

In my compression experiments, I do not observe evidence of strain localization into a small number of compaction bands such as often arise during compression of low-density and/or “re-entrant” foams (e.g. [35, 69, 93]), example images demonstrating this are given in Fig. 3.6. The deformation is globally homogeneous at the specimen-scale. At the spatial resolution of the DIC algorithm using subset size of approximately 16 px (see 3.3.2), inhomogeneities in the displacement field arising from strain localization, e.g. due to local cell buckling (cf. [48, 84, 147]), are an order of magnitude smaller than displacement due to the globally applied deformation and a similar scale as the displacement noise floor, see Fig. 3.7. Due to this high level of homogeneity relative to many other low-density open-cell foams, components

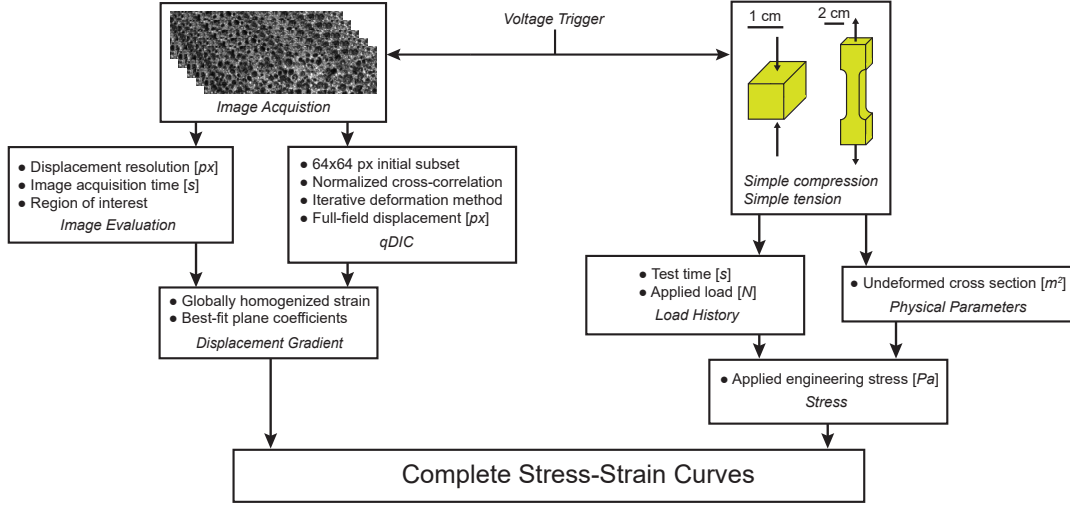


Figure 3.5: Outline of the experimental data-flow for quality-factor-based digital image correlation (qDIC) data, which is reduced to axial and lateral engineering strain data, and force data, which is used to compute the axial engineering stress. Experiments are initiated via a voltage signal provided when the imposed displacement history is begun, ensuring that time-synchronized load and images datasets are collected.

of the global displacement gradient tensor are extracted from these displacement fields for each deformation step in each experiment via a global plane-fitting procedure, which is described in detail in [27], and the components du_1/dx_1 and du_2/dx_2 are identified as the axial engineering strain and lateral engineering strain, respectively. For comparison, the strain maps based on an optimal-9 differentiation filter (e.g., see Table 1 of [27]; based on [200,228]) for a single compressive strain level ($e_{11} = 28\%$, $VSG = 80 \text{ px}^2$) are shown in Fig. 3.8, demonstrating the random nature of the diffuse localization. The corresponding axial engineering stress is computed from load-cell data and the previously-measured, referential cross-sectional area.

A summary of the experimental data-flow used to reduce experimental data to axial engineering strain, lateral engineering strain, and axial engineering stress data is outlined in Fig. 3.5. The axial engineering stress versus axial engineering strain and lateral engineering strain versus axial engineering strain data are plotted as solid lines in Fig. 3.9 for the (A) low, (B) moderate, and (C) high density foams. The black curves represent the average of the nine instances of each experiment, and the shaded areas surrounding the solid curves represent one standard deviation above and below the average, which indicate good repeatability.

Arrows indicate loading and unloading, demonstrating that hysteresis is minimal and that the strain-rate is sufficiently low such that the experimentally-measured behavior represents the equilibrium, elastic response of the material.

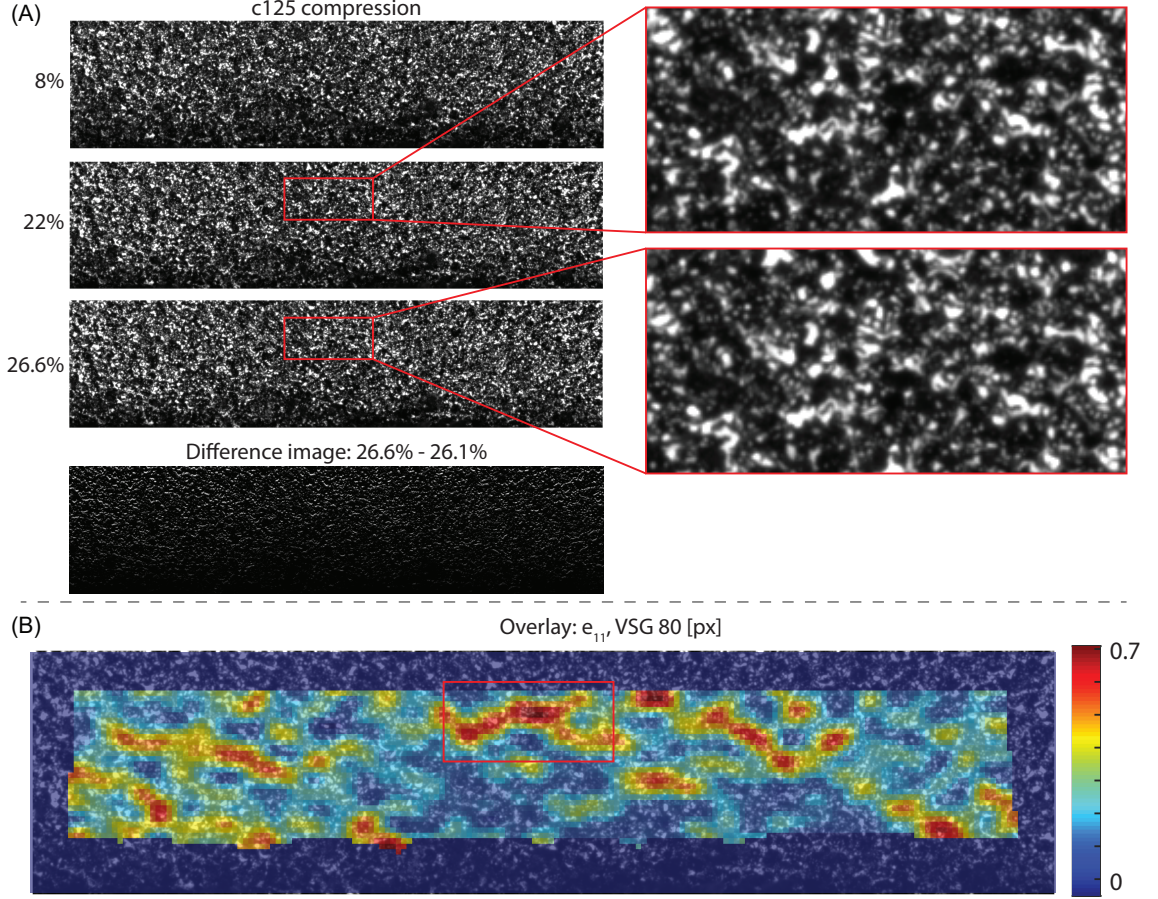


Figure 3.6: Example images for the moderate density foam, showing relatively little compaction band development or other extensive localization. (A) Successive images of the speckled surface of the foam during linear elastic and plateau deformation regimes. The difference image shows little evidence of compaction band formation at 26% applied strain for a 0.5% increment in strain. (B) The strain map (optimal-9 spatial differentiation kernel, $VSG = 80 \text{ px}^2$) for the 26.6% overall deformation case, showing inhomogeneity but no single extensive localization band. Mean strain in this image closely matches the plane-fit based measurement.

3.3.2 Digital image correlation parameters

The digital image correlation details for typical cases of each experimental DIC configuration are given in Table 3.1. Noise-floor resolution is defined as the standard deviation of the

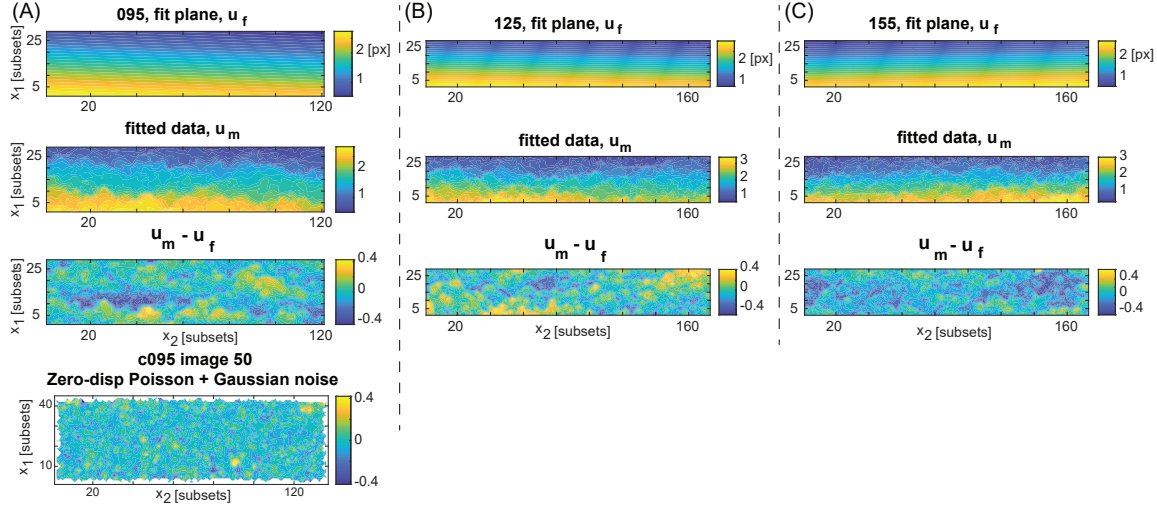


Figure 3.7: Example plane fitting results for the (A) low, (B) moderate, and (C) high density foams for an incremental displacement step from 26.1% to 26.6% global strain. Plots show contours of the fit plane, the experimental data and the point-wise difference between the best-fit plane and the experiment. Noise profile (bottom left) is obtained via qDIC between two copies of the 26.6% deformation image with added Poisson and white Gaussian noise with variance of 0.75% of the full-scale range of the image, mimicking the noise profile typical to my quasistatic imaging procedure.

DIC displacement reconstructed from sequential, nominally zero-displacement images. To document the strain computation using a global plane fit, the goodness-of-fit parameters (R^2 and root mean square error) for the beginning, middle, and end of the test from the plane fits to a typical experiment are provided in Table 3.2. This is an alternative to using the virtual strain gauge (VSG) measurement ($VSG = (SW - 1)dm + SS$, where SW is the strain window size, e.g. $SW = 9$ for optimal-9, dm is the step size between subset centers, and SS is the minimum subset size), typical for strain-kernel-based methods, since the VSG provides little practical information in the context of the global plane fitting technique employed.

3.4 Constitutive theory

Note that the vast majority of modeling and finite element analysis development was undertaken by my collaborator Ms. Xiuqi Li and her advisor Dr. David Henann; despite this, the modeling effort has been integral to the development and execution of my focus, the experiments, and thus I include a relatively detailed summary. In large part I have compiled

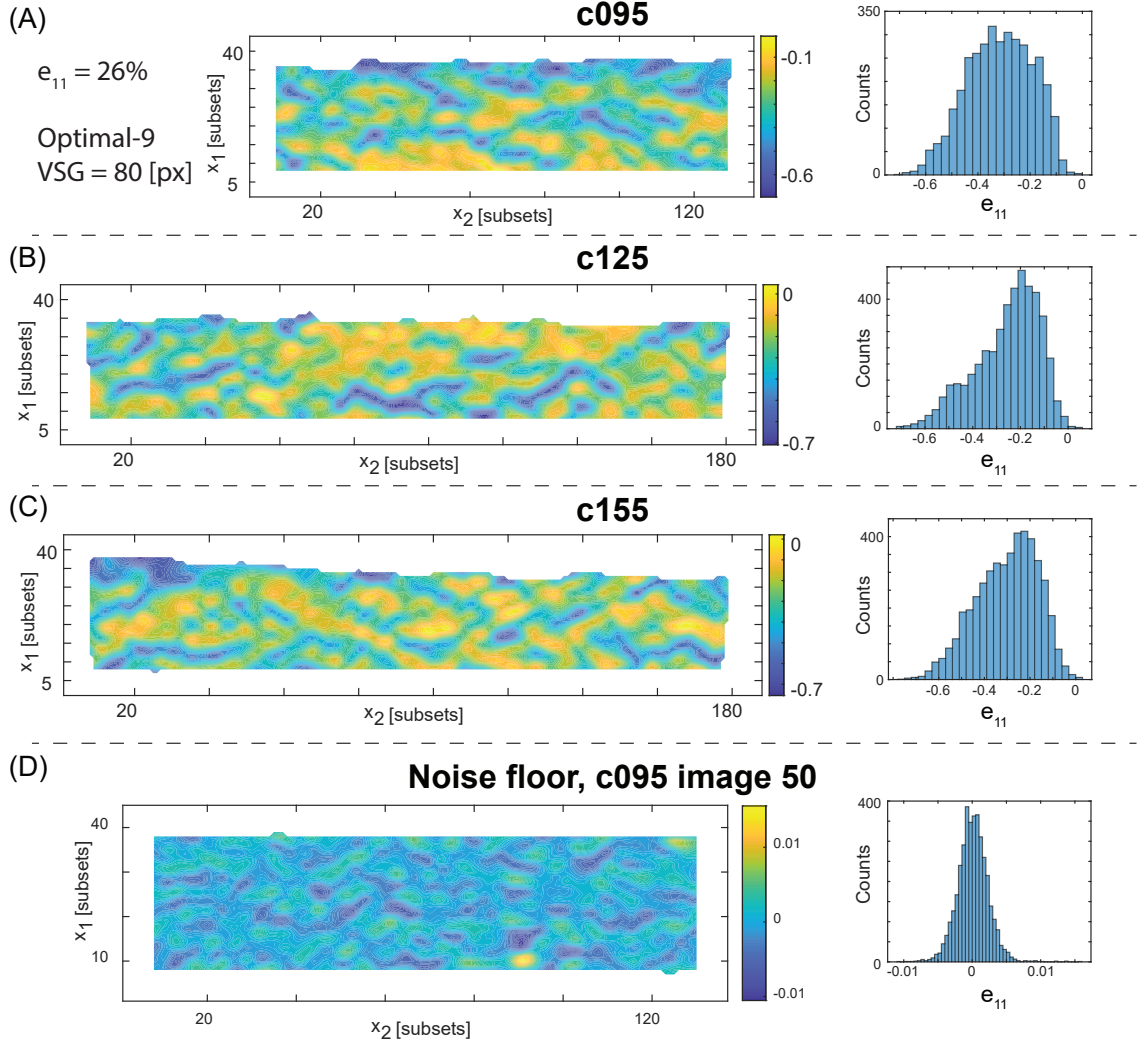


Figure 3.8: Example strain maps for (A) low, (B) moderate, and (C) high density XRD foam at approximately 26% global compression. While some degree of inhomogeneity exists, the distribution of strain is roughly skewed Gaussian-like and single-peaked, indicating the generally random nature of the underlying deformation and limited extent of localization [48]. (D) Shows a noise-floor derived from the zero-displacement noisy simulation of Fig. 3.7.

and written this summary based on notes provided to me by Ms. Li, and a significant portion of the account of the model I present herein constituted an initial draft of the theory sections of [28], although I have included edits, updates, and clarifications from the final manuscript of [28] as needed. The manuscript of [28] itself is the canonical version of the quasistatic model at the time of writing and provides a more comprehensive outlook on the theory and calibration procedures, in addition to a systematic analysis of model stability.

In this section, I describe our hyperelastic constitutive model for the elastic behavior of

Table 3.1: Typical digital image correlation specification details for the experimental calibration and validation test cases. In most cases, values reported are from the high density specimen 3 experiment 1 data and are typical.

Specification	Compression	Tension	Shear 0% Precompression	Shear 25% Precompression	Inhomogeneous Tension
Technique	Single Camera q-factor Based Digital Image Correlation				
Pre-filtering	Gaussian, [3px, 0.2]				
Min. Subset Size [px]	16				
Min. Step Size [px]	8				
Correlation criterion	FFT-based ZNCC				
Interpolation	Bicubic splines				
Camera noise [%]	0.30	0.25	0.25	0.25	0.25
Image size [px ²]	1200x350	800x1900	1300x1200	950x1200	1100x2000
Measurement points	6560	23750	24375	17813	34375
Number of images	260 (0.033 Hz)	250 (0.040 Hz)	180 (0.033 Hz)	200 (0.033 Hz)	235 (0.143 Hz)
Pixel-to- μm conversion	1 px = 10 μm	1 px=8 μm	1 px=8 μm	1 px=8 μm	1 px = 8 μm
Resolution: spatial (est.)	8 px = 80 μm	8 px = 64 μm	8 px = 64 μm	8 px = 64 μm	8 px = 64 μm
Resolution: noise [px]	0.05	0.0075	0.046	0.033	0.05

Table 3.2: Typical plane fit parameters for strain computation in compression and tension experiments. Values reported are from the moderate density specimen 2 experiment 3 data and are typical. Perfect R^2 is 1.00 (R^2 values above 0.99 are rounded down) and lower root mean square error (RMSE) is a better fit.

Image number –	Compression		Tension	
	R^2	RMSE	R^2	RMSE
10	u_1 : 0.95	u_1 : 0.93	u_1 : 0.99	u_1 : 0.29
	u_2 : 0.97	u_2 : 0.57	u_2 : 0.99	u_2 : 1.72
125	u_1 : 0.99	u_1 : 1.71	u_1 : 0.99	u_1 : 1.34
	u_2 : 0.88	u_2 : 4.59	u_2 : 0.99	u_2 : 4.00
230	u_1 : 0.97	u_1 : 1.71	u_1 : 0.75	u_1 : 1.29
	u_2 : 0.07	u_2 : 4.54	u_2 : 0.74	u_2 : 2.32

isotropic, elastomeric foams and apply the model to the XRD foam. The calibrated model fits are shown in Fig. 3.9. To begin discussing a material model, a self-consistent mathematical framework for defining the governing equations is necessary to develop the constitutive relations. Since elastomeric foams exhibit large deformations with coupled deviatoric and volumetric responses, a hyperelastic finite deformation formulation in a specialized invariant basis (i.e. that due to [227]) is adopted. In this basis space, the physical meaning of each invariant is more clearly associated with a particular deformation mode, which improves the translation of experimental data into an intuitive and properly calibrated model for predictive simulation.

3.4.1 Basic kinematics

We consider a body B identified with the region of space it occupies in a fixed reference configuration and denote an arbitrary material point within B as $\mathbf{X}$. The referential body B then undergoes a motion $\mathbf{x} = \chi(\mathbf{X}, t)$ to the deformed body $\mathcal{B}_t$ at each time t . The deformation gradient is given by $\mathbf{F} = \nabla \chi$, such that the ratio of referential to deformed volume is strictly greater than zero, i.e., $J = \det \mathbf{F} > 0$.¹ The displacement field is denoted as $\mathbf{u}(\mathbf{X}, t) = \chi(\mathbf{X}, t) - \mathbf{X}$, and the left and right Cauchy-Green deformation tensors are $\mathbf{B} = \mathbf{F}\mathbf{F}^\top$ and $\mathbf{C} = \mathbf{F}^\top \mathbf{F}$, respectively. The deformation tensors admit the spectral decompositions $\mathbf{B} = \sum_{i=1}^3 \lambda_i^2 \mathbf{l}_i \otimes \mathbf{l}_i$ and $\mathbf{C} = \sum_{i=1}^3 \lambda_i^2 \mathbf{r}_i \otimes \mathbf{r}_i$, where $\{\lambda_i | i = 1, 2, 3\}$ are the principal stretches, $\{\mathbf{l}_i | i = 1, 2, 3\}$ are the principal directions of $\mathbf{B}$, and $\{\mathbf{r}_i | i = 1, 2, 3\}$ are the principal directions of $\mathbf{C}$. Finally, for later use, we introduce the logarithmic (Hencky) finite-strain tensor in the deformed body: $\mathbf{E} = \sum_{i=1}^3 (\ln \lambda_i) \mathbf{l}_i \otimes \mathbf{l}_i$.

3.4.2 Isotropic free-energy function

As in many general hyperelastic material models for finite deformation, the constitutive response is characterized by the functional form of the strain-energy density function, ψ , that

¹*Notation:* The symbols ∇ , Div and Curl denote the gradient, divergence, and curl with respect to the material point $\mathbf{X}$ in the reference body B ; grad , div , and curl denote these operators with respect to the point $\mathbf{x} = \chi(\mathbf{X}, t)$ in the deformed body $\mathcal{B}_t$. We write $\text{tr} \mathbf{A}$, $\det \mathbf{A}$, and $\text{dev} \mathbf{A}$ to denote the trace, determinant, and deviatoric part of a tensor $\mathbf{A}$, respectively.

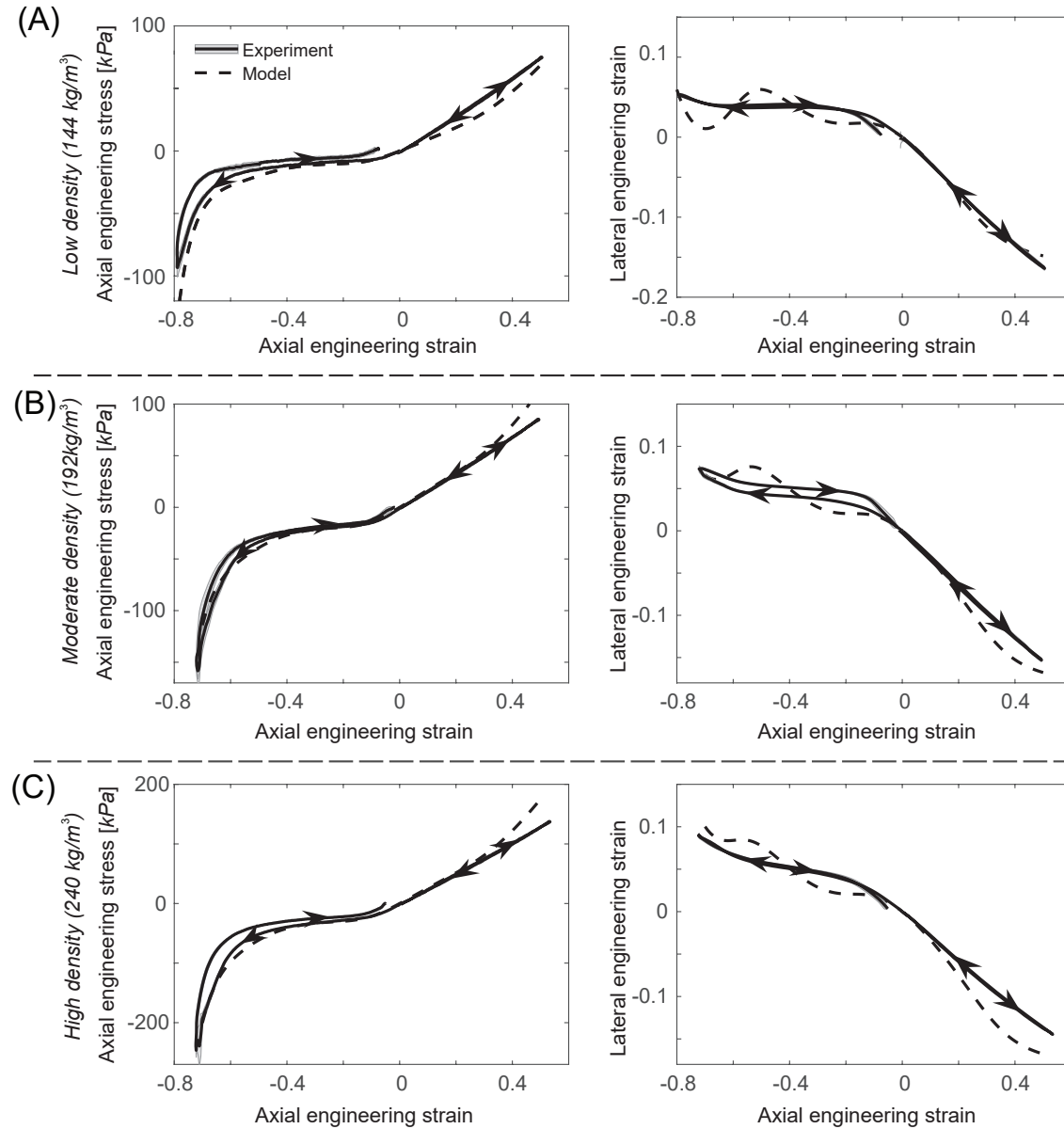


Figure 3.9: (Left) Axial engineering stress versus axial engineering strain curves and (right) lateral engineering strain versus axial engineering strain curves for polyurethane-based, open-cell elastomeric foams of (A) low (144 kg/m³), (B) moderate (192 kg/m³), and (C) high (240 kg/m³) density with the same matrix composition. The solid lines with shaded error areas (one standard deviation, nine experiments) indicate experimental data, and the dashed lines are constitutive model fits. Note that for (C) the stress axis differs in range from (A) and (B). For all densities, note both the plateau regime in compression in both the stress and lateral strain histories as a function of axial strain, and the small lateral strain magnitudes under large axial strain magnitudes as characteristic features of deformation in open-cell elastomeric foams.

depends on the local deformation gradient tensor, $\mathbf{F}$. In particular, for an isotropic material, ψ is an isotropic function of $\mathbf{C}$, the right Cauchy-Green deformation tensor. Thus, for an isotropic material, three invariants $\mathbf{C}$ are necessary and sufficient to span the basis space of deformation states and replace $\mathbf{F}$. Numerous invariants are available and are employed, as convenient, to describe finite deformations. The most typical are principal invariants of $\mathbf{C}$, i.e., $I_1 = \text{tr } \mathbf{C}$, $I_2 = (1/2)((\text{tr } \mathbf{C})^2 - \text{tr } (\mathbf{C}^2))$, $I_3 = \det \mathbf{C} = J^2$ as in [15, 19, 21, 128, 157, 229] or the principal stretches $\{\lambda_1, \lambda_2, \lambda_3\}$ of $\mathbf{V}$, where $\mathbf{F} = \mathbf{V}\mathbf{R}$ is a polar decomposition into deformation and rotation tensors. For example, the Ogden-type descriptions of elastomers such as [12, 13, 230] are principal-stretch-based. The typical principal invariant basis arises straightforwardly from the eigen-state of the deformation, but the physical meaning of each invariant is not easily interpreted and capturing experimental data for compressible foams is difficult, since it is not straightforward to isolate the effect of each invariant (for example, see the discussion of [231]). Similarly, the principal stretch bases admits a straightforward spectral decomposition, but the $\{\lambda_1, \lambda_2, \lambda_3\}$ basis is unwieldy for modeling purposes. To avoid these difficulties, the continuum formulation for the free-energy function adopted is based upon invariants of the Hencky (i.e. natural or logarithmic) strain tensor $\mathbf{E}$, as proposed in [227].

3.4.3 Invariant Basis

The log-strain invariants of [227], denoted $\{K_1, K_2, K_3\}$, more accurately describe moderate strains (i.e. 5% to 25% strains) and describe physical modes of deformation that are experimentally tractable, thus reducing measurement error. To wit, the first invariant, $K_1 = \text{tr } (\mathbf{E}) = \ln(J)$; $K_1 \in (-\infty, \infty)$ where $J = \det(\mathbf{V}) = \det(\mathbf{F})$ as above, gives the magnitude and sign of dilatation in the material with positive values for dilation and negative for contraction, i.e. it is an amount-of-dilatation measure. The second invariant, $K_2 = |\text{dev } (\mathbf{E})| = \sqrt{\text{dev } (\mathbf{E}) : \text{dev } (\mathbf{E})}$; $K_2 \in [0, \infty)$, describes the magnitude of constant-volume distortion in the material, i.e. it is a magnitude-of-distortion measure (note that by defining the tensorial direction of deviatoric strain as $\mathbf{N} = \text{dev } (\mathbf{E})/K_2$, such that $\mathbf{N}$ is a deviatoric and unit-magnitude tensor, the logarithmic strain may be expressed as

$\mathbf{E} = \frac{1}{3}K_1\mathbf{1} + K_2\mathbf{N}$). The third invariant defines the type of distortion in the material via $K_3 = 3\sqrt{6}\det(\mathbf{N})$, i.e. it is a mode-of-distortion measure. The pre-factor $3\sqrt{6}$ was chosen so that $K_3 \in [-1, 1]$. K_3 is constant for a given distortion mode, e.g $K_3 = -1$ in compression, $K_3 = 0$ in shear, and $K_3 = 1$ in tension. The experiments to characterize a material described by a free-energy function in this basis space (i.e., $\psi(K_1, K_2, K_3)$) is designed to specify and/or measure amount-of-dilatation and/or magnitude-of-distortion under a fixed K_3 . Finally, as shown by [227], the Cauchy stress using these invariants given through the derivatives of the free-energy function $\tilde{\psi} = \psi(K_1, K_2, K_3)$ by

$$\mathbf{T} = J^{-1} \left[\frac{\partial \tilde{\psi}}{\partial K_1} \mathbf{1} + \frac{\partial \tilde{\psi}}{\partial K_2} \mathbf{N} + \frac{\partial \tilde{\psi}}{\partial K_3} \frac{1}{K_2} \mathbf{Y} \right] \quad (3.4.1)$$

where $\mathbf{Y} = 3\sqrt{6}\mathbf{N}^2 - \sqrt{6}\mathbf{1} - 3K_3\mathbf{N}$.

3.4.4 Decomposition of the free-energy function

To develop a functional form of the free energy function, $\tilde{\psi}$, the characteristics of the mechanical response are determined, and appropriate terms extracted to describe key features. In the free energy decomposition used to capture the observed behavior of open-cell foam, in particular the coupled response between invariants, $\tilde{\psi}$ is additively decomposed into three terms in the form

$$\psi(K_1, K_2, K_3) = G_0 [X(K_1)K_2^2 + L(K_2, K_3)] + Bf(K_1), \quad (3.4.2)$$

where G_0 and B are the ground state shear and bulk moduli, respectively, and $X(K_1)$, $L(K_2, K_3)$, and $f(K_1)$ are phenomenological fitting functions. The interpretation for each term is as follows. The $G_0X(K_1)K_2^2$ term captures the coupled distortional and volumetric response. The $G_0L(K_2, K_3)$ term is the higher-order distortional response, which we assume is uncoupled from the volumetric response, but the inclusion of K_3 allows for mode-dependency in higher-order distortions. Finally, $Bf(K_1)$ describes the purely volumetric response. More detail is available in [28]. Certain restrictions are required on these functions to ensure that the solution to the free-energy is admissible over the greatest range possible. Specifically,

the following are needed: $X(K_1 = 0) = 1$ and remains positive in the intended K_1 range; $L(K_2 = 0, K_3) = \partial L / \partial K_2|_{K_2=0} = 0$, and that, following [227], $\partial L / \partial K_3$ goes to zero as order K_2^3 or higher as K_2 goes to zero. Moreover, we require that $\partial L / \partial K_2 > 0$ for all $K_2 > 0$ and $-1 \leq K_3 \leq 1$, so that the function $L(K_2, K_3)$ increases monotonically with K_2 ; $d^2 f / dK_1^2 > 0$ for all K_1 , i.e., that df/dK_1 is a monotonically increasing function of K_1 .

3.4.5 Specialization of the free-energy function

To fit to the experimental data collected in Section 3.2 for low, moderate, and high density XRD foam, the phenomenological fitting functions are chosen as follows for each term in the free energy decomposition, Eq. 3.4.2. In future, alternative forms may be developed for other materials. The coupled response function $X(K_1)$ is iteratively calibrated to

$$X(K_1) = \frac{1}{2} (X'_1 + X'_2) K_1 + \frac{\Delta_K}{2} (X'_1 - X'_2) \ln \left[\frac{\cosh((K_1 - K_1^0)/\Delta_K)}{\cosh(K_1^0/\Delta_K)} \right] + 1. \quad (3.4.3)$$

where X'_1 is the initial slope of $X(K_1)$, $\Delta_K > 0$ the parameter denoting the K_1 -range across which X'_1 transitions to X'_2 , where X'_2 is a small value compared to X'_1 to capture the experimental data and chosen such that constraints on $X(K_1)$ and df/dK_1 are satisfied, and $K_1^0 < 0$ is the position in K_1 of the transition to the plateau region. The higher-order distortional response is taken to be given by

$$L(K_2, K_3) = C_0 K_2^p + C_1 (1 + K_3) K_2^q, \quad (3.4.4)$$

where $C_0 > 0$ and $C_1 > 0$ are constant parameters and $p > 2$ and $q > 2$ are higher-order exponents. Finally, the purely volumetric response function is fitted to

$$\frac{df}{dK_1} = \frac{J^{C_2} - 1}{C_2} + C_3 J \left[J^{-r} - \left(\frac{1 - J_{\min}}{J - J_{\min}} \right)^r \right], \quad (3.4.5)$$

where $J = \exp(K_1)$ is the volume ratio and $J_{\min}$, r , C_2 , and C_3 are constant parameters. We note that the parameter $J_{\min}$ represents the minimum allowable value of J and that df/dK_1 diverges as J approaches $J_{\min}$ from above. The parameter $J_{\min}$ is limited to be

3.5 Validation experiments and simulations

In this section, I discuss the validation of the predictive capability of our hyperelastic constitutive theory by comparing experimental measurements to model predictions (model implementation and finite element simulations conducted by Ms. Xiuqi Li) in settings involving inhomogeneous deformation. To probe the model in a variety of situations, i.e., compression, shear, and tension-dominated situations, we consider three specific types of validation tests: (1) spherical and conical indentation, (2) shear both without and with a fixed amount of pre-compression, and (3) tension of a specimen with circular holes. The constitutive model of Section 3.4 was numerically implemented in Abaqus/Standard [232] using a user-material (UMAT) subroutine. Throughout this section we obtain model predictions in inhomogeneous deformation scenarios using finite-element calculations in Abaqus/Standard. Similarly to the theory section, Section 3.4, reports for finite element results are my summarized interpretation of notes provided by Ms. Li; refer to [28] for a more thorough discussion of modeling aspects.

3.5.1 Spherical and conical indentation

First, I consider the indentation validation case, for both spherical and conical indenter geometries. Both indentation configurations involve compression-dominated deformation directly beneath the indenter. As discussed in Section 3.2, indentation experiments are conducted on foam specimens with a square $25.4 \times 25.4 \text{ mm}^2$ cross section and a height of $H = 12.9 \text{ mm}$. One square face of each specimen is constrained by a bonded aluminum backer, and the specimen is indented on the opposite unconstrained face. Spherical indentation is performed using a steel indenter tip with a radius of $r = 3.97 \text{ mm}$, and conical indentation is performed using an aluminum indenter tip with a base angle of $\theta = 30^\circ$. Photographs of the experimental setup prior to deformation for spherical and conical indentation are shown in Fig. 3.10(A) and (C), respectively. Experiments are performed at an indenter displacement-rate of $\dot{\delta} = 2.0 \times 10^{-3} \text{ mm/s}$ for spherical indentation and $\dot{\delta} = 2.5 \times 10^{-3} \text{ mm/s}$ for conical indentation. The maximum indenter displacements achieved during spherical and conical indentation are approximately 4 mm in all cases. The experimentally-measured

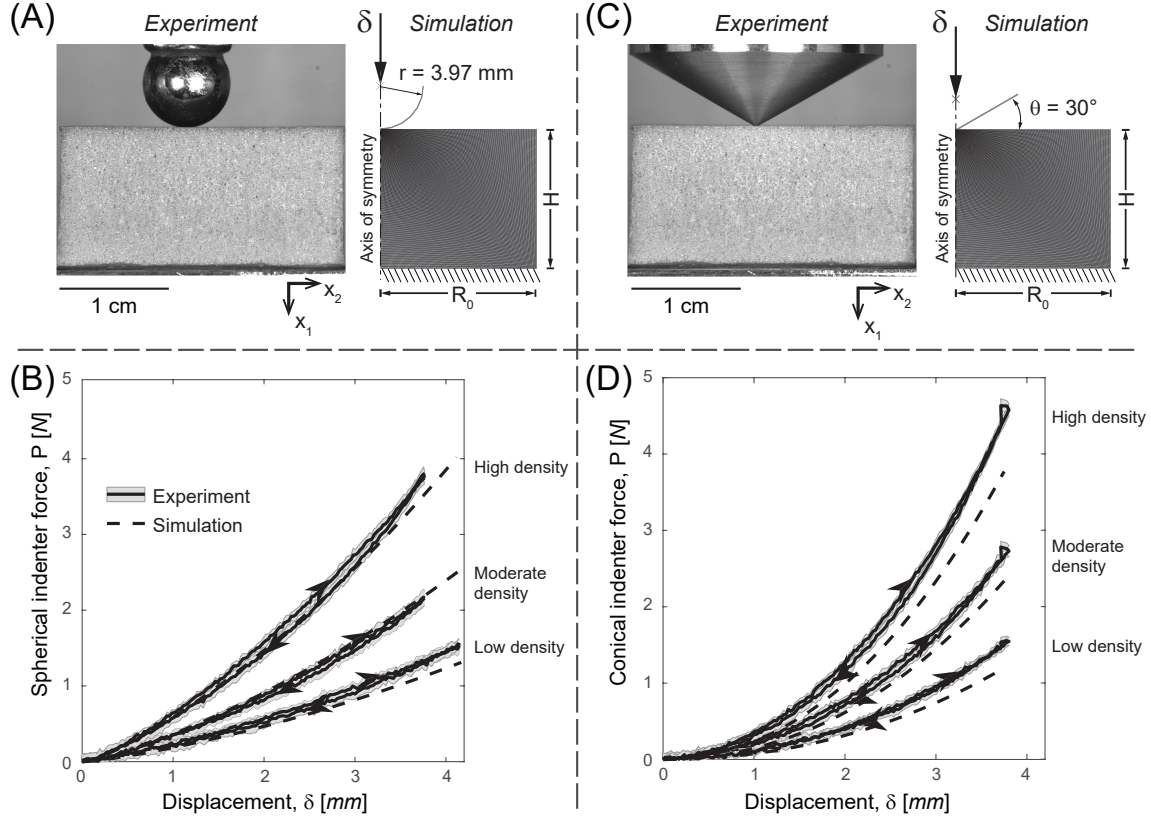


Figure 3.10: Spherical and conical indentation validation tests. Snapshots of the experimental setup prior to deformation and the corresponding finite-element configurations for (A) spherical and (C) conical indentation. Indenter force P versus indenter displacement δ for (B) spherical and (D) conical indentation. Solid lines with shaded error regions represent experimental data, and dashed lines are the predictions from the constitutive model.

indenter force P versus displacement δ relations for spherical and conical indentation are shown as solid lines in Figs. 3.10(B) and (D), respectively. In both cases, data for all three foam densities are shown. As in Fig. 3.9, the shaded regions surrounding the solid curves represent one standard deviation above and below the mean across all nine instances of the experiment, and the arrows indicate loading and unloading. Hysteresis is minimal, demonstrating that the chosen indenter displacement-rates are sufficiently slow.

For the corresponding finite-element simulations we idealize the geometry as axisymmetric for computational efficiency, which involves idealizing the cuboid foam specimen as a disk of radius R_0 and height $H = 12.9$ mm with outer radius $R_0 = 14.4$ mm. The axisymmetric finite-element configurations for spherical and conical indentation are shown in Figs. 3.10(A) and (C), respectively. The foam substrate is meshed using 17,236 Abaqus-CAX4H elements

(four-node, axisymmetric, quadrilateral, constant-pressure elements) and the indenters are idealized as rigid surfaces. Contact between the foam substrate and the indenter is set to no slip in both cases. To model the fixed-base boundary condition all displacement components are prescribed to be zero on the bottom face of the mesh. The left face of the mesh is coincident with the axis of symmetry, and hence, the horizontal displacement is set to zero along this face. The right face of the mesh is unconstrained. The calculated indenter force P versus displacement δ relations given the model parameters identified in Table 3.3 are included as dashed lines for all three foam densities in Figs. 3.10(B) and (D). Generally, there is good agreement between the experimentally-measured and simulated responses, especially for the case of spherical indentation. Idealization and no-slip assumptions were checked in preliminary experiments and are quite accurate.

3.5.2 Simple-shear-like deformation without and with pre-compression

Next, I conducted experiments to validate aspects of the model related to shear-dominated deformations. I performed two types of experiments, carried out on foam cuboids with a square cross-section of side-length $W = 30$ mm and a height of $H = 12.9$ mm. First, I subject foam cuboids to a simple-shear-like deformation, in which one square face is translated parallel to the other by a shear displacement δ . In these experiments, the square faces are constrained by bonded aluminum backers, and the shear displacement is applied by moving one backer at a rate of $\dot{\delta}/H = 2.0 \times 10^{-4}$ 1/s to a maximum shear displacement of $\delta/H = 0.5$ and followed, symmetrically, by unloading while the other backer remains fixed and the distance between backers is held at H . Notably, all rectangular side-faces remain unconstrained, so that edge-effects are present, and deformation is simple-shear-like and inhomogeneous. In the second type of experiment, we consider simple-shear-like deformation with a fixed uniaxial pre-compression as an example of coupled distortional/volumetric deformation. The simple-shear-like experiment with pre-compression is carried out in two steps. First, the specimen is compressed in displacement control such that the final distance between the constrained faces is $0.75H$. Specimens are then allowed to rest for at least 40 minutes while the pre-compression is held fixed in order to allow complete relaxation of viscoelastic material

behavior. Second, the specimen is deformed by a shear displacement δ at the same rate of $\dot{\delta}/H = 2.0 \times 10^{-4} \text{ 1/s}$ to a maximum shear displacement of $\delta/H = 0.375$ and then unloaded, while the distance between the constrained faces is held fixed at $0.75H$. In both types of experiments we utilize a fixture akin to a double-lap shear design, as mentioned in Section 3.2 and pictured in Fig. 3.11(A), in which two specimens are tested simultaneously in order to maintain symmetry, and therefore, the resultant shear force P is identified as half the force applied to the moving center platen. The measured normalized shear force P/W^2 versus shear displacement δ relations for all three foams densities and both without and with pre-compression are shown as solid lines in Fig. 3.11(C). As before, the shaded regions denote the range of one standard deviation above and below the mean experimental measurement, and the arrows denote loading and unloading, demonstrating minimal hysteresis. Two features are notable in the force-displacement relations. First, in shear-dominated deformation, no plateau region is observed, and the slope of the force-displacement relation increases slightly as deformation progresses. Second, when pre-compression is applied, the force-displacement relation becomes more compliant.

For the corresponding finite-element simulations, a fully three-dimensional model is utilized consisting of 97,200 Abaqus-C3D8H elements (eight-node, three-dimensional, hexahedral, constant-pressure elements) is shown in Fig. 3.11(A). For simple-shear-like deformation *without* pre-compression all displacement components are prescribed to be zero on the right face of the mesh. On left face the vertical displacement u_2 is prescribed, while the other displacement components are constrained to zero. The remaining faces are traction-free. The deformed mesh after a shear displacement of $\delta/H = 0.5$ is also shown in Fig. 3.11(A). For simple-shear-like deformation *with* pre-compression, pre-compression is applied by prescribing $u_1 = -0.25H$ and $u_2 = u_3 = 0$ on the right face of the mesh, while holding all displacement components at zero on the left face of the mesh. The deformed mesh after pre-compression is shown in 3.11(B). Then, a shear displacement is applied by fixing $u_1 = -0.25H$ and $u_2 = u_3 = 0$ on the right face of the mesh and prescribing $u_2 = \delta$ and $u_1 = u_3 = 0$ on the left face of the mesh. The deformed mesh is shown in Fig. 3.11(B). The calculated shear force P versus shear displacement δ relations are included as dashed lines in Fig. 3.11(C) for each

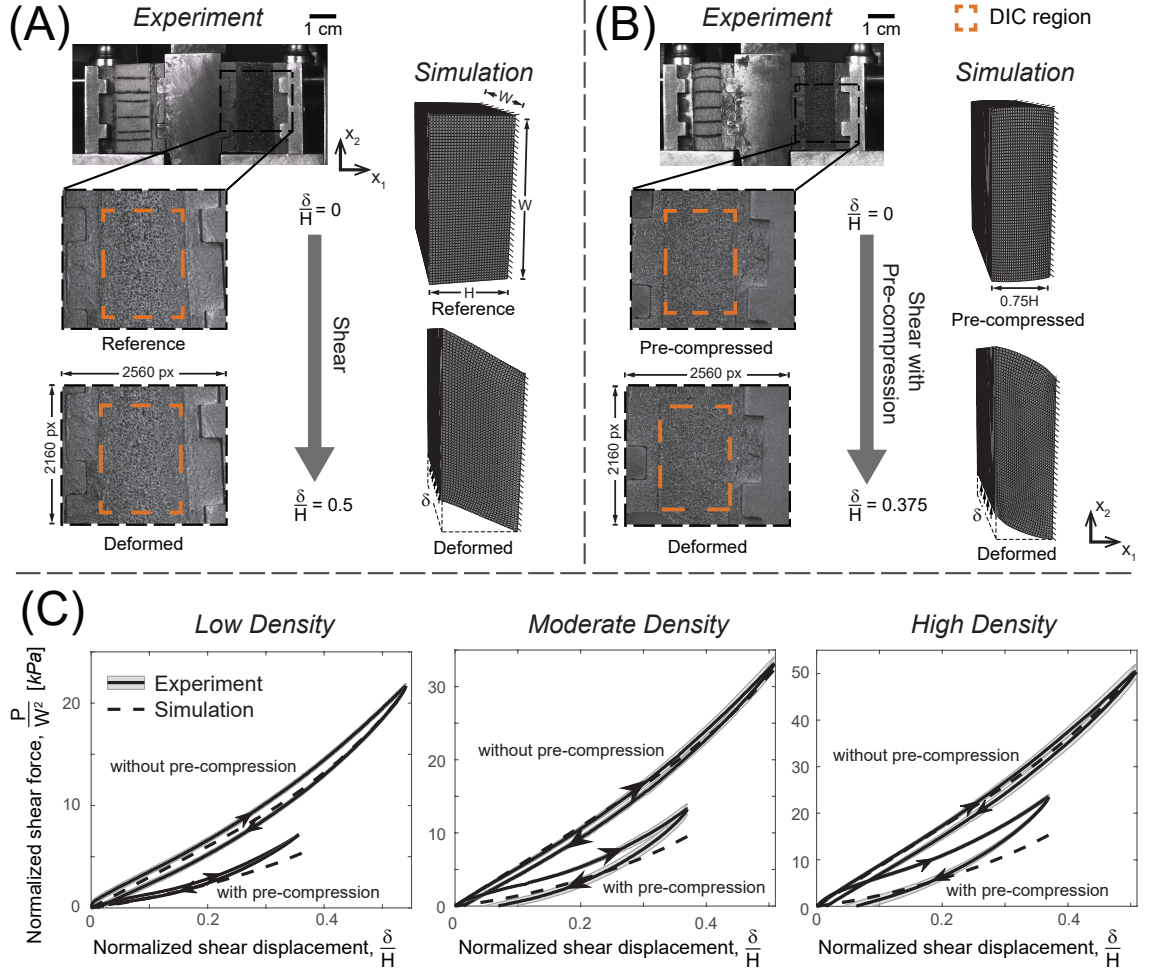


Figure 3.11: Simple-shear-like validation tests both without and with uniaxial pre-compression. (A) Snapshot of the double-shear fixture with two specimens prior to deformation and the experimental region-of-interest used for DIC imaging (ca. $1300 \times 1200 \text{ px}^2$) in both the reference and the deformed states for the case without pre-compression. The corresponding finite-element configuration is shown on the right, also in both the reference and the deformed states. (B) Experimental images and finite-element configuration for the case with pre-compression (DIC region ca. $950 \times 1200 \text{ px}^2$), displaying the pre-compressed and the deformed, sheared states. (C) Normalized shear force P/W^2 versus normalized shear displacement δ/H both without and with pre-compression and for all three densities. Solid lines with shaded error regions represent experimental data, and dashed lines are simulated predictions of the constitutive model.

density both without and with pre-compression. For all densities, the model captures the response both without and with pre-compression. In particular, our model is able to capture the lack of a plateau region in the response during shear-dominated deformation and the increased compliance with pre-compression.

In addition to P versus δ relations, I also compared experimentally-measured and numerically-simulated displacement fields on the front face of the specimen. The DIC regions-of-interest are outlined with dashed lines in the reference, undeformed state and the deformed state at the maximum level of shear displacement for simple-shear-like deformation without and with pre-compression in Figs. 3.11(A) and (B), respectively. Contours of the two-dimensional, DIC-calculated displacement field, $u_1(x_1, x_2)$ and $u_2(x_1, x_2)$, on the front face of the specimen in a representative experiment on the moderate density foam are shown in the left columns of Figs. 3.12(A) and (B). In Fig. 3.12(A), the fields represent the displacement from the reference state to the deformed state, the fields in Fig. 3.12(B) for simple-shear-like deformation with pre-compression represent the displacement from the pre-compressed state to the deformed, sheared state. This exception in the definition of the displacement field is only made in Fig. 3.12(B) and, although somewhat imprecise, we denote the displacement components as u_1 and u_2 for notational convenience. The corresponding simulated displacement fields are shown in the right columns of Figs. 3.12(A) and (B), demonstrating that the salient features of the experimental displacement fields are captured by the model. In particular, inhomogeneous deformation occurring due to edge-effects are clearly apparent in the u_1 -fields in both experiments and simulations. Overall, the magnitudes and distributions of both the u_1 - and u_2 -components of the displacement field are consistent between experiment and simulation.

3.5.3 Tension of a specimen with circular holes

Finally, I conducted a validation test to evaluate the predictive capability of the constitutive model in a tension-dominated setting. Given the favorable comparisons of model predictions and experiments across all three densities in the cases of indentation and shear, I focus only on the high density foam in this final test. To obtain inhomogeneous deformation fields, I took inspiration from the work of [233] and cut circular holes in my tension specimens. Specifically, I began with the dogbone geometry used in the simple tension characterization experiments described in Section 3.2 and laser-cut six, non-overlapping circular holes, each with a radius of 2.5 mm, at arbitrarily-chosen locations in the gage-section. This creates a

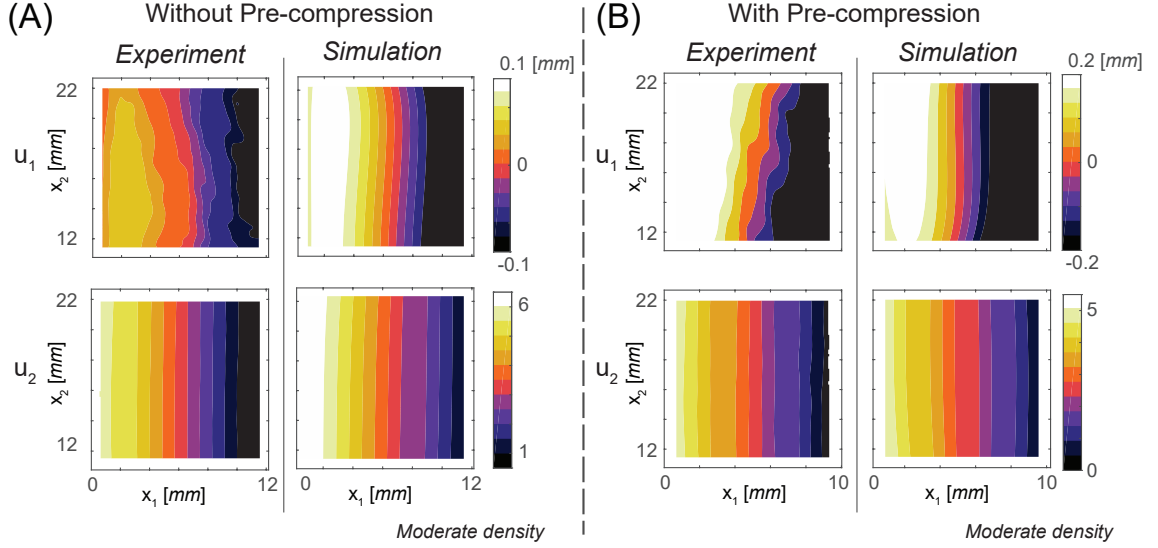


Figure 3.12: Contours of the two-dimensional displacement field on the front face of the specimen in the DIC region-of-interest from both experiment and simulation for simple-shear-like deformation (A) without pre-compression and at a shear displacement of $\delta/H = 0.5$ and (B) with pre-compression and at a shear displacement of $\delta/H = 0.375$ for the moderate density foam. For simple-shear-like deformation with pre-compression, the fields represent the displacement from the pre-compressed state to the deformed, sheared state.

purposefully asymmetric geometry, which is shown in Fig. 3.13(A). These modified specimens are then subjected to a tension-testing procedure analogous to that described in Section 3.2 for homogeneous simple tension experiments. The bottom grip is fixed, and the top grip moves upward at a displacement rate of $\dot{\delta} = 2.0 \times 10^{-3}$ mm/s. The measured tension force P as a function of the top grip displacement δ is shown in Fig. 3.13(B) as a solid line. As before, the shaded area denotes the range of one standard deviation above and below the average experimental measurement, and the arrows denote loading and unloading, demonstrating minimal hysteresis.

In the corresponding finite-element simulations, we modeled only the region between the grips, consisting of the gage-section and the section of the specimen between the grips and the gage-section. The specimen is modeled in three dimensions, matching the experimental geometry, and meshed with 185,216 Abaqus-C3D8H three-dimensional elements, as shown in Fig. 3.13(A). The front and back faces of the mesh as well as the side faces and the interior faces of the holes are traction-free, whereas the bottom of the mesh is fully constrained to zero displacement. On the top surface of the mesh, the vertical displacement u_2 is prescribed,

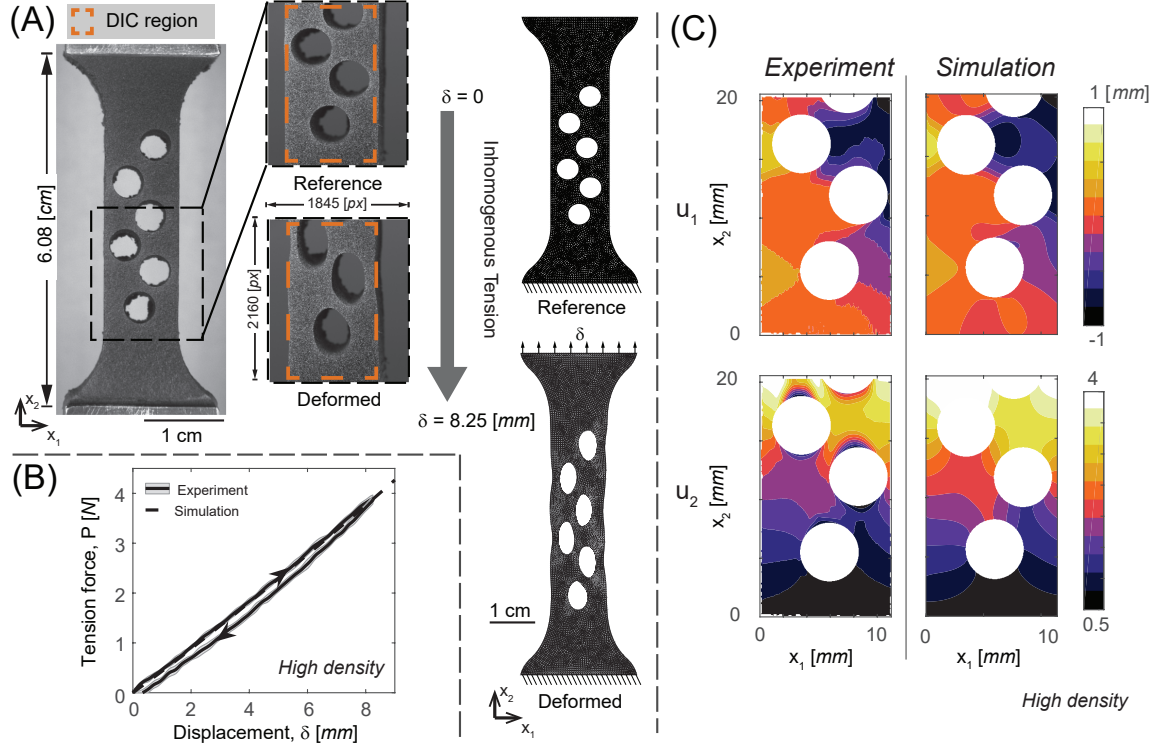


Figure 3.13: A tension-dominated validation test for the high density foam consisting of tension of a specimen with circular holes, in which a standard dogbone geometry is modified with six asymmetrically-placed 2.5 mm radius holes. (A) Snapshot of the experimental specimen prior to deformation and the experimental region-of-interest used for DIC imaging in both the reference and the deformed states (DIC region ca. 1100 x 2000 px²) as well as the corresponding finite-element configurations. (B) The top-grip force P versus displacement δ , in which solid lines with shaded error regions represent experimental data, and dashed lines are simulated predictions of the constitutive model. (C) Contours of the two-dimensional displacement field on the front face of the specimen in the DIC region-of-interest from both experiment and simulation.

while the other displacement components are constrained to zero. Fig. 3.10(B) shows the numerically-predicted force-displacement response as a dashed line, showing excellent agreement with the experimentally-measured relation.

As in the shear validation case, we also compare experimentally-measured and numerically-simulated displacement fields on the front face of the specimen. The DIC region-of-interest is shown both in the reference state as well as the deformed state after a top-grip displacement of $\delta = 8.25$ mm in Fig. 3.13(A), outlined with dashed lines, and contours of the two-dimensional, DIC-calculated displacement field, specifically $u_1(x_1, x_2)$ and $u_2(x_1, x_2)$, on the front face of the specimen in a representative experiment are plotted in Fig. 3.13(C). The corresponding

simulated front-face displacement fields are plotted alongside the experimental results in Fig. 3.13(C). There is strong qualitative agreement between the experimentally-measured and simulated displacement fields throughout the region-of-interest. The notable differences, specifically, the displacement gradients near the tops and bottoms of the holes in the experimental results, arise as an artifact of incorporating internal material-void boundaries within an Eulerian framework for the hybrid, cumulative-iterative displacement summation scheme used in the qDIC algorithm (see [27]) and subsequent plotting in the reference configuration and is thus an artifact of the experiment rather than a model discrepancy.

3.6 Discussion

3.6.1 Comparison to other models

Note again that the alternative model implementations and resultant stress-strain and axial-lateral strain data were provided by Ms. Li and Dr. Henann. The summary presented forthwith is an abbreviated interpretation and is based on the notes of Ms. Li and my own observations, whereas [28] contains a complete description of the model implementations.

In this section, we compare the ability of the proposed model to capture experimental data with that of other important hyperelastic models for porous elastomers in the literature. To accomplish this comparison, the compression and tension data of the moderate density foam is fit to four representative models for elastomeric foam: the Blatz-Ko hyperelasticity model [15], Danielsson *et al.* micro-mechanical model [19], a void-space homogenization-based hyperelastic model of Shrimali *et al.* [21], and an Ogden-like phenomenological, stretch-based hyperelastic model of Storåkers [13]. For the Blatz-Ko model the ground-state shear modulus is the only fitting parameter, which is determined from the compression and tension experiments. The Danielsson *et al.* model uses the ground-state shear modulus and also adds a porosity-dependent parameter, which was set to the $\phi_0 = 1 - J_{min}$ of the moderate density case. Two parameterizations (Ogden-2 and Ogden-4) of the Ogden-type model were fit using the nonlinear fitting procedure provided in Abaqus.

Each model is compared to the experimental data for stress-strain and lateral-axial strain. Resultant fits are shown in Fig. 3.14 with experimental results, the proposed model, and the comparison cases for stress-strain (Fig. 3.14(A) and (C)) and lateral-axial strain (Fig. 3.14(B) and (D)). The Blatz-Ko and Danielsson *et al.* model results shown in Fig. 3.14(A) and (B) capture the small strain behavior (see insets), but fail to capture finite-deformation response. Both exhibit no plateau in compression, and overestimate the overall stress level and lateral strain level beyond roughly 10% axial engineering strain.

Two Ogden-type models were fit to the data. These are the most commonly utilized phenomenological hyperelasticity model for highly-compressible foams and are built in to Abaqus [232] as the “hyperfoam” material. The two-term Ogden model, termed Ogden-2, has 6 parameters to fit. Although Ogden-2 captures the overall stress level, as shown in Fig. 3.14(C), small strain response acuity is sacrificed. To attempt to capture the initial slope with the Ogden-type model, a four-term model (i.e. Ogden-4) was used, wherein the small strain data was more heavily weighted in the fitting procedure. To ensure that the nonlinear fitting procedure for experimental data in Abaqus converged favorably, the lateral strain response is fixed to zero, see Fig. 3.14(D). There are eight fitted parameters in total for Ogden-4. Although the small strain response of Ogden-4 more closely matches the experimental observations than does Ogden-2, the Ogden-4 stress-strain response is non-monotonic in compression, which implies the existence of an instability. The Ogden-2 model passes a loss-of-ellipticity stability test in most experimentally-accessible regions of the K -invariant basis space, whereas the Ogden-4 model fails the stability test in the majority of accessible K -space. For this class of elastomeric, open-cell foam it is typically not practical to obtain a fit that behaves well in stress-strain response, lateral-axial strain response, and stability using the Ogden-type model (sc. although Ogden-2 here fits well it was carefully chosen and the result is not robust; the fit is highly sensitive to initial conditions, and small deviations in experimental data can lead to large mismatches or loss-of-convergence in the parameter fitting scheme).

To evaluate model performance for controlled multi-modal loading, the predictions from each model are compared to experimental data in shear without and with pre-compression in

Fig. 3.14(E) and 3.14(F). The Blatz-Ko, Danielsson *et al.*, and Shrimali *et al.* [21] models fail to predict or dramatically underestimate the decrease in normalized shear force-displacement response under pre-compression. This strong coupling between deviatoric and volumetric deformation is an important aspect of the complete constitutive response, which has also recently been observed in expanded polystyrene (EPS) foam [44]. Due to the onset of instability in the Ogden-4 model the simulation of shear without and with pre-compression was limited to $\delta/H = 0.3$ and $\delta/H = 0.08$, respectively. The predictions made by Ogden-2 model are relatively consistent with experiments with pre-compression, but not without, while Ogden-4 model underestimated the softening with applied pre-compression.

3.7 Concluding remarks

3.7.1 Summary of quasistatic experiments and models

In this chapter, I presented an extensive new set of experiments on three densities of an the “Poron XRD” open-cell polyurethane-based elastomeric foam. Experiments consisted of tension, compression, shear with and with pre-compression, spherical and conical indentation, and inhomogeneous tension. All experiment were quasi-static, isothermal and were conducted on with three experiments per specimen and three specimens per test condition. A novel finite-deformation, hyperelastic continuum model based on a strain energy density formulation using Hencky strain invariants (i.e. K -invariants of [227]) was developed by collaborators in conjunction with my experiments and calibrated parameterizations for the experimental data for each density are given. In compression and tension, the fitted model for each of the three densities of XRD foam captures the nonlinear stress versus axial strain and lateral strain versus axial strain responses, in particular, the tension/compression asymmetry featuring a nearly-flat plateau regime in compression. The model predictions were shown to be consistent with my experimental data in the inhomogenous validation cases across compression, shear, and tension dominated settings. All simple compression/tension experimental data and resulting fitted model data for the XRD foam investigated have been made available to the community via GitHub (<https://github.com/FranckLab>) and as attached to [28] as

research data. The user-material subroutine implementation of the constitutive model and input files for each validation case assembled by Ms. Li and Dr. Henann are also available (<https://github.com/HenannResearchGroup>).

3.7.2 Limitations

The experimental program described in this chapter and our hyperelastic model were developed to capture the equilibrium response of open-cell, isotropic elastomeric foams with relatively high void volume fraction. As such, the model has several operational limitations arising from assumptions made in development. First, many elastomeric foam materials exhibit strain-rate-dependent, viscoelastic material behavior, whereas the proposed model is rate-independent. The expansion of the experimental program to address this limitation is the topic of Chapter 4. Second, the polymeric matrix material of the foam may be temperature sensitive, in some cases dramatically so since it is relatively near the glass transition temperature under normal working conditions, and this effect may be coupled with the rate-sensitivity of the matrix material. A brief foray into the temperature sensitivity of XRD is discussed in Chapter 5. All experiments in Chapters 3 and 4 were conducted under approximately isothermal conditions at room temperature. Accordingly, the model also assumes isothermal conditions, and the effect of variable temperature is not captured. To use the model at elevated or reduced temperature, a new set of experiments at the temperature of interest would be required. Third, depending upon the microstructure, elastomeric foam materials may exhibit strain localization into compaction bands under uniform, homogeneous external loading. The XRD foam at quasistatic rates did not exhibit localization, and localization behavior is not accounted for in the model. Initial measurements of a “localizing” foam are shown in Chapter 5. As a final note, all calibration experiments for this study were conducted on a single class of material over a moderate range of densities, although we expect that materials exhibiting generally similar constitutive responses can be well fit by the proposed model, this has not been experimentally validated.

3.7.3 Future perspectives

There are numerous avenues for improvement of the proposed model. We expect that improvements will be made to the phenomenological fitting functions utilized in the present work, either driven by additional experimental data for different elastomeric foam materials or motivated by micromechanical arguments. Furthermore, we envision that the model framework may be extended to incorporate the effects of strain-rate and temperature sensitivity by adapting existing finite viscoelasticity frameworks for monolithic elastomers, e.g., [234, 235].

Describing and capturing localization arising from microstructural defects is a key aspect of deformation in very-low-density foams, and developing a model that accounts for this behavior will be key to successfully predicting the deformation response of foamed polymers more broadly. In a more basic sense, future work to understand important phenomena associated with elastomeric foams, e.g., the plateau and transitional regions in compression, distortional-volumetric coupling, and the marked increase in dissipation in shear with pre-compression, which occurs even at quasistatic loading rates, would be a valuable asset toward complete continuum model development.

Finally, as an alternative to hyperelastic continuum models [12, 13, 15, 28], explicit homogenizations [20, 21, 158], or direct RVE-based micromechanics [19, 79, 236, 237] recent progress in multi-scaling techniques from the computational heterogeneous material modeling literature may prove valuable for porous elastomers. Unit cell-based techniques include modeling approaches based on representative volume elements (RVE), representative unit cells (RUC), and statistical volume elements (SVE) (e.g., [238–241]). These have been used extensively to study heterogeneous materials [213], heterogeneous elastomers (e.g. filled rubbers [165, 238, 242]), and to a lesser extent porous materials [243]. Recently, schemes such as self consistent clustering analysis (SCA) [244] for order reduction or micromorphic modeling [245] for homogenization have been introduced. Although typically used for metals and/or hard composites they hold promise for translation to porous elastomers. In particular, the tools of SCA, e.g. offline clustering-based discretization and calibration-free online

stages for efficient computations [244], the ability to capture localization processes [246], and applications for non-linear history dependent finite-strain [247, 248] (cf. viscoelastoplasticity in polymer foams) make it a particularly exciting avenue to explore.

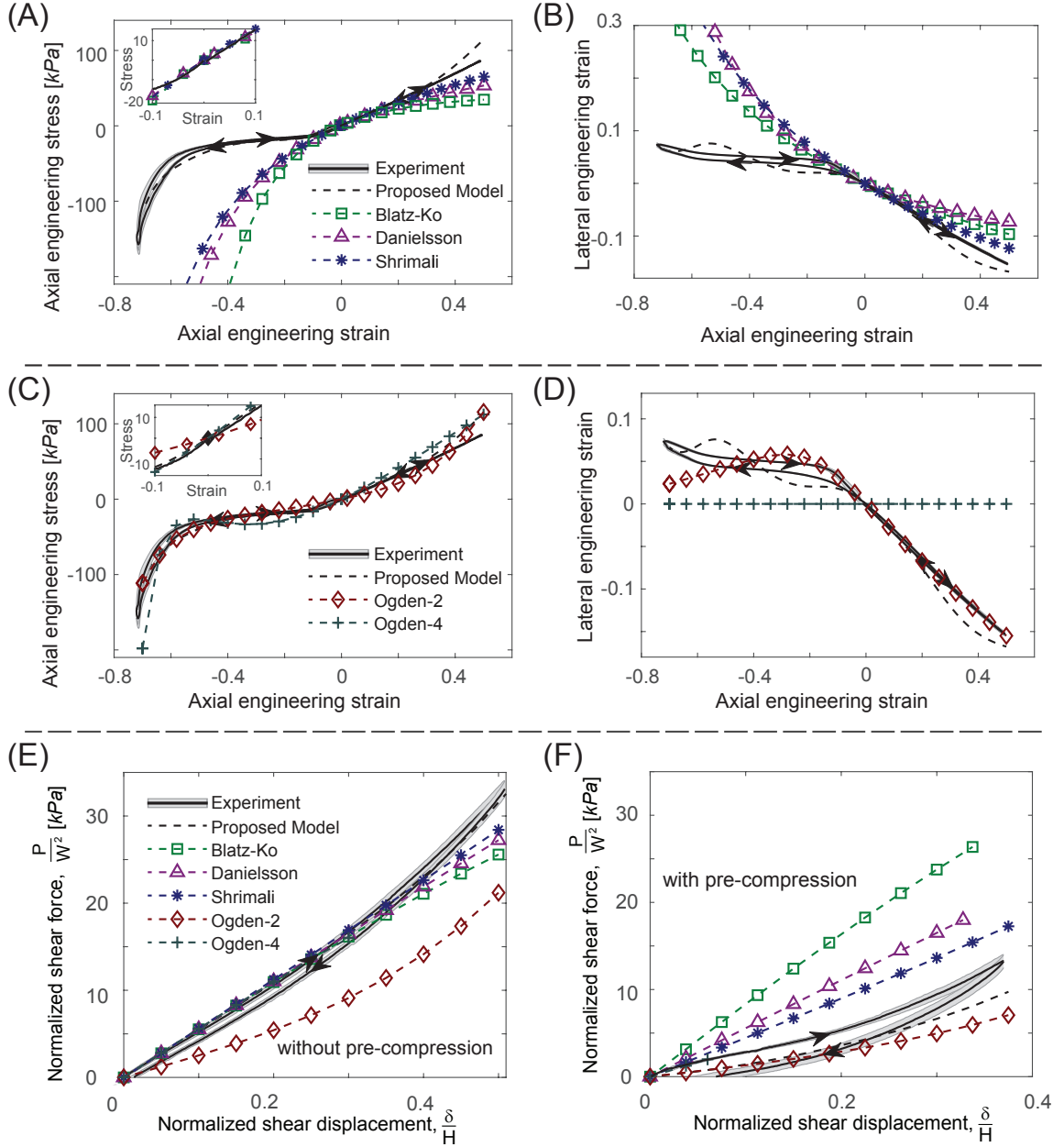


Figure 3.14: Comparison of fits obtained using hyperelastic models for porous elastomers in the literature, as in Fig. 1.7, with the proposed approach for the moderate density XRD foam included as well, to demonstrate the benefits of our new model. (A) Axial engineering stress and (B) lateral engineering strain versus axial engineering strain fits for the proposed model, the Blatz-Ko ([15]) model, the Danielsson *et al.* ([19]) model, and the Shrimali *et al.* [21] model. (C) Axial engineering stress and (D) lateral engineering strain versus axial engineering strain fits for a two-term Ogden-Storakers fit (Ogden-2) and a four-term Ogden-Storakers fit (Ogden-4) [12, 13]. (*Insets:* (A) and (C)) Close-up of the small-strain regime. Normalized shear force P/W^2 versus normalized shear displacement δ/H for the simple-shear-like deformation both (E) without and (F) with pre-compression for all models considered as well as experimental data for the moderate density XRD foam.

Chapter 4

Elastomeric Foam Characterization Under Elevated Rates

4.1 Introduction

As discussed in Chapter 1, the rate dependent nature of elastomeric foams is critically important for design and use of these materials in practice. Diverse experiments over a wide range of strain rates have been conducted to attempt to capture time-dependencies in elastic foam, and can largely be grouped into three categories on the bases of strain rate: low rate “non-quasistatic”, in which the response displays non-negligible hysteresis, for example due to viscous effects in the matrix ($\dot{\epsilon}_{applied} \approx 10^{-3} - 10^0$ 1/s); intermediate rate or “impact-like” rates, in which the response is hysteretic and other mechanism, e.g. glassy transition of ligaments [249] and microinertia [250], may be active ($\dot{\epsilon}_{applied} \approx 10^0 - 10^3$ 1/s); and, high rate or “ballistic-like” rates, in which shock-front propagation mechanics dominate the overall response [39, 149] (i.e. macroscopic force equilibrium no longer exists) and internal gas viscosity plays a roll (at high rates the impact velocity, rather than effective strain rate, is typically used since the strain rate is highly nonuniform due to the shock front, for sake of comparison these rates roughly correspond to $\dot{\epsilon}_{applied} \approx 10^3 - 10^7$ 1/s and higher) [33]. For an excellent overview of the behavior of dynamic compression of cellular materials, albeit largely drawn from the foamed metal literature, refer to the recent article of Sun and Li [7]. The

focus of this Chapter is on the non-quasistatic and intermediate rate response of the highly rate-sensitive XRD polymer foam when structural shock mechanisms are not activated.

No established criterion exists for segregating truly quasistatic and slow non-quasistatic experiments for elastomeric foam; however, the natural distinction in modeling is whether viscous dissipative effects have been included (i.e., rate-independent vs. rate dependent modeling). For elastic polymer foams a broad selection of the available models include rate-sensitivity and/or viscoelasticity as it is a common feature among polymers suited for impact protection and foamed material applications [8, 17, 111]. For example, [17, 31, 62, 79, 111, 113, 120, 122, 128, 129, 134, 157, 161, 236, 251, 252] all employed viscoelasticity to address dissipation in polymer foams. Although experimental treatments differ, at slow non-quasistatic rates standard screw-driven or, less commonly, servo-hydraulic load frames are nearly universal. Often, a fixturing process beyond the standard compression platens or tension grips is used to probe a specific strain state (e.g., [44, 135, 138]) or for model validation (e.g., [28, 129]); for schematics of selected examples see Fig. 1.6).

For past intermediate rate efforts, a wide selection of equipment has been used to impose deformation. Energetic impacts are relatively common to access this strain rate, for example pneumatically driven impacts (e.g. [17]) or drop-weight systems [42, 71, 140, 141]. A flywheel-driven impactor was used by Viot to impose uniaxial and hydrostatic deformations [131]. Small-strain rheological testing, a wide-spread technique in soft polymer and gel testing, has also been used for polymer foams [9, 142], although the effectiveness is somewhat limited since the highly non-linear response domain is inaccessible. High-speed load frames, one of the techniques used in this work, have also been used to good effect [57, 87, 143].

The purpose of this chapter is to introduce the three experimental techniques I designed, or oversaw the design of, for non-quasistatic and intermediate rate experiments and show that by employing a multi-mechanism (i.e., Prony-series-like Generalized Maxwell or Maxwell-Wiechert) rheological model the data generated at non-quasistatic rates can be accurately captured. Additionally, evidence is presented to indicate a strain-rate dependent change in deformation characteristics, from non-localizing to localizing, as intermediate rates are encountered. It is organized as follows. In Sec. 4.2, the foam samples are introduced and the

implementations of the three testing apparati are described. A brief description of the model extension to capture viscous effects is given in Sec. 4.3. The results of non-quasistatic rate tests are presented and discussed in the context of the extended model in Sec. 4.4. Section 4.5 contains the impact-like rate experimental results and discussion of the relevant observations. Finally, Sec. 4.6 summarizes the dynamic testing effort and mentions future outlooks in this direction.

4.2 Experimental Program

In this section I outline the experimental program for elevated rate characterization of the XRD foam, which largely mirrors that of Chapter 3. It includes specimen and experimental details for elevated rates, briefly introduces the design of two new experimental systems and the analysis adapted for the requirements of higher rate experiments.

4.2.1 Foam samples

Specimens were in large part saved during the quasistatic experimental program (i.e., consult Chapter 3, Sec. 3.2 for details). A small number of specimens were replaced, as needed, but prior to final data collection, from identical stock material. Replacements were typically needed due to damage during experimental validation tests. For example, although I did not address failure modes in this work, maximum strain to failure in tension at $\dot{\epsilon}_{applied} = 10\text{ 1/s}$ is (observationally) lower than at quasistatic rates, possible due to strain localization and rate dependent glassy transition the matrix material or similar effects [249]. The highest-rate compression experiments were reserved until last, since specimen setup slightly modified the specimens (see Sec. 4.2.2).

For all elevated rate experiments a 3.25 hr or more wait time between repeated tests or alignment on a given specimen was maintained. At the current stage of development, elevated rate data collection has been limited to primary calibration experiments (i.e., uniaxial compression and tension). Modeling efforts to capture rate dependence are limited to data collected at on the low-speed load frame ($\dot{\epsilon}_{applied} = 10^{-3}\text{ 1/s}$ to $\dot{\epsilon}_{applied} = 10^{-1}\text{ 1/s}$) and

the impact-rate experiments remain observational. For these highest-rate experiments the applied strain maxima are approximately -65% for compression and 20% for tension. As in the quasistatic case, all experiments were conducted at room temperature and ambient humidity.

4.2.2 Experimental setup

To impose the desired strain ranges and rates three experimental systems were designed and implemented, although only two of these machines were deployed to collect primary data. In order of increasing strain rate, the first system was the same as outlined in Chapter 2 Sec. 2.4.1 and elsewhere, with minimal reconfiguration, and operated at a higher crosshead speed. In the initial design of this load frame, considerations were made to allow the machine to operate at strain rate ranges up to about $\dot{\epsilon}_{applied} = 10^{-1} \text{ 1/s}$ for the typical tension and compression geometries I have employed. Second, Mr. Jialiang Tao lead the design and implementation of a new "high-rate" electro-mechanical load frame to operate at rates of up to about $\dot{\epsilon}_{applied} = 10^2 \text{ 1/s}$ for these specimen geometries. Third, Ms. Lauren Chanen, Mr. Jialiang Tao, and I designed and built a drop-weight system for compression experiments between $\dot{\epsilon}_{applied} = 10^1 \text{ 1/s}$ and $\dot{\epsilon}_{applied} = 10^3 \text{ 1/s}$. Due to the limitations of this system, in addition the slow-to-fast experimental approach for calibration data, no primary experiments were conducted on this system, and the discussion is limited to a description of the setup.

For initial extension of the experimental program to elevated non-impact rates (i.e., "non-quasistatic", the range between $\dot{\epsilon}_{applied} = 10^{-3} \text{ 1/s}$ and $\dot{\epsilon}_{applied} = 10^{-1} \text{ 1/s}$) the low-rate load frame was simply operated with elevated actuator rates, up to an upper limit of 0.0045 m/s. Since this is screw-driven stepper-motor system the displacement control is precise and contains few artifacts (for example, due to inertia or feedback-loop control schemes) at elevated rates, up to the velocity and acceleration limitations for the camera and load cell. At the rates required for tension at $\dot{\epsilon}_{applied} = 10^{-1} \text{ 1/s}$, which was the most extreme case for this system, actuator velocity reversal (i.e. constant positive velocity switching to constant negative velocity) is accomplished in approximately 0.05 s for an experiment with total test time of 10 s. The load cell and camera were operated using synchronized timing

at up to 25 Hz via a LabView interface, or approximately 250 data points for the fastest testing condition. This yielded both sufficient control and temporal resolution to effectively utilize this system for experiments between $\dot{\epsilon}_{applied} = 2 \times 10^{-4}$ 1/s (or slower if needed) and approximately $\dot{\epsilon}_{applied} = 10^{-1}$ 1/s. Faster rates yielded significant turn-around artifacts from the linear actuator and reduced sampling for synchronized load and images. Individually, the camera and load cell can operate at higher rates, limited by the maximum full-frame camera framing rate of 60 Hz, but details of synchronization caused stuttering, i.e. skipped frames, at rates higher than approximately 30 Hz.

For faster impact-like strain rates of $O(10$ 1/s) (viz. up to and namely the $\dot{\epsilon}_{applied} = 10$ 1/s of the fastest dataset presented here) a new load frame plus high-speed camera system was implemented (the “high-rate load frame”). See the schematic in Fig. 4.1 for a visual overview. In principal and operation, this new system is similar to the low-rate load frame and camera system, but implementation and componentry were designed to operate at much higher rates. An electromagnetic linear actuator (0150-1220, LinMot USA, Inc., Lake Geneva, WI) is used in place of the screw-driven stepper motor. The linear actuator enables higher actuation velocities (up to about 2.7 m/s) and linear accelerations (up to about 240 m/s²) at the cost of decreased positional accuracy, to about ± 0.05 mm as compared to the single-step accuracy of about 0.006 mm of the low-rate load frame. A low-noise 440 N range dynamic load cell (Model 1050V3, Dytran Instruments Inc., Chatsworth, CA) with rise time less than 6 μ s (i.e. suitable for excitation frequencies up to approximately 80 kHz) was mounted to the x -, y -translation stage and configuration to record transmitted load through the specimen. A high-speed camera (FastCam Mini AX200, Photron USA Inc., San Diego, CA) with full-sensor framing rates up to 6400 Hz on a 1024 x 1024 px² monochrome sensor and 32 GB of internal memory was used for imaging the sample for digital image correlation (DIC). A macro lens (Milvus Makro-planar, Carl Zeiss Inc., Thornwood, NY) with 2x tele-converter and close-up lens set were used in conjunction with two LED light panels (SpectroLED, Genaray) for imaging and illumination. Due to the strict lighting requirements for high-speed imaging and relatively diffuse light provided by the LED light panels, cross-polarization was not used. The camera and load cell were triggered from a data acquisition board (USB-6211, National Instruments,

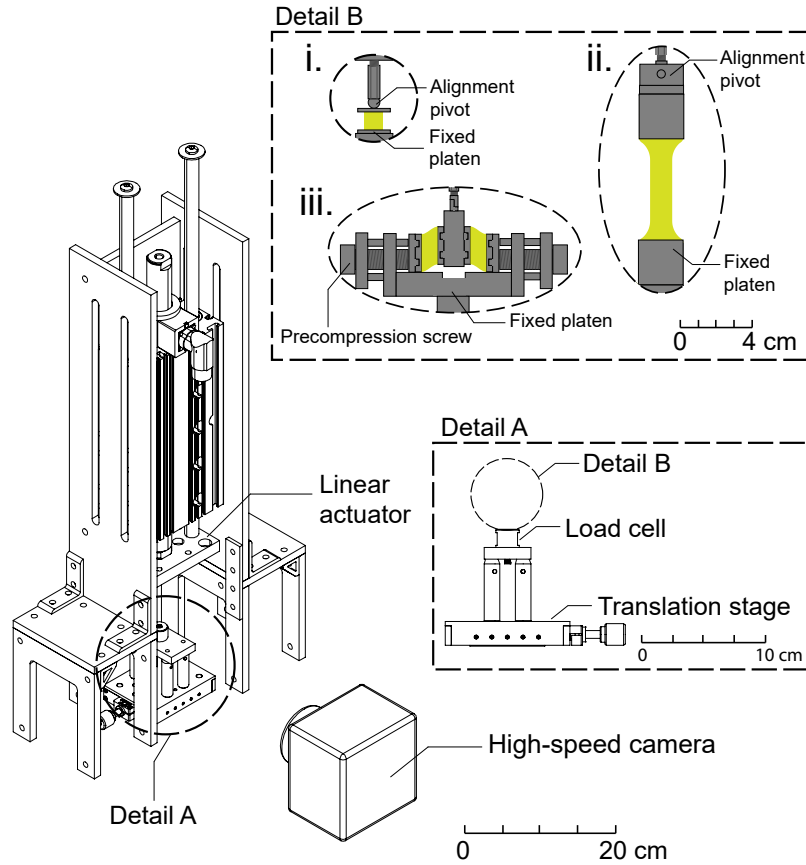


Figure 4.1: Schematic showing the high-rate linear actuator-based load frame and high-speed camera. A custom steel frame holds the actuator at an adjustable height above the specimen stage. (*Detail A*) The specimen stage for attaching specimen holding insets (e.g., compression platens or tension grips). (*Detail B*) Three available insets for (i.) compression, (ii.) tension, and (iii.) double-lap shear.

Austin, TX) when a control signal was received from the motor. Figure 4.2 shows a schematic diagram of this process. Although triggering is simultaneous, small random delays in the hardware processing and differences in sampling between the camera (typically 6400 Hz) and load cell (typically 72 kHz) rate necessitate slight timing adjustment and resampling in post-processing.

In this setup, two changes were made to the protocol used for the low-speed low frame that we demonstrated have little effect on the resultant data. The two procedural change were: first, the load cell was mounted underneath the specimen (as in the drop-weight system diagram of Fig. 4.4) rather than in series with the actuator; and, second, a small dot of epoxy resin was used affix the compression specimen to the bottom platen. Regarding the first,

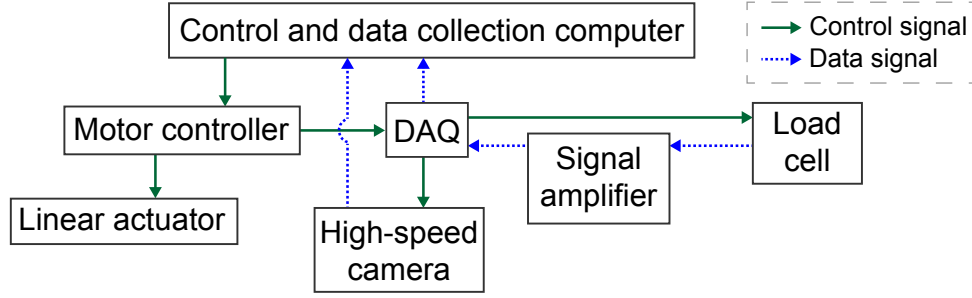


Figure 4.2: Diagram of the control and data signals for high rate experiments. Data collection is triggered simultaneously, but differences in sampling rate and variable hardware delays in the camera initialization process necessitate slight timing adjustment and resampling in post-processing. As for the low-rate frame, primary data collected are the camera images (full-field data) and the total applied force (scalar data).

although linear elastic theory does not strictly hold [7, 33], it serves as an estimate of the validity of placing the load cell on the transmitted side of the specimen. The stress P-wave speed is then $C_p = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$, using the quasistatic parameters for low density foam as a worst-case and assuming an effective Poisson's ratio of 0.05, $C_p \approx 24.7 \text{ m/s}$. The traveling wave reaches the load sensor in ca. $0.5 \times 10^{-4} \text{ s}$, about 0.05% of the total test time, and thus approximate force equilibrium holds. Regarding the second, the epoxy spot size is typically less than 15% of the bottom surface of the otherwise well lubricated specimen, and although a small amount of frictional confinement-type behavior is observed in incremental displacement plots, they are not inconsistent with low levels of friction observed in the lower-rate cases.

As a final point regarding the high-rate load frame, machine performance is less pristine than the low-rate case, particularly in tension-type experiments. As a graphical representation of this limitation, Fig. 4.3 shows the reconstructed axial engineering strain magnitude and rate for both compression (A) and tension (B). For example experiments, strain and strain rate are measured via DIC as a function of test time at a target 101/s strain rate. Strain rate is computed directly from strain history via a simple central differencing algorithm. For compression, the actuator starts out of contact with the top platen, thus the strain rate quickly rises and remains at approximately the target value. During unloading the strain rate slows to the fastest rate allowed by the material response, since the top platen is not fixed to the actuator or specimen, and a power-law type decay is observed. For tension, the

specimen is always fixed to the actuator, hence the constant-acceleration rise followed by loading at the target rate. A constant acceleration is briefly stabilized in unloading, and followed by a noisy domain. The final 0.01 s and strain rate reversal occur as the controller attempts to return to setpoint after the controlled motion is terminated, but the actuator is still travelling with non-negligible inertia, leading to oscillations.

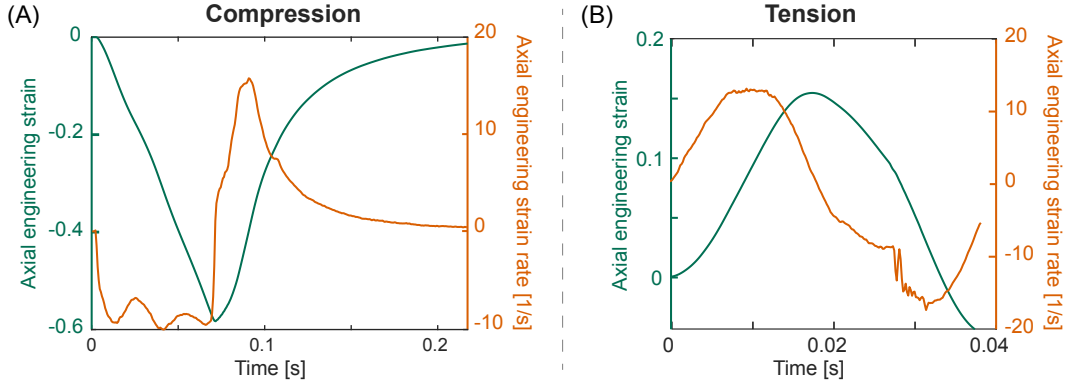


Figure 4.3: Strain and strain rate as a function of time for the high-speed load frame at a target 10 1/s strain rate. (A) For compression, the strain rate quickly rises and remains at approximately the target value; during unloading the strain rate is governed only by the fastest rate allowed by the material response. (B) For tension, a constant-acceleration region is followed by loading at the target rate and a partially controlled unloading.

The third experimental system, the drop-weight system, enabled testing at higher rates (from approximately $\dot{\epsilon}_{\text{applied}} = 10^0$ up to $\dot{\epsilon}_{\text{applied}} \approx 10^3$ 1/s) than the load-frame based designs, with certain limitations. For a complete discussion of the system see [253]. Since this system was not used to collect primary data, the discussion here will be brief. The system uses a linear impact from a dropped impact bar (the incident “striker bar”) to deform the specimen. A high-speed camera (shared with the high-rate load frame) records the kinematics of the event and a high-sensitivity 500 lbf dynamic load cell (9721B500, Kistler, Novi, MI) records transmitted force, see the diagram of Fig. 4.4 (lighting and alignment systems not shown, for clarity). Approximate total strain and strain rate desired for a given experiment and specimen material are *a priori* estimated with 1D stress wave dynamics to select drop heights, and striker bar length and material (see [253]). An adjustable height drop frame with guide tubing for the striker enabled total drop heights of up to about $h = 5.5$ m. Assuming negligible dissipation due to, e.g., tube friction and aerodynamic drag, the impact

velocity is $\mathbf{v} \approx \sqrt{2g_0h}$ where g_0 is the local gravitational acceleration constant, and total impact energy is $K_{drop} \approx \frac{1}{2}m\mathbf{v}^2$. The lowest section of tubing is tiltable along two axes such that, when used in conjunction with two orthogonal alignment cameras and two specimen translation stages, precise alignment between the drop axis and center axis of the specimen is achieved. Specimens are geometrically identical to the compression-type specimens described in Chapter 3, although the speckling procedure was modified to account for differences in imaging at high rates.

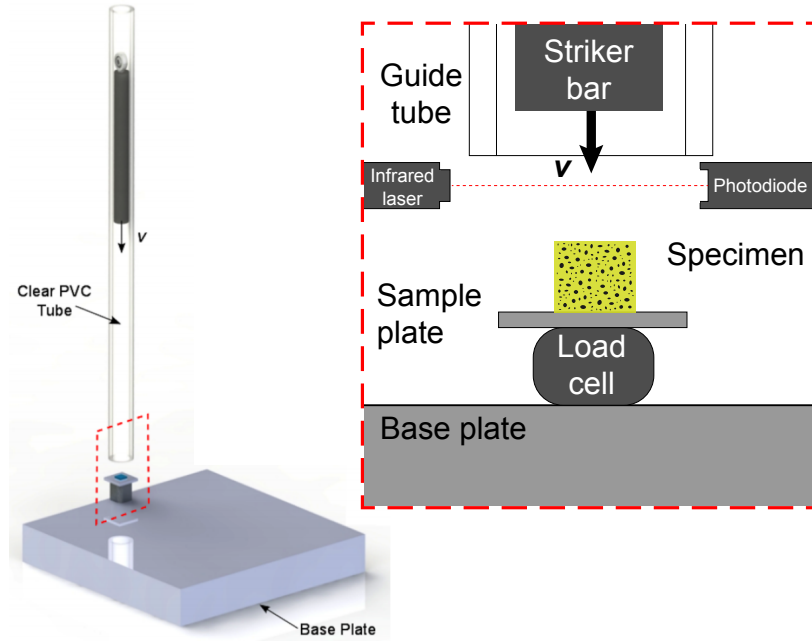


Figure 4.4: Diagram of the major components of the drop-weight system. An incident striker bar of known mass and material properties is dropped from a known height, h , and impacts the specimen with $\mathbf{v} \approx \sqrt{2g_0h}$ where g_0 is the local gravitational acceleration constant. The kinematic field is recorded using a high-speed camera and high-intensity lighting (not shown) for motion reconstruction via topology-based particle tracking [254], while the transmitted force is measured with a high-sensitivity dynamic load cell. The data recording is triggered by an infrared photogate.

For intermediate rate experiments, the camera resolution is reduced and frame rate increased to capture the event, which necessitated three additional changes. First, due to increased light needed at faster framing rates (typically 13.5 kHz, 760 x 640 px²), the lighting was changed to a high-intensity 4-channel LED light source (LED4D067, Thorlabs Inc., Newton, NJ) coupled to a liquid light guide and cross-polarized. Second, with the reduced

resolution available at higher rates DIC become impractical and we thus use a particle tracking technique to resolve high spatial resolution at lower pixel counts. Specifically, the topology-base particle tracking (TPT) of Patel *et al.* [254] is adapted for the 2D images collected by the high-speed camera. Third, in order to use TPT, two new speckling techniques were introduced that generate distinct individual particles that can be localized for TPT tracking.

As with changes to camera setup, the triggering solution was also modified for the drop-weight system. Typical impact times are $250\ \mu\text{s}$ to $750\ \mu\text{s}$, and thus a rapid response time and accurate timing with respect to the event is important. Since no motor signal is available, an infrared photogate was designed and built to trigger the camera and load cell data collection. The photogate is arranged just above the specimen, such that the striker bar breaks the gate beam and a falling trigger signal begins data collection with several images and force datapoints captured prior to the striker bar impact (see Fig 4.4).

4.3 Viscous model overview

In this section I briefly describe the update to the model to capture viscous effects in the XRD foam. The design and implementation of this model, as before, is largely the work of Dr. David Henann and Ms. Xiuqi Li, and I have included this interpretation for completeness. Currently, the model has been fit to non-quasistatic experiments for a single density of foam, but immediate future work to extend the effort to additional densities and impact-like rates is underway.

4.3.1 Elastic formulation

The model uses a multi-mechanism decomposition of the response into an equilibrium response and three non-equilibrium branches in a non-linear Maxwell-Weichert fashion. The equilibrium response is taken to follow the non-linear hyperelastic free energy density function extensively described in [28] and more briefly in Chapter 3. To summarize, the model uses finite deformation kinematics and invariants of the Hencky strain tensor [227] with three

phenomenological fitting functions to capture the rate-independent response. The kinematics are described by the polar decomposition of the deformation gradient ($\mathbf{F}$) into deformation ($\mathbf{V}$) and rotation ($\mathbf{R}$) tensors as $\mathbf{F} = \mathbf{V}\mathbf{R}$, with the volume ratio of material from reference to deformed configuration given by $J = \det \mathbf{F} > 0$. The logarithmic, or Hencky, strain is $\mathbf{E} = \ln \mathbf{V}$, for which an admissible orthogonal basis denoted $\{K_1, K_2, K_3\}$ is given by $K_1 = \text{tr} \mathbf{E} = \ln J$ which quantifies the dilation of the material, $K_2 = |\text{dev}(\mathbf{E})| = \sqrt{\text{dev}(\mathbf{E}) : \text{dev}(\mathbf{E})}$, which quantifies the magnitude of distortion in the material, and $K_3 = 3\sqrt{6}\det\left(\frac{\text{dev}\mathbf{E}}{K_2}\right)$, which indicates the mode distortion (i.e., compression, tension, shear, etc.). Using these invariants, the free energy density is formulated as

$$\psi(K_1, K_2, K_3) = G_0 [X(K_1)K_2^2 + L(K_2, K_3)] + Bf(K_1), \quad (4.3.1)$$

where G_0 and B are the ground state shear and bulk moduli, respectively, and $X(K_1)$, $L(K_2, K_3)$, and $f(K_1)$ are phenomenological fitting functions that are described elsewhere (Chapter 3 and [28]). This free energy density function forms the elastic structure of the model and is used for both the equilibrium response and the elastic component of the non-equilibrium (nonlinear Maxwell-type) mechanisms.

4.3.2 Viscoelastic rheological material model

The model extension for viscoelasticity and capturing of dissipation effects is a multi-mechanism decomposition of the material with a branch for each timescale investigated, following for example [235, 255, 256]. It is a Maxwell-Wiechert-type model consisting of a non-linear spring-like element in parallel with three Maxwell-type non-linear elements, as diagrammed in Fig. 4.5. For a given branch α of the model, the following definitions hold. The elastic distortion is $\mathbf{F}^{e(\alpha)}$ with $J^{e(\alpha)} = \det \mathbf{F}^{e(\alpha)} > 0$. The elastic distortion admits the polar decomposition $\mathbf{F}^{e(\alpha)} = \mathbf{V}^{e(\alpha)}\mathbf{R}^{e(\alpha)}$ into its deformation and rotation tensors. The elastic Hencky strain is given by $\mathbf{E}^{e(\alpha)} = \ln \mathbf{V}^{e(\alpha)}$ with invariants $\{K_1^{e(\alpha)}, K_2^{e(\alpha)}, K_3^{e(\alpha)}\}$. The viscous distortion is given by $\mathbf{F}^{v(\alpha)}$ with $J^{v(\alpha)} = \det \mathbf{F}^{v(\alpha)} > 0$. The viscous distortion rate is $\mathbf{D}^{v(\alpha)} = \text{Sym} \left(\dot{\mathbf{F}}^{v(\alpha)} \mathbf{F}^{v(\alpha)-1} \right)$. The non-equilibrium Cauchy stress is by definition symmetric

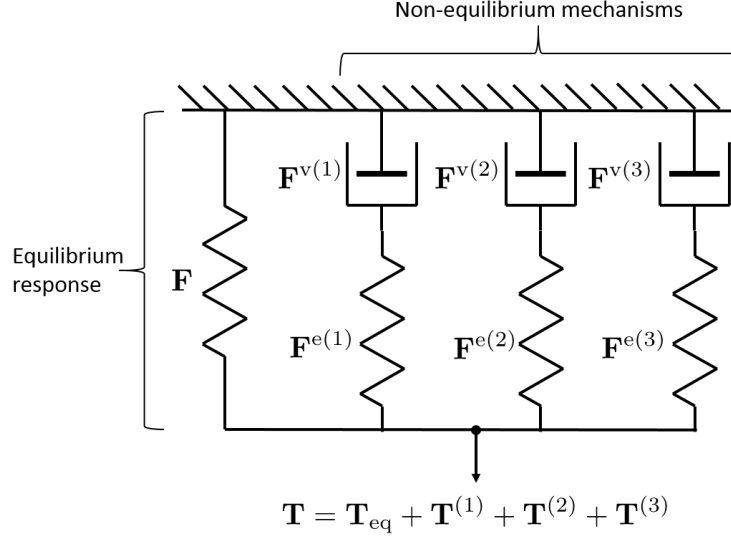


Figure 4.5: A schematic of the rheological Maxwell-Wiechert-type non-linear material model (with illustration credit to Dr. David Henann). Each branch corresponds with a strain rate regime. The phenomenological fitting functions are preserved across springs and each non-equilibrium viscous response is associated with one fitting parameter. The total stress response, $\mathbf{T}$, is given by the additive decomposition of the stress in each branch, and the deformation by $\mathbf{F} = \mathbf{F}^{e(\alpha)}\mathbf{F}^{v(\alpha)}$ where $\alpha = 1, 2, 3$ are the mechanisms.

$\mathbf{T}^{(\alpha)} = \mathbf{T}^{(\alpha)T}$. Finally, $\phi_{neq}^{(\alpha)}$ is the non-equilibrium free energy density. With these defined, the elastic component of the non-equilibrium Maxwell-like element α is

$$\psi_{neq}^{(\alpha)} = G_{neq}^{(\alpha)} \left[X(K_1^{e(\alpha)}) K_2^{e(\alpha)2} + L(K_2^{e(\alpha)}, K_3^{e(\alpha)}) \right] + B_{neq}^{(\alpha)} f(K_1^{e(\alpha)}), \quad (4.3.2)$$

where $G_{neq}^{(\alpha)}$ and $B_{neq}^{(\alpha)}$ are the non-equilibrium, ground-state shear and bulk moduli. The phenomenological fitting functions X , L and f and their material parameters remain the same as for the equilibrium spring, except for the parameters C_1 and p for which new parameters C_{1neq} and p_{neq} are introduced. The ratio of elastic to bulk moduli is taken to be constant across all non-equilibrium branches, such that a single fitting parameter only is required for each elastic contribution to the non-equilibrium branches. The Cauchy stress is given by

$$\mathbf{T}^{(\alpha)} = J^{e(\alpha)-1} \mathbf{R}^{e(\alpha)-1} \frac{\partial \psi_{neq}^{(\alpha)}}{\partial \mathbf{E}^{e(\alpha)}} \mathbf{R}^{e(\alpha)T}. \quad (4.3.3)$$

Finally, to describe the time-dependent behavior, the evolution of the viscous distortion

term $\dot{\mathbf{F}}^{v(\alpha)}$ is

$$\dot{\mathbf{F}}^{v(\alpha)} = \mathbf{D}^{v(\alpha)} \mathbf{F}^{v(\alpha)}, \quad (4.3.4)$$

where $\mathbf{F}^{v(\alpha)}(t = 0) \equiv \mathbf{I}$. The viscous distortion rate then takes the form

$$\mathbf{D}^{v(\alpha)} = \frac{1}{2\eta^{(\alpha)}} \text{dev}(\mathbf{M}^{e(\alpha)}) + \frac{1}{9\kappa^{(\alpha)}} \text{tr}(\mathbf{M}^{e(\alpha)}) \mathbf{I}, \quad (4.3.5)$$

where the Mandel stress tensor is $\mathbf{M}^{e(\alpha)} = J^{e(\alpha)} \mathbf{R}^{e(\alpha)T} \mathbf{T}^{(\alpha)} \mathbf{R}^{e(\alpha)}$. The material parameters $\eta^{(\alpha)}$ and $\kappa^{(\alpha)}$ are the shear and bulk viscosities, respectively. For a given mechanism, α , the relaxation times are assumed to be equal, such that $\tau^{(\alpha)} = \eta^{(\alpha)} / G_{neq}^{(\alpha)} = \kappa^{(\alpha)} / B_{neq}^{(\alpha)}$, and, as with elastic parameters, only one fitting parameter is needed for each mechanism.

In total, to capture the non-quasistatic response over approximately four decades of strain rate eight additional parameters are required. As with the base quasistatic model a streamlined step-by-step fitting procedure to allows the model to be robustly calibrated for a given material. The model has been applied for the high-density XRD foam case and, as demonstrated in Fig. 4.8, admirably captures the material response at elevated rates.

4.4 Non-quasistatic rate results

Calibration experiments were conducted across three globally applied low strain rates and for the three bulk densities, low (144 kg/m³), moderate (192 kg/m³), and high (240 kg/m³). Strain rates were selected at orders of magnitude spaced between the quasistatic rate used in Chapter 3 and both the upper limit of both the low-speed load frame and non-quasistatic response regime, namely $\dot{\epsilon}_{applied} = 10^{-3}$ 1/s, $\dot{\epsilon}_{applied} = 10^{-2}$ 1/s, and $\dot{\epsilon}_{applied} = 10^{-1}$ 1/s.

4.4.1 Experimental observations

In preliminary experiments over the range of strain rates outlined above no change in deformation mechanisms or increase in localization were observed (in contrast to the impact-like rate experiments, as discussed later), and thus the same post-processing procedure as described in Chapter 3 is used to collect stress, axial strain, and transverse strain.

Experimental data for the low, moderate, and high density foams are shown in Figs. 4.6, 4.7, and 4.8, respectively.

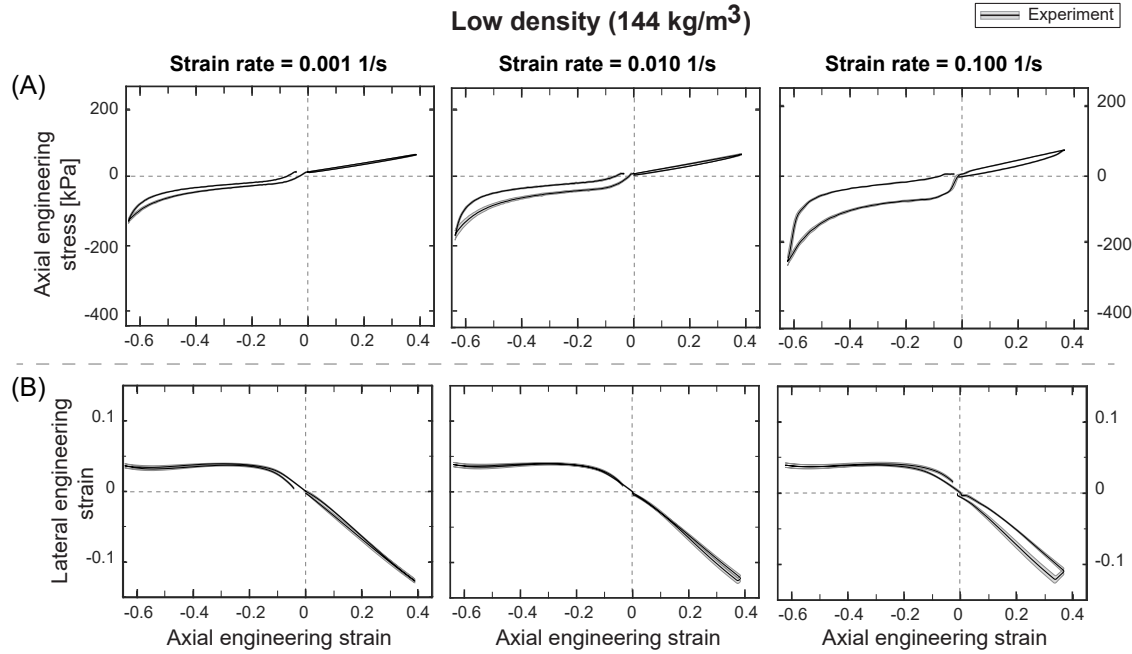


Figure 4.6: The data for compression and tension of the low density XRD foam at non-quasistatic rates is shown for experiments with shaded area one standard deviation. (A) The stress-strain responses showing increasing dissipation and stiffness at elevated rates. (B) The axial-lateral strain response demonstrating relatively little rate-sensitivity for Poisson's function and very limited anisotropy in elastic recovery.

Similar trends are observed across all densities for stress strain response in this range of strain rates. Notably, there is strain-rate stiffening in the initial response and overall higher force levels in the loading portions of the curves. The onset of densification occurs earlier at higher strain rates. Most striking, although expected from this material, the energy dissipation dramatically increases with increasing rate. Interestingly, increasing density also appears to be associated with increasing dissipative losses (i.e., stress-strain hysteresis). This is consistent with the supposition that at these low, non-quasistatic rates and simple deformation modes, the dissipation mechanism is dominated by viscous effects in the base polymer – with increased relative density more matrix polymer is available to dissipate input energy. Quantification of each of these effects will be important to understand the complete response of the material, but remains future work at the time of writing.

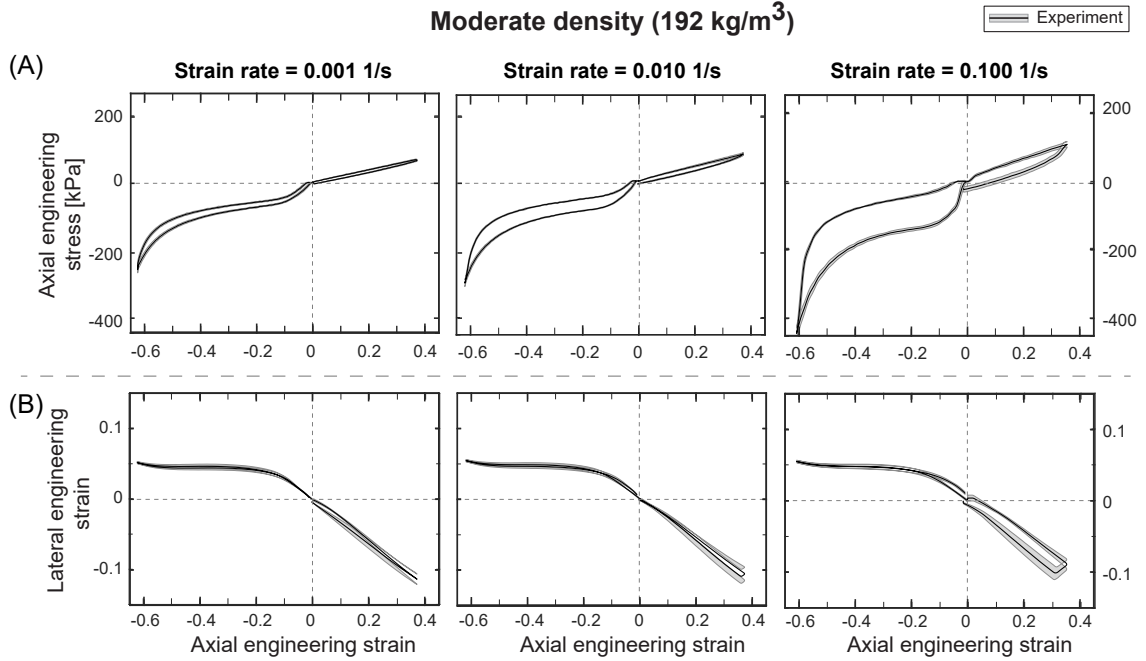


Figure 4.7: The data for compression and tension of the moderate density XRD foam at non-quasistatic rates is shown for experiments shaded area one standard deviation. (A) The stress-strain responses showing increasing dissipation and stiffness at elevated rates. (B) The axial-lateral strain response demonstrating relatively little rate-sensitivity for Poisson's function and very limited anisotropy in elastic recovery.

4.4.2 Fitted parameters

The calibrated parameter list used for the equilibrium spring and non-equilibrium phenomenological fitting functions are given in Table 4.1 for the high density XRD foam. In total 14 parameters are used. Four are density dependent and 10 are density independent, although only one density is currently fitted. For this strain rate sweep, the equilibrium response is taken to be given by the lowest rate, $\dot{\epsilon}_{\text{applied}} = 10^{-3} \text{ 1/s}$.

Table 4.1: Material parameters for the high density XRD foam equilibrium branch and non-linear springs in the dissipative branches. In total, fourteen parameters are utilized, with four density-dependent parameters and ten density-independent parameters.

Density	Density-dependent				Density-independent									
	G_0 [kPa]	B [kPa]	$J_{\min}$	C_1	K_1^0	Δ_K	X'_1	X'_2	C_0	p	q	r	C_2	C_3
High	90.0	259.2	0.22	1.9	-0.21	0.2	3.7	0.45	0.23	4	5	3	11.5	0.026

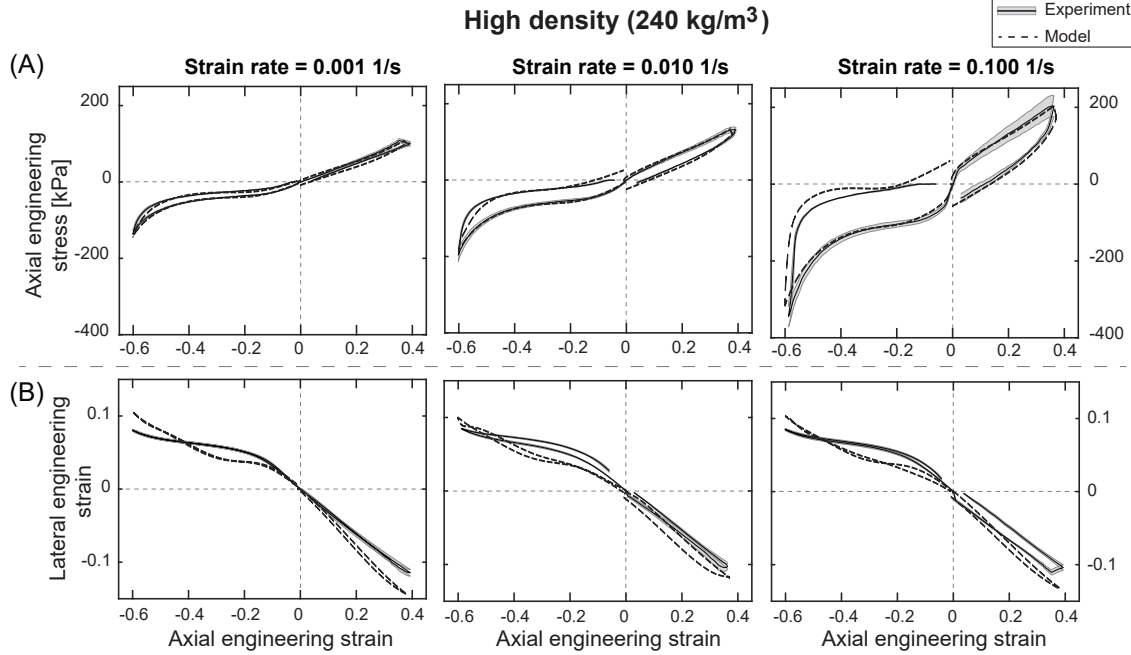


Figure 4.8: The data for compression and tension of the high density XRD foam at non-quasistatic rates is shown for experiments (solid lines with shaded area one standard deviation) and the calibrated model response (dashed lines). (A) The stress-strain responses showing increasing dissipation and stiffness at elevated rates. (B) The axial-lateral strain response demonstrating relatively little rate-sensitivity for Poisson's function and very limited anisotropy in elastic recovery.

Table 4.2: Non-equilibrium branch fitting parameters for the high density Poron XRD foam. Eight independent additional parameters are needed although all eleven are provided here for ease of use.

Density	Non-equilibrium										
	G_{neq}^1	B_{neq}^1	τ^1	G_{neq}^2	B_{neq}^2	τ^2	G_{neq}^3	B_{neq}^3	τ^3	c_{neq}	p_{neq}
High	28	80.6	60	50	144	8	800	2300	0.2	2	4

The additional parameters for the Maxwell-type branches necessary to capture dissipative and strain rate sensitivity effects (at the continuum scale) are given in Table 4.2. Only eight fitted parameters are needed (one per spring and per dashpot, plus two additional tunable parameters in L for the non-equilibrium branch springs), however, 11 are listed for the convenience of the reader, keeping in mind that $G_{neq}^{(\alpha)}/B_{neq}^{(\alpha)} = G_0/B$ and $\tau^{(\alpha)} = \eta^{(\alpha)}/G_{neq}^{(\alpha)} = \kappa^{(\alpha)}/B_{neq}^{(\alpha)}$. Density dependence has not yet been evaluated thoroughly for this set of parameters.

4.5 Intermediate Rate Results and Discussion

To characterize the XRD foam at a use-case-realistic rate and transition from the non-quasistatic to impact-like intermediate rate, specimens were tested in compression and tension at $\dot{\epsilon}_{\text{applied}} = 10\text{ }1/\text{s}$.

4.5.1 Experimental datasets

For tension experiments, the complete mean and one standard deviation (shown in grey) of axial engineering stress versus axial stretch (left) and lateral stretch versus axial stretch (right) are given in Fig. 4.9. The trends of increasing stiffness and dissipation with density, i.e., by comparing Figs. 4.9(A) for low, (B) for moderate and (C) for high density, and rate are preserved (cf. Figs. 4.6, 4.7, and 4.8). No evidence of mode-transition behaviors (e.g., glassy-transition with localized reduction in strain at high rate) are observed at impact-like rate uniaxial tension. Compression data are given in Fig. 4.10(A) for low, (B) for moderate, and (C) for high density. Loss of contact occurred (marked by starred points on the stress-stretch curve) and was estimated using the raw voltage signal obtained from the load cell. Again, the compression results are generally consistent with the observations across densities and rates in the non-quasistatic rate regime. A noteworthy exception to this is the occurrence of nonmonotonicity in the onset of the plateau regime, which is indicative of compaction band or crush zone formation [10].

4.5.2 Strain localization in intermediate rate experiments

The quasistatic and low-rate strain measurement used plane-fitting to globally homogenize the deformation response and reduce noise under the assumptions of uniform continuum behavior for the material, as verified in Chapter 3 Sec. 3.3.1. However, at higher rates a transition from uniform compression or widely diffuse buckling during the plateau to a compaction-band dominated mode is expected (for example see [33]) and as indicated by the nonmonotonic nature of the stress response. Thus, throughout the intermediate rate analysis,

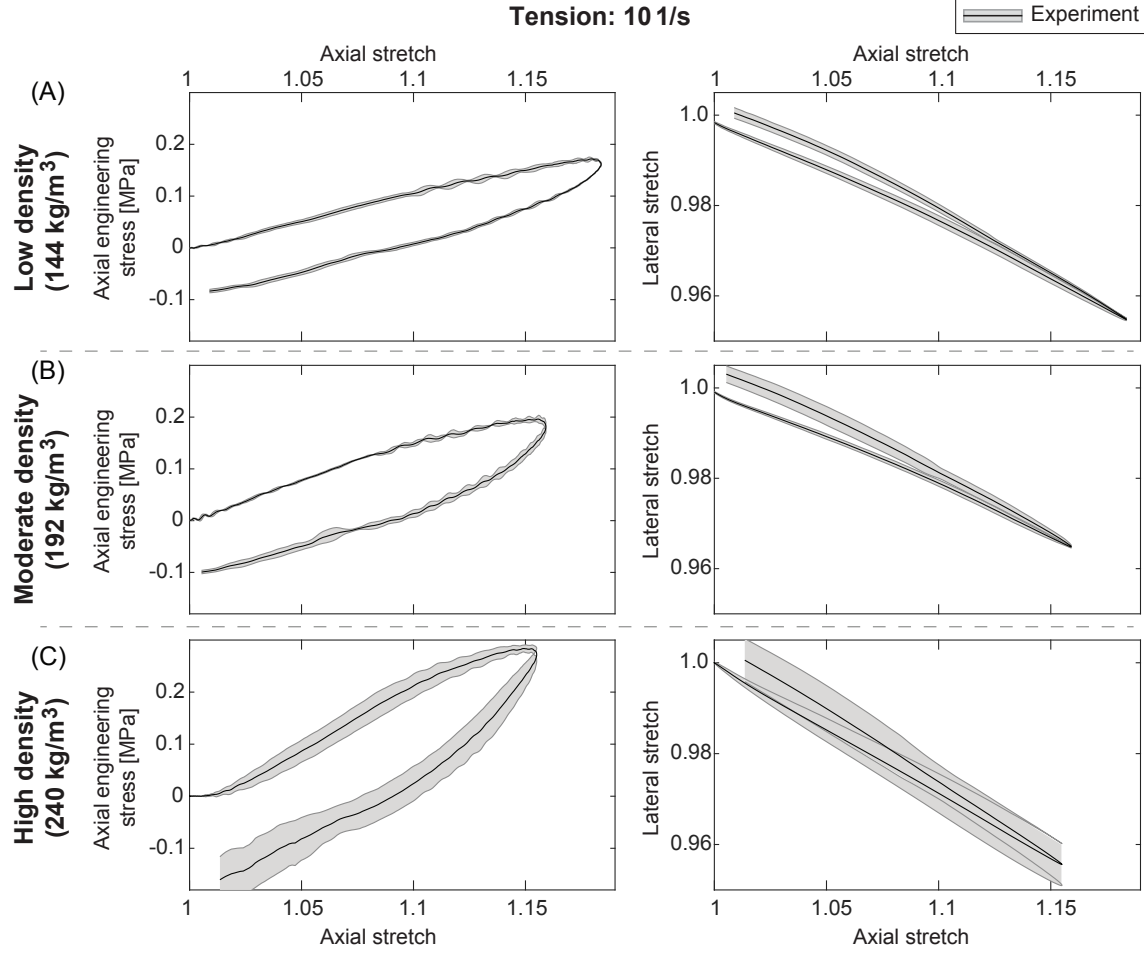


Figure 4.9: The axial engineering stress versus axial stretch and lateral stretch versus axial stretch data for intermediate rate ($\dot{\epsilon}_{\text{applied}} = 10 \text{ 1/s}$) tension of three densities of XRD foam. Shaded area indicate one standard deviation. (A) The low-density foam. (B) The moderate density foam. (C) The high density foam.

instead of making use of the global plane fitting, displacement gradients and strains fields are computed via an optimized 9-tap convolutional spatial differentiation kernel (based on the work of [200] and [228], see also Chapter 3 Sec. 3.3.1). This gives a balance between noise level and spatial resolution with a virtual strain gauge size of $VSG = 80 \text{ px}^2$, and enables visualization of strain concentrations. In addition, histograms of strain fields can also be used to identify localization [48].

To observe localization that occurs at this impact-like rate (in compression only) refer to Fig. 4.11(A), which shows an example case of the low density foam stress-strain response with insets for incremental strain maps. In each case the strain increment is the step culminating

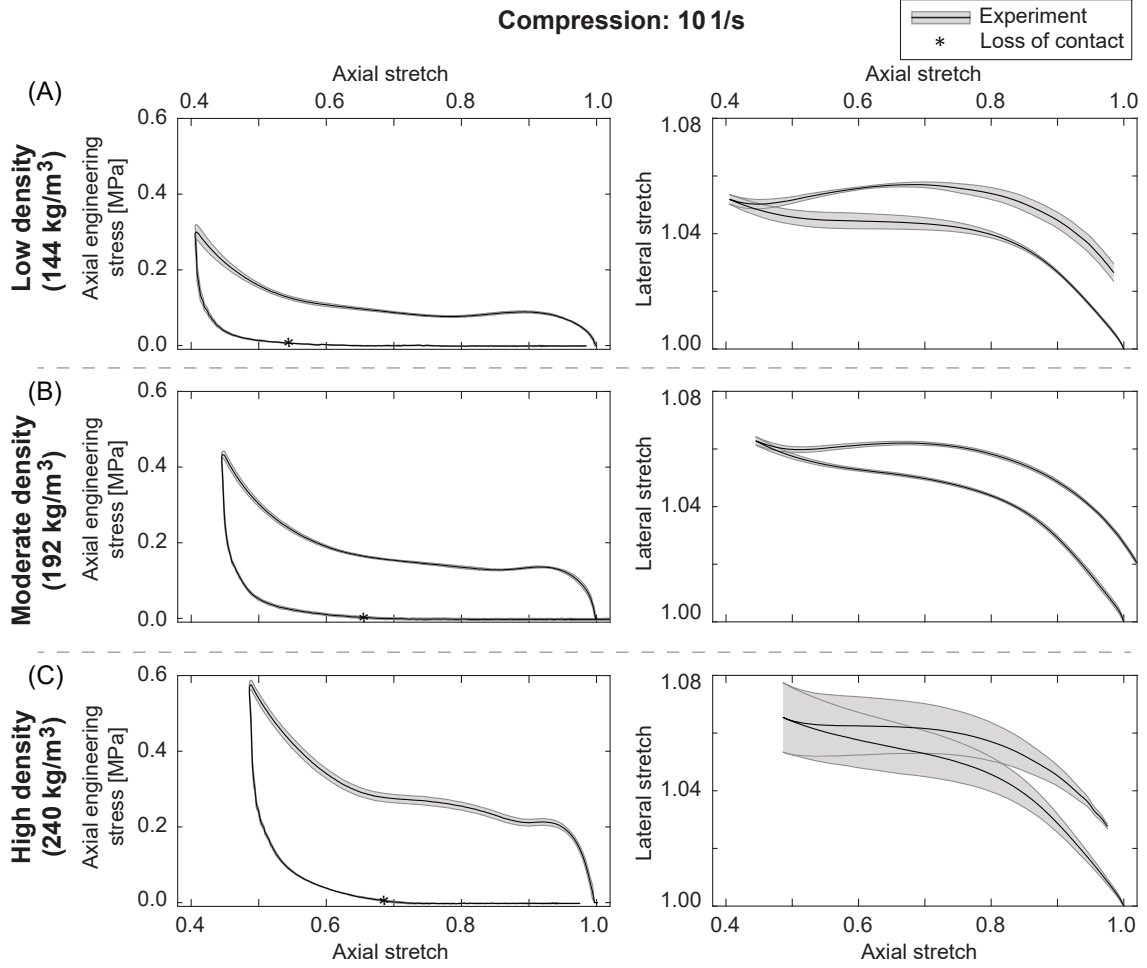


Figure 4.10: The axial engineering stress versus axial stretch and lateral stretch versus axial stretch data for intermediate rate ($\dot{\epsilon}_{\text{applied}} = 10 \text{ 1/s}$) compression of three densities of XRD foam. Shaded area indicates one standard deviation and stars indicate approximate loss of contact. (A) The low-density foam. (B) The moderate density foam. (C) The high density foam.

in the quoted global mean strain, with the corresponding incremental mean strain magnitudes between 0.57% and 0.67%. The strain maps indicate that in the initial “linear” regime strain is relatively uniform but a small band is beginning to develop. As compression continues to just beyond the “knee” in the stress response, a distinct compaction band has developed in the field of view. In the plateau regime several hotspots begin to appear, but the compaction band is no longer accepting additional strain and is now a region of low (incremental) strain surrounded by slightly higher strain for this increment. Finally, in the densification regime, the strain is again relatively diffuse, with further developed hotspots. The histograms of strain for each of these levels are shown in Fig. 4.11(B) and tell a similar story: at 5%

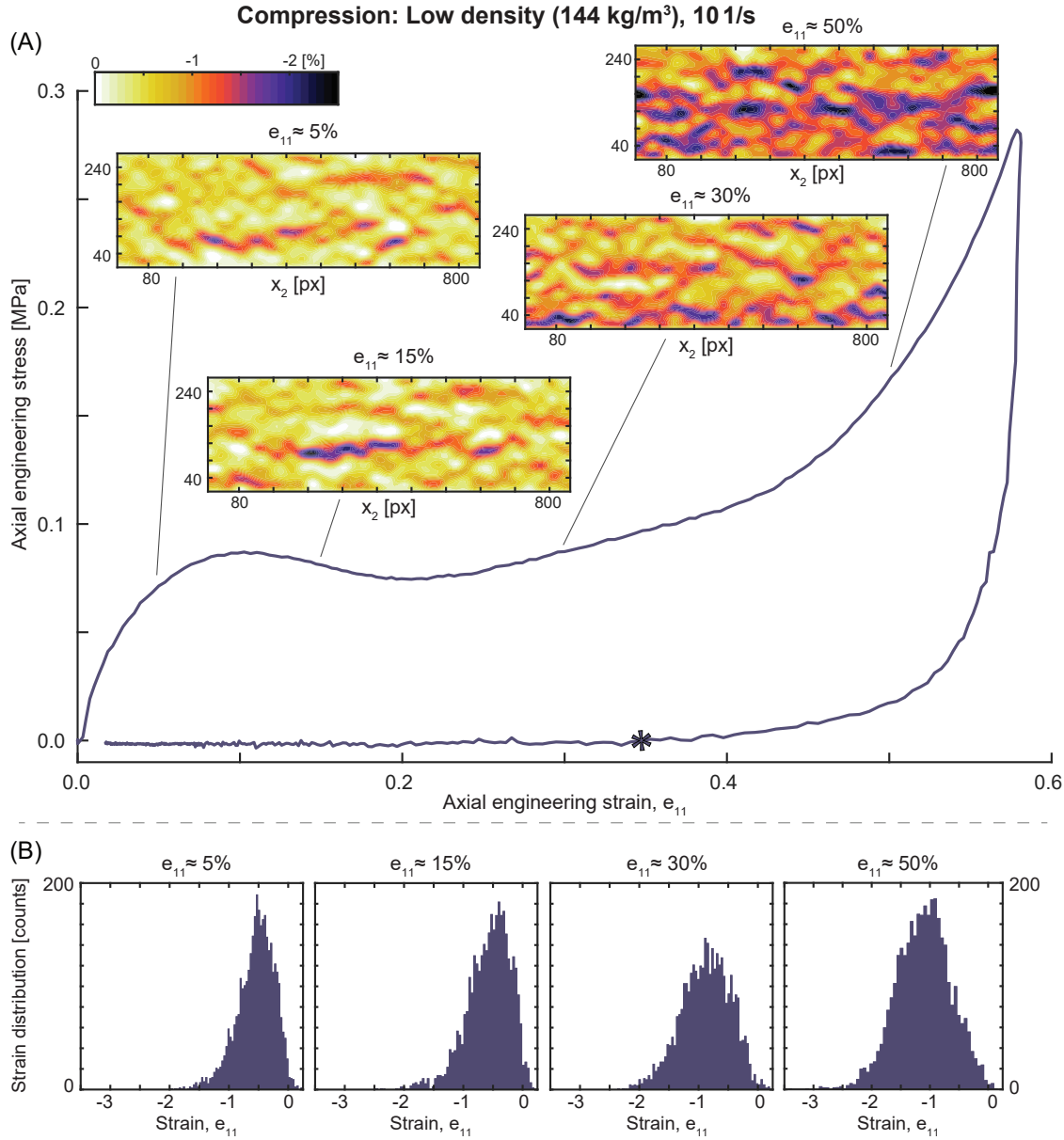


Figure 4.11: A representative intermediate rate compression experiment on low density XRD foam, demonstrating the onset of localization at the impact-like rate. (A) Stress-strain plot with maps of incremental strain for four deformation regimes. (B) Histograms of strain distribution for the four points along the stress-strain curve.

strain the peak is relatively Gaussian but tails to the left, at 15% strain a weak secondary peak develops, and the distribution broadens at 30% and 50% strain. The overall peak shifts slightly towards larger incremental strains (i.e. larger change in strain per fixed time step for imaging). Given the relatively fixed strain rate, this is likely indicative of strain localization also occurring outside the field of view used to compute strain (typically the

bottom 35% of the specimen, see Fig. 3.4) that becomes compacted prior to the mid-plateau and densification sample points.

4.6 Concluding remarks

4.6.1 Summary of elevated rate experiments

In this chapter I described three experimental systems for testing foams at elevated strain rates, and demonstrated the effectiveness of two of these techniques by measuring the response of three densities of “Poron XRD” across five decades of strain rate. For low-rate testing the required modifications to the quasistatic testing procedure were outlined. A new high-rate load frame designed for experiments up to 100 1/s was implemented. A drop-weight system was designed and built for compression experiments up to 10^3 1/s. I show experimental data for tension and compression for low rates with a generalized Maxwell rheological model extension fit to the high density case, in addition to data for impact-like rates on the new high-speed load frame. Finally, the appearance of strain localization at higher rates was discussed.

4.6.2 Limitations and future directions

With the expanded capabilities enabled by intermediate and high rate test facilities for foams, numerous directions to explore are available. In this work, I have only studied one type of polymer foam, and expanding the testing to additional types is a straightforward next step. Prior to additional testing, though, further development of the high-speed load frame – particularly toward strain rate stabilization in tension – will be important. Expanding testing to the drop weight system will allow higher compressive strain rates, albeit in a relatively uncontrolled fashion, which may be important to understand the full spectrum of material responses.

My elevated rate experiments were limited to compression and tension. As described in Chapters 1 and 3, many other modes are available to choose from and additional experiments

using these modes will be necessary to validate the rheological extension to our hyperelastic model. Adapting the testing fixtures used for quasistatic validation is another straightforward short-term future direction.

Finally, using full-field imaging to investigate localization and the interplay of localization with geometrical and material parameters is an extensive and exciting prospect that I have only briefly touched on in this work. Sweeping through intermediate strain rates may reveal critical conditions for localization, localization modes in polymer foams, or additional mechanisms important to consider when modeling the material over many decades of strain rate. When paired with sophisticated image analysis, DIC, and/or post-hoc μ CT the tools for new fundamental insight into the coupling between strain rate and localization soft polymer-based foam may now be available.

Chapter 5

Preliminary Studies and Summary

5.1 Preliminary studies

Three studies preliminary to proposed future work from Chapters 3 and 4 were conducted. These consisted of a localization measurement on a low-density localization-prone polyurethane foam, a temperature sensitivity study on moderate density XRD foam, and an initial effort to use nanoindentation to obtain ligament-level material properties of XRD foam.

5.1.1 Localization measurement

To visualize and investigate the presence of localization, a candidate low density, open-cell, polyurethane foam (“soft”, Team Wendy, Cleveland, Ohio) known to form compaction bands at relatively moderate strains and slow strain rates (see [34]) was tested in compression. The compression test follow the standard procedure discussed in Chapter 3, with the exceptions that the inherent contrast of the material itself was sufficient for qDIC motion tracking – and thus no paint was applied – easing visual observation of the underlying deformation, and that wait time was prolonged due to the slower relaxation timescale of this material. The maximum target axial engineering strain magnitude was 20%. Deformation gradient and strains were computed via the method discussed in Chapter 4 using an optimal-11

filter. Figure 5.1 shows the inhomogeneous spatial distribution of local strain (visualized using the vertical component of the deformation gradient, F_{22}) on the specimen surface and highlights the co-localization of a compaction event with a band of compressive strain with an amplification factor of four over the global mean strain (19.98% mean specimen strain over the DIC region).

5.1.2 Temperature sensitivity

Since we expect, based on the results of, e.g., [257], or on contemporary experiments on polymer foams (although not necessarily polyurethane-based), e.g., [53–56, 129], that temperature sensitivity will play an important role in the mechanical response, preliminary experiments to assess this effect were conducted. Preliminary data for temperature sensitivity analysis was gathered using spherical indentation experiments at an equivalent nominal strain rate of 5×10^{-2} 1/s. The specimen thickness was about 13 mm and was indented to 4.17 mm at 0.21 mm/s for about 40 s total test time, the slowest strain rate where overall thermodynamics were a sufficiently slow timescale that heat loss during the experiment was small. To conduct experiments the procedure for spherical indentation outlined in Chapter 3 was used, with modifications. The specimen stage was altered to insulate the specimen from the ambient environment and incorporate both a temperature-matched stage plate and calibrated infrared temperature probe for real-time temperature tracking (see Fig. 5.2(A)). Specimens were allowed to equilibrate at either just above (for hot tests) or just below (for cold tests) the target temperature. Specimens were quickly installed from the temperature-controlled environment onto the stage and tested as soon as the temperature reached $\pm 1^\circ\text{C}$ of the target. For the extreme tests, temperature changes were on the order of $3\text{--}5^\circ\text{C}$ over the test duration.

The data collected, shown in Fig. 5.2, illustrates the dramatic effect of temperature on both the apparent stiffness and the dissipation, which can be seen in the amount of hysteresis, on the XRD foam. In particular, there is a large increase in apparent stiffness and hysteresis for temperatures below room temperature (i.e., 21°C), Fig. 5.2(B) as the anticipated glass transition temperature (e.g. around -20°C per [257]) is approached. Similarly

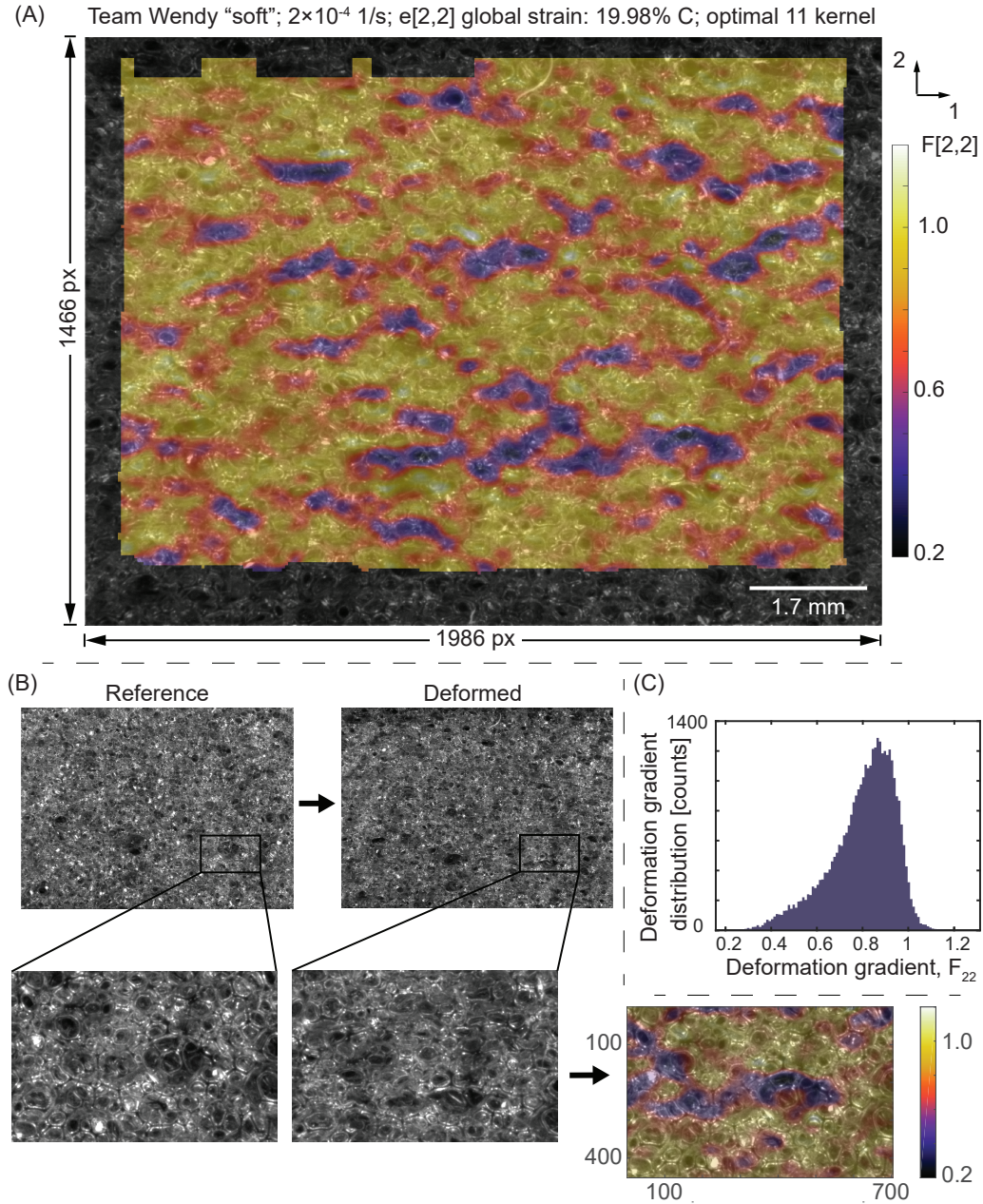


Figure 5.1: Preliminary experiments demonstrating highly developed localization in a “soft” low-density open-cell polyurethane foam. (A) Globally applied compressive strain is 20%, whereas the axial component of the deformation gradient tensor (F_{22}) varies from about 10% equivalent local tensile strain to about 80% equivalent compressive local strain (mean strain is 19.98%). (B) A typical microstructural event consisting of pore closure and collapse. During this event, a small region undergoes approximately 80% strain, and the strain overlay inset demonstrates the co-localization of strain region with microstructural collapse. (C) Histogram of strains showing a distinct “shoulder” but no clear second peak for this moderate global mean strain level.

of note is the reduction in hysteresis and softening of the material at elevated temperatures (Fig. 5.2(C)). No noteworthy difference in the material behavior under indentation is observed for temperatures of 40°C and 55°C. Finally, to assess self-heating under cyclic loading, five compression experiments at an effective strain rate of 10^{-1} 1/s (see Chapter 4) were conducted in immediate sequence while the temperature was monitored; as shown in Fig. 5.2(D) a slight upward trend is plausible but inconclusive. With this equipment and in these test conditions dissipative heating is not confidently detectable. Similar future experiments conducted in an environmental chamber and observed with thermal imaging, might prove a powerful tool toward understanding thermal, dissipative, and localization effects.

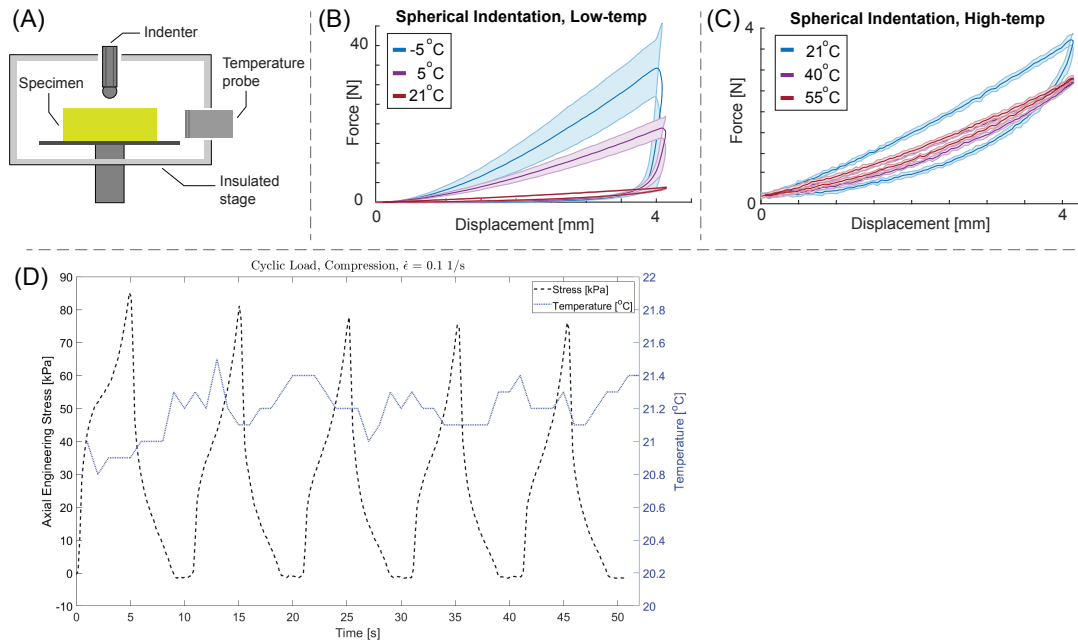


Figure 5.2: Preliminary temperature sensitivity results for moderate density XRD foam using spherical indentation force-displacement, highlighting the strong temperature dependence in these foams. (A) Schematic of the indentation adaptation for temperature experiments. (B) Reduced temperature results. (C) Elevated temperature results. (D) Temperature change as a function of cyclic loading for a standard compression specimen.

5.1.3 Nanoindentation

A brief study was conducted to directly measure the matrix material properties. While informative as a preliminary effort, more work is needed to produce robust data; however, I will outline the progress to date. To begin, specimens of high density XRD foam were excised

from the stock material. These specimens were then vacuum-impregnated with a low viscosity, low temperature epoxy (Epoxy Embedding Medium Kit, Sigma Aldrich, St. Louis, MO) and polished with a series of wet-use sandpapers. Once prepared, the embedded specimens were tested at the Nanoscale Imaging and Analysis Center at the University of Wisconsin-Madison (TI-950 Triboindenter, Hysitron, Eden Prairie, MN). A $1.1\ \mu\text{m}$ conispherical indenter tip was used with a trapezoidal load function featuring 60 s load and unload periods and a 30 s hold period at $75\ \mu\text{N}$ peak force. This loading process resulted in approximately 250 nm of vertical displacement (see Fig. 5.3), which is smaller than the typical ligament size upon which indents were placed, as was the tip size.

Load-displacement data was analyzed with the built-in post-processing available with the instrument, following the basic technique of [258], where the slope of the unloading curve ($S = \frac{dP}{dh}|_{P_{max}^+}$, where P is the applied load, and h is the indentation depth) and cross section of the tip ($A = A(h)$) are used to determine the reduced modulus via $E_r = \frac{\sqrt{\pi}}{2} \frac{S}{\sqrt{A}}$. The hardness is simply given by the maximum indentation force over the cross section of the indenter tip at maximum displacement, i.e., $H = \frac{P_{max}}{A_{max}}$. Resulting matrix material hardnesses measured for the high density XRD foam are given in Table 5.1. Unexpectedly high values for reduced modulus of O(1 GPa) were measured, cf. [257] from which we expect O(1 MPa) modulus. This may be a result of specimen preparation, substrate effects, time-dependence, or limitations of assumptions made in the post-processing and a more detailed investigation is needed before reporting moduli. While variability is relatively low given the assay and specimen characteristics, more work is necessary to verify these numbers hold across specimens, indentation rates, and maximum loads; the influence of specimen preparation, hydration, and temperature are unaccounted for at present. In addition, nano-dynamical mechanical analysis can be conducted in future to obtain time-dependent parameters (storage and loss moduli, loss tangent), or more detailed analysis could be applied (e.g., via a finite element model updating scheme) to account for nonlinear material properties of the polyurethane.

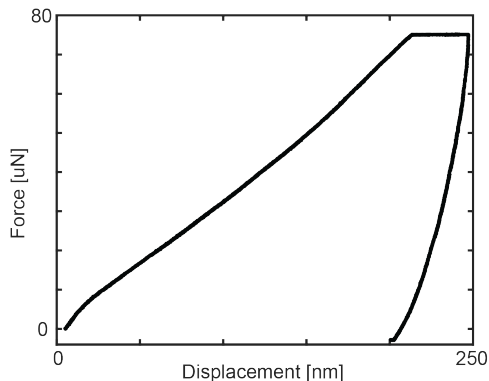


Figure 5.3: Preliminary representative nanoindentation curve for XRD matrix material. Note the relaxation during the constant-force hold period.

Table 5.1: Matrix material properties obtained from nanoindentation on individual ligaments of a high density XRD foam.

n	Hardness [GPa]
1	0.0562
2	0.0698
3	0.0841
Total	0.070 ± 0.014

5.2 Summary

Elastic polymer foams find widespread application in protective equipment, cushioning, and other applications where the soft, dissipative, and large volumetric strain materials are useful, but a complete, predictive hyperelastic model remains elusive. The base, or matrix, material is a elastic polymer solid, and cells or pores are typically gas-filled void space – so called low density elastomeric foams have void fractions over 70% and up to 99%+. Void spaces can either be closed cell – prohibiting mass transfer between void space in the foam – or open cell. In terms of mechanical response, elastomeric foams typical display tension-compression asymmetry, non-linear Poisson’s function, deviatoric-volumetric coupling, and rate- and temperature-sensitivity. In addition, many foams are anisotropic and have locally non-affine deformation characteristics. These effects arise due to both microstructural features and matrix material properties.

In this work, I described the development of both physical and computational tools to evaluate and characterize foam behavior and outlined a new predictive hyperelastic continuum model calibrated and validated with a series of quasistatic and dynamic rate experiments. This development was accomplished in four phases: first, development of a robust displacement tracking technique (“qDIC”); second, quasistatic tension, compression, and shear of a nonlocalizing, isotropic foam; and, third, multi-rate system development and

experiments for viscoelasticity. The experimental platforms consisted of custom load frames instrumented for digital image correlation (DIC). In Chapter 2 I described the development of a new DIC algorithm that uses the iterative deformation method and quality factors for cross-correlation to address the challenges of measuring full-field strain on foam. In Chapter 3 stress-strain and axial-transverse strain from tension and compression experiments were used to calibrate a novel continuum model co-developed with collaborators. This model was validated using shear with and without pre-compression, indentation, and inhomogeneous tension. In Chapter 4 two new experimental systems for elevated rate testing and a series of higher strain-rate experiments were conducted. These were used to calibrate time-dependency in an extended model and investigate a transition toward localization-driven deformation behavior at impact-like rates. Taken together, these efforts yield a straightforward workflow for characterizing elastomeric foam for both model implementation and use in engineering applications.

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