Holographic description of black holes and cosmic inflation in asymptotically anti de Sitter backgrounds

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- Roy, S., and D.A. Lowe. **Holographic description of asymptotically AdS_2 collapse geometries.** Phys. Rev. D 78, 124017 (2008).

- Roy, S., and D.A. Lowe. **Chiral Geometries of 3d Gravity.** Phys.Lett.B668:159-162,2008.

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Dedication

I dedicate this thesis to *Baba* and *Ma*. Any accomplishment of mine is due, more than anyone or anything else, to the life of countless sacrifices and hardships they gracefully undertook in order for me to pursue my aims and aspirations unencumbered.

Contents

List of Figures

Chapter 1

Introduction and Outline

Gravity, described very well classically by the Einstein's general theory of relativity, has forever eluded a complete, consistent quantum mechanical description. This is total contrast to all other interactions in nature which are described by the most elegant of the quantum field theories namely, non-abelian gauge theories. Nothing brings out this visceral hostility between gravity and quantum mechanics to the fore like black holes do, which are objects that arise in general relativity as the unavoidable end point of gravitational collapse. As demonstrated in the past [1, 2, 3], quantum phenomena in (evaporating) black hole backgrounds endanger unitarity and causality, *the* most sacred tenet in all of physics. Furthermore despite being the *classically unique* end point of gravitational evolution, black holes were shown to have both temperature and (quantum mechanical) entropy [4, 1] which scale with the area of the event horizon, in contrast to entropy of local (quantum) theories which scale with volume. Ever since all out efforts of the theoretical fundamental physics community for decades has been devoted to reconcile this conflict by embedding gravity from the very beginning in a manifestly quantum framework. Superstring theory [5, 6, 7, 8, 9], the fundamental degrees of freedom of which are extended objects (strings, membranes) instead of point particles has distinguished itself as the leading (perhaps the only) this consistent quantum completion of gravity through a series of successes in answering questions which eluded any explanation in classical general relativity - for example, successfully accounting for the (quantum) black hole entropy [10] and resolving certain special singularities of general relativity, the orbifold singularities [11, 12] in string perturbation theory. In addition to being a theory of quantum gravity, (super)string theory also unifies gravity with all other interactions which are described by quantum gauge theories in an ultimate *Theory of Everything*. Despite such a huge promise and tantalizing glimpses of how black holes could possibly be fit into a unitarity respecting quantum framework, string theory itself is not a fully complete theory to start with. It is still being *discovered* in phases and there is a very long way to go before it can redeem its ambitious claims as *the* final theory of nature. The perturbative formulation of superstring theory has severe limitations when one encounters inherently nonperturbative phenomena such as spacelike singularities that lie at the heart of neutral black holes and those which appear at the big bang/ big crunch in cosmology. Fortunately, after the successful identification of nonperturbative (solitonic) degrees of freedom of superstring theory [13], the theory has undergone a period of revolutionary changes culminating in the realization that the perturbative string theories are all various limits of an mysterious 11-dimensional theory, dubbed *M-theory* which has as its low-energy dynamics supergravity interacting with the 2-dimensional ($M2$) and 5-dimensional membrane ($M5$). The intense investigation of black holes using these nonperturbative degrees of freedom, $D-branes$ then led to the biggest breakthrough in string theory till date, the Anti de Sitter/Conformal Field theory ($\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$) correspondence [14, 15, 16, 17]. This correspondence implies that string theory in asymptotically (locally) anti de Sitter spaces is isomorphic to some conformal field theory (a gauge theory) living on the conformal boundary of the anti de Sitter space. Since the dynamics of the nonperturbative regime of string theory is still now covered in mystery, for the first time, via the dual conformal field theory, we have fully nonperturbative definition of string theory/quantum gravity, albeit in asymptotically anti de Sitter spaces. Since then the duality has been generalized to many cases where the gravitational background is not anti de Sitter and dual gauge theory is nonconformal which is known in general as the gauge/gravity duality.

The aim of this dissertation is to address nonperturbative issues in quantum gravity in Anti de Sitter spacetimes by means of the manifestly holographic framework of Anti de Sitter/Conformal Field theory ($\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$) correspondence (in Lorentzian signature) The outline of the dissertation is as follows. In Section 2 we review, in some detail, the properties of Anti de Sitter space: various coordinate charts, conformal structure, properties of geodesics, the boundary value problem and finally characteristics of black holes in asymptotically AdS spaces. In Section 3, we introduce the duality between fields propagating in AdS and the operators in the conformal field theory supported on the "conformal boundary" of AdS. The map [18, 19, 20, 21, 22] of local bulk insertions in terms of "smeared out" boundary CFT operators is constructed in the strictly semiclassical limit. We begin with pure AdS space in various coordinate systems where it is demonstrated how (approximate) bulk locality emerges from the dual CFT. The best result is obtained for the case of Rindler coordinates one is able to minimize the support of the smeared CFT operators to a very compact region of the *complexified* boundary. Holographic maps are also constructed for various AdS black holes. We show how bulk degrees of freedom in regions of bulk behind horizons are encoded in the dual CFT and what is the CFT interpretation of the black hole singularity.

In Section 4 we model a scenario of eternal inflation by embedding "landscape" of string theory vacua in the manifestly holographic framework of AdS/CFT duality. Bubble geometries with de Sitter interiors within an ambient Schwarzschild anti-de Sitter black hole spacetime are investigated and a characterization for the states in the dual CFT on boundary of the asymptotic AdS which code the expanding dS bubble are obtained. These can then in turn be used to specify initial conditions for cosmology. This scenario naturally interprets the entropy of de Sitter space as a (logarithm of) subspace of states of the black hole microstates. Consistency checks are performed and a number of implications regarding cosmology are discussed including how the key problems or paradoxes of conventional eternal inflation are overcome.

Finally in Section 5, we turn to pure AdS gravity in (2+1)-dimensions which is classically equivalent to Chern-Simons theory. Recent investigations suggest that the dual CFT is holomorphically factorizable. By analyzing classical gauge field solutions, it is shown that the measure of the Chern-Simons path integral cannot factorize in a chiral way if it represents a sum over physically sensible states.

Chapter 2

Classical geometry of Anti de Sitter space

2.1 Introduction to Anti de Sitter space

Anti de Sitter space is at the focal point of the latest and perhaps the biggest breakthrough in Superstring theory, which is currently the only candidate for a unified theory of everything - namely the Anti de Sitter/ Conformal field theory duality (AdS/CFT). So in this inaugural chapter we acquaint ourselves with the classical geometry of this very special spacetime which will then set the scene for the introducing the duality. The general references for this chapter are [23, 24, 25].

A $(d+1)$ dimensional Anti de Sitter space (AdS_{d+1}) is defined as the uniformly negatively curved maximally symmetric spacetime. A maximally symmetric spacetime in $(d+1)$ dimensions is one which is defined which has the largest possible set of isometries i.e. largest number of Killling vectors i.e. $d+1+(d+1)d/2 = (d+1)(d+2)/2$. For such a highly symmetric spacetime, the following relation holds between the Riemann curvature tensor and the metric tensor,

$$R_{abcd} = K(g_{ac}g_{bd} - g_{ad}g_{bc})$$

where K is constant which is negative for AdS_{d+1}. So this implies the Ricci tensor is a multiple of the metric tensor,

$$R_{ab} = g^{cd}R_{acbd} = K\,d\,g_{ab}.$$

Such spacetimes are called *Einstein Spacetimes***.** The constant K is then expressed in terms of the curvature scalar as,

$$K = \frac{R}{d(d+1)}$$

or equivalently the curvature scalar is a (negative for AdS_{d+1}) is a constant all over spacetime,

$$R = K\,d(d+1).$$

In a physical context, more precisely in Einstein's General Relativity this manifold is arises as a solution to the vacuum Einstein field equation with a (negative) cosmological constant, Λ,

$$R_{ab} - \frac{1}{2} g_{ab} R + g_{ab} \Lambda = 0.$$

Again contracting indices one gets,

$$R = 2\frac{d+1}{d-1}\Lambda$$

Equivalently one can think of AdS spacetime as a solution to Einstein field equation with sources, the stress-energy tensor of which is given by,

$$T_{ab} = -\frac{1}{8\pi G_N}\Lambda g_{ab}$$

where G_N is the Newton's universal gravitational constant. Since for such a source the energy density,

$$T^{00} = -\frac{\Lambda}{8\pi G_N} g^{00}$$

is clearly negative for $\Lambda < 0$. This was the main reason why this spacetime was long considered uninteresting for describing physically relevant background spacetime. However following the developments in extended supergravities revealed that theories in which the $O(N)$ group is gauged have anti-de Sitter space as their ground or most symmetric state. Parallel developments in general relativity, namely the proof of the *positive mass theorem* [26] for (asymptotically) AdS spaces. These developments proved that asymptotically anti-de Sitter solutions are stable even though the potentials that appear in the theories are unbounded below.

Now before we march ahead any further it will be appropriate to get acquainted with some notations which are prevalent in the research literature, the negative constant, K is often written in terms of an inverse length squared by introducing the characteristic length scale of AdS space, l, also called the AdS radius of curvature as follows,

$$K = -\frac{1}{l^2}.$$

To complete the discussion, we express in terms of the AdS radius, the curvature scalar,

$$R = -\frac{d(d+1)}{l^2}$$

and the cosmological constant,

$$\Lambda = -\frac{d(d-1)}{2\,l^2}.$$

There is another mathematical way of introducing the Anti de Sitter spacetime as a homogeneous space of the group manifold $SO(d-1,2)$ and hence as a quotient,

$$AdS_{d+1} \equiv \frac{SO(d,2)}{SO(d,1)}.$$

This is best understood as follows. The isometry group of AdS space is $SO(d,2)$ and the action of $SO(d,2)$ on AdS_{d+1} is transitive i.e. any point in AdS_{d+1} can be moved to any other point via a $SO(d,2)$ symmetry transformation (active transformation language). The little group or stabilizer group i.e. the residual isometry group if a particular point on AdS_{d+1} is unmoved under the symmetry operation is $SO(d,1)$ - such residual transformations which have a fixed point are called *orbits* around that fixed point. Pictorially we can think of constructing or tracing over the whole $SO(d,2)$ group manifold as a sum over of all points in AdS_{d+1} and their respective orbits, i.e.

$$\begin{aligned} SO(d,2) &= \sum_{p\in AdS_{d+1}} SO(1,d) \text{ orbits "around" the point } p. \\ &= AdS_{d+1} \times SO(d,1) \end{aligned}$$

This coset space definition of AdS spacetime also allows us to introduce the first set of coordinate system in it, namely the embedding coordinates $(X_{-1}, X_0, X_1, \ldots, X_d)$ i.e. those coordinatizing higher dimensional $(d+2)$ dimensional flat spacetime with signature $(d,2)$ i.e. $\mathbb{R}^{d,2}$ in which we defined the "AdS_{d+1} hyperboloid" as a hyperbolic hypersurface,

$$-X_{-1}^2 - X_0^2 + \sum_{i=1}^{d} X_i^2 = -l^2$$

This hypersurface has the exact symmetries or isometries as AdS_{d+1} and most importantly provides not just about the local structure of the AdS_{d+1} hyperboloid i.e. the metric, but even the global topology which is $\mathbb{R}^d \times S^1$. This is pathological from a physical perspective since the time direction is a circle i.e. the manifold is not simply connected and one has closed time like curves which violate causality, the most sacred tenet in physics. One easily circumvents this obstacle by "unwrapping" the time direction i.e. precisely speaking working with the *covering space* instead which is simply connected. This covering space of the "AdS_{d+1} hyperboloid" is AdS_{d+1} spacetime. So the topology of AdS_{d+1} is then simply $\mathbb{R}^{d+1}$.

Now we shall introduce a set of coordinate charts (both global and local) parametrizing AdS_{d+1} using the embedding coordinates.

2.2 AdS Coordinate Charts

2.2.1 Global charts

2.2.1.1 Static coordinates

These are defined by,

$$\begin{aligned} X_{-1} &= \sqrt{l^2 + r^2} \cos \frac{t}{l} \\ X_0 &= \sqrt{l^2 + r^2} \sin \frac{t}{l} \\ X_i &= r\omega_i,\ i = 1, \dots, d \end{aligned}$$

where the $\{\omega_i : \sum_i \omega_i^2 = 1\}$ coordinatize a S^{d-1}. The pullback metric of this map on the AdS hyperboloid is,

$$ds^2 = \left(1 + \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{l^2}\right)} + r^2 d\Omega_{d-1}$$

with the coordinate ranges $t/l \in [0, 2\pi)$, $r \in (0, \infty)$. One needs to unwrap the t circle to obtain AdS spacetime i.e. make $t \in (-\infty, \infty)$ with $g_{ab}(t + 2\pi l, r, \{\omega_i\}) = g_{ab}(t, r, \{\omega_i\})$.

2.2.1.2 The τ, χ, ω_i coordinates

These are defined by,

$$\begin{aligned} X_{-1} &= l \cosh \chi \cos \tau \\ X_0 &= l \cosh \chi \sin \tau \\ X_i &= l \sinh \chi \omega_i,\ \ i = 1, \dots, d \end{aligned}$$

where the ω's parametrize a S^{d-1}. Then the pullback metric on the AdS hyperboloid is,

$$ds^2/l^2 = d\chi^2 - \cosh^2 \chi \left(d\tau^2 + \tanh^2 \chi \, d\Omega_{d-1}\right)$$

with the coordinate ranges being, $\tau \in [0, 2\pi)$, $\chi \in (0, \infty)$. For AdS space we simply make $\tau \in (-\infty, \infty)$ with $g_{ab}(\tau + 2\pi, \chi, \{\omega_i\}) = g_{ab}(\tau, \chi, \{\omega_i\})$.

2.2.1.3 The $\tau, \rho, \{\omega_i\}$ coordinates

These are closely related to the $\tau, \chi, \{\omega_i\}$ coordinates,

$$\begin{aligned} X_{-1} &= l \sec \rho \cos \tau \\ X_0 &= l \sec \rho \sin \tau \\ X_i &= l \tan \rho \, \omega_i,\ i = 1, \dots, d \end{aligned}$$

i.e the two coordinates sets are related by a redefinition of χ,

$$\sec \rho = \cosh \chi$$

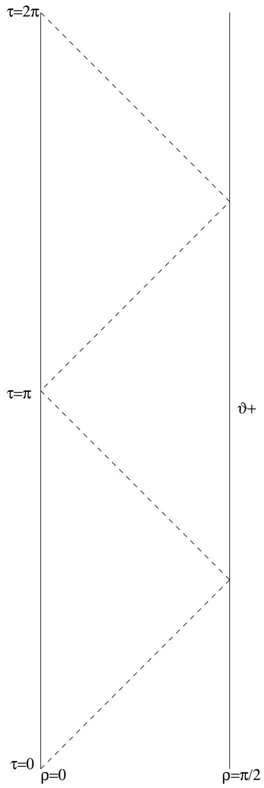

Figure 2.1: Conformal diagram from AdS_{d+1}. $\mathfrak{J}+$ is the timelike spatial infinity at $\rho = \pi/2$. The diagram is continued infinitely in both vertically up and down directions. Each point in the diagram has a S^d which has been suppressed.

The metric then looks like,

$$ds^2 = \frac{l^2}{\cos^2 \rho} \left(-d\tau^2 + d\rho^2 + \sin^2 \rho d\Omega_{d-1} \right)$$

with $\rho \in [0, \frac{\pi}{2})$. Note that time direction has been unwrapped i.e. $-\infty < \tau < \infty$. This chart has the advantage that in this chart the noncompact AdS_{d+1} can be compactified i.e. "infinity can be brought in" by simply adding the "point at infinity" $\rho = \frac{\pi}{2}$ and hence allows us to see the asymptotic structure of AdS space. So for $d > 1$, AdS_{d+1} space now looks like a infinite cylinder - the axis of the cylinder being the time direction, the radial direction being that of ρ and at each point in the cylinder we have a S^{d-1}. For $d = 1$ case i.e. AdS_2 we have two disconnected asymptotic infinities at $\rho = \pm\frac{\pi}{2}$. The conformal diagram of AdS spacetime is produced in Fig.2.1.

2.2.2 Local charts

2.2.2.1 Poincare coordinates

These are defined in terms of the global coordinates as follows:

$$\begin{aligned}
X_0 &= \frac{l}{2z}\left(l^2+z^2+\mathbf{x}^2-t^2\right)\\
X_{-1} &= l\frac{t}{z}\\
X_d &= \frac{1}{2z}\left(-l^2+z^2+\mathbf{x}^2-t^2\right)\\
X_i &= l\frac{x_i}{z}
\end{aligned}$$

and the metric is then,

$$ds^2=\frac{l^2}{z^2}\left(dz^2-dt^2+d\mathbf{x}^2\right)$$

where $\mathbf{x}=(\{x_i\},i=1,...d-1)$ and the coordinate ranges are $-\infty<x_i,t<\infty$ and $0<z<\infty$. In these coordinates, the "point at infinity" or conformal "boundary" i.e. $\rho=\frac{\pi}{2}$ is at,

$$z=0$$

while we have a horizon at $z=\infty$ where the norm of time translation ∂_t becomes lightlike. As a result it only covers a part of the AdS$_{d+1}$ manifold (The regions covered by the Poincare patch is shown in Fig.2.2). However this coordinate chart is still important since it makes manifest the subgroups $ISO(d-1,1)$ i.e. boundary Poincare group and "Dilatation subgroup" $SO(1,1)$ i.e

$$(t,\mathbf{x},z)\rightarrow(\lambda t,\lambda\mathbf{x},\lambda z)\,,\lambda>0$$

of the symmetry group $SO(d,2)$ of AdS$_{d+1}$. The eigenvalues of the Dilatation operator furnish the energy spectrum of the boundary CFT.

2.2.2.2 Rindler or BTZ-like coordinates

This coordinate chart is exclusively for $d=2$. We introduce these by he definitions,

$$\begin{aligned}
X_{-1} &= \pm\ r'\cosh\theta\\
X_1 &= r'\sinh\theta\\
X_0 &= l\sqrt{\frac{r'^2}{l^2}-1}\sinh\frac{t'}{l}\\
X_2 &= \pm l\sqrt{\frac{r'^2}{l^2}-1}\cosh\frac{t'}{l}
\end{aligned}$$

and the metric appears as,

$$ds_I^2=-\left(\frac{r'^2}{l^2}-1\right)dt'^2+\frac{dr'^2}{\frac{r'^2}{l^2}-1}+r'^2d\theta^2$$

The above metric covers only the region $X_{-1}^2-X_1^2\geq 0\cap X_0^2-X_2^2\leq 0$ (region I) and we have four patches considering each sign permutation once $(I++, I+$

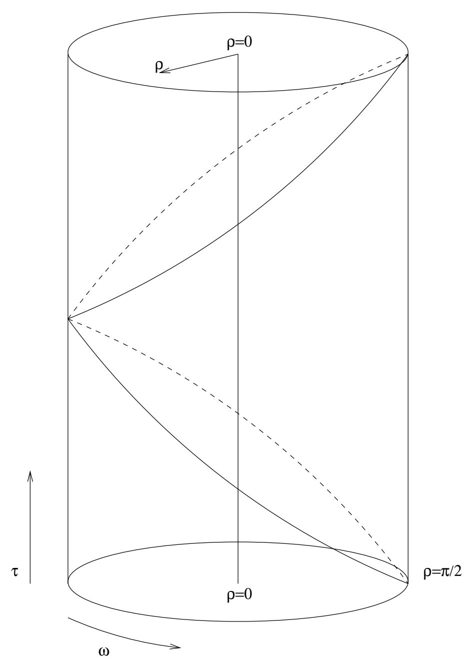

Figure 2.2: Poincare patches : AdS$_{d+1}$ spacetime is a cylinder (continued infinitely in the vertical directions). The ρ coordinate parametrizes the radial direction and time, τ parametrizes the axial direction. The S^d directions are coordinatized by ω's. The Poincare patches are obtained by diagonally chopping the cylinder as shown. Then the Poincare patch $r > 0$ is the region of the cylinder confined between the slices and the section of the boundary $\rho = \pi/2$ contained within the slices constitute the the conformal diagram of Minkowski space (diamond). We need another such patch to cover the complementary part of the AdS (cylinder).

$-, I-+, I--)$. So we need cover the two other complementary regions which are $X_{-1}^2 - X_1^2 \geq 0 \cap X_0^2 - X_2^2 \geq 0$ (region II) and $X_{-1}^2 - X_1^2 \leq 0 \cap X_0^2 - X_2^2 \geq 0$, by the similar coordinate definitions for region II and region III respectively,

$$\begin{aligned} X_{-1} &= \pm \; r' \cosh\theta \\ X_1 &= r' \sinh\theta \\ X_0 &= \pm l\sqrt{1 - \frac{r'^2}{l^2}} \cosh\frac{t'}{l} \\ X_2 &= l\sqrt{1 - \frac{r'^2}{l^2}} \sinh\frac{t'}{l} \end{aligned}$$

$$\begin{aligned} X_{-1} &= r' \sinh\theta \\ X_1 &= \pm r' \cosh\theta \\ X_0 &= \pm l\sqrt{\frac{r'^2}{l^2} + 1} \cosh\frac{t'}{l} \\ X_2 &= l\sqrt{\frac{r'^2}{l^2} + 1} \sinh\frac{t'}{l} \end{aligned}$$

and the metric,

$$ds^2_{II/III} = -\left(1 \mp \frac{r'^2}{l^2}\right) dt'^2 + \frac{dr'^2}{1 \mp \frac{r'^2}{l^2}} + r'^2 d\theta^2.$$

So we in all we have $3 \times 4 = 12$ patches to cover the AdS hyperboloid as shown in Fig.2.3.

2.3 Geometric properties of AdS

Now that we have acquainted ourselves with a number of coordinate systems in AdS space, the next thing is look at is particle motion i.e. geodesics and the fields in AdS i.e. initial value or boundary value problems in AdS.

2.3.1 Geodesics and trajectories

Anti-de Sitter space acts like a confining box for test particles for non-vanishing mass. Although heuristic, this is most clearly visible when one looks at the "Newtonian potential" in Global coordinates,

$$V(r) = -\frac{1}{2} g_{00} = 1 + \frac{r^2}{l^2_{AdS}}$$

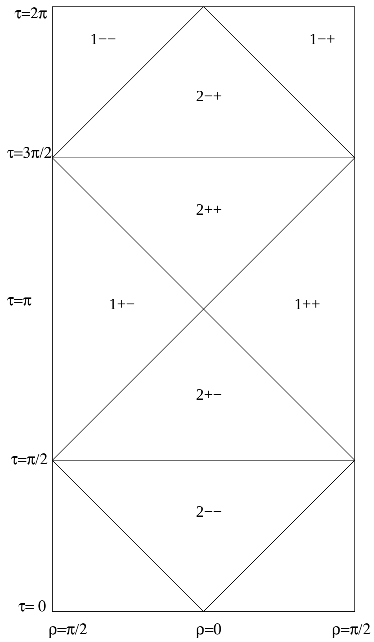

Figure 2.3: Cross section of the AdS cylinder showing 8 of the 12 BTZ-like coordinate patches.

So test particles with non-vanishing mass are "turned back" and they return to their origin (recall the AdS is homogeneous space so any point can be brought to the origin of the global coordinate system) in global time duration,

$$\Delta t = \pi l \quad or \quad \Delta\tau = \pi$$

In fact, all the timelike geodesics from any point in AdS space (to either the past or future) reconverge to an image point, diverging again from this image point to refocus at a second image point, Q and so on. The future timelike geodesics from some origin, say P therefore never reach the timelike infinity $\rho = \pi/2$.

In contrast null geodesics, can reach timelike infinity in finite coordinate time and form the future null cone of the point P. Thus unlike flat space, there exist points which are in the timelike future of P but cannot be connected to P by timelike geodesics. See Fig. 2.4.

2.3.2 Global (Non)Hyperbolicity

AdS space is has a topology $\mathbb{R} \times S^3$ (or $\mathbb{R} \times \mathbb{R}^3$) i.e. it has time-like boundary at $\rho = \pi/2$. The effect of this is that the Cauchy problem is not well defined. While one can find families of spacelike surfaces (such as the surfaces t = constant) which cover the space completely, each surface being a complete cross-section of the space- time and one can find null geodesics which never intersect any given surface in the family. Cauchy data on such a spacelike surface determines the evolution of the system only in a region which is bounded by a null hypersurface called a Cauchy horizon. Any attempt to predict beyond this region is prevented by fresh information coming in

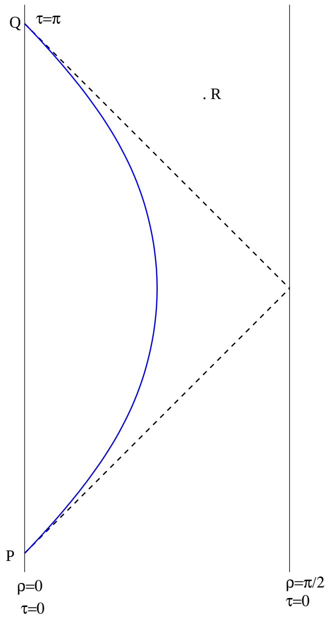

Figure 2.4: Geodesics in AdS. The dotted lines are outgoing and incoming null geodesics while the blue line is a timelike geodesic. The timelike geodesics emanating from P reconverge at the image point Q and so on. The points like R which are timelike separated from P but outside the light cone (dotted lines) can never be reached from P through geodesics.

(or leaking out) from the timelike infinity. In contrast, spacetimes which have a Cauchy hypersurface, the initial conditions on which determines the future (and the past) uniquely and in particular globally hyperbolic spacetimes are those which are homeomorphic to $\mathbb{R} \times \Sigma$ where Σ is a three-dimensional manifold, and for each a $t \in \mathbb{R}$, $t \times \Sigma$ is a Cauchy surface for the manifold. Clearly AdS is not a globally hyperbolic spacetime.

In addition there is an associated but more technical issue. Rigorous field quantization procedures require the Cauchy data to have a compact support. As a consequence of global hyperbolicity, Cauchy data on any hypersurface will then possess this property. However it is plain clear that for AdS space, even if initial value data is has a compact support in one spacelike hypersurface will evolve in a manner that it becomes noncompact on other spatial hypersurfaces.

2.3.3 Einstein-Static Universe: Cauchy completion of AdS

Since the boundary of AdS extends in the timelike direction labeled by τ , we need to specify a boundary condition on the R × Sp at $\rho = \pi/2$ in order to make the Cauchy problem well-posed on AdS. This is accomplished for conformally invariant fields in AdS by mapping it to the Einstein Static Universe (Cylinder) [25, 39].

Einstein's static universe (ESU) is a closed (i.e. has a topology $\mathbb{R} \times S^d$ i.e. positive spatial curvature), and contains uniform dust and a positive cosmological constant with value precisely$\Lambda = 4\pi G\rho$, where G is Newtonian gravitational constant, ρ is the energy density of the matter in the universe The radius of curvature of space of the Einstein universe is equal to

$$R = \Lambda^{-1/2} = \frac{1}{\sqrt{4\pi G\rho}}.$$

The Einstein universe is one of Friedmann's solutions to Einstein's field equation for dust with density ρ, cosmological constant Λ, and radius of curvature R. It is the only non-trivial static solution to Friedmann's equation. The ESU metric in global coordinates is given as,

$$ds^2_{ESU} = K^{-1}\left(d\tau^2 - d\rho^2 + \sin^2\rho\, d\Omega_d^2\right);\ \infty > \tau > -\infty, \pi \geq \rho \geq 0, K = -\frac{2R}{d(d+1)}.$$

This metric appears to be conformal to the global AdS metric with the conformal factor $\Omega = \sec\rho$,

$$g_{AdS} = \Omega^2 g_{ESU},$$

but globally the ranges of ρ implies that AdS covers only half of the ESU i.e. only the upper hemisphere of the spatial section S^d. Similarly Minkowski space $\mathbb{M}^{1,d}$ can also be conformally mapped to the Einstein static cylinder (see the following figure which shows the Einstein cylinder cut along $\rho = \pi$). The absence of a global Cauchy surface in AdS and associated leakage of initial data specified a fixed τ slice i.e. the upper hemisphere can now be understood as information crossing the equator of the S^d of the conformal extension to ESU. Mind you, the ESU itself is globally hyperbolic because the spatial sections, S^d which are the Cauchy surfaces are compact. Then when one imposes the right boundary conditions on the equator the Cauchy problem becomes well-defined in AdS. There a couple of distinct prescriptions to accomplish this. The first one called as the "open"/"transparent" boundary condition prescription. Here one first notices that the specification of Cauchy data on $\tau = 0$ slice of ESU is equivalent to specifying data on the pair of *partial* surfaces, $\Sigma_1 \equiv \{\tau = 0, 0 < \rho \leq \pi/2\}$ and $\Sigma \equiv \{\tau = \pi, \pi/2 < \rho \leq \pi\}$. Now $\Sigma_1 \cup \Sigma_2$ can be used as a surface on which the "effective initial value data" can be specified for AdS by conformally mapping from the ESU (refer to Fig.2.5). The net effect is then to recycle charges like energy, angular momentum etc. which is being lost through the timelike boundary i.e. the equator and having a well-defined but rather peculiar conservation law. But one would perhaps want a more familiar looking conservation law, where one would define conserved charges by integrating over just *one* spacelike hypersurface and not a *pair* as prescribed above. This is achieved by a second boundary condition prescription, namely the "closed"/"reflective" boundary condition. Here we expand fields propagating in the ESU in a basis which have a vanishing flux of data across the equator of the ρ-sphere, so the light ray is actually reflected back inwards at $\rho = \pi/2$. This requires admitting only those fields which have a sufficiently rapid asymptotic fall-off behavior such that the conserved currents vanish at $\rho = \pi/2$. This criterion in translates into a lower bound for the masses of excitations which can be

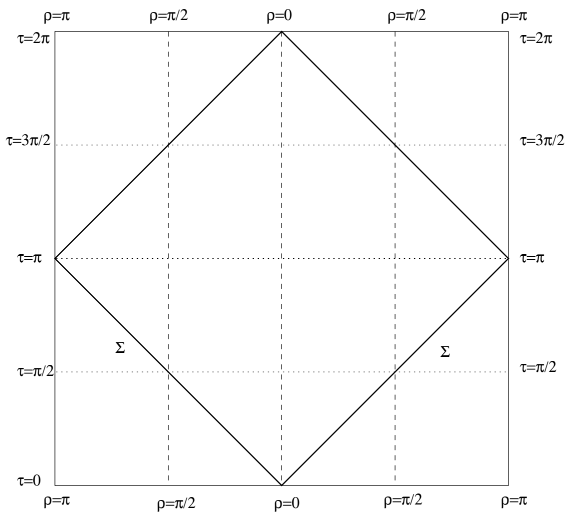

Figure 2.5: The ESU cylinder cut along $\rho = \pi$ and unwrapped and flattened. The Minkowski space is then given by the rhombus shaped region (bounded by bold null lines) while AdS hyperboloid is the rectangular region $\rho \leq \pi/2$ bounded by vertical dashed lines. The Σ's provide the compact support necessary to define a retarded Green's function for AdS.

supported (consistently quantized) in AdS, known as the *Breitenlohner-Freedman* bound:

$$m^2 l_{AdS}^2 \geq -\frac{d^2}{4}.$$

2.4 AdS Black holes

2.4.1 Schwarzschild AdS (SAdS) black holes

The Einstein equations with a negative cosmological constant admit black hole solutions which are asymptotic to anti-de Sitter space. A spherically symmetric static AdS black hole or a Schwarzschild AdS black (SAdS_{d+1}) hole is given by the metric [27],

$$ds^2 = -\left(1 - \frac{2M}{m_{Pl}^{d-1} r^{d-2}} + \frac{r^2}{l_{AdS}^2}\right) dt^2 + \left(1 - \frac{2M}{m_{Pl}^{d-1} r^{d-2}} + \frac{r^2}{l_{AdS}^2}\right)^{-1} dr^2 + r^2 d\Omega_{d-1}^2$$

where m_p is the Planck mass and the Newton's constant $G = 1/m_{pl}^{d-1}$. The Penrose diagram for these black holes is in figure 2.6. So this metric has a event horizon at $r = r_+$, where $1 - \frac{2M}{m_{Pl}^{d-1} r_+^{d-2}} + \frac{r_+^2}{l_{AdS}^2} = 0$. The corresponding Hawking temperature obtained from the conical singularity method is,

$$\beta^{-1} = \frac{1}{4\pi}\left(\frac{d-2}{r_+} + \frac{d}{l_{AdS}^2} r_+\right).$$

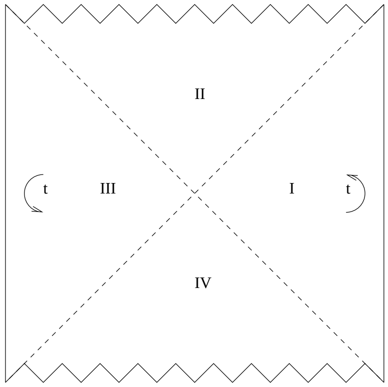

Figure 2.6: Penrose diagram of an extended AdS-Schwarzschild black hole. Region I covers the region that is outside the horizon from the point of view of an observer on the right boundary. Region II is an identical copy and includes a second boundary. Regions III and IV contain spacelike singularities. The diagram shows the time and radial directions, over each point there is a sphere S^{d-1}. This sphere shrinks as we approach the singularities.

This temperature (β^{-1}) has a minimum when,

$$r_+ = r_0 = l_{AdS}\sqrt{\frac{d-2}{d}}$$

or, equivalently when,

$$\beta^{-1} = T_0 = \frac{\sqrt{d(d-1)}}{2\pi} l_{AdS}^{-1}.$$

This implies for $r > r_0$, the black hole mass,

$$M = \frac{m_{Pl}^{d-1}}{2}\left(1 + \frac{r_+^2}{l_{AdS}^2}\right) r_+^{d-2}$$

increases with β^{-1}! This is in contrast with flat space, for which the temperature is inversely proportional to black hole and the canonical ensemble for these AdS black holes ($T > T_0$) exist. In fact it has been shown [27], that for,

$$T > T_2 = m_{Pl}^{1/2} l_{AdS}^{1/2}$$

black holes have positive specific heat and and exist in equilibrium with Hawking radiation forever. These are called *Eternal* black holes. For temperatures $T < T_0$ thermal radiation can exist in equilibrium without collapsing into a black hole.

2.5 BTZ black hole

General Relativity in (2+1)-dimensions has long been considered an arena for investigating issues in gravity without the additional complications of gravitational dynamics in higher dimensions. The reason for this is that in (2+1)-dimensions there are no local propagating degrees of freedom in the bulk i.e. there are no gravitational waves in the bulk and the dynamics arises purely from global topology - defects and boundaries [28, 29] (for a comprehensive review consult [30]). Although the theory does not have gravitational waves i.e. no gravitational attraction, interestingly, for the case of negative cosmological constant i.e. in asymptotically Anti- de Sitter space. it does have black hole solutions [31, 32]! One might be puzzled by the fact that if there is no gravitational attraction, how can one have a black hole. It is clearly a black hole: it has an event horizon and (in the rotating case) an inner horizon, it appears as the final state of collapsing matter, and it has thermodynamic properties much like those of a (3+1)-dimensional Kerr black hole. We review the basic features of this special black hole spacetime here.

In three spacetime dimensions, the full curvature tensor is completely determined by the Ricci tensor,

$$R_{\mu\nu\rho\sigma} = g_{\rho[\mu}R_{\nu]\sigma} + g_{\sigma[\nu}R_{\mu]\rho} - \frac{1}{2}g_{\mu[\rho}g_{\sigma]\nu}R.$$

For a spacetime with a cosmological constant, Einstein field equations,

$$R_{\mu\nu} = 2\Lambda g_{\mu\nu},$$

imply that in $2+1$-d, that the curvature tensor is completely fixed globally,

$$R_{\mu\nu\rho\sigma} = \Lambda(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}).$$

i.e. every spacetime is locally AdS (or dS or flat) and the whole spacetime is expressible as a collection of such neighborhoods appropriately patched together. In particular there are *no curvature singularities*. This special coincidence in $2+1$-d i.e. absence of local propagating d.o.f. was the reason why it was considered that gravity in $2+1$-d spacetime is *trivial*.

Since every spacetime in $2+1$-d with a negative cosmological constant is locally AdS$_3$, one can obtain the BTZ black hole by a quotient of AdS$_3$. As we have seen before one can express the AdS$_3$ metric in Rindler ("freely falling") coordinates (we make the change in notation, $l_{AdS} \to l$ to avoid clutter),

$$ds^2_{AdS_3} = -\frac{r'^2 - l^2}{l^2}dt'^2 + \frac{l^2}{r'^2 - l^2}dr'^2 + r'^2 d\phi'^2,\ (t', \phi') \in (-\infty, \infty), r' > l$$

Making the following coordinate changes,

$$\begin{aligned} \frac{r'^2}{l^2} &= \frac{\hat{r}^2 - r_-^2}{r_+^2 - r_-^2},\ \hat{r} > r_+, \\ \begin{pmatrix} t'/l \\ \phi' \end{pmatrix} &= \frac{1}{l}\begin{pmatrix} r_+ & -r_- \\ -r_- & r_+ \end{pmatrix}\begin{pmatrix} \hat{t}/l \\ \hat{\phi} \end{pmatrix} \end{aligned}$$

and the quotienting along the $\hat{\phi}$ dimension,

$$\hat{\phi} \sim \hat{\phi} + 2\pi,$$

one arrives at the BTZ metric,

$$\begin{aligned} ds^2_{BTZ} &= -\frac{(\hat{r}^2 - r_+^2)(\hat{r}^2 - r_-^2)}{\hat{r}^2 l^2} d\hat{t}^2 + \frac{\hat{r}^2 l^2}{(\hat{r}^2 - r_+^2)(\hat{r}^2 - r_-^2)} d\hat{r}^2 \\ &\quad + \hat{r}^2 (\frac{r_+ r_-}{\hat{r}^2 l} d\hat{t} - d\hat{\phi})^2. \end{aligned} \quad (2.1)$$

This metric looks like a dimensional reduction of the well known Kerr black hole in asymptotically flat spacetimes. Clearly $\hat{r} = r_+$ and $\hat{r} = r_-$ are event horizons. The "mass" and "spin" of the BTZ are given by,

$$\begin{aligned} Ml^2 &= r_+^2 + r_-^2 \\ Jl &= 2r_+ r_-. \end{aligned}$$

To obtain the metric for the degenerate $r_+ = r_-$ i.e. extreme, $J = \pm Ml$ case, can begin from the Poincare patch of AdS$_3$,

$$ds^2 = \frac{1}{z^2}(dz^2 + dw^+ dw^-), \ z > 0$$

with $w^\pm = t \pm x$, make the following set of coordinate changes,

$$\begin{aligned} y &= \frac{l}{\sqrt{\hat{r}^2 - r_+^2}} e^{-\frac{r_+}{l}(\frac{\hat{t}}{l} - \hat{\phi})} \\ w^+ &= \hat{\phi} + \frac{\hat{t}}{l} - \frac{l r_+}{\hat{r}^2 - r_+^2} \\ w^- &= \frac{l}{2\hat{r}} e^{-\frac{2r_+}{l}(\frac{\hat{t}}{l} - \hat{\phi})} \end{aligned}$$

and again as before mod out the $\hat{\phi}$ dimension,

$$\hat{\phi} \sim \hat{\phi} + 2\pi,$$

one arrives at the black hole metric,

$$ds^2_{BTZ} = -\frac{(\hat{r}^2 - r_+^2)^2}{\hat{r}^2 l^2} d\hat{t}^2 + \frac{\hat{r}^2 l^2}{(\hat{r}^2 - r_+^2)^2} d\hat{r}^2 + \hat{r}^2 (\frac{r_+^2}{\hat{r}^2}\frac{d\hat{t}}{l} - d\hat{\phi})^2, \hat{r} \in (r_+, \infty), \hat{\phi} \in [0, 2\pi). \quad (2.2)$$

Clearly, $\hat{r} = r_+$ is an event horizon. The Penrose diagram of the BTZ spacetime is shown in figure 2.7.

The resemblance with the flat space Kerr black hole is very apparent in the Penrose diagram. So more explicitly, the BTZ is realized by modding out along a pair of dimensions generated by the Killing vector of the AdS$_3$ spacetime. So that Killing vector for the generic BTZ case is,

$$\xi = \frac{r_+}{l} M_{12} - \frac{r_-}{l} M_{03} - M_{13} + M_{23}$$

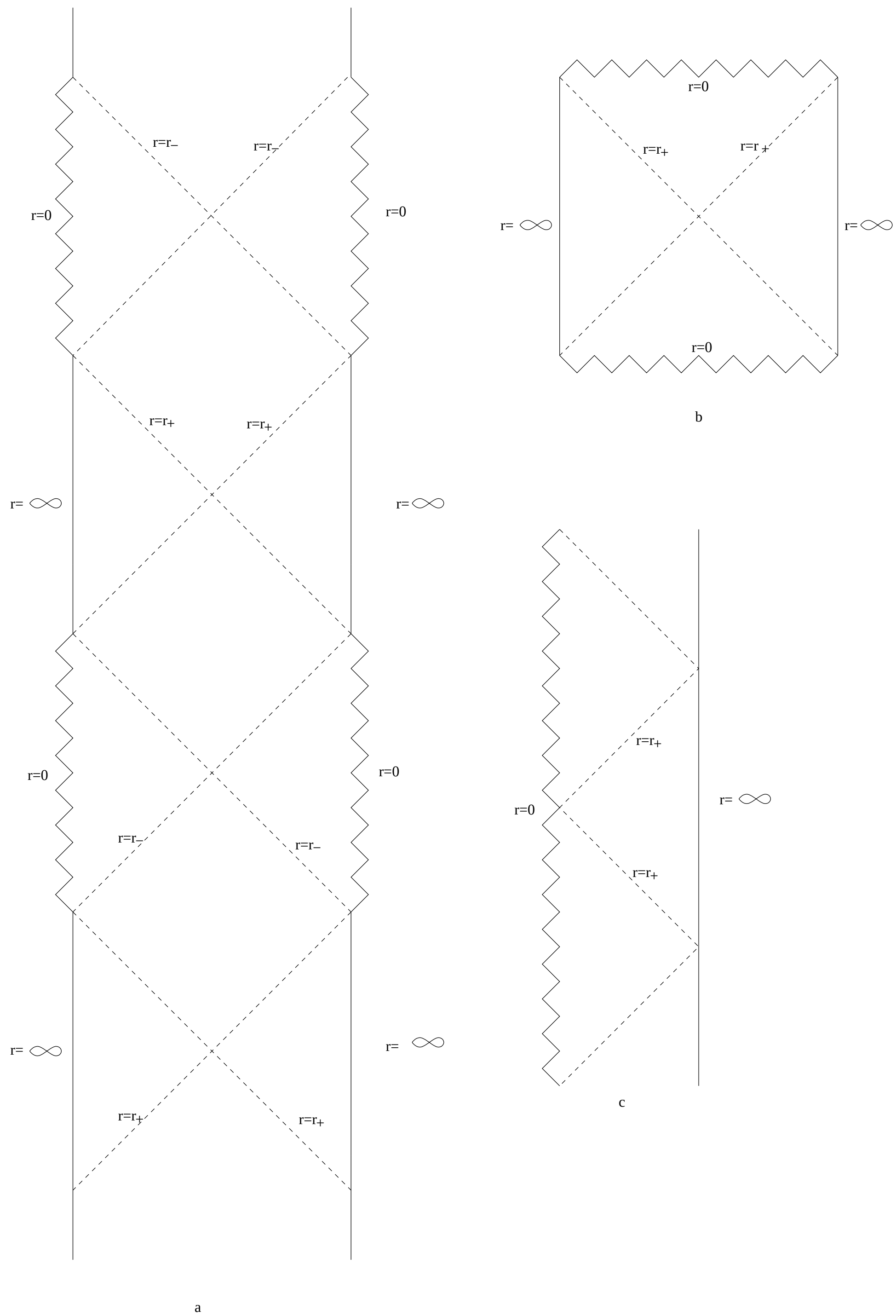

Figure 2.7: Penrose conformal diagram of the BTZ black hole. (a) The generic BTZ black hole; (b) the static ($J = 0$) black hole; and (c) the extreme ($J = \pm Ml$) black hole.

where,

$$M_{ab} = x_{[b}\partial_{a]}$$

are the $SO(2,2)$ Killing vectors of the embedding space $\mathbb{R}^{2,2}$ of AdS$_3$,

$$x_{-1}^2 + x_0^2 - x_1^2 - x_2^2 = l^2.$$

Now as a ramification of this quotienting one must make sure to delete the portions of the AdS$_3$ spacetime where one might form closed timelike curves, i.e., when the norm of ξ becomes timelike,

$$\xi \cdot \xi > 0.$$

This is the region $\hat{r} \leq 0$. So we realize that the singularity in the BTZ is not a curvature singularity but in a more general sense that geodesics end abruptly here and on cannot predict time-evolution from hereon. Just to complete this group theoretic analysis, one must add that the quotienting also reduces the symmetries as compared to that of AdS$_3$ i.e. $SO(2,2)$ This is because only those symmetries, the generators of which Poisson-commute with ξ can stay intact. This reduced symmetry group is of the BTZ is $\mathbb{R} \times SO(2)$.

With this we have covered almost all aspects of classical general relativity in asymptotically AdS spacetime and the ground is prepared for the introduction of the AdS$_{d+1}$/CFT$_d$ duality in the following chapter.

Chapter 3

The AdS/CFT duality

The Maldacena conjecture [14, 15, 16] states that type IIB Superstring theory in $\mathrm{AdS}_5 \times S^5$ is isomorphic to $\mathcal{N}=4$ $SU(N)$ Super-Yang Mills theory supported on the conformal boundary of AdS_5 i.e. $\mathbb{R} \times S^3$. The exact map between the parameters of both sides are,

$$\begin{aligned} g_s &= g_{YM}^2 \\ 4\pi g_s N &= \left(\frac{l_{AdS}}{l_s}\right),^4 \end{aligned}$$

where g_s and l_s are the string coupling and the string length respectively while g_{YM}^2 and N are the Yang Mills coupling and the rank of the gauge group respectively. l_{AdS} is the familiar AdS radius. The $\mathcal{N}=4$ $SU(N)$ Super-Yang Mills theory in addition to being maximally supersymmetric is also a conformal field theory i.e. the quantum β function vanishes. The AdS/CFT correspondence was historically motivated by comparing stacks of elementary branes with corresponding gravitational backgrounds in string or M-theory (A nice review is provided in [33]). For example, the correspondence between a large number N of coincident D3-branes and the 3-brane classical solution leads, after an appropriate low-energy limit is taken, to the duality between $\mathcal{N}=4$ supersymmetric $SU(N)$ gauge theory and Type IIB strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$.

In general this duality or isomorphism is extended to general spacetimes i.e. a theory of quantum gravity in AdS_{d+1} background is dual to a conformal field theory (CFT) supported on the conformal boundary of AdS_{d+1} i.e. a *nongravitational* (conformal) quantum field theory in $\mathbb{R}^{1,d-1}$. In the absence of a nonperturbative formulation of string theory which has in addition to perturbative "stringy" i.e. one dimensional degrees of freedom, a zoo of extended objects such as Dirichlet branes and orientifold planes of diverse dimensions, one can think of the dual gauge theory as a complete nonperturbative definition of (super)string theory. In the limit when the AdS radius is much larger than the string length with string coupling fixed, i.e.

$$\frac{l_{AdS}}{l} \gg 1$$

string theory is well approximated by a supergravity (SUGRA) theory in the AdS space. But the map between the parameters on then implies the rank of the gauge group N becomes very large. Thus the large N limit of gauge theories describe the supergravity excitations in AdS space. If we express the Yang-Mills coupling like,

$$g_{YM}^2 = \frac{\lambda}{N}$$

where λ is called the 't Hooft coupling after 't Hooft who introduced the $1/N$ expansion for quantum gauge theories [34]. In this respect the AdS/CFT duality is a concrete example of 't Hooft's proposal. We shall be concerned with this particular limit i.e. the gravity limit $N \to \infty$.

A precise definition of this duality in the SUGRA limit is stated as follows. We associate to the SUGRA compactification on $\mathrm{AdS}_{d+1} \times M$ a conformal field theory living on a space conformal to the d-dimensional boundary $\mathcal{B}$ of the AdS factor. To each SUGRA field Φ_i there is a corresponding local operator $\mathcal{O}_i$ in the conformal field theory. The relation between SUGRA theory in the bulk and field theory on the boundary is:

$$Z_{SUGRA}[\{\Phi_i\}] = e^{iI_{eff}[\{\Phi_i\}]} = \langle T e^{i\int_{\mathcal{B}} \Phi_{0i}\mathcal{O}_i}\rangle_{CFT} \tag{3.1}$$

Here I_{eff} is the effective SUGRA action in the bulk, Φ_{0i} is the field Φ_i restricted to the boundary, and T is the time-ordering symbol in the field theory on $\mathcal{B}$. The expectation value on the right hand side is taken in the boundary field theory, withΦ_{0i} treated as a source term. So given a boundary field we solve for the corresponding bulk field and use it to relate the bulk effective action to boundary correlation functions. Although the Maldacena duality was first formulated in a Euclidean signature AdS [14, 15, 16], we shall work entirely in Lorentzian signature AdS version [17, 35, 36, 37] where a slight reformulation is necessary. But before we proceed any further we must pause and survey some generic features of quantum fields propagating in fixed AdS background which will then help us in formulating the duality in a way which we shall use for the rest of the treatise.

3.1 Quantum Fields in AdS

We shall consider a scalar field ϕ since this will be enough for the purposes of this treatise. The massive scalar field propagating in a fixed AdS (or asymptotically AdS in many future cases) background is described by the equation,

$$\frac{1}{\sqrt{g}}\partial_a\left(\sqrt{g}g^{ab}\partial_b\Phi\right) - m^2\Phi^2 = 0 \tag{3.2}$$

which can be derived from the action integral,

$$I[\Phi] = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2}g(\partial\Phi,\partial\Phi) - \frac{1}{2}m^2\Phi^2\right)$$

and of course, g is the AdS$_{d+1}$ metric (or at least asymptotically locally AdS$_{d+1}$ form) and for convenience we shall use the global coordinates (t, r, Ω). So after substituting this form of the metric in eqn... 3.2 one obtains the following,

$$-\frac{1}{1+r^2/l_{AdS}^2}\partial_t^2\Phi + \frac{1}{r^{d-1}}\partial_r\left(r^{d-1}\left(1+r^2/l_{AdS}^2\right)\partial_r\Phi\right) + \frac{1}{r^2}\Delta^2_{S^{d-1}}\Phi - m^2\Phi = 0$$

where $\Delta_{S^{d-1}}$ is the Laplace-Beltrami operator in S^{d-1}. Next we separate variables by inserting the ansatz,

$$\Phi(t, r, S^{d-1}) = e^{-i\omega t}Y_{l,\{m\}}(S^{d-1})f(r)$$

where $Y(S^{d-1})$ are the spherical harmonics on S^{d-1},

$$\Delta^2_{S^{d-1}}Y_{l,\{m\}}(S^{d-1}) = -l^2(l^2+d-2)Y_{l,\{m\}}(S^{d-1}).$$

So the radial equation is,

$$\frac{1}{r^{d-1}}\frac{d}{dr}\left(r^{d-1}\left(1+r^2/l_{AdS}^2\right)\frac{df}{dr}\right) + \left(\frac{\omega^2}{1+r^2/l_{AdS}^2} - \frac{l^2(l^2+d-2)}{r^2} - m^2\right)f = 0$$

which in the asymptotic limit, $r \to \infty$, acquires a homogeneous form,

$$r^2\frac{d^2f}{dr^2} + (d+1)r\frac{df}{dr} - m^2l_{AdS}^2f = 0.$$

This has two independent solutions,

$$f(r) \sim r^{-\Delta_\pm},\ \Delta_\pm = \frac{d}{2} \mp \sqrt{\frac{d^2}{4} + m^2l_{AdS}^2}. \tag{3.3}$$

These two different asymptotic scalings for a field propagating in AdS is a generic result independent of the coordinate sets used. For example, in Poincare coordinates, one obtains again for an ansatz of the form,

$$\Phi(t, z, \mathbf{x}_{d-1}) = e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\chi(z),$$

the asymptotic scalings for component $\chi(z)$,

$$\chi(z) \sim z^{\Delta_\pm}.$$

The solution corresponding to asymptotic scaling exponent Δ_- is called a normalizable mode since for such solutions one can define a regular inner product. For a fixed a spacelike slice Σ, AdS$_{d+1}$ with coordinates $\mathbf{y}$ and an orthogonal, timelike coordinate t. Given two solutions u_1 , u_2 to the scalar wave equation, define the inner product:

$$(u_1, u_2) \equiv i\int_\Sigma d^d\mathbf{y}\sqrt{g}g^{00}\left(u_1^*\partial_t u_2 - \partial_t u_1\, u_2^*\right)$$

Hence these can be quantized straightforwardly. The solution corresponding to asymptotic scaling power Δ_+ is called a nonnormalizable mode since such solutions diverge as one approaches infinity and one can't construct a well-defined scalar product (Hilbert space norm) and these should be thought of some non-fluctuating classical value or expectation value of the field.[1]

These two modes fall under distinct irreducible representations of the AdS_{d+1} isometry group $SO(d,2)$, which as we have seen in the previous chapter is the Conformal group of Minkowski space $\mathcal{M}^{d,1}$. The exponents $\Delta_\pm$ are then simply the eigenvalue corresponding to the Dilatation operator, $\hat{D} = it\partial_t + i\mathbf{x}\cdot\nabla$ (in Poincare basis) i.e. the scaling/conformal dimensions of the dual field $\Phi_{0,b}$ in the Conformal Field Theory living at the conformal boundary.

Now that we have familiarized ourselves with the scaling behavior of fields in AdS we are in a state to make the duality precise. In order to define the path integral in eqn. 3.1, we must specify the behavior of fields at the AdS boundary. The boundary conditions can be specified by including nonnormalizable modes. These modes are precisely the modes that are identified in eqn. 3.1 with classical sources $\Phi_{0,b}$ coupling to boundary operators.

$$\Phi_{0,b} = \lim_{r\to\infty} \frac{\Phi_{non-normalizable}}{r^{-\Delta_+}},$$

and as such these modes should be frozen or non-fluctuating. Normalizable solutions to the wave equation are then used in the mode expansion of operators and the construction of a Fock space. The resulting Hilbert space of states is identified with the Hilbert space of the dual boundary CFT,

$$\mathcal{O} = \hat{a}\, u + \hat{a}^\dagger u^*, \quad u = \lim_{r\to\infty} \frac{\Phi_{normalizable}}{r^{-\Delta_-}}.$$

Here $\hat{a}$, $\hat{a}^\dagger$ are creation and annihilation operators. To reemphasize, nonnormalizable modes correspond to operator insertions in the boundary gauge theory; from the AdS point of view they provide non-trivial boundary conditions. The normalizable modes fluctuate in the bulk; quanta occupying such modes have a dual description in the boundary CFT Hilbert space.

However in [38], it was further argued that in fact both scaling behaviors are at par. The same bulk scalar lagrangian in fact corresponds to *two* distinct CFTs for corresponding to the two different asymptotic fall offs of bulk field. This is because as was demonstrated in [39] there are two different AdS-invariant quantizations of the scalar field with m^2 in the range,

$$-\frac{d^2}{4} < m^2 < -\frac{d^2}{4} + 1$$

One Lagrangian – in this case that of a scalar field of given mass squared m^2 – can give rise to two different quantum field theories in AdS space, depending on the

[1]By contrast in the interior, the normalizable solution is well-behaved while the nonnormalizable solution is ill-behaved.

choice of boundary condition by adding extra boundary terms to the bulk action, $I[\Phi]$. Then the generating functional for correlators in the theory with one choice of operator dimension is a Legendre transform of the generating functional in the theory with the other choice. So one can have the normalizable boundary field to correspond to the CFT vacuum expectation value of a scalar operator with conformal dimension same as the asymptotic fall-off, say $\mathcal{O}_{\Delta_-}$,

$$\phi_{0,\,normalizable} = \frac{1}{d-2\Delta_-}\langle \mathcal{O}_{\Delta_-}\rangle \tag{3.4}$$

3.2 AdS/CFT, Holography and Black holes

On the basis of the universal (horizon) area law scaling behavior of entropy of black holes it has been suggested that any consistent (quantum) unified theory i.e. one which incorporates gravity and matter must obey the *Holographic Principle* [40, 41, 42] - i.e. the fundamental degrees of freedom of a quantum gravity theory must be described by a local *nongravitational* field theory in one less (space) dimension - which is called the holographic dual. Thus AdS/CFT duality is an exact realization of the Holographic principle. Here the quantum gravity theory is type IIB superstring theory in $\mathrm{AdS}_5 \times S^5$ while the holographic dual is a $SU(N)$ gauge theory (which is also superconformal invariant). As mentioned in the previous section this development is a breakthrough since conventional means of quantizing gravity including early string theory were not fruitful since there was no norperturbative description. Black holes for example are in a sense nonperturbative objects and perturbative string theory is insufficient to describe black hole physics. So now that we have this dual gauge theory we can start addressing nonperturbative phenomenon in gravity - (quantum) physics of black holes and cosmology. The two key (and in fact closely related) mysteries concerning black holes which has been the subject of intense investigation for over four decades and still eluding a transparent resolution are the information loss puzzle [2] and entropy puzzle [4, 1]. The information loss puzzle which can be stated as loss of knowledge of the quantum state of the matter-energy which collapsed (to give rise to hole in the first place) after the black hole evaporates quantum mechanically i.e. local field theory in the presence of black hole directly imply loss of unitarity. Similarly the entropy puzzle which implies the degrees of freedom of a quantum theory of gravity scales with area rather than with volume as in local quantum field theory. So the holographic nature of the AdS/CFT duality makes a decisive verdict that quantum mechanical unitarity is preserved since the underlying theory is gauge theory but locality is lost since the dual theory is supported on a spacetime with one less spatial dimension.

Despite such a precise definition of the duality eqn. 3.1, the exact understanding of how a local bulk spacetime (gravity) emerges from an underlying gauge field theory with one less spatial dimension is still incomplete. This is because the duality in the form of 3.1 can only provide information about diffeomorphisms invariant observables which are nonlocal in the bulk in terms of (correlation functions) of *local* boundary operators. Issues related to locality and causality in the emergent spatial dimension (the *holographic* dimension). However, many of the quantum gravity questions we would like to address are not simply related to local boundary correlators. These questions include:

- How does a quasi-local bulk spacetime emerge from the CFT[2]?
- How does the region behind a causal horizon get encoded in the CFT?
- What is the CFT description (or perhaps resolution) of a black hole singularity?

To address the above questions one has to develop a set of tools for recovering local bulk physics from the CFT which we do in the next section.

3.3 Local bulk operators in AdS/CFT: The HKLL map

In a series of papers [18, 19, 20] the Lorentzian AdS/CFT map was reformulated to express local operators in the bulk in terms of non-local operators on the boundary. This map was done in the leading semiclassical approximation – meaning both large N and large 't Hooft coupling, λ considering only free scalar fields in AdS.

As we have seen before the general lesson of AdS/CFT is that the boundary ($z \to 0$) scaling behavior of fields propagating in AdS background

$$\lim_{z\to 0} \Phi(z, x) \sim z^{\Delta}\Phi_0(x),$$

imply the "boundary field", Φ_0 corresponds to conformal dimensions, Δ of some operator in the dual CFT, say $\mathcal{O}$,

$$\Phi_0(x) \leftrightarrow \mathcal{O}(x).$$

We shall restrict ourselves to *normalizable* modes[3]. Since the boundary value of the field can used to construct the on-shell field in the interior of the bulk, this identification then implies the existence of a map between local fields in the bulk and $non - local$ operators in the CFT,

$$\Phi(z, x) \leftrightarrow \int dx' K(x'|z, x)\mathcal{O}(x').$$

The kernel of integration, $K(x'|z, x)$ will be called the $Smearing\ function$[4]. Bulk-to-bulk correlation functions, for example, are then equal to correlation functions of the corresponding non-local operators in the dual CFT,

$$\langle\Phi(z_1, x_1)\Phi(z_1, x_1)\rangle_{SUGRA} \leftrightarrow \int dx'_1 dx'_2 K(x'_1|z_1, x_1)K(x'_2|z_2, x_2)\langle\mathcal{O}(x'_1)\mathcal{O}(x'_2)\rangle_{CFT}.$$

[2]Specifically how does one recover locality in the holographic or emergent fifth dimension.

[3]c.f. eqn. 3.4.

[4]Similar constructions were initiated in [36, 43, 44].In these works, one can view the construction of a local bulk operator as an integral over the entire boundary of AdS. Vanishing of commutators of local bulk operators at spacelike separation relies on delicate cancellations in this approach.

Just to be clear, this construction is performed in the limit,

$$\begin{aligned} l_s, l_p &\to 0 \qquad \text{in the bulk} \\ N, \lambda &\to \infty \qquad \text{in the boundary} \end{aligned}$$

Here l_s and l_p are the bulk string and Planck lengths, while N and λ are parameters for some kind of 't Hooft large-N expansion in the CFT whose details won't matter for us (for example, for $d = 3$, N could be the Chern-Simons level number).

Since smearing functions define the map by which, in the semiclassical limit, local bulk excitations are encoded on the boundary they are crucial for spacetime. The semiclassical limit tightly constrains behavior at finite N. For example, as we will see, smearing functions can be used to count the number of independent commuting operators inside a volume in the bulk, even at finite N. They can also be used to study bulk locality and causality: for example understand the causal structure of a black hole from the boundary point of view.

Let us summarize the principal features and implications of this smearing function construction as developed in [18, 19, 20]:

1. **Global coordinates:** The global basis smearing function can be chosen to have support on boundary points that are spacelike separated from the bulk point. This is illustrated in figure 3.1. The exact form of the smearing function depends on the dimension: for even-dimensional AdS it's given by,

$$\begin{aligned} K(\tau, \Omega|P) &= c_{d\Delta} \lim_{\rho\to\pi/2} (\sigma(x|P)\cos\rho)^{\Delta-d}\theta(\text{spacelike}) \\ c_{d\Delta} &= \frac{(-1)^{(d-1)/2} 2^{\Delta-D}\Gamma(\Delta - \frac{d}{2} + 1)}{\pi^{d/2}\Gamma(\Delta - d + 1)}. \end{aligned}$$

   for odd-dimensional AdS it's given by,

$$\begin{aligned} K(\tau, \Omega|x') &= a_{d\Delta} \lim_{\rho\to\pi/2} (\sigma(x|x')\cos\rho)^{\Delta-d} \log\big(\sigma(x|x')\cos\rho\big)\theta(\text{spacelike}), \\ a_{d\Delta} &= \frac{(-1)^{(d-2)/2} 2^{\Delta-d}\Gamma(\Delta - \frac{d}{2} + 1)}{\pi^{1+\frac{d}{2}}\Gamma(\Delta - d + 1)}. \end{aligned}$$

   and $\sigma(x = \tau, \rho, \Omega|x', \rho', \Omega')$ is the AdS-covariant distance function,

$$\sigma(x|x') = \frac{\cos(\tau - \tau') - \sin\rho\sin\rho'\cos(\Omega - \Omega')}{\cos\rho\cos\rho'}$$

2. **Smearing functions in the Poincare´e patch**: For even dimensional AdS the smearing function can be taken to have support at spacelike separation in the Poincar´e patch.

$$\begin{aligned} \phi(P) &= \int dT d^{d-1}X\, K_{\text{Poincare}}(T, X|P)\phi_0^{\text{Poincare}}(T, X) \\ K_{\text{Poincare}} &= 2c_{d\Delta} \lim_{Z\to 0} (\sigma(T, Z, X|P)Z)^{\Delta-d}\,\theta(\text{spacelike})\,. \end{aligned}$$

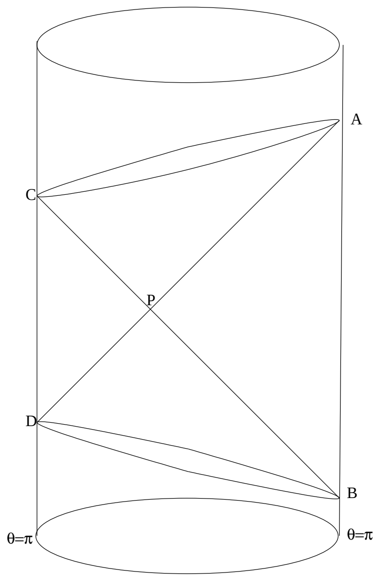

Figure 3.1: Global Smearing function: AdS is the (infinite) cylinder. For a local bulk insertion at P, the (compact) region of support of the global smearing function is the one within the spacelike lightcone (projection of this spacelike cone on $\theta = \pi$ section is the triangle PAB and PCD).

For odd dimensional AdS the smearing function has support on the entire Poincar´e boundary:

$$\begin{aligned}\Phi(P) &= \int dT d^{d-1}X\, K_+ \Phi_{0+}^{\text{Poincare}} + \text{c.c.},\\ K_+ &= a_{d\Delta}(\sigma Z)^{\Delta-d}\Big|_{T\to T-i\epsilon} \log|\sigma Z|\,.\end{aligned}$$

A function analytic in the lower half T plane gives vanishing result when integrated against $\Phi_{0+}^{\text{Poincare}}$. So we can even take K_+ to be given by the rather peculiar $i\epsilon$ prescription

$$K_+ = \frac{1}{2} a_{d\Delta}(\sigma Z)^{\Delta-d}\Big|_{T\to T-i\epsilon} \log(\sigma Z)\Big|_{T\to T+i\epsilon}\,.$$

3. **Rindler/BTZ-like coordinates**: This is perhaps the more interesting case since in these coordinates there is a acceleration horizon and it is very easy to make a spinless BTZ (by compactifying ϕ) which has a black hole event horizon. To construct a smearing function one must analytically continue the spatial coordinates of the boundary theory to imaginary values. This in fact enables one to find a smearing function with support on a compact region of

the complexified geometry. The explicit result for AdS_3 is given by,

$$\begin{aligned}\Phi(r,\phi,t) &\sim \frac{(\Delta-1)\,2^{\Delta-2}}{\pi r_+^2}\int_R dxdy \lim_{r'\to\infty}\left(\frac{\sigma}{r'}\right)^{\Delta-2}\Phi_0\left(\phi+i\frac{Ry}{r_+},t+\frac{R^2x}{r_+}\right)\\ &= \frac{(\Delta-1)\,2^{\Delta-2}}{\pi r_+^\Delta}\int_R dxdy\left(\frac{r}{r_+}\left(\cos y-\sqrt{1-\frac{r_+^2}{r^2}}\cosh x\right)\right)^{\Delta-2}\\ &\qquad\qquad \Phi_0\left(\phi+i\frac{Ry}{r_+},t+\frac{R^2x}{r_+}\right)\end{aligned}$$

In these expressions $\Delta = 1+i\sqrt{m^2-1}$. However by analytically continuing $m^2 \to -m^2$ we can take Δ to coincide with the conformal dimension in AdS. Since $\sigma > 1$ in the domain of integration this analytic continuation is straightforward.

The interesting insights which are obtained by constructing these smearing functions are as follows:

1. By representing bulk operators as operators on the boundary with compact support – in fact with support that is as small as possible – we can have bulk operators whose dual boundary operators are spacelike separated. Such bulk operators will manifestly commute with each other just by locality of the boundary theory. This statement will continue to hold at finite N. [45]

2. A very interesting fact that is trivially expressed in such a setup was that the vanishing of commutators of a pair of local bulk insertions which are (spacelike) separated purely in the z-direction *implied* that the corresponding smeared CFT operators must commute although they have large overlapping support on the boundary. This is shown in figure 3.2.

3. UV/IR connection: Since the size of the smeared operator is determined by the radial position of the bulk point, this is the sharpest statement of the radius/scale duality [46, 47]. In Rindler coordinates, the smearing integral can be reduced to a compact region of the complexified geometry, whose size shrinks to zero as the bulk point approaches the boundary. This makes scale-radius duality manifest. For example, a bulk point at radius r gets smeared over a range of time δt on the boundary given by

$$\cosh\frac{r_+\delta t}{2R^2} = 1\Big/\sqrt{1-\frac{r_+^2}{r^2}}.$$

This is just the elapsed time between the point on the boundary which is lightlike to the future of the bulk point at the same value of ϕ, and the point on the boundary which is lightlike to the past at the same ϕ.

4. The bulk scalar two-point function is singular when bulk points are coincident or are lightlike separated. Since the smearing functions are finite and have

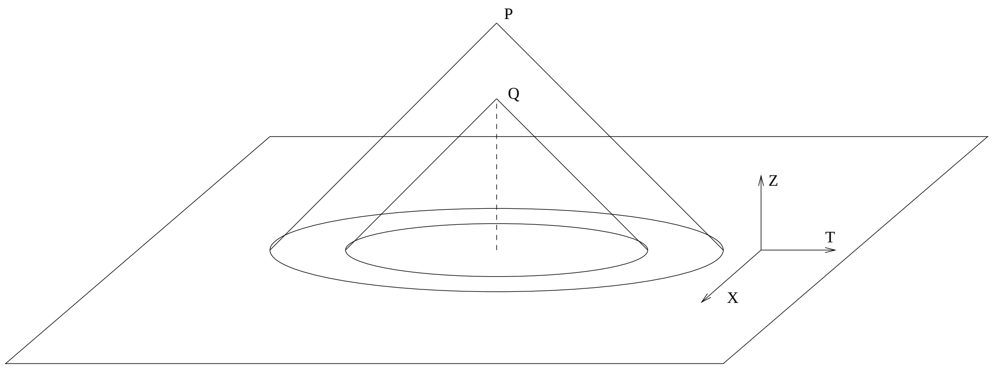

Figure 3.2: Diagram showing the overlap of the smearing regions corresponding to bulk insertions P and Q separated only in the Z direction i.e. having identical transverse coordinates. These bulk insertions are spacelike separated but their smearing functions have non-vanishing overlap (the overlap of the two circles on the T, X plain which is the boundary). The vanishing of bulk commutator demands that despite the overlap the respective dual CFT (smeared) operators commute at $N \to \infty$

compact support, it's easy to see that this singularity can only arise from UV singularities in the boundary theory. This means that regions inside the bulk are not related to a boundary theory with a conventional UV cutoff, so there is no UV/IR relationship in the sense of relating bulk IR and boundary UV cutoffs.

5. Implications for finite N: Hitherto we have focused purely in the $N \to \infty$ regime i.e. the $l_{Pl} \to 0$ or pure classical supergravity. But how does the picture modify at finite N. At finite N one does not expect a semiclassical bulk spacetime anymore. In [20, 21] it was shown that even at finite N, the same smearing function in Poincare coordinate provide the unique covariant way to map a primary field in the CFT to a scalar field in the bulk. But these bulk operators fail to commute even at spacelike separations and respect the holographic bound.

3.4 Smearing functions for AdS$_2$ collapse geometries

In the present section we generalize these results to an asymptotically AdS$_2$ spacetime with a null collapsing shockwave. The outline of this section is as follows. In subsection 3.4.1 we review the coordinate system(s) employed for this Vaidya spacetime and provide the Penrose diagram. We begin by constructing the boundary to bulk map for points outside the black hole horizon in section 3.4.2. First, in subsection 3.4.2.1 we consider a massless scalar, where the result is identical to the pure AdS$_2$ case when written using global null coordinates, despite the time-dependent geometry. In subsection 3.4.3 we deal with the slightly more complicated massive case where we need to propagate the field across the shock using appropriate matching conditions. The smearing function again is only nonvanishing in the spacelike separated region on the boundary.

The results are extended to points inside the horizon in subsection 3.4.4. It is still possible to express local bulk fields in terms of CFT operators on the single boundary at infinity, provided these are analytically continued to complex values of the boundary coordinates. Thus we accomplish the aim of completely characterizing a local theory propagating in a *time-dependent* bulk geometry in terms of the analytic boundary theory.

3.4.1 The AdS$_2$ Vaidya spacetime

We consider a 2d black hole which can be obtained by a dimensional reduction of a BTZ formed by the gravitational collapse of a null dust in 3d asymptotically AdS spacetime [48, 49] given by the metric,

$$ds^2 = -f(r)dv^2 + 2dvdr = -f(r)dt^2 + \frac{dr^2}{f(r)} \qquad (r > 0)\,, \tag{3.5}$$

where

$$dt = dv - \frac{dr}{f(r)}\,,$$

and

$$f(r) = \begin{matrix} r^2 - r_0^2, & v \geq 0 \\ 1 + r^2, & v < 0 \end{matrix}\;.$$

The two-dimensional action is the Jackiw-Teitelboim gravity theory [50, 51]. Such a black hole is formed by doing cylindrical reduction on a spacetime with a negative cosmological constant Λ as a result of the gravitational collapse of a null dust shell. We set $\Lambda = -1$ from now on.

Tortoise coordinates (t, r_*) are defined as,

$$t = v - \int \frac{dr}{f(r)} \tag{3.6}$$

and

$$r_* = \int_\infty^r \frac{dr}{f(r)} = \begin{matrix} \frac{1}{2r_0} ln\left|\frac{r-r_0}{r+r_0}\right|, & v \geq 0 \\ \tan^{-1} r - \pi/2 & v < 0 \end{matrix}\;.$$

Here the constant of integration $-\pi/2$ is chosen to make $r_* \to 0$ as $r \to \infty$ from both sides of the shock. In order to distinguish between the local coordinates defined in the regions $v \geq 0$ and $(v < 0)$, we shall use the superscript $+$ and $-$ respectively. The range of the local coordinates are $-\infty < r_*^+ \leq 0$ for points outside the horizon $(r > 0)$ and $-\pi/2 < r_*^- < 0$. Inverting the above relation, we have $r_*^\pm$ in terms of r,

$$r = -r_0 \coth(r_*^+ r_0) = -\cot r_*^-\,. \tag{3.7}$$

The Penrose diagram is shown in figure 3.3.

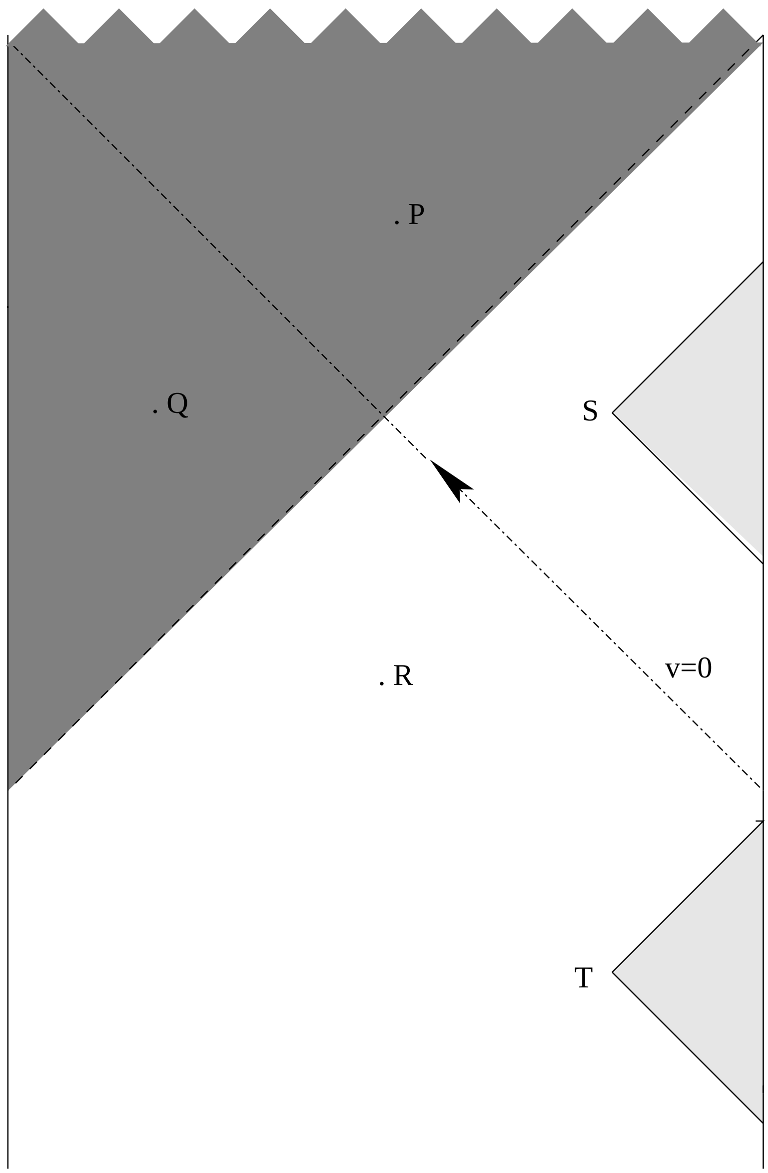

Figure 3.3: Penrose diagram

3.4.2 Boundary/Bulk duality outside the horizon

3.4.2.1 Zero Mass

A free massless scalar minimally coupled to Jackiw-Teitelboim gravity, propagating in such a black hole spacetime satisfies the equation of motion,

$$\frac{1}{\sqrt{-g}}\partial_a(\sqrt{-g}g^{ab}\partial_b\phi) = 0\,.$$

3.4.2.2 Smearing function for point having support to either the past or future of the shock

In tortoise coordinates (3.6) the wave equation is simply

$$(-\partial_t^2 + \partial^2_{r_*^{\pm}})\phi = 0\,.$$

Now Green's theorem is,

$$\phi(x) = \int dS'\sqrt{h}\eta^a\left(\partial'_a\phi(x')G(x,x') - \phi(x')\partial'_a G(x,x')\right)$$

where h is the induced metric and η is the normal derivative on the boundary at infinity. We begin by considering points such as S and T in figure 3.3. If we can find a Green's function $G(r'_* - r_*, t' - t)$ which has support inside the spacelike cone extending from the bulk point to the boundary, e.g.

$$G \propto \theta(r'_* - r_* - |t - t'|)\theta(r'_* - r_*)\,,$$

then we can extract the desired smearing function. We obtain such a Green's function using contour integration,

$$(-\partial_t^2 + \partial^2_{r_*})G(r'_* - r_*, t' - t) = \delta(r'_* - r_*)\delta(t' - t)\,. \tag{3.8}$$

In momentum space,

$$(\omega^2 - k^2)G(k,\omega) = 1\,.$$

So formally,

$$G(r'_* - r_*, t' - t) = \int \frac{d\omega dk}{(2\pi)^2}\frac{1}{\omega^2 - k^2}e^{i(k\Delta r_* - \omega\Delta t)}$$

where $\Delta r_* = r'_* - r_*$, $\Delta t = t' - t$. Now lets do the k integration first by closing the contour in the upper half-plane,

$$\begin{aligned}
G(r'_* - r_*, t'-t) &= -\int \frac{d\omega}{2\pi} e^{-i\omega\Delta t} \int \frac{dk}{2\pi} \frac{1}{(k-i\epsilon)^2 - \omega^2} e^{ik\Delta r_*} \\
&= \frac{1}{i}\theta(\Delta r_*) \int \frac{d\omega}{2\pi} e^{-i\omega\Delta t} \frac{e^{i\omega\Delta r_*} - e^{-i\omega\Delta r_*}}{2\omega} \\
&= \frac{1}{4}\theta(\Delta r_*)\left(\mathrm{sgn}(\Delta r_* + \Delta t) + \mathrm{sgn}(\Delta r_* - \Delta t)\right) \\
&= \frac{1}{2}\theta(\Delta r_*)\theta(\Delta r_* - |\Delta t|)\,.
\end{aligned}$$

Now the normalizable modes have the asymptotic fall off,

$$\phi(r_*, t) \sim r_* \phi_0(t)$$

as $r_* \to 0$ since for a massless minimally coupled scalar the scaling dimension is $\Delta = \frac{1}{2} + \sqrt{\frac{1}{4} + m^2R^2} = 1$. So according to to Green's theorem,

$$\phi(r_*, t) = \int dt' \phi_0(t') G(0 - r_*, t' - t) = \frac{1}{2}\int_{t+r_*}^{t-r_*} dt' \phi_0(t')$$

(recall that $r_* < 0$) and hence the smearing function is simply the Green's function with compact support on spacelike separated region on the boundary.

3.4.2.3 Smearing function for point with support to both past and future of shock

In this section we consider points such as R from figure 3.4. For convenience we will use null coordinates for the calculation. For a null ray

$$2dr/f(r) - dv = 0$$

Introduce new coordinate u

$$2r_* - v = u$$

where u is a constant over the ray. At the shock $v = 0$, we have the relation

$$u = 2r_*$$

So according to (3.7) the values of u (on the same null ray) across the shock are related by,

$$r_0 \coth(u^+ r_0/2) = \cot(u^-/2)$$

or,

$$u^- = h(u^+) = 2\cot^{-1}(r_0 \coth(u^+ r_0/2))\,.$$

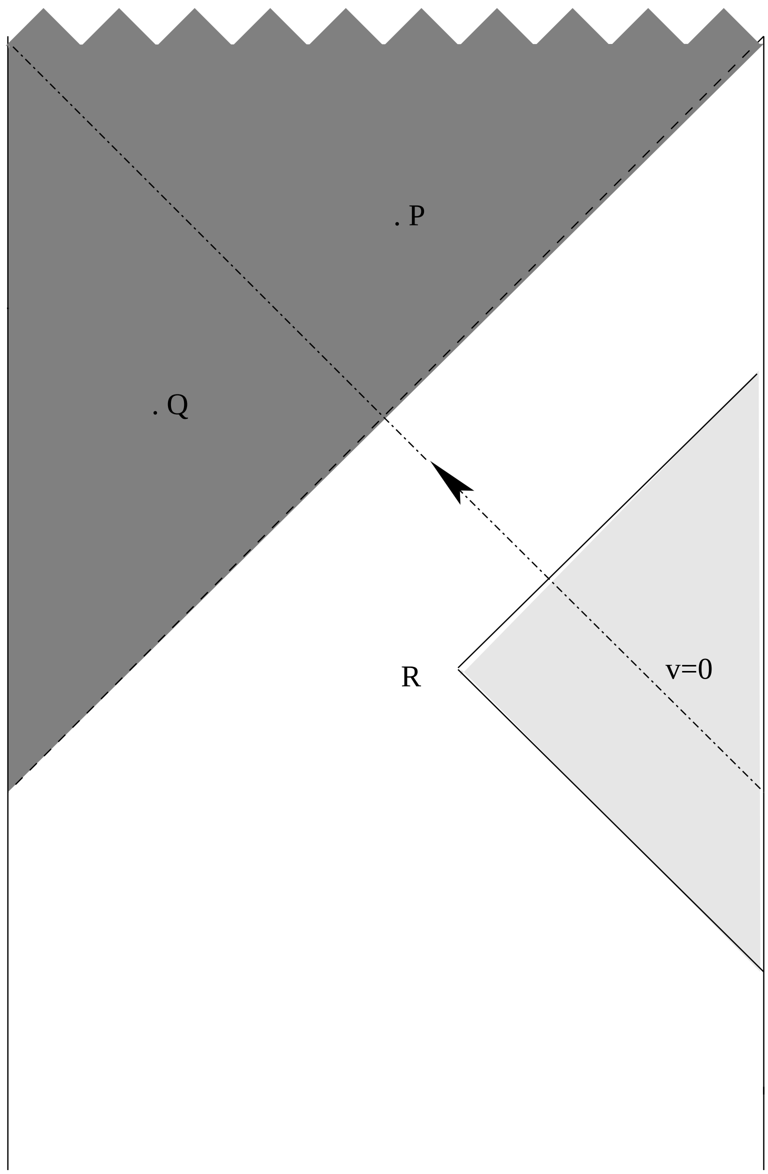

Figure 3.4:

So now we can make a **global** null coordinate,

$$u = \begin{cases} u^-, & v < 0 \\ h(u^+), & v > 0 \end{cases} .$$

In u,v parameters, the metric is,

$$ds^2 = F(u,v)dudv$$

where the conformal factor is,

$$F(u,v) = \begin{cases} f\left(r(u,v)\right) = \csc^2\left((u+v)/2\right), & v < 0 \\ f\left(r(h^{-1}(u),v)\right)\frac{dh^{-1}(u)}{du} = \frac{2r_0^2 \csc h^2\left(\coth^{-1}((\cot(u/2))/r_0)+vr_0/2\right)}{(1+r_0^2)\cos u-(1-r_0^2)}, & v > 0 \end{cases} .$$

Spacelike infinity $r_* = 0$ is where

$$u^{\pm} + v = \theta(v)h(u) + \theta(-v)u + v = 0\,.$$

In the global u, v parameters, the Green's function equation for a massless minimally coupled scalar is,

$$2\partial_u\partial_v G(u-u', v-v') = \delta(u-u')\delta(v-v')$$

So, we obtain

$$G(u-u', v-v') = \frac{1}{2}\theta(v'-v)\theta(u'-u)\,.$$

This is a **global** Green's function since the coordinate u is continuous across the shock. Again this Green's function is nonvanishing only in the spacelike separated region on the right boundary and hence furnishes a smearing function which has a compact support on the timelike boundary at infinity,

$$\begin{aligned}
\phi(u,v) &= \frac{1}{2}\int_{-\infty}^{0} dv' \left(G(u-u', v-v')\partial_{u'+v'}\phi(u',v')\right)|_{u'=-v'} + \\
&\qquad \frac{1}{2}\int_0^\infty dv' \left(\frac{dh}{du}(v')\right)^{1/2} \left(G(u-u', v-v')\partial_{h(u')+v'}\phi(u',v')\right)|_{h(u')=-v'} \\
&= \int_{-\infty}^{\infty} dv' \frac{1}{2}\left(\theta(-v')\theta(u+v')\theta(v-v') + \theta(v')\theta(u-h^{-1}(-v'))\theta(v-v')\right) \\
&\qquad \left(\frac{dh}{du}(v')\right)^{1/2} \phi_0(v')\,.
\end{aligned}$$

Recall that normalizable fields have asymptotic behavior $\phi(u,v) \sim (u+v)\phi_0(v)$ where $\lim_{u\to -v}\phi_0(v) \neq 0$ for $v < 0$ while for $v > 0$, we have $\phi \sim (h(u)+v)\phi_0(v)$ with $\lim_{h(u)\to -v}\phi_0(v) \neq 0$ again. So the smearing function for massless fields in Kruskal coordinates is,

$$K\left(v'|u,v\right)=\frac{1}{2}\left(\theta(-v')\theta(u+v')\theta(v-v')+\theta(v')\left(\frac{dh}{du}(v')\right)^{1/2}\theta(u-h^{-1}(-v'))\theta(v-v')\right).$$

The expression clearly demonstrates the fact that the smearing function cannot be expressed in terms of a AdS_2-covariant distance function since the metric is discontinuous (non-analytic) across the shock and cannot be covered by a single analytic coordinate patch although locally it is pure AdS_2 everywhere. The $u^{\pm}, v$ coordinates parametrize null geodesics and $u^{\pm}$ suffers a jump across the shock. As a result, in terms of these coordinates the smearing function has a discontinuity across the shock. Nevertheless the smearing function has support only for points on the right boundary spacelike separated from the bulk point, showing the construction of [18, 19, 20] generalizes to time-dependent bulk geometries.

3.4.3 Non-vanishing mass

The massless minimally coupled field is insensitive to the conformal factor in the metric in the u, v coordinates which leads to a very simple Green's function in these coordinates. On the other hand, the massive scalar does feel the conformal factor,

$$\frac{1}{\sqrt{-g}}\partial_a(\sqrt{-g}g^{ab}\partial_b G(x,x'))-m^2G(x,x')=\frac{1}{\sqrt{-g}}\delta(x-x')\,.$$

Now, lets go back to local parameters $r_*^{\pm}$, $t^{\pm}$ since that is more suitable for calculation,

$$ds^2=\begin{cases}\csc^2 r_*^-(-dt^{-2}+dr_*^{-2}), & v<0\\ r_0^2\csc h^2(r_*^+r_0)(-dt^{+2}+dr_*^{+2}), & v>0\end{cases}$$

and are related by continuity across the surface of the shock by,

$$\begin{aligned} r &= -r_0\coth(r_*^+r_0)=-\cot r_*^- \\ t^+ &= v-r_*^+(r_*^-)=t^-+r_*^--r_*^+(r_*^-)\,. \end{aligned} \tag{3.9}$$

The Green's function for the $v<0$ region with support over the spacelike separated boundary region is the same as the one for pure AdS_2 with support over space like separated region on the right boundary and was worked out in[18] for arbitrary non-vanishing mass,

$$G^-(r_*'^- - r_*^-, t'^- - t^-)=\frac{1}{2}P_{\Delta-1}(\sigma)\theta(r_*'^- - r_*^-)\theta(r_*'^- - r_*^- - |t'^- - t^-|)$$

where $\Delta=\frac{1}{2}+\sqrt{m^2+\frac{1}{4}}$ and the AdS_2 covariant distance function,

$$\sigma(x,x')=\frac{\cos(t'^- - t^-)-\cos r_*^-\cos r_*'^-}{\sin r_*^-\sin r_*'^-} \tag{3.10}$$

and the $P_{\Delta-1}(\sigma)$ is the Legendre function.

For the $v > 0$ region, the relevant Green's function is the one for a point outside the horizon of a BTZ black hole with support on the spacelike region of the right boundary. This is again just the pure AdS_2 global Green's function transformed to the set of local coordinates[18],

$$G^{+}(x,x') = \frac{1}{2}P_{\Delta-1}(\sigma)\theta(\rho'-\rho)\theta(\rho'-\rho-|\tau-\tau'|)$$

where ρ,τ coordinates are defined in terms of the old r, t^{+} by,

$$\begin{aligned} \frac{\cos\tau}{\sin\rho} &= -\ \frac{r}{r_0} \\ \frac{\sin\tau}{\cos\rho} &= \ \tanh t^{+} r_0 \end{aligned} \tag{3.11}$$

and the metric is then,

$$ds^2 = \csc^2\rho(-d\tau^2 + d\rho^2)\,.$$

In these coordinates we define the boundary field $\phi_0(\tau)$,

$$\phi_0(\tau) = \lim_{\rho\to 0}\frac{\phi(\tau,\rho)}{\sin^{\Delta}\rho}$$

and for the $v < 0$ region,

$$\phi_0(t^{-}) = \lim_{r_*^{-}\to 0}\frac{\phi(t^{-},r_*^{-})}{\sin^{\Delta} r_*^{-}}\,.$$

3.4.3.1 Matching conditions across the shock

- Continuity of the field

The field is continuous across the shock,

$$\phi(v=0^{+},r) = \phi(v=0^{-},r).$$

- Continuity of the normal derivative of the field

By integrating the d'Alembertian over a Gaussian surface straddling $v = 0$ and noting that there are no sources of the ϕ-field on $v = 0$,

$$\begin{aligned} \int d^2x\sqrt{-g}\left(\frac{1}{\sqrt{-g}}\partial_a(\sqrt{-g}g^{ab}\partial_b\phi) - m^2\phi\right) &= 0 \\ \eta_a(\sqrt{-g}g^{ab}\partial_b\phi(x))|^{v=\epsilon} - \ \eta_a(\sqrt{-g}g^{ab}\partial_b\phi(x))|^{v=-\epsilon} &= 0 \end{aligned}$$

where η_a is the outward unit normal vector to $v = \mathrm{const.}$. In the global coordinates (r, v), the normal derivative is just $\partial_r\phi$ and then the condition is simply,

$$\partial_r\phi(v=0^{+},r) = \partial_r\phi(v=0^{-},r)\,.$$

3.4.3.2 Smearing function for a point with its right spacelike separated regions extending to either sides of the shock

In this case we have a local bulk insertion for a point like R in figure 3.4. Our strategy is to first use Green's theorem to express the normalizable bulk field at x^- (the superscript indicates it is in the $v<0$ region) in terms of an integral over spacelike infinity of the $v<0$ region *and* the surface of the shock $v=0$, which bounds the spacelike separated region to the right of x^-

$$\begin{aligned}\phi(x^-) &= \int dt'^- \left((2\Delta-1)G^-(x^-,x'^-)\sin^{\Delta-1} r'^-_* \phi_0(t'^-)\right)|_{r'^-_*\to 0} \\ &\qquad + \int dr'^-_* \left(\phi(x'^-)\left(\overleftrightarrow{\partial}_{r'^-_*} - \overleftrightarrow{\partial}_{t'^-}\right)G^-(x^-,x'^-)\right)_{r'^-_*+t'^-=0} .\end{aligned}$$

Then using the matching conditions across the shock we convert the integral over the shock to an integral over the timelike infinity of the $v>0$ region

$$\int dr'^-_* \left(\phi(x'^-)\left(\overleftrightarrow{\partial}_{r'^-_*} - \overleftrightarrow{\partial}_{t'^-}\right)G^-(x^-,x'^-)\right)_{r'^-_*+t'^-=0}$$

$$= \int d\tau' \phi_0(\tau') \left(\int dr'^-_* K^+(\tau'|\rho,\tau)\left(\overleftrightarrow{\partial}_{r'^-_*} - \overleftrightarrow{\partial}_{t'^-}\right)G^-(x^-,x'^-)\right)\Bigg|_{\substack{\rho+\tau = r'^-_* + t'^- = 0 \\ \rho'=0}}$$

where $\rho = \rho(r'^-_*, t'^-)$, $\tau = \tau(r'^-_*, t'^-)$ and K^+ is the smearing function for a field in the $v>0$ region. So, the full smearing function is,

$$\begin{aligned}K(x|x') &= \theta(-t'^-)(2\Delta-1)\left(G^-(x^-,x'^-)\sin^{\Delta-1} r'^-_*\right)|_{r'^-_*\to 0} \\ &\quad + \theta(\tau')\left(\int dr'^-_* K^+(\tau'|\rho,\tau)\left(\overleftrightarrow{\partial}_{r'^-_*} - \overleftrightarrow{\partial}_{t'^-}\right)G^-(x^-,x'^-)\right)\Bigg|_{\rho+\tau=r'^-_*+t'^-=0}\end{aligned} \tag{3.12}$$

The first term is simply the AdS_2 global smearing function as worked out in [18],

$$K_1 = \theta(-t'^-)(2\Delta-1)\left(G^-(x^-,x'^-)\sin^{\Delta-1} r'^-_*\right)|_{r'^-_*\to 0}$$

$$\theta(-t'^-)\frac{2^{\Delta-1}\Gamma(\Delta+1/2)}{\sqrt{\pi}\Gamma(\Delta)}\left(\frac{\cos(t'^- - t^-) - \cos r^-_*}{\sin r^-_*}\right)^{\Delta-1}\theta(-r^-_* - |t'^- - t^-|). \tag{3.13}$$

The second term in (3.12), the integral along the shock, is a bit more involved. Explicitly (dropping the overall $\theta(\tau')$),

$$K_2 = \int dr'^-_* K^+(\tau'|\rho,\tau)(\overleftrightarrow{\partial}_{r'^-_*} - \overleftrightarrow{\partial}_t)G^-(r^-_*, t^-|r'^-_*, t'^-) \tag{3.14}$$

where

$$K^{+}(\tau'|\rho,\tau) = \frac{2^{\Delta-1}\Gamma(\Delta+1/2)}{\sqrt{\pi}\Gamma(\Delta)}\left(\frac{\cos(\tau'-\tau)-\cos\rho}{\sin\rho}\right)^{\Delta-1}\theta(-\rho-|\tau'-\tau|)$$

and

$$G^{-}(r_*'^{-}-r_*^{-},t'^{-}-t^{-}) = \frac{1}{2}P_{\Delta-1}(\sigma)\theta(r_*'^{-}-r_*^{-})\theta(r_*'^{-}-r_*^{-}-|t'^{-}-t^{-}|)$$

where $\Delta = \frac{1}{2}+\sqrt{m^2+\frac{1}{4}}$ and the AdS$_2$ covariant distance function is given in (3.10). K_2 is more conveniently expressed as a sum of two contributions

$$K_2 = K_{2I} + K_{2II}$$

where

$$\begin{aligned}
K_{2I} &= \theta(\tau')\int dr_*'^{-}K^{+}(\tau'|\rho,\tau=-\rho)(\partial_{r_*'^{-}}-\partial_t)G^{-}(r_*^{-},t^{-}|r_*'^{-},t'^{-})|_{t'^{-}=-r_*'^{-}} \\
&= \left(-2\sin\frac{\tau'}{2}\right)^{\Delta}\left(\frac{2^{\Delta-2}\Gamma(\Delta+1/2)}{\sqrt{\pi}\Gamma(\Delta-1)}\right)\times \\
&\quad \int dr_*'^{-}\left(\frac{\sin(\tau'/2+\rho)}{\sin\rho}\right)^{\Delta-1}\theta(-\rho-|\tau'+\rho|)\left(\frac{\cos r_*'^{-}-\cos t^{-}}{\sin r_*^{-}\sin^2 r_*'^{-}}\right)\times \\
&\qquad \frac{\sigma P_{\Delta-1}-P_{\Delta-2}}{\sigma^2-1}\theta(\Delta r_*^{-})\theta(\Delta r_*^{-}-|r_*'^{-}-t^{-}|) \quad + \\
&\quad \frac{1}{2}\theta(\tau')\theta\left(-\frac{r_*^{-}+t^{-}}{2}\right)\left(K^{+}(\tau'|\rho,\tau=-\rho)\,P_{\Delta-1}(\sigma)\right)|_{r_*'^{-}=\frac{r_*^{-}-t^{-}}{2}=-t'^{-}}
\end{aligned}$$

while

$$\begin{aligned}
K_{2II} &= \int dr_*'^{-}\left(G^{-}(r_*^{-},t^{-}|r_*'^{-},t'^{-})(\partial_{r_*'^{-}}-\partial_{t^{-}})K_1(\rho,\tau|\tau')\right)|_{\substack{\tau=-\rho \\ t'^{-}=-r_*'^{-}}} \\
&= \left(-2\sin\frac{\tau'}{2}\right)^{\Delta}\left(\frac{2^{\Delta-3}\Gamma(\Delta+1/2)}{\sqrt{\pi}\Gamma(\Delta-1)}\right)\times
\end{aligned}$$

$$\int dr_*'^{-}P_{\Delta-1}(\sigma)\theta(\Delta r_*^{-})\theta(\Delta r_*^{-}-|-r_*'^{-}-t^{-}|)\frac{\sec^2 r_*'^{-}}{1/r_0+r_0\tan^2 r_*'^{-}}\frac{\sin^{\Delta-2}(\tau'+\rho/2)}{\sin^{\Delta}\rho}\theta(-\rho-|\tau'+\rho|)$$

where on the shock (3.9) reduces to

$$\tan\rho = \tan\rho(r_*'^{-}) = r_0\tan r_*'^{-}\,.$$

So, the full smearing function in this case looks complicated as the bulk geometry changes across the shell from pure AdS$_2$ to a AdS$_2$ black hole, but nevertheless has support only on the right boundary region spacelike separated from the bulk point.

3.4.4 Points inside the horizon

As has been pointed out in[18] for operator insertions at points inside the future Rindler horizon in pure AdS_2, the smearing function has support on both boundaries i.e. boundaries of the left and right Rindler wedges. For integer conformal dimension Δ,

$$\phi(P) = \int_{-\infty}^{\infty} dt\, K^R_{Rindler}(t|P)\phi_0^{Rindler,R}(t) + (-)^\Delta K^L_{Rindler}(t|P)\phi_0^{Rindler,L}(t) \tag{3.15}$$

where

$$\begin{aligned} K^R_{Rindler}(t|P) &= \frac{2^{\Delta-1}\Gamma(\Delta+1/2)}{\sqrt{\pi}\Gamma(\Delta)} \lim_{r\to\infty}\left(\frac{\sigma}{r}\right)^{\Delta-1}\theta(\sigma-1) \\ K^L_{Rindler}(t|P) &= \frac{2^{\Delta-1}\Gamma(\Delta+1/2)}{\sqrt{\pi}\Gamma(\Delta)} \lim_{r\to\infty}\left(-\frac{\sigma}{r}\right)^{\Delta-1}\theta(-\sigma-1) \end{aligned}$$

and $\sigma = \sigma(t, r|P)$ is the AdS_2 invariant distance. [5]

However the integral over the left Rindler boundary can be mapped back to the right Rindler boundary using the identification,

$$\phi_0^{Rindler,L}(t) = \phi_0^{Rindler,R}(t+i\pi)\,.$$

This identification was arrived at after noting that a complex change of Rindler coordinates takes one from the left to the right Rindler wedge,

$$t \to t + i\pi.$$

So combining these information we have a smearing function which has compact support entirely on the *complexified* right boundary.

$$\phi(P) = \phi(P) = \int_{-\infty}^{\infty} dt\, K^R_{Rindler}(t|P)\phi_0^{Rindler,R}(t) + (-)^\Delta K^L_{Rindler}(t|P)\phi_0^{Rindler,R}(t+i\pi)\,. \tag{3.16}$$

A similar result for the BTZ black hole was also derived in [20].

Now let us extend these results to the shockwave geometry. In this case we no longer have the same global isometries used in the derivation of the expressions (3.15) and (3.16). However for bulk points to the future of the shell such as point P in figure 3.3, by analytic continuation, we can pretend to be in an eternal AdS_2 black hole and the expression (3.16) carries over [20].

New features appear when we study points to the past of the shock, but inside the horizon, such as Q in figure 3.3. For this point we first use the spacelike Green's

[5] For non-integer Δ, the result is a bit more complicated and can be found in[18]. The upshot is that a local bulk operator inside the black hole horizon is delocalized over both the left and right boundaries.

function to express the bulk scalar in terms of an integral on $v = 0^-$ and the portion of the boundary $r_*^- = 0$ on the $v < 0$ side

$$\phi(Q) = \int dt'^- K_1(Q|t')\phi_0(t'^-) + \int dr_*'^- \left(\phi(x'^-)\left(\overleftrightarrow{\partial}_{r_*'^-} - \overleftrightarrow{\partial}_{t'^-}\right) G^-(Q, x'^-)\right)_{r_*'^- + t'^- = 0} .$$

We divide the integral on the shock into two parts, one outside and the other inside the horizon $r = r_0$ ($r_*^- = \tanh^{-1} r_0 - \pi/2$)

$$\begin{aligned}\phi(Q) \quad = \quad & \int dt'^- K_1(Q|t')\phi_0(t'^-) + \int dr_*'^- \left(\phi(x'^-)\left(\overleftrightarrow{\partial}_{r_*'^-} - \overleftrightarrow{\partial}_{t'^-}\right) G^-(Q, x'^-)\right)_{v=0, r<r_0} \\ & + \int dr_*'^- \left(\phi(x'^-)\left(\overleftrightarrow{\partial}_{r_*'^-} - \overleftrightarrow{\partial}_{t'^-}\right) G^-(Q, x'^-)\right)_{v=0, r>r_0}\end{aligned}$$

where K_1 is same as in (3.13). Now we use the matching conditions to express the integral over the shock *outside the horizon* to an integral over the boundary $r_*^+ = 0$ just as was done for the point R. Finally for the integral over the $v = 0^-$ surface inside the horizon ($r < r_0$) side we again first match it to quantities just across the shell i.e. $v = 0^+$ and then express it in terms of an integral over analytically continued (complexified) right boundary just as done for the point P. The final expression is,

$$\begin{aligned}\phi(Q) \quad = \quad & \int dt'^- K_1(Q|t')\phi_0(t'^-) + \int d\tau' K_2(Q|\tau')\phi_0(\tau') \\ & + \int d\tau' K_{3I}(Q, \tau')\phi_0^{Rindler,R}(\tau') + \int d\tau' K_{3II}(Q, \tau')\phi_0^{Rindler,R}(\tau' + 3i\pi)\end{aligned} \tag{3.17}$$

where K_2 is given by (3.14) while K_3's are

$$\begin{aligned}K_{3I}(Q, \tau') \quad &= \quad \int dr_*^- \left[K^R_{Rindler}(x'^-|\tau')\left(\overleftrightarrow{\partial}_{r_*'^-} - \overleftrightarrow{\partial}_{t'^-}\right) G^-(Q, x'^-)\right]_{v=0, r>r_0} \\ K_{3II}(Q, \tau') \quad &= \quad \int dr_*^- \left[(-)^\Delta K^L_{Rindler}(x'^-|\tau')\left(\overleftrightarrow{\partial}_{r_*'^-} - \overleftrightarrow{\partial}_{t'^-}\right) G^-(Q, x'^-)\right]_{v=0, r>r_0} .\end{aligned}$$

This is a highly nontrivial result. The bulk theory describes propagation in a non-analytic geometry. The boundary CFT nevertheless inherits analytic amplitudes from boundary locality. By exploiting the analyticity of the boundary theory, the expression (3.17) shows how propagation in the bulk geometry is reconstructed. Bulk non-analyticity shows up only in the smearing function factors.

3.4.5 Conclusions

In this section we looked at a model time-dependent black hole geometry which has a *single* asymptotic AdS_2 boundary and constructed a bulk-boundary map which represents local on-shell bulk field operators as non-local boundary operators with compact support but on the boundary. We used analyticity of the boundary CFT as a tool to construct such maps for points inside the black hole horizon. The smearing function is very similar to the one for a pure AdS_2, for points outside the horizon, the function has support over only spacelike separated boundary points. For points inside the horizon the integral is delocalized over the single boundary at infinity, and requires a complex time contour.

One can obtain a more general time-varying geometry by collapsing a series of such null shells. The present construction should generalize straightforwardly to this case, using the general time dependent Vaidya metric [48]. It will also be interesting to generalize this construction to AdS bubble solutions relevant to cosmology, such as those studied in [52, 53, 54].

3.4.6 APPENDIX

3.4.6.1 Coordinate changes

For $v > 0$ region, we used v, r set, t^+, r_*^+ set and also τ, ρ set to suit our purpose. Here we review their transformations into each other and also with the t^-, r_*^- coordinates (used in the $v < 0$) region at $v = 0$.

$$r = -r_0 \coth(r_*^+ r_0) = -\cot r_*^-$$

which implies,

$$r_*^+ = -\frac{1}{r_0} \coth^{-1}\left(\frac{r}{r_0}\right) \tag{3.18}$$

and,

$$r_*^+|_{v=0} = \frac{1}{r_0} \coth^{-1}\left(\frac{\cot r_*^-|_{v=0}}{r_0}\right) \tag{3.19}$$

Similarly,

$$t^+ = v - r_*^+(r) = v + \frac{1}{r_0} \coth^{-1}\left(\frac{r}{r_0}\right) \tag{3.20}$$

implies,

$$t^+|_{v=0} = -\frac{1}{r_0} \coth^{-1}\left(\frac{\cot r_*^-|_{v=0}}{r_0}\right) \tag{3.21}$$

Finally,

$$\rho = \sin^{-1}\left(\frac{1}{\coth^2 r_0 r_*^+ \cosh^2 r_0 t^+ - \sinh^2 r_0 t^+}\right)^{1/2} \tag{3.22}$$

so $\rho \to 0$ as $r_*^+ \to 0$ i.e. $r \to \infty$ and on the surface of the shock,

$$\rho|_{v=0} = \sin^{-1}\left(\frac{\sinh^2 r_0 r_*^+}{\cosh 2r_0 r_*^+}\right)^{1/2} \tag{3.23}$$

and similarly,

$$\tau = \sin^{-1}\left(\frac{1}{\cosh^2 r_0 r_*^+ \coth^2 r_0 t^+ - \sinh^2 r_0 r_*^+}\right)^{1/2} \tag{3.24}$$

and,

$$\tau|_{v=0} = -\rho|_{v=0} \tag{3.25}$$

Finally, the relation between the t^-, r_*^- and τ, ρ coordinates,

$$\frac{\cos\tau}{\sin\rho} = -\frac{r}{r_0} = \frac{1}{r_0}\cot r_*^- \tag{3.26}$$

$$\sin(\tau+\rho) = \tanh(r_*^- + t^-)r_0 \tag{3.27}$$

Chapter 4

AdS$_4$ Holography and (Punctuated) Inflation

4.1 Introduction

Making contact with realistic cosmology remains the fundamental challenge for any candidate unified theory of matter and gravity. Precise observations [71, 72] indicate that the current epoch of the acceleration of universe is driven by a very mild (in Planck units) negative pressure, positive energy density constituent - “dark energy”. Likewise there is strong evidence that the Big Bang phase of the universe was preceded by a exponentially accelerated (inflation) phase [73, 74] driven by a negative pressure, positive energy fluid. In most cosmological models this inflationary stage is brought about by a scalar field coupled to gravity, slowly “rolling down” an almost flat potential hill, and upon reaching the bottom of the hill gives rise to the big-bang stage. In most such models inflation is inevitably “future eternal” i.e. there are always some residual regions which keep on rapidly inflating. The steady state picture that emerges is a fractal multiverse structure with many “pocket universes” causally separated by continually inflating sterile regions. These “pocket universes” might possess all possible different “fundamental constants” of nature (including a cosmological constant). So in this scenario one now talks about “environmental constants” of nature instead.

AS mentioned previously that (Super)string theory is the currently leading candidate for an unified theory of fundamental interactions. So it should be compatible with realistic cosmology if it has to be anything more than an attractive toy model. It is great news that string theory appears to have an enormous “landscape” of vacua including many long-lived metastable states with positive cosmological constants [75] forming a quasicontinuum [76, 77, 78, 79] which are highly relevant for realistic cosmology. These metastable phases ultimately decay to neighboring stable anti-de Sitter phases via “rolling down the hill” and quantum mechanical tunneling and this is exactly what one needs to attempt a fundamental physics model of the (inflationary) multiverse. So it is important to develop tools in the fundamental theory which describe such transitions.

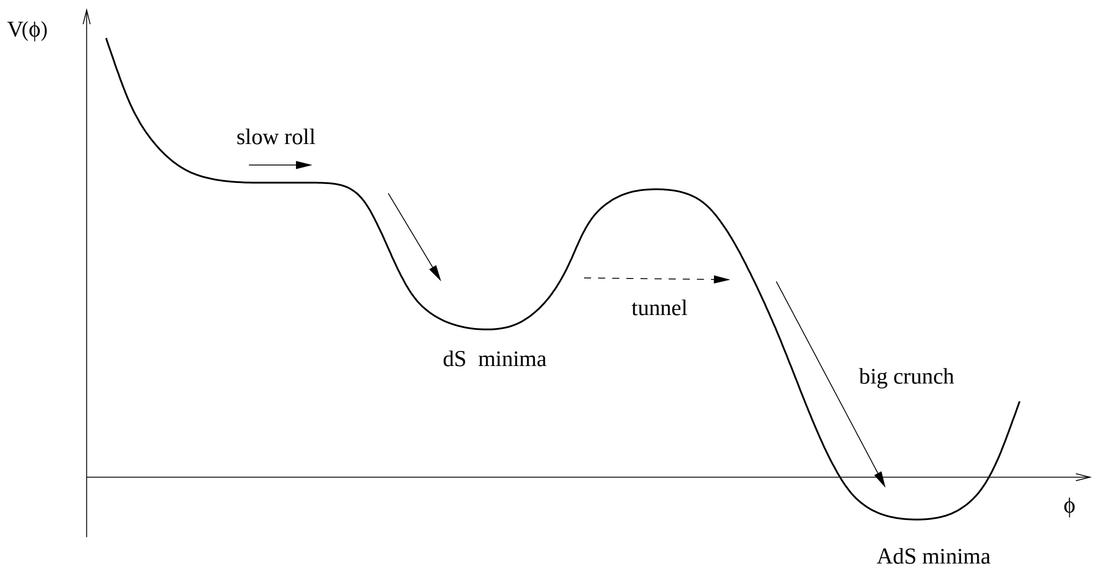

Figure 4.1: The potential landscape showing the dS metastable and AdS stable vacua. The arrows indicate the time evolution of a possible worldline.

The AdS/CFT formulation of String theory [24] is the potentially nonperturbative definition of superstring theory in asymptotically AdS spaces and is an perfect setting to embed the string theory landscape. The neighborhood of the landscape that we study is summarized in figure 4.1.

The first step within this approach was taken in [52] which looked at classical AdS-Reissner-Nordstrom/dS domain wall type geometries (an inflating dS bubble interior patched to an asymptotic AdS black hole spacetime) in the thin wall approximation. Classical bubble spacetimes without charge were further investigated in [53] and subsequently in [54]. The latter has an added attractive feature that the mysterious Gibbons-Hawking entropy of de Sitter space could be identified as a (logarithm of) subspace of black hole microstates. The aim of the present paper is to build on the work of [54] by attempting to identify the CFT states which code these inflating states in a thermal ensemble, and further explore the consequences of this scenario.

The chapter is organized as follows. In section 4.2, we review the semiclassical consistency condition of [54] which arises from interpreting de Sitter entropy within a unitary framework for quantum gravity. This amounts to the condition that the number of states available must be bounded below by the Gibbons-Hawking entropy if states corresponding to a truly semiclassical region of de Sitter space exist. . Then in section 4.3, we review the classical bubble spacetimes. Keeping in mind the dual CFT as the underlying quantum description we are forced to rule out the whole class of time-symmetric bubble solutions based on the consistency condition. In particular, this rules out tunneling solutions of the type described in [80]. In section 4.4, we focus on the CFT picture of the time-asymmetric solutions compatible with the aforementioned consistency condition and their signatures in the dual CFT. We model the effect of the radiation reflecting off the shell by reducing the problem to that of a moving mirror in a black hole background. In section 4.5 we discuss consistency checks of such a scenario. In section 4.6we draw various conclusions regarding the implications for such a picture for cosmology and how several problems/paradoxes that plague the eternal inflation scenario - namely the measure problem, the youngness paradox and the Boltzmann brains paradox are overcome in this setting.

4.2 Quantum De Sitter state

In [54] it had been advocated that the Gibbons-Hawking entropy of dS space can be correctly interpreted as the logarithm of the dimension of quantum gravity state (Hilbert) space. This proposition was based on numerical observations of the entropy of the present universe, dominated by a contribution from the CMB photons. We review this rationale briefly here once again. The entropy of CMB photons see, for example, [81] is,

$$S_{CMB} \sim 10^{88}$$

with wavelength, $\lambda_{CMB} \sim 1$ mm, while the Gibbons-Hawking entropy of dS space with the present day cosmological constant is,

$$S_{GH} \sim 10^{123}.$$

So with a field theory cut off with about $\epsilon = 1$ TeV, the total number of states in effective field theory associated with CMB photons is $(\lambda_{CMB}\epsilon)^3 S_{CMB} \sim 10^{138}$, way too large to be accounted by the de Sitter entropy.

From past experience with AdS/CFT we know that the notion of a cut off in the bulk effective field theory (EFT) is complex and in particular dependent on the respective quantum (CFT) state, unlike the conventional uniform cut off in usual effective field theories [20, 19, 18, 22]. So an EFT in the bulk with an adaptive step size cutoff which allocates up to 10^{35} states per CMB photon would saturate the Gibbons-Hawking entropy of dS space with the present day cosmological constant. Hence we are justified in saying that Gibbons-Hawking de Sitter entropy does count (logarithm of) the number of possible states in EFT, albeit with a clever cut-off [54].

AdS/CFT also taught us that generic states in the quantum theory (CFT) have no nice bulk space time interpretation. But in addition to just a smooth space time geometry (metric) once also requires large families of observables to reproduce the entire span of semiclassical physics on this smooth geometry. This observation coupled with the above numerical bound on the entropy of the universe leads us to the following hypothesis for the de Sitter state in a quantum theory of gravity:

- **Semiclassical Consistency Hypothesis (SCH):** *The set of microstates representing a semiclassical de Sitter region must number at least* $e^{S_{GH}}$ *.*

4.3 Inflation in AdS/CFT: dS/SAdS domain walls

Let us briefly review the geometric set up which can be found in great detail in [82, 52, 53]. We have a domain wall geometry by patching together a (interior) dS-region with cosmological constant (c.c.) $\Lambda_{dS} = \frac{3}{l_{dS}^2}$ and an (exterior) AdS-Schwarzschild geometry with c.c set to $\Lambda_{AdS} = \frac{3}{l_{AdS}^2}$ and hole mass M across a thin spherical shell (parametrized $r = R(\tau)$ where τ is the proper time of the shell. The world volume metric of the shell is then given by the equation,

$$ds^2 = -d\tau^2 + R(\tau^2)d\Omega^2$$

The geometry is specified by the following metric and the coordinate ranges,

$$ds^2 = \begin{cases} -f_{dS}(r)dt_{dS}^2 + \frac{dr^2}{f_{dS}(r)} + r^2 d\Omega^2, & f_{dS}(r) = 1 - \frac{r^2}{l_{dS}^2}, \qquad r < R(\tau) \text{ (inside)} \\ -f_{SAdS}(r)dt_{SAdS}^2 + \frac{dr^2}{f_{SAdS}(r)} + r^2 d\Omega^2, & f_{SAdS}(r) = 1 - \frac{2GM}{r} + \frac{r^2}{l_{AdS}^2},\ r > R(\tau) \text{ (outside)} \end{cases}$$

Here r is a global coordinate and the times inside and outside the bubble are related by equating the invariant distance along the shell from inside and outside.

$$\left(-f_{dS}(r)dt_{dS}^2 + \frac{dr^2}{f_{dS}(r)}\right)_{r=R(\tau)} = -d\tau^2 = \left(-f_{SAdS}(r)dt_{SAdS}^2 + \frac{dr^2}{f_{SAdS}(r)}\right)_{r=R(\tau)} \tag{4.1}$$

The consistent junction conditions relate the discontinuity of the extrinsic curvature across the shell to the stress energy tensor of the shell (defining $\kappa = 4\pi G\sigma$, with σ the surface energy density of the shell),

$$\sqrt{\dot{R}^2 + f_{SAdS}(r)} - \sqrt{\dot{R}^2 + f_{dS}(r)} = \kappa R$$

which determine the shell trajectory (substituting $r = R(\tau)$) can be rewritten

$$\dot{r}^2 + V_{eff}(r) = 0 \tag{4.2}$$

so the trajectory is that of a zero net energy particle moving in potential,

$$V_{eff}(r) = -\frac{\left(\frac{1}{l_{dS}^2} + \kappa^2 - \frac{1}{l_{AdS}^2}\right)^2 + \frac{4}{l_{dS}^2 l_{AdS}^2}}{4\kappa^2} r^2 + 1 + \frac{GM\left(\frac{1}{l_{dS}^2} + \frac{1}{l_{AdS}^2} - \kappa^2\right)}{\kappa^2}\frac{1}{r} - \frac{G^2M^2}{\kappa^2 r^4} \tag{4.3}$$

in one space dimension. The solutions can be broadly classified into two categories - one when the parameters are such that the potential has a maximum that is positive and when it is negative. In the first case the zero energy particle is reflected off the barrier and the bubble traces out a time symmetric trajectory, in some choice of coordinates, while in the second case the particle rolls over to the top of the potential hill to the other side and the bubble traces out a time asymmetric trajectory , figure 4.2.

As shown in [53] for all the time symmetric cases:

$$r_{bh} < r_{dS}$$

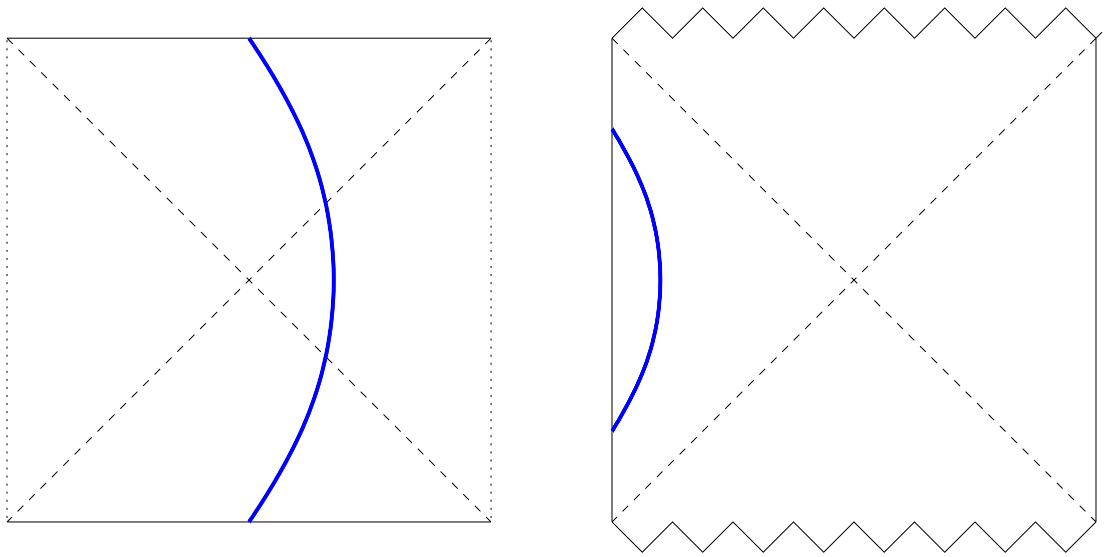

Figure 4.2: A Time symmetric bubble trajectory: The left diagram is dS spacetime and the right diagram is SAdS spacetime. The blue curve is the bubble trajectory. The domain wall geometry is made up of the region to the left side of the blue curve in the dS spacetime diagram and to the right side of the blue curve of the AdS spacetime diagram.

i.e. the black hole horizon radius is less than the de Sitter horizon length and this implies,

$$S_{GH} > S_{BH} \, ,$$

with S_{BH} the Bekenstein-Hawking entropy of the black hole.

Now let us consider a black hole such that $r_{bh} > l_{AdS}$ so that the black hole is well-described by the canonical ensemble in the CFT. The available microstates, $e^{S_{BH}}$, must exceed the microstates of the internal dS bubble, $e^{S_{GH}}$, according to the SCH. There are simply not enough microstates in the dual CFT to constitute the whole range of semiclassical phenomena in the interior of the bulk dS bubble. So the SCH rules out all such time symmetric bubble configurations in the quantum picture. In particular no such false vacuum dS bubbles can be formed by quantum mechanical tunneling from AdS spacetime. Thus we see in the AdS/CFT framework, the type of tunneling events considered in [80, 83] do not occur. Once any kind of local tunneling process allows arbitrarily large regions of semiclassical spacetime to be formed, we quickly violate unitarity, so this is an important new constraint that arises from the CFT viewpoint, that would not be seen via a straightforward semiclassical gravity analysis.

We are left with the time asymmetric situation where the bubble emerges from the past singularity $R(\tau = 0) = 0$ and keeps on expanding indefinitely i.e. $r(\tau \to \infty) \to \infty$ to simulate an eternally inflating bubble of de Sitter space inside an asymptotic AdS space. The conformal diagram of such a geometry is shown in figure 4.3.

This can be ensured if the outward normal points to increasing r when $r \to \infty$

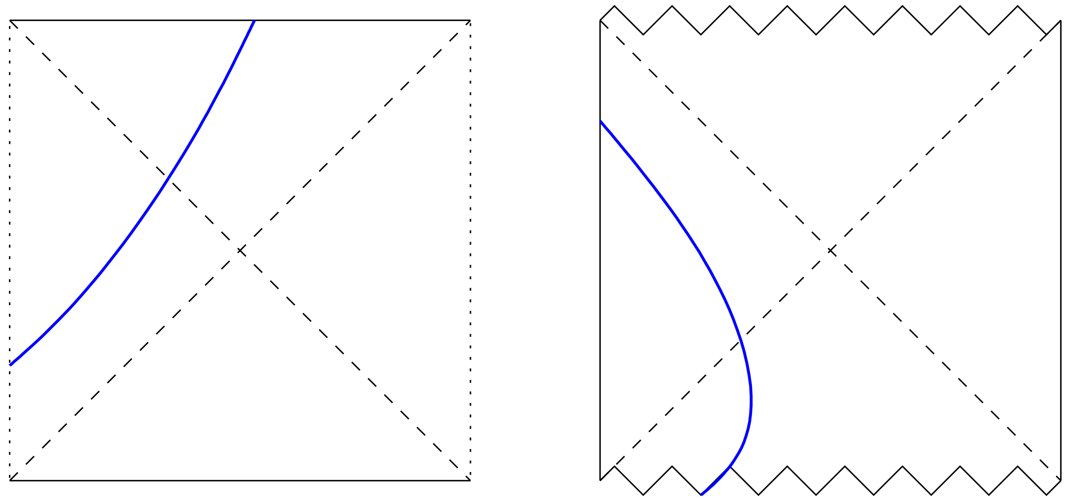

Figure 4.3: Time asymmetric trajectory : The interior dS region is shown on the left, and the exterior SAdS region is shown on the right. The domain wall geometry is made up of the regions to the left of the blue curve in the dS diagram and to the right of the SAdS diagram joined across the bubble wall (blue curve).

i.e. by having $\beta_{dS} > 0$ which gives

$$\kappa^2 > \frac{1}{l_{dS}^2} + \frac{1}{l_{AdS}^2}\,.$$

For these time asymmetric cases Gibbons-Hawking dS entropy can be accounted for by the black hole microstates (i.e. one can arrange for $r_{bh} > l_{dS}$) by having

$$\left(2GMl_{AdS}^2\right)^{1/3} > l_{dS}.$$

After taking into account back reaction as was shown in [54], the left asymptotic boundary of the bubble exterior ceases to exist as this region is unstable to perturbations and a singularity develops. In the bubble interior, any worldline eventually tunnels back into the stable AdS vacuum, but the tunneling portion of the bubble undergoes a big crunch [84].

4.4 THE CFT PICTURE

4.4.1 Schwarzschild anti-de Sitter black hole

The basic setup we consider is shown in figure 4.5. A large black hole is formed from the collapse of a null shell sent in from infinity into empty AdS_4 spacetime. This process is expected to be described by a pure state within a single conformal field theory. Under time evolution, the state is thermalized, and late-time correlation functions will be well-approximated by those in the canonical ensemble, at temperature corresponding to the Hawking temperature of the black hole, or alternatively in the microcanonical ensemble at fixed energy.

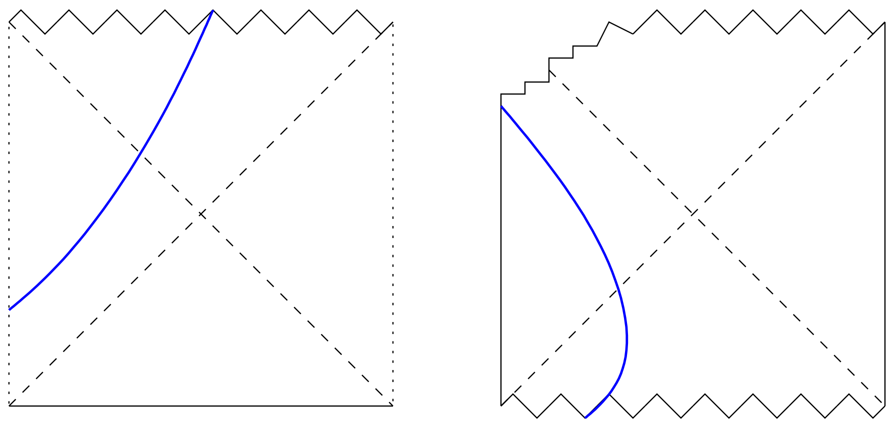

Figure 4.4: Penrose diagrams after taking into account perturbations. The left diagram shows the bubble trajectory in dS spacetime. The region to the right of the bubble wall is excised. Eventually any given worldline tunnels back into an AdS region, leading to a big crunch. The diagram on the right shows the trajectory through AdS spacetime. The region to the left of the wall is excised. Perturbations lead to a Cauchy horizon with a null singularity meeting the bubble wall at infinity.

The conformal field theory is unitary and can be viewed as living on the left conformal boundary $\mathbb{R}\times S_2$. Since the spatial directions are compact the CFT entropy in these statistical ensembles will be finite and given by the Bekenstein-Hawking entropy of the black hole S_{BH}. For timescales of order the Poincare recurrence time $e^{S_{BH}}$, the correlators will deviate wildly from the thermal correlators [85, 86, 87, 88, 89, 90]., but ultimately will settle down again to quasi-thermal correlators. This is represented in figure 4.5 by singularities separating well-behaved semiclassical regions with a SAdS black hole exterior.

Note that some of these regions apparently have two disconnected boundary regions at infinity [91] (such as the second region from the top in figure 4.5). However, in line with the setup in the previous paragraphs, these asymptotic regions are not to be viewed as completely independent of each other, but rather as representing analytic continuation of modes with finite energy from a single exterior region, dual to states in a single CFT. Ultimately these bulk geometries are approximate descriptions of the time evolution of a pure state in a single CFT.

In general states emerging from the white hole singularities in figure 4.5 will contain all excitations present in the thermal ensemble. In particular, the bubble solutions of section 4.3 will appear with nonvanishing probability. Since the Poincare recurrences go on forever, we have an infinite number of times to produce bubbles with realistic cosmological interiors, including inflation, while at any given time only a finite number of CFT states need be considered. These transitions populate the string theory landscape in a *timelike* manner instead of a *spacelike* manner as in standard eternal inflation [92, 93, 94, 95, 96, 97, 98, 99]. We refer to this scenario as *punctuated eternal inflation.* As we will see, many of the attractive features of eternal inflation survive in this picture, while at the same time, problems of the cosmological measure and many other issues are ameliorated.

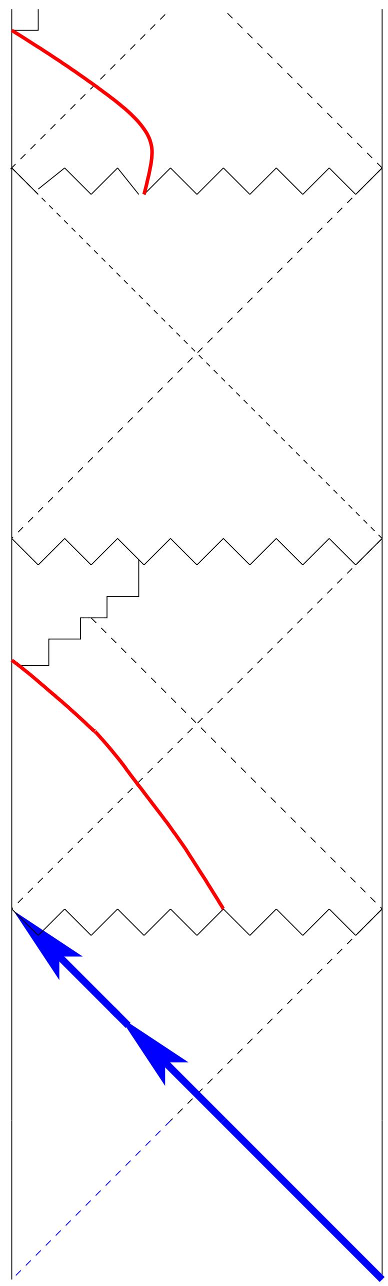

Figure 4.5: Punctuated Eternal Inflation: Starting with empty AdS space, dual to the vacuum state of the CFT, a light-like shell, shown by the blue line, collapses into a black hole. A bubble with dS interior, shown by the red curve, appears out of the spacelike singularity. The left side of the red line i.e. the bubble wall is a piece of dS space. This sequence repeats, following the quasiperiodical Poincare recurrences in the dual gauge theory.

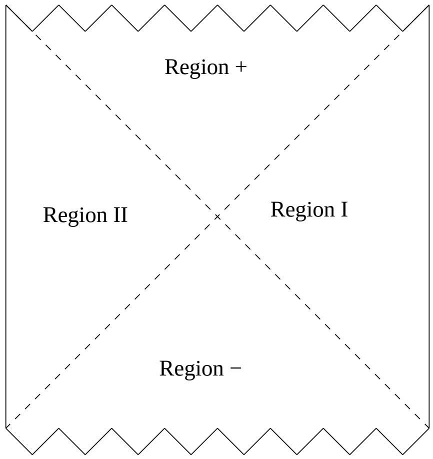

Figure 4.6: The two asymptotic regions of SAdS

4.4.2 Bubble states in the CFT

It is helpful to begin by first considering modes of a scalar field with finite global energy, and the relevant vacuum state. Let us review Unruh's construction [100] adapted to the eternal SAdS geometry. The geometry includes two asymptotic regions related via analytic continuation. Any finite frequency Kruskal modes can be continued analytically across the horizon and used to provide a complete basis in the full SAdS manifold. Likewise, Rindler modes can be patched together to define global modes. For the Rindler modes, positive frequencies are defined with respect to asymptotic timelike killing vectors, and positive frequency for global modes is defined with respect to the null killing vector at the horizon when the metric is written in Kruskal-like coordinate[100].

The two types of vacuum state are related by

$$|0\rangle_{Kruskal} = \prod_{\omega} \exp(e^{-4\pi GM\omega}\, b^{\dagger}_{1\omega} b^{\dagger}_{2\omega})|0\rangle_1 \times |0\rangle_2$$

where $1, 2$ indicate the left/right asymptotic regions of SAdS and $b_{1/2}$ ($b^{\dagger}_{1/2}$) are the respective Rindler annihilation (creation) operators. The Kruskal annihilation operator can expressed in terms of the creation and annihilation operators on the left and right regions as follows,

$$a^{\dagger Kruskal}_{\omega} = b^{\dagger}_{1\omega} + e^{-4\pi GM\omega} b_{2\omega}\,. \tag{4.4}$$

Thus we see that a finite energy mode well-localized in one asymptotic region will necessarily have an exponentially suppressed component in the other asymptotic region. These finite frequency bulk modes may then be mapped to operators in the conformal field theory via the standard bulk to boundary map of [15, 16], generalized to the eternal black hole background.

Now we have so far made the convenient approximation that the bubble wall is thin. The essential details of the construction are not expected to change if we generalize to consider walls of finite thickness, that might be built from finite frequency

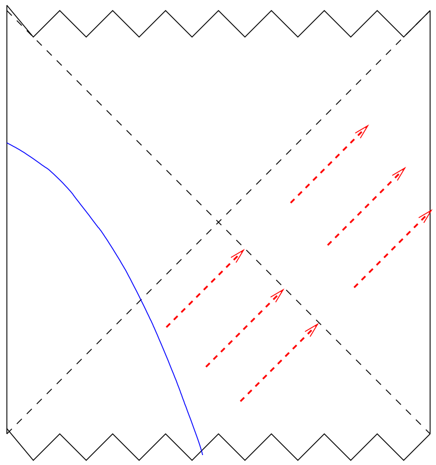

Figure 4.7: Radiation coming off the bubble walls

modes. Moreover, we expect operators creating and annihilating bubbles to obey a similar equation to (4.4). This is good news for representing such operators in the CFT, since they may be reconstructed using the positive and negative frequency modes in only the right asymptotic region, which may then be mapped into the CFT. In addition to this leading order effect, further details of the bubble state can be extracted from the radiation coming off the bubble wall to the left asymptotic region (depicted by the red dashed arrows in the figure 4.7).

To study the effects of the radiation coming off the bubble we model the bubble wall as a perfect mirror i.e. impose purely reflecting boundary conditions for the (scalar) field propagating in the SAdS background and restrict ourselves spherical symmetry. The computation is sketched in Appendix 4.8. Further specification of the bubble would require the particular details of the metastable string theory vacuum giving rise to the potential shown in figure 4.1. Nevertheless, for a given potential and choice of initial state in the gravity dual, one has a well-defined (one-to-many) mapping into the CFT.

At this point let us collect a few facts on the spectrum of bubble states, which will be useful in the subsequent sections. One could compute frequencies of normalizable excitations around the bubble geometry by solving the gravity equations of motion. However this spectrum is expected to receive important corrections due to back-reaction and quantum effects. The CFT enables us to take these effects into account. Even without all the details of the CFT dual that incorporates a bulk potential of the form figure 4.1, some qualitative features of the spectrum can be determined. The domain wall states considered in the present scenario are viewed as a subset of black hole microstates. These are dual to states in the CFT$_3$ on the spacetime $\mathbb{R} \times S_2$, therefore the entire spectrum of CFT states will be discrete. We can schematically write the microcanonical density matrix to be,

$$\rho(E) = \sum_{a,b,\ldots} |\psi_{a,b,\ldots.}\rangle\langle\psi_{a,b,\ldots}|$$

where

$$\mathbf{H}|\psi_{a,b,....}\rangle = E|\psi_{a,b,....}\rangle$$

are energy eigenstates with *discrete* internal labels $a, b, ...$. For large total energy E this can be well-approximated by the density matrix of the canonical ensemble

$$\rho(\beta) = \sum e^{-\beta \mathbf{H}} |\psi_{a,b,\cdots}\rangle\langle\psi_{a,b,\cdots}|$$

where β is the black hole inverse temperature. This may then be directly mapped into the canonical ensemble in the CFT at inverse temperature β. This will be our basic working definition of the ensemble of states that contains inflating bubbles of the type shown in figure 4.7. As we will see, this is already sufficient to provide interesting bounds on the measures of cosmological interest.

Of course the canonical ensemble also contains many states that do not represent cosmological solutions of interest. One could construct a more refined ensemble by selecting on black holes with expanding interiors. Local bulk correlators can be constructed in these spacetimes (including points inside the bubble) by generalizing the two-dimensional construction of [22]. Correlators of massless fields will exhibit a characteristic power law falloff with geodesic separation inside the bubble, and this geodesic separation depends directly on the vacuum energy inside the bubble. Using the methods of [18, 19, 20] these correlators may be mapped into integral transforms of correlators in the CFT. This yields a straightforward, though calculationally tedious way of selecting the vacuum energy inside the bubbles.

Another, more direct way, to select on expanding bubble states is to use characteristic radiation coming off the bubble (as shown in figure 4.7) to refine the specification of the CFT state. As mentioned above, this is explored in a simplified model in appendix 4.8. On the gravity side the solution is that of a black hole with a set of quasinormal modes excited. The amplitudes for these modes will be determined by the bubble initial state, and the detailed properties of the potential of figure 4.1. This can be represented by a time-dependent density matrix in the CFT, that will, in general, depend on the vacuum energy inside the bubble.

4.5 Consistency checks

The semiclassical consistency hypothesis of section 4.2 by construction produces quantum versions of de Sitter space with a sensible semiclassical limit as $\hbar \to 0$ and $S_{GH} \to \infty$. In this section we perform some consistency checks on the approach to the semiclassical limit, with a view to keeping S_{GB}, S_{BH} and $\hbar$ finite to use as a physically sensible regulator for eternal inflation. The detailed scenario we have in mind for the remainder of the paper is shown in figure 4.1, where a worldline begins in a rapidly inflating phase, that later settles into a metastable de Sitter vacuum, that will be modeled as the late-time development of bubbles solutions such as those shown in figure 4.7.

We begin by studying the late-time history of this worldline. From the original work [101, 84] we know that a worldline in the dS bubble tunnels back to the stable

AdS vacuum. This event is accompanied by extra excitation energy that quickly leads to a big crunch. Related tunneling solutions within the AdS/CFT context have recently been studied in [102]. From an estimate of the tunneling rate we can we know how long this worldline lasts in proper time. This time is (see, for example [75])

$$\tau_{decay} \sim e^{S(\phi)} e^{S_{GH}}, \tag{4.5}$$

where $S(\phi) < 0,\ |S(\phi)| \gg 1$ and is just the $O(4)$ invariant Euclidean action for the tunneling trajectory from dS to AdS space. This may be converted into a timescale in the CFT by mapping to Schwarzschild time in the exterior AdS region. This calculation is performed in appendix 4.7, eqn. (4.17).

$$t_{decay,SAdS} \approx B - \frac{Al_{AdS}^2}{l_{dS}} e^{-e^{cS_{GH}}},\ 0 < c < 1$$

where A, B are constants defined in appendix (4.7). Thus the decay time is typically shorter than a Planck time in SAdS coordinates, from the moment when the bubble wall reaches the null singularity in the exterior region, as shown in figure (4.4). For such short timescales we should not trust the semiclassical gravity description, though the CFT description is valid. We therefore conclude that this tunneling process happens long after conventional semiclassical physics has broken down in the dS bubble.

The Poincare recurrence time for de Sitter space $\tau_{Poincare,\, dS} \sim e^{S_{GH}}$ is already much longer than the tunneling timescale (4.5). This eliminates all the issues one encounters when the decay time exceeds the recurrence time, for instance breakdown of a classical general relativity by reversal of the arrow of time in a semiclassical regime as discussed in [103].

The finest time resolution we can reasonably hope to resolve in the bulk is

$$\Delta t_{SAdS} = l_{pl}$$

the Planck time. Translating this back into de Sitter proper time (as shown in appendix 4.7) we obtain

$$\tau_{dS} = l_{dS} \log\left(\frac{Al_{AdS}^2}{l_{pl} l_{dS}}\right). \tag{4.6}$$

As we will later see, this timescale will typically be $l_{dS}10^d$ where d is some number of order 1. Thus we see that in this scenario our semiclassical de Sitter bubble survives for a far shorter than one might have imagined based on tunneling probabilities, before the discreteness of the CFT makes its presence felt. Nevertheless, as we will see, for reasonable parameter choices, our present cosmological history can be embedded in this scenario.

Now let us address the question of whether for times smaller than (4.6) the CFT can accurately reproduce correlators inside the dS bubble. This issue arises in studies of the black hole information problem, where the CFT must encode the details of the internal state of the black hole. As shown in [104, 105], for effective field theory in black hole backgrounds, the departure from locality on observations made over times less than the scale in Schwarzschild coordinates $\Delta t_{SAdS} \sim S_{BH}\beta_H$ is less than the scale $e^{-S_{BH}}$ (for example the amplitude of Hawking emission) following information theoretic arguments first invoked in [106]. Recall that the AdS$_5$/CFT$_4$ map implies $S_{BH} \propto N^2$ and hence $e^{-S_{BH}} \sim \frac{1}{e^{N^2}}$ represents effects which are non-perturbative in $1/N$ in the dual gauge theory. For the $d = 3$ case the dual CFT is expected to be a generalization of the CFT studied in [107]. There the CFT was realized as a Chern-Simons theory with Chern-Simons level number k playing the role of gauge coupling, g^2_{YM}.

The observer in the dS bubble will only be able to measure, for example, energy with an intrinsic error of $1/l_{dS}$ given the above discussion. This is already a much larger error than the error potentially allowed from a construction of bulk amplitudes from the exact CFT. Therefore we conclude there are no immediate obstructions to reproducing semiclassical physics inside the bubble from the exact CFT description, for proper times less than (4.6).

4.6 Implications for cosmology

In this section we examine a number of standard issues in cosmology within this scenario.

4.6.1 Bound on the number of e-foldings of inflation

Suppose we make a bubble with $\Lambda_{dS} = \Lambda_{today} \sim (10^{-3}eV)^4$ and say it developed from a GUT scale bubble excitation $\Lambda_{GUT} = (10^{16}GeV)^4$ following the time evolution sketched in figure (4.1). Let us denote the maximum number of e-foldings in the high-scale phase by n_{max}. If we use the Gibbons-Hawking entropy to count states in the different de Sitter phases, then balancing the entropy before and after high-scale inflation, we have

$$\begin{aligned} \text{No. of Hubble volumes } \times S_{GUT} &\leq S_{today} \\ n_{max} &\leq \frac{1}{3}\log\frac{\Lambda_{GUT}}{\Lambda_{today}} \\ n_{max} &\leq 86. \end{aligned} \tag{4.7}$$

So we see the present scenario reproduces the bound on the number of efolds derived in [108]. These kinds of bounds are discussed further in [109].

The logic presented here is rather different to that of [108]. Here we match the number of available microstates within the ensemble of states that evolves toward a single large Λ_{today} bubble to derive the bound. In [108] they instead consider the entropy of a fluid in de Sitter and match the entropy of the stiffest fluid, that of a gas of black holes, in order to reach the bound (4.7). Nevertheless it is satisfying the bound of [108] is reproduced when de Sitter spacetime is embedded in a complete unitary theory of quantum gravity, such as the AdS/CFT duality. The present scenario offers a number of modifications and refinements to the *Cosmological Complementarity* principle presented in [110].

4.6.2 Natural minimization of the cosmological constant

A fundamental puzzle in modern cosmology is smallness of the cosmological constant compared to the scales of fundamental physics. In terms of Hubble parameters, there is a huge hierarchy

$$\frac{H_{today}}{H_{inflation}} \propto \sqrt{\frac{\Lambda_{today}}{\Lambda_{GUT}}} \sim 10^{56}\,.$$

The present scenario gives us a natural explanation for decrease of the cosmological constant. Let us consider the ensemble of microstates that can evolve to a single large bubble, which we will refer to as the grand ensemble. For timescales smaller than (4.6) we treat this ensemble as a closed system. Semiclassical evolution of the bubble implies that it keeps growing for as long as possible. The classical singularity theorems of Hawking and Penrose [111] imply the initial state began at high density. As the bubble expands, the number of states is ultimately bounded by $e^{S_{BH}}$, the number of black hole microstates in the external AdS spacetime.

We model the GUT-scale phase of inflation by a similar ensemble with $\Lambda_{dS} = \Lambda_{GUT}$, which we refer to as the small ensemble. With each e-folding an entire Hubble volume worth observables are added to the small ensemble. We should view the GUT-scale region as contained within the grand ensemble of the previous paragraph, since ultimately it evolves toward it. Therefore we can apply the second law of thermodynamics to the small ensemble and find that the entropy (the log of the number of available microstates) increases as the bubble expands. This, of course, can only be consistent if $\Lambda_{today} < \Lambda_{GUT}$. Extending this argument by relating the cosmological entropy in a background with time-dependent Hubble parameter [112] to the microscopic entropy, one finds

$$\dot{S} \geq 0 \Rightarrow \dot{H} \leq 0\,.$$

A closely related argument has been put forward by Strominger [113] based on a $c-theorem$ by viewing the de Sitter regions as dual to CFTs. He proposed our universe is represented as a renormalization group (RG) flow between two conformal fixed points where the forward time-translation in the bulk was interpreted as an IR-to-UV RG flow of a putative dual CFT. The Hubble parameter,

$$H = \frac{\dot{a}}{a} \tag{4.8}$$

was proposed to play the special role of the (inverse of) central charge of the dual CFT,

$$c = \frac{1}{HG} \tag{4.9}$$

because for matter obeying the null energy condition, the Einstein equation implies,

$$\dot{H} \leq 0 \Rightarrow \dot{c} > 0 \,.$$

In the context of the present scenario, the conformal symmetry of the de Sitter phase arises as an emergent symmetry of the subset of black hole microstates corresponding to the expanding bubbles, over a restricted range of timescales. Nevertheless, because the gravity problem can be understood in terms of a RG flow, it suggests that these de Sitter solutions do correspond to stable (or at least quasi-stable) fixed points. It remains an open problem how to better identify these fixed points more directly once embedded in the consistent AdS/CFT framework.

4.6.3 Unitarity and eternal inflation

One of the features of the standard eternal inflation scenario is that once started, there is always a region on a given spacelike slice where high-scale inflation is occurring. Essentially baby universes branch off in these highly quantum regions. It is difficult to imagine how such processes can preserve unitarity in a quantum description. If the number of microscopic degrees of freedom changes with time, then unitarity is necessarily violated. Certainly if such branching processes occurred locally, we can invoke the results of [114] that show that such violations of unitarity would lead to Planck scale violations of energy and momentum conservation in the bulk.

Aside from the issue of unitarity, standard eternal inflation has the attractive feature that the probability distribution of inflating regions should eventually reach a stationary state. This has the potential to lead to some very interesting predictions, however as we will review later, the infinite nature of the spacelike slices in this stationary state has so far stymied this effort.

Another useful feature of eternal inflation is that it provides a natural mechanism for populating the landscape of string theory vacua. There is always some region on a given spacelike slice in a position to roll down to one of the stable or metastable vacua of string theory. This phenomena then opens the door to anthropic arguments playing a role in our understanding of the observed parameters of nature.

In the bubble scenario, unitary is no longer an issue, since the states are realized as states in a unitary conformal field theory. As we have seen, this regulates both the spacelike and timelike extent of a cosmological region that may be treated semiclassically. However as far as we have seen, there is room to describe a large expanding universe for a large number of Hubble times.

As we have seen, the semiclassical consistency hypothesis of section (4.2) rules out tunneling to semiclassical expanding bubbles. Because baby universes cannot form from smooth initial data, the problems with bulk unitarity described in [114] are avoided. Instead expanding bubbles only emerge from punctuation points, i.e. the spacelike singularities of black/white holes. Unlike standard eternal inflation, a time-independent probability distribution does not emerge.

However Poincare recurrences in the CFT lead to quasi-periodic recurrence. Thus rather than in the spacelike direction, we have infinite extent in the timelike direction. The randomness of the recurrences can provide a mechanism to populate whatever stable and metastable vacua are present, so this attractive feature of eternal inflation is preserved.

4.6.4 Measure on the space of initial data

Now as emphasized before we can work in a microcanonical ensemble since the spacetime goes through many *punctuated* phases and has sufficient time to explore the entire phase space. So at the fundamental level, the measure on the space of initial data should be set by ergodicity. In particular, all bubble states at equal energy will be equally probable. The precise probability distribution will depend on the details of the spectrum of black hole microstates in the particular CFT dual to the gravity background with potential shown in figure 4.1. However a number of general properties of these distributions can be determined simply from a knowledge of the black hole entropy, which must bound the bubble entropy, and the interpretation of bubble entropy described in section 4.2 and in [54].

For single bubbles, one could approximate the probability distribution as

$$P_{bubble}\,(l_{dS}) = \frac{e^{S_{GH}(l_{dS})-q(l_{dS},M)}}{e^{S_{BH}(M)}}, \tag{4.10}$$

where M is the fixed mass of the large AdS black hole that dominates the microcanonical ensemble, and $q(l_{dS}, M) \geq 0$ represents a kind of form factor that parametrizes the likelihood of the black hole having such bubble solutions as microstates. Qualitatively, we expect $\lim_{l_{dS}\to l_{pl}} q(l_{dS}, M) = 0$ since the formation of small bubbles near the singularity should be a local process, independent of M. However $\lim_{l_{dS}\to r_{BH}} q(l_{dS}, M) \gg 1$, where r_{BH} is the black hole horizon radius, reflecting the fact that no bubble states larger than the black hole are allowed. This means large single bubbles are more likely, with a typical size $l_{pl} < l_{dS} < r_{BH}$.

Now let us consider the effect of multiple bubbles. For very large bubbles, only a single bubble will fit inside the black hole, so the question becomes whether there is more entropy in a soup of small bubbles. Let us approximate the collection of small bubbles by a gas of particles in thermal equilibrium with the anti-de Sitter Schwarzschild black hole. The entropy of this gas will be bounded above by the entropy of a gas of massless particles (which for the sake of simplicity we can take to be conformally coupled). This one of the systems studied in the work by Hawking and Page [27]. They find that for $M > m_{pl}^2 l_{AdS}$ that the black hole entropy dominates over the entropy of the massless particles. Here $m_{pl}^2 = 1/G$, with m_{pl} the Planck mass. Assuming the form factor $q(l_{dS}, M)$ does not fall off very rapidly, the entropy of single large bubbles will therefore dominate over that of multiple small bubbles, for sufficiently large black holes. The probability distribution for different values of l_{dS} can then be well-approximated by (4.10) assuming the gravity theory has access to a full landscape of string theory vacua with many different values of l_{dS} possible, so that $q(l_{dS}, M)$ is a smooth function.

4.6.5 Resolution of cosmological paradoxes

In the present scenario time development is future eternal, and at any given time the region along a spacelike slice undergoing interesting cosmology (i.e. a large region in an expanding phase) is of finite extent. It will be useful to count the number of potential observers within a given causal diamond inside this region. To do this we compute the entropy per Hubble volume inside the expanding bubble as a function of time. Here we have in mind a bubble with an interior rolling down the potential as shown in figure 4.1, so for now on we will generalize to a Robertson-Walker metric for the interior

$$ds^2 = -dt^2 + a(t)^2(dr^2 + d\Omega^2)\,.$$

Denoting the densities by ρ_ψ, ρ_γ and ρ_Λ for the densities of matter, radiation and cosmological constant respectively, the Friedman equations are

$$\dot{a}(t) - a(t)\sqrt{\frac{8\pi l_{pl}^2}{3}\left(\rho_\psi + \rho_\gamma + \rho_\Lambda\right)} = 0$$

$$\dot{\rho}_\psi(t) + 3\frac{\dot{a}(t)}{a(t)}\rho_\psi(t) = 0$$

$$\dot{\rho}_\gamma(t) + 4\frac{\dot{a}(t)}{a(t)}\rho_\gamma(t) = 0$$

$$\dot{\rho}_\Lambda(t) = 0$$

The entropy density for any component is

$$s_i = \frac{(\rho_i + P_i)}{T_i} = \frac{(1+w_i)\rho_i}{T_i}$$

where $P = w\rho$ is the equation of state ($w = 0, \frac{1}{3}, -1$ for matter, radiation and, cosmological constant respectively). Now the temperature of each component is given by,

$$\frac{1}{T_i} = \frac{\partial S_i}{\partial E_i} = \frac{\partial s_i}{\partial \rho_i}$$

and the entropy density as a function of the ρ is then,

$$\frac{s_i}{s_{i0}} = \left(\frac{\rho_i}{\rho_{i0}}\right)^{\frac{1}{1+w_i}}$$

and similarly the energy density of each component is,

$$\frac{\rho_i}{\rho_{i0}} = \left(\frac{a(t)}{a_0}\right)^{-3(1+w_i)}$$

So the entropy per Hubble volume from matter and radiation is,

$$S_H(t) = \sum_i s_i\left(\frac{1}{H}\right)^3 = s\left(\frac{a_0}{\dot{a}(t)}\right)^3 \tag{4.11}$$

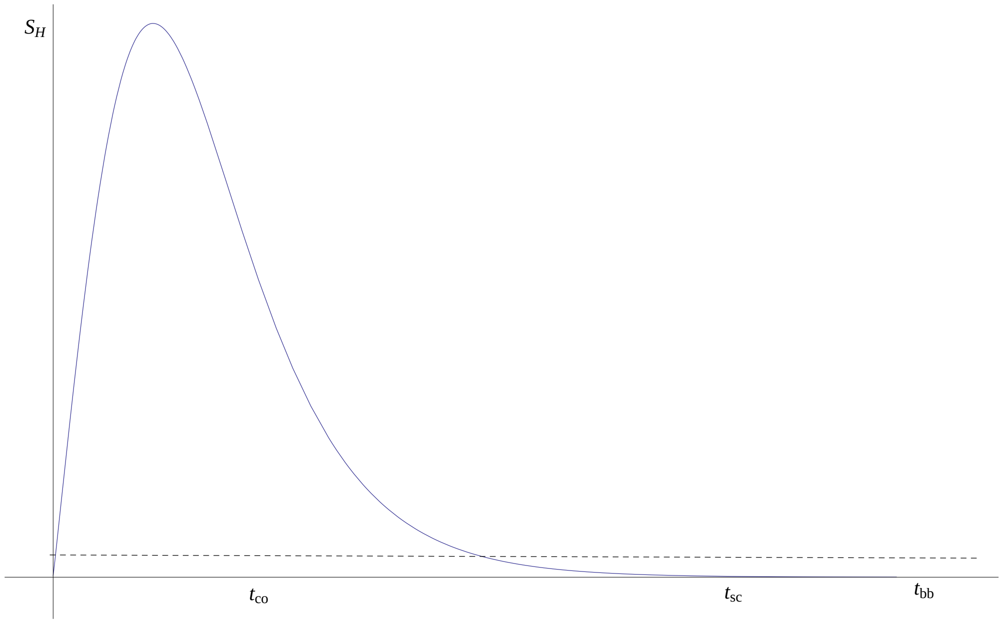

Figure 4.8: Entropy of matter and radiation per Hubble volume as a function of time. This provides an estimate of the number of potential observers as a function of time.

where $s = \sum_i s_{i0}$ and we sum over only matter and radiation components, and take this as a measure of the number of potential observers in the bubble as a function of time. In addition, there is a small contribution from Hawking radiation associated with the cosmological horizon

$$S_\Lambda = \frac{\pi^2}{15} T_{dS}^3 \frac{1}{H_{dS}^3} \sim O(1) \tag{4.12}$$

This component is constant with time, and dominates at late times . This leads to the issue of so-called Boltzmann Brain observers, that we discuss momentarily.

The entropies are shown in figure 4.8. The peak of the entropy curve is located at $\ddot{a} = \sum_i (\rho_i + 3P_i) = 0$ i.e. at

$$\rho_\psi + 2\rho_\gamma - 2\rho_\Lambda = 0 \tag{4.13}$$

which typically happens for $t < l_{dS}$ when there is a balance between matter, radiation and cosmological constant energy densities. This will be less than the timescale t_{sc} when semiclassical physics breaks down (4.6) if we have a region where gravity is a good description inside the bubble. Also shown on the plot is the timescale when Boltzmann brains dominates t_{BB}, which will always be much larger than the timescale (4.6). This figure will play a key role in seeing how various cosmological paradoxes are resolved.

4.6.5.1 Cosmological measure

The theory of inflation has had many successes in terms of explaining a number of observed phenomena within our Hubble volume, such as providing natural explanations of the horizon problem, the flatness problem and the spectrum of cosmic microwave background perturbations. Extending the theory of inflation using semiclassical approaches to quantum gravity leads to eternal inflation [92, 93, 94, 95, 96, 97, 98, 99], which describes a multiverse of pocket universes. The goal then becomes how to ascribe probabilities to histories through this multiverse, i.e. the cosmological measure problem.

In eternal inflation the "pocket universes" are infinite in number. Without any regularization, probabilities, being a ratio of two infinities, are ill-defined. Even after regularization there is no guarantee that the answer could not depend on the regularization method. Current attempts to define the measure tend to be coordinate dependent and and hence ambiguous ([115] and references therein).

This issue is resolved in the present scenario: a natural regulator that preserves unitarity is introduced. A probability measure for different cosmologies can then be extracted by evaluating (4.10) for a given conformal field theory, which computes the likelihood of expanding bubbles as a function of their parameters.

As emphasized in [116, 117] what is most relevant for observation are probabilities conditioned on the existence of observers. This can be incorporated into the discussion in the following way. The density of observers can be expected to be proportional to the entropy density of matter (and perhaps also radiation). However entropy density alone is not a good guide, because one could have a high density, rapidly expanding region of spacetime. In this case the number of degrees of freedom in causal contact with each other can remain small, so complex systems, such as observers, will not form.

Therefore we propose to count the amount of matter and radiation per Hubble volume as an approximate measure of the number of degrees of freedom in causal contact with each other. This has the advantage of being a locally defined quantity along a given worldline. Moreover it reduces to the volume of a causal diamond when the cosmological constant dominates the expansion.

The result is a world-line measure that determines the likelihood an given observer will measure a particular set of landscape parameters, as well as other cosmological observables. For example, if we condition on the the observer being in an expanding region where semiclassical gravity operates, then we conclude from (4.10) and figure 4.8 that the most likely observer will exist at time $\tau = \tau_{peak}$.

4.6.5.2 Cosmic Coincidence Problem:

Another fundamental puzzle of cosmology is why the energy density of dark energy is comparable to the energy of the matter today

$$\rho_\Lambda \sim \rho_{Matter} \, . \tag{4.14}$$

This is resolved by the world-line measure described in the previous section. A typical observer will appear near the peak of the entropy per Hubble volume, where the densities are related by (4.13).

This computation is similar in spirit to the argument of [118]. There they generalize Weinberg's argument [117] for the most likely cosmological constant and treat both the cosmological constant and the density contrast at the time of recombination are random variables. They conclude that most galaxies form at the time of cosmological constant dominance, providing an anthropic explanation of (4.14). The world-line measure of the previous section provides a much simplified version of this computation, relying only on $H(t)$.

4.6.5.3 Youngness Paradox and Oldness Paradox

One of the more popular cosmological measures in standard eternal inflation assigns likelihood proportional to volume of a given pocket universe. In this picture, a region of large vacuum energy expands very rapidly and comes to dominate the volume along a family of spacelike hypersurfaces (see for example [115]. Thus for any finite time truncation the pockets with Planck scale expansion dominate in an overwhelming manner. This would then make our universe which has been around for billions of years a very special old pocket [98].

The cosmological measure described in section 4.6.5.1 also resolves this paradox. A typical bubble will actually be one with a very large number of states according to (4.10). Since this state evolves from a singular state, an observer will see higher density looking toward the past, but such an observer will typically sit near the coincidence point (4.13).

4.6.5.4 Boltzmann Brains

If the bubble were to be arbitrarily long-lived, it might happen that the area under the curve in figure 4.8 associated with Hawking radiation could dominate, versus the area under the matter/radiation peak. This leads to a potential oldness paradox in the present scenario, which is often referred to as the Boltzmann Brain paradox.

In the usual picture of eternal inflation expansion is eternal. This leads to Boltzmann brain observers that form at late times from thermal or vacuum fluctuations (see for example [103]). Again volume weighting of the expanding regions leads to an infinite number of Boltzmann brains in comparison with ordinary observers living around the time of cosmic coincidence. Since Boltzmann brains would infinitely dominate over ordinary observers, observations like ours would be atypical. Again a suitable solution to the measure problem is needed to cure the putative catastrophe of Boltzmann brains.

In the present scenario, semiclassical physics breaks down long before Boltzmann brains become an issue, and the problem is avoided. If the entropy of a Boltzmann brain is $S_{BB} \gg 1$ the timescale for formation will be of order

$$\tau_{BB} = l_{dS} e^{S_{BB}} \tag{4.15}$$

This is to be compared with the timescale at which semiclassical physics breaks down (4.6). Having a separation of scales between l_{dS} and l_{pl} so that semiclassical physics is valid requires some fine tuning, but these scales appear inside a log in (4.6). A much higher degree of fine tuning is needed to make this time larger than (4.15). Therefore most worldlines will not see dominance of Boltzmann brain observers.

If one does manage to find a region of the landscape where (4.15) is less than (4.6), one still avoids eternal expansion of the bubble because the worldline eventually tunnels back to the stable AdS region as described in section 4.5 on timescale given in (4.5). We conclude that while there may be regions of the landscape where Boltzmann brain observers could dominate, extreme fine tuning is required. A detailed specification of the CFT would be needed to make sharper statements.

In the scenario where an expanding bubble lives inside an exterior SAdS geometry, there is a different type of Boltzmann brain that can appear in the exterior region. These are potentially more dangerous. Consider the history of such an observer who is created by a thermal fluctuation outside the Schwarzschild black hole. Such an observer will follow a timelike geodesic, and will enter the black hole horizon and hit the singularity on a timescale of order r_{bh}. Semiclassical gravity should be a good approximation in this exterior region, and modulo Poincare recurrences, this region is eternal. Therefore we expect a substantial number of such observers to appear and this number could overwhelm the number of ordinary observers.

In this scenario, the stable AdS vacuum state is supersymmetric, so atoms and other complex bound states will not appear. Thus the kind of complex quasi-stable bound state corresponding to a Boltzmann brain may simply not appear in this region. Rather such an observer would likely need a large supersymmetry breaking bubble to surround her, which leads us back to the original scenario.

Alternatively, for observables of interest to us, we can condition probabilities by the requirement that observers live in an expanding phase. This rules out this dangerous class of Boltzmann brain observers. While it is straightforward to impose this condition in the limit $l_{pl} \to 0$ where semiclassical gravity is valid, it remains to be seen whether this distinction is possible in the full quantum regime.

4.6.5.5 Problems of time

The time coordinate is not a physical observable in a diffeomorphism invariant formulation of gravity. Usually the way around this is to use a rolling scalar field to define an invariant notion of time. However this then raises issues with recovering locality and causality. These issues are reviewed in [119].

In the AdS/CFT context, time on the conformal boundary provides a global time coordinate. The CFT comes equipped with a well-defined norm, and time evolution in the conformal field theory is unitarity. The CFT norm induces a well-defined measure on the space of bubble cosmologies, of which (4.10) is a simple example. Less clear is how bulk locality and causality emerges, but this is a topic under active investigation.

Another problem of time is how the arrow of time arises in a CPT invariant theory such as gravity. Let assume the history shown in figure (4.5) effectively explores the microcanonical ensemble in the CFT. In addition to expanding bubble states, by CPT invariance we will also have an equal number of time-reversed collapsing bubble states. However any given macroscopic bubble solution that gives an expanding or contracting region de Sitter region will necessarily be time asymmetric [54]. Thus in this scenario the arrow of time arises from spontaneous symmetry breaking: the underlying theory is CPT invariant, but the solutions break this symmetry.

4.7 Appendix A

In this appendix we find the relation between comoving time inside the dS bubble, and Schwarzschild time in the AdS region. The proper time along the bubble wall τ satisfies (4.1) with the radial coordinate $r(\tau)$ determined by (4.2). This allows us to determine $t_{SAdS}(r)$ at the bubble wall via

$$dt_{SAdS} = dr\left(\frac{1}{f_{SAdS}(r)}\left(\frac{1}{f_{SAdS}(r)} - \frac{1}{V_{eff}(r)}\right)\right)^{1/2}$$

Integrating in the approximation, $r \to \infty$,

$$t_{SAdS}(r) = B - A\frac{l_{AdS}^2}{r}, \quad A = \sqrt{1 + \frac{4\kappa^2 l_{AdS}^2 l_{dS}^4}{\left(l_{AdS}^2 + l_{dS}^2(\kappa^2 l_{AdS}^2 - 1)\right)^2 + 4 l_{AdS}^2 l_{dS}^2}} \qquad (4.16)$$

with B an integration constant. We see the bubble reaches $r = \infty$ at finite Schwarzschild time $t_{SAdS} = B$.

In the interior, the r coordinate becomes timelike for $r > l_{dS}$, while t_{dS} approaches a constant on the bubble wall $t_{dS} = C$. Now let us transform to comoving time τ_{dS} in the de Sitter patch, working with flat spatial slices

$$ds^2 = -d\tau_{dS}^2 + l_{dS}^2 e^{2\tau_{dS}} d\vec{x}^2$$

The comoving time τ_{dS} is related to r by

$$\tau_{dS} = l_{dS}\log\left(\frac{r}{l_{dS}}\right)$$

so inserting this into (4.16) we obtain

$$t_{SAdS} = B - \frac{A l_{AdS}^2}{l_{dS}} e^{-\frac{\tau_{dS}}{l_{dS}}} \qquad (4.17)$$

Now the semiclassical approximation is not expected to continue to hold very close to the null singularity induced by back-reaction(4.4). If we assume the semiclassical approximation holds to $t_{SAdS} = B - l_{pl}$ this translates into breakdown at comoving de Sitter time

$$\tau_{dS} = l_{dS}\log\left(\frac{A l_{AdS}^2}{l_{pl} l_{dS}}\right).$$

4.8 Appendix B

Working in the semiclassical probe approximation we consider a massless (conformal) scalar field propagating in this bubble geometry with the purely reflective boundary conditions at the bubble wall and zero flux at the AdS spacelike infinity. For computational convenience we specialize to $(1+1)$-d. Since all space times are conformal to flat (Minkowski) space time in $(1+1)$-d, this is conformally mapped to the propagation of a scalar field in a flat background with moving mirror [45].

1+1d spacetime with movable boundary (perfect mirror) whose trajectory parametrized by $\tau(u)$, the time at which the mirror intersects the null coordinate, $u \equiv t - x$,

The modes solutions are,

$$\phi_\omega = e^{-i\omega v} + e^{-i\omega p(u)}$$

where

$$p(u) = 2\tau(u) - u.$$

Then transform to the mirror rest frame (U,V) with mirror at the origin,

$$V = v, \qquad U = p(u)$$

or,

$$ds^2 = du dv = \partial_U p^{-1}(U)\, dU dV$$

So using equation (6.136/137) [45], the energy flux in u, v frame is a clean,

$$T_{uu} = \frac{1}{12\pi} p'^{1/2} \partial_u^2 (p'^{-1/2}). \tag{4.18}$$

Now lets move on to pure AdS$_2$. The metric is,

$$\begin{aligned} ds^2 &= -(1+r^2)dt^2 + \frac{dr^2}{1+r^2}, \quad t, r \in (-\infty, \infty) \\ &= -\sec^2 x\, (dt^2 - dx^2),\ x \equiv \tan^{-1} r \in [-\pi/2, \pi/2] \\ &= -\sec^2 \frac{v-u}{2}\, du dv, \quad u, v \equiv t \mp x \end{aligned}$$

As before we now shift to a frame where the mirror is at rest at the origin (i.e. $U = p(u)$ and $V = v$) and we can (conformally) relate this to the mirror at rest in flat space case,

$$ds^2_{AdS} = -\sec^2 \frac{v-u}{2} du dv = -\sec^2 \left(\frac{V - p^{-1}(U)}{2} \right) \partial_U p^{-1}(U)\, dU dV = C(U,V) dU dV \tag{4.19}$$

Then the renormalized energy-momentum tensor expectation in the U, V vacuum is,

$$T_{UU} = -\frac{1}{12\pi} F_U[C(U,V)], \qquad T_{VV} = -\frac{1}{12\pi} F_V(C(U,V))$$

$$T_{UV} = \frac{RC(U,V)}{96\pi}$$

where the operator F is defined as $F_x(y(x)) = y^{1/2}\partial_x^2(y^{-1/2})$ and R is the AdS$_2$ Ricci scalar. Now reverting to u, v coordinates,

$$T_{uu} = \left(\frac{\partial U}{\partial u} \right)^2 T_{UU} = T^{(AdS)}_{uu} + T^{(m)}_{uu}$$

\

where $T^{(AdS)}_{uu} = -\frac{1}{12\pi}F_u[\sec^2\frac{v-u}{2}]$ is the piece due to a static mirror and due to MOTION of the mirror (c.f. [120]),

$$T^{(m)}_{uu} = -\frac{1}{12\pi}\left(\frac{dp}{du}\right)^{1/2}\frac{d}{du}\left(\frac{dp}{du}\right)^{-1/2} + \frac{1}{12\pi}\left(\frac{dp}{du}\right)^{-1/2}\sec\left(\frac{v-u}{2}\right)\frac{d}{du}\left(\frac{\frac{d}{du}\left(\frac{dp}{du}\right)^{1/2}}{\frac{dU}{du}\sec(\frac{v-u}{2})}\right) \tag{4.20}$$

$$T^{(m)}_{vv} = T^{(m)}_{uv} = 0 \tag{4.21}$$

So, the entropy(density) due to radiation reflected from the moving mirror,

$$s = \frac{1+d}{d}\left(T^{00(m)}\right)^{\frac{d}{d+1}} = 2\cos^2\frac{v-u}{2}\left(T^{(m)}_{uu}\right)^{1/2} \tag{4.22}$$

Here of course number of space dimensions, $d = 1$.

The temperature of the this reflected radiation is,

$$\beta^{-1}_{rad} = b\left(T^{00(m)}\right)^{1/2} = 2b\cos^2\frac{v-u}{2}\left(T^{(m)}_{uu}\right)^{1/2}. \tag{4.23}$$

where b is some universal constant.

The contribution to mass of the AdS space is given by the integral over a spacelike slice $t =$ fixed,

$$\begin{aligned} M_{radiation} &= \int_{t=\text{fixed}} dr\, T^{0(m)}_{0} \\ &= \int_{t=\text{fixed}} dr g_{00} T^{00(m)},\ g = g_{AdS_2} \qquad (4.24) \\ &= -\int_{-\pi/2}^{0} du \sec^4 u\, \cos^4 u\, T^{(m)}_{uu}(u, v=-u) \\ &= -\int_{-\pi/2}^{0} du\, T^{(m)}_{uu}(u, v=-u) \qquad (4.25) \end{aligned}$$

For a uniformly accelerating (Rindler) trajectory with acceleration parameter α which is close enough to the situation of the exponentially expanding bubble,

$$\begin{aligned} x(\tau) &= \alpha^{-1}\cosh\alpha\tau \\ t(\tau) &= \alpha^{-1}\sinh\alpha\tau \end{aligned}$$

we have,

$$u(\tau) = -\alpha^{-1}e^{-\alpha\tau}, \quad v(\tau) = \alpha^{-1}e^{\alpha\tau} = -\alpha^{-2}u^{-1}(\tau)$$

Now if we go to the rest frame of this Rindler observer i.e. where the observer stays at the origin,

$$V(v) - U(u) = 0,$$

With $V(v) = v$ and $U(u) = -\alpha^2 u^{-1}$ this condition is satisfied. So for uniformly accelerating mirror we have,

$$p(u) = -\alpha^{-2} u^{-1}.$$

For this case, the mass contribution from radiation,

$$M_{radiation} \sim finite - \frac{1}{12\pi} \lim_{r \to 0} \ln r + \frac{\pi}{48\alpha^2} \lim_{r \to \infty} \ln \left(-\frac{1}{r} \right).$$

So the mass contribution is finite but one needs both UV and IR cut-offs as expected.

Analogously for the AdS-Sch black hole the stress-energy tensor splits into two different components

1. Hawking radiation of the black hole.

2. The contribution from the reflection from the moving mirror. The reflected radiation just alters the black hole mass as argued in the above calculation by a finite amount, so for a parametrically huge mass black hole this effect is not appreciable.

Chapter 5

Pure AdS$_{2+1}$ gravity as a Chern-Simons theory

The (BTZ) black hole metrics are locally isomorphic to the maximally symmetric solution, namely anti de Sitter space and share the same asymptotic symmetry group - the conformal group in 2-dimensions [55]. The infinite set of Virasoro charges of this asymptotic conformal group parametrize all asymptotically AdS metrics[56]. In particular for BTZ metrics, the Virasoro charges $(L_0, \bar{L}_0)$ are simply linear combinations of the mass (M) and spin (J) of the black hole $(L_0 = \frac{Ml+J}{2}, \bar{L}_0 = \frac{Ml-J}{2})$.

There is a long history of efforts formulating gravity in general d-dimensions as a gauge theory by combining the vierbein and spin connection into a single $ISO(d-1,1)$ gauge connection, since small diffeomorphisms can be expressed as local Lorentz rotations and translations [57] on shell. However this program was abortive since the Einstein-Hilbert action could not be expressed in terms of the gauge connection. This obstacle was circumvented for the $d = 2+1$ case [58, 59] for general nonvanishing cosmological constant and the Einstein-Hilbert action was expressed as a Chern-Simons action for the $ISO(2,1)$ connection for zero cosmological constant, and a $SO(2,2)$ connection for negative cosmological constant. Since $SO(2,2) \equiv SL(2,\mathbb{R}) \times SL(2,\mathbb{R})/\mathbb{Z}_2$ (the $\mathbb{Z}_2$ acts as -1 on the $SL(2,\mathbb{R})$ connections), the action can be written as a sum of two Chern-Simons terms with independent $SL(2,\mathbb{R})$ gauge connections. By adding an additional topological term to the Einstein action[59] the coefficients of the left/right Chern-Simons terms become independent. Based on this motivation, it has been conjectured the path integral for pure $2+1$-d gravity with negative cosmological constant factors holomorphically [60, 61, 62, 63, 64, 65, 66, 67]. Also in other extensions of $2+1$-d AdS gravity with a gravitational Chern-Simons terms there are sectors where the left (or right) gauge connection can be pure gauge i.e. geometries that are chiral [68, 69]. In such cases again the path integral is expected to be holomorphic factorizable. Of course there are several issues - the classical equivalence of Einstein's theory and the Chern-Simons theory might not extend to the quantum realm in such a simple manner. This is certainly true for large diffeomorphisms when the vierbein is noninvertible and the metric interpretation is unclear.

The aim of this section is to study geometries where one of the CS gauge fields is set to be globally trivial (anti-de Sitter) with the other gauge field being nontrivial

(BTZ-like). If holomorphic factorization holds at the level of the Chern-Simons path integral, such geometries should have a meaningful metric interpretation. We find that such metrics generically have naked closed timelike curves (CTCs) and hence do not respect causality. The plan of the section is as follows. In section 3.1 we review the Chern-Simons formulation of the BTZ solution and note the relationship between the holonomies and the casimirs (mass and spin). In section 3.2, we construct "hybrid" metrics made of a left($^+$) connection of a (M, J) BTZ solution and a right($^-$) connection of a (m, j) BTZ solution. The hybrid metric after a suitable change of coordinates is shown to be another BTZ solution with charges $(\frac{M+m}{2} + \frac{J-j}{2l}, \frac{(M-m)l}{2} + \frac{J+j}{2})$. In section 3.3 we set the right connection to pure AdS$_3$ and note that these geometries are super-rotating BTZ solutions which necessarily have naked CTCs thus violating causality. The right connection when set to zero instead of pure AdS$_3$ gives rise to singular metrics.

5.1 Chern-Simons formulation of the BTZ black hole

To start with, let us very briefly review the Chern Simons formulation of Lorentzian 2+1d gravity with negative cosmological constant $\Lambda = -\frac{1}{l^2}$. The details can be found in [59, 56, 69]. The vierbein, e^a and the spin connection one form ω^a are combined into a $SO(2,2)$ gauge connection one form,

$$A = e^a P_a + \omega^a J_a$$

with the algebra,

$$[J_a, J_b] = \epsilon^c_{ab} J_c\,,\ [P_a, P_b] = \frac{1}{l^2}\epsilon^c_{ab} J_c\,,\ [J_a, P_b] = \epsilon^c_{ab} P_c\,.$$

The $SO(2,2)$ generators can be split into two $SL(2,\mathbb{R})$ copies,

$$T^{\pm}_a = \frac{1}{2}(J_a \pm l P_a)$$

satisfying

$$[T^+_a, T^+_b] = \epsilon_{ab}{}^c T^+_c\,,\ [T^-_a, T^-_b] = \epsilon_{ab}{}^c T^-_c\,,\ [T^+_a, T^-_b] = 0.$$

Here $\epsilon^{012} = 1$ and $\eta = \text{Diag}\ (-1, 1, 1)$. The gauge connection accordingly factorizes to,

$$\begin{aligned} A &= A^{a+} T^+_a + A^{a-} T^-_a \\ A^{\pm} &= \omega \pm \frac{e}{l}. \end{aligned}$$

The Einstein-Hilbert action with a negative constant term can then be cast as the difference of two Chern-Simons terms,

$$I = I_{CS}[A^+] - I_{CS}[A^-],$$

where the Chern-Simons action is,

$$I_{CS}[A] = \frac{k}{4\pi}\int Tr(dA \wedge A + \frac{2}{3}A \wedge A \wedge A),$$

where the trace is over the 2-dimensional representation of $SL(2,\mathbb{R})$,

$$T_0 = -i\frac{\sigma^2}{2},\ T_1 = -i\frac{\sigma^3}{2},\ T_2 = \frac{\sigma^1}{2},$$

and the Chern-Simons coupling constant or level number,

$$k = -2l.$$

Following [31, 32, 56] we choose the convention $8G = 1$. The solutions to Einstein's equations are given by flat connections

$$dA^{\pm} + A^{\pm} \wedge A^{\pm} = 0.$$

In fact the above pair of flat connection equations of motion can also be obtained by any action which is the difference of two Chern-Simons actions with *different* Chern-Simons couplings i.e. $k^+ \neq k^-$. This corresponds to adding a topological term to the Einstein action with a coupling proportional to the difference $(k^+ - k^-)$, which does not change the local equations of motion, but will be important if the full path integral is considered.

Flat connections are classified according to their holonomy, i.e. every flat connection can be gauge transformed to the form

$$A^{\pm} = U^{\pm -1} dU^{\pm} \tag{5.1}$$

where $U \in SL(2,\mathbb{R})$ is a gauge transformation element. The connection is globally pure gauge only if U is single valued. The holonomy, w, for a connection is given by the Wilson loop operator along noncontractible cycles

$$W^{\pm} = \exp(\oint A^{\pm}) = \exp(w^{\pm})\,.$$

The local pure gauge condition is a reflection of the fact that in 2+1d any solution of Einstein equation with a negative cosmological constant is locally anti-de Sitter space, which is the maximally symmetric solution. Globally distinct solutions can be obtain by orbifolding the maximally symmetric AdS$_3$ spacetime, i.e. identifying points along an orbit of some killing vector, thus introducing nontrivial cycles. This leads to black hole solutions [31, 32] with mass M and spin J given by the metric,

$$ds^2 = -N(r)^2 dt^2 + N(r)^{-}2dr^2 + (r^2 N^{\phi} dt + d\phi)^2$$

where,

$$N^2(r) = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}$$

and

$$N^{\phi} = -\frac{J}{2r^2}$$

and $t \in (-\infty, \infty)$, $r \in (0, \infty)$ and $\phi \in [0, 2\pi)$. Now quotienting the maximally symmetric AdS$_3$ space reduces the number of symmetries (or Killing vectors) by demanding bonafide tensor fields respect single-valuedness after the periodic identifications. The necessary and sufficient condition for which is that tensor fields must commute with the Killing vectors belonging to the identification subgroup (along the orbits of which the identifications were made). This is true only for $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial \phi}$ for the BTZ. Thus the symmetry group of BTZ is not $SO(2,2)$ but $R \times SO(1)$[31].

The $SO(2,2)$ gauge connection was worked out in [70] but these have the unpleasant feature that the connections for the extreme BTZ are singular. So we use a different gauge in which the left and right connections appear as

$$A^{+0} = -N(r)\left(\frac{dt}{l} - d\phi\right), \; A^{+1} = \frac{1-\frac{Jl}{2r^2}}{N(r)}\frac{dr}{l}, \; A^{+2} = -\left(\frac{J}{2r} + \frac{r}{l}\right)\left(\frac{dt}{l} - d\phi\right),$$

$$A^{-0} = -N(r)\left(\frac{dt}{l} + d\phi\right), \; A^{-1} = -\frac{1+\frac{Jl}{2r^2}}{N(r)}\frac{dr}{l}, \; A^{-2} = \left(\frac{J}{2r} - \frac{r}{l}\right)\left(\frac{dt}{l} + d\phi\right)$$

and the corresponding gauge transformation elements are obtained by solving (5.1)

$$\begin{aligned}
U^{\pm} &= e^{\theta_{0\pm}T_0}e^{\theta_{1\pm}T_1}e^{\theta_{2\pm}T_2} \\
\theta_{0\pm} &= 0 \\
\sinh\theta_{1\pm} &= \frac{N(r)}{\sqrt{M \pm \frac{J}{l}}} \\
\cosh\theta_{1\pm} &= \frac{\frac{J}{2r} \pm \frac{r}{l}}{\sqrt{M \pm \frac{J}{l}}} \\
\theta_2\pm &= \sqrt{M \pm \frac{J}{l}}\left(\phi \mp \frac{t}{l}\right).
\end{aligned}$$

Clearly these transformation are not suitable for the extreme case $J = \pm Ml$. For $J = Ml$, the nonsingular gauge group element is,

$$\begin{aligned}
U^{-} &= e^{\theta(T_0+T_2)}e^{\theta_1 T_1}, \\
\theta &= -ln\left|\frac{r}{l} - \frac{Ml}{2r}\right|, \\
\theta_1 &= -\left(\phi + \frac{t}{l}\right).
\end{aligned}$$

Finally the Wilson loop operator along a constant t, ϕ loop,

$$W^{\pm} = \begin{pmatrix} \cosh\pi\sqrt{M \pm \frac{J}{l}} & e^{-\theta_1}\sinh\pi\sqrt{M \pm \frac{J}{l}} \\ e^{\theta_1}\sinh\pi\sqrt{M \pm \frac{J}{l}} & \cosh\pi\sqrt{M \pm \frac{J}{l}} \end{pmatrix}.$$

So the eigenvalues are e^{λ}, $\lambda = \pm\sqrt{M \pm \frac{J}{l}}$ and the holonomy matrices turn out to be,

$$w_{\pm} = \begin{pmatrix} \pm\pi\sqrt{M \pm \frac{J}{l}} & 0 \\ 0 & \mp\pi\sqrt{M \pm \frac{J}{l}} \end{pmatrix}$$

This gives the quadratic Casimirs,

$$Tr(w_+^2 + w_-^2) = 4\pi^2 M$$
$$Tr(w_+^2 - w_-^2) = 4\pi^2 \frac{J}{l}\,.$$

5.2 Hybrid Geometries

In this section we are interested in metrics resulting from combining the left and right $SL(2,\mathbb{R})$ connections for different BTZ solutions, say we combine a left connection for a mass M, spin J and a right connection for mass m and spin j BTZ metric. The metric for such a hybrid geometry is then,

$$ds^2 = g_{tt}(r)dt^2 + 2g_{t\phi}(r)dtd\phi + g_{rr}(r)dr^2 + g_{\phi\phi}(r)d\phi^2,$$

with,

$$\begin{aligned}
\frac{g_{tt}}{l^2} &= \frac{M+m}{4} + \frac{Jj}{8r^2} - \frac{r^2}{2l^2} - \frac{1}{2}N_1(r)N_2(r) \\
\frac{g_{t\phi}}{l^2} &= -\frac{j+J-ml+Ml}{4l} \\
\frac{g_{rr}}{l^2} &= \left(\frac{1-\frac{Jl}{2r^2}}{2N_1(r)} + \frac{1+\frac{jl}{2r^2}}{2N_2(r)}\right)^2 \\
\frac{g_{\phi\phi}}{l^2} &= \frac{m+M}{4} - \frac{jJ}{8r^2} + \frac{J-j}{2l} + \frac{r^2}{2l^2} + \frac{1}{2}N_1(r)N_2(r)
\end{aligned} \tag{5.2}$$

where $N_1(r) = \sqrt{-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}}$ and $N_2(r) = \sqrt{-m + \frac{r^2}{l^2} + \frac{j^2}{4r^2}}$. In these coordinates the metric or the geometry looks highly unintuitive since the metric components are quartic in r and the determinant seems to have multiple non-coincident zeroes making it hard to analyze the location of horizons, surfaces bounding regions with CTCs, etc. for the general M, m, j, J. To gain insight we switch over to a different set of coordinates as follows. It has been shown [56] that the most general solution to Einstein equations with negative cosmological constant in (2+1)-dimensions with asymptotically anti-de Sitter boundary conditions is of the form

$$ds^2 = \frac{lL(w)}{2}dw^2 + \frac{l\bar{L}(\bar{w})}{2}d\bar{w}^2 + \left(l^2e^{2\rho} + \frac{L(w)\bar{L}(\bar{w})}{4}e^{-2\rho}\right)dwd\bar{w} + l^2d\rho^2, \tag{5.3}$$

for arbitrary functions $L(w)$ and $\bar{L}(\bar{w})$. Here $w = \phi + \frac{t}{l}$, $\bar{w} = \phi - \frac{t}{l}$ are the boundary coordinates and ρ is a radial coordinate. $\phi \in [0, 2\pi)$ is an angular coordinate, while t

is the time coordinate. In particular the BTZ solution is a special case with constant metric coefficients,

$$L(w) = L_0 = \frac{Ml+J}{2}$$
$$\bar{L}(\bar{w}) = \bar{L}_0 = \frac{Ml-J}{2}$$

So after effecting the following change of coordinates,

$$w = \phi + \frac{t}{l}$$
$$\bar{w} = \phi - \frac{t}{l}$$

$$\rho = \frac{1}{2}\ln\left|\frac{\left(N_1 + \frac{r}{l} + \frac{J}{2r}\right)\left(N_2 + \frac{r}{l} - \frac{j}{2r}\right)}{4}\right|$$

we arrive at the metric in the form of (5.3) with,

$$\begin{aligned} L(w) &= \frac{Ml+J}{2} \\ \bar{L}(\bar{w}) &= \frac{ml-j}{2}\,. \end{aligned} \tag{5.4}$$

So it is just another BTZ black hole metric with ADM mass, M_{ADM} and spin, J_{ADM} given by,

$$\begin{aligned} M_{ADM} &= \frac{M+m}{2} + \frac{J-j}{2l} \\ J_{ADM} &= \frac{(M-m)l}{2} + \frac{J+j}{2}\,. \end{aligned}$$

A crucial observation is that if the extremality bounds $Ml \geq |J|$ and $ml \geq |j|$ are satisfied then the resulting black hole also satisfies an extremality bound,

$$M_{ADM}^2 l^2 - J_{ADM}^2 = (Ml+J)(ml-j) \geq 0\,. \tag{5.5}$$

5.3 Chiral geometries

5.3.1 Left BTZ - Right AdS$_3$ geometries

For this case we set $m = -1$, $j = 0$ i.e. pure AdS$_3$ space instead of a black hole right connection. So the resulting black hole has mass and spin,

$$\begin{aligned} M_{ADM} &= \frac{M-1}{2} + \frac{J}{2l} \\ J_{ADM} &= \frac{M+1}{2}l + \frac{J}{2} \end{aligned}$$

which clearly violates the extremality condition $M_{ADM}^2 l^2 - J_{ADM}^2 = -(Ml+J)l$ i.e. the regions having CTCs are naked since there are no horizons to mask them. Clearly for such a spacetime causality breaks down.

5.3.2 Left BTZ - Right zero geometries

In this case we set,

$$A^- = 0$$

and keep the A^+ same as in the last case. So, the metric in this case becomes

$$ds^2 = \frac{Ml + J}{4} l(d\phi - \frac{dt}{l})^2 + \frac{(1 - \frac{Jl}{2r^2})^2}{4(N_1(r))^2} dr^2$$

Clearly the metric determinant vanishes, it is non-invertible. So these holomorphic geometries do not make sense either.

5.4 Conclusion and Outlook

If the path integral for pure gravity involves something like a holomorphically factorized path integral over Chern-Simons gauge fields, we have seen that large families of seemingly sensible gauge connections give rise to geometries with naked closed time-like curves. Eliminating these connections using some kind of physical state conditions that respect holomorphicity would conversely eliminate the basic vacuum solution, pure AdS. It seems then that physical state conditions must violate holomorphicity, by for example, requiring that the inequality (5.5) be satisfied. Alternatively it may simply be the case that the Chern-Simons formulation of pure gravity is not a good starting point for the quantization of the theory. The study of limits of string theory compactifications that give rise to pure gravity should provide useful clues to aid further progress.

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