For my graduate research, I have focused on the creation of high resolution weak lensing mass maps in order to study dark matter substructure in clusters of galaxies at high redshift (z>0.5). Galaxy clusters are the largest virialized structures in the universe, and are meaningful environments for tests of cosmology. Mass substructure in clusters of galaxies is important for several reasons: as a test of hierarchical Cold Dark Matter, as a way of reducing the scatter in the mass calibration of clusters for dark energy studies, and as environments for galaxy and active galactic nuclei evolution.

Gravitational lensing is a powerful tool for measuring mass in astronomical objects. According to General Relativity, very massive objects bend the light of more distant objects around them, thus distorting the shape of these background objects. In the weak lensing regime, these shape distortions are small (few percent level), and thus only detected as a systematic alignment about the lensing cluster. Therefore, rigorous shape detection, extraction, and statistical analysis are required to generate mass maps.

Starting with Hubble Space Telescope data, I carefully correct for the HST’s Point Spread Function ellipticity pattern and stack the images for each cluster. After extracting the stack data, I perform a weak lensing analysis of the cluster. To characterize the level of substructure, I apply a cluster finding variable aperture filter. Finally, I fit an ensemble of NFW profiles to the detected matter peaks.
Dark Matter Substructure in High Redshift Clusters of Galaxies

A dissertation presented
by
Paul M. Huwe

to
The Physics Department

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
Physics

Brown University
Providence, State of Rhode Island and Providence Plantations
May 2013
This dissertation by Paul M. Huwe is accepted in its present form by the Department of Physics as satisfying the dissertation requirement for the degree of Doctor of Philosophy.

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- Evolution of Dark Matter, Dark Energy
- High Resolution Gravitational Lensing
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---

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- American Association of Physics Teachers (AAPT)
I would like to give my deepest thanks to Professor Ian Dell’Antonio for his guidance, mentorship, and patience as my thesis advisor. Throughout my graduate career, he not only helped me to be a better scientist, but he also provided great leadership, counsel, and support. He has truly been the Strider to my Frodo in my quest for knowledge.

I want to thank my committee members, Prof. Richard Gaitskell and Prof. Savvas Koushiappas for their time, effort, and patience in reviewing my work.

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For the past four years, I have been able to fill my house, Maison Pi, with friends from the physics department. They truly made Pi a home for me during my graduate studies, and I am eternally grateful. I therefore thank all the Maison Pi denizens: Shawna, Andy, and Kodiak Fischer, Helen and Hachiko Hanson, Mike Luk, Juliette Alimena, and Lisa Goldberg. I would also like to thank all my friends both in the department and not for their support and companionship.

I would like to thank my family for their love and support. I thank my brothers Arthur and Trevor for keeping me grounded while providing me great friendship and amusement. I thank Gary for being the father I didn’t have for most of my life. My mom I thank for her unending support and teaching me by example that no matter what life throws at me, through hard work and determination I can achieve anything.

Finally, I thank my wife Beth, to whom this thesis is dedicated. Without her love, support, and patience (and patience and patience), these past several years would have been much less vibrant and enjoyable and incredibly more difficult. I am a better person for having known her, and it is with all my love that I thank her.
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In present cosmological models, roughly 84% of the matter in the universe is dark [62]. The study of the distribution of this dark matter is an active and popular field of cosmological research. In this thesis, we examine how dark matter is distributed in high redshift (z > 0.5) clusters of galaxies. By utilizing precision weak gravitational lensing, we map out the mass in these clusters [49].

1.1 Modern Cosmology

The standard model of modern cosmology is built upon the cosmological principle, which states that “there is nothing special about our location in the universe” [65]. This statement yields two important observable consequences: on large scales
(>100Mpc), our universe is isotropic and homogeneous. Isotropy means that there is no preferred direction to the universe - it looks roughly the same in all directions. Homogeneity stipulates that there are no preferred locations in the universe - it looks the same regardless of an observer’s location. Studies have shown both of these axioms to be true on large scales (Fig. 1.1.1) [2, 37].

1.1.1 Expanding Universe

An additional bedrock observation of modern cosmology is that the universe is expanding. In 1929, Edwin Hubble discovered a linear velocity to distance relationship for galaxies, indicating this expansion (Fig. 1.1.2) [40]. From these observations, he derived Hubble’s Law:

\[ v = H_0 D \]  

(1.1)

where \( v \) is the galaxy’s velocity, \( D \) the distance to it, and \( H_0 \) is the Hubble constant (the most recent published value is 67.80 ± 0.77 km/s/Mpc)[62].

As photons travel through an expanding space, their wavelengths expand as well. This wavelength stretching results in a redshift of the observed photons. The relationship for redshift, \( z \), emitted wavelength, \( \lambda_o \), and observed wavelength \( \lambda_e \), is as follows:

\[ 1 + z = \frac{\lambda_o}{\lambda_e} \]  

(1.2)

1.1.2 Friedmann Cosmology

Starting with the cosmological principle, Friedmann, Lemaître, Robertson, and Walker (FLRW) derived the following metric to describe the distance between
Figure 1.1.1: Slices through the SDSS 3-dimensional map of the distribution of galaxies. Earth is at the center, and each point represents a galaxy, typically containing about 100 billion stars. Galaxies are colored according to the ages of their stars, with the redder, more strongly clustered points showing galaxies that are made of older stars. The outer circle is at a distance of two billion light years. The region between the wedges was not mapped by the SDSS because dust in our own Galaxy obscures the view of the distant universe in these directions. Both slices contain all galaxies within -1.25 and 1.25 degrees declination [2].
Figure 1.1.2: Hubble’s original 1929 data which indicated the linear velocity - distance relation [40].

space-time events in an expanding universe:

\[ ds^2 = c^2 dt^2 - a^2(t) d\Sigma^2. \]  
(1.3)

In this metric, \( dt \) is the standard time coordinate, \( a(t) \) is the scale factor, and \( d\Sigma^2 \) represents the metric for a uniform curvature 3-D space. \( d\Sigma^2 \) can be written as:

\[ d\Sigma^2 = \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2, \]  
(1.4)

where \( r \) is the comoving coordinate and \( d\Omega^2 \) represents the standard spherically symmetric angular part of the metric:

\[ d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2. \]  
(1.5)

\( k \) is the curvature of the space, with \( k < 0 \) corresponding to a negative curvature universe, \( k > 0 \) for a positive curvature universe, and \( k = 0 \) indicates a flat universe.

The co-moving expansion parameter or scale factor, \( a(t) \), in Eq. 1.3 describes the
relative expansion of the universe. It can be expressed in terms of the proper
distance by:

\[ d(t) = a(t)d_0, \]  \hspace{1cm} (1.6)

where \( d(t) \) is the proper distance at epoch \( t \), and \( d_0 \) is the distance at the present
day. Convention is that the present day scale factor is set to unity,

\[ a_0 = a(t_0) = 1. \]  \hspace{1cm} (1.7)

The relationship between redshift and the scale factor is then:

\[ a(t) = \frac{1}{1 + z}. \]  \hspace{1cm} (1.8)

Finally, one can utilize the scale factor to define the Hubble Parameter:

\[ H \equiv \frac{\dot{a}(t)}{a(t)}, \]  \hspace{1cm} (1.9)

which is a more general version of Hubble’s Law.

Utilizing the FLRW metric, Eq. 1.3, Friedmann derived a simplified form of
Einstein’s field equations from which two independent equations can be derived:

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} \left( \rho c^2 + 3P \right) \]  \hspace{1cm} (1.10)

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2}. \]  \hspace{1cm} (1.11)

In these equations, \( \rho \) refers to the density of a homogeneous fluid, and \( P \) is the
Corresponding fluid pressure. Eq. 1.11 is referred to as the Friedmann equation,
while Eq. 1.10 is the acceleration equation. The fluid equation in a space-time
described by the FLRW metric is:

$$\dot{\rho}c^2 = -3 \frac{\dot{a}}{a} (\rho c^2 + P).$$

(1.12)

Upon defining a critical density parameter,

$$\rho_{cr} = \frac{3H_0^2}{8\pi G},$$

(1.13)

the Friedmann equation, Eq. 1.11, can be rewritten as:

$$\left(\frac{H}{H_0}\right)^2 = \frac{\rho}{\rho_{cr}} - \frac{kc^2}{a^2H_0^2}.$$  

(1.14)

We can introduce critical density fractions for the components of this universe. For each component (radiation, matter, and dark energy), we define the critical density, \(\Omega\) to be

$$\Omega = \frac{\rho}{\rho_{cr}},$$

(1.15)

and similarly define a curvature term

$$\Omega_k = -\frac{kc^2}{a^2H_0^2}. $$

(1.16)

With these, the Friedman equation can be rewritten as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k.$$  

(1.17)

Given that we observe at present time \((H_0, a_0)\), it is convenient to cast the critical density parameter as functions of current values. To do so, we define an equation of state relating \(P\) to \(\rho\):

$$P = w \rho c^2.$$  

(1.18)
The equation of state parameter, $w$, can be defined for each component of the energy density:

- non-relativistic matter: $w \approx 0$
- radiation: $w = \frac{1}{3}$
- dark energy: $w = -1$,

under the assumption that dark energy is a cosmological constant.

Substituting these state parameters into Eqs. 1.18, 1.10, and 1.11, we find the following dependencies (which indicate growth rates):

$$\rho_r \propto a^{-4}$$
$$\rho_m \propto a^{-3}$$
$$\rho_k \propto a^{-2}.$$  

With these, we can recast the Friedmann equation with present day density parameters.

$$\left(\frac{H}{H_0}\right)^2 \approx \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \Omega_\Lambda.$$  

Recent observations derive estimates for the present day density parameters[62]:

- matter: $\Omega_{m,0} = 0.308 \pm 0.010$
- radiation: $\Omega_{r,0} \approx 10^{-4}$
- dark energy: $\Omega_\Lambda = 0.692 \pm 0.010$
- spatial curvature: $\Omega_{k,0} = -0.0005^{+0.0065}_{-0.0066}$

As both the spatial curvature and radiation energy densities are much less than 1, the Friedmann equation for our universe can be further simplified to

$$\left(\frac{H}{H_0}\right)^2 \approx \frac{\Omega_{m,0}}{a^3} + \Omega_\Lambda.$$  

7
Distances

Given the expansion of space-time, distance is no longer a simple Euclidean measure. Though there are several distances used in cosmology - luminosity distance, comoving distance, etc. - we focus on one particular distance in our studies: the angular diameter distance. This distance relates the diameter of an object (normal to the viewer) with the angle it subtends in the sky. For small objects,

\[ D_A \approx \frac{d}{\theta}. \]  

(1.24)

In a flat, matter and dark energy dominated universe, the angular diameter distance is given by:

\[ D_A = \frac{c}{H_0(1+z_e)} \int_{z_0}^{z_e} \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda}}}. \]  

(1.25)

Magnitude

In astronomy, the brightness of an object is typically given in a logarithmic measure termed the magnitude. While there are a few magnitude systems, this thesis is only concerned with the ABMAG system, as that is the preferred system for the Hubble Space Telescope. The ABMAG system defines the relationship between the magnitude and the flux density per unit frequency, \( f_\nu \):

\[ \text{ABmag} = -2.5 \log_{10} f_\nu - 48.60. \]  

(1.26)

All magnitudes in this thesis will be in the ABMAG system.

1.2 Dark Matter Structure

Most of the matter in the universe is dark. The dark matter density parameter of the universe is \( \Omega_{c,0} \approx 0.26 \) - roughly 84% of the matter energy density of the...
Cold Dark Matter (CDM) is therefore of paramount importance to our understanding of the universe. Of particular importance to this thesis is that observations of the distribution of CDM can inform theories of structure formation.

1.2.1 Growth of Perturbations

The dark matter distribution in the present universe is highly anisotropic, with small clumps, large clumps, and voids. In the early universe, however, the initial dark matter distribution is roughly uniform. The modern day pattern can be achieved if small deviations from homogeneity were present in the early universe.

Applying perturbation theory to the linearized Einstein equations, one can calculate that initial density perturbations of the form

\[ \delta = \frac{\delta \rho}{\rho} \ll 1, \]  

will grow as follows:

- matter dominated era: \( \delta \propto a \)
- radiation dominated era: \( \delta \propto a^2 \)
- dark energy dominated era: \( \dot{\delta} = 0 \)

Therefore, perturbation growth is most prevalent during the matter dominated era.

The spherical tophat collapse (STC) model, or isotropic tophat collapse model, can be used to approximate how an over-dense region collapses into a bound mass. Proposed in 1972 by Gunn and Gott, the STC model is a simple and useful method for detailing Halo formation [33]. It begins with the assumptions that the density inside an over-dense perturbation in space is roughly uniform and that space is itself roughly uniform. Beginning with a region of comoving size \( R_0 \), background density \( \rho \), and initial density \( \delta_i \), we can solve for a mass value,

\[ M_0 = \left( \frac{4\pi R_0^3}{3} \right) \rho(1 + \delta_i) \approx \left( \frac{4\pi R_0^3}{3} \right) \rho, \]

9
where the size of this region, $R(z)$, changes as the universe evolves.

The evolution of the over-density can be solved for by considering the region inside $R_0$ as a “miniature” FLRW universe with $\Omega_0 > 1$. In the matter dominated era, this produces the parameterized solution:

$$\frac{R(z)}{R_0} = \frac{(1 + z) (1 - \cos(\theta))}{(5/3)|\delta_0|} \quad \text{and} \quad \frac{1}{1 + z} = \left(\frac{3}{4}\right)^{\frac{2}{3}} \frac{(\theta - \sin(\theta))^{\frac{2}{3}}}{(5/3)|\delta_0|},$$

(1.30)

where

$$\delta_0 = \frac{\delta_i}{1 + z_i}. \quad (1.31)$$

This gives rise to a cycloidal solution for $R(z)$ that expands and then contracts. As a result, the mean density inside the over-density reaches a minimum at turnaround before contraction (see Fig. 1.2.1). The turnaround from expansion to contraction occurs at $\theta = \pi$, where the density ratio is $9\pi^2/16 \approx 5.55$. This ratio of average density within the region to the density outside the region at turnaround is a constant of the model. If the STC model is evolved to the theoretical limit of $2\pi$, the over-density would collapse to a point. This obviously is not physical, and before this can occur, other physics take hold.

Virial physics stops the collapse from reaching a singularity. The virial theorem states that the average kinetic energy of a stable system is a constant function of the average potential energy of that system. Particularly, for a gravitationally bound system

$$2\langle T_{\text{Tot}} \rangle = -\langle V_{\text{Tot}} \rangle, \quad (1.32)$$
Figure 1.2.1: A schematic of the evolution of the mean density of a perturbation in the spherical tophat collapse description in a matter-dominated Universe. At high redshifts, the perturbation is small and the density falls as the mean density of the Universe \( \rho \propto (1+z)^{-3} \). As the perturbation becomes non-linear, the spherical tophat model predicts a minimum density at the ”turnaround” redshift (which depends on the size of the perturbation through the evolution of the horizon size). After that, virialization occurs and the density stabilizes at 8 times the density at turnaround. (Note that the density evolution during virialization is not precisely described in this schematic diagram). The density of the perturbation is expressed in units of the mean matter density of the Universe today.
which yields,

\[ \langle T_{\text{Tot}} \rangle = \frac{3GM^2}{5R_{\theta=\pi}}. \tag{1.33} \]

To calculate the final density, we first note that the process of virialization will result in an over-density radius that is lower by a factor of two than its size at turnaround. This means that the volume of the halo decreases by a factor of 8, which increases the density by the same factor. In addition, the background density has decreased due to universal expansion. From Eq. 1.30, we can determine the ratio of the background density at turnaround to that at virialization to be approximately 4. This means that at virialization, the mean density will be \((9\pi^2/16) \times 8 \times 4 = 18\pi^2 \approx 178\) times the background density. This property has given rise to common use variables \(r_{200}\) and \(M_{200}\) which indicate the radius that encloses the matter in this collapse and the mass in that radius. The STC model’s strength is that it provides an analytic approximation to an over-density perturbation’s evolution into a bound massive object in an expanding universe.

Detractors of the STC model point to the overall simplicity and overgeneralization of the physics involved. However, the STC model provides a useful guideline to the relevant physical scales of collapsed over-densities.

The STC model describes the nonlinear evolution of a perturbation. However, the redshift at which perturbations become nonlinear depends on the physical scale of the perturbation. Cosmic inflation freezes in a scale-invariant spectrum of fluctuation length scales into the gravitational potential on scales larger than the horizon. The later-time growth of these perturbations is governed by the time that the fluctuations re-enter the horizon. Because the horizon size grows with time, smaller scale fluctuations enter the horizon first. These fluctuations have more time to grow, and therefore become nonlinear (and undergo collapse and virialization) first. In this way, structure formation is hierarchical, with small structures collapsing into halos first, and then merging into larger halos.
1.2.2 Halo Profiles

Though the STC model provides a clean distribution for the halo, it does not define well the distribution of dark matter within the halo. This model implies cores of constant density, which is unrealistic. Several functions have been proposed that better describe halo profiles seen in N-body simulations, with the most popular being the Navarro, Frenk, and White (NFW) Profile \([58]\). (Although the Einasto profile is gaining in popularity \([22]\).) The NFW profile is given by

\[
\rho_s = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2},
\]

where the critical density for closure of the universe at the redshift of the halo, \(\rho_c\), and the characteristic over-density of the halo, \(\delta_c\), are

\[
\rho_c = \frac{3H^2(z)}{8\pi G}
\]

and

\[
\delta_c = \frac{200}{3} \frac{c^3}{\ln(1 + c) - c/(1 + c)}.
\]

The halo profile is characterized by the concentration parameter

\[
c = \frac{r_{200}}{r_s},
\]

which describes how sharply the profile is shaped. The scale radius, \(r_s\), is a characteristic radius of the cluster. The mass enclosed within \(r_{200}\) for a NFW halo is

\[
M_{200} = \frac{800\pi}{3} \rho_c r_{200}^3.
\]

It is important to note that the NFW profile is an average descriptor, and does not necessarily truly describe individual peaks. It has been commonly used in previous cluster lensing analyses, however, and thus we use it to provide a basis of
comparison with previous studies.

Simulations of the large scale dark matter distribution in the present day universe depict a complex arrangement of knots, sheets, and filaments. As cosmological simulations have improved in resolution, they have increasingly been able to resolve structure and substructure. Millennium XXL and Horizon Run 3 simulations use > $3 \times 10^{11}$ particles, over a large volume ($3h^{-1}$ Gpc and $11h^{-1}$ Gpc respectively) to achieve a fine mass simulation (mass resolution of $8.5 \times 10^{9}h^{-1}M_{\odot}$ and $1.25 \times 10^{11}h^{-1}M_{\odot}$) of large scale structure in the universe [3, 47]. While simulations like the Phoenix Project and Aquarius Project simulate galaxy clusters and galaxies ($M_{200}$ ranges of $5.5 \times 10^{14} - 2.4 \times 10^{15}M_{\odot}$ and $8.2 \times 10^{11} - 1.9 \times 10^{12}M_{\odot}$ respectively) with detailed mass resolutions of > $6.5 \times 10^{5}M_{\odot}$ for the Phoenix Project and > $1.7 \times 10^{3}M_{\odot}$ for the Aquarius Project [3, 73]. The results of these simulations form some of the theoretical predictions motivating the work in this thesis.

1.3 CLUSTER SUBSTRUCTURE

In the CDM paradigm, structure forms hierarchically - smaller structures form first, with larger structures forming later (as depicted in Fig. 1.3.1) [51]. A consequence of this paradigm is that clusters at higher redshifts should have more substructure, as subhalos have had less time to become virialized into the cluster core.

Theoretical studies of cluster formation have focused on N-body simulation of dark matter halo interaction. Gao et al. analyzed several large simulations to investigate substructure features of dark matter halos [27]. They noted that substructure is more abundant as a function of increasing redshift (Fig. 1.3.2), lower concentration (Fig. 1.3.3), and earlier formation time (Fig. 1.3.3).

By analyzing halos in the Millennium-II simulation, Boylan-Kolchin et al. derived
Figure 1.3.1: "Merger tree" of hierarchical dark matter structure formation. Time flows down the plot, with $t_f$ being the formation time [51].

A function to describe the abundance of substructures in massive halos [9]:

$$N(> \mu \equiv m_{\text{sub}}/M_{200}) = \left( \frac{\mu}{\mu_1} \right)^{-0.94} \exp \left[ - \left( \frac{\mu}{\mu_{\text{cut}}} \right)^{1.2} \right],$$  

with $N$ being the fraction of substructures that have a mass of $m_{\text{sub}}$ or greater. Gao et al. found the following fit parameters for clusters at various redshifts [27]:

$$z = 0.0 : \mu_1 = 0.0092, \mu_{\text{cut}} = 0.07$$
$$z = 0.5 : \mu_1 = 0.0118, \mu_{\text{cut}} = 0.06$$
$$z = 1.0 : \mu_1 = 0.0130, \mu_{\text{cut}} = 0.05$$
$$z = 2.0 : \mu_1 = 0.0140, \mu_{\text{cut}} = 0.02$$  

(1.40)

Simulations also show that most substructure mass is located far from the cluster radius (Fig. 1.3.4). This is expected, as subhalos close to the core lose much of their mass to strong gravitational tidal stripping, while the more distant subhalos undergo much less tidal stripping due to being farther away.
Figure 1.3.2: Substructure mass fraction for simulated halos in the mass range $[1 - 3] \times 10^{13} h^{-1} M_\odot$. The mass fraction increases with increasing redshift [27].
Figure 1.3.3: Left. Substructure mass fraction for simulated halos in three NFW concentration groupings. Lower concentration halos have higher mass fractions. Right. Substructure mass fraction in three formation time groupings. Later formation times have higher mass fractions [27].

Figure 1.3.4: Cumulative substructure mass fraction for simulated halos in the mass range as a function of radial distance from the core [73].
In the CDM paradigm, subhalos often merge with larger halos. This merger history has the potential to complicate substructure measures and thus dark matter statistics. Studies by Stewart et al. and Boylan-Kolchin et al. show significant merger rates in the high redshift regime (Fig. 1.3.5) [9, 74].

In all, these simulation results predict substructure properties to sufficient precision that, although substructures typically make up only $\sim 10\%$ of a typical cluster’s mass, their statistics are an important diagnostic test of CDM structure formation and cosmology[9, 14, 16, 26–28, 73].
1.3.1 Measuring Substructure

There are several methods to measure substructure in clusters of galaxies. X-Ray observations of clusters probe gas that is assumed to be in virial equilibrium with the gravitational potential \([79, 80]\). The velocity distribution of galaxies in a cluster along with the variations in the velocity dispersion as a function of position in a cluster can reveal the mass distribution \([29, 30]\). Spatial over-densities of galaxies can trace out the mass structure of a cluster \([31, 48]\). Finally, gravitational lensing, where background light is bent due to matter, can reveal a map of the projected cluster mass \([46, 70]\). Of these techniques, only gravitational lensing makes no assumptions on the physical state of the baryons nor the dynamical state of the system. This makes lensing a natural analysis technique for studying the dark matter distribution, and it is therefore the technique used in this thesis.
One important consequence of Albert Einstein’s Theory of General Relativity (GR) is gravitational lensing \[23\]. In GR, massive bodies curve space-time around them, and light paths follow geodesics in this curved space-time. This leads to distortions in the observational properties of objects behind the lensing mass. From these distortions, the dark matter content of the lensing mass can be measured.

2.1 Theory

As we observe distant objects the sky in two dimensions, the gravitational lensing theory described here focuses on 2-D mass distributions. The schematics of gravitational lensing are shown in Fig. 2.1.1. This two dimensional lensing
approximation holds in the limit that the distances \(D_d, D_{ds},\) and \(D_s\) are much greater than the “thickness” of the lens. For a typical cluster and background galaxy arrangement, these angular distances are measured in Gpc. The virial radius of a cluster of galaxies is of order of one Mpc, so the thin lens approximation is appropriate in cluster lensing.

We begin with the formula for the deflection angle of light from the source:

\[
\hat{\alpha}(\xi) = \frac{4G}{c^2} \int \frac{d^2 \tilde{\xi} \Sigma(\tilde{\xi})}{|\tilde{\xi} - \xi|^2},
\]

where \(\Sigma(\tilde{\xi})\) is the surface mass density of the lens, the 2-D projected mass density at the lens plane.

Assuming a circularly symmetric lens, Eq. 2.1 reduces to

\[
\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2 \xi}.
\]
From the geometry in Fig. 2.1.1, we note that in this small angle limit,

\[ D_{ds} \hat{\alpha} = D_s \alpha. \]  

(2.3)

This allows us to define the Lens Equation:

\[ \beta = \theta - \alpha = \theta - \frac{D_{ds}}{D_s} \hat{\alpha}. \]  

(2.4)

One property of particular importance in gravitational lensing is the convergence, \( \kappa \).

\[ \kappa = \frac{\Sigma}{\Sigma_{cr}}, \]  

with the critical surface mass density defined as

\[ \Sigma_{cr} = \frac{c^2}{4\pi G D_d D_{ds}}. \]  

(2.6)

The convergence is the dimensionless surface mass density, and it quantifies the strength of a lens: \( \kappa \geq 1 \) is the strong lensing regime, and \( \kappa \ll 1 \) is weak lensing. \(^1\)

Utilizing the convergence and Eq. 2.3, we can rewrite Eq. 2.1 for the scaled deflection angle

\[ \alpha = \frac{1}{\pi} \int d^2 \theta' \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2}. \]  

(2.7)

The scaled deflection angle can be written in terms of the gradient of a potential,

\[ \alpha = \nabla \psi. \]  

(2.8)

\(^1\)While strong lensing is a more sensitive probe of substructures, it is limited to the core of a halo. For this thesis, we are more interested in large substructures found away from the cores of cluster, hence the use of weak lensing.
The deflection potential,

\[ \psi(\theta) = \frac{1}{\pi} \int d^2\sigma' \kappa(\sigma') \ln |\theta - \sigma'|, \quad (2.9) \]

is the 2-D analogue of a Newtonian gravitational potential. This potential satisfies the Poisson equation for lensing:

\[ \nabla^2 \psi(\theta) = -2\kappa(\theta) \quad (2.10) \]

### 2.2 Observables

Background sources are distorted by the gravitational lensing process. These distortions manifest themselves in 3 ways: deflection, magnification, and shear. The total distortion of background sources can be determined from the following Jacobian Matix:

\[ \mathcal{A}(\theta) = \frac{\delta \beta}{\delta \theta} = \begin{pmatrix} \delta_{ij} - \frac{\delta^2 \psi(\theta)}{\delta \theta_i \delta \theta_j} \\ \end{pmatrix} \]

\[ = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (2.11) \]

where \( \gamma \) is the complex shear.

The deflection is the angular displacement due to lensing. It is the lowest order, and most obvious effect of lensing. However, it is not useful as a measurement property of the lens as observers have no way of knowing a priori the original source locations.

The magnification of a source is the ratio of the observed flux to the source’s true flux. It is given by

\[ \mu = \frac{1}{\det(\mathcal{A})} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2}. \quad (2.13) \]
Like the deflection, magnification has no unique observable - it requires prior knowledge of the flux, size, and spatial distribution of sources.

The inability to measure these two effects via weak lensing alone highlights a weakness of the technique - the mass sheet degeneracy. In weak lensing, the convergence solution is degenerate under the following transformation:

$$\kappa \rightarrow \kappa' = \lambda \kappa + (1 - \lambda) \quad (2.14)$$

Therefore, neither of these methods can uniquely determine the lensing mass.

Fortunately, there exists another observable property: the shear. The lensing shear is the stretching of an image in the direction orthogonal to the radius of the sphere to the source. In Eq. 2.12, we introduced the complex shear, $\gamma = \gamma_1 + i \gamma_2$, with the components:

$$\gamma_1 = \frac{1}{2} \left( \frac{\delta^2 \psi(\theta)}{\delta \theta_1 \delta \theta_1} - \frac{\delta^2 \psi(\theta)}{\delta \theta_2 \delta \theta_2} \right) \quad (2.15)$$
$$\gamma_2 = \frac{\delta^2 \psi(\theta)}{\delta \theta_1 \delta \theta_2}$$

Like the deflection and magnification, the shear also suffers from a degeneracy,

$$\gamma \rightarrow \lambda \gamma. \quad (2.16)$$

However, we can define a reduced shear that eliminates this issue:

$$g = \frac{\gamma}{1 - \kappa}, \quad (2.17)$$

yielding a unique observable for researchers to study. The Jacobian matrix
(Eq. 2.12) in terms of reduced shear is

\[
A = \begin{pmatrix}
1 - g_1 & -g_2 \\
-g_2 & 1 + g_1
\end{pmatrix}
\]  \hspace{1cm} (2.18)

In the weak lensing limit, \( \gamma \ll 1 \) and \( \kappa \ll 1 \). Therefore, the reduced shear given by Eq. 2.17 is approximately equal to the shear.

2.3 Analysis Techniques

There are several steps in the analysis of the data for a weak lensing study of dark matter substructure. First and foremost, a method to extract the reduced shear signal is required. Second, the shear signal of detected peaks must be converted into masses. Finally, the significance of each peak detection must be ascertained in order to distinguish it from spurious detections.

2.3.1 Kaiser-Squires-Broadhurst Algorithm

In 1993 and 1995, Kaiser, Squires, and Broadhurst (KSB) developed a method for calculating shear signals from images [45, 46]. To utilize this method, images need to be processed, and the positions and intensity profiles of every source galaxy must be measured.

Beginning with the intensity distribution of an object, \( I(x) \), second order, or quadrupole, moments can be obtained as follows:

\[
Q_{ij} = \frac{\sum I(x)W(x)x_i x_j}{\sum I(x)W(x)}, \hspace{1cm} (2.19)
\]

where \( W(x) \) is a weighting function. The position vector \( x \) is measured from the centroid of the object, eliminating the first order moments.

Weighted second order moments measurements are subject to two forms of error, pixelization and intensity measurement shot noise. Objects measured by CCDs are subject to the finite resolution scale of CCD pixels. This pixelization can limit the
accuracy of shape measurements for small galaxies. Therefore it is important to be cognizant of this effect during background source selection. Intensity is a measure of the number of photons collected in a pixel during an exposure. Photons arrive at the detector in random intervals, with the arrival time probability governed by a Poisson distribution. This generates shot noise in the intensity measurement, which must be taken into account when analyzing the data [6, 10, 38].

The weight function serves several purposes. It must be selected so that it forces the sums in Eq. 2.19 to converge. As real images have noise, bad pixels, hot pixels, and cosmic rays, a weight function can help ameliorate these defects. It is also important to provide less weight to outer pixels, which have lower signal to noise and are thus far more sensitive to artifacts in the data. With these in mind, selection of a proper weight function is crucial to order measurement accuracy.

Within the suite of standard astronomical analysis software packages are two programs of use for modeling shapes, Source Extractor and Ellipto [6, 7]. Each of these programs offers a different option for second order moment weight functions. Source Extractor has a windowed mode, where it weights its measurements with a circular Gaussian profile. Ellipto utilizes an elliptical Gaussian profile.

The choice of weight function for an object depends on its expected observed shape. The objects of interest in moment measurements are stars and galaxies. Stellar profiles are typically circular, and are thus best weighted by Source Extractor. For galaxies, ellipticals are typically expected to hold to a de Vaucouleurs profile, while spirals are normally represented by an elliptical exponential profile. In either case, Ellipto is the preferred weight profile for galaxies [6].

Once the moments are determined from Eq. 2.19, complex ellipticities of the objects can be introduced:

$$\chi = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}$$  (2.20)
\[ \epsilon = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\sqrt{Q_{11}Q_{22} - Q_{12}^2}}, \]  

(2.21)

where \( \chi \) is often referred to as the polarization. In the weak lensing limit, \( \chi \approx 2\epsilon \), so there is an easy mapping between the two. The position angle of a source is given by:

\[ \phi = \frac{1}{2} \arctan \left( \frac{\epsilon_2}{\epsilon_1} \right). \]  

(2.22)

Having measured the shape of a background galaxy, the next step is to equate that ellipticity to a shear. The KSB theory provides the transformation of a source’s intrinsic quadrupole moments, \( Q^{(i)} \), to the lensed result via

\[ Q = AQ^T = AQ^T, \]  

(2.23)

where \( A \) is the lensing Jacobian defined in Eq. 2.18. Applying this transformation yields

\[ \epsilon = \frac{\epsilon^{(i)} + g}{1 + \epsilon^{(i)}g^*}, \]  

(2.24)

which simplifies to

\[ \epsilon \approx \epsilon^{(i)} + g \approx \epsilon^{(i)} + \gamma \]  

(2.25)

in the weak lensing limit.

As galaxies are intrinsically elliptical, with a range of ellipticities, they are poor sources of shear information on an individual basis. As a group, however, galaxies are randomly aligned, and the expectation value of their ellipticities averages to zero,

\[ \langle \epsilon^{(i)} \rangle = 0. \]  

(2.26)

An important note regarding this simplification is the assumption that the
distribution of background galaxies is uncorrelated. The dominant noise for galaxy populations is the intrinsic scatter of their shapes. The RMS ellipticity for galaxies is $<\sigma_\epsilon> \approx 0.3$, varying slightly with the distribution in redshift, magnitude, and color [34, 39, 63]. Therefore, with a large sample of randomly aligned background galaxies, the average intrinsic ellipticity signal washes out, leaving just the average shear:

$$\langle \epsilon \rangle = \langle \gamma \rangle. \quad (2.27)$$

To obtain a mass map of the lens, the shear must be converted to the convergence. The relationship between the shear and convergence can be obtained through equations 2.15 and 2.9:

$$\gamma(\theta) = \frac{1}{\pi} \int d^2\theta' D(\theta - \theta') \kappa \theta', \quad (2.28)$$

with

$$D(\theta) = \frac{\theta_2^2 - \theta_1^2 - 2i\theta_1^2\theta_2^2}{|\theta|^4}. \quad (2.29)$$

To obtain an expression for the convergence, we take the Fourier transform of Eq. 2.28

$$\kappa(r) = \frac{1}{\pi} \int d^2r' \text{Re}[D^*(r - r')\gamma(r')]. \quad (2.30)$$

For use with actual data, it is more practical to recast Eq. 2.30 in terms of a sum:

$$\kappa(r) = \frac{2}{n\pi} \sum_{k=1}^{n} \text{Re}[D \ast (r - r_k)(2\epsilon_k)], \quad (2.31)$$

where $n$ is the total number of background galaxies. As with Eq. 2.19, a weight function is needed for this sum. There are two problems that dictate the shape of the weight function. First, the kernel, $D$, diverges at $r = 0$ and, thus it over-weights
the ellipticity (which is dominated by the random shape noise) of galaxies very close to the point where $\kappa$ is being calculated. At radii much larger than the virial radius, the shear due to the mass falls asymptotically to zero, and including additional galaxies beyond that radius would increase the noise and not the signal. In practice, the exact choice of the weight function is not critical, as long as it trends to zero on scales smaller than typical galaxy separations and on scales larger than typical cluster virial radii. With a proper weight function, $W(r)$, Eq. 2.31 becomes

$$\kappa(r) = \frac{1}{n \pi} \sum_{k=1}^{n} W(r - r_k) \frac{\epsilon^T_k(r - r_k)}{|r - r_k|^2},$$  

(2.32)

where $\epsilon^T_k(r - r_k)$ is the tangential ellipticity of galaxy $k$ with respect to $r$,

$$\epsilon^T_k(r - r_k) = \epsilon_{1k} \cos(2\varphi) + \epsilon_{2k} \sin(2\varphi),$$  

(2.33)

with

$$\cos(2\varphi) = \frac{(x - x_k)^2 + (y - y_k)^2}{(r - r_k)^2}$$  

(2.34)

$$\sin(2\varphi) = \frac{2(x - x_k)(y - y_k)}{(r - r_k)^2}$$

The complementary ellipticity to the tangential ellipticity is given by

$$\epsilon^c_k(r - r_k) = \epsilon_{1k} \sin(2\varphi) + \epsilon_{2k} \cos(2\varphi).$$  

(2.35)

This ellipticity is often referred to as the B-mode polarization, with $\epsilon^T_k(r - r_k)$ being the E-mode polarization. While the E-mode is used for mass map creation, the B-mode is used as a statistical control. The lensing shear imparts no signal to the B-mode peaks, thus any strong SNR B-mode peaks indicate biases in the galaxy catalog that may require correction.
2.3.2 Errors

Weak lensing shear observations are affected by sources of error. Examples of intrinsic error are shape alignment and shot noise. Extrinsic error derives from the point spread function (PSF) size and anisotropy, instrument limitations, catalog selection biases and dilution, and shape measurements. This thesis seeks to minimize these effects, with methods for doing so outlined in the appropriate sections.

2.4 Observational Techniques

Researchers must give careful consideration to observations for weak lensing studies. For optimal weak lensing detection, observers want to maximize SNR. Some observing methods to help accomplish this are: maximizing faint object flux levels, optimizing seeing, spreading observations over several pointings, and observing in multiple color band filters.

Weak lensing SNR is driven by the number of background source galaxies. The longer an observer can observe an area of the sky, the more faint objects they can detect to sufficient SNR for lensing. Another important effect on number of resolved background galaxies is the seeing. The seeing refers to the size of the PSF, which determines the object size detection threshold. With minimized seeing, observers can resolve smaller objects, thus increasing their galaxy totals.

It is not ideal to take all this data in one exposure, however. Overly long exposures have several problems. First, the longer the exposure, the more objects that become saturated. CCD detectors have a maximum charge they can store, so when they become saturated, the extra flux is effectively thrown away. This can greatly distort shape measurements as the intensity profile will be incorrect, having been cropped in the center by CCD saturation. Second, during observations, random noise events can occur. The most dominant source of this comes from cosmic rays hitting the detector. With just one exposure, the information from
many pixels can be ruined by cosmic rays saturating the pixel, satellite pass-bys, or electronic glitches. Taking multiple images allows for the rejection of bad pixels data without marring the total image data. Lastly, telescopes move through space and thus to remain focused on an object, they must adjust their orientation. While engineers attempt to minimize any smearing due to this adjustment, telescope guiding is never perfect. Shorter exposures are much less sensitive to this sort of shape smearing.

Finally, it is advantageous to observe in multiple color bands. Doing so enables observers to determine the redshift of their background galaxies. Redshift information is used in determining which galaxies are background vs. foreground, and in ascertaining peak masses in the NFW model. Although these photometric redshifts are not as accurate as spectroscopic redshifts, they are far cheaper in terms of telescope time and easier to obtain for faint galaxies.

2.5 APERTURE MASS AND SCHIRMER METHOD

Ground based cluster surveys often utilize weak lensing. By generating lensing maps of large swaths of the sky, these surveys can look for mass peaks. To do so, they hunt for mass concentrations via the aperture mass [46, 69]:

\[ M_{\text{ap}}(\theta) = \int d^2\theta' \kappa(\theta') U(|\theta - \theta'|), \]  

(2.36)

where \( U(|\theta|) \) is a compensated filter function, meaning

\[ \int d\theta \, \theta U(|\theta|) = 0. \]  

(2.37)

The aperture mass can be recast in term of the tangential shear, given by the relation Eq. 2.28:

\[ M_{\text{ap}}(\theta) = \int d^2\theta' Q(|\theta'|) \gamma(\theta'; \theta), \]  

(2.38)
with

$$Q(\theta) = \frac{2}{\theta} \int_0^\theta d\theta' \theta' U(\theta') - U(\theta). \quad (2.39)$$

For use with data, this is typically cast into a sum

$$M_{ap}(\theta) = \frac{1}{n} \sum_i \epsilon_i^T (\theta) Q(|\theta_i - \theta|), \quad (2.40)$$

where \( n \) is the mean number density of galaxies.

It is important to note that the aperture mass method does not provide the actual mass of the cluster. By definition, the aperture mass is a local measurement involving only the shears of galaxies within an angle of \( \theta \) of the center. Even with this limitation, it is a powerful method for extracting significant mass peaks, and survey researchers utilize other methods for mass estimation \[35, 53, 55, 68–71\].

One popular aperture mass filter was proposed by Schirmer et al. as part of the Garching-Bonn Deep Survey (GaBoDS) \[68\]. This filter is designed to best highlight NFW profiles (see Sec. 1.2.2), and is given by

$$Q(x) = \frac{1}{(1 + e^{a-bx} + e^{-c+dx})} \frac{\tanh (x/x_c)}{\pi \theta_0^2 (x/x_c)}, \quad (2.41)$$

with

$$x = \frac{\theta}{\theta_0}. \quad (2.42)$$

This filter has a lot of tunable parameters, with the following settings for NFW
Figure 2.5.1: The weight profile of the NFW halo finding filter developed by Schirmer. The quantity $x$ is the normalized filter radius [36].

The shape of the filter is given in Fig. 2.5.1. The Schirmer filter is one of the best high redshift NFW halo finders, and is used regularly for high redshift cluster finding surveys [60].

This thesis is not about finding clusters in survey data, but rather finding...
substructure peaks in clusters. Using the mass aperture filter method on observations of a cluster will pick out substructure NFW peaks, provided there is sufficient background source density. To find a 5% substructure peak of a $10^{14} \, M_\odot$ cluster at redshift $z = 0.5$, one can take advantage of Section 1.2.2 to get a rough estimate of the background source density needed. First, we calculate the $r_{200}$ for the cluster, using both Eq. 1.38 and Eq. 1.35:

$$M_{\text{clus}}^{200} = \frac{800\pi}{3} \rho_c (r_{200}^{\text{clus}})^3 = 10^{14} M_\odot,$$

which implies

$$r_{200}^{\text{clus}} = \sqrt[3]{0.54 \text{Mpc}^3} = 0.82 \text{Mpc}.\quad (2.45)$$

Then we compare the mass ratios in order to obtain a radial ratio:

$$\frac{M_{\text{sub}}^{200}}{M_{\text{clus}}^{200}} = \left( \frac{r_{\text{sub}}^{200}}{r_{\text{clus}}^{200}} \right)^3 \quad (2.46)$$

yielding

$$r_{200}^{\text{sub}} = 0.3 \text{Mpc}, \quad (2.47)$$

which at $z = 0.5$, subtends roughly $48''$.

Finally, we use the following relation to solve for number density [5]:

$$\theta \geq \sqrt{\frac{N}{n\pi}}, \quad (2.48)$$

where $n$ is the background source density and $N$ is the number of galaxies needed for a shear measurement. To estimate $N$, we use the RMS ellipticity for galaxies
from Section 2.3.1:

\[
\sigma_\gamma \approx (0.25)\gamma \approx \frac{\langle \sigma_\epsilon \rangle}{\sqrt{N}}
\]

\[
N \approx \left( \frac{16\langle \sigma_\epsilon \rangle}{\gamma} \right)^2
\]

\[N \approx 144,\]

where we estimate for a 25% shear error due to shape noise, and assign a conservative mean interior shear of 0.1. We can now use Eq. 2.48 to find the source density:

\[
n \geq \frac{N}{\theta^2\pi}
\]

\[
n \geq 72 \frac{\text{gal}}{\text{arcmin}^2}.
\]

This galaxy density generally cannot be achieved on the ground, due to the atmospheric PSF - it requires a space telescope.

Another valuable feature of the Schirmer filter is that it separates peaks by aperture size. By varying \(\theta_0\) in Eq. 2.41, peaks of various scales can be searched for independent of peaks of other scales. Several popular weak lensing filters search for all peaks at once, which can make characterizing individual peaks difficult \[24, 76, 78\]. This is particularly true for substructure peaks, where the total lensing signal is dominated by the cluster peak.

To date, this filter has only been used for ground based surveys, where it is mass limited to clusters of moderate or greater mass \((> 10^{14} \, M_\odot)\). In this thesis, we propose to utilize it for searching for substructure peaks in clusters at high redshift. In addition to its benefits in substructure search, we can take advantage of its separable aperture sizes in order to better characterize found substructure peaks.
2.6 NFW Fitting

As aperture mass methods do not provide the actual mass of halos, a method is needed to estimate peak mass. For this, we utilize the NFW profile outlined in Section 1.2.2.

To estimate peak mass, we fit the average shear in annuli to the shear for an NFW profile (described in Eq. 1.34). Wright and Brainerd calculated NFW shear as a function of radius[82]:

\[
\gamma_{nfw}(x) = \begin{cases} 
\frac{r_s \delta_c \rho_c}{\Sigma_c} g_<(x), & x < 1 \\
\frac{10}{3} + 4 \ln \left( \frac{1}{2} \right), & x = 1 \\
\frac{r_s \delta_c \rho_c}{\Sigma_c} g_>(x), & x > 1 
\end{cases}
\] (2.51)

with

\[
g_<(x) = \frac{8 \arctanh \sqrt{1 - x}}{x^2 \sqrt{1 - x^2}} + \frac{4}{x^2} \ln \left( \frac{x}{2} \right) - \frac{2}{(x^2 - 1)} + \frac{4 \arctanh \sqrt{1 - x}}{(x^2 - 1) (1 - x^2)^{1/2}}
\] (2.52)

\[
g_>(x) = \frac{8 \arctan \sqrt{\frac{x - 1}{1 + x}}}{x^2 \sqrt{x^2 - 1}} + \frac{4}{x^2} \ln \left( \frac{x}{2} \right) - \frac{2}{(x^2 - 1)} + \frac{4 \arctan \sqrt{\frac{x - 1}{1 + x}}}{(x^2 - 1)^{3/2}},
\]

where \(x = r/r_s\), the critical density (\(\rho_c\)) is given by Eq. 1.35, the halo overdensity (\(\delta_c\)) as described in Eq. 1.36, and the critical surface mass density (\(\Sigma_c\)) from Eq. 2.6.

One issue with this shear fitting method is that the NFW profile, and thus its tangential shear (Eq. 2.51), depends on two parameters, which can be chosen to be \(r_{200}\) and the concentration parameter, \(c\) (Eq. 1.37). In the presence of infinite SNR, a two parameter fit can be made. However, in the presence of noisy data, the shear profile isn’t adequately constrained enough to achieve a two parameter fit. Therefore, there is a need for an additional component to reduce the fit to one
parameter.

Ideally, we would find a mapping between the Schirmer filter radius that best matches halo and subhalo peaks with the NFW scale radius, or virial radius of those peaks. To date, we have been unable to find such a mapping. Fortunately, there exists a popular way to reduce this to a one parameter fit in the literature. By utilizing a concentration relation proposed by Bullock et al. in simulation studies [12, 72], the concentration of a cluster can be described as a function of redshift:

\[
c = \frac{A}{(1 + z)^B} \times \left( \frac{M_{200}}{2.0^{12} h^{-1} M_\odot} \right)^C,
\]

where A, B, and C are tunable parameters. Values of these well suited for clusters of redshift less than 2 were established in simulations by Duffy et al. (Figure 2.6.1) [18]:

\[
A = 6.71 \pm 0.12 \\
B = 0.44 \pm 0.05 \\
C = -0.091 \pm 0.009.
\]

While this relation works well for isolated mass peaks, it is important to note that subhalos in halos have a higher concentration then they would were they the same mass outside of a halo. Accordingly, we recognize that our NFW fit method may not properly describe the concentration of subhalos.

With this prior, we perform a one parameter fit on \( r_{200} \) and use that to determine \( M_{200} \), and thus \( c \). While this prior is true on average, there can be significant scatter in the fit. This is similar to the weakness of the NFW profile being merely an average descriptor for the cluster mass profile. However, this mass estimator is still valuable because it provides a simple means of comparison to masses derived via other techniques.
Figure 2.6.1: Concentration - mass relations at redshifts of 0, 1, and 2 for various concentration fitting algorithms. The Duffy et al. parameters provide the best fit to the data (solid tan line) \[18\].
Weak lensing observations are typically made from two types of optical observatories: ground-based and space-spaced. The choice of which imager is used for research is largely dictated by properties of the data, as each telescope type has different strengths and weaknesses.

The advantages for ground-based imagers are threefold: they often have a large field-of-view (FOV) with which to create mosaics from, they are on larger telescopes, and it is much easier to obtain large amounts of exposure time. Large telescopes can have a FOV in the tens of arcminutes on a side (Suprime - 34’ × 27’, Mosaic - 40’ × 40’, and ODI - 1° × 1°). Due to decreased construction and operation costs, ground-based observatories are much more numerous than space based ones. Observing in the atmosphere, however, limits the resolution of
ground-based imaging. These telescopes are well suited for observations of wide areas of sky at depth.

At present, there is one optical space imager, the Hubble Space Telescope (HST, Fig. 2). Hubble is a 2.4 meter telescope in a low earth orbit, with a period of $\sim 97$ minutes. The HST has three optical instruments of interest: the Wide Field and Planetary Camera 2 (WFPC2 - removed), the Advanced Camera for Surveys Wide Field Channel (ACS WFC), and the Wide Field Camera 3 (WFC3). In contrast to the ground-based telescopes listed above, each of the HST imagers has a tiny FOV (WFPC2 - $150'' \times 150''$, ACS WFC - $202'' \times 202''$, and WFC3 - $164'' \times 164''$). As the Hubble is the only space-based imager, it is very difficult to obtain observing time. The big advantage for the HST, however, is that because it is in space, it can resolve more distant and faint objects.

For studies of clusters of galaxies beyond redshift $z > 0.5$, deep observations are
Figure 3: Number density of resolved background galaxies in a single deep pointing as a function of redshift for three imaging conditions. The cutoff at $z < 0.35$ is due to the FOV of the HST imagers [17].

As shown in Fig. 3, Hubble has a better background source density at high redshift than even the best ground-based seeing allows. In addition, the FOV cutoff for the HST does not hinder high redshift cluster observations, as the Hubble imagers capture more than a Mpc$^2$ area for a $z > 0.5$ cluster in a single pointing. As stated, getting observing time on the Hubble Space Telescope is quite difficult. Fortunately, the Space Telescope Science Institute (STScI) maintains a public archive of all previous HST observations. Accordingly, this thesis is a study of high redshift clusters with extant archival HST images.
3.1 Advanced Camera for Surveys

Primarily motivated by the PSF correction scheme detailed in Sec 3.2, the clusters in this thesis were observed with ACS. The Advanced Camera for Surveys was installed on the Hubble during servicing mission 3B, in March 2002. It is the most commonly used optical instrument in the HST archive, and has been used for many high redshift cluster observations.

The ACS has three independent channels, HRC, SBC, and WFC. The High Resolution Channel (HRC) was used for detailed images of small images, with a FOV of $29'' \times 25''$ and excellent pixel resolution, $0.028''/\text{pix} \times 0.025''/\text{pix}$. Although the HRC had the best resolution of the three instruments, its FOV was too small for cluster studies. The Solar Blind Channel (SBC) is an ultraviolet camera, and thus not useful for optical observations of large samples of high redshift galaxies that are typically much fainter in the UV than the optical. The final and most popular instrument, the Wide Field Camera (WFC), is the imager used for this thesis.

3.1.1 Wide Field Camera

There are primarily three reasons why the WFC is the most commonly used ACS instrument. First, having the largest FOV of the Hubble imagers at $202'' \times 202''$, the WFC enables much more efficient use of Hubble observing time when studying extended objects such as clusters. Second, the plate scale resolution of $(0.05''/\text{pix})^2$ allows for extremely high resolution studies. Finally, the WFC has a wide spectral response ($\sim 3500\text{Å} - 11,000\text{Å}$) which gives researchers a great range of filter options for observations. These properties make the WFC a highly versatile instrument.

When processing data from any telescope, researchers must be aware of all the significant sources of error for that particular instrument. For the Wide Field Camera, errors largely come from two sources: the optical components and the electronic components. STScI have produced two data correcting software packages (Multidrizzle and CALACS) that address some of these errors.
Electronic Features and Errors

The Wide Field Camera utilizes two adjacent 4096 x 2048 pixel\(^2\) charge-coupled devices (CCDs), separated by \(\sim 50\) pixels. The CCDs are sensitive from ultraviolet to near-infrared wavelengths, and have good quantum efficiency (QE) across that spectrum, as indicated in Fig. 3.1.1. These CCDs do not suffer from quantum efficiency hysteresis, where the QE depends on the exposure history.

Each CCD is readout by two amplifiers, to minimize noise. The signal electrons are then converted to data numbers (DN) in the analog-to-digital converter (ADC). The ADC is 16 bit, thus the maximum depth for a pixel is 65,535 DN. The gain controls the electron to DN conversion, and the WFC can be operated at gains of 1, 2, 4, or 8 electrons/DN. The WFC was designed to operate with a gain of 2, and that default gain is the only value presently supported.

With these electronics, there are many potential sources for error. The CALACS
program, produced by STScI, is used to correct for some of these errors. CALACS performs the following tasks: bias subtraction, dark current subtraction, and flat fielding.

CALACS begins by flagging bad pixels in reference files. Additional bad pixels are flagged during the remainder of its processing. The next correction CALACS performs involves removing the amplifier bias from the image. CALACS removes the linear bias as measured in the overscan region of the CCDS (the region that is outside the exposed area). The smaller, nonlinear bias effects are address in the third step, dark current correction.

Dark current is when pixels generate much higher rates of electrons than normal. For the WFC CCDs, these pixels are separated into two groups: low current (< 0.02 electrons/sec) pixels to be corrected, and high current pixels to be discarded. In images, these high current pixels are typically saturated, and thus cannot be scaled back to obtain the photonic data. The highest current pixels are dubbed super-hot as they generate so many electrons that during readout they saturate the remaining pixels in the column. This leads to the creation of hot columns, which must be removed from the data along with the hot pixels. To calibrate for the dark current, CALACS takes four 1000 second dark images (images with the shutter closed) each day. These images, along with a high SNR image created from the previous two months worth of dark images, are properly scaled for gain and exposure time, and then subtracted from the processed image. The dark images also measure the secondary bias, thus removing it in this process.

Finally, images are flat-fielded by CALACS. These flat-field images are generally created by using a uniform luminosity across the entire CCD, in order to measure the relative sensitivity of each pixel. For a ground-based telescope, this is usually done by imaging an illuminated screen each night before observing. As the HST lacks this ability, STScI implemented a two step approach to creating accurate flats. First, they performed a standard flat-field measure of the CCDs before launch.

While in space, HST periodically images the globular cluster 47 Tucanae. The
stars in the cluster (that are not variable) are used to measure each pixel’s sensitivity. In addition to profiling the relative sensitivity of the CCD pixels, these flats also model the long wavelength fringing. The WFC CCDs are susceptible to fringing for wavelengths longer than $7500\text{"Å}$, which only manifests itself in the narrowband filter, F892N. To correct for both these CCD properties and create its final image, CALACS appropriately scales the flat-field image and applies it to the data. Fig. 3.1.2 shows the effect of the CALACS pipeline.

There are several insignificant electronic noise sources that CALACS does not correct for:

- The read noise of the CCD amplifiers. This noise is $\sim 5 - 6$ electrons per image. This contributes a random background noise which is smaller than the sky background except in short ($<100$s) exposures with the bluest filters.

- Cross-talk interference between these amplifiers during readout. This is minimized at gain equals 2, yielding a cross-talk of $\sim 1$ electron per image.

- Time dependent shutter shading, as the shutter opens and closes. This contributes a $< 0.3\%$ effect in exposures longer than 0.5 seconds.
Figure 3.1.3: Spiral Galaxy M100 imaged by HST before (Left) and after (Right) the installation of corrective optics [1].

Given that our observations are typically ~1000 seconds long, with a sky background in the $10^2 - 10^3$ electron range, none of these effects are noticeable.

The final common electronic noise source that CALACS does not correct is the charge transfer efficiency (CTE). This is addressed in section 3.2.

OPTICAL FEATURES AND ERRORS

The Hubble Space Telescope was launched with a flawed mirror. This incorrectly ground mirror caused a spherical aberration in the telescope images (see Fig. 3.1.3). Replacement of the mirror in orbit was impossible, so all future instruments had to be designed with corrective optics. The WFC does so with a minimum of instrumentation, but this results in a small geometric image distortion that must be corrected (Fig. 3.1.4). This distortion is corrected with STScI’s Multidrizzle software.

The Multidrizzle package can perform three tasks with images: image registration, cosmic ray and bad pixel rejection, and drizzling. We utilize the multidrizzle package exclusively for geometry correction of the optical distortion, along with
Figure 3.1.4: Distortion map of the nonlinear components of the WFC [32].
cosmic ray and bad pixel rejection. Fig. 3.1.5 shows the effect of these corrections on an CALACS processed image. The drizzle stacking in Multidrizzle is far too imprecise in astrometry (order of pixels compared to a PSF size of $\sim 1.5$ pixels) to be used for weak lensing purposes, and thus we manually stack our images [77].

There are three further optical properties to discuss. The first, the point spread function, will be covered in section 3.2. The second is the drift rate and jitter of the HST. The HST has two fine guidance cameras that it uses to lock on to guide stars when observing objects. If two suitable objects are unavailable, it locks on to one and performs a gyro controlled roll. This roll increases the HST drift rate, as illustrated in Table 3.1.1. Weak lensing observations are typically $\sim 1000$ seconds

<table>
<thead>
<tr>
<th>Type</th>
<th>per 1000s exposure (arcsec (pixel))</th>
<th>per orbit (96 min) (arcsec (pixel))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Stars - Maximum</td>
<td>0.002 (0.03)</td>
<td>0.01 (0.2)</td>
</tr>
<tr>
<td>1 Star - Typical</td>
<td>0.0041 (0.08)</td>
<td>0.024 (0.47)</td>
</tr>
<tr>
<td>1 Star - Maximum</td>
<td>0.0080 (0.16)</td>
<td>0.046 (0.92)</td>
</tr>
</tbody>
</table>

**Table 3.1.1:** The drift rates for the HST for both 1-star and 2-star guiding modes [32].
Table 3.1.2: The wide-band filters for the Advanced Camera for Surveys on the Hubble Space Telescope. For each filter, the central wavelength, filtered wavelength width, and roughly matching telescope filter for comparison [77].

<table>
<thead>
<tr>
<th>Name</th>
<th>Central Wavelength (Å)</th>
<th>Width (Å)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F435W</td>
<td>4297</td>
<td>1038</td>
<td>Johnson B</td>
</tr>
<tr>
<td>F475W</td>
<td>4760</td>
<td>1458</td>
<td>SDSS g</td>
</tr>
<tr>
<td>F555W</td>
<td>5346</td>
<td>1193</td>
<td>Johnson V</td>
</tr>
<tr>
<td>F606W</td>
<td>5907</td>
<td>2342</td>
<td>Broad V</td>
</tr>
<tr>
<td>F625W</td>
<td>6318</td>
<td>1442</td>
<td>SDSS r</td>
</tr>
<tr>
<td>F775W</td>
<td>7764</td>
<td>1528</td>
<td>SDSS i</td>
</tr>
<tr>
<td>F814W</td>
<td>8333</td>
<td>2511</td>
<td>Broad I</td>
</tr>
<tr>
<td>F850LP</td>
<td>9445</td>
<td>1229</td>
<td>SDSS z</td>
</tr>
</tbody>
</table>

long, and 0.1 pixel level centroiding precision is required for proper moment measurements, so care must be used if clusters have been imaged in 1 star guiding mode. The jitter for a typical Hubble observation is $\sim 0.003 - 0.005$ arcsec, so it does not significantly affect the shapes of measured objects.

The final optical property is the set of filters used for weak lensing observations. Filters are used to measure fluxes in particular wavelength bands. This is important for determining the spectral energy distributions and ultimately the redshifts of galaxies. The wide spectral response ($\sim 3500\text{Å} - 11,000\text{Å}$) of the HST allows for a range of wideband filters (Table 3.1.2 and Fig. 3.1.6). Wideband filters, as opposed to narrowband filters, are used in lensing studies because they enable efficient photometry of large populations of background sources given their much higher throughput.

**Final Throughput**

All of these properties put together define the sensitivity limits of the Hubble Advanced Camera for Surveys. The total throughput of the WFC is shown in
Figure 3.1.6: Transmission percentages for the wide-band filters of the Advanced Camera for Surveys on the Hubble Space Telescope as a function of wavelength [77].
Fig. 3.1.7, and the throughput of the three filters utilized in this thesis (F555W, F775W, and F814W) are displayed in Fig. 3.1.8.

Given the throughput, the limiting magnitude for a given exposure can be calculated. Fig. 3.1.9 shows the time needed to resolve an ABMAG=26 extended source to a SNR of 5. The exposures for the cluster observations for this thesis all have a minimum duration of 1000 sec, thus ensuring high SNR resolution of faint objects. Through various stacking techniques, we can reach fainter magnitudes with good SNR as well, provided sufficient exposures are available. Fig. 3.1.10 shows the limiting magnitude of the WFC in a 10 hour exposure, effectively setting the practical magnitude limit for HST observations [77].

3.2 PSF CORRECTION - JEE METHOD

The point spread function (PSF) is the response of the telescope optics and the atmosphere to an infinitely compact light source. Although the Hubble Space Telescope has no atmosphere to contend with, the optical components of the system induce a PSF on images with a characteristic scale (defined by the FWHM) of about 0.07” (1.5 pixels). While this PSF is much smaller than ground-based seeing, correcting for this maximizes the lensing resolution attainable with HST studies. Typically, PSFs are difficult to model, as they are dominated by the random interference generated by air movement in the atmosphere. Fortunately, the Hubble PSF is much simpler. It consists of two components - a long term steady degradation of materials due to age and radiation exposure, and a short term oscillatory effect. This oscillatory effect is a “breathing” mode of the HST PSF, and it is generated by the orbit of the HST. As the HST moves towards the sun, it warms and expands. As it moves away from the sun, it cools and contracts.

In 2007, Jee et al. devised a method for modeling this PSF [42]. They performed
Figure 3.1.7: Total transmission percentages for the Advanced Camera for Surveys on the Hubble Space Telescope as a function of wavelength. This combines the effects of the CCDs and optical elements [77].
Figure 3.1.8: Transmission percentages for this thesis’s wide-band filters of the ACS on the Hubble as a function of wavelength [77].

Figure 3.1.9: Time required for ACS WFC extended sources to obtain ABMAG=26 at a SNR of 5. The bumps correspond to the wavelengths covered by the two broadest and highest transmission filters, F606W and F814W [77].
a principal component analysis (PCA) of all the star rich observations done by Hubble, particularly the flat-field observations of globular cluster 47 Tucanae. Using these PCA solutions, they created hundreds of PSF Template files for each wideband ACS filter (Table 3.1.2). These template files describe the PSF at each observation, and thus map the PSF changes with time. The PSF differs significantly in this time cycle as can be seen in Fig. 3.2.1. This PCA approach better preserves the PSF shape, as shown in Figures 3.2.2 and 3.2.3. The Jee PSF correction scheme provides better results than the popular Tiny Tim software package (Fig. 3.2.4).

In addition, the Jee PSF templates also incorporate the charge transfer efficiency correction. Some pixels in the WFC will have delayed charge release during readout, causing a charge trail behind objects. While this is not a large effect, it has the potential to distort measurements for background galaxies. In a study of CTE, Jee et al. performed measurements of the CTE and found it to be an $\sim 0.01$ ellipticity effect [43]. Jee et al. folded the CTE effect, which is a time-varying function due to

![Figure 3.1.10: The SNR 5 magnitude limit for point sources in a 10 hour long exposure [77].](image)
Figure 3.2.1: Left: 2002 PSF Template File. Right: 2006 PSF Template File. Significant shape and orientation differences can be seen between the two PSF files. This highlights the time dependency of the HST point spread function [42].

Figure 3.2.2: Models of PSF fitting for a single star. (a) The original uncorrected star. (b) Wavelet decomposition with $\sim 150$ basis functions. (c) Shapelet decomposition with $\sim 80$ basis functions. (d) Jee PCA method representation. The Jee method best fits both the core and wings of the stellar profile [42].
Figure 3.2.3: Radial profiles for the PSFs displayed in Fig. 3.2.2. Again the Jee PCA decomposition best matches the stellar profile [42].

Figure 3.2.4: *Left:* Image processed by Tiny Tim software. *Right:* Same image with Jee PSF Template applied. The Jee method better corrected for the PSF in the image [42].
ongoing CCD degradation, into the PSF template files they created.

The Jee method must be applied to every image individually, and several steps are required to utilize the PSF correction scheme. First, an image must be individually drizzled in order to correct for the WFC camera distortion. Jee et al. determined that the parameters that best preserved the data and minimized noise and aliasing are a Lanczos3 drizzling kernel at the WFC pixel scale and fraction.

After processing the image, we must select several bright, isolated, and unsaturated stars distributed across the image [52]. Once stars are selected, we measure the second order moments of each star in the image, as outlined in section 2.3.1. Then for each PSF template file for the corresponding filter, we perform a weighted $\chi^2$ fit of the second order moments of the stars and the moments of the template file’s PSF at the star’s location:

$$\chi^2 = \sum \left[ \frac{(Q_{11}^{\text{star}} - Q_{11}^{\text{psf}})^2}{\sigma_{Q_{11}^{\text{star}}}^2} + \frac{(Q_{22}^{\text{star}} - Q_{22}^{\text{psf}})^2}{\sigma_{Q_{22}^{\text{star}}}^2} + \frac{(Q_{12}^{\text{star}} - Q_{12}^{\text{psf}})^2}{\sigma_{Q_{12}^{\text{star}}}^2} \right].$$  (3.1)

This selection technique is displayed in Fig. 3.2.5.

Minimizing Eq. 3.1 selects the matching PSF Template file for this exposure. Fig. 3.2.6 shows that only 10 stars are needed to match to the correct PSF Template with a 97% success rate. With the template file chosen, we can correct for the PSF. This technique will be discussed in Section 4.1.
Figure 3.2.5: *Left*: Bright, isolated, and unsaturated stars chosen from an exposure. *Right*: The positional PSFs for the matching template file. The shape and orientation match is apparent [42].

Figure 3.2.6: The rate for successfully fitting to the proper PSF Template file as a function of the number of stars used in the fit [42].
To begin our study, we needed to find a template cluster. We began that search by setting four criteria for our sample selection: observations must be of sufficient depth in a broadband filter to expect a resolved background galaxy density of >80 per square arcminute, redshift greater than 0.5, sufficient area coverage to subtend 1 Mpc at the cluster redshift, and no other known clusters projected to within 5 arcminutes of any part of the HST field (to prevent contamination of the lensing signal from unrelated mass structures). After compiling this list (see Table 6.2.1), we chose the WARP J0216.5-1747 Cluster as our prototype.

This cluster was discovered in 2002 as part of the Wide Angle ROSAT Pointed
Survey, and its X-ray properties have been measured by several groups [13, 61, 67]. These studies established a redshift of 0.578, an X-Ray luminosity of $2.8 \times 10^{44}$ erg/s, and a temperature of 6 keV for WARP J0216.5-1747. This cluster has only one set of optical images, all from the Hubble archive. It was observed in Cycle 12 (2003) by Jorgensen as part of a program to study the morphology of galaxies in mid-high redshift clusters[44]. The cluster observations consist of 16 pointings in the F775W filter, at 3 different roll angles. The total exposure time is $\sim 20$ ksec, although the overlap is not complete (Fig. 7, with a detailed zoom of the cluster in Fig. 8). For the current cosmology, the mosaic covers an area 3.5 Mpc$^2$ at redshift 0.578.

We selected this cluster as our prototype for several reasons. First and foremost, it met the four criterion outlined above for clusters in our sample. Second, it had sufficient imaging depth for a high resolution weak lensing analysis without containing too many pointings. This was important as a large mosaic would have greatly increased the processing time at each stage of the analysis pipeline development, thus greatly extending the software development timescale. Third, this cluster was deemed unremarkable in the X-Ray analyses, thus any results we found would not be based on a previously known exotic feature of the cluster. Fourth, the X-Ray luminosity and temperature implied a lower mass cluster, making it a challenging test for this technique. Finally, this cluster had no previous lensing analysis, therefore encouraging us to publish our generated lensing maps whatever the result.

4.1 PSF CORRECTION

We began work on this cluster by downloading it from the HST archive system, MAST. The archive provides files in various processing stages (raw, CALACS applied, Multidrizzled, etc.), along with sets of optimal calibration files and files
Figure 7: HST mosaic image of the cluster WARP J0216.5-1747, showing the three roll angles for the exposures and the overlap region containing the cluster center.
existing at the time of observation. Although these calibration files allow us to completely reanalyze the raw images, we found it best to use the CALACS preprocessed images (utilizing the optimal calibration files as determined by the STScI pipeline).

Starting with these files, we Multidrizzled each image according to the our implementation of the Jee PSF-fitting technique as described in section 3.2. Once made, stars for PSF fitting had to be found in each image. To begin, we measured all the bright objects in the image with Source Extractor, utilizing the circular Gaussian fitting mentioned in section 2.3.1 for optimal stellar shape measurements. Then we filtered any objects that were too extended in intensity profile or too elliptical, as the Hubble PSF ellipticity is never more than 5%, and typically less than 2% (see Fig. 3.2.1). Of the remaining objects, we removed all with full width at half maximum (FWHM) that were much larger or much smaller than the HST PSF size (∼ 1.5 pixels). Then we viewed the objects to identify bright stars, which are easy to pick out because of the prominent diffraction spikes (see Figure 8). Once
selected, these stars were matched to each F775W PSF template file in the method outlined in section 3.2. The decision over which PSF file best matched an image was difficult, as often several template files had similar fits. This was due to some PSF Template files being quite similar, even if taken years apart (see Fig. 4.1.2). To check these PSF similarities, we created an image using the second best fitting template files for each.
Figure 4.1.2: Two similar PSF template files from years apart. Results from data processed with the 2003 template did not differ significantly from results those processed with the 2006 template file. 


These results showed no significant changes from those obtained with the best fit PSF template results. This signifies both that the HST PSF does not vary greatly, and that the measured shear signal is not strongly dependent on the exact choice of template when there are multiple similar PSF maps. Or, in other words, the space of PSF template maps is oversampled.

Fig. 4.1.1 shows the PSF matching method results for one of the WARP images and its fit PSF template file. The PSF correction shows both better centroiding and dispersion in the stellar shapes. For this image, the initial stellar ellipticity centroid was $\langle e \rangle \approx (-6.4 \times 10^{-3}, 4.3 \times 10^{-3})$, with a dispersion of $\langle |e|^2 \rangle^{1/2} \approx 1.1 \times 10^{-2}$. The residuals improve both the centroiding of the stars ($\langle \delta e \rangle \approx (1.9 \times 10^{-3}, 1.4 \times 10^{-3})$) and the dispersion ($\langle |\delta e|^2 \rangle^{1/2} \approx 3.1 \times 10^{-3}$). Each image, its corresponding template fit, and the $\chi^2$ of the fit (see Eq. 3.1) are shown in Table 4.1.1.

With all the matching template files found, we PSF corrected each image. We did this by the PSF Convolution method introduced by Fischer and Tyson[24]. This method applies an image-based circularization kernel to the images before galaxy shapes are measured, as opposed to applying a position and size-based correction on
<table>
<thead>
<tr>
<th>Image</th>
<th>Matched PSF Template</th>
<th>$\chi^2$ ($\times10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>j8pi07b3q</td>
<td>F775W_2002-04-18_13_03_42</td>
<td>1.5</td>
</tr>
<tr>
<td>j8pi07bfq</td>
<td>F775W_2004-09-06_19_08_07</td>
<td>2.1</td>
</tr>
<tr>
<td>j8pi07bjq</td>
<td>F775W_2003-05-26_22_11_16</td>
<td>4.5</td>
</tr>
<tr>
<td>j8pi07c3q</td>
<td>F775W_2002-04-18_14_55_48</td>
<td>6.1</td>
</tr>
<tr>
<td>j8pi08a8q</td>
<td>F775W_2004-12-14_13_39_47</td>
<td>1.8</td>
</tr>
<tr>
<td>j8pi08adq</td>
<td>F775W_2002-04-18_14_18_56</td>
<td>6.4</td>
</tr>
<tr>
<td>j8pi08asq</td>
<td>F775W_2004-09-06_19_08_07</td>
<td>5.8</td>
</tr>
<tr>
<td>j8pi08zyq</td>
<td>F775W_2002-04-18_14_46_35</td>
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<tr>
<td>j8pi09osq</td>
<td>F775W_2003-01-17_20_39_35</td>
<td>7.1</td>
</tr>
<tr>
<td>j8pi09ozq</td>
<td>F775W_2003-11-28_10_19_46</td>
<td>6.5</td>
</tr>
<tr>
<td>j8pi09p5q</td>
<td>F775W_2003-05-26_22_11_16</td>
<td>5.5</td>
</tr>
<tr>
<td>j8pi09pfq</td>
<td>F775W_2003-02-27_01_18_18</td>
<td>3.6</td>
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<td>j8pi10nmq</td>
<td>F775W_2004-09-06_19_08_07</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4.1.1: The matched PSF Template file for each image in the WARP J0216.5-1747 mosaic. The $\chi^2$ is normalized to the flux of the brightest fits star (see section 3.2).
Figure 4.1.3: The PSF postage stamps corresponding to the 11 stars used in the fit for Fig. 4.1.1

the measured galaxy shapes, as is typically done in implementations of the KSB algorithm based on the Imcat software[46]. The circularization is chosen so that application on stellar images restores their circular shape, and therefore results in removal of the PSF ellipticity in all objects in the image. The PSF Template files prescribe a $31 \times 31$ postage stamp of the Gaussian modeled PSF (see Fig. 4.1.3) at each pixel position in an image. Circularizing with a Gaussian is straightforward, all it requires is a simple rotation of the Gaussian by 90° and the application of it to the image.

The Template files are not collections of PSF postage stamps, as there would need to be $\sim 16$ million of them per file, which would create very large files (\$13$GB). Not only would such a file be slow to process, a library of thousands of these files would result in great storage and transfer difficulties. Instead, the
template files simply contain the PCA coefficients for the solution at each pixel.

To circularize each image, we perform the following. For each pixel, we first get the corresponding PSF postage stamp from the matched template file. We then rotate that stamp, multiply it by the intensity value of the image pixel, and then paste that scaled stamp into an initially empty new image. After doing this for every pixel, we create a completely circularized image. An example of the results of this process is depicted in Fig. 4.1.4.

It is important to note that there are a couple caveats with this PSF Convolution technique. First, while this circularization preserves the flux of images, as the postage stamps are normalized to a total flux of 1, the imaged size of individual objects increases (as can be seen in Fig. 4.1.4). This requires careful attention when any calculations regarding size are needed. The second important point is that the smearing caused by this circularization technique will dilute the shear signal of background sources. PSF correction techniques have a trade-off between signal dilution and increased noise - the former typically associated with image-based PSF correction, while the latter is associated with catalog-based PSF correction. In
addition to preferring signal dilution to increased noise, we chose image-based PSF correction because circularization automatically accounts for the different smearing of individual objects. Catalog-based KSB reconstructions correct object shapes strictly by their size, and thus can be less accurate on an object by object basis.

4.2 Stacking

Once all images are circularized, we can create a final stacked image from them. As mentioned in Section 3.1.1, Multidrizzle is too imprecise as a stacking solution for weak lensing studies. It is useful, however, for establishing the general alignment of each image in its own canvas file. This file provides us the rotation and shift information for each image, which we use as follows. First, we Multidrizzle stack all the images from the CALACS output stage, while keeping all the created intermediary files. We have to stack the CALACS only images because Multidrizzle will only operate on such images - it will not stack already Multidrizzled, circularized, or otherwise processed images. Within the intermediary files are the canvases which have the CALACS output rotated and shifted into place. Next, we take the circularized image and subtract the sky background as determined by Source Extractor. Then we rotate this image and place it in the canvas file over the Multidrizzle output, thus creating a canvas file with the properly circularized image in place.

With proper canvas files made, we can correct the astrometry to the precision required for weak lensing studies. We begin by extracting all the bright objects from each canvas with Source Extractor. Utilizing Pyraf, the updated python version of IRAF, we first use Geomap to calculate the transformations of the objects in each file compared to the reference image[8, 75]. As the HST has a FOV of only 202”×202” and no atmosphere to contend with, the transformations consist only of shifts and rotations. These transformations are then applied to the images via the Geotran package. The various stages of these file transformations is depicted in
With the completed canvas files, we can proceed to the final stacking stage. We utilize the *Imcombine* package in *Pyraf* to median combine the canvassed images. During this process, we allow for sigma clipping, where pixels that have values that are inconsistent with the remaining images are discarded. This removes spurious pixel values (due to hot pixels and cosmic rays) that have been missed in the process to this point, ensuring optimal shape preservation. We can test the astrometry of the stacked results by analyzing the shapes of all the stars utilized in the PSF template matching process. Fig. 4.2.2 highlights both a bad stack alignment and the alignment chosen for this analysis. The final stack of WARP J0216.5-1747 can be seen in Fig. 7.

### 4.3 Schirmer Filters

With the final stacked image, we are able to process the data and make aperture mass maps of the cluster. To make a master catalog of objects, we begin by extracting the shape information with *Source Extractor* for all objects. We then use *Ellipto* to model each galaxy. Following that, many cuts are made on the data:

- Objects detected with any errors are removed.
- Objects with ABMAG equal to or less than (brighter) than the cluster galaxies are discarded. The 12 brightest cluster galaxies are at an average ABMAG of $\sim 21.4$, so we made the ABMAG cut at 22.4, to also remove the more moderately bright cluster galaxies. This cut was made once, before measurement of the tangential shear, to avoid ”optimizing” the selection for signal and thus biasing our results.
- Objects with FWHM significantly less than the PSF size (noise objects) are deleted.
Figure 4.2.1: One WARP image through several steps. Top Left: Circularized image. Top Right: Image after rotation. Bottom Left: Rotated image set in place on pre-stack canvas. Bottom Right: Canvassed image after astrometry correction.
Figure 4.2.2: Ellipticity plots of the stacked stars used for PSF template selection. **Left:** Poor astrometry revealed by ellipticity offsets and scatter. **Right:** Well centroided ellipticities and tight scatter indicate good astrometry in the stacked result.

These cuts highlight a downside to our prototype cluster selection: all of the data is in the same filter. By enforcing an ABMAG cut, we are taking advantage of the fact that on average, magnitude will scale with distance. Magnitude cutting is imprecise, as at a fixed distance, galaxies can span a range of fluxes. Because of this, we will have some faint cluster galaxies contaminating our sample. While we account for foreground galaxies unrelated to the cluster in our calculation of the critical density, we do not have a good way to fit the contribution of faint cluster members. This causes our cluster masses to be underestimated by 10-15% depending on the (unknown) slope of the cluster mass function. We note that this contamination’s effect will be on absolute masses, not mass ratios, so it will not affect substructure mass ratio values. This method is, however, straightforward to generalize to fields with multi-band data by using galaxy colors to exclude galaxies with colors consistent with cluster members[54].

With the cuts made, we use the Fiatreview package to display the stack image with the source shape fits overlaid (see Fig. 4.3.3). In this program, we hand-remove stars, extremely elliptical objects, sources with poor shape fits, overlapping objects, and any spurious detections. As with the cluster ABMAG cut, this is done once,
before measurement of tangential shears.

The master catalog produced from these steps has a total of 1421 objects over \( \sim 17 \) square arcminutes, which yields a galaxy source density of \( \sim 84 \) galaxies per square arcminute. The spatial distribution of the galaxies can be seen in Fig. 4.3.1. There are no large density abnormalities that would result in false measurements.

The background galaxies in the catalog are distributed over a magnitude range of \( 22.4 < \text{ABMAG} < 26 \) and are plotted versus Elliptical Gaussian apodized size \( (I_{xx} + I_{yy}) \) in Fig. 4.3.2. The relatively shallow depth of the catalog is due to the results having been median stacked. While median stacking helped ensure stability in our shape measurements along with eliminating bad pixels missed elsewhere, it limited the effective depth of the catalog. We found that, given the stacking procedure we utilized, median stacking provided the best results for this study. Changes to the stacking procedure to allow full depth studies will be discussed in section 6.1.1.

With the master catalog, we are then able to create aperture mass maps and detection sigma maps. To create these maps, we wrote software to apply the Schirmer Filter Method (see section 2.5) to the WARP J0216.5-1747 data. The programs perform the following:

- Loop over a range of Schirmer Filter scales.
- Sample the image at a block-size of 50 pixels to reduce computation time.
- Generate aperture mass and Monte Carlo variance data for SNR maps.
- Calculate detection significance of SNR map features.

We follow the convergence SNR map generation method as outlined in Kubo et al. 2007 and as adopted by many other studies [50]. For every block of pixels, we calculate the filtered aperture mass using a modified version of the Fiatmap software. In order to better quantify the lensing signal, we generate SNR maps. The
Figure 4.3.1: The background source density for the WARP J0216.5-1747 cluster. *Left:* Source density per square arcminute. *Center:* Galaxies per square 0.1 arcminute. *Right:* Cluster Mosaic.
Figure 4.3.2: The Elliptical Gaussian apodized size ($I_{xx} + I_{yy}$) and Magnitude for the background galaxies in the WARP J0216.5-1747 master catalog that are used for shear determination.
Figure 4.3.3: The ellipticity shape and size for all detected objects overlaid on the cluster mosaic. We utilize this program to hand-select objects that need to be removed.
Table 4.3.1: The number of random maps required to obtain a $\sigma$ level of detection significance.

<table>
<thead>
<tr>
<th>Number of Maps</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>3.890592</td>
</tr>
<tr>
<td>100000</td>
<td>4.417173</td>
</tr>
<tr>
<td>1000000</td>
<td>4.891638</td>
</tr>
<tr>
<td>10000000</td>
<td>5.326724</td>
</tr>
</tbody>
</table>

noise in the mass reconstruction is dominated by shape noise, and this can be ameliorated by running a series of Monte Carlo mass reconstructions\cite{56}. We generate 100 random maps wherein we shuffle the galaxy shapes while using the same positions. We calculate the variance of these maps in order to generate a stable estimation of the noise. Finally, we divide the aperture mass map by the noise map to generate a SNR map.

The creation of detection significance maps follows a similar strategy. It begins with the same aperture mass map above. Then, again, we implement Monte Carlo mass reconstructions by shuffling galaxy shapes and orientations. Instead of calculating the variance, however, we directly compare the pixel value of the random maps with the data map. By doing so with many realizations, we can determine the detection significance for peaks in the aperture mass map given the background source distribution from the stack. The attainable sigma of detection is a function of the number of random maps utilized, with the conversions listed in Table 4.3.1.

With both the SNR and Significance maps, we can identify dark matter peaks present in our weak lensing maps. To determine the matching Schirmer Filter for a found peak, we examine the peak through several filter radii and select the filter that maximizes the SNR and Significance. For example, in Fig. 4.3.4, we show the SNR and Significance for a mass peak in five consecutive Schirmer Filters. The peak maximizes in both SNR and Significance at a filter radius of 900 pixels. We utilize this method to characterize all peaks with significant detection (Significance $> 4\sigma$).
4.4 Results

4.4.1 Aperture Mass Maps

For the WARP J0216.5-1747 cluster there are two significant dark matter peaks. We detect the primary cluster peak at the center of the mosaic. This peak saturates our Significance at $\sim 4.4\sigma$ over the filter range 2700pix to 3400pix. Within this range, the aperture mass SNR peaks in filter 3100pix (see Fig. 4.4.1).

In addition to the primary peak, there are two other peaks with apparent high sigma detection. These two features correspond to voids in the dark matter map. We disregard both these features as both are just off the edge of the image. Their signals are potentially driven by the geometry of the image and low local source density, rather than the distribution of mass near the cluster.

As mentioned in section 2.3.1, the B-mode SNR map is an important check of bias in the catalog. The B-mode map for this filter radius is shown in Fig. 4.4.2. There is one region in the B-mode map with ($|\sigma > 3|$). The peak corresponds in the
The Schirmer Filter results for the primary mass peak of the WARP J0216.5-1747 cluster. The matching filter radius is 3100 pixels (~ 1.2 Mpc). Left: Detection Significance of the primary peak, it is saturated at $4.4\sigma$. Right: Shear SNR of the peak, which maximizes at $\sim 4.1$.

image to the location of a star right at the edge of the pixel array. This causes a scattered light glint, wherein light extends from the star due to reflection in the CCD. The extra light can distort the light profiles of galaxies near the star, thus producing an anomalous signal.

The second peak found in WARP J0216.5-1747 is a dark matter substructure in the lower right portion of the mosaic. This peak has a Significance of $>4.2\sigma$ in the 900 pixel radial filter size. This is where the peak is also maximized in SNR, as seen in Fig. 4.3.4. The complete aperture mass SNR and Significance maps can be seen in Fig. 4.4.3.

Due to the small (900 pixel) filter radius, the B-mode SNR map for the substructure characteristic radius contains several strong ($|\sigma| > 3$) peaks. This is largely due to fluctuations in the variance map. At this filter radius size, not many galaxy positions are used for each pixel in the Monte Carlo method, and fluctuations in the galaxy numbers dominate the noise variance estimates. Figure 4.4.4 shows both these maps in full and with zooms around the substructure location. The strong peaks in the B-mode map align with valleys in the noise map. The low values
Figure 4.4.2: The B-mode map for the 3100 pixel filter radius (primary peak filter size). The single peak is likely due to noise and too far from the cluster’s primary peak to affect that detection.
in the valleys artificially inflate the B-mode SNR, causing the high peaks. Therefore we can conclude that the B-mode peaks are due to artificial variance depression, and not oddities in the source catalog. The substructure peak, however, is not located in an area with low noise nor near a B-mode peak, confirming the validity of its detection significance. When dealing with low filter radius objects, this check to make certain a shear signal peak is not artificially inflated is very important.

In our HST images, we only consider galaxies with characteristic size (as measured by the FWHM) greater than the PSF size. As a result, the typical galaxy in our shape sample is 3.7 times the size of the PSF (and has a shape measured over an isophotal radius roughly 2.5 times the FWHM). Even for these galaxies, however, the effect of the PSF on the measured shears is not negligible. In particular, the effect of the ellipticity of the PSF in determining the systematic shears of galaxies needs to be accounted for. We can estimate the size of the effect by comparing the typical shear associated with gravitational lensing with the typical ellipticity of a galaxy. For the observations of WARP J0216.5-1747, the pre-correction stellar PSFs have mean ellipticity equal to $1.0 \pm 0.3\%$ (and a scatter in individual measurements
Figure 4.4.4: B-mode SNR and noise maps for the 900 pixel Schirmer Filter Radius. Because the filter radius is so small, the noise map has significant variation, causing peaks in the B-mode SNR map. Both maps are binned to 50 pixels, and in all four images, the black circle indicates the high detection significance area of the substructure peak. Top Left: B-mode SNR map. Top Right: Variance mean noise map. The color scaling is exaggerated to highlight the features. Bottom Left: Zoom of B-mode map around the substructure peak location. No significant B-mode peaks nearby. Bottom Right: Zoom of noise map. The substructure peak is not in a low variance area.
of 1.1%. The induced ellipticity on galaxies from this systematic stellar ellipticity depends on the size/PSF ratio for the galaxies we used. For a mean size ratio of 3.6, the effective induced ellipticity would be $0.3 \pm 0.1\%$. To obtain a measure of the effect of ignoring this ellipticity on our signal, we compare the magnitude of the induced ellipticity to the mean tangential ellipticity of the galaxies. For the main cluster peak of WARP J0216.5-1747, the mean ellipticity of galaxies inside the maximal significance aperture is 1.44%. Therefore, not correcting for the stellar PSF ellipticity would bias the mass signal measured for the cluster by about 20%.

4.4.2 NFW Fits

With the aperture mass peaks characterized, the final step is to fit them to NFW profiles to get an estimate of the mass. Following the method outlined in section 2.6, we fit the peaks in the following fashion. Beginning with the primary cluster peak, we use the 50 pixel × 50 pixel bin that maximizes the detection significance as a starting area. We find the pixel in this bin that maximizes the lensing signal, and use it for the cluster center.

To compare the shear signal with theory, we need to obtain the critical surface mass density, $\Sigma_{\text{cr}}$ from Eq. 2.6, which we would ideally calculate from the redshift distribution of the background galaxies. Because the WARP data is all in one filter, we do not have photometric redshift measurements for the background sources, and must rely on another method to estimate the redshifts. This issue is very common in weak lensing analyses, as imaging in the requisite number of filters for photometric redshifts is difficult to obtain, particularly on the Hubble Space Telescope. Accordingly, we (as well as many previous lensing studies) approximate the lensing distribution by matching the lensing background object’s magnitude to that of the objects in a deep photometric catalog (typically UDF or COSMOS)[15, 41, 52, 59]. We begin this matching method by selecting the UDF results for this photometric filter, F775W, and sorting the UDF galaxies into magnitude bins. Then we match the magnitude of each source galaxy to a magnitude bin and randomly select a UDF
object from that bin. We then use the redshift information from the randomly selected UDF galaxy. After matching all of the background sources in this way, we can estimate a critical surface mass density for the cluster. We repeated this process 100 times in order to generate an estimate of the error in this selection process. It should be noted that the contamination from faint cluster galaxies will bias this relation by assigning incorrect high redshifts to these galaxies, leading to a lower mass estimate than the true mass. For WARP J0216.5-1747, we obtain

$$\Sigma_{cr} = 8.1^{+1.1}_{-1.0} \times 10^{15} M_\odot/Mpc^2. \quad (4.1)$$

With the critical surface mass density, we can compare the theoretical predictions with our data. To do so, we bin the galaxy shapes in annuli about the primary cluster out to a set cut-off radius, and perform a $\chi^2$ fit of the average shear in those bins to the predicted shear. With the primary peak fitted, we then fit the substructure peak in a similar manner, with one notable addition - the projected shear signal from the primary peak is subtracted from the substructure shear at each background galaxy as follows:

$$e_{\text{sub}}^T(gal) = e_{\text{sub}}^T(gal) - e_{\text{pri}}^T(gal) \cos(2\Delta\theta), \quad (4.2)$$

where $\Delta\theta$ is the difference between the angle from the primary peak to the galaxy and the angle from the substructure to the galaxy. Once the substructure peak is fit, we do not go back to modify the primary cluster. By experimenting with cut-off radii for the primary peak, we discovered that the fit does not change appreciably if we extend the cut-off to beyond the radius where the substructure lies ($\sim 700$ kpc).

Table 4.4.1 lists the NFW fit results. We fit to a single parameter $(r_{200})$, utilizing Eq. 2.53 to derive the concentration and Eq. 1.38 to derive $M_{200}$. As a result, the values of the other NFW parameters depend entirely on our $r_{200}$ estimate. Because $M_{200}$ scales as $r_{200}^3$, the mass estimates are unfortunately more uncertain the the $r_{200}$ values.
To compare our cluster mass measurement, we utilize the Mass-Temperature scaling relation in Bryan and Norman[11]. Bryan and Norman performed hydrodynamic simulations of galaxy clusters and compared the luminosity weighted temperature to the halo mass as a function of halo mass and redshift. They find a good fit to the $M \propto T^{3/2}$ relation expected for virialized clusters. They calculate the normalization of the relation and its evolution with redshift (including the normalization at $z = 0.5$, very close to the redshift of our clusters). Based on the simulations, they find an intrinsic scatter of $\sim 15\%$ on the mass at fixed x-ray temperature, which is comparable to the 20% scatter found for real X-ray selected clusters. We use the Bryan and Norman relation to estimate the expected cluster mass as an indicator of whether our mass estimates are reasonable, but given the large uncertainty in our mass measurements we do not make an attempt to independently constrain the mass-temperature relation based on our own measurements.

The temperature of this cluster, 6keV, roughly corresponds to a fit mass of $7 \pm 1.5 \times 10^{14} \, M_\odot$ [67]. Our mass fit is lower than that inferred from the X-rays, possibly due to the bias induced by the inclusion of cluster member galaxies in our background sample.

In addition to the cluster core, we separately detect a substructure peak with $34^{+39}_{-21}\%$ the mass of the primary cluster peak at high significance ($\sigma \approx 4.2$). Despite the large measurement uncertainties on the mass ratio, this is intriguing as we expect large substructures to be rare in these clusters (see Eq. 1.39). Of course, in addition to the large fit error, one caveat to the substructure detection is that due to insufficient knowledge of the redshift distribution of the galaxies, we cannot rule out the possibility that the substructure is a foreground dark matter peak. Therefore, more observations or clusters are needed to better inform the impact of this result on substructure simulations.

With this work, we have shown our overall method to be successful in detecting
<table>
<thead>
<tr>
<th>Peak</th>
<th>Concentration</th>
<th>$r_{200}$ (Mpc)</th>
<th>$M_{200} \times 10^{14}$ M$_\odot$</th>
<th>Detection Significance $(\sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>$3.74^{+0.60}_{-0.58}$</td>
<td>$1.00^{+0.33}_{-0.32}$</td>
<td>$2.03^{+2.4}_{-1.1}$</td>
<td>4.4 (saturated)</td>
</tr>
<tr>
<td>Substructure</td>
<td>$4.12^{+0.84}_{-0.66}$</td>
<td>$0.70^{+0.27}_{-0.31}$</td>
<td>$0.70^{+1.05}_{-0.57}$</td>
<td>4.2</td>
</tr>
</tbody>
</table>

**Table 4.4.1:** The NFW fit values with 1-$\sigma$ variance for both the primary and substructure peaks along with the detection significance in WARP J0216.5-1747. The substructure has $30^{+47}_{-26}\%$ the mass of the primary cluster peak.

Cluster substructure. We were able to obtain very tight PSF correction, enabling a high resolution lensing analysis of this cluster. By separating the peaks via characteristically sized filters, we are able to individually detect both the cluster peak and substructure peak to high significance. In addition, we showed that lower filter sizes are valuable for isolating and emphasizing substructure peaks.
5

Application to ZwCl 1332.8+5043 and MACS J0257.1-2325

5.1 ZwCl 1332.8+5043

The next cluster in our study, ZwCl 1332.8+5043, was discovered by Zwicky in 1961[84]. X-ray studies of the cluster indicate a measured luminosity of $L_x = 2.4 \times 10^{44}$ erg/s and temperature of $4.31 \pm 0.28$ keV, at redshift 0.62[64, 67, 67, 80]. ZwCl 1332.8+5043 was imaged with WARP J0216.5-1747 in Jørgensen’s Cycle 12 (2003) galaxy morphology study. The total exposure is $\sim 20$ ksec, split across 16 pointings over two minimally overlapping roll angles. These observations cover an area of 4.0 Mpc$^2$ in the current cosmology.
The mosaic for this cluster is displayed in Fig 5.1.1 with a cluster center zoom in Fig. 5.1.2. The alignment of the mosaic is rather unusual in that the cluster center is near the edge of one roll angle, and there is a large gap right next to the cluster between the two roll angles. Jorgensen chose this alignment because in that gap are two very bright stars that would have ruined the exposures.

We initially selected the cluster due to its redshift, observing depth, and mosaic size. We again chose a smaller mosaic for processing and analysis efficiency. While the pattern is problematic for weak lensing analyses given the gap in coverage near the cluster center, we felt this cluster would be a good test of our method in non-ideal imaging conditions.

As with the WARP cluster we began by finding the corresponding PSF Template files for the images. The fitting results for one of the ZwCl 1332.8+5043 images is displayed in Fig 5.1.3. Before PSF correction, the centroid and dispersion for this image were \( <e> \approx (6.2 \times 10^{-3}, 4.0 \times 10^{-3}) \) and \( <|e|^2 \>^{1/2} \approx 1.5 \times 10^{-2} \), respectively. After PSF correction, the centroid improved to \( <e> \approx (-1.8 \times 10^{-3}, 3.1 \times 10^{-4}) \), while the dispersion improved to \( <|e|^2 \>^{1/2} \approx 3.7 \times 10^{-3} \). This shape improvement is similar to that of WARP J0216.5-1747 and the scatter in ellipticity is comparable to the measurement error, indicating that our PSF correction scheme works well. Table 5.1.1 lists all the matched PSF template files with their corresponding \( \chi^2 \) fit values. With the PSF templates selected, we circularized, rotated, canvassed, and median stacked the images.

We extracted sources from the stack via Source Extractor, and modeled each with Ellipto as discussed in section 2.3.1. As in WARP J0216.5-1747, this cluster was observed in only one filter. Accordingly, the same cuts are made in the catalog as were for the WARP cluster: FWHM, error during detection, and ABMAG. As the dozen brightest galaxies in this cluster were at an ABMAG of \( \sim 20.1 \), the ABMAG
Figure 5.1.1: The Hubble mosaic of cluster ZwCl 1332.8+5043
Figure 5.1.2: Zoom of the interior $\sim 228'' \times \sim 80''$ of the ZwCl 1332.8+5043 cluster. Diffraction spikes and scattered light from the bright stars in the gap are visible.

Figure 5.1.3: Left: Whiskerplot displaying stellar shapes for a ZwCl 1332.8+5043 image. Shear sticks are magnified by $\sim 5 \times 10^4$. The mean stellar ellipticity error is $\langle \delta e \rangle \approx 3.8 \times 10^{-3}$. Middle: Matched PSF template whiskerplot. Right: Red plus signs represent the initial stellar ellipticities, while the black dots are the PSF corrected ellipticities.
<table>
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</table>

*Table 5.1.1:* ZwCl 1332.8+5043 matched PSF Template files. The $\chi^2$ is normalized to the flux of the brightest fits star (see section 3.2).
cutoff was set at 21 to also remove the more moderately bright members. Finally, as with WARP J0216.5-1747, the objects were inspected to remove overlapping objects, objects with poor shape fits, extremely elliptical objects, stars, and any other spurious detections.

The final background galaxy catalogue contains 1248 objects over ~ 21 square arcminutes, resulting in a background source density of ~ 60 sources per square arcminute. This source density is lower than that for the WARP cluster, due to many more objects being poorly fit or detected with errors, and like the WARP cluster, the source density is suppressed by median stacking. The background source density maps (Fig. 5.1.4) show no large over-density bias in the catalog, and the magnitude and Elliptical Gaussian apodized size ($I_{xx} + I_{yy}$) distribution contains no abnormalities (see Fig. 5.1.5).

5.1.1 Results

Aperture Mass, Detection, and B-mode Maps

We find two peaks in our analysis of the ZwCl 1332.8+5043 cluster. Both peaks are at the center of the cluster in an extended signal area, which is prominent in both aperture mass maps. The first of the two peaks saturates our detection ($\sigma > 4.4$) in the 2900 pixel map, with an aperture mass SNR of ~ 4.1 (see Fig. 5.1.6). Outside of the cluster core, no other features are detected with a significance of at least 4.

The second center peak’s maximum lies in the 2400 pixel filter radius map. As shown in Fig. 5.1.7, this peak has a detection significance of $\sigma \approx 4.3$, and an aperture mass SNR of 4.1. As with the 2900 pixel filter map, there are no $4\sigma$ features away from the cluster core.

Fig. 5.1.8 shows the B-mode maps for both filter sizes. The odd alignment required for the imaging produces a prominent ($|\sigma| > 3$) peak right at the junction.
Figure 5.1.4: ZwCl 1332.8+5043 background source density. *Left:* Source density per square arcminute. *Center:* Galaxies per square 0.1 arcminute. *Right:* Cluster Mosaic.
Figure 5.1.5: The Elliptical Gaussian apodized size ($I_{xx} + I_{yy}$) and Magnitude for the background galaxies in the ZwCl 1332.8+5043 master catalog.
Figure 5.1.6: The Schirmer Filter results for the 2900 pixel (∼ 1.0 Mpc) filter radius. Left: Detection Significance of a cluster core peak, it is saturated at 4.4σ. Right: Shear SNR of the same peak, which maximizes at ∼ 4.1.
Figure 5.1.7: The Schirmer Filter results for the 2400 pixel (\( \sim 0.84 \) Mpc) filter radius. *Left:* Detection Significance of the second cluster core peak, it is detected at \( \sim 4.3\sigma \). *Right:* Shear SNR of the same peak, which maximizes at \( \sim 4.1 \).
Figure 5.1.8: The B-mode maps for the two core cluster peak characteristic filter radii. Both maps display a B-mode peak at the junction of the two roll angles, caused by the geometry of the mosaic. Left: 2900 pixel filter radius. Right: 2400 pixel filter radius.

As expected, this peak indicates a potential bias in the weak lensing analysis resulting from the gap in the cluster mosaic. Even with this B-mode peak, however, it is encouraging that we are able to detect not only the cluster peak, but that it is bimodal. We do so at both greater than 4 SNR and greater than 4σ detection significance, despite the geometry of the imaging.

NFW Fits

We perform our NFW fitting as outlined in section 4.4.2. We begin by estimating the critical surface mass density, $\Sigma_{cr}$. As this cluster was imaged in the same filter at the WARP cluster (F775W), we used the same UDF catalog for our background
source comparisons. Doing so, we obtained a critical surface mass density of

$$\Sigma_{cr} = 7.2^{+1.3}_{-0.73} \times 10^{15} \text{M}_\odot/\text{Mpc}^2.$$  \hfill (5.1)

The peaks in this cluster are separated by \(\sim 16.3\) arcsec, which corresponds to \(\sim 113\) kpc at the cluster redshift, putting them inside the Einstein Radius (\(\sim 32\) arcsec). This is not enough separation for our NFW fitting code to distinguish the masses of each peak, as our NFW fit code is tailored for fitting small substructures in a cluster environment. Accordingly, we obtain a mass estimate for the cluster alone. We do so by setting the center of the cluster to the aperture mass peak between the bimodal peaks. With that we obtain the fit data outlined in Table 5.1.2.

While we detected no significant substructure outside of the core in the cluster ZwCl 1332.8+5043, we were able to identify a bimodal core to greater than 4\(\sigma\) detection significance. This is particularly interesting as Vikhlinin et al. cites ZwCl 1332.8+5043 as an example of a merger cluster based on the X-ray morphology, so it is encouraging to see the same result emerging from our lensing analysis[80]. They published a \(M_{500}\) of \(2.73 \pm 0.27 \times 10^{14} \text{M}_\odot\) and a temperature of \(4.31 \pm 0.28\) keV. According to the scaling relation derived by Bryan and Norman, this corresponds to a fit mass of \(4.5 \pm 0.9 \times 10^{14} \text{M}_\odot\). Our NFW fit mass is consistent with these results[11], if on the low end of the M-T relation prediction (once again, contamination from faint cluster galaxies in the “background” sample may be responsible for the difference).

The results for ZwCl 1332.8+5043 demonstrate the success of our algorithm in

<table>
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Table 5.1.2: The NFW fit for the cluster ZwCl 1332.8+5043 with 1-\(\sigma\) variance. As we are unable to fit each bimodal peak separately, we fit to the high aperture mass SNR between the peaks.
detecting subpeaks in a challenging mosaic alignment and in the cluster core, where it was not designed to work. Our results also highlight the limitation of our mass-measuring technique - we were unable to fit each of the two peaks separately. Doing so will require a more detailed simultaneous joint mass fitting algorithm.

5.2 MACS J0257.1-2325

The final cluster in our study is MACS J0257.1-2325. First cataloged in the ROSAT All-Sky Survey, this cluster is at redshift 0.505\[81\]. The X-Ray Luminosity of the cluster is \(L_x = 13.7 \pm 0.3 \times 10^{44} \text{erg/s}\), and it has a measured temperature of \(10.5 \pm 1.0 \text{keV}\), indicating it is the most massive cluster of this thesis\[21, 81\]. The archival Hubble images of this cluster come from two studies: a Cycle 12 (2004) study of dark matter by Ebeling, and a Cycle 14/15 (2006 and 2007) survey of supernovae by Gal-Yam\[19, 20, 25\]. The observations were taken in two filters, with \(~4.5\text{ksec}\) exposure over 6 pointings in the F555W filter, and \(~8.9\text{ksec}\) over 14 pointings in the F814W filter.

The images were all at one roll angle, thus covering an area of 1.6 Mpc\(^2\). Figures 5.2.1 and 5.2.2 display the entire mosaic along with a zoom of the cluster core, respectively. The cluster is centered in the mosaic, with very well defined strong lensing arcs. We selected this cluster because, in addition to meeting the same criteria as the previous two did, it had imaging in two filter bands. This enabled us to better determine our background objects.

We selected the matching PSF template files with no change in procedure, and a sample fit is shown in Fig. 5.2.3. This fit corrected the centroiding and dispersion of the selected stars from \(<e> \approx (-3.4 \times 10^{-3}, 1.1 \times 10^{-2})\) and \(<|e|^2>^{1/2} \approx 1.3 \times 10^{-2}\), to \(<e> \approx (-6.1 \times 10^{-4}, 1.4 \times 10^{-3})\), and \(<|e|^2>^{1/2} \approx 3.4 \times 10^{-3}\). The \(\chi^2\) fit information is displayed in Table 5.2.1.
Figure 5.2.1: The MACS J0257.1-2325 HST mosaic.
Figure 5.2.2: The interior $\sim 120'' \times \sim 90''$ of the MACS J0257.1-2325 cluster. Strong lensing arcs are visible about the center.

Figure 5.2.3: Left: Whiskerplot displaying stellar shapes in a MACS J0257.1-2325 image. Shear sticks are magnified by $\sim 5 \times 10^4$. The mean stellar ellipticity error is $\langle \delta e \rangle \approx 4.0 \times 10^{-3}$. Middle: Matched PSF template whiskerplot. Right: Red plus signs represent the initial stellar ellipticities, while the black dots are the PSF corrected ellipticities.
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**Table 5.2.1:** MACS J0257.1-2325 matched PSF Template files. The $\chi^2$ is normalized to the flux of the brightest fits star.
Figure 5.2.4: The difference between ABMAG in F555W filter and ABMAG in F814W filter versus the F814W ABMAG. The red line at 1.16 indicates the cluster cutoff - objects below the line are more blue and thus kept.

Following the PSF template file selection, we circularized, rotated, canvassed, and median stacked the images in their respective filters. After extracting the objects in each stack, we compared their colors to determine background objects. We began by measuring the magnitude of each object in the blue (F555W) and red (F814W) filters, and taking the difference between them. We selected several cluster galaxies from the image and measured $\text{ABMAG}_B - \text{ABMAG}_R$ for them, resulting in an average value of 1.16. We removed every object redder than the bright cluster galaxies. The color-magnitude plot is displayed in Fig. 5.2.4.
The physical reasoning for this color selection lies in the galaxy populations of clusters. On average, galaxies in clusters are ellipticals with low star formation rates and old, red stars. Background galaxies, typically being in lower density regions than the cluster, are often spiral galaxies, which are actively forming stars. Therefore, on average, background galaxies will have more flux in the blue wavelength band. There is scatter in this method, and with more filters this process can be used to finely discern between cluster and background galaxies. However, this is a more reliable measure than cutting on ABMAG [54].

After color selecting background objects, we perform our usual size cuts and inspections of the galaxies. Our final catalog has 505 sources over \( \sim 12 \) square arcminutes, for a density of \( \sim 42 \) galaxies/square arcminute (see Fig. 5.2.5). This density is lower than the other clusters because the exposure time in each filter is much smaller. In particular, the blue filter had exposure times of almost half those in the previous clusters. Only objects with significant detection in both colors are used in the final catalog due to the color selection scheme. Therefore, the short blue exposures limits the number density of usable galaxies. Of course, the higher mass of the cluster offsets the lower density of background galaxies used in the lens reconstruction.

5.2.1 Results

Aperture Mass, Detection, and B-mode Maps

We find two significant mass peaks for this cluster, the primary cluster peak and one large substructure peak. The primary cluster saturates our detection significance \( (> 4.4\sigma) \) over a wide range of filter radii - from 1100 pixels to \( >4000 \) pixels. As with the primary peak for the WARP J0216.5-1747 cluster, in order to select a characteristic filter radius, we choose the one that maximizes the aperture mass SNR in this saturated set. The corresponding filter radius is 3800 pixels, yielding a SNR of \( \sim 6.6 \). The detection significance and aperture mass SNR maps
Figure 5.2.5: The background galaxy density for the MACS J0257.1-2325 cluster. *Left:* Source density per square arcminute. *Center:* Galaxies per square 0.1 arcminute. *Right:* MACS J0257.1-2325 Mosaic.
Figure 5.2.6: The Schirmer Filter results for the primary peak in MACS J0257.1-2325. The filter radius is 3800 pixels (\(\sim 1.2\) Mpc). Left: Detection Significance of the cluster core peak. It saturates at \(> 4.4\sigma\) over a large region and in many consecutive filter radii. Right: Shear SNR of the same peak, which maximizes at \(\sim 6.6\).

are displayed in Fig. 5.2.6. In this filter, there is also a void that has a \(> 4\sigma\) detection significance, but we disregard it as it is on the edge.

We do detect a substructure peak \(\sim 48''\) away from the cluster core. With an aperture mass SNR of \(\sim 4.6\), it is detected at \(\sim 4.3\sigma\) significance in the 2400 pixel filter radius (see Fig. 5.2.7). There are also three edge effect voids with detection significance \(> 4\sigma\) in this filter. As in previous cases, we disregard these detections as they are too close to the image edge to be reliable.

Figure 5.2.8 shows the B-mode SNR maps for both of the cluster peaks. Each map has a feature with \(\sigma > 3\), but the peaks are near an edge and not near the cluster or substructure. In addition, the SNR for the B-mode features is well below the aperture mass SNR for the mass peaks (\(\sim 6.6\sigma\) and \(\sim 4.3\sigma\)), so we do not see them as indicative of major problems in the catalog.
Figure 5.2.7: The Schirmer Filter results for the substructure peak in MACS J0257.1-2325. The filter radius is 2400 pixels (≈ 0.76 Mpc). Left: Detection Significance of the substructure peak. It is detected at ≈ 4.3σ. Right: Shear SNR of the same peak, which maximizes at ≈ 4.6.

Figure 5.2.8: The B-mode SNR maps for cluster MACS J0257.1-2325. Although both have peaks, the features are nearer the edge, and thus likely due to edge effects rather than strong catalog bias. Left: 3800 pixel filter radius. Right: Shear 2400 pixel filter radius.
NFW Fits

As with the ZwCl 1332.8+5043 cluster, our code is insufficient to fit both peaks. The substructure is too close to the core of this cluster (~300 kpc) for us to fit it. Accordingly, we only fit the peak for the primary structure.

As with the previous clusters, we generate a critical surface mass density from the background sources and a reference catalog. As our master catalog is in the F814W filter, we use the COSMOS source and photo-Z catalog for our magnitude matches as COSMOS has data in this filter, while the UDF does not. Doing so, we obtain

\[ \Sigma_{cr} = 6.1^{+0.94}_{-0.52} \times 10^{15} \text{M}_\odot / \text{Mpc}^2. \]  

(5.2)

This leads to the fit in Table 5.2.2.

As expected from the 10.5 ± 1.0 keV, we fit a large mass to MACS J0257.1-2325, 1.6^{+0.57}_{-0.45} \times 10^{15} \text{M}_\odot. The cluster’s temperature is just beyond the scaling relation from Bryan and Norman, which cuts off between 10-10.5 kev. At the cutoff point, their relation predicts a fit mass of 2.5 ± 0.5 \times 10^{15} \text{M}_\odot.

In addition to the temperature scaling fit, we can compare our results to that of a published strong lensing study of this cluster. Zitrin et al. included MACS J0257.1-2325 in a strong lensing study of cluster cores in 12 high redshift clusters[83]. In their study, they use a parametric model of the mass to predict the positions of the strong lensing arcs and multiply imaged galaxies they detect. In our analysis, we do not use lensing arcs, and while we may have some multiply imaged galaxies in our sample, they would make up a tiny fraction of the background.

<table>
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<th>( M_{200} ) (( \times 10^{14} ) \text{M}_\odot)</th>
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**Table 5.2.2:** The NFW fit with 1-\( \sigma \) variance for the primary peak of MACS J0257.1-2325. We do not fit the substructure peak as we are unable separate it.
catalog. Accordingly, our techniques have little overlap both in data and measured quantities, and thus are nearly independent of one another.

Their mass map is displayed in Fig. 5.2.9 alongside our results from 2400 pixel filter radius SNR map. Zitrin et al. do not publish variance maps of their strong lens reconstruction, so we cannot easily quantify the level of agreement between the two maps. However, our map is qualitatively in strong agreement with theirs, as not only do they detect the substructure peak in the same place, the cores also share similar general features such as the left-right asymmetry in the cluster core mass distribution. We cannot distinguish the smaller peaks in the core that Zitrin et al. do, because we do not use strong lensing in our analysis, and therefore do not have as localized constraints on the mass distribution.

Although the substructure peak was too close to the cluster core for us to fit to an NFW profile, we were still able to detect its existence to high significance. Additionally, our cluster core map showed further peaks that we could possible separate with better source density. Finally, the qualitative agreement between our weak lensing results and published strong lensing results is a confirmation of the validity of our technique, even for central regions of clusters.
Conclusions and Further Directions

Our method successfully found substructure peaks in two of our three clusters, and a bimodal center possibly indicating a merger in the third. We were able to detect these peaks to high significance ($> 4\sigma$), independent of the cluster peak. In one cluster, WARP J0216.5-1747, we fit a NFW profile to the substructure peak, indicating the presence of a $30^{+47}_{-26}\%$ subhalo. The large uncertainties in the substructure mass estimate indicate the need for a much larger statistical sample of substructures. To our surprise, our method characterized substructure within or near the core of the other two clusters. Although bright stars nearby the ZwCl 1332.8+5043 cluster impair our ability to conclusively state we found a bimodal peak, our results are consistent with the active merger classification the cluster has been given. For our final cluster, MACS J0257.1-2325, our substructure and
primary peak spatial findings matched those derived by Zitrin et al., who performed a strong lensing analysis of the cluster. While our technique cannot probe the core of a cluster as well as strong lensing can, our results were consistent with the published strong lensing mass reconstructions.

We have demonstrated that our technique is successful at characterizing large substructure in clusters of galaxies. We were able to find substructure down to \( \sim 30\% \) of the cluster peak mass, and able to measure peak masses with reasonably good precision despite the uncertainties. Although the clusters analyzed in this thesis form too small a sample to draw conclusions about substructure prevalence in clusters, the results so far point to great promise for our matched filter technique.

6.1 Future work

The research in this thesis does highlight further work that is required before we can draw cosmological conclusions from cluster substructure measurements.

Although the NFW fitting code can successfully measure masses for substructures far from the main cluster core and in the limit where the induced ellipticities are small, it does not work for structures close to the cluster core or closely separated large substructures. The next step is to create a joint NFW fitting code that properly accounts for the degeneracy in the fitting parameters.

While we are sigma limited in measuring the characteristic Schirmer filter radius for massive cluster peaks, our technique well determines which filter radius best characterizes substructure peaks. We want to expand this characterization by determining the filter radius to peak radius mapping. This expansion requires the creation of cluster substructure simulations to establish the radial mapping.

Another future addition to this technique is to measure the mean bias in total mass determination introduced by cluster galaxies. This can be achieved by comparing the fits of clusters where the cluster galaxies have been filtered by magnitude, color-magnitude, and color-color selection. To perform this study,
clusters with multiple filter data sets must be analyzed. Once measured, this bias can then be extrapolated to cluster data sets containing only one or two filters.

Although we selected clusters with no obvious other structure along the line of sight, contamination is still a concern. This is a difficult problem to constrain, because redshift surveys (the most promising technique for measuring line-of-sight contamination) are very expensive. Luckily, the CLASH and Frontier Fields cluster samples will have both deep spectroscopy and many-band imaging that can be used to obtain photometric redshifts. For those clusters, as well as the other clusters with deep redshift information, we will be able to determine the contamination fraction. Because those clusters are not selected based on their foreground (or background) structure, we will be able to apply the contamination fraction for this subsample of HST-observed clusters to our whole sample.

To perform a large study of clusters, we need to quantify, as a function of the cluster mass and the source density, how close to the main peak and at what level of substructure we will actually be able to detect. For this, many substructure simulations must be analyzed by our technique. This work can be combined with the Schirmer filter radial mapping, as both projects have the same simulation requirements.

When constructing a future sample, care must be given to avoid bias towards more substructure in our selection. HST-observed clusters that researchers use to study the high-z universe are biased to having more substructure due to its positive effect on the strong lensing cross-section. Accordingly, previous X-ray and lensing results must be utilized to properly weight the sample towards unbiased cluster distributions.

6.1.1 Astrodrrizzle

The final aspect of our technique we want to improve is the stacking. Our present stacking technique is time consuming and doesn’t take advantage of the HST pipeline for bad pixel removal. As indicated in section 3.1.1, Multidrizzle is too
imprecise for lensing studies, which is why we presently stack by hand.

Last year, STScI announced a new stacking program, Astrodrizzle. It has all the advantages of Multidrizzle - drizzle stacking algorithm, bad pixel removal in the stack, and full utilization of all HST calibration data while stacking - while eliminating the one disadvantage; it can stack to a precision sufficient for lensing studies. This makes Astrodrizzle the ideal stacking program for our research.

Upon obtaining the program, we immediately began writing a software pipeline to utilize Astrodrizzle with our PSF correction technique. Once completed we reanalyzed the WARP J0216.5-1747 cluster. In doing so, we noticed that we got spurious results and upon investigation, we discovered an odd systematic. Figure 6.1.1 shows the ellipticity pattern of the PSF selection stars in both our original stack and the Astrodrizzle stack. There is an odd systematic in the Astrodrizzle stack that we need to correct. Our hypothesis is that it is a charge transfer efficiency effect, as all HST image files are now CTE corrected for use with Astrodrizzle. We surmise that the CTE correction STScI is applying causes us to select incorrect template PSF files, as the Jee et al. templates assume that the images have not been CTE corrected.

Figure 6.1.1: Left: Ellipticities of PSF matching selected stars in the stack of WARP J0216.5-1747. Right: The ellipticity pattern for the same stars after stacking with Astrodrizzle and PSF correction. There is an odd bias in the stellar shapes.
We will investigate this effect further and seek to correct for this systematic. Astrodrizzle will enable us to do much larger cluster mosaics in many filters and expand our study greatly, as it removes the laborious stacking process we presently do. It is therefore a top priority project to complete.

6.2 Expansion to General High Redshift Cluster Study

While our substructure detection for WARP J0216.5-1747 is exciting, we need to analyze many more clusters to significantly probe cosmology. The next major step in our research program is to extend our study to a large group of high redshift clusters. We have generated a list of galaxy clusters presently in the Hubble archive sufficient for weak lensing substructure studies (see Table 6.2.1). Additionally, STScI has granted large Hubble time allocations to CLASH and Frontier Fields. Each group is obtaining deep imaging of their target clusters in multiple filters - 8 for Frontier Fields and 16 for CLASH. Although the clusters for these programs are mostly at $z < 0.5$, they will form ideal calibration datasets for our techniques and methods.

It is our intention to analyze each of these clusters to better inform the evolution of substructure in high redshift galaxies. We will analyze the clusters with multiband imaging first, in order to minimize the foreground contamination issue seen in this thesis. We also would try to integrate strong lensing results into our study, to map the cores of galaxy clusters.

With the successful analysis of the clusters in this thesis, we are excited about utilizing the techniques detailed here in order to probe the CDM paradigm. This larger study will provide valuable groundwork for future studies on next generation space imagers.
<table>
<thead>
<tr>
<th>Cluster</th>
<th>Observed Area (Mpc)</th>
<th>Redshift</th>
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<td>Depth (mag)</td>
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**Table 6.2.1:** The cluster sample with existing HST images that satisfies the following four criteria: $z > 0.5$, observed area > 1 Mpc diameter about the center, sufficient observing depth for minimum required background source density, and no other clusters within 5 arcsec.
References


Colophon

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