The quantities and properties of the dark constituents of the universe reveal much about the past and future development of the cosmos. It is thought that dark matter provides 85% of the mass in the universe, and that it is certainly the dominant component in large structures from galaxies to superclusters. Dark matter density, clustering, and quantity have repercussions for the universe’s geometry, its past and future evolution (and ultimate fate), as well as for the more specific question of how structure has formed. Weak gravitational lensing is one of the only tools available to measure dark matter densities directly; most other methods measure observables that have a poorly known, and often debated, relationship to the dark mass. Although its direct sensitivity to mass distributions provides an obvious advantage for weak lensing, because it is a statistical measurement whose signal strength typically has been of the same order of magnitude as systematic and random noise, until recently it has been difficult to adequately and convincingly control for these. For this reason, weak lensing data collection has normally been specifically designed to mitigate these factors. The unique requirements of optimal weak lensing observation combined with limited telescope time have meant that there are relatively few datasets that have been used for weak lensing. There is a huge body of past and future data designed for other types of studies for which weak lensing analysis has been considered inappropriate.
However, weak gravitational lensing techniques have matured. In this work for the first time, we have applied weak lensing methods to a dataset that was not engineered for weak lensing studies. The NOAO Deep Wide-Field Survey (NDWFS) optical dataset includes almost 9 square degrees over 27 subfields with a range of best seeing from $0.66''$ to $1.20''$ in three different filters ($I$, $R$, and $Bw$). We use this dataset to show that weak lensing analysis may now be successfully applied to data formerly not considered suitable. We find that detections of cluster-sized mass densities from this dataset perform comparably to detections from previous surveys. We correlate our detections with the Chandra XBoötes results for extended sources.
Measuring Weak Gravitational Lensing in General Purpose Imaging Surveys

by

Wessyl R. Kelly

M. Sc., Brown University, 2001
B. A., Washington University, 1997

Submitted in partial fulfillment of the requirements
for the Degree of Doctor of Philosophy in the
Department of Physics at Brown University

Providence, Rhode Island
May 2011
This dissertation by Wessyl R. Kelly is accepted in its present form by the Department of Physics as satisfying the dissertation requirement for the degree of Doctor of Philosophy.

Date ____________  ____________________________
Ian Dell’Antonio, Director

Recommended to the Graduate Council

Date ____________  ____________________________
Ian Dell’Antonio, Reader

Date ____________  ____________________________
David Cutts, Reader

Date ____________  ____________________________
Gregory Tucker, Reader

Approved by the Graduate Council

Date ____________  ____________________________
Sheila Bonde
Dean of the Graduate School

iii
Wessyl Kelly was born in Walnut Creek, CA, in the United States. She received a B.A. in Physics from Washington University in Saint Louis, MO, in August of 1997. After working at the University of Illinois, Urbana-Champaign developing undergraduate web-based science curricula, she came to Brown University and completed her M.Sc. in Physics in 2001. She has been working with the Observational Cosmology and Gravitational Lensing Group under Professor Ian Dell’Antonio. This dissertation, “Measuring Weak Gravitational Lensing in General Purpose Imaging Surveys”, is the culmination of research begun in 2006.
To Jortha
Acknowledgments

Heartfelt thanks to Ian Dell’Antonio for his constant guidance and enthusiasm; I am very fortunate to have had an advisor whose determination matched and whose patience exceeded my own. Many thanks to Dean Hudek for his sound advice and unfailing encouragement, no less than for employing me when I needed to pay the bills, and for keeping me in stitches all the while. Hossein Khiabanian’s excellent suggestions helped guide my early progress, and Richard Cook’s input was as valuable during later stages; my sincere friendship and appreciation to both of them. Thanks also to Jeffrey Kubo whose initial image stacks were instrumental in shaping this project, and to David Herrera (NOAO Kitt Peak) who spent unknown hours retrieving the NDWFS data from the archives.

Finally, without the support of my family on so many levels, most particularly of Dara and Jim, I would not have achieved this. Gratitude and love to them all.
Contents

List of Tables x

List of Figures xi

1 Preliminaries 1

2 Cosmology 4
   2.1 Fundamentals 4
   2.2 Evolution of Structure 11
   2.3 Dark Matter 14
      2.3.1 Observational evidence for dark matter 17
      2.3.2 Front-runners for CDM 28
   2.4 Cluster Cosmology 34
      2.4.1 Cluster formation 36
      2.4.2 Finding clusters 38

3 Gravitational Lensing 50
List of Tables

A.1 Cluster detections. .............................................. 131
List of Figures

2.1 The distribution of galaxies in part of the two-degree-field galaxy red-shift survey (2dFGRS), drawn from a total of 141,402 galaxies. From Ref. [209]. .............................................................. 5

2.2 The Hubble diagram for the High-Z SN search and the Supernova Cosmology Project. The plots show the residual of the distances relative to a flat universe with $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. From Ref. [234]. .................. 8

2.3 The measured spectrum of M82 as might be observed at different redshifts. From Ref. [23]. ................................................................. 10

2.4 Uniform spectrum of the CMBR compared to Planck blackbody spectrum. The FIRAS experiment on the COBE satellite measured the spectrum at 34 equally spaced points along the blackbody curve. Uncertainties are a small fraction of the line thickness. From Ref. [105]. .................. 12
2.5 For the Coma cluster, histogram of the galaxy distribution found by wavelet analysis, left: with redshifts $3000 \text{ km s}^{-1} \leq cz \leq 28,000 \text{ km s}^{-1}$ (step size 500 km s$^{-1}$), and right: with redshifts $4000 \text{ km s}^{-1} \leq cz \leq 10,000 \text{ km s}^{-1}$ (step size 100 km s$^{-1}$). From Ref. [110].

2.6 Fit of exponential disk and spherical halo to rotation curve of NGC 3198. From Ref. [269].

2.7 A foreground-reduced Internal Linear Combination (ILC) map of the Cosmic Microwave Background Radiation anisotropy, based on the five-year WMAP data. From Ref. [138].

2.8 The WMAP three-year power spectrum (in black) compared to other measurements of the CMB angular power spectrum, including Boomerang [148], ACBAR [164], CBI [221], and VSA [92]. From Ref. [137].

2.9 Two different representations of the HI gas density of the possible “dark galaxy” VIRGOHI 21. Left: ALFALFA observations of the field around VIRGOHI 21 and NGC 4254 superposed on an optical image. From Ref. [125]. Right: WSRT data superposed on a color image from the Isaac Newton Telescope. From Ref. [193].
2.10 The Bullet Cluster. *Top:* Color image of the merging cluster 1E 0657-558. *Bottom:* Chandra x-ray image of the cluster, 500 ks exposure. In both panels, green contours are the weak-lensing convergence reconstructions, with the outer contour level at $\kappa = 0.16$ and increasing in steps of 0.07. The white bar indicates 200 kpc at the distance of the cluster. From Ref. [71].
2.11 Axion exclusion limits in the plane spanned by the mass of the axion $m_{\text{axion}}$ and the axion-photon-photon coupling constant $g_{a\gamma\gamma}$. The diagonal yellow region (Axion Models) and the solid diagonal line (KSVZ E/N=0) indicate, respectively, the regions predicted by most invisible axion models and by the KSVZ model in particular. The cosmological hot dark matter constraint is shown by the short vertical line (HDM) within this region [36, 123, 124], and the astrophysical limit from studies of stars on the horizontal branch by the horizontal dashed orange line (HB Stars) [218, 219, 220]. The telescope search for $a \rightarrow \gamma\gamma$ decays reported in Ref. [118] excludes the orange region (Telescope), and microwave cavity axion searches exclude the gray regions (Microwave Cavity) [22, 23, 98]. Solar axion searches exclude the regions above the blue lines (upper dotted: Lazarus, et al., [170]; lower dotted: Tokyo Helioscope [196]; and solid: CAST [14, 228, 286]. The cyan region (Bragg Reflection) is excluded by searches for axion conversion in the Coulomb field of nuclei in a crystal lattice [25, 39, 202], and the green region (Laser Experiments) is excluded by searches for a “light shining through a wall” event [63]. From Ref. [253].
2.12 Limits on the WIMP mass and interaction cross section from several current and future direct searches. Limits plotted are WARP (magenta [37]), KIMS (blue [176] and dotted blue [175]), CRESST II (cyan [167]), EDELWEISS II (light green [20]), ZEPLIN II (dark green [173]), CDMS (solid red [10] and dotted red [235]), XENON10 (green [15]), and DEAP/CLEAN (dotted black [189]). Dotted lines represent projected experimental limits. DAMA/LIBRA is the filled yellow region ([232] and references therein), and the IceCube annihilation limits (to $b\bar{b}$: upper, and to $W^+W^-$: lower) are the light blue regions [3]. Favored parameter spaces of various SUSY models are shown in gray (from light to dark: Baltz & Gordolo [27], Trotta et al.: 95% CL [263], Baltz & Gordolo [28], and Trotta et al.: 63% CL [263]), and by black crosses (Battaglia et al. [32]). Figure generated using the WIMP Dark Matter plotter by R. Gaitskell and J. Felmini at http://dendera.berkeley.edu/plotter.

2.13 A high resolution mosaic of the central region of the Coma cluster taken by the Hubble Space Telescope. From Ref. [65].
2.14 Illustration of the sensitivity of the cluster mass function to the cosmological model. *Left:* The measured mass function and predicted models (with only the overall normalization at $z = 0$ adjusted) computed for a $\Omega_M = 0.25, \Omega_\Lambda = 0.75$ cosmology. *Right:* Data and the models for a $\Omega_M = 0.25, \Omega_\Lambda = 0$ cosmology. When the overall model normalization is adjusted to the low-$z$ mass function, the predicted number density of $z > 0.55$ clusters is in strong disagreement with the data, and therefore this combination of $\Omega_M$ and $\Omega_\Lambda$ can be rejected. From Ref. [272].

2.15 *Left:* Optical image of Centaurus cluster. *Right:* Corresponding x-ray image. From Ref. [143].

2.16 Interferometric images of the Sunyaev-Zel’dovich effect for a sample of galaxy clusters over a large redshift range. From Ref. [64].

2.17 SZ component map extracted from four simulated noisy Olimpo SZ-cluster survey maps. The SZ cluster signal, subdominant at all observed frequencies, now appears clearly. From Ref. [216].

2.18 The strength of the SZ effect depends on the total mass of a cluster. The vertical axis shows the SZ decrement, which is the difference between the cosmic microwave background signal and the SZ signal. The green line is the SZ decrement for a cluster of lower mass compared to that of the red line. From Ref. [256].
2.19 Redshift vs. radius for galaxies around the CAIRNS clusters. The caustic pattern is evident as the funnel-shaped regions with high density. From Ref. [224].

2.20 Left: This Hubble Space Telescope image shows several blue, loop-shaped objects that actually are multiple images of the same galaxy. They have been duplicated by the gravitational lens of the cluster of yellow, elliptical and spiral galaxies—called CL 0024+1654—near the photograph’s center. From Ref. [73]. Right: The reconstructed total mass density in CL 0024+1654 at redshift $z = 0.39$. From Ref. [266].

2.21 Simulation of the cosmic shear in a Cold Dark Matter (CDM) model. The light blue regions represent overdensities such as groups and clusters of galaxies. Dark regions correspond to voids. The line segments represent the amplitude and the direction of the lensing shear produced by these structures. From Ref. [146].

2.22 Weak lensing map (contours) superposed over an optical image of a galaxy cluster near the strongly gravitationally lensed quasar RXJ0911. From Ref. [152].

3.1 Cosmic shear field (white ticks) superposed on the projected mass distribution from a cosmological $N$-body simulation: overdense regions are bright, underdense regions are dark. From T. Hamana in Ref. [107].
3.2 Diagram of a lensing system showing a “thin lens” mass distribution, the optical axis, the deflection angle, and the apparent position of the distorted background source galaxy. 53

3.3 Left: Distortion due to the convergence and the complex shear for a circular source of known size. For such a source, the two are easily distinguished. From Ref. [257]. Right: The tangential and cross-components of the shear, for an elliptical image with $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0$, and three different directions $\phi$ with respect to a reference point. The angle $\alpha$ is the angle between the major axis of the image and the direction tangent to $\phi$. Inspired by M. Bradač from Ref. [237]. 60

3.4 Left: Single resolution ML convergence map (pixel scale 1.5'); Middle: Single resolution direct method convergence map (pixel scale 1.5'); and Right: Multiresolution convergence map of DLS field F2. Pixel scale 0.75' within the box, and 1.5' elsewhere. From Ref. [155]. 73

3.5 Limiting magnitudes and coverage for several surveys. Black triangles indicate completed surveys. Red triangles indicate ongoing or future surveys. Based on Figure 1 in [90]. 78

4.1 Detections from the real catalog compared to average detections of the randomized catalogs, top: in the original NDWFS stack, and bottom: in the DLS stack.

4.2 Example of the masking process, left: chip, and right: mask. Pixels that are white in the mask file are excluded when the chip is processed.

4.3 Star selection display from disstar. The selected star locus is indicated by the heavier points. From Ref. [280].

4.4 Objects made elliptical by anisotropies in the PSF become round when convolved with their conjugate moments. Left: Before PSF correction. Right: After PSF correction. Crosses indicate objects with ellipticities that are too small to see at this scale.

4.5 Scatter plot of cuts made on subfield J1426p3346 (R, 0.77″).

4.6 Stacked image for subfield J1434p3421 (I, 0.79″). The green box in the center indicates the close-up field of view used in a, b and c of Figure 4.7.

4.7 Small central region of data image of J1434p3421 (I, 0.79″) (green rectangle in Figure 4.6) showing the Source Extractor catalog of objects (a) before filtering and hand cleaning, (b) after automatic filtering, and (c) after both filtering and cleaning. The cleaned, filtered catalog shown in c is the input for the mass density reconstruction program fiatmap.
4.8 Top: The original stacked image of subfield J1437p3532 ($Bw$, 0.84$''$).

Bottom: The final mass density map of this subfield after processing and analysis.

5.1 Average number of objects detected per map above a given flux threshold for sets of $N$ maps, $N=\{100, 500, 1000, 2000, 5000\}$.

5.2 Average number of objects detected per map above a given flux threshold for different sets of 100 random maps.

5.3 Number of detections in actual data map (solid circle) and the average number of detections from the randomized maps (open circle) for each subfield as a function of seeing. Marker color indicates the filter used for that subfield: blue = $Bw$-band, red = $R$-band, and green = $I$-band. Error bars represent $\pm 1\sigma$.

5.4 Sample plot of detections from actual map (red) and of average spurious detections from the 500 random maps (blue) for subfield J1437p3532 ($Bw$, 0.84$''$). This plot shows two detections above noise levels in the actual data.

5.5 Variance in number of objects detected per map above a given flux threshold for sets of $N$ maps, $N=\{100, 500, 1000, 2000, 5000\}$.
5.6 Number of source galaxies used in mass map construction as a function of seeing for the three colorbands. We see a general trend toward fewer galaxies with worse seeing for all bands as expected. Lower counts in the first three subfields (seeing $0.77''$, $0.77''$, and $0.79''$) are attributable to an initial tendency to overfilter potential faint galaxies during the final catalog generation. The on-average higher source detection rates in $Bw$ are discussed in the text. 120

5.7 Plots of detections from real map (solid red circles), average detections over 500 randomized maps with error bars ($\sigma$), and detections from a randomly selected randomized map (open green circles) as a function of increasing DETECT_THRESH for six representative subfields. 122

5.8 Detections from the Chandra XBoötes survey. Smoothed ($\approx 60''$), processed image showing locations of 43 extended sources. The circles marking clusters are 10 times the size of the detected source. From Ref. [154]. 124

5.9 Field positions of NDWFS cluster “detections” (red circles), and XBoötes extended source detections (blue circles) [154]. The size of the circle represents the relative maximum flux for the NDWFS data, and the relative flux $S_{14}$ in units of $10^{-14}$ erg/(cm s) for the x-ray data. 125

5.10 Field positions of matched objects (within $\sim 6'$ of each other). Symbols and colors as defined in Figure 5.9. 126
Chapter 1

Preliminaries

Current cosmological research is focused on a handful of parameters that together describe the past, present and future behavior of the universe. Some of these same parameters also constrain the properties and interactions of its constituent matter and energy, and hence, of the evolution of structure from early density fluctuations. One particularly interesting area of inquiry is that of galaxy cluster cosmology. Cluster mass and population counts, in particular, are studied for many reasons. First, accurate cluster mass and population studies yield measurements of the overall mass density of the universe, as well as the amplitude of the initial density perturbation spectrum of the early universe. Second, clusters are the largest virialized structures, and—in the ΛCDM model—the youngest. The ΛCDM model, also called the concordance model, currently best explains the known observations, and it predicts that
structure has formed hierarchically, from small to large scales. Third, multiple independent and complementary measurements may be applied to clusters to better constrain their properties. For example, there are many methods for identifying clusters and deriving their masses, including measuring optical richness, velocity dispersion of member galaxies, x-ray emission spectra of hot intracluster gas, spectral distortions of the CMB, and weak lensing effects on background galaxies. The combination of results from these diverse methods on the same clusters allow for better calibration and tighter constraints on cosmological parameters.

Except for weak lensing, each of the aforementioned techniques suffers from the same difficulty with correctly relating a measured observable to the total mass of a cluster. Doing so usually requires \textit{a priori} suppositions relating the luminosity and mass, assumptions of hydrostatic or thermal equilibrium, or knowledge of a cluster’s mass profile and dynamical state. In contrast, the technique of weak gravitational lensing measures total mass directly, but suffers from intrinsically small signal-to-noise and large systematic errors. To maximize the former and minimize the latter in weak lensing studies, typically data collection is first carefully engineered, and then multiple steps are taken to process and extract results. Many such surveys in the past have been specifically optimized for weak gravitational detection, but far more data come from surveys not created with weak lensing in mind. In the current work, we investigate the consequences of ignoring this first step—that of careful observational design—by using a dataset from the NOAO Deep Wide-Field Survey that was not
intended for weak lensing analysis. By comparison with x-ray data, results show that when applied to general data, weak lensing performs on par with other cluster selection methods.

This dissertation is structured as follows. Chapter 2 reviews the theory and current state of the field of cosmology. First, it introduces the fundamentals, the ΛCDM model and its variations, and the candidates for dark matter. Then, a more detailed discussion follows of cluster properties, of observational evidence for dark matter, and of current projects to detect or elucidate its nature. In Chapter 3, we describe the general theory of gravitational lensing, the statistical methods of weak gravitational lensing, and the application of weak lensing to observational data. In Chapter 4, we discuss the NOAO Deep Wide-Field Survey data, and describe the Deep Lens Survey data processing pipeline used here to process it. In Chapter 5, we discuss our results and present directions for future research.
Chapter 2

Cosmology

The problem of understanding the cosmos and its origins has preoccupied some of the best thinkers of every civilization. Despite the truly ancient roots of this field, only in the past two decades have technological advances introduced an age of “precision cosmology”. In this chapter, we review the basic characteristics of the universe, its history and structure, and some of the tools used to study it.

2.1 Fundamentals

We know that our universe is currently expanding. This is evident by the redshift observed in the spectra of distant galaxies, that is greater the more distant the source. An expanding universe implies that at early times the universe was very hot and very dense, and that it went through several transitions as it cooled. The epochs between these transitions left their imprints on the observable universe today. We
Figure 2.1: The distribution of galaxies in part of the two-degree-field galaxy redshift survey (2dFGRS), drawn from a total of 141,402 galaxies. From Ref. [209].

have indications that our universe is homogeneous and isotropic on scales larger than approximately 100 Mpc [209], as illustrated in Figure 2.1. Expansion, homogeneity, and isotropy imply a space-time that is described by a Robinson-Walker metric whose line element is

\begin{align*}
-c^2d\tau^2 &= -c^2dt^2 + a^2(t)dx^2 \tag{2.1} \\
dx^2 &= \frac{dr^2}{1 - kr^2} + r^2d\Omega^2 , \tag{2.2}
\end{align*}

where \(dx\) is a time-independent three-dimensional spatial metric, and \(a(t)\) is the scale factor and carries the time dependence. In Equation 2.2, \(d\Omega^2\) is a unit solid angle given by \(d\theta^2 + \sin^2(\theta)d\phi^2\). The curvature, \(k\), is constant and therefore describes a spherical \((k > 0)\), Euclidean \((k = 0)\), or hyperbolic \((k < 0)\) geometry. To precisely
characterize the expansion, the scale factor must be known. Using the Robinson-Walker metric in Einstein’s field equations yields the Friedmann-Lemaître equations of motion for the scale factor $a$ in terms of density $\rho$ and pressure $p$

\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{a^2} \quad (2.3)
\]

\[
\ddot{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right).
\]

The rate of expansion of the universe, $\dot{a}/a$, is called the Hubble parameter, and is denoted $H(t)$. We also define the Hubble constant, $H_0$, which is the rate of expansion today at time $t_0$. Its value is measured to be around $71.0 \pm 2.5 \text{ km s}^{-1}\text{Mpc}^{-1}$ [168]. The Hubble constant is often expressed in terms of a reduced Hubble constant, $h = 0.71$, where

\[
H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1}.
\]

From Equation 2.3, it is clear that the condition for a flat universe, $k = 0$, is equivalent to requiring that it has the critical density

\[
\rho_c \equiv \frac{3H^2}{8\pi G}.
\]

(2.4)

In this work, we assume that $k \approx 0$, which is a safe enough assumption for our purposes. For the expansion rate of a specific epoch, we use the equation of state (which relates the pressure $p$ to the density $\rho$) of the component that dominates during this period. By assumption, we can treat this component alone as a perfect fluid, with

\[
p = w\rho c^2
\]

(2.5)
for some value $w$. Then, the density $\rho \propto a^{-3(1+w)}$ and the scale factor $a \propto t^{(2/3(1+w))}$, as long as $w > -1$. Different forms of matter and energy may be broadly categorized according to their equations of state, that is, according to the value $w$. For example, for a relativistic particle (e.g. photons or light neutrinos), $w = 1/3$, $\rho \propto a^{-4}$ and $a \propto t^{1/2}$. For a non-relativistic particle (e.g. baryons or cold dark matter), $w \sim 0$, $\rho \propto a^{-3}$ and $a \propto t^{2/3}$. A cosmological constant ($w = -1$) or a dark energy ($w < -1/3$) accelerates the cosmic expansion—an acceleration which has been observed (see for example recent results from supernova surveys in Figure 2.2) [21, 30, 160, 213, 214, 222, 223, 234, 251, 259, 262].

The density of any constituent, $X$, is conveniently expressed as a fraction, $\Omega_X$, of the critical density:

$$\Omega_X \equiv \frac{\rho_X}{\rho_c}. \quad (2.6)$$

Depending on the epoch and the context of interest, $X$ commonly represents photons, neutrinos, or relativistic particles in general ($\Omega_\gamma$, $\Omega_\nu$, or $\Omega_R$, respectively), baryonic matter ($\Omega_b$), non-baryonic matter or dark matter ($\Omega_{mb}$ or $\Omega_{DM}$), curvature ($\Omega_k$), or vacuum energy ($\Omega_\Lambda$, i.e. dark energy or a cosmological constant). Measurements support that $\rho_{TOT}$ is extremely close to the critical density, that is $\Omega_{TOT} \sim 1$ or $k \sim 0$. Current values of the so-called “matter budget” the density ratio of relativistic particles $\Omega_R < 0.001$, baryonic matter $\Omega_b \sim 0.04$, dark (non-baryonic) matter $\Omega_{DM} \sim 0.22$, curvature $\Omega_k \sim 0$, and vacuum energy $\Omega_\Lambda \sim 0.73$ [168]. Clearly, without the dark matter component, we have a significant “missing mass” problem (around 85%
Figure 2.2: The Hubble diagram for the High-Z SN search and the Supernova Cosmology Project. The plots show the residual of the distances relative to a flat universe with $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. From Ref. [234].
of matter is unaccounted for). Baryons alone are far from sufficient to explain the observed gravitational effects we observe. In this work, we are concerned primarily with this dark matter component—which we label $\Omega_{\text{CDM}}$ since we later argue that it must be cold.

**Redshift**

Before continuing, a few words must be devoted to *redshift*, since it is the single most useful and pervasive tool used in astronomical and cosmological studies. The redshifts of known emission lines have been mentioned above as the clearest evidence of the expansion of the universe. For example, observations show that the $\alpha$ emission line of hydrogen ($\lambda_{\text{H}\alpha} \sim 6563$ Å) will be shifted toward larger wavelengths the farther the light travels from its point of origin to reach an observer. This is because, as the light travels, $\Delta a(t)/a(t) > 0$. In terms of the wavelengths at the time observed (today) and the time emitted, the redshift $z$ may be expressed as

$$1 + z = \frac{\lambda_0}{\lambda_{\text{em}}}.$$  \hspace{1cm} (2.7)

As space expands, the wavelength of radiation effectively stretches. Thus, the redshift may also be expressed in terms of the scale factor $a(t)$ at observation and emission,

$$1 + z = \frac{a(t_0)}{a(t_{\text{em}})}.$$  \hspace{1cm} (2.8)

Observationally, these expressions are extremely useful. In most cases, $a(t)$ has had the same form throughout a photon's trip from source to detector, i.e. the universe has
Figure 2.3: The measured spectrum of M82 as might be observed at different redshifts. From Ref. [23].

been dominated by the same matter or energy component during this time. Thus, for most measurements, Equations 2.7 and 2.8 provide a direct method for calculating the distance to the source. Figure 2.3 illustrates the effects of redshift using the spectrum from galaxy M82 [23].
2.2 Evolution of Structure

In the standard cosmological model, the universe began with a hot big bang. Initially, all its constituents were in thermal equilibrium in a dense, relativistic plasma. The first important feature about this model is that as the universe expanded and cooled, the average thermal energy of the plasma eventually became smaller than nuclear binding energies, allowing light nuclei (D, $^3$He, $^4$He, $^7$Li) to form in a process called **primordial nucleosynthesis**. Further expansion and cooling eventually meant that thermal processes also no longer overcame atomic binding energies, allowing electrons to be captured into neutral hydrogen atoms and other stable light elements. This transition, called **recombination**, eliminated Thomson scattering between photons and free electrons. Once these scattering events became rare, the photons propagated freely, hence this event is also referred to as **decoupling**. When the photons decoupled they preserved a snapshot of the matter distribution at the time, called the **last scattering surface**. Inhomogeneities in the matter density were recorded in the energies and densities of this scattered radiation. This radiation has since cooled and redshifted, and its imprint is now detected in microwave frequencies at a temperature of $\sim 2.73 \text{K}$, whence the current nomenclature of the cosmic microwave background radiation (CMBR; see Figure 2.7).

The second critical characteristic of the model is that since the density of each

---

*Here we will not be concerned with developments preceding the Planck scale, when quantum gravitational effects are important. Strictly speaking, the rest of this thesis assumes that the universe has sufficiently cooled and rarefied such that the laws of General Relativity hold.*
constituent scales by a different power of $a(t)$ as described above, the dynamics of different components have dominated the expansion at different times. Hence, the universal expansion has different epochs demarcated by brief periods when the densities of two components were near equal. The first such period occurred when the radiation density, which scales as $a^{-4}$, approximately equaled the matter density, which scales as $a^{-3}$. This is called the epoch of matter-radiation equality. It occurred after primordial nucleosynthesis, but before radiation decoupling. Later there was another transition from matter to vacuum energy domination. Further, measurements
of the curvature indicate that the universe is very close to (or exactly) flat, and therefore curvature has never been the dominant constituent driving the expansion (for a review of the relationship between geometry and expansion, see the Introduction in Linder, 2008 [177]). It seems that, as long as the vacuum energy is stable and cannot decay into other forms, the universe will remain vacuum energy dominated for the rest of its existence.

Out of initial matter overdensities developed the large scale structure observed today as clusters of galaxies, superclusters, walls and voids. The current small size of the primary anisotropies in the CMBR ($\Delta T/T \sim 10^{-5}$) indicates that the inhomogeneities in the baryon-photon plasma at last scattering had very small amplitudes. 

*Gravitational instability* requires that overdense regions, hindered by self-gravitation, expand more slowly than $H(t)$, thereby gradually increasing the region’s overdensity compared to average. In contrast, underdense regions expand at a rate closer to $H(t)$, therefore becoming increasingly less dense than average. However, density perturbations on scales less than the comoving horizon grow very slowly until after matter-radiation equality. When matter becomes dominant, perturbations grow linearly with the scale factor $a$. Later, after matter-radiation decoupling, baryons are freed from radiative interactions and fall into overdense regions, further increasing the amplitude of these modes. As the overdensity fluctuations in a region become high, linear perturbation theory no longer applies. When the region becomes sufficiently overdense, as defined by the Jeans mass,$^\dagger$ the gravitational self-attraction exceeds the

$^\dagger$The Jeans instability in an expanding universe describes the competing forces of collapse and
Hubble expansion. Then infall overcomes expansion outward and we say that this
region—or structure—has decoupled from the Hubble flow. (Note that in structure
formation this means that structures decouple, and later virialize, from the inside
out.) At this stage, further evolution depends on the details of the structure itself,
and the structure becomes increasingly non-linear. Currently, only structures larger
than approximately $8h^{-1}$ Mpc are still in the linear perturbation regime. Practically
speaking then, clusters of galaxies are the largest, and youngest, systems to have
decoupled and virialized.

2.3 Dark Matter

Four components of the dark universe are known to exist; three of these significantly
impact the dynamics we observe. The first component, dark energy, affects dynamics
only on scales close to the horizon. Evidence for it primarily derives from observa-
tions that the universal expansion is accelerating; acceleration of $H(t)$ requires that
the substance currently dominating the cosmos has a negative equation of state, as
discussed in Section 2.1. Dark energy is defined as such a substance. Measurements
indicate it must contribute as much as 70% of the critical density at the current epoch
[13, 21, 72, 101, 181, 212, 223, 251, 252]. The exact nature and origins of the dark
energy remain to be explained. For the purposes of structure formation and cluster

dominating the cosmos has a negative equation of state, as
discussed in Section 2.1. Dark energy is defined as such a substance. Measurements
indicate it must contribute as much as 70% of the critical density at the current epoch
[13, 21, 72, 101, 181, 212, 223, 251, 252]. The exact nature and origins of the dark
energy remain to be explained. For the purposes of structure formation and cluster

---

expansion on the mass in a region of space. The Jeans mass is the amount of matter in a region
of a given size that is necessary to overcome both its internal pressure and the rate of expansion
of the universe, such that it begins to collapse.
dynamics, with which we are concerned here, the other three dark components are of considerably more interest.

The second of the dark components, baryonic dark matter, is the most familiar, but the smallest contributor to the dark matter budget. Baryonic dark matter consists of ordinary matter, and is “dark” only in the sense that its thermal emission is too faint or too cold to see from earth. Gas giant planets, stellar remnants, cool gas and dust, as well as black holes fall within this category of so-called Massive Compact Halo Objects (MACHOs). However, from both theory and observations, it is clear that primordial nucleosynthesis only gave us enough baryonic matter—luminous or otherwise—to comprise around 4% of the critical density [77, 78], and most of these baryons remain in the diffuse intergalactic and intercluster media [109]. Therefore, the amount in “dark” gravitationally condensed baryonic objects is far from enough to resolve the missing mass question.

Standard neutrinos constitute the next dark component since they interact very weakly with radiation and other matter. The amount they contribute to the overall density depends on the rest masses and the number of neutrino species. In the standard model of particle physics, there are three neutrinos and their three antineutrinos. Their effect on structure formation, among other things, depends on the relative timing of neutrino decoupling, matter-radiation equality, and when neutrinos became non-relativistic. Current experimental bounds put their total mass $\sum m_\nu$ under $0.67\text{ eV}$ (95%CL) and their number of species $N_{\text{eff}} = 4.4 \pm 1.5$ (65%CL) [162].
This implies, first, that the contribution by neutrino mass is not sufficient to supply all the dark matter of the universe. Second, neutrinos must have been highly relativistic when they decoupled from the thermal plasma of the early universe. In such a scenario, they contribute a *hot dark matter* component, which, if large enough, would cause structures to form first at the largest scales (cluster sizes). Substructures such as galaxies and star systems would only arrive later as a result of fragmentation. Observations of AGN and galaxies at high redshift seem to rule out that hot dark matter of any form significantly affected early structure formation. Thus, although neutrinos certainly exist and certainly are dark, they are not the dark matter we seek.

By far the most interesting dark component for the purposes of this study is the still-to-be-identified *non-baryonic cold dark matter*. Although many dark matter candidates have been proposed, a consensus is emerging that cold dark matter (CDM) must comprise the largest part of the dark matter in the universe. CDM is so named because it was already non-relativistic when it decoupled from the primordial plasma. Further, it interacts electromagnetically very weakly, and thus, it primarily interacts through gravitational attraction. Current measurements put the dark matter contribution to the universal density budget at around 22%, most of which is thought to be CDM [168]. As such, we discuss CDM in somewhat greater detail.
2.3.1 Observational evidence for dark matter

Many independent results now support the idea that a large fraction of the matter in the universe consists of dark matter, and specifically, of non-baryonic cold dark matter. Most of these investigations, however, have had to make assumptions about the composition or the dynamical state of the systems studied in order to be able to relate observed luminosities to the total mass. Unfortunately, the dynamics of galaxies and clusters of galaxies are as yet poorly understood. Worse, as noted, dark matter itself seems likely to comprise much of the matter in these systems, and its properties are indeed what we wish to elucidate. Therefore, assumptions about dynamics or composition automatically introduce uncertainties into results about dark matter. Taken together though, evidence supports a growing consensus that this “missing mass” makes up approximately 22% of the total matter/energy budget (in a flat universe), and approximately 85% of the total mass budget [168]. We report some of this evidence below.

Galaxy motion in clusters

The earliest evidence of dark matter was documented by Fritz Zwicky in 1933 based on observations of the Coma cluster [287, 288]. He observed that, based on the visible luminous mass in the cluster, the virial theorem predicted much slower velocities for the outer member galaxies than were measured. More specifically, assuming a mass-to-light ratio approximately equal to that of our sun, there appeared to be around
400 times more mass than could be seen in stars and galaxies. This was the first instance of the "missing mass" problem, which has since become well-established in other clusters and in systems of other sizes as discussed below.

**Rotation curves of spiral galaxies**

Close to 40 years later, Vera Rubin and collaborators studied the Doppler effect on the spectra of stars in edge-on spiral galaxies [208]. Since at that time galactic mass was presumed baryonic (and therefore luminous), the measured mass as a function of radius was expected to decrease at high radii, and thus the rotational velocity of the galaxy would approximately follow a Keplerian $r^{-\frac{1}{2}}$ curve. Rotation curves derived from Doppler measurements showed evidence of much higher velocities at large radii.
than predicted by the luminous matter (see Figure 2.6). In order to attain such high speeds, the mass distribution must be significantly denser and more extended than could be seen. These were the first indications that spiral galaxies are surrounded by a roughly spherical halo of cold dark matter that extends well beyond the visible stellar distribution. The halo deepens and deforms the gravitational potential well from its classic shape without significantly increasing electromagnetic emissions.

Combined with current estimates of baryonic cold dark matter, spiral galaxy rotation curves further support the case for non-baryonic CDM. Studies of the Milky Way indicate that MACHOs and other faint baryonic matter comprise no more than 20% of the halo mass of our galaxy [12, 261]. Assuming the Milky Way is a typical
spiral galaxy, we can extrapolate that, although they contain a greater fraction of baryons than do clusters, spiral galaxies provide further evidence of “missing mass”. Some significant amount of new and unknown matter must be present [129].

**Low Surface Brightness galaxies**

Low surface brightness (LSB) galaxies are unique systems of study that solidly confirm the existence of dark matter. An LSB galaxy is typically defined as a diffuse system whose central surface brightness in the $B$-band, $\mu_0$, is at least one magnitude fainter than the night sky, or more concisely, $\mu_0 \gtrsim 23$ mag/arcsec$^2$. Evidence suggests that lower surface brightness usually corresponds to higher $M/L$ ratios. Most LSB galaxies appear to be extremely dominated by dark matter, with large $M/L$. Thus, they are considered to be better laboratories for exploring the properties and behaviors of dark matter than are higher luminosity systems. Furthermore, they are abundant and ubiquitous. Population estimates (with very wide error bars) show that LSB galaxies may constitute as much as 90% of all galaxies [206, 207]. Many LSB galaxies are scattered across the sky as field galaxies, where they are isolated from the mergers and tidal forces found in groups and clusters [49, 199, 227]. Most LSB galaxies seem to be disks, including the dim dwarf galaxies that are satellites within the halo of the Milky Way. For these, velocity profiles and mass measurements may be obtained from rotation curves in the same way as for the brighter spiral galaxies discussed above [129, 208, 269]. Even when there is no discernible rotation, a mass profile may still be
derived by using the virial theorem to relate the observed velocity dispersion to the velocity profile [68, 114, 165, 166, 247, 254]. Such studies show that LSB galaxies are dominated by non-luminous matter even at small radii, and that the mass of some of them is more than 95% dark [48, 50, 87, 88, 89, 215].

**Large Scale Structure**

Observed large scale structures dictate that not only must there be large proportions of non-baryonic matter in the universe, but also that this dark matter must be cold. If hot (relativistic) dark matter had dominated the overdense regions in the early universe, free-streaming would have allowed it to smooth out density perturbations on small scales. In this case, large structures would form earliest, and smaller structures would derive only from subsequent fragmentation, called “top-down” formation. Hence, there should be no small-scale structure at high redshifts. However, galaxies and QSOs are already quite clearly present at $z \geq 6.4$ [185, 277], and more recently a gamma ray burst has been detected from a star as distant as $z \sim 8.2$ [258]. Thus a large fraction of the matter density must be non-relativistic (cold) in order to attain the hierarchical—that is, bottom-up—structure formation observed.

**CMBR predictions**

The cosmic microwave background radiation (CMBR)—and more specifically, its anisotropies in temperature and polarization—is a rich source of information about
Figure 2.7: A foreground-reduced Internal Linear Combination (ILC) map of the Cosmic Microwave Background Radiation anisotropy, based on the five-year WMAP data. From Ref. [138].

cosmic evolution. In the primordial plasma, gravity, pressure, diffusion and the universal expansion all influenced the matter and energy densities. A fluid dynamical treatment, in general, admits oscillatory as well as growing or decaying solutions to perturbations in the density field. For perturbations in the baryon density smaller than the Silk damping scale,‡ photons have a sufficiently large mean free path to diffuse out of the overdensity [246]. When radiation decouples from baryonic matter at recombination, oscillations that are attaining maximum or minimum amplitudes at the sound horizon are frozen in as baryon over- or underdensities and as temperature inhomogeneities of the now free-streaming radiation. The photons of the CMBR originate on this last scattering surface. From any single vantage point (like one on Earth), the background inhomogeneities look like angular anisotropies on the sky, and

‡Silk damping arises from a net efflux of photons out of hotter, overdense regions, smoothing out anisotropies in both the baryonic and radiative components. The Silk scale is determined by the mean free path of photons at that density, and the Silk mass is defined as the mass enclosed on the Silk scale.
may be described in terms of spherical harmonics. As the universe has aged, these primary anisotropies have been modified by secondary processes, such as reionization, and the Sunyaev-Zel’dovich (refer to Subsection 2.4.2) and integrated Sachs-Wolfe effects (for a review, see [99]). The power at each angular scale depends on the exact value of the cosmic parameters that regulate these processes, specifically $\Omega_b$ and $\Omega_{DM}$ among others. Clearly, if baryons constitute the only matter in the universe, then Silk damping requires that only large-scale perturbations ($M \gtrsim 10^{12} M_\odot$) survive. From measurements of the power spectrum of the CMB there is overwhelming evidence that the primary matter component cannot be baryonic. Some form of dark matter is required to supply the density fluctuations on smaller scales that survive beyond

Figure 2.8: The WMAP three-year power spectrum (in black) compared to other measurements of the CMB angular power spectrum, including Boomerang [148], ACBAR [164], CBI [221], and VSA [92]. From Ref. [137].
decoupling (see Figure 2.8). Further, CDM particles in particular are necessary to match the evidence provided by large scale structure as a function of redshift. Recently, WMAP has published its seventh-year results, achieving significantly improved constraints from the third acoustic peak [168]. The power in the even-numbered peaks compared to that in the odd-numbered peaks reflects the ratio of baryonic to dark matter densities. The latest CMB measurements by WMAP constrain the baryonic mass to $\Omega_b = 0.0449 \pm 0.0028$ and the dark matter to $\Omega_{DM} \approx 0.222 \pm 0.026$ [168].

**Dark galaxies**

According to cold dark matter models, and the prediction of hierarchical structure formation, we expect a vast number of galaxies to populate the universe on small scales. This is termed the satellite problem because these galaxies should be extremely numerous as satellites of our own galaxy, and have been observed only in very small numbers. [159, 201]. However, evidence points to greater baryon loss with decreasing mass [81, 188], supporting the hypothesis that these smaller galaxies are, to a large degree, dark. A dark galaxy is defined as a galaxy that consists of the dark matter halo and, for the purposes of observations from earth, effectively no stars. Baryons inhabit the dark halo only as a diffuse cloud of hydrogen gas. Though by nature difficult to detect, technological improvements in recent years have placed some such objects within observational reach through radio detection. In 2005, Minchin et al. reported to have found a possible dark galaxy in the Virgo cluster (Figure 2.9) [194].
The object was first detected at Jodrell Bank Observatory by the VIRGOHI neutral hydrogen survey [84]. There had been prior claims of HI gas clouds as potential indicators of dark galaxy candidates, but each of these later proved to have a faint optical counterpart or in fact to be associated with nearby visible galaxies (see, for example, Refs. [52, 115, 156, 190, 229, 238, 239]). Continuing observations of this newest candidate, dubbed VIRGOHI 21, so far have been unable to rule it out as the first instance of a dark galaxy [47, 97, 125, 153, 195]. However, data points along the presumed rotation curve tend to be sparse and bunched, so some debate remains as to whether they do indeed reflect a rotation curve. More complete observations may be able to settle the question. If the data represent a true rotation curve, VIRGOHI 21 has a mass of approximately $10^{11} M_\odot$ (around a factor of ten less than the Milky Way). Indeed, to alleviate the satellite problem, we need objects considerably less
massive than this, but cold dark matter models do not exclude wholly dark galaxies of considerable size [85]. The accumulating evidence of VIRGOHI 21 as a large dark galaxy argues for the possible existence of numerous dark subhalos as well, that are too small to have been detected thus far. Additional solid evidence of dark galaxies may eventually support the dominance of a cold dark matter component in most structures.

The Bullet Cluster

Some of the most compelling evidence of dark matter comes from the study of the Bullet Cluster (1E 0657-56) that was published in September 2006 by Clowe et al. [71]. Clowe and his collaborators charted the gravitational potential of this system of two colliding galaxy clusters via weak lensing measurements. The collision is taking place almost directly perpendicular to our line of sight (i.e. in the “plane” of the sky), affording a particularly useful view. Studies have shown that most of the baryonic matter in galaxy clusters resides in the diffuse intracluster gas, and relatively little comprises the stellar and galactic populations (see David et al. [83] and references therein). Hence, without collisionless dark matter, we would expect gravitational lensing to show potential wells that approximately coincide with the densest x-ray emissions from the hot gas. This is the scenario predicted by theories of modified gravity (Modified Newtonian Dynamics, [192]). Clowe et al. compared the centers of the lensing mass peaks to the visible light distribution [70, 71], and to the gas
Figure 2.10: The Bullet Cluster. *Top:* Color image of the merging cluster 1E 0657-558. *Bottom:* Chandra x-ray image of the cluster, 500 ks exposure. In both panels, green contours are the weak-lensing convergence reconstructions, with the outer contour level at $\kappa = 0.16$ and increasing in steps of 0.07. The white bar indicates 200 kpc at the distance of the cluster. From Ref. [71].

x-ray emission [182]. Their results are displayed in Figure 2.10. Because the clusters are moving supersonically with respect to one another, a shockwave in the x-ray emitting gas clearly lags behind the matter concentrations, whereas the gravitational potential more closely follows the visible cluster centers. Such a correlation of the gravitational potential with the visible light undermines theories of modified gravity, and dramatically supports cold dark matter as the primary mass component on cluster scales (but see [18] for a dissenting view).
2.3.2 Front-runners for CDM

Many particles have been proposed to explain the cold dark matter component. There are a few plausible extensions to the Standard Model of particle physics that predict such candidates, but so far, these particles remain hypothetical. A description of most of them is beyond the scope of this thesis. Below, we discuss the most promising CDM candidates, which include the axion and Weakly-Interacting Massive Particles (WIMPs) in general, with particular consideration of the neutralino.

The Axion

The axion was initially proposed to explain the lack of CP violation in strong interactions [210, 211]. The axion has a well-understood profile as it is currently postulated. Today, it would have a small mass in the range \( m_a \sim 10^{-6} - 10^{-4} \) eV, but would have been massless in the early universe above what is called the Peccei-Quinn symmetry breaking scale [274, 276]. According to current theory, axions would be extremely numerous and very weakly coupled. As such, they effectively would never have been in thermal equilibrium; they decoupled long before matter-radiation equality. As bosons, they would form a very dense Bose condensate after the QCD phase transition. These predicted properties make axions a natural candidate for cold dark matter.

Various experiments have gradually been ruling out regions of axion parameter space. Only recently, however, have several experiments begun probing the regions
considered most likely to uncover the axion. The searches underway almost all use a detection mechanism based on the Primakoff effect or its inverse. The inverse Primakoff process generates photons when a pseudoscalar, like the axion, scatters off charged particles in a strong electromagnetic field. Solar searches, such as CAST [19, 4], and the Tokyo Axion Helioscope [144], are sensitive to the x-rays expected from axions in the solar plasma interacting with the massive solar magnetic fields. Other experiments use a strong magnetic field to create microwave radiation from the axions predicted to be in the galactic halo as relics of the Big Bang (ADMX [22, 54, 98]). Still other experiments use both the Primakoff process and its inverse to regenerate photons. These methods create axions from a laser traveling through a strong magnetic field parallel to the beam’s polarization direction. The photon beam is then physically obstructed by a wall, and another strong magnetic field on the other side is applied to regenerate photons from the presumably pure axion beam passing through the wall [63]. Other experiments measure the induced rotation (ellipticity) of a linearly polarized laser beam when it passes through a strong transverse magnetic field (PVLAS [283], BMV [225], GammeV [69], OSQAR [217]). Of all of these experiments, only early PVLAS experiments detected a possible axion signal. More recently, the PVLAS collaboration attributed the earlier result to instrumental artifacts [284]. Figure 2.11 shows regions in the plane of axion-photon-photon coupling vs. the axion mass that have been excluded by these and other experiments.
Figure 2.11: Axion exclusion limits in the plane spanned by the mass of the axion $m_{\text{axion}}$ and the axion-photon-photon coupling constant $g_{a\gamma\gamma}$. The diagonal yellow region (Axion Models) and the solid diagonal line (KSVZ E/N=0) indicate, respectively, the regions predicted by most invisible axion models and by the KSVZ model in particular. The cosmological hot dark matter constraint is shown by the short vertical line (HDM) within this region [36, 123, 124], and the astrophysical limit from studies of stars on the horizontal branch by the horizontal dashed orange line (HB Stars) [218, 219, 220]. The telescope search for $a \rightarrow \gamma\gamma$ decays reported in Ref. [118] excludes the orange region (Telescope), and microwave cavity axion searches exclude the gray regions (Microwave Cavity) [22, 33, 98]. Solar axion searches exclude the regions above the blue lines (upper dotted: Lazarus, et al., [170]; lower dotted: Tokyo Helioscope [196]; and solid: CAST [14, 228, 286]. The cyan region (Bragg Reflection) is excluded by searches for axion conversion in the Coulomb field of nuclei in a crystal lattice [25, 39, 202], and the green region (Laser Experiments) is excluded by searches for a “light shining through a wall” event [63]. From Ref. [253].
Weakly-Interacting Massive Particles

Besides the axion, a whole category of particles has been proposed to satisfy the requirements of cold dark matter. Generally named weakly-interacting massive particles, or WIMPs, these particles also require extensions of the Standard Model of particle physics. The first key property of WIMPs is that they interact with baryonic matter only via the weak force. This means that they are electromagnetically neutral and have no color. The second key property is that they decouple from thermal processes in the early universe when they are non-relativistic. When this is true, there is a period between matter-radiation equality and recombination during which the density perturbations of the WIMPs can grow compared to those of the baryon-photon plasma. These overdense regions seed the bottom-up hierarchical structure formation when at decoupling the baryons become free to fall into the resulting potential wells.

Of the WIMP candidates proposed so far, the most popular have been inspired by the idea of supersymmetry (SUSY), in which every particle in the Standard Model has a supersymmetric partner. It is postulated that SUSY particles carry a quantum number called $R$-parity, which is commonly assumed to be conserved. Neutralinos are the proposed mixed eigenstates of the supersymmetric partners of the neutral gauge bosons ($\gamma$, $Z^0$, and the neutral Higgs particle, $H^0$). In many SUSY models, including the minimal supersymmetric standard model (MSSM), the lightest of these neutral eigenstates is also the lightest supersymmetric particle (LSP) of the model. If $R$-parity is conserved, then the LSP is the final stable product of the decays of other
SUSY particles. For this reason, some consider this neutralino LSP to be the most likely candidate for CDM.

Currently, there is a variety of experiments looking for evidence of WIMPs. Many are in their second or third generations, having undergone multiple upgrades over the course of years. Experiments are classified as either direct or indirect searches according to their method of detection. Direct detection experiments seek a signal from a nucleus recoiling from an elastic collision with a WIMP. Because WIMPs interact so weakly and so infrequently, such experiments demand large target masses, extremely low backgrounds, stability over long periods of operation, and sensitivity to the small recoil energies. Most such experiments therefore are located deep underground and use cryogenic crystals (CDMSII [9], DAMA/LIBRA [40], CRESST [16, 17], EDELWEISS [59, 231], HDMS [158], KIMS [176]), superheated liquid bubble chambers (PICASSO [86], COUPP [35]), low pressure gas ionization tracks (DRIFT [61]), or noble liquid targets (XENON [15], ZEPLIN [174], DEAP/CLEAN [189, 51], WARP [60]). In contrast, indirect searches look for WIMP annihilation products emanating from the core of the earth, sun or galaxy where concentrations of WIMPs should be higher and annihilations therefore relatively more common. Some examples of these signature products and their corresponding searches include gamma rays (GLAST [200]), neutrinos (IceCube [117], ANTARES [8]), and positrons and antiprotons (PAMELA [46]). The annihilation product and the details of its detection may enable researchers to distinguish between different WIMP models. Additionally, in
Figure 2.12: Limits on the WIMP mass and interaction cross section from several current and future direct searches. Limits plotted are WARP (magenta [37]), KIMS (blue [176] and dotted blue [175]), CRESST II (cyan [167]), EDELWEISS II (light green [20]), ZEPLIN II (dark green [173]), CDMS (solid red [10] and dotted red [235]), XENON10 (green [15]), and DEAP/CLEAN (dotted black [189]). Dotted lines represent projected experimental limits. DAMA/LIBRA is the filled yellow region ([232] and references therein), and the IceCube annihilation limits (to $b\bar{b}$: upper, and to $W^+W$: lower) are the light blue regions [3]. Favored parameter spaces of various SUSY models are shown in gray (from light to dark: Baltz & Gordolo [27], Trotta et al.: 95% CL [263], Baltz & Gordolo [28], and Trotta et al.: 63% CL [263]), and by black crosses (Battaglia et al. [32]). Figure generated using the WIMP Dark Matter plotter by R. Gaitskell and J. Felinni at http://dendera.berkeley.edu/plotter.
a class of their own, collider experiments have the potential of detecting supersymmetric candidates. The DØ Tevatron experiments have already ruled out portions of the parameter space for the mass of a possible dark photon [2]. Experiments at the CERN Large Hadron Collider, such as ATLAS [24] and CMS [67], will reach energies at which a SUSY dark matter particle can either be discovered or conclusively ruled out. Figure 2.12 illustrates some of the masses and interaction cross-sections that have or will soon be probed.

2.4 Cluster Cosmology

The first attempts to measure the large scale, three-dimensional distribution of matter in the universe were based on observations of the 2D projected densities of galaxy clusters [6]. Then, for some years modeling and observations shifted to the galaxies themselves to try to constrain the distribution of the DM and its interaction profile. Recently, however, interest has returned to cosmology on cluster scales. Scientists are particularly interested in galaxy clusters because of the multiple techniques available at these scales to observe the components of the matter distribution (stars, intracluster gas, and dark matter).

As the largest gravitationally bound systems, galaxy clusters in principle offer a surprising number of tests of cosmology. In practice, however, clusters are hard to define, and even harder to identify exactly. For our purposes, a cluster is taken to have the following properties:
• More than 50 member galaxies that are not more than two magnitudes fainter than the third brightest member.

• Total mass of \( \gtrsim 10^{14} M_\odot \).

• Intracluster medium filled with hot gas (\( T \gtrsim 10^7 \text{K} \)).

• Diameter \( D \sim 2h^{-1}\text{Mpc} \).

• Most recent structures to decouple from the Hubble expansion, and virialize.

• Dominated by dark matter (mass contribution from galaxies \( \sim 5\% \), intracluster gas \( \sim 10\% \), and dark matter \( \sim 85\% \)).

• Mostly populated by early-type galaxies with low star formation rates.

Note that this last item is a matter of expediency. It arises for the simple reason that this study uses weak lensing techniques, which are relatively insensitive to higher redshift clusters where galactic star formation would still be active. So defined, these properties specifically make clusters the impressive laboratories they are for investigating dark matter, structure formation, and cosmological parameters. Many methods are therefore being used to locate clusters and identify their member galaxies.
Figure 2.13: A high resolution mosaic of the central region of the Coma cluster taken by the Hubble Space Telescope. From Ref. [65].

2.4.1 Cluster formation

Predictions of the evolution of structure depend strongly on the details of the cosmological model [120]. Although several models have been proposed in which cold dark matter does not dominate the overall mass component (including alternative theories of gravity, hot dark matter, warm dark matter, etc.), cluster observations are difficult to reconcile with these. A model with a large fraction of non-relativistic, weakly interacting matter seems necessary. The concordance model, mentioned previously with $\Omega_b \sim 0.04$, $\Omega_{\text{CDM}} \sim 0.22$, and $\Omega_\Lambda \sim 0.73$ [168], is such a model, and does a good job of explaining the basic characteristics of cluster evolution with redshift [26, 45, 271].

As introduced in Section 2.2, in a universe whose mass component is dominated
by cold dark matter, structure formation proceeds hierarchically from the bottom up. Smaller scale density perturbations have larger amplitudes at a fixed time, and therefore decouple from the Hubble expansion first. As the universe ages, larger and larger perturbations decouple and begin non-linear collapse. Adjacent smaller overdensities coalesce in the resultant potential wells. (Baryon overdensities follow dark matter after recombination. See Subsection 2.3.1.) The process determines the cluster mass function, \( n_M(M, z) \), defined as the galaxy cluster abundance with mass greater than \( M \) in a comoving volume element at redshift \( z \). Several important parameters affect this evolution. Primary among them are: 1) the rms linear mass
fluctuations on scales of $8h^{-1}\text{Mpc}$, $\sigma_8$, which constrain the amplitude of the power spectrum of the initial fluctuations; 2) the matter density fraction, $\Omega_M$; and 3) the dark energy equation of state determined by $w$ from Equation 2.5, which in turn specifies $\Omega_\Lambda$. Simulations demonstrate the dependence of cluster formation on these quantities, and continue to improve both in precision and complexity [44, 53, 134]. However, clusters have not yet been observed in sufficient numbers and with sufficient accuracy to distinguish between most numerical models. The more clusters are found and studied, the better we will be able to constrain these critical parameters.

2.4.2 Finding clusters

Early studies of galaxy clusters were, of course, optical. After Zwicky’s observations of galaxy velocities in the Coma cluster, the first definitive cluster catalog was created by George Abell from the Palomar Sky Survey photographic plates [6]. He devised a set of selection criteria—on which ours are based—and documented 4073 visibly overdense regions. However, then as now, optical identification of clusters suffers from projection effects, which can cause distant background and foreground galaxies to be counted as part of a cluster. Further, even actual proximity does not guarantee that galaxies are gravitationally bound, and including unbound objects introduces significant errors in measured cluster properties. Nonetheless, despite these drawbacks we now have assembled a considerable catalog of known clusters. Below, we discuss several of the methods used to find clusters and study their masses, such as x-ray and
Sunyaev-Zel’dovich radiation from intracluster gas, velocity dispersions of member galaxies, galaxy infall, and strong and weak gravitational lensing.

**X-ray measurements**

X-ray surveys detect emissions directly from the hot gas that accumulates in the deep gravitational well of a cluster. The intergalactic gas was left behind from inefficient star and galaxy formation processes, or has been re-ejected from galaxies during supernova events. Hence, the majority of the baryonic mass of a cluster exists outside its galaxies. The gas is compressed and heated by the cluster’s gravitational potential, such that it emits a bremsstrahlung spectrum as a function of its temperature. Because accurate luminosity and temperature profiles can be collected with long exposures, this method allows for “precision” measurements, with low backgrounds and high signal-to-noise. Also, by using x-ray emissions, cluster identification is much less sensitive to projection effects, since individual galaxies are typically not strong x-ray sources (although measurements are sometimes contaminated by active galactic nuclei (AGN), which do emit strongly in x-ray). If the gas were completely isothermal, in principle its temperature would yield a simple way to measure the cluster mass, however spectrum measurements require more x-ray photons than are often available. In addition, the intracluster gas may be far from isothermal. X-ray measurements are sensitive to the dynamical relaxation of the gas, and thus the presence and distribution of convection currents—which commonly occur—can severely erode
the usefulness of the mass calculations. The radial dependence of temperature and
the details of temperature inhomogeneities complicate measurements to such a degree
that often one relies on luminosity instead of spectral data. The luminosity, however,
as always requires a scaling relation to link it to the temperature, further complicating
the correlation of direct x-ray measurements to the cluster mass.

Cluster gas concentrations can also be used indirectly to reveal a cluster’s presence. The microwave spectrum of the cosmic background radiation is isotropic and
virtually that of a perfect blackbody at $T \sim 2.73$ K. CMB photons passing through
the hot gas of a cluster inverse Compton-scatter, thereby reducing the flux of CMB
photons at some frequencies and increasing it at others. This spectral distortion in the
CMBR, called the Sunyaev-Zel’dovich (SZ) effect, can be measured as an decrement
or increment in particular bandwidths [255]. The magnitude of the distortion depends
on the scattering rate which in turn depends on the density of the gas. Integrating
over the surface area of the cluster and its volume yields a total gas mass and its
Figure 2.16: Interferometric images of the Sunyaev-Zel’dovich effect for a sample of galaxy clusters over a large redshift range. From Ref. [64].
Figure 2.17: SZ component map extracted from four simulated noisy Olimpo SZ-cluster survey maps. The SZ cluster signal, subdominant at all observed frequencies, now appears clearly. From Ref. [216].

Figure 2.18: The strength of the SZ effect depends on the total mass of a cluster. The vertical axis shows the SZ decrement, which is the difference between the cosmic microwave background signal and the SZ signal. The green line is the SZ decrement for a cluster of lower mass compared to that of the red line. From Ref. [256].
(mass-weighted) temperature. Once again, however, the gas mass must be calibrated to the mass of the total baryonic and dark matter components before it can give us the total mass of the cluster. Since the SZ effect does not depend on the distance to a cluster, in principle it can detect clusters out to arbitrarily high redshift. On the other hand, this insensitivity to distance also means that it cannot easily distinguish between two different clusters that overlap in projection. The Sunyaev-Zel’dovich effect therefore requires observations from other methods to distinguish separate clusters, and to obtain a measurement of their total masses.

**Velocity dispersion measurements**

Dispersion measurements of the velocities of member galaxies are a common tool for obtaining the cluster mass. For a system that is in dynamic equilibrium, the virial theorem applies, and the mass of the cluster is related to the square of the velocity dispersion of its member galaxies along the line of sight. The method’s accuracy is determined by how well the system under study satisfies the equilibrium condition. It also clearly relies on correct identification of cluster members, and is sensitive to overlap. Furthermore, the assumption of virialization limits the application of this method to measurement of the mass within some virial radius, when the assumption applies at all.
Galaxy infall method

The galaxy infall (or caustics) method is a recently developed technique that circumvents this question of dynamic equilibrium. The method requires no knowledge of the distance to a cluster, nor any a priori criteria for identifying which galaxies are cluster members. For a region of sky thought to contain a cluster, the redshifts of all galaxies in the region are measured and plotted as a function of angular distance on the sky from the presumed cluster center. The redshift data of all the cluster members contain an offset due to the cosmic expansion proportional to the cluster’s distance from the observer. However, in addition, each object’s redshift is increased or decreased with respect the expansion redshift by an amount proportional to the component of the peculiar velocity of the galaxy (with respect to the cluster center) along the line of sight. This means there will be a maximum and minimum line-of-sight velocity component which depends on the total mass of the cluster and on the angular distance from the cluster center. This yields a funnel shape in a plot of redshift vs. angle, as illustrated in Figure 2.19. The funnel edges are termed its caustics, and reflect the congregation of galaxy redshifts along these lines due to the redshift space distortion. The mass of the cluster and its galaxy members well outside of the cluster’s virial radius are then determined from the thickness of the funnel and the detailed shape of its caustics. In some sense, this method claims the advantage of self-selecting true cluster members, however clumpiness in the cluster mass distribution, which can create large scatter in the peculiar velocities, will pollute the data.
Overlapping structures along the line of sight also add spurious signals. These contaminants can sometimes reach the point of obscuring the caustic lines due to actual member galaxies. On the other hand, its ability to probe regions definitively outside the virial radius still makes this method attractive.

**Gravitational lensing**

Strong gravitational lensing uses the distortions, magnifications and positions of multiple images of a background source to reconstruct the density profile of a cluster...
lens. It is a method that may be applied independent of the dynamical state of the cluster, and it requires no calibration of an intermediate observable; strong lensing measures mass directly if redshifts are known. However, in order to generate high distortions and multiple images from the same source, the gravitational potential well must be sufficiently deep. For this reason, strong lensing is typically only sensitive to the innermost, densest regions of a cluster-size object. It neglects the more diffuse outer regions of a cluster, which comprise a significant percentage of the total cluster mass. It typically uses space-based imaging that is free from atmospheric distortions. Further, the strong lensing signal depends on the relative distances from observer to lens and lens to source, and on the alignment of a suitable source behind the cluster. These factors naturally constrain the number of clusters to which the technique may be applied.

Weak lensing uses the same principles of image distortion as strong lensing. However, whereas strong lensing uses multiple images to determine visible differences in position, magnification and shape, weak lensing measures deformations that are tiny and require statistical methods to extract them from the noise. Like strong lensing, weak lensing results do not depend on assumptions about the dynamical state of a system. Indeed, the primary restriction on weak lensing surveys—the absence of large and bright foreground objects—is completely uncorrelated with the galaxy cluster distribution. Weak lensing has the further advantage that for their greater breadth, datasets are used from the many ground-based telescopes, and these are far
Figure 2.20: *Left:* This Hubble Space Telescope image shows several blue, loop-shaped objects that actually are multiple images of the same galaxy. They have been duplicated by the gravitational lens of the cluster of yellow, elliptical and spiral galaxies—called CL 0024+1654—near the photograph’s center. From Ref. [73]. *Right:* The reconstructed total mass density in CL 0024+1654 at redshift $z = 0.39$. From Ref. [266].

more widely available than images from the HST. Because the signal is so small, weak lensing analysis is highly susceptible to all manner of noise, so ground-based images present their own challenges. Atmospheric effects, telescope optics, CCD imperfections, and transient events in the field of view, among other factors, all degrade the signal quality. Traditionally, weak lensing observations have been strictly tailored to minimize these influences. It is interesting to ask, then, how well weak lensing methods can be applied to datasets optimized for other purposes. As instruments and experiments increasingly are designed to accommodate the widest range of scientific inquiry, the answer to this question will become increasingly important. Hence, one goal of the present work is to establish the robustness of weak lensing methods when
Figure 2.21: Simulation of the cosmic shear in a Cold Dark Matter (CDM) model. The light blue regions represent overdensities such as groups and clusters of galaxies. Dark regions correspond to voids. The line segments represent the amplitude and the direction of the lensing shear produced by these structures. From Ref. [146].

Figure 2.22: Weak lensing map (contours) superposed over an optical image of a galaxy cluster near the strongly gravitationally lensed quasar RXJ0911. From Ref. [152].
applied to general observational datasets.

In the following chapter, we introduce the general formalisms of gravitational lensing. Then we refine our focus to examine weak lensing in detail. We discuss its theory, as well as its applications. Lastly, we consider weak lensing techniques, present their strengths and limitations, review earlier and ongoing weak lensing research efforts, and place our own study within this context.
Chapter 3

Gravitational Lensing

*Gravitational lensing* occurs when the apparent sizes and shapes of distant objects are distorted as light rays pass through the gravitational potentials of intervening masses. This phenomenon was proven conclusively during the solar eclipse of 1919 when Eddington verified that the observed position of a star was consistent with the deflection predicted by General Relativity. Modern lensing observations require light from very bright, very distant sources (usually galaxies, or QSOs) to pass through an intervening region of high mass density (usually a galaxy, or a galaxy group or cluster). In the regime of strong gravitational lensing, as defined below, the image of a background source is highly distorted, usually into large arcs and multiple images. In such cases, the positions and magnification ratios of the images are used to reconstruct the original unlensed signal. Hence, the lens serves as a powerful probe of its own distribution of mass. As mentioned in Subsection 2.4.2, strong gravitational lensing
is sensitive only to very dense regions, therefore a large portion of the sky cannot be probed by it.

Younger than many other observational techniques, weak lensing has come of age in the past decade. It shares strong lensing’s ability to measure total mass without assumptions about dynamical states or luminosity, however weak lensing may be applied to the whole sky, not just to the densest regions around large visible structures. Weak lensing detects very small distortions of each background object. Without the multiple images provided by a strong lens, there is no way to distinguish the distortions of a particular object’s image from its intrinsic shape. Therefore, weak lensing relies on a statistical approach. It assumes that over a large number of intrinsically elliptical objects viewed projected on the sky, in the absence of any lensing distortions the average projected tensor ellipticity is zero. Thus, in a given area of sky, any coherent alignment in ellipticity among galaxies reflects lensing caused by intervening matter. Through observations of large numbers of these background objects, small deviations from zero average ellipticity in a region of space yield a shear field, that is, a field of the magnitudes and directions of the average projected ellipticities (see an example in Figure 3.1). The shear field is then used to reconstruct a map of the foreground matter density. Since dark matter constitutes some 85% of the mass in the universe, this is one of the best and most direct measures of its quantity and distribution. Further, weak lensing studies may be compared with optical and x-ray data to better correlate luminosity to mass in different cosmic environments.
Below, we describe how weak lensing is used.

### 3.1 Lensing Basics

In its most general form, lensing involves a mapping of points in a source plane ($S$) to points in an image. The transformation from one to the other is then related to the lensing mass. We assume that for systems of interest, the extended lensing mass distribution can be accurately represented by a thin lens approximation. In this approximation the distribution of the lensing mass along the line of sight (LOS) is small compared to the distances from observer to lens, and from lens to source. For most galaxy- and cluster-sized lenses, these conditions apply. For a thin lens,
Figure 3.2: Diagram of a lensing system showing a “thin lens” mass distribution, the optical axis, the deflection angle, and the apparent position of the distorted background source galaxy.

we thus also refer to the lensing plane \((L)\). We define the \(\hat{z}\) direction to be along the observer-lens line of sight, and the lensing plane to be the plane normal to \(\hat{z}\) containing the center of mass of the lensing distribution. In this plane, we also define a two-dimensional position vector \(\xi\). Projected onto the lensing plane, the three-dimensional mass distribution becomes a surface mass density, \(\Sigma(\xi)\). We also define the source plane normal to the \(\hat{z}\)-axis at distance \(D_S\), and the 2D position vector \(\eta\) in the source plane. The relation we ultimately seek is then a tensor transformation of the 2D image of an object of given shape and size in the source plane to one in the lens plane.

In the thin lens limit, we can approximate the deflection of the light path by two
straight lines with vector deflection angle $\hat{\alpha}$ which depends on the (vector) observation angle $\theta$, as shown in Figure 3.2. For small angles, we write down the following relationships

$$\theta D_S = \beta D_S + \hat{\alpha} D_{LS}$$

(3.1)

$$\alpha D_S = \hat{\alpha} D_{LS}$$

(3.2)

which combine to give the *lensing equation*

$$\beta = \theta - \alpha(\theta)$$

(3.3)

Here, $D_L$, $D_{LS}$, and $D_S$ are the angular diameter distances from observer to lens, from lens to source, and from observer to source, respectively. The angle $\beta$ is defined to be between the observer-lens line of sight and the observer-source line of sight in the absence of the lens; $\theta$ is the angle between the observer-lens LOS and the apparent LOS to the source image after deflection by the lens; $\hat{\alpha}$ is the deflection angle and $\alpha$ is the *reduced* deflection angle caused by the presence of the lens.

The deflection angle may be written in terms of the position vector $\xi$ as an integral over the surface mass density $\Sigma(\xi)$ in the lens plane

$$\hat{\alpha}(\xi) = \frac{4G}{c^2} \int d^2\xi' \Sigma(\xi') \frac{\xi - \xi'}{|\xi - \xi'|^2},$$

(3.4)

where again $G$ is the gravitational constant and $c$ is the speed of light in vacuum. For a circular and constant surface mass density, using Equations 3.2 and 3.4, and that $\xi = D_L \theta$ from Figure 3.2, the reduced deflection angle is given by

$$\alpha(\theta) = \frac{4\pi G D_L D_{LS}}{c^2 D_S} \Sigma(\theta).$$

(3.5)
From this expression we obtain the critical mass density,

$$\Sigma_{cr} \equiv \frac{c^2}{4\pi G} \frac{D_S}{D_LD_{LS}},$$  \hspace{1cm} (3.6)

and then $\alpha$ may be rewritten as

$$\alpha = \frac{\Sigma}{\Sigma_{cr}} \theta.$$

(3.7)

For a general lens, the quantity $\Sigma(\theta)/\Sigma_{cr}$ is called the convergence and is denoted by $\kappa(\theta)$. The convergence is a useful parameter to characterize the strength of the lens, or alternatively, the depth of the lensing potential. Whenever $\Sigma(\theta) > \Sigma_{cr}$, or $\kappa > 1$, then multiple images of background sources can result. This is the strict definition of the strong lensing regime. In this study, we are concerned instead with the case of $\kappa \ll 1$; we will return to this case in detail.

To obtain the mapping from the source to the lens plane, it is often more convenient to work with gravitational potentials, rather than mass densities. We define the two-dimensional projection $\psi(\theta)$ of the 3D lensing potential $\Phi(\theta, z)$ as

$$\psi(\theta) = \frac{2}{c^2} \frac{D_{LS}}{D_L D_S} \int dz \, \Phi(\theta, z),$$  \hspace{1cm} (3.8)

with $\theta$ defined as above. The reduced deflection angle can then be expressed in terms of $\psi$ as

$$\alpha(\theta) = \nabla_{\theta} \psi(\theta).$$  \hspace{1cm} (3.9)

Then for an arbitrary point, the lensing equation (Equation 3.3) describes a mapping that may be represented by its Jacobian matrix:

$$\mathcal{A} = \frac{\partial \beta}{\partial \theta} = \left( \delta_{ij} - \frac{\partial^2 \psi(\theta)}{\partial \theta_i \partial \theta_j} \right).$$  \hspace{1cm} (3.10)
Defining an element $\psi_{ij} \equiv \frac{\partial^2 \psi(\theta)}{\partial \theta_i \partial \theta_j}$, we write the convergence in terms of the transformation

$$\nabla_\theta^2 \psi = \psi_{11} + \psi_{22} = \text{Tr} \psi_{ij} = 2 \frac{\Sigma(\theta)}{\Sigma_{cr}} = 2 \kappa(\theta).$$

(3.11)

We also define the complex shear, $\gamma \equiv \gamma_1 + i \gamma_2$, in terms of the elements of the Jacobian

$$\gamma_1(\theta) = \frac{1}{2} (\psi_{11} - \psi_{22}) = \gamma(\theta) \cos[2 \varphi(\theta)],$$

$$\gamma_2(\theta) = \psi_{12} = \psi_{21} = \gamma(\theta) \sin[2 \varphi(\theta)].$$

The shear results in distortions of the image shape according to the modulus of the shear $|\gamma| = \sqrt{\gamma_1^2 + \gamma_2^2}$. In the last expressions, $\varphi$ is the angle between the principal axis of the shear and the coordinate axis, $\tan \varphi = \frac{\gamma_2}{\gamma_1}$; that is, $\varphi$ defines the direction of the tidal forces due to the lens distribution. The Jacobian matrix may then be reformulated in terms of the convergence and the complex shear

$$\mathcal{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} = (1 - \kappa) \mathbf{I} - |\gamma| \begin{pmatrix} \cos 2 \varphi & \sin 2 \varphi \\ \sin 2 \varphi & -\cos 2 \varphi \end{pmatrix}.$$ 

(3.12)

Although the transformation Equation 3.12 is useful conceptually, in the following sections we will rewrite it in terms of values that may be measured directly from observational data.

In the next section, the convergence and shear take on primary roles. When $\kappa \ll 1$ and $|\gamma| \ll 1$, strong distortions cannot be detected on individual objects. This is called the weak lensing regime. Distinguishing a lensing signal from the random
shapes and orientations of field galaxies becomes a matter of statistical detection, and for that, large numbers of objects and excellent systematic error correction are necessary.

3.2 Weak Lensing

By the broadest of descriptions, weak gravitational lensing measures the 2D projected shapes of tens (or hundreds) of thousands of distant galaxies, detects correlations between their alignments on the sky, and uses these values to reconstruct either the 2D surface mass distribution or the 2D gravitational potential field of the lensing matter. If redshift data are also available, such that the distances to sources can be estimated accurately, a 3D deprojection of the matter field can be obtained (see for example [126, 127, 157, 186]).

As stated previously, in the absence of intervening mass, a large random sample of galaxies should have no intrinsic correlation among their shapes and orientations in projection on the sky. By design, this assumption usually holds well for weak lensing measurements. This is because weak lensing surveys must be relatively deep—since the weak lensing signal diminishes for \( z \sim 0.5 \), and relatively wide to capture the structures of interest while guaranteeing sufficient field objects to satisfy the statistics. Therefore, most of the resolved sources are well separated in real space, and not usually subject to orientation effects that could arise during formation or tidal interactions of large scale structures. In short, we expect the average projected
ellipticity $\langle |\epsilon| \rangle \approx 0$ over our sample.

To describe the projected ellipticity of each image object, we first define the surface brightness $I(\theta)$ of the object at angular position $\theta$. Then, the center of the object may be written as

$$\bar{\theta} \equiv \frac{\int d^2 \theta I(\theta) q_I[I(\theta)] \theta}{\int d^2 \theta I(\theta) q_I[I(\theta)]}, \quad (3.13)$$

where $q_I(I)$ is an appropriate weight function. We may then define the second brightness moments of the object with respect to some conveniently chosen basis axes $i, j \in \{1, 2\}$ as

$$Q_{ij} = \frac{\int d^2 \theta I(\theta) q_I[I(\theta)] (\theta_i - \bar{\theta}_i) (\theta_j - \bar{\theta}_j)}{\int d^2 \theta I(\theta) q_I[I(\theta)]}. \quad (3.14)$$

With these relations, the size and shape may be defined in terms of the second moments:

$$\varepsilon \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}. \quad (3.15)$$

The quantity $\varepsilon$ is called the complex ellipticity. The brightness moments of the source and its image are related by the transformation matrix at position $\bar{\theta}$ as

$$Q^{(s)} = A(\bar{\theta}) Q A^T(\bar{\theta}). \quad (3.16)$$

Defining the reduced shear

$$g(\theta) \equiv \frac{\gamma(\theta)}{1 - \kappa(\theta)}, \quad (3.17)$$

the Jacobian of the lens equation (Equation 3.12) can be rewritten as

$$A = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}, \quad (3.18)$$
where again \(\{1,2\}\) label the chosen coordinate axes. Then, it can be shown that the transformation between the source and the image ellipticities does not depend on the shear and convergence independently, but rather only on the reduced shear \(g\). Specifically, we obtain

\[
\varepsilon^{(s)} = \begin{cases} 
\frac{\varepsilon - g}{1 - g^*\varepsilon} & \text{for } |g| \leq 1 \\
\frac{\varepsilon - g\varepsilon^*}{\varepsilon^* - g^*} & \text{for } |g| > 1
\end{cases},
\]

(3.19)

and the reverse transformation in which \(\varepsilon \leftrightarrow \varepsilon^{(s)}\) and \(g \leftrightarrow -g\). In the case of weak lensing, since \(\kappa \ll 1\) and \(|\gamma| \ll 1\), then also \(|g| \ll 1\) and

\[
\langle 2|\gamma| \rangle \approx g \approx \langle |\varepsilon| \rangle.
\]

(3.20)

Hence, for weak lensing, the average measured ellipticity over a field of galaxy images gives a good approximation of the local shear, which in turn enables us to reconstruct either the 2D gravitational potential or the 2D mass density of the lens.

Rather than defining the lensing parameters with respect to arbitrary Cartesian axes \(\{1,2\}\), usually we are more interested in the components of the shear and ellipticity with respect to a point on the sky (the center of a cluster, for example). When this is the case, as shown in Figure 3.3, taking \(\phi\) to specify a given direction, we define the \textit{tangential} and \textit{cross components} of the ellipticity as

\[
\varepsilon_t = -\Re[\varepsilon e^{-2i\phi}] \quad \varepsilon_x = -\Im[\varepsilon e^{-2i\phi}],
\]

(3.21)

and entirely analogous relations hold for the shear. It is important to note that the
transformation matrix $\mathcal{A}$ of Equation 3.18 and the reduced shear $g$ are invariant when $\kappa \rightarrow \kappa' = \lambda \kappa + (1 - \lambda)$ and $\gamma \rightarrow \gamma' = \lambda \gamma$ for any scalar value $\lambda$. Thus the convergence and shear may only be determined to within some value of $\lambda$. This is known as the “mass sheet degeneracy”. Effectively, it corresponds to a reduction of the projected surface mass density by a factor of $1 - \kappa$ and the addition of a sheet of uniform mass density in the lensing plane, which does not alter the transformation from the source to the image plane. In practice, this means that analyzing the shear alone is not sufficient to measure the absolute mass of a system.

There are several common ways of breaking the mass sheet degeneracy, most requiring measurement of some additional parameter. There is one means, however, that uses only the lensing data itself, and which relies on the magnification of source
galaxies by the intervening lens. The magnification, given by

\[ \mu = (\text{det } \mathbf{A})^{-1} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2}, \]  

depends on the convergence and the shear independently. Magnification by a lens has two effects. It magnifies sources such that faint galaxies can be detected that otherwise could not, and it also expands the apparent separation between the galaxies. In combination, the result is a bias that can either increase or decrease the observed number density of the source galaxies. By comparing population counts in a lensed region to those of appropriately chosen unlensed fields, one derives a scale to determine the absolute mass of a lens. For the purposes of the current study, we do not use the magnification, since we are concerned here with detecting cluster lenses, rather than with their actual mass.

### 3.3 Using Weak Lensing

Four years after his remarkable “missing mass” observations of the Coma cluster, Zwicky proposed that so-called extragalactic nebulae could provide gravitational lenses to help weigh the universe. Gravitational lensing has been maturing theoretically and observationally ever since. Throughout the 1960s, theoretical work progressed describing the deflection of light as it propagates through an inhomogeneous universe (e.g. [119, 230, 285]). In 1979, Walsh et al. observed the first multiply
imaged QSO [273], and later Lynds & Petrosian (1986) and Soucail et al. (1987) published the first evidence of strongly distorted galaxies [179, 250]. Weak gravitational lensing requires statistical analysis and reconstruction methods to obtain the mass density or the gravitational potential from the small signal in the shear. In 1990, Tyson, Wenk & Valdes published the first observations of weakly distorted galaxies around two different galaxy clusters [264]. In 1993, Kaiser & Squires proposed a method to derive the 2D projected mass distribution by linear inversion [149], and many variants of their method have been employed since [102, 104, 236, 265]. More recently, Bartelmann et al. published a maximum likelihood reconstruction method to obtain the 2D gravitational potential [31], which also has been expanded and modified in subsequent work [56, 155, 243]. These advances, coupled with refinements in telescope technologies and observational techniques over the past two decades, have allowed weak gravitational lensing to become a robust and widely used tool.

A discussion of some of the primary advantages and limitations of weak lensing techniques is in order. Since weak lensing signals are very small, it is also necessary to review the major sources of systematic error that must be corrected in order to obtain accurate shape measurements.

### 3.3.1 Reconstruction methods

Two types of techniques are most commonly used to reconstruct a potential or mass distribution from observations. The first category, usually termed *direct methods*,
derives an estimate of the shear field based on observed distortions of galaxy elliptic-
ities, and then inverse transforms this field to obtain the 2D projected surface mass
density (or the deflection potential) of the area of interest. The second category, often
referred to as indirect methods, parameterizes the convergence on a set of grid points,
and calculates a best fit of the model to the observed shear field. Both methods yield
relatively good agreement with simulated datasets. On actual observations, different
implementations perform better on some datasets than on others, according to the
details of how the technique is applied.

Direct methods

The first direct methods were pioneered by Kaiser & Squires [149]. Kaiser & Squires
(KS) calculated the convergence at each point $r$ on the sky due to the source ellip-
ticities measured at some distance $\theta$ from $r$. They showed that, in the limit of a
continuous ellipticity field, the projected mass density (proportional to the conver-
gence) could be obtained by convolving the ellipticity resulting from a point mass
with the actual field function, and then integrating over all space. Specifically,

$$\Sigma(r) = -\frac{2}{\pi} \int d^2\theta' \chi_i(\theta' - r)\varepsilon_i(\theta') \frac{\varepsilon_i(\theta')}{(\theta' - r)^2}. \quad (3.23)$$

For the case of a finite number of galaxy ellipticities, KS rewrote this as a sum over
the galaxies divided by the surface number density of galaxies. The problem with
this method is that for small radii, the integral diverges in the presence of random
shape noise. KS replaced Equation 3.23 with

\[ \Sigma(r) = \int d^2 \theta' W(\theta' - r) \chi_i(\theta' - r) \varepsilon_i(\theta') , \]  

(3.24)

introducing a weight function of the form

\[ W(\theta) = \int \frac{d^2 k}{(2\pi)^2} T(k) J_2(k\theta) , \]  

(3.25)

where \( J_2 \) is the second-order Bessel function of the first kind, and the integral is taken over the Fourier modes \( k \). The transfer function \( T(k) \) is chosen to be, for example, a two-dimensional Gaussian filter, \( T(k) = \exp(-k^2 \Theta^2/2) \). Such a weight, or “smoothing window” as described by KS, suppresses the divergence at small radii, and incorporates the necessary \( 1/\Theta^2 \) form of the mass falloff far from the point. Within the smoothing window, the noise is on the order of the rms galaxy ellipticity divided by the root of the number of galaxies. The smoothing window may be adjusted accordingly to reduce the noise but at the cost of resolution. The KS method has been extremely successful and widely applied. Many subsequent refinements of KS use different weight functions to keep noise under control, and one (Fahlman et al. discussed below [102]) uses a variation of the method to obtain a radially dependent mass profile without introducing a smoothing window at all.

Tyson & Fischer (TF) [265] improved upon Kaiser & Squires by adding a high radius cutoff to the weight function. In KS, the sum of tangential ellipticities is taken over all galaxies out to the edge of their field. However, beyond a certain radius from the chosen center, inclusion of additional galaxies adds little to the signal but
continues to add to the noise. To avoid summing over a region like this, where signal-to-noise is inherently too low, Tyson & Fischer used a weight (apodizing) function with lower and upper cutoff radii

\[ K(\theta) = \frac{1}{\theta} e^{-\theta^2/2\theta_{\text{max}}^2} \left[ 1 - e^{-\theta^2/2\theta_{\text{min}}^2} \right]. \]

(3.26)

TF applied this expression to a single cluster (Abell 1689) and therefore chose values of $\theta_{\text{max}}$ and $\theta_{\text{min}}$ appropriate to their field ($\theta_{\text{max}} = \text{half the field size and } \theta_{\text{min}} = 14''$). Note that they used this function to find the shape—and thus the center—of the cluster, rather than for the mass density itself. They found that a $1/\theta$ dependence gave higher signal-to-noise than the $1/\theta^2$ necessary to approximate the mass. When applied, the $1/\theta$ enhances the detection of diffuse mass concentrations over compact ones. In a study of a single cluster, they can compensate for this distortion simply by choosing the proper parameters. This functional form would not be applicable to a field of multiple clusters such as ours. And since we are interested in maintaining a functional form directly related to the mass, we use the more conventional $1/\theta^2$ dependence. Excepting this detail, the Tyson & Fischer method resembles the one used in the present work.

Schneider applied a variant of KS to the problem of detecting previously unknown objects via a weak lensing map [236]. Schneider wrote the convergence $\kappa$ in terms of the shear. Then, he used $\kappa$ to obtain a surface mass density $m(\mathbf{r})$ at a point $\mathbf{r}$ in terms of the tangential shear and a weight, or window function, $Q(\theta)$. He writes this
formulation as

\[ m(r) = - \int d^2 \theta \gamma_i(\theta; r) Q(|\theta|), \quad (3.27) \]

\[ Q(\theta) \equiv \frac{2}{\theta^2} \int_0^\theta d\theta' \theta' w(\theta') - w(\theta), \quad (3.28) \]

where \( w(\theta) \) is another weight function. Schneider’s first aim was to find the function \( Q \) that maximizes signal-to-noise for a given mass distribution. He found that the ideal \( w \), and hence the ideal \( Q \), depends on the signal itself, which is not a priori known. However, he used these results to construct \( w \) and \( Q \) empirically. First, he required that \( w(\theta) = 0 \) and thus \( Q(\theta) = 0 \) for all \( \theta \) outside some radius \( R \). This radius he called the “aperture size” or “filter scale”. Further, he required that \( Q(\theta) = 0 \) for \( \theta < \nu_1 R \) and thus \( w(\theta) = 1 \) within this radius, and that \( w \) should behave approximately as \((1/\theta - \text{const})\) over a broad range of radii. At radius \( R \), both \( w \) and \( Q \) go to zero. Schneider proposed three functional forms for \( w \) on the intervals \([0, \nu_1 R], [\nu_1 R, \nu_2 R], \) and \([\nu_2 R, R] \). These functions for \( w(\theta) \) are characterized by five parameters \((a, b, c, \nu_1 \) and \( \nu_2 \)). Integration yields corresponding expressions for \( Q(\theta) \). Schneider used this \( Q \) in an expression for the signal-to-noise, which he then maximized by choosing the most appropriate values for his five parameters. For this maximally efficient weight function, the aperture size \( R \) determines the region over which the galaxy ellipticities are summed. At each point over a field of galaxies, Schneider applied the same aperture radius \( R \) and used the shear and \( Q \) to obtain the surface mass density at that location. For points within \( 2R \) of each other, some of the same galaxies contribute to the calculated mass densities of both points, such that
each value is not an independent measure. None the less, this method still yielded mass values that are less correlated with the values at other points than in most other KS implementations.

Fischer & Tyson [104] used a mass reconstruction taken directly from Seitz & Schneider [242] to measure the mass distribution of cluster RXJ 1347.5-1145, which is the most luminous x-ray cluster known. They defined the convergence at a field position \( r \) in terms of the tangential ellipticity \( \varepsilon_i \) of the \( i \)-th galaxy, and \( W(\theta; s) \), a function of the angular distance \( \theta \) from \( r \) and the single smoothing scale \( s \):

\[
\kappa(r) = -\frac{1}{n\pi} \sum_{i=1}^{N} \frac{W(\theta; s)\varepsilon_i(r)}{\theta^2},
\]

\[
W(\theta; s) = 1 - \left( 1 + \frac{\theta^2}{2s^2} \right) e^{-\theta^2/2s^2}.
\]

The argument in favor of this construction was that a single smoothing scale seems more “natural” than the contrivance of introducing several different parameters. Although true in some sense, such a restriction limits the usefulness of the weighting functions available. Indeed, an argument can be made that more naturalness is not particularly important in light of the inherent artificiality of the weight function itself.

Fahlman et al. [102] modified the Kaiser & Squires method and used it to analyze the mass of cluster MS 1224. They used KS to first locate the centroid of their mass distribution. Having the center of the cluster, they then developed a KS-like method, but for which no weight function was required at all. For \( r \), the distance from the centroid, they defined a one-dimensional function of the radius, called the densitometry \( \zeta(r) \), as the average mass density of the disk inside radius \( r' \), minus the
average mass density of an annulus of inner radius \( r' \) and outer radius \( r_{\text{max}} \). Their choice of \( r_{\text{max}} \) was dictated by the properties of the cluster and their data field. They then observed how the densitometry changed as the radius \( r' \) took on values between 0 and \( r_{\text{max}} \). The densitometry provided a lower bound on the projected mass density, and could be related to the tangential shear by

\[
\bar{\sigma}(r') > \zeta(r') = \frac{1}{(1 - r^2/r_{\text{max}}^2)} \int_{r'}^{r_{\text{max}}} \gamma_t d(\ln r).
\]  

(3.31)

They then compared the values of the densitometry for different \( r' \) to the analogous profile due to an spherical isothermal distribution of known mass. From this comparison they obtained an estimate of the mass of the cluster. The method of Fahlman et al. has the interesting advantage of no artificially introduced weight function or smoothing scale. However, the annulus for a given \( r' \) is completely contained within the annuli for all smaller \( r' \). Thus, the densitometry at different \( r' \) is even more highly correlated than is the convergence found by standard direct methods.

The reconstruction method used in this thesis is a direct method most closely related to that of Tyson & Fischer [265] described above. We are concerned with the ability of our analyses to detect clusters, rather than with measuring the absolute mass of the clusters once they have been found. Thus, we restrict ourselves to calculating the convergence from the tangential ellipticities, and represent only relative overdensities in our “mass” maps. Summing over the tangential ellipticities of \( N \)
galaxies in a field, the convergence is given by

$$\kappa(\theta) = \frac{1}{N} \sum_{\text{field gal}} \frac{\epsilon_t}{r^2} e^{-r/r_{\text{out}}} \left( 1 - e^{-r/r_{\text{in}}} \right). \quad (3.32)$$

Here, $\epsilon_t$ is the tangential ellipticity of the galaxy whose centroid is $r$ away from our point $\theta$. (We have exchanged the roles of $r$ and $\theta$.) We multiply $\epsilon_t$ by inverse exponential functions to suppress contributions from galaxies at small and large $r$. This algorithm is used by the flatmap program discussed in Subsection 4.2.3.

**Indirect methods**

Although direct methods have the advantages of simplicity and speed, they are limited in a handful of critical ways. Their most extreme failing is arguably the difficulty of incorporating additional information into the mass reconstruction. Magnification information in particular is often available to break the mass sheet degeneracy. At other times, the presence of strong lensing effects, in principle, could be used to more fully constrain the lens distribution. However, there is no satisfactory way to include them in the direct formalism. Another unattractive aspect of direct methods is the somewhat subjective weight functions (smoothing scales) discussed above, for which there is no very well-defined measure of a good choice. Indirect methods address several of the weaknesses of the direct methods.

Indirect methods, also called Maximum Likelihood (ML) or inverse methods, have
been used for decades in other fields of study, for example, optics, image reconstruction, signal processing, etc. Much of the theory and implementation of these techniques in astronomy is based on a huge body of work coming to us from other fields.

The basic premise is to start with a parameterized “generic” function (the deflection potential or the convergence in our case) defined at each point in a grid. Also necessary is a measured or simulated field of objects and their properties of interest, such as ellipticity or shear. Then a minimum $\chi^2$ fit is performed to find the parameters that yield the best match to the measured data. Most indirect methods have thus far been applied to relatively small real or simulated data fields (on the order of single cluster sizes). The majority choose to model the deflection potential $\psi$, primarily because fitting of $\psi$ is local; it depends only on adjacent pixels. In contrast, a few researchers instead have modeled the convergence directly. This requires considerably more computing power since the fit of the convergence at any point depends on the fit at all other points both on and off the grid. Although in fitting the potential some resolution is lost when taking the derivative to obtain the convergence, fitting the convergence directly is extremely inefficient by comparison and yields correlated values. All ML methods share a characteristic ease with which additional constraints can be imposed. For example, to fit both the shear and the magnification, there are simply two $\chi^2$ terms ($\chi^2_\gamma$ and $\chi^2_\mu$) to be minimized simultaneously.

In theory, continued iteration of the fitting procedure should bring a function closer and closer to its ideal form. However, in practice, we must take into account
the noise of real data. Iterations beyond some optimal point begin fitting the noise, rather than the large scale structure. To prevent this, earlier implementations of ML fitting used a stopping number to limit the iterations. More recently, ML methods have all been implemented with some kind of regularization function which balances the requirements of simplicity and smoothness of the fit function with those of the best fit demanded by the data (noise and all). Although some methods have been proposed to determine the optimal stopping point in iterative methods, regularization of ML methods has largely superseded the purely iterative methods at this time.

The most commonly used regularization is the Maximum Entropy formulation. Briefly, to the typical expression for $\chi^2$ a regularization term is added that depends on the relative entropy of a given state compared to its expected value. This expected value is the prior. A uniform prior is frequently the starting point, but any knowledge about the potential or mass density function can be included in the prior to improve the result. The regularization term is added to the minimization function with a regularization coefficient that usually acts as a free parameter. For a more complete discussion, see Bridle et al. [56] and references therein.

Early implementations of indirect methods divided their fitting data into a grid, and computed the average value of, for example, the shear, in each block. Then, the function was fit to this block average value. The practice effectively imposed an arbitrary smoothing scale on the “raw” data. These studies usually performed the additional step of varying the data grid resolution and choosing the grid size
that gave the best minimum $\chi^2$ results [31, 56, 267]. Of course, by smoothing the original data in this way, information is lost. Seitz et al. [243] first introduced the idea of using the ellipticity data from individual galaxies as the matching criteria. This required an interpolation step in which the effects of the potential function had to be calculated at the off-grid galaxy locations. Although it is certainly computationally more intensive, most agree that this method is preferable to the inherent smoothing introduced by the grid-based averaging.

Simultaneous to the introduction of fitting to individual galaxies, Seitz et al. provided the first example of a variable entropic regularization term, and a so-called moving prior determined by the data itself. The effect is that in dense data regions, higher resolution will be achieved, while overfitting in regions of low galaxy populations will be avoided. Subsequent studies have further refined these ideas, while also incorporating the new galaxy fitting interpolation procedure. Marshall et al. [183] applied the formalism of an earlier paper [56] to actual and simulated data. They used a method similar to that of Seitz et al. to fit to interpolated galaxy ellipticity values. They also introduced an intrinsic correlation function (ICF) to map the stronger or weaker correlations between convergence values at different points. High correlation meant less information available from those regions, which in turn contributed to lower resolution there. Although in principle a good idea, Marshall et al. reported no significant difference in results from this treatment and the single-resolution analysis they studied. More recently, Khiabanian & Dell’Antonio achieved better success.
Figure 3.4: *Left:* Single resolution ML convergence map (pixel scale 1.5′); *Middle:* Single resolution direct method convergence map (pixel scale 1.5′); and *Right:* Multiresolution convergence map of DLS field F2. Pixel scale 0.75′ within the box, and 1.5′ elsewhere. From Ref. [155].

[155]. Like Marshall et al., they fitted to individual galaxy ellipticities, and used a regularization function that defined the resolution of a region according to the number of galaxies (and therefore the amount of information) available there. They also used a moving prior, that was initially uniform, but was the output of the last iteration for subsequent fitting. Their fit was to the shear of a 4 deg$^2$ region of the wide-field optical data of the Deep Lens Survey. Figure 3.4 shows a comparison of their final multiresolution convergence map, a single-resolution ML map and a single-resolution direct reconstruction map of the same region.

### 3.3.2 Primary sources of error

Because of its statistical nature, signal-to-noise is always a primary concern in weak lensing studies. In order to extract the signal, noise sources must be identified and mitigated carefully.
The strongest source of noise derives from the fact that we are trying to extract small elliptical distortions imposed on intrinsically elliptical objects. On average, the apparent intrinsic ellipticities of galaxies are $0.2 - 0.4$ depending on their redshift, whereas the distortion typically is on the order of $0.01$. This random noise can be combated by increased galaxy number densities.

After the intrinsic shape noise of the galaxies, anisotropies in the point spread function introduce the next important source of noise. The point spread function (PSF) describes the widening and blurring of light from a point-like source as it passes through the atmosphere and the telescope optics before reaching the detector. The PSF effectively determines the resolution achieved by the system. In weak lensing, anisotropies typical of the point spread function distort the galaxy ellipticities and must be removed. Usually, the PSF anisotropy is on the order of a few percent on one arcminute scales. As will be described in more detail in Chapter 4, the apparent size and shape of the stars in an image may be used to measure the PSF at these positions. These data points are fit to a model of the PSF, which is then used to calculate its inverse transform at all points in the field. This transformation is applied to the shape of each galaxy to effectively remove the PSF distortion, as illustrated in Figure 4.4.

Actual alignments in ellipticity should occur at some frequency over a given area of sky. These arise from background galaxies that are in fact close enough to each other to experience mutual gravitational interactions. Fortunately, such interacting
systems typically contribute much less than 1% to the measured convergence, and thus can be safely ignored in cluster searches.

There are several sources of noise, which normally plague weak lensing studies, that are not an issue in the present study. That we can ignore these particular systematics arises from the fact that we are concerned exclusively with locating clusters (i.e. finding mass overdensities) rather than with measuring the mass of these structures per se. Magnification bias occurs because the magnification of sources by weak lensing allows detection of galaxy populations at higher redshifts than would otherwise be possible with the same instrument. However, the effect is only significant for the case of high mass clusters, which are not relevant in our study. Another common source of error arises from the calibration of redshift measurements. Redshifts are needed to break the mass sheet degeneracy discussed at the end of Section 3.2, which again has no relevance for our work. Since we are not interested in the absolute mass of our structures, we have no need to normalize our mass density field.

There are other effects whose amplitudes are considerably smaller than our signal and we therefore neglect these sources of noise. Any errors in our measurements of the second moments of the ellipticities will be random in nature and far smaller than the random noise introduced by the intrinsic shapes of the source galaxies themselves. Cosmic shear amplitudes are only around 0.1% on megaparsec scales, and are expected to be small enough due to extended structures along our line of sight that our thin lens approximation holds (see Section 3.1). Without redshifts, there remains
the possibility that multiple separate small structures along the line of sight will be incorrectly interpreted as a cluster detection. We address the case of false positives in Chapter 5. Further, not only have we assumed that the lensing mass distribution can be accurately modeled by a two dimensional lensing plane, in our method we have implicitly assumed the source galaxies can be modeled on a 2D source plane. This is equivalent to asserting that they all lie at the same redshift, and that, therefore, our angular diameter distances $D_L$, $D_S$, and $D_{LS}$ and our deflection angles are well-defined. Clearly, source galaxies lay in an unknown range of redshifts, and this indeed introduces errors in our measurements. To diminish these effects in our study, we have used a deep wide-field survey, such that we have high numbers of source galaxies and we average over all detected redshifts. We anticipate that the residual effect will be small compared to the other sources of noise.

Usually, particular aspects of the design of a weak lensing study are chosen specifically to minimize certain sources of noise. For example, atmospheric dispersion is reduced when observations are made near zenith and in the $I$-band. Some instrumental effects, such as telescope flexion and imperfect guiding, are also minimized when data are taken near zenith. In addition, weak lensing studies often restrict observations to nights with above average seeing (usually $<0.9\arcsec$), and when the optics and detector arrays have been well-characterized, distortions may be corrected analytically. In this project, we specifically wanted to test the resilience of weak lensing analysis on datasets that were not optimized in these ways. Hence, all these extrinsic
factors were present in our data. Some of them were resolved through common methods like flat-fielding, zero-subtraction, and mapping of the PSF as described above. Others, however, such as the range of seeing and the multiplicity of filter bands used, remained to test our method.

3.4 Weak Lensing Studies

In this decade, weak lensing is entering its heyday. Telescopes and digital camera arrays have recently reached the level where large regions of sky may be mapped deeply for weak lensing analysis. In this context, “large” means much larger than individual cluster scales. In weak lensing, the field galaxies are the data, and the broader and deeper the field, the more complete the dataset. A number of next generation, ground- and space-based observatories have been proposed, and several are funded and underway. Weak lensing experiments that have recently come online, or soon will, are providing data from the deepest and widest fields surveyed to date.

3.4.1 Past and present

The CFHT (Canada-France-Hawaii Telescope) Legacy Survey released its first data in November 2009. This large collaboration supports four missions: a supernova survey, a deep survey, a wide synoptic survey, and a very wide shallow survey. The third, the CFHTLS-Wide, is studying large scale structure and the matter distribution in the universe using weak lensing measurements and photometric redshifts. It covers
Figure 3.5: Limiting magnitudes and coverage for several surveys. Black triangles indicate completed surveys. Red triangles indicate ongoing or future surveys. Based on Figure 1 in [90].
170 deg$^2$ total in four non-contiguous fields and five filters, reaching a maximum magnitude of $\sim 24.5$ in the $i'$-band. The camera used for the CFHTLS-Wide data was the $1 \times 1$ deg$^2$ MegaCam, which has a pixel scale of 0.187$''$ and a median seeing of 0.7$''$. Results have already been published, including photometric redshift studies [7, 57, 79], studies of cosmic shear [96, 108], and studies of the neutrino mass via weak lensing [260].* Data acquisition and analysis continues for multiple lines of research.

Another CFHT project is the second Red Sequence Cluster Survey (RCS2) [282]. Its predecessor, RCS1, used the red sequence of early-type galaxies to locate clusters over a field of 90 square degrees in two filters, and then acquired follow-up data and applied weak lensing techniques to measure the mass of these clusters as a function of redshift [139, 140, 141, 142]. The RCS2 employs the same technique, this time to image close to 1000 deg$^2$ in three filters using the MegaCam camera to a depth of $\sim 24$ mag in $R$. Its purpose is to identify clusters out to a redshift of $z \sim 1$ in order to probe dark energy and the current equation of state of the universe (Section 2.1, Equation 2.5). Preliminary results of cluster richness versus redshift have already been published [116], and weak lensing analyses from the RCS2 are expected.

The COSMOS Project is the widest survey ever taken by the Hubble Space Telescope. Hubble surveyed more than 194,000 galaxies out to redshift $z \sim 5$ over a 2 square degree equatorial field on the ACS camera [186]. One exposure by the ACS

---

*These studies use the fact that any hot dark matter component affects the observable properties of the universe differently than do CDM or dark energy. Hence, the cosmic shear, measured via weak lensing, may be used to constrain the possible total mass of the three neutrino generations.
achieves 0.05" resolution and magnitudes as faint as 27 in the F606W filter. The primary goal of COSMOS is to chart the development of large scale structure in order to constrain cosmological parameters. These data are unusual among the newest weak lensing observations due to the narrow field of view, but the unmatched depth and resolution of the images offer other interesting features. Weak lensing results soon to be published by Schrabback et al. [240] offer independent constraints on values of $\sigma_8$, $\Omega_M$, $w$, and the deceleration parameter, $q_0$. Other COSMOS studies have applied weak lensing to investigate the scaling relation of x-ray emission to cluster mass [171, 172], and to other questions.

The Garching-Bonn Deep Survey (GaBoDS) began in 2002 with weak lensing observations using the Wide-Field-Imager (WFI) on the European Southern Observatory/Max Planck Gesellschaft 2.2m telescope. It later expanded to become a data-mining project of the ESO archives to find high quality, lensing images, and now includes 15 square degrees of data from multiple observations [131, 136, 275, 281] over three well-separated regions. The data include five filters ($U$, $B$, $V$, $R$ and $I$) which range in limiting magnitude from 26.2 to 24.3. The pixel scale on the WFI is 0.238 arcsec, while data from other sources varied. Weak lensing studies of the GaBoDS images have verified cluster detections [244], investigated galaxy bias [248] and Lyman-break galaxies [135], probed cosmic shear [132], and used shear selection to locate mass concentrations and potential dark matter halos [187, 245].

The Subaru Weak-Lensing Survey is another project whose goal is to identify and
study cluster halos through shear selection. Conducted with the Subaru Telescope’s Suprime-Cam (pixel scale 0.202 arcsec), researchers have compiled a catalog of candidate halos from an area of 16.72 deg\(^2\) of convergence maps. The median seeing of the data used was 0.65”, excluding images with seeing > 0.9” [197]. Spectroscopic measurements have been used to confirm detections and to obtain reliable redshifts. Initial results have also been compared to published x-ray detections from the XMM-LSS results (X-ray Multi-mirror Mission - Newton satellite, Large Scale Structure), and indicate good agreement between the weak lensing and x-ray data [122, 198, 205].

Figure 3.5 displays the relative widths and depths of several past and future weak lensing surveys. Of primary interest here are the datasets from the NOAO Deep Wide-Field Survey (NDWFS) and the Deep Lens Survey (DLS). In the present work, we analyze anew the fields of the NDWFS by adapting the weak lensing data processing pipeline from the DLS. We discuss the NDWFS in detail in Chapter 4, and the Deep Lens Survey merits a thorough description here before we proceed.

Similar to the RCS and Subaru surveys, the Deep Lens Survey was designed to study the density of matter in the universe as a function of redshift. It comprised five separate fields of approximately 4 deg\(^2\) each: two in the Northern Hemisphere taken in Arizona on the Mayall 4m telescope, and three in the Southern Hemisphere taken on the Blanco 4m telescope at Cerro Tololo, Chile. The survey method was to produce a shear-selected catalog of galaxy clusters, and to seek any correlation between the galaxies and large-scale structure. The DLS sky regions and its observing schedules
were chosen to optimize the data for weak lensing analysis. The DLS needed both accurate shape measurements and accurate redshifts for its galaxies and clusters in order to accomplish its goals. Four filter bands were used, three of which (Harris B, V and Sloan z') were used exclusively to obtain photometric redshifts. Shape measurements were taken exclusively in the R-band, and only on nights with excellent atmospheric conditions (seeing FWHM less than 0.9 arcsec). The photometric data, less demanding of observing conditions, were taken when shape measurements could not be done. Final observations were made in 2007. Many results have been published, while analysis continues. Geller at al. compared the 2D projected matter distribution from DLS weak lensing to the 3D distributions obtained from redshifts [112, 113]. Many studies have correlated DLS shear-selected clusters with optical, spectroscopic and x-ray data [1, 278, 280]. One cluster halo was detected purely from the weak lensing data [279]. The masses as calculated using the photometric redshifts and the weak lensing data have also been compared to x-ray mass estimates [241]. These studies all used the direct reconstruction method we use in the present work on the NDWFS data to obtain the mass density maps. Yet another study applied a regularized maximum likelihood reconstruction to find halo candidates [163].

The differences between the NDWFS and DLS datasets are worth noting. The fields imaged by the DLS are five in number and are non-contiguous, whereas the combined subfields of the NDWFS provide a contiguous field of almost 9 square degrees. The DLS observing pattern was dithered far more extensively than that of
the NDWFS, resulting in much greater overlap between subfields and more complete coverage of the region, but less uniform depth overall. With very little overlap between subfields, regions along the edges of the NDWFS subfields have poor data quality, but the central regions of each subfield have far more uniform depth than do the DLS final images. As mentioned previously, the DLS observing schedule was optimized such that its shape measurements were always performed under the best possible sky conditions, and always in a single filter. In the NDWFS, we have a collection of image data of highly variable quality for each subfield, and we choose the exposures in the bandpass with the best seeing for that subfield. Further, the number of exposures taken differs greatly from subfield to subfield. In sum, science images of adjacent subfields could be in any of three filters, taken under seeing conditions from 0.77" to 1.19". This large range of seeing duplicates the conditions with which the all-sky surveys, and other upcoming wide-field surveys will have to contend. Newer surveys will not have the luxury of making observations under only the best circumstances.

3.4.2 Upcoming projects

The Large Synoptic Survey Telescope (LSST), an 8.4m, 9.6 square degree field, ground-based telescope, will survey half the sky every lunation in 6 different bands [145][178]. Observations are planned for a period of five to ten years, with 12 visits per year per region, achieving a limiting magnitude of around 24 per visit (or approximately 26 mag overall). The LSST will be among the widest, deepest, and fastest
Figure 3.6: *Left:* The NOAO Deep Wide-Field Survey. (Credit: B. Jannuzi, A. Dey, NDWFS team/NOAO/AURA/NSF.) *Right:* The Deep Lens Survey. (Credit: T. Tyson, I. Dell’Antonio, D. Wittman, DLS team/NOAO.)

instruments ever built. Its camera has a pixel scale of 0.2 arcsec, and it hopes to achieve a seeing of 0.6 – 0.7 arcsec. Its goal is to acquire data that can be used in a wide range of current astronomical research. The LSST has four main targets: the constituents of the dark universe, near earth objects, transient phenomena, and galactic structure. In pursuit of the dark matter and energy, it will accumulate quantities of very wide and very deep weak gravitational lensing data, which will be used in conjunction with precision photometric redshifts to constrain the cluster mass density with redshift. The same imaging and redshift data will also be used to track baryon acoustic oscillations, and thousands of supernovae events. These measurements will generate tighter constraints on many cosmological parameters—including on the nature of dark energy and dark matter—than previously possible. However,
one characteristic of a wide synoptic survey like the LSST is that observations will occur under all sky conditions and that the observing program will not be tailor-made for weak lensing analysis. Therefore, the ability demonstrated in the present study to extract the best possible weak lensing signal from general survey data will also be needed for the LSST data. Operations of the LSST are not slated to begin until 2015 at the earliest.

Coming online in September 2011, the Dark Energy Survey (DES) has goals similar to those of the LSST. It will use similar methods to investigate the dark energy: galaxy cluster counts, weak lensing shear tomography, supernovae, and baryon acoustic oscillations. The DES will include a wide-field survey taken from the existing Blanco 4m telescope at the Cerro Tololo Inter-American Observatory (CTIO). The new DECam camera, built at Fermilab, will collect weak lensing data over a 5000 deg$^2$ field of the South Galactic Cap [5, 29]. The camera will have a pixel scale of 0.27 arcsec, and simulations project a median delivered seeing of $\sim 0.7''$—close to that of the CTIO site itself. The DES will cover a broad field but its exposure times will be short; its best measurements will extend to around 24th magnitude (five times brighter than the NDWFS dataset used in the present study). Just as for the LSST project, weak lensing analysis will again need to be applied to data not optimized for the technique.

A similar survey in the Northern Hemisphere will be performed by the Pan-STARRS project through the Institute for Astronomy at the University of Hawaii
Observing on the first (PS1) of the four planned telescopes began in March 2009 at the LURE Observatory on Haleakala. Currently, the Pan-STARRS collaboration is completing installation of the second telescope, and working to bring PS1 to full survey status. Like the LSST and the DES, Pan-STARRS will have a multi-pronged approach, and of these only the Galaxy and Cosmology projects will include a prominent role for weak lensing. Also like the DES, Pan-STARRS will use relatively short exposures to cover approximately 6000 deg$^2$ at a depth of around 24 mag. It will also probe an ultra-deep region of around 1200 deg$^2$ to 27th magnitude in the $g$-band, and less deeply in three additional bands to obtain accurate photometric redshifts. The redshifts will be used to obtain cluster density counts, and to reconstruct the three-dimensional mass distribution from weak lensing mass maps.

As part of NASA’s Physics of the Cosmos program (previously, the Beyond Einstein project), NASA and the DOE are funding the Joint Dark Energy Mission (JDEM) [11, 270]. These agencies will be working together to construct a wide-field space telescope that will be able to measure the expansion history of the universe and its rate of change. Several revisions of the proposals have occurred, including both large- and medium-sized projects with commensurate budgets, a variety of science goals, and differing priorities regarding techniques, measurements, and instrumentation. Details are still solidifying, but it is thought that JDEM will share many of the same goals and methods as other next-generation experiments, but with the considerable advantages of space-based observations. NASA expects a launch date
for JDEM in 2015.

While details of the JDEM project remain in flux, the European Space Agency is pursuing a similar project. EUCLID, a merger in May 2008 of the Dark Universe Explorer (DUNE) and the Spectroscopic All-sky Cosmology Explorer (SPACE) projects, will be launched into space with a 1.2m Korsch telescope and a camera with 0.2 arcsec pixel scale and a field-of-view of $1.0 \times 0.5\,\text{deg}^2$. EUCLID will orbit at L2, the second Sun-Earth Lagrangian point. Over approximately five years, it intends to use weak lensing in the optical and IR bands over 20,000 square degrees of sky to probe the nature of dark energy and dark matter [74, 75, 169]. It will also be studying baryon acoustic oscillations and will perform a deeper survey covering a 40 deg$^2$ region. Unlike the projects discussed earlier, whose images are destined for a slew of different applications and analyses, EUCLID has been optimized for its two primary cosmological measurements. EUCLID is slated to launch and begin its first mission sometime in 2017.

In this chapter, we have covered the theory and practice of weak lensing studies, including the most common mass reconstruction methods and the major sources of error. We also presented some of the past and future observations most relevant to our own study. In the next chapter, we describe in detail the DLS data processing pipeline, and its application to the NDWFS dataset.
Chapter 4

The Cluster Population Study

We next examine in detail the NDWFS dataset, the DLS image processing pipeline—including generation of the mass overdensity maps from the calculated convergence fields, and our methods for error estimation.

4.1 The NOAO Deep Wide-Field Survey

The NOAO Deep Wide-Field Survey (NDWFS) is one of the most useful existing datasets for studying weak lensing systematics. This survey covers, in the optical and near-infrared, a total of 18.6 square degrees in two fields. Final optical images of the Boötes field were released in October 2004 and are available through the NOAO archive.* For this field there are also extensive data available across the electromagnetic spectrum. There is a matched x-ray catalog taken by NASA’s Chandra X-ray

*For more information about the third data release and for access to the archives, see http://www.noao.edu/noao/noaodeep/DR3/dr3-data.html.
Observatory (henceforth referred to as XBoötes) [154], as well as data in the radio (VLA FIRST [34], WSRT [91]), mid-infrared (Spitzer ISS [100]), far-infrared (Spitzer MIPS [249]), and ultraviolet wavebands (GALEX [184]).

The NDWFS and XBoötes data are notable for their width and depth in the optical, near-infrared, and x-ray regions of the spectrum. The data are being used to study large-scale structures by probing properties of high redshift systems ($z \sim 1 - 4$), for example, galaxy clusters, luminous star-forming galaxies, and red envelope galaxies, such as old ellipticals and dusty protogalaxies (see e.g. [58, 80, 133]). In addition, the AGN and Galaxy Evolution Survey (AGES [76]) has obtained redshifts for over 10,000 galaxies and quasars in the Boötes field.

Although a few other surveys have surpassed the NDWFS in one of either area or depth, the optical Boötes data simultaneously possess exceptional width and exceptional depth compared to previous surveys. This makes the data extremely attractive for a weak lensing search for clusters. By enabling us to average over a far larger projected volume of space, the greater size and depth of the NDWFS field should lead to more detections of galaxy clusters. Since weak lensing measures all matter, not just luminous mass, when systematics are well-controlled, weak lensing can provide the best calibration of other datasets that are based on measurements of luminous matter alone. Precisely because other analyses have already been done on the data of the NDWFS and its matched datasets, a comparison of the results given by each method will further constrain the systematics in all of them.
Finally, the NDWFS Boötes dataset is useful to lensing methods precisely because its data collection was not optimized for weak lensing analysis. The data of the upcoming LSST and NASA’s Physics of the Cosmos Missions (such as the Joint Dark Energy Mission, or JDEM) will be used for weak lensing studies, but—because of their broad scientific goals—they will also not be weak lensing optimized. Just like the NDWFS data, LSST data will vary significantly between filters in depth and seeing, depending on atmospheric conditions and on the particulars of the automated observing schedule. Likewise, although not subject to atmospheric distortions, the space-based JDEM observations will still benefit from the calibrations and systematic error reductions obtained in the current study.

The NDWFS optical data were taken by the MOSAIC-1 camera on the Kitt Peak National Observatory Mayall 4m telescope. The camera contains 8 separate CCD chips. In each subfield, images were taken in each of three filterbands: $B_w$, $I$ and $R$. Data collection occurred on an observing schedule determined well in advance, which resulted in variable seeing and extinction between subfields and filters. Each band for each subfield was imaged multiple times: in the $B_w$ filter for approximately 1200 seconds per exposure, and in the $I$ and $R$ filters for 600 seconds per exposure. A spatial offset between exposures of approximately 50 arcseconds insured coverage of regions that otherwise might have fallen in the gaps between the CCD arrays. An average of ten exposures were taken per subfield per filter, although this number varied widely depending on bandpass and sky conditions.
The NDWFS Boötes field consists of 27 subfields over a region approximately 3.5 degrees by 2.9 degrees. For our analysis we included 22 of the 27. Five were omitted because adequate data for these were unavailable. For each subfield, we used only the filter with the best seeing, hence we have a mosaic of contiguous subfields, but in different optical wavelengths and of different quality. Therefore, this project allowed us to test the robustness of weak lensing analyses on inhomogeneous data.

We retained the preliminary data reduction steps done by Buell Jannuzi and Arjun Dey, and other collaborators at NOAO. Specifically, the \texttt{mscred} package [268] of IRAF\textsuperscript{1} and its several tasks, such as \texttt{ccdproc}, \texttt{zerocombine}, \texttt{flatcombine}, and \texttt{rmpupil}, were used by Jannuzi and Dey to perform crosstalk correction, zero subtraction, flat fielding, and pupil removal, respectively. To this pre-processed data from Kitt Peak, we then applied the DLS procedures. The DLS pipeline is described below.

\section{4.2 The DLS Pipeline}

We used an analysis based on the data processing pipeline developed by the Deep Lens Survey collaboration, in which the Observational Cosmology and Weak Gravitational Lensing laboratory at Brown University has been very involved (DLS [280]). The DLS data was taken using the same equipment as the NDWF survey. Kitt Peak’s

\textsuperscript{1}The Image Reduction and Analysis Facility is a standard utility for astronomical data reduction and analysis developed and maintained by the NOAO.
Mayall 4m telescope using the MOSAIC-1 camera. Unlike that of the NOAO Deep Wide-Field Survey, the DLS dataset was optimized to measure the weak lensing signal in a single ($R$) band of non-contiguous fields. The DLS looked at five fields, each one smaller than the field of the NDWFS. Data processing and the first lensing analyses on the DLS are largely complete [280], and represented the state-of-the-art in weak lensing techniques at the time of publication. As such, the processing pipeline that was developed for stacking and analyzing the DLS images, with some modification, is an excellent tool to apply to the NDWFS dataset.

For weak lensing measurements, object distortions must be meticulously controlled. For this reason, we first determined the degree to which the DLS stacking procedure would improve shape measurements on the NDWFS images. The NDWFS stacking was performed by the `mscred.mscstack` task of IRAF. The DLS procedure includes steps to calculate and use the PSF to mitigate any remaining optical and atmospheric distortions throughout the field, as will be discussed in detail below. Data for three subfields were used (J1426p3456 ($I$, 0.97$''$), J1426p3531 ($I$, 1.34$''$), and J1426p3456 ($I$, 1.32$''$)). The NOAO stacked images were compared to the same images stacked using the DLS protocol. We compared object detections above noise (as measured by average spurious detections in randomized object fields) in each stack (Figure 4.1). We determined that shape distortions were significantly reduced (stars were rounder) by applying the full DLS stacking procedures to the NDWFS data. Having demonstrated that the DLS stacking algorithm gave better results, we
4.2.1 Generating the stacked image

The DLS stacking protocol consists of a sequence of tasks, performed by the dls\texttt{catalog}, dls\texttt{star}, dls\texttt{make}, and dls\texttt{combine} programs, with help from a handful of supporting scripts. To improve the signal-to-noise and provide complete coverage of the imaged region, we combine multiple individual exposures. However, several steps must be performed on each chip in each exposure to ensure that the combined stack represents an accurate image of the sky. These steps include corrections for photometry, for sky background, for distortions due to atmospheric, instrumental and projection effects, and for chance deviant or corrupt pixel values. Of the aforementioned programs, each has a different function in applying these corrections and constructing the final stacked image.

To begin preparing the original images for DLS stacking, the first task is to decompose each exposure file into its eight constituent chips. Taking the set of 7 - 25 exposures for a given subfield, the dls\texttt{catalog} program outputs a set of data files for each chip of each exposure. The next tasks are to subtract the sky background, and identify, locate, and measure all objects with adequate signal-to-noise in each chip to be used for photometric and astrometric corrections. Dls\texttt{catalog} calls the Source Extractor (SE) program [43] to do this, which generates a first catalog of all the suitable objects in the field of view. This catalog file contains the list of detected
Figure 4.1: Detections from the real catalog compared to average detections of the randomized catalogs, *top*: in the original NDWFS stack, and *bottom*: in the DLS stack.
objects with their positions, moments and magnitudes. Dlscatalog then applies the Ellipto program [42] which employs a more sophisticated algorithm to recalculate the shape measurements made by SE, as discussed in more detail below.

Source Extractor (SE) is a common utility used to identify and characterize objects in a CCD image of the sky. It examines the CCD image file pixel by pixel. First, the program performs a sky-subtraction on each chip separately (auxiliary to the NDWFS sky-subtraction which was performed on the whole exposure). It does this by making a sky map of the intensities of background (i.e. non-object) pixels as a function of position. It measures and removes any slowly varying intensity component over the field of view as an initial correction to the photometry of the chip. Next, Source Extractor applies user-input parameters to find pixel patterns that satisfy specified criteria.

Of primary importance are the detection threshold (DETECT THRESH), the minimum detection size, and the splitting condition. For each pixel, SE determines whether the pixel intensity exceeds the specified detection threshold. Then it looks for adjacent pixels that also exceed the detection threshold. When it determines that multiple contiguous pixels satisfy the detection criterion, it counts these pixels and assesses whether the group exceeds the specified minimum size. A group of contiguous pixels that passes these first two tests we define as a “primal” object. Next, SE re-examines each of these primal objects, seeking multiple intensity peaks within the object’s boundary. Source Extractor uses the input splitting criterion to deblend
such peaks into separate smaller objects. Upon separation, SE tries to remove from each the background contribution of the other(s), thereby determining the boundary of the newly created object and each pixel’s intensity. The secondary objects thus created are then evaluated against the original size and detection criteria, and those that are too small or too faint are rejected.

The objects remaining in the catalog are single-peaked detections that meet the size and intensity requirements. Source Extractor next measures and records the properties of these objects. It locates each object’s centroid, it records the object’s size, and it determines its ellipticity by calculating its second moments. It also evaluates the object’s intensity using different methods. It calculates an isophotal flux (Flux\_Iso) by summing the contributions of each pixel within the object boundary (i.e. the isophote specified by the detection threshold). It calculates a circular aperture flux (Flux\_Aper) based on one or more radii specified by the user. It records the peak flux: the maximum valued pixel within the object boundary (Flux\_Max). Finally, it calculates a total flux, using the gradient of the intensity to estimate and include the portion of the object’s signal that might have been too faint to detect outside of the object boundary. The program records these values in a catalog file, along with any error code generated while processing each object.

Although Source Extractor measures objects’ ellipticities, because the later stages of our analysis depend so tightly on these shape measurements, we use another program, Ellipto, to calculate the ellipticities in a more sophisticated manner. Ellipto
begins with the ellipticity and size of an object calculated by Source Extractor. It then tries to convolve the object’s signal with an elliptical Gaussian distribution. When the size and shape of the Gaussian matches those of the object, signal-to-noise is maximized. Ellipto adjusts the Gaussian kernel until the fit converges to a single “best” value. If the fit fails to converge, or if it converges to a value that is far from the original Source Extractor input, an error code is generated. Ellipto calculates the second moments of the Gaussian fit, and records these and any error code in the catalog file. It is the shapes, sizes, magnitudes, and positions recorded in these catalogs that the \texttt{dlsmake} and \texttt{dlscombine} programs use later to properly combine the exposures into the final stacked image as described below.

We employ Source Extractor and Ellipto in conjunction twice to generate object catalogs, and Source Extractor alone a third time during our data analysis. Each time we choose different input parameters for Source Extractor according to our specific detection goals. In the case of our initial extraction of objects from the field, Source Extractor uses a high detection threshold. This ensures that the objects detected—on which depend the calculations and corrections of the photometry, astrometry, and the point spread function—have well-defined shapes and are well-measured. Later uses of SE and Ellipto have different aims and are discussed later in the process.

A digital image taken by a CCD chip is always expected to have a certain number of “bad” pixels. Data from these pixels must be omitted from science images, because their counts are inaccurate. Such data corruption arises from many different sources.
Some pixels are bad because of electronic artifacts, such as “dead pixels”, crosstalk between adjacent pixels, or saturation effects near very bright foreground stars. Other bad pixels result from transient physical anomalies, such as satellites or cosmic rays that pass through the field of view during an exposure, or reflections or dust along the optical path of the telescope. A bad pixel mask file specifies the pixels on each chip for each exposure that should not be included in the final data stack. For bad pixels caused by the instrumentation or by saturation bleeds, we use the masks created by the NOAO team for the original data. Then we visually inspect each chip for transients, and add these to the bad pixel masks (see Figure 4.2). When the restacking procedure is performed, the pixels on a given chip specified in the bad pixel mask file are not included in the data in the final image.

Next, we calculate the corrections that will enable us to combine individual exposures correctly. We use dlsstar to display a size versus magnitude plot of all the objects in the initial catalog. Dlsstar attempts to identify the star locus for each chip automatically and displays its selections. The plots are then visually inspected and the star selection is modified by hand when necessary (see Figure 4.3). The moments of the selected stars reflect distortions of the PSF over the field of view. Dlsmake generates a 4th order polynomial fit to the spatial variations of these moments, to be used in a later step to calculate and apply the PSF correction. All the objects in the original chip catalog are compared and matched to the objects for the same chip in other exposures. Data from the matched objects are then compared to the standard
Figure 4.2: Example of the masking process, *left*: chip, and *right*: mask. Pixels that are white in the mask file are excluded when the chip is processed.
USNO catalog for that region of sky. This comparison is used to output a polynomial transformation which maps any chip coordinates onto the coordinate system of the reference catalog. Offsets of individual objects can result from inhomogeneous “stretching” by the atmosphere, telescope optics, and/or the CCD instrumentation, and resemble the effects of the PSF. Indeed, the transformations obtained from the object matching are next used to extract these effects from the inhomogeneities in the PSF, and the moments of the stars are recalculated. The transformations calculated thus also account for the projection effects inherent in any mapping of a curved surface (the sky) onto a flat plane (the CCD array). Because of the dithering of our pointings, each exposure incorporates its own projection effects on its own image plane. Having the mapping of each flat exposure onto the spherical coordinates of the reference catalog, we then use the known central point of our subfield, to reproject the sky from each exposure onto the tangent plane of the central coordinates. Dlsmake records all these corrections in various files for use in the final step of the stacking procedure.

Lastly, dlscombine is called by the dlsmake program. The dlscombine program performs the actual restacking of the images based on the corrections calculated by dlsmake as described above. Dlscombine first corrects for the PSF by convolving the shape of each object with the conjugate moments of the PSF distortion at the object’s position. Hence, an object made elliptical due only to the PSF anisotropy is made round again (see Figure 4.4). Then, dlscombine uses the calculated astrometry to
generate for each pixel of the final image, a list of contributing pixels from the various input exposures. Each input pixel may contribute to one or more output pixels in varying amounts, for example, the \((i, j)\) pixel in the input exposure may contribute 60\% of its value to the \((i', j') = (1, 1)\) pixel and 40\% of its value to the \((i', j') = (1, 2)\) pixel in the final stack. Therefore, for each output pixel, the input values will be appropriately scaled. At this point, a second scaling based on the relative photometry of each input chip is recorded for later use in the calculation of the final output pixel value and its noise. In the last stage, \texttt{dlscombine} loops over the contributing pixel values twice. On the first iteration, it uses the list of contributing pixel values and the photometric scaling to calculate the output pixel’s average value and the variance.
Figure 4.4: Objects made elliptical by anisotropies in the PSF become round when convolved with their conjugate moments. *Left:* Before PSF correction. *Right:* After PSF correction. Crosses indicate objects with ellipticities that are too small to see at this scale.
in the the values. On the second pass, it compares each input value to the average plus or minus the variance; it excludes values which fall outside this interval, and recalculates the average and the variance without these outliers. This final average is then recorded as the value for that pixel in the final stacked image, while the variance is recorded in a separate map reflecting the error in each final pixel value. This procedure is repeated for all the pixels in the final image. The output is the corrected “fits” image,‡ with controlled systematics and distortion, which provides us with a “true” representation of the sky and is ready for weak lensing analysis.

4.2.2 Detecting and measuring background galaxies

With our science images in hand, we apply Source Extractor and Ellipto to our data for the second time. This time, the detected objects and their shapes are to be used to measure the weak lensing signal in the subfield. As such, our primary concern is the completeness of the object catalog. Therefore, we use a relatively low detection threshold and a relatively small minimum detection area when calling SE. Ellipto is applied as before to improve the moments calculated by Source Extractor. The catalog that results needs cleaning to remove any remaining undesirable objects, false detections, objects polluted by adjacent bright foreground stars, and objects near to the edge of the subfield where data were sparse and the noise high. The cleaning occurs in two parts: first, by automatic filtering and then, by hand.

‡“Flexible Image Transport System”: an image format commonly used in astronomy.
The first step of cleaning is simple filtering. Depending on certain factors like bandpass and seeing, a catalog output by SE for one subfield typically consists of between 130,000 to 200,000 objects (see Figure 4.7a). Preliminary cuts are made using a utility that allows us to eliminate objects with certain undesirable values. Typically we perform four cuts. (1) The first cut omits objects that have generated a non-zero error code during processing. By requiring that our objects have no error code, we insure that we only use objects whose ellipticities are well-defined and well-fit by an elliptical Gaussian distribution. This requirement often eliminates up to half of our initial catalog. Next, we view the collection of objects in a scatter plot of size versus magnitude. Applying the filtering utility again, we next remove (2) objects whose size is below the PSF-limited star locus, (3) objects that are very large and/or bright, and (4) objects that are very faint. These simple filters provide a quick way to exclude from our catalog extended objects (such as foreground galaxies and stars), very faint and/or very small objects (below the PSF and therefore not useful), and very bright, small objects (such as distant stars). After these preliminary cuts, an example of which is shown in Figure 4.5, typically our catalogs contain 20 to 50 thousand objects.

It remains only to inspect the subfield visually to eliminate any undesirable objects not caught by our filtering. We display the surviving objects against the stacked image file (see Figure 4.7b). The objects are closely examined against several criteria: that they are well-defined, extended but small, well removed from any bright foreground
Figure 4.5: Scatter plot of cuts made on subfield J1426p3346 ($R$, 0.77″).
star or galaxy, and in regions with good signal-to-noise (i.e. sufficiently far from subfield edges). Objects that do not meet all of these criteria are removed by hand. The final catalogs contain 10 to 30 thousand objects per subfield (see Figure 4.7c), and these then are the input for the fiatmap utility, which reconstructs the weak gravitational signal from them.

### 4.2.3 Constructing the mass map

To reconstruct the mass density map from the ellipticities, we use the direct inversion mapping application fiatmap (for details on direct techniques, refer to Subsection...
Figure 4.7: Small central region of data image of J1434p3421 (I, 0.79") (green rectangle in Figure 4.6) showing the Source Extractor catalog of objects (a) before filtering and hand cleaning, (b) after automatic filtering, and (c) after both filtering and cleaning. The cleaned, filtered catalog shown in c is the input for the mass density reconstruction program fiatmap.
3.3.1. **Fiatmap** is based on the earlier application **invlens** by J. Anthony Tyson and Francisco Valdes [104, 279]. As input, the **fiatmap** program takes an inner radius $r_{\text{in}}$, an outer radius $r_{\text{out}}$, a block size $b$, and a catalog of objects and their associated ellipticities (the “fiat” file). It outputs a digital image (the “fits” file). For each pixel in the output image, for every object in the field, the object’s weighted tangential ellipticity is added to the pixel’s value. Our weight function, $e^{-r/r_{\text{out}}}(1 - e^{-r/r_{\text{in}}})$, rolls off contributions from objects nearer than $r_{\text{in}}$ and farther than $r_{\text{out}}$. The block size $b$ effectively specifies the resolution of the final map; for $b = 100$, a $100 \times 100$ block of pixels in the original sky image contributes data to one pixel in the final map. In addition, a weight file is also necessary to diminish the contributions of objects near the edges of the original image which have very low S/N. **Fiatmap** uses the empirical formula given for this purpose in Ref. [104]. Thus, the contribution of a given galaxy depends not only on its distance from the pixel being calculated, but also on the quality of the measurement of its shape. This process is repeated for each pixel in the output map, and the pixel’s value is then written to a data file. The output image is a map of intensities which are related to the mass density at each point. For each of the subfields, we tested different values for $r_{\text{in}}$, $r_{\text{out}}$, and $b$. We determined that the same parameters provided the best compromise between detailed and large scale features for all the subfields. Hence, all of our maps were generated using $r_{\text{in}} = 700$, $r_{\text{out}} = 3000$, and $b = 100$. An example for one subfield is shown in Figure 4.8.
Figure 4.8: Top: The original stacked image of subfield J1437p3532 ($Bw$, 0.84″).
Bottom: The final mass density map of this subfield after processing and analysis.
4.3 Detecting Galaxy Clusters

Each mass density map generated by the \texttt{fiatmap} program is then analyzed for mass overdensities using a detection procedure similar to the one described above for finding and measuring background galaxies. We again apply the Source Extractor program, but with new detection criteria. First, we use the same relatively low detection threshold as in our second application to ensure completeness of our detections. Since we are also interested in how completeness affects our detection rates, we also generate maps with detection thresholds ranging from very low ($0.40$) to very high ($2.00$). The minimum detection area is chosen to be relatively high; clusters are large structures ($\sim 1 \text{arcmin}^2$), and in our density maps one pixel covers a $20'' \times 20''$ region of sky. All other parameters remained as before (see Subsection 4.2.2). Source Extractor output catalogs of the positions and sizes of the cluster-like objects for each detection threshold and each subfield. These served as our final data for the cluster analysis.

In addition to the actual mass density map, we generated multiple statistically similar maps from each subfield (for a similar method, see [163]). The \texttt{fiatmap} program allows a “randomized” map to be created along with a real map. To the position of each object in the actual catalog, the size and ellipticity of a different object is assigned. Hence, each location in the original catalog is also the site of an object in the randomized catalog, but has a different object located there. Thus, any alignment between adjacent objects in the original is broken, and a collection of many such random maps provides a measure of the likelihood of spurious alignments.
occurring. Each of these maps was then analyzed according to the procedure just
described for the actual map. Comparison of the real map of the subfield to the
statistical properties of the random maps gave us a measure of our errors. In the
following chapter, we present our results. We discuss them in detail, and compare
them to those of the Chandra XBoötes survey.
Chapter 5

Results and Discussion

In both number and distribution, detections from our weak lensing analysis generally agree with other results for the Boötes field. They are also consistent with typical values of efficiency and completeness from other weak lensing surveys. In particular, our results support the evidence from surveys such as the Chandra XBoötes survey, that the Boötes field is an area of relatively low mass density, with few truly massive galaxy clusters. It seems then that weak lensing analysis has potential to be useful when applied to deep and wide surveys even of inconsistent seeing quality.

However, when identifying our objects with specific x-ray detections, we do not see the correspondence we expected between our highest projected mass density objects and the highest x-ray flux objects of the XBoötes survey. There are a few factors that might contribute to this discrepancy. One contribution may be due to the selection effects inherent in the different measurements. X-ray flux falls off with
distance as $1/d^2$ which skews x-ray detections toward structures at lower redshift. This means that the objects with highest x-ray flux may be at redshifts closer than where weak lensing is most sensitive ($z \sim 0.2 - 0.5$). Further, we expect some of our detections to be due to superpositions along the line of sight. Cluster x-ray emission is generated by bremsstrahlung from the hot dense intracluster gas. The observed x-ray spectrum depends on the gas temperature and density. Without needing to assume anything about the gas density’s relationship to the total mass density, it is clear that multiple objects distributed along the line of sight can provide a high lensing signal, while emitting relatively little x-ray flux. Certainly, some of our detections may not show x-ray counterparts because their mass concentrations are either smaller than the detection threshold of XBoötes or they are simply not x-ray bright. Other explanations might be a failure in our PSF corrections, or filter induced variations in sensitivity. These possibilities will be discussed in greater detail below. These possibilities do not, however, explain why over half the x-ray sources (23 out of 43) had no apparent weak lensing counterpart, despite what should have been sufficiently high completeness in our detections. In the following sections, we first present our error analysis, and then we offer our results in the context of the extended x-ray source catalog from the Chandra data.
5.1 Estimating the Error

We use an error analysis method similar to that of Kubo et al. [163] to estimate our signal-to-noise. We use fiatmap’s random map generator in which object ellipticities are randomly reassigned to another object’s position. We generate 500 maps for each different choice of our Source Extractor detection threshold for each subfield. An analysis of the average number of detections, of its variance over the sample of 500, and of the dependence on the detection threshold gives us our expected rate of false positive detections.

The randomized mass density maps, generated as described at the end of Section 4.3, contain no real clusters. Therefore, upon analysis with Source Extractor, any detections of mass overdensities in these maps reflect chance alignments in the background ellipticities, and give us a measure of false positive detections. In order to assess the significance of our detections in the actual maps, we need a statistically significant number of randomized maps to establish how often such chance alignments are expected. We compare the number of objects detected above a certain flux for a range of values for the Source Extractor DETECT\_THRESH parameter. As described in Subsection 4.2.1, the detection threshold specifies the necessary minimum value of each pixel contiguous within the specified detection area. (Note that the SE detection threshold is unrelated to the S/N detection threshold, which is a commonly used value and will be encountered later in Section 5.2.) We allow the detection threshold to range from 0.40 to 2.00 in increments of 0.10, with the additional value
of 0.95. Detection rates are compared for sets of 100, 500, 1000, 2000, and 5000 randomized maps for subfield J1434p3421 ($I, 0.79''$). A plot of these detections is shown in Figure 5.1. We see that the average number of detections as a function of DETECT\_THRESH converges for sets of 500 random maps or more. Further, we verify that our random maps are truly random by examining the number of detections in different 100-map subsets of the group of 5000. We find that detection rates are virtually identical for any group of 100 maps selected, as displayed in Figure 5.2. As another check, at the detection threshold of 0.95, we plot the detections in our
Figure 5.2: Average number of objects detected per map above a given flux threshold for different sets of 100 random maps.

real maps, and the average detections plus or minus 1σ errors for the corresponding randomized maps, against the seeing of the subfield. Figure 5.3 shows this result; colors indicate the bandpass of the data used for that subfield. No correlation is found between counts in actual and randomized maps, nor does there seem to be any dependence on seeing. In Figure 5.4, we show an example of a plot displaying the average number of spurious detections from the randomized maps (in blue) for subfield J1437p3532 ($Bw, 0.84''$), and the detections from the actual data map (in red). The mean squared variance of chance detections over the sample of random maps,
Figure 5.3: Number of detections in actual data map (solid circle) and the average number of detections from the randomized maps (open circle) for each subfield as a function of seeing. Marker color indicates the filter used for that subfield: blue = $Bw$-band, red = $R$-band, and green = $I$-band. Error bars represent ±1σ.

As shown in Figure 5.5, this method allows us to estimate the fraction of spurious detections in each subfield, without needing a specific way to measure the noise itself. This is highly advantageous when working with a dataset of quality that is expected to vary from subfield to subfield. However, the disadvantage is that the sources of noise cannot be identified, quantified and mitigated individually. Examples include possible variations in the redshift distribution of the background galaxies, intrinsic galaxy alignments, large scale structures that
Figure 5.4: Sample plot of detections from actual map (red) and of average spurious detections from the 500 random maps (blue) for subfield J1437p3532 (Bw, 0.84″). This plot shows two detections above noise levels in the actual data.
Figure 5.5: Variance in number of objects detected per map above a given flux threshold for sets of $N$ maps, $N=\{100, 500, 1000, 2000, 5000\}$.

defeat our thin lens approximation, and variations in the PSF on scales smaller than our applied corrections. Additional error analysis indeed will be a necessary future step in order to tease out a complete interpretation of our results.

In Figure 5.6, we plot the number of background galaxies used to create the final mass maps for each subfield. This is the number of sources from the original stacked image which remain after the filtering and cleaning steps. We see the expected decrease in number counts as seeing increases. Note that the first three subfields (seeing 0.77", 0.77", and 0.79") have fewer sources than would have been expected
Figure 5.6: Number of source galaxies used in mass map construction as a function of seeing for the three colorbands. We see a general trend toward fewer galaxies with worse seeing for all bands as expected. Lower counts in the first three subfields (seeing 0.77″, 0.77″, and 0.79″) are attributable to an initial tendency to overfilter potential faint galaxies during the final catalog generation. The on-average higher source detection rates in $Bw$ are discussed in the text.

from the trend apparent in the rest of the points in Figure 5.6. We attribute this to more stringent screening of borderline galaxy candidates in these three fields, which were processed first. The plot also manifests an interesting characteristic in that the subfields in $Bw$ on average seem to have more source galaxies going into their map reconstruction step. Although $Bw$ exposures were longer than the others (usually 1200 s compared to 600 s for $I$ and $R$), the $Bw$ images also had the lowest average
number of exposures per stack ($Bw$ subfields averaged 8.1 exposures per stack, $R$ subfields averaged 12.8 per stack, and $I$ subfields averaged 20 per stack). We therefore cannot readily link the higher source galaxy count to an effect of our data-taking or processing. Further, we see no evidence in our x-ray matched data to suggest that we are detecting clusters more accurately in the $Bw$ subfields. Since this filter is more sensitive to bluer wavelengths, one might have expected it to be more sensitive to clusters at the low redshifts most easily detected in x-ray data. However, there is no significant difference between the numbers or masses of clusters detected in $Bw$ subfields compared to others, nor do we seem to have matched x-ray objects in those subfields more effectively (see results in Figures 5.9 and 5.10). In Figure 5.7, we display a representative sample of plots for different subfields as a function of the Source Extractor detection threshold value (0.40, 0.50, 0.60,... 2.00, 0.95), showing the actual detections, the average number of detections in 500 randomized maps, and the estimated error in each subfield. Also plotted are the detections from one arbitrarily selected map from the 500 randomized mass maps for each subfield. The plots reflect that only around 30% of our subfields show evidence of detection peaks conclusively above noise levels.

### 5.2 Lensing vs. X-ray

We compare our detection results with those of the Chandra XBoötes catalog of extended sources displayed in Figure 5.8 [154]. A list of our detection positions,
Figure 5.7: Plots of detections from real map (solid red circles), average detections over 500 randomized maps with error bars (1σ), and detections from a randomly selected randomized map (open green circles) as a function of increasing DETECT\_THRESH for six representative subfields.
fluxes, and projected areas is provided in the appendix (Table A.1). Figure 5.9 displays both the NDWFS and XBoötes detections. Since redshifts for the XBoötes extended sources are not yet available, we instead use the published x-ray flux as a proxy (albeit an imperfect one). In the figure, the relative sizes of the NDWFS circles represent the peak flux of the object, or in other words, the pixel reflecting the highest concentration of mass. The relative sizes of the x-ray circles represent the flux of the object, $S_{14}$, in units of $10^{-14}$ erg/(cm s). We made no attempt to calibrate the two sets, but the circles provide an easy means to compare relative mass density or intensity within each dataset, respectively. A common criterion used in the literature for two objects to match is that their centroids lie within $\sim 3'$ [113, 163]. Using this criterion, we find nine matches between our data and the Chandra x-ray data. However, for these purported matches, we find little correspondence between the measured relative mass density of the object and its x-ray flux. In fact, we see that the second brightest object in the x-ray catalog corresponds to the third least massive of our matched objects. Relaxing our matching criterion to include objects within $\sim 6'$ of each other (see Ref. [163]), we increase our matches to 19. Now, we match the brightest source in the x-ray catalog with the most massive of our matched objects. Unfortunately, the most massive of our matched objects is only as massive as at least eight of our other detected “clusters” (see Table A.1). Hence, it is not clear without further analysis whether the match of the brightest x-ray object is significant in this case. In Figure 5.10, we display the matched x-ray and weak lensing object
Figure 5.8: Detections from the Chandra XBoötes survey. Smoothed (≈ 60′′), processed image showing locations of 43 extended sources. The circles marking clusters are 10 times the size of the detected source. From Ref. [154].

A feature of the catalog in Figure 5.9 that is worthy of comment is our choice to use the Source Extractor detection threshold of 0.95. In the approximately 7.6 square degrees in our 22 subfields, we obtain 127 detections, or ∼ 16.7 peaks/deg². This is a relatively low detection threshold compared to what is commonly seen in the literature. Our intention was to tolerate higher rates of false positives in favor of
Figure 5.9: Field positions of NDWFS cluster “detections” (red circles), and XBoötes extended source detections (blue circles) [154]. The size of the circle represents the relative maximum flux for the NDWFS data, and the relative flux $S_{14}$ in units of $10^{-14}$ erg/(cm s) for the x-ray data.
Figure 5.10: Field positions of matched objects (within $\sim 6'$ of each other). Symbols and colors as defined in Figure 5.9.
completeness. The advantage of using as our error the rms variance in the detection counts of our randomized maps is that we do not estimate the signal-to-noise directly for each peak. Instead, we claim that a detection rate 1σ above the average random detections gives us a more definite measure of actual peak numbers. To illustrate, for a subfield map with nine detections, and an average detection rate of 6 peaks per randomized map, plus or minus 1 peak (corresponding to 1σ), we are 68% confident that two of our peaks are real, although we do not know specifically which ones.

Methods for estimating detection significance in weak lensing studies vary considerably in the literature, leading to wide ranging values of efficiency and completeness. Ray-tracing in $N$-body simulations indicates that even in the ideal limit of no intrinsic galaxy ellipticity, efficiency never exceeds 85%. In such a case, then 15% of detections will still result from projection effects along the line of sight [128]. Hamana et al. [121] predict completeness of 56 - 63% for a galaxy density that is on average somewhat higher than ours (source galaxy number density 30 arcmin$^{-1}$, average $z \sim 1$), and using a signal-to-noise detection threshold of $S/N > 4$. They show that the result is highly sensitive to redshift. In weak lensing surveys, a signal-to-noise ratio of 3.5 - 4.0 is most commonly used for selection. This has generated detection rates of 3.5 peaks/deg$^2$ [111], 6.0 peaks/deg$^2$ [122, 198], 8.3 peaks/deg$^2$ [233], and 11 - 15 peaks/deg$^2$ [130], to mention just a few. In another comparison of weak lensing to x-ray data, Dietrich et al. [94] show that while one identified x-ray underluminous
cluster (bright in optical, faint in x-ray) is very massive (A 315), another underluminous x-ray cluster has no discernible lensing signal at all (A 1456). We expected our low threshold to ensure more complete matching of the XBoötes detections, even at the risk of spurious signals. Although our detection rates were as expected, we need additional error analysis, and redshift data, to determine which of our individual peaks are true clusters, and why in these cases there is little correspondence between mass and x-ray luminosity.

5.3 Future Directions

Many questions remain incompletely answered about the NDWFS dataset and our application of weak lensing techniques to it. Additional analyses are required, and primary among these is further examination of the sources of error. More importantly, redshift information can clear up numerous ambiguities. For example, with spectroscopic redshift data, we will be able to identify multiple lensing structures along the line of sight at different redshifts. Lens redshifts will also enable us to better gauge whether mismatches in x-ray and WL signal strength are due to different sensitivities in different wavelengths. With photometric redshift data for background galaxies, measured variations in their redshifts will allow a more sophisticated mass reconstruction. Another interesting test will be to repeat the current analyses for the remaining bandwidths in the NDWFS subfields. With a greater range of seeing, and
more data in each color, we will be able to perform a much more revealing investigation of the relationship between cluster detections, wavelength sensitivities, and seeing. Further, our innovative application of direct mass reconstruction methods to the NDWFS data leads inevitably to the possibility of applying indirect, maximum likelihood procedures instead. The ability of ML methods to beat down noise when regularized with an appropriate smoothing function begs the question as to whether, especially in noisy data fields, the ML methods might not prove even more informative. These may provide the basis of future work.
Appendix A

Data Table
Table A.1: Cluster detections.

<table>
<thead>
<tr>
<th>Subfield</th>
<th>Filter</th>
<th>Seeing (arcsec)</th>
<th>RA (deg)</th>
<th>DEC (deg)</th>
<th>Flux Iso</th>
<th>Flux Aper</th>
<th>Flux Best</th>
<th>Flux Max</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1426p3311</td>
<td>I</td>
<td>1.18</td>
<td>216.707809</td>
<td>33.061210</td>
<td>0.180261</td>
<td>0.303640711</td>
<td>0.380119369</td>
<td>0.00706474</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.703024</td>
<td>33.159900</td>
<td>0.709197</td>
<td>1.000276348</td>
<td>0.892811961</td>
<td>0.00940494</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.706375</td>
<td>32.981818</td>
<td>0.591894</td>
<td>0.220312937</td>
<td>0.556980512</td>
<td>0.00989355</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.581113</td>
<td>33.346322</td>
<td>0.205030</td>
<td>0.114635730</td>
<td>0.325746861</td>
<td>0.00711338</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.264412</td>
<td>33.315114</td>
<td>0.146241</td>
<td>2.51189E-28</td>
<td>0.219765745</td>
<td>0.00718200</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.384541</td>
<td>33.324857</td>
<td>0.996025</td>
<td>0.730869722</td>
<td>0.987824970</td>
<td>0.00903229</td>
<td>125</td>
</tr>
<tr>
<td>1426p3346</td>
<td>R</td>
<td>0.77</td>
<td>216.423707</td>
<td>33.675356</td>
<td>2.772900</td>
<td>2.728726446</td>
<td>2.348767344</td>
<td>0.01846730</td>
<td>265</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.677986</td>
<td>33.989310</td>
<td>0.122844</td>
<td>2.51189E-28</td>
<td>0.269475943</td>
<td>0.00768223</td>
<td>19</td>
</tr>
<tr>
<td>1426p3421</td>
<td>I</td>
<td>0.99</td>
<td>216.675612</td>
<td>34.268439</td>
<td>0.303416</td>
<td>0.200705836</td>
<td>0.384875264</td>
<td>0.01212710</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.688157</td>
<td>34.536541</td>
<td>0.250208</td>
<td>0.377537417</td>
<td>0.419913657</td>
<td>0.00749570</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.742992</td>
<td>34.361746</td>
<td>0.141871</td>
<td>2.51189E-28</td>
<td>0.246581222</td>
<td>0.00871250</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.288100</td>
<td>34.490640</td>
<td>0.215189</td>
<td>0.339695354</td>
<td>0.331344678</td>
<td>0.00735328</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.363388</td>
<td>34.393615</td>
<td>0.570361</td>
<td>0.965339329</td>
<td>0.528493926</td>
<td>0.00711879</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.482816</td>
<td>34.406462</td>
<td>0.375610</td>
<td>0.335861327</td>
<td>0.515608419</td>
<td>0.00840270</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.513703</td>
<td>34.546669</td>
<td>0.185340</td>
<td>2.51189E-28</td>
<td>0.302691343</td>
<td>0.00792241</td>
<td>30</td>
</tr>
<tr>
<td>1426p3456</td>
<td>I</td>
<td>0.97</td>
<td>216.682659</td>
<td>34.784298</td>
<td>0.116849</td>
<td>0.071082059</td>
<td>0.291393594</td>
<td>0.00643732</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.482219</td>
<td>35.175218</td>
<td>0.416046</td>
<td>0.486452008</td>
<td>0.591234815</td>
<td>0.00851310</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.477700</td>
<td>34.968698</td>
<td>0.196702</td>
<td>0.519995997</td>
<td>0.440920204</td>
<td>0.00717108</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.581107</td>
<td>35.108957</td>
<td>0.093675</td>
<td>2.51189E-28</td>
<td>0.266342211</td>
<td>0.00634816</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.300493</td>
<td>34.846233</td>
<td>0.873097</td>
<td>0.810438380</td>
<td>1.134697244</td>
<td>0.01045590</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>216.392762</td>
<td>35.059206</td>
<td>0.670395</td>
<td>0.787710200</td>
<td>0.834295690</td>
<td>0.00879837</td>
<td>78</td>
</tr>
<tr>
<td>1428p3236</td>
<td>R</td>
<td>0.86</td>
<td>217.354867</td>
<td>32.713030</td>
<td>0.363466</td>
<td>0.682213014</td>
<td>0.918756502</td>
<td>0.00658564</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>217.276366</td>
<td>32.681979</td>
<td>0.663931</td>
<td>1.129275744</td>
<td>1.694806028</td>
<td>0.00706509</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>217.436563</td>
<td>32.563044</td>
<td>1.310100</td>
<td>1.157812708</td>
<td>0.905899458</td>
<td>0.01049050</td>
<td>182</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>217.150577</td>
<td>32.635181</td>
<td>0.943015</td>
<td>1.198615805</td>
<td>1.543405354</td>
<td>0.00910989</td>
<td>132</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>217.080964</td>
<td>32.492997</td>
<td>0.152421</td>
<td>0.605340875</td>
<td>0.455654866</td>
<td>0.00678602</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>217.304699</td>
<td>32.735829</td>
<td>0.396749</td>
<td>0.755370465</td>
<td>0.732487133</td>
<td>0.00772304</td>
<td>32</td>
</tr>
<tr>
<td>1428p3311</td>
<td>I</td>
<td>1.05</td>
<td>217.100704</td>
<td>33.212775</td>
<td>0.423567</td>
<td>0.530298013</td>
<td>0.862422344</td>
<td>0.00684377</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>217.214168</td>
<td>33.330084</td>
<td>0.234625</td>
<td>0.110978794</td>
<td>0.273302323</td>
<td>0.00702622</td>
<td>24</td>
</tr>
<tr>
<td>Subfield</td>
<td>Filter</td>
<td>Seeing (arcsec)</td>
<td>Position</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>----------------</td>
<td>----------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>RA (deg)</td>
<td>DEC (deg)</td>
<td>Flux Iso</td>
<td>Flux Aper</td>
<td>Flux Best</td>
<td>Flux Max</td>
<td>Area</td>
</tr>
<tr>
<td>1428p3346</td>
<td>I</td>
<td>0.93</td>
<td>217.229568 33.536522</td>
<td>2.308310</td>
<td>1.983170255</td>
<td>2.881111062</td>
<td>0.01694210</td>
<td>269</td>
<td></td>
</tr>
<tr>
<td>1428p3421</td>
<td>R</td>
<td>1.19</td>
<td>217.237159 34.351682</td>
<td>0.933979</td>
<td>1.197071249</td>
<td>2.250090572</td>
<td>0.00888949</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>1431p3236</td>
<td>I</td>
<td>0.77</td>
<td>218.151996 32.572599</td>
<td>0.746356</td>
<td>0.659173895</td>
<td>0.735055277</td>
<td>0.00984275</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>1431p3311</td>
<td>Bw</td>
<td>1.05</td>
<td>217.738686 33.061787</td>
<td>0.984773</td>
<td>1.092747949</td>
<td>1.220252229</td>
<td>0.00991850</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>1431p3346</td>
<td>I</td>
<td>1.05</td>
<td>217.661724 33.601208</td>
<td>0.454346</td>
<td>0.699584212</td>
<td>0.766513839</td>
<td>0.00921198</td>
<td>71</td>
<td></td>
</tr>
</tbody>
</table>

Table A.1 (continued)
<table>
<thead>
<tr>
<th>Subfield</th>
<th>Filter</th>
<th>Seeing (arcsec)</th>
<th>RA (deg)</th>
<th>DEC (deg)</th>
<th>Flux Iso</th>
<th>Flux Aper</th>
<th>Flux Best</th>
<th>Flux Max</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1431p3421</td>
<td>R 1.03</td>
<td>218.027690</td>
<td>34.277855</td>
<td>0.395642</td>
<td>0.540405798</td>
<td>0.511540472</td>
<td>0.009820104</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>1431p3456</td>
<td>I 0.90</td>
<td>217.910199</td>
<td>35.041040</td>
<td>0.149944</td>
<td>0.271044126</td>
<td>0.568015256</td>
<td>0.00523823</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>1434p3311</td>
<td>Bw 0.98</td>
<td>218.699597</td>
<td>33.085615</td>
<td>0.365952</td>
<td>0.324758106</td>
<td>0.40922967</td>
<td>0.00490784</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>1434p3346</td>
<td>R 0.88</td>
<td>218.767323</td>
<td>33.751733</td>
<td>0.340388</td>
<td>0.549287859</td>
<td>0.545808130</td>
<td>0.00640267</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>1434p3421</td>
<td>I 0.79</td>
<td>218.818769</td>
<td>34.507064</td>
<td>1.009530</td>
<td>0.878860610</td>
<td>0.946630200</td>
<td>0.0148240</td>
<td>117</td>
<td></td>
</tr>
</tbody>
</table>

Note: Table A.1 (continued)
<table>
<thead>
<tr>
<th>Subfield</th>
<th>Filter</th>
<th>Seeing (arcsec)</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1434p3456 Bw</td>
<td>1.00</td>
<td>218.826291</td>
<td>34.878230</td>
</tr>
<tr>
<td>1434p3531 Bw</td>
<td>0.87</td>
<td>218.830952</td>
<td>35.360864</td>
</tr>
<tr>
<td>1437p3347 R</td>
<td>0.87</td>
<td>219.413329</td>
<td>33.594383</td>
</tr>
<tr>
<td>1437p3422 Bw</td>
<td>0.80</td>
<td>219.312998</td>
<td>34.270910</td>
</tr>
<tr>
<td>1437p3457 Bw</td>
<td>0.85</td>
<td>219.499208</td>
<td>34.985129</td>
</tr>
</tbody>
</table>

Table A.1 (continued)
<table>
<thead>
<tr>
<th>Subfield</th>
<th>Filter</th>
<th>Seeing (arcsec)</th>
<th>RA (deg)</th>
<th>DEC (deg)</th>
<th>Flux _Iso</th>
<th>Flux _Aper</th>
<th>Flux _Best</th>
<th>Flux _Max</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1437p3532</td>
<td>Bw</td>
<td>0.85</td>
<td>219.245001</td>
<td>35.602682</td>
<td>1.041050</td>
<td>1.111629338</td>
<td>1.292528119</td>
<td>0.00854261</td>
<td>221</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>219.571438</td>
<td>35.617343</td>
<td>0.717843</td>
<td>0.859409203</td>
<td>0.875628725</td>
<td>0.00919579</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>219.380292</td>
<td>35.371962</td>
<td>0.142787</td>
<td>0.190985326</td>
<td>0.242080607</td>
<td>0.00488309</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>219.475659</td>
<td>35.531412</td>
<td>0.113976</td>
<td>0.336449587</td>
<td>0.289574290</td>
<td>0.00421743</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>219.120694</td>
<td>35.721411</td>
<td>0.175965</td>
<td>0.293143319</td>
<td>0.352241077</td>
<td>0.00518678</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>219.165864</td>
<td>35.684473</td>
<td>0.241543</td>
<td>0.224905461</td>
<td>0.453398418</td>
<td>0.00535454</td>
<td>62</td>
</tr>
</tbody>
</table>
Bibliography


[143] “Imagine the Universe!”, HEASARC, ASD, NASA.


