

Dynamic Shear and Void Collapse in Polymethylmethacrylate

By

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Dedication



“When I heard the learn’d astronomer,
When the proofs, the figures, were ranged in columns before me,
When I was shown the charts and diagrams, to add, divide, and measure them,
When I sitting heard the astronomer where he lectured with much applause in
the lecture-room,
How soon unaccountable I became tired and sick,
Till rising and gliding out I wander’d off by myself,
In the mystical moist night-air, and from time to time,
Look’d up in perfect silence at the stars.”

— Walt Whitman

*To Luana,
who, from time to time, pulls me back to see the stars.*

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1.1 Overview

Understanding the behavior of materials under dynamic loading is critical to a number of engineering applications, including aerospace, protective armor, high-speed machining, and the testing of energetic materials. A key goal of this research is the development of thermomechanical constitutive models that capture the stress-strain response of materials at a wide range of strain rates, including energy localization, damage, and failure.

Under quasi-static loading conditions where the strain rate is low (typically $\dot{\epsilon} < 10^{-1}/\text{s}$), materials remain in stress equilibrium, where inertial and wave propagation effects may be neglected. Such tests generate mechanical properties commonly found in engineering reference books, including uniaxial tensile testing (Young's modulus, yield strength, ultimate tensile strength), multi-point bending (flexural modulus and flexural stress-strain response), torsion testing (shear modulus, shear yield strength, and ultimate shear strength), indentation (hardness), and compact tension testing (fracture toughness and fatigue crack growth data). Many engineering applications, however, operate in dynamic regimes where mechanical properties are highly sensitive to the manner in which loading occurs. For example, many engineering materials exhibit strain-rate stiffening, in which the elastic stiffness increases with loading rate. Mechanical measurements taken under quasi-static conditions are generally insufficient to properly model the response of materials under dynamic conditions; consequently, other experimental techniques are needed.

Over the past century, a variety of experimental techniques that characterize the dynamic mechanical response of materials have been developed, as represented in Fig. 1.1. As the desired testing strain rate increases, experiments generally shift from configurations in which loads are imposed by hydraulic or servo-hydraulic machines to impact experiments

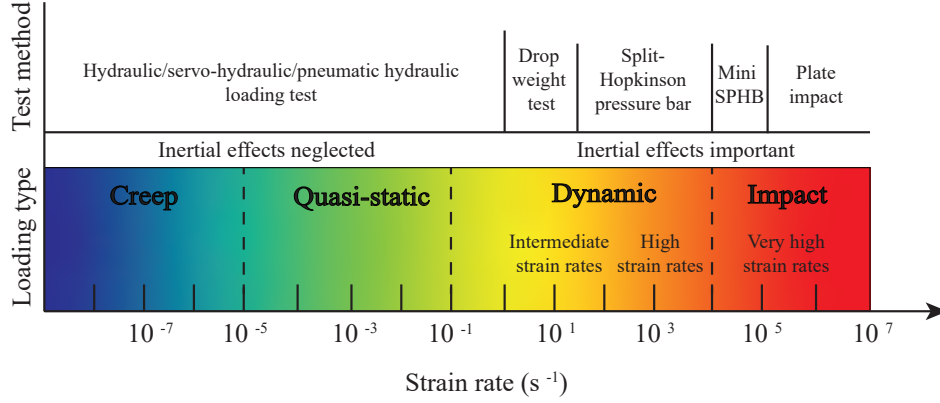


Figure 1.1: Strain rates generated by various test apparatuses.

that dynamically load samples via stress waves. Such tests generally require high-speed diagnostics, including imaging, velocimetry, and high-frequency electrical signals from stress and strain gauges, to capture transient and non-uniform mechanical responses. In order of increasing strain rate, several notable dynamic techniques are as follows. Drop weight tests ($\dot{\epsilon} \sim 1/\text{s}$), where vertically falling masses strike samples. Split-Hopkinson (or Kolsky) bar experiments ($\dot{\epsilon} \sim 10^2 - 10^4/\text{s}$), in which impact between metal rods generates stress pulses that load a small sample in compression, tension, or torsion (Hopkinson, 1914; Kolsky, 1949). The use of miniature Hopkinson bars, with diameters commonly ≤ 1 mm, can generate strain rates up to $10^6/\text{s}$. This range overlaps nicely with plate impact experiments, where strain rates of $10^5 - 10^7/\text{s}$ are possible. Here, light gases (e.g., helium, nitrogen), powder charges, explosives, laser ablation, or electromagnetic coils are used to propel sabots carrying a flyer plate. Impact occurs between the flyer and a target plate, thereby generating high strain rates and normal stresses typically ranging from 500 MPa to 100 GPa. Pressures beyond 1 TPa are achievable in impressive impact facilities such as the Sandia Z Machine (Lemke et al., 2011). Plate impact via gas-driven flyers is the primary experimental platform used in this work, and the theoretical basis is discussed at length in Chapter 2. While the preceding list of dynamic mechanical tests is primarily concerned with quantifying material behavior under relatively simple stress states (i.e., uniaxial stress or uniaxial strain), there exists a variety of other experiments that impose

highly complex, non-uniform, and multi-axial states of stress and strain. Rather than characterizing individual material properties, such experiments are commonly used to validate material models under dynamic loading. Examples include Taylor cylinder impact and expanding ring/cylinder tests. In general, much of this thesis is concerned with tests that aim to simplify boundary conditions and states of stress under dynamic loading in order to better provide clarity on the nature of deformation and failure in one material of interest: PMMA.

1.2 Material behavior of PMMA

Poly(methyl methacrylate) (PMMA)—also known as acrylic in common parlance and as the commercial names Plexiglas, Perspex, Lucite, and Acrylite—is a lightweight thermoplastic and among the most ubiquitous polymers in existence. PMMA is perhaps best known as a common substitute for glass in windows and eyewear, though it sees myriad scientific and engineering applications as well. Examples include positive photoresists in photolithography, microfluidic devices, attenuators in explosives gap testing, and window materials in plate impact experiments. PMMA is a commonly tested material in shock physics, with a host of Hugoniot and equation of state data available in the literature. Consequently, PMMA is often considered a model material for experimental validation. Despite this, the material behavior of PMMA remains an ongoing research topic, and further work is needed to understand its dynamic behavior. As an example, Fredenburg and Chisolm (Fredenburg & Chisolm, 2014) observed rounded shock wave profiles in impact experiments on cerium oxide. Rounded shock profiles are a known consequence of viscoelastic effects in polymers and may therefore render the use of PMMA as a window material problematic when studying certain materials. Modeling high-strain-rate viscoelastic effects in PMMA is therefore an ongoing research topic, but several challenges remain in interpreting its continuum response.

PMMA is an amorphous, glassy polymer, meaning it lacks a well-defined periodic

structure and exists below its glass transition temperature under ambient conditions. Above the glass transition temperature, the mechanical response becomes more compliant, or rubbery. T_g for PMMA at atmospheric pressure is generically 100-110 °C, depending on the material pedigree. Typical of many polymers, the thermomechanical response of PMMA is highly dependent on temperature, pressure, and strain rate. Under ambient conditions, in uniaxial tension, it is brittle at all strain rates, ductile at low compressive strain rates, and brittle at high compressive strain rates (Holmquist et al., 2016). The strain-rate and temperature-dependent behavior under compressive loading is well represented by Fig.1.2. For low strain-rates, the stress-strain response may be separated

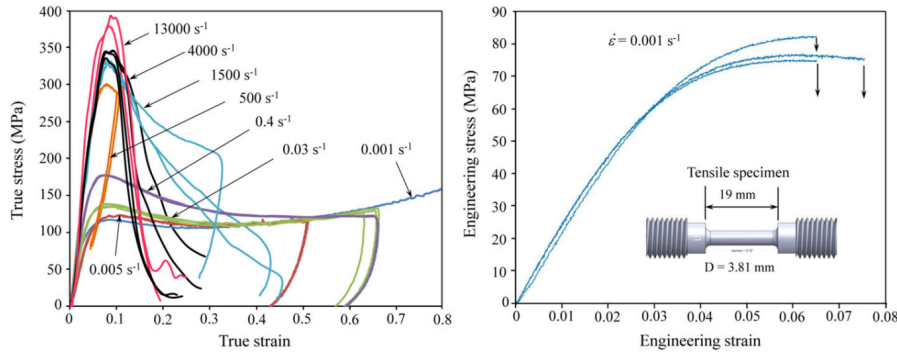


Figure 1.2: (a) Strain-rate dependence of PMMA under compressive loading (b) Temperature dependence under compression for a constant strain-rate.¹

into nonlinear viscoelastic, inelastic strain-softening, and inelastic strain-hardening regions. One of the first explanations for plastic deformation in amorphous polymers dates back to Eyring (Eyring, 1936), who hypothesized nearly a century ago that plastic flow is mediated by molecular rearrangement that accommodates material deformation. In Eyring’s theory, the restriction of chain mobility for increasing strain rate or decreasing temperature results in increased resistance to deformation. Robertson (Robertson, 1966) later considered chain straightening to be the dominant plasticity mechanism in the glassy regime. Argon (Argon, 1973) introduced the “double kink” model, which was highly successful in capturing the pressure, strain-rate, and temperature dependence of plastic flow. Boyce (Boyce et al., 1988) later expanded on this framework to model the effects of

¹Reprinted from (Holmquist et al., 2016). Reproduced with permission from Springer Nature.

strain rate, pressure, temperature, true strain softening, and strain hardening. Here, a glassy polymer must be stressed sufficiently to overcome the intermolecular resistance to segmental rotation. At this point, chain alignment occurs, reducing the configurational entropy, and plastic flow occurs. This corresponds to the strain-softening region of the stress-strain curve. As the material continues to be strained, entropic resistance leads to strain hardening at high plastic strain levels. As described by Oleinik (Oleinik et al., 2007), there has been a gradual drift away from chain straightening as a limiting kinetic stage of the process towards a description similar to non-polymer solids, such as crystalline materials, where plasticity is mediated by the nucleation and propagation of dislocations. In this view, chain straightening is undoubtedly part of the deformation process, but is not seen as a limiting stage of the overall plastic response. Rather, there are localized plastic deformation sites with large elastic fields around them, fields akin to the solutions of Eshelby's famous inclusion problem (Eshelby, 1957). In this view, plasticity below the glass transition temperature in glassy polymers is predominantly mediated by the activation of densely packed microscopic shear bands (P. D. Wu & van der Giessen, 1995). Under uniaxial strain, shear bands form on planes of maximum resolved shear stress, thereby resulting in ductile behavior. Finally, fracture occurs at larger strains by a chain-scission mechanism (Gearing & Anand, 2004). Shear-yielding and viscoplastic deformation in amorphous polymers has been clearly described in a thermodynamically consistent framework by several researchers: (Ames et al., 2009; Anand & Gurtin, 2003; Anand et al., 2009; Arruda et al., 1995; Boyce et al., 1988; Clayton, 2026; B. E. Clements, 2012; P. D. Wu & van der Giessen, 1995).

1.2.1 High strain-rate response

Under quasi-static loading, viscoelastic effects are primarily accounted for only in the deviatoric response. At higher strain rates, large volume changes result in bulk viscoelastic effects such as wavefront rounding, precursor decay, and unloading hysteresis.

Such effects are observable in PMMA up to around 4 GPa of pressure, as in the plate impact experiments by Barker and Hollenbach (Barker & Hollenbach, 1970) and shown in Fig. 1.3. Schuler and Nunziato were among the first to model both bulk and deviatoric viscoelastic effects in PMMA with a nonlinear Maxwell type model (Schuler & Nunziato, 1974) to reproduce experimental shock and release wave profiles.

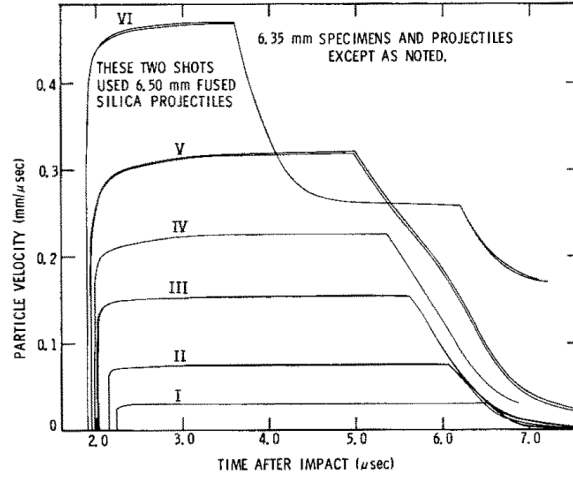


Figure 1.3: Shock wave profiles in PMMA, which show significant rounding.²

Several experiments have also characterized the highly nonlinear viscoplastic response of PMMA by shear strength measurements. The dynamic strength of PMMA has been measured by lateral stress gauges by Millett and Bourne (Millett & Bourne, 2000), Bat'kov (Bat'kov et al., 1996), and Jordan (Jordan et al., 2020). These results suggest that the shear strength of PMMA is highly pressure dependent until around 6 GPa, after which there is a considerable loss in strength, as shown in Fig. 1.4. The predominant explanation for this is thermal softening induced due to shock heating as well as viscoelastic and viscoplastic dissipation. Clayton has recently captured this effect in a new constitutive model (Clayton, 2026). Here, shear strength decays considerably beyond 10 GPa and effectively vanishes beyond 15 GPa, though the material here has neither melted nor fractured.

Compared to continuum modeling at low to intermediate strain-rates (i.e., $10^{-3} - 10^3$ /s),

²Reprinted from (Barker & Hollenbach, 1970), with the permission of AIP Publishing.

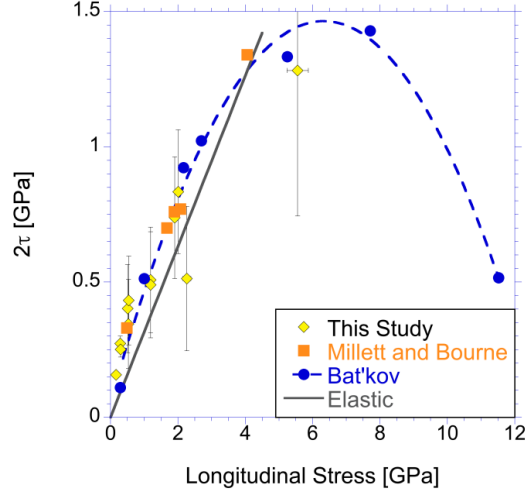


Figure 1.4: Shear strength of PMMA as measured with lateral stress gauges.³

high-strain rate (i.e., $10^3 - 10^7/\text{s}$) models of PMMA are generally less micromechanical in origin and are instead phenomenologically oriented. Common high-strain-rate applications include plate impact experiments, explosives testing, ballistics, and aerospace collisions. With such extreme deformation, these problems are often modeled in Eulerian or Arbitrary Lagrangian-Eulerian (ALE) schemes. The complexity inherent in implementing history-dependent polymer models in such frameworks has led to a dearth of viscoelastic models in the shock modeling community (Cearley et al., 2024). Commercially available hydrocodes typically have a limited number of plastic and damage models that generally depend on strain, strain rate, and temperature. Two common choices are the Johnson-Cook plasticity (G. R. Johnson, 1983) and damage model (G. R. Johnson & Cook, 1985), models which were originally designed to capture plastic flow and failure in metals. Here, the yield stress σ_Y and the plastic failure strain ε_f^p are expressed in terms of the product of strain hardening, strain-rate hardening, and thermal softening terms. In the context of amorphous polymers, the Johnson-Cook model is unable to capture the transient, nonlinear, and history-dependent behavior of polymers, leading to the absence of several key features, such as strain softening (in this context, “structural softening” is also used) under isothermal conditions and viscoelastic relaxation. Moreover, these implementations

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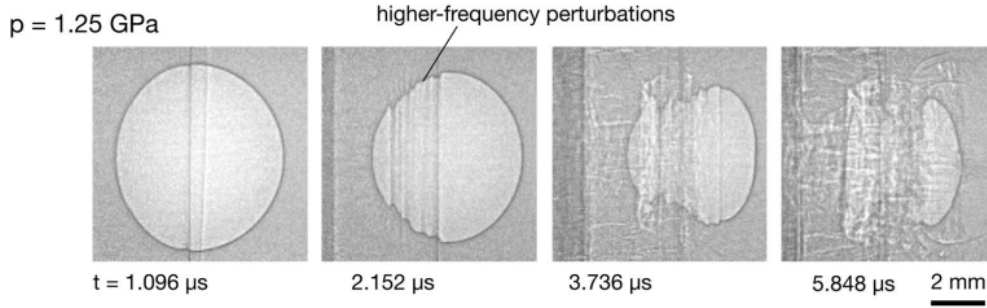


Figure 1.5: Radiographs of a shock-loaded spherical pore in PMMA at a shock pressure of 1.25 GPa⁴.

typically do not formally account for continuum thermodynamic constraints (Clayton, 2026). Nonetheless, it is not wholly inappropriate to apply the Johnson-Cook model to PMMA in certain contexts. Holmquist (Holmquist et al., 2016) was the first to apply the Johnson-Cook plasticity and damage models to PMMA. Here, the model showed generally good agreement with Taylor cylinder, rod-impact, and ballistic-penetration experiments. Since then, Holmquist’s parameterization has been used in several dynamic studies of PMMA, including dynamic void collapse (Lawlor et al., 2024; Rai et al., 2020). In these studies, the classic competition between strain hardening and thermal softening is observed, giving way to thermal localization in the shape of adiabatic shear bands in the vicinity of collapsing pores. An example of these experiments and simulations is shown in Fig. 1.5. Failure via shear banding is readily generated by the Johnson-Cook model under a variety of loading conditions, but this does not necessarily explain the actual failure modes in the material. Here, additional direct experimental observation of failure is warranted.

1.2.2 Dynamic failure

In spherical pore collapse experiments on PMMA, perturbations were observed along the surface of collapsing pores, which was given as evidence for the development of shear

⁴Reprinted from "Collapse dynamics of spherical cavities in a solid under shock loading," by E. M. Escauriza et al., 2020, Scientific Reports, 10, 8455. Licensed under CC BY 4.0 (<http://creativecommons.org/licenses/by/4.0/>).

bands that later led to fracture (Escauriza et al., 2020); experimental radiographs and simulation results are shown in Fig. 1.5. Kinematic fields have also been quantitatively measured in the vicinity of collapsing spherical pores and observed in thin bands of comparatively large amounts of equivalent Tresca strain (Lawlor et al., 2024). The magnitude, however, was quite small, with a reported maximum of around $\gamma_{Tresca} = 0.07$, whereas the simulations produced plastic strains in excess of $\bar{\epsilon}^p = 1.0$. Simulations produced shear band widths in the range of $10\text{-}100\mu m$, which is far greater than the experimental resolution of $7\mu m/px$. Here, it was argued that filtering due to the DIC virtual strain gauge size could explain the reduction in the measured strains in the vicinity of shear localization.

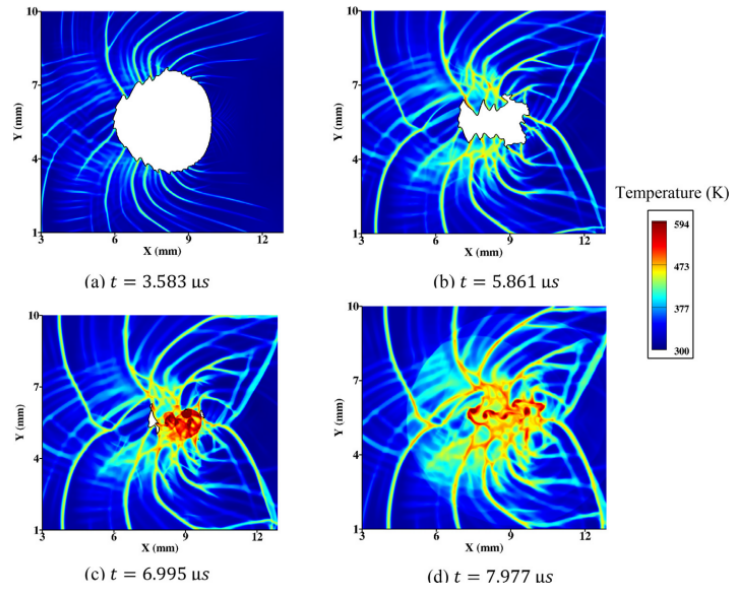


Figure 1.6: Simulations of the temperature fields surrounding a collapsing pore in PMMA, which demonstrate substantial shear banding during the collapse process.⁵

⁵Reprinted from Journal of the Mechanics and Physics of Solids, Vol. 143, Nirmal Kumar Rai et al., Mechanics of shock induced pore collapse in poly (methyl methacrylate)(PMMA): Comparison of simulations and experiments, 104075, Copyright (2020), with permission from Elsevier.

Low velocity impact experiments on PMMA previously demonstrated shear band widths of approximately $150\ \mu\text{m}$ and shear strains of up to $\gamma = 1.2$ (Archer & Lesser, 2011). In these experiments, a ductile-brittle transition was observed, whereby at low strain rates ($< 0.01/s$), diffuse shear bands form and the response is ductile. At higher strain rates ($> 10/s$), shear bands begin to coalesce, Mode II cracks propagate rapidly, and the response is brittle. An image of a Mode II crack surrounded by shear bands and micro-tensile cracks is shown in Fig. 1.7. This matches what one typically expects for large-scale shear band formation, as in impact tests in polycarbonate (Malhotra & Guduru, 2020) or in ductile metals (Guduru, 2001). It is therefore unclear whether adiabatic shear bands can actually be observed in PMMA under plate-impact conditions, where inertial confinement may suppress fracture. This brittle response is somewhat surprising, given the high degree of confinement, which in other materials might be sufficient to suppress damage. Chen and Ravichandran were among the first to demonstrate this concept in aluminum nitride (Chen & Ravichandran, 1996).

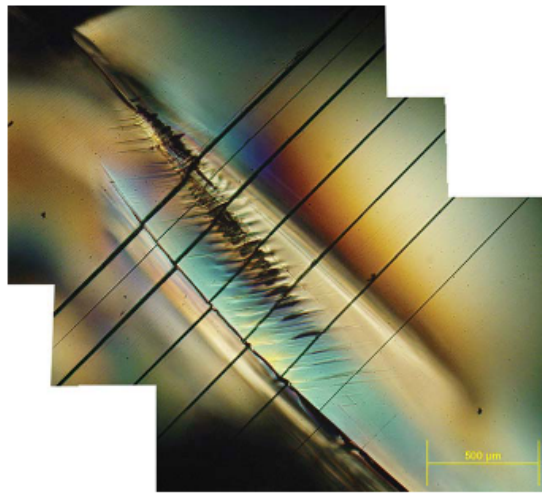


Figure 1.7: “PMMA prior to complete Mode II fracture viewed under crossed polars. Black lines at 45° are surface marks for surface displacement reference. Shear band is identified by offset in reference lines. Shallow micro-tensile cracks form at 45° to shear bands. Scale bar = $500\mu\text{m}$.”⁶

⁶Reproduced with permission from (Archer & Lesser, 2011), “Shear Band Formation and Mode II Fracture of Polymeric Glasses”, *Journal of Polymer Science Part B: Polymer Physics*; published by John Wiley and Sons. Copyright ©1999-2026 John Wiley & Sons, Inc or related companies.

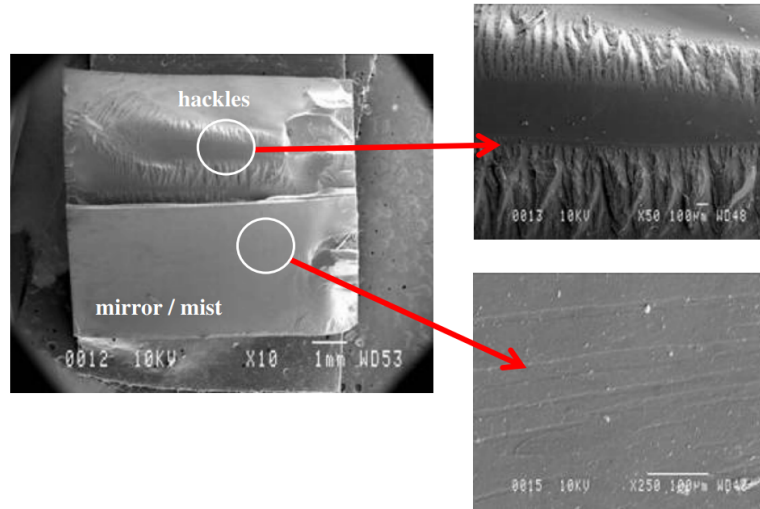


Figure 1.8: Hackled surfaces characteristic of brittle failure in unconfined samples.⁷

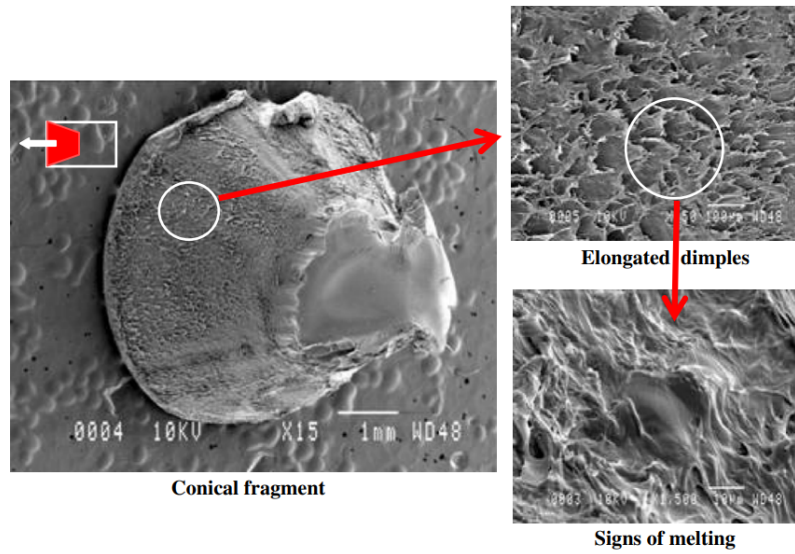


Figure 1.9: Ductile failure in confined samples, showing local melting in a shear band.⁸

⁷Adapted from (Rittel & Brill, 2008). Journal of the Mechanics and Physics of Solids, 56(4), D. Rittel and A. Brill, Dynamic flow and failure of confined polymethylmethacrylate, 1401–1416, Copyright (2008), with permission from Elsevier.

⁸Adapted from (Rittel & Brill, 2008). Journal of the Mechanics and Physics of Solids, 56(4), D. Rittel and A. Brill, Dynamic flow and failure of confined polymethylmethacrylate, 1401–1416, Copyright (2008), with permission from Elsevier.

In split-Hopkinson bar experiments on PMMA, Rittel (Rittel & Brill, 2008) artificially confined the material by placing metal sleeves over PMMA samples. Post-mortem fractography revealed that while the Hopkinson bar tests for unconfined samples indeed exhibited surfaces expected for brittle failure, such as hackled surfaces, confined samples showed signs of local melting and substantial plastic flow. Brittle and ductile failure surfaces from these experiments are shown in Fig. 1.8 and Fig. 1.9, respectively. Rittel also noted that

With the right combination of boundary conditions and sample geometry, both brittle and ductile failure modes are possible in PMMA over a range of strain rates. Whether shear bands can propagate throughout the entire sample in a plate impact experiment, or whether the material fails due to another damage mechanism, is an outstanding question that requires further investigation.

1.3 Objectives

Despite being one of the most widely used and studied engineering materials in high-strain-rate contexts, the thermomechanical response of PMMA under dynamic loading warrants further investigation. Key outstanding questions include:

- How does the shearing response change with strain-rate and pressure?
- What mechanisms control dynamic failure at the mesoscale? More specifically, does the competition between pressure and strain-rate lead to brittle or ductile damage (or both) in plate impact scenarios, and are shear bands likely to emerge?
- Are aspects typically associated with low strain-rates, like viscoelastic relaxation and strain softening, relevant in modeling the high-strain rate response of glassy polymers?
- What other constitutive ingredients are required to capture the dynamic shearing

response and failure processes?

This thesis aims to shed light on these questions by first characterizing the dynamic shear response of PMMA in pressure-shear plate impact experiments. Following this, shock-loaded pore collapse is investigated to explore the effect of heterogeneity on dynamic failure. Continuum-level simulations are conducted to give key insights into the material behavior of PMMA under these extreme loading conditions.

1.4 Organization of Thesis

This thesis is organized into 8 chapters. Chapter 2 presents a brief overview of the theoretical basis for plate impact experiments. Specifically, the theories of elastic stress waves and shock waves, as well as the pressure-shear plate impact experiment, are introduced.

Chapter 3 describes the characterization of a newly constructed plate impact facility at Brown. This chapter highlights validation tests on single crystals of Y-cut quartz and on tungsten carbide. Experimental challenges in preparing samples, troubleshooting gun mechanics and diagnostics, and interpreting data are also discussed.

Chapter 4 presents pressure-shear experiments on PMMA at pressures ranging from 2.5 to 10 GPa and total strain rates of $10^5 - 10^6 \text{ s}^{-1}$. Material failure at high strain-rates and agreement with the literature is discussed.

Chapter 5 covers two constitutive models for PMMA under high-strain-rate loading. The first comprises the complete Mie-Gruneisen equation of state, adapted from (Malhotra et al., 2022), and a Johnson-Cook strength model, adapted from (Holmquist et al., 2016). The second model is a modification of the viscoelastic and viscoplastic Glassy Amorphous Polymer (GAP) model as presented in (B. E. Clements, 2012). These two models lead to strikingly different interpretations of PSPI experiments, which are discussed in detail.

Chapter 6 details shock-loaded cylindrical pore collapse experiments conducted at two synchrotron light facilities: (1) the Dynamic Compression Sector at the Advanced Photon Source as part of Argonne National Laboratory and (2) the European Synchrotron Radiation Facility (ESRF) in Grenoble, France. Quantitative measurements of kinematic fields in the vicinity of collapsing pores are made, giving key insights into material localization and failure.

Chapter 7 presents finite element simulations of shock-loaded pore collapse for the two primary constitutive models outlined in Chapter 5. These models again give rise to different interpretations on the nature of failure mechanisms inherent in the collapse process in the regime where material strength dominates.

Chapter 8 concludes the work with final thoughts on the results and directions for future work.

Theory of stress waves and plate impact experiments

2.1 Overview

Under dynamic loading, the thermomechanical response of materials is highly complex, and properties (such as the elastic stiffness, yield strength, and fracture toughness) typically depend on the loading rate, strain history, pressure, and temperature. *In situ* measurements of dynamic loading applications, such as armor penetration, vehicle crashes, or testing of energetic materials, are often insufficient to disentangle competing mechanisms that lead to material deformation and failure. At their core, impact experiments typically rely on basic geometries that greatly simplify the analysis of dynamic material behavior. The experiments in this thesis all use single-stage gas guns that propel projectiles down a barrel at high velocities, ranging from 100 m/s to over 1 km/s. Flyer plates attached to projectiles impact target plates and generate extreme pressures and high-strain rates. In these scenarios, uniaxial strain or plane strain conditions prevail, thereby reducing the complexity of the problem to 1D or 2D. Also characteristic of these experiments are extreme pressures of ~ 1 GPa, short time durations of $1 - 10\mu s$, and high strain-rates of $10^4 - 10^6/s$. The setup of these experiments and the extreme conditions that they generate are predicated on the theory of wave propagation in solids. To better understand the design, purpose, and limitations of these experiments, the theoretical backbone of plate impact will be explained.

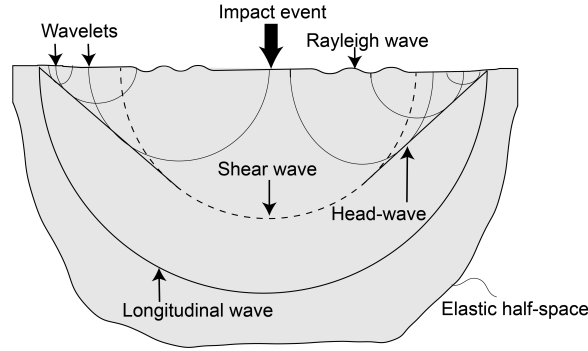


Figure 2.1: Impact on an isotropic elastic half-space induces the propagation of a longitudinal wave, a shear wave, and Rayleigh waves. Diffracted wavelets from the free surface form the head-wave.

2.2 Theory of plate impact experiments

2.2.1 Elastic wave propagation in solids

A wave propagating through a solid medium is characterized by two general speeds: the wave velocity, which is the speed at which the wave propagates, and the particle velocity, which is the velocity of small discrete material elements. As described by (Meyers, 1994), several types of waves are possible in an elastic material:

- Longitudinal waves, where particle velocity is in the same direction as the wave velocity
- Shear, or transverse, waves, where particle velocity is orthogonal to the wave velocity
- Rayleigh, or surface, waves
- Interfacial waves between two media
- Bending (or flexural) waves

A schematic of waves propagating from a site of impact is shown in Fig. 2.1. Here, an idealized impact event creates complex states of stress in a material: longitudinal and shear waves propagate into the material, Rayleigh waves propagate along the surface, and diffracting waves from free surfaces can create and often limit experimental time windows

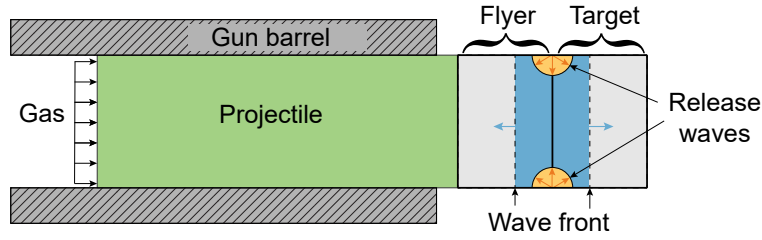


Figure 2.2: A prototypical plate impact experiment consists of high-pressure gas driving a projectile with a flyer plate attached to the front, which then strikes a target plate.

in impact experiments. In Fig. 2.1, the geometric envelope of countless diffracted wavelets being continuously generated along the free-surface makes up the head-wave. This is, of course, a demonstration of Huygens' principle, which states that every point that a wave moving along a boundary reaches will itself become a new, secondary point source. The simultaneous superposition of all these waves further obscures analyses in an already challenging experimental environment. Designing a plate impact experiment, therefore, requires careful consideration of the target geometry.

This thesis is primarily concerned with longitudinal waves and shear waves, which are generated via plate impact experiments. Consider a cylindrical flyer plate with initial velocity V_0 that is launched at a cylindrical target plate by means of a gas gun launcher. Both plates are modeled as linear elastic materials. In this case, uniaxial strain (and 3D stress) conditions are maintained until the arrival of release waves from the traction-free boundaries. At impact, the momentum of the flyer will impart a stress field on the flyer and target plates, both of which may be modeled as elastic half-spaces. A basic schematic of the experiment is shown in Figure 2.2.

In the absence of body forces, the field equations for linear elasticity are:

$$\begin{cases} \frac{\partial \sigma_{ij}}{\partial x_j} = \rho_0 a_i & \text{Balance of momentum} \\ \sigma_{ij} = \mathbb{C}_{ijkl} \varepsilon_{kl} & \text{Hooke's Law} \\ \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) & \text{Strain-displacement relation} \end{cases} \quad (2.1)$$

where σ_{ij} is the stress, x_i are components of the position vector, ρ_0 is the density, a_i is the acceleration, $\mathbb{C}_{ijkl}$ is the 4th order elastic tensor, ε_{ij} is the infinitesimal strain tensor, and u_i is the displacement vector. Substitution of the displacement strain relation into Hooke's law and subsequently into the momentum balance generates the wave equation

$$\mathbb{C}_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} = \rho_0 \frac{\partial^2 u_i}{\partial t^2}. \quad (2.2)$$

In the case of uniaxial strain for an isotropic material, the problem simplifies to the 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c_L^2 \frac{\partial^2 u}{\partial x^2}. \quad (2.3)$$

The stress wave pulses propagate through the plates at a speed c_L , also known as the longitudinal wave speed, defined as

$$c_L^2 = \frac{E(1 - \nu)}{\rho_0(1 + \nu)(1 - 2\nu)} \quad (2.4)$$

where E is the Young's modulus, and ν is Poisson's ratio. The solution to 2.3 is

$$\sigma \pm (\rho c_L) u = \text{constant along the characteristics } \frac{dx}{dt} = \mp c_L \quad (2.5)$$

This derivation may be repeated for shear waves by considering transverse particle velocity. Here, we seek solutions to the (transverse) wave equation

$$\frac{\partial^2 u}{\partial t^2} = c_s^2 \frac{\partial^2 u}{\partial x^2}. \quad (2.6)$$

where c_S is the shear wave speed defined as

$$c_S^2 = \frac{\mu}{\rho_0} \quad (2.7)$$

where μ is the shear modulus. As in the longitudinal case, solutions are given by

$$\tau \pm (\rho c_s)v = \text{constant along the characteristics} \frac{dx}{dt} = \mp c_S \quad (2.8)$$

where τ is the shear stress. An important result here is that the stress may be related to the velocity by the impedance ρc . Measuring particle velocity is therefore the primary goal of many plate impact experiments, as described for the pressure-shear plate impact experiment in Section 2.4.

2.2.2 Plastic waves

At sufficient strains, plastic waves are generated in the material. To see this, consider the uniaxial stress-strain relation:

$$\sigma_{11} = \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} E \varepsilon_{11} \quad (2.9)$$

$$\sigma_{22} = \sigma_{33} = \frac{\nu}{1 - \nu} \sigma_{11} \quad (2.10)$$

This is not a hydrostatic state of stress, and the deviatoric stresses may therefore induce yielding. Let the material be elastic-perfectly plastic with a uniaxial yield stress of σ_Y and be described by the Tresca yield criterion: $\sigma_3 - \sigma_1 = \pm \sigma_Y$. The relation between the longitudinal stress and σ_Y is then

$$\sigma_{11} = \frac{1 - \nu}{1 - 2\nu} \sigma_Y \equiv \sigma_H \quad (2.11)$$

where σ_H is the Hugoniot Elastic Limit (HEL) of the material. When a material is dynamically loaded above the HEL, plastic waves will be generated, and a linear elastic

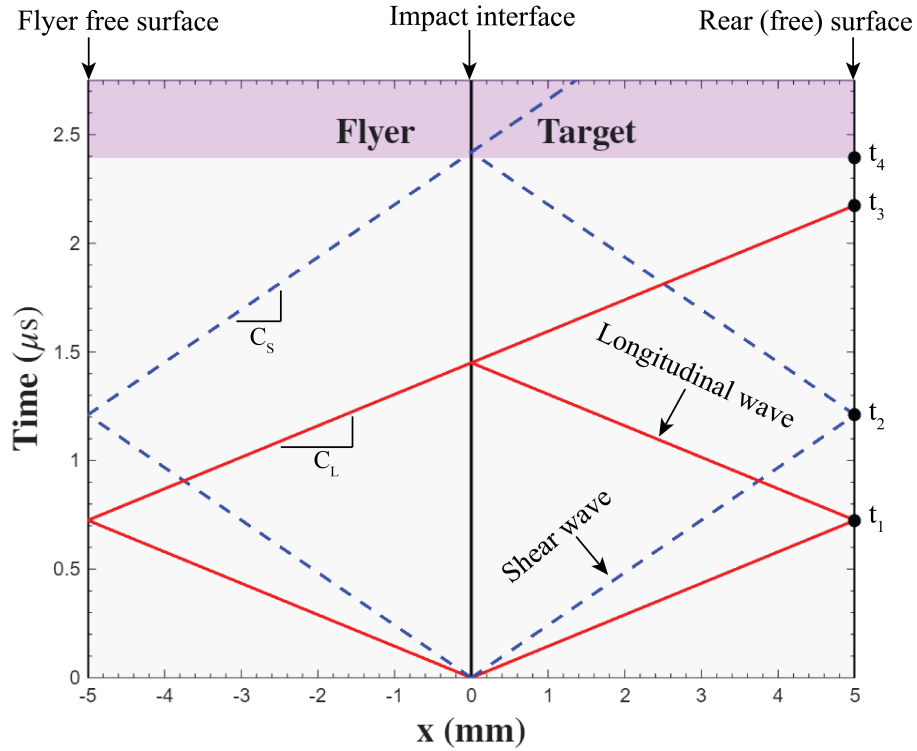


Figure 2.3: An $x - t$ diagram depicting a generic impact event between a flyer and a target. Longitudinal and (slower) shear waves propagate into both plates. At t_1 , the longitudinal wave arrives at the rear surface, where measurements are made. At t_2 the shear wave arrives. At t_3 , the unloading waves from the free surfaces of the flyer and target simultaneously arrive at the free surface. At t_4 boundary waves from the periphery of the plates arrive, generating a highly complex stress-state, thereby ending the experimental time window.

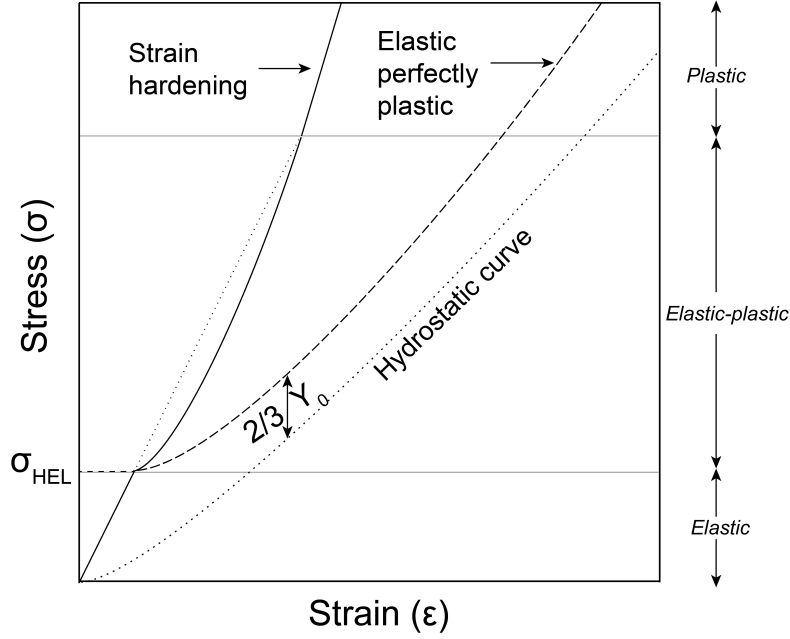


Figure 2.4: A stress-strain curve depicting the elastic, elastic-plastic, and fully plastic regions in a compressively loaded material.

assumption is no longer valid. The propagation speed of these plastic waves is related to the slope of the stress-strain curve at a given value of strain:

$$c_p^2 = \frac{1}{\rho_0} \frac{d\sigma_{11}}{d\varepsilon_{11}}. \quad (2.12)$$

The implication of this is that an elastic wave, known as the elastic precursor, will propagate into the material with speed c_L and will be followed by a fan of plastic waves with varying wave speeds. Lastly, note that the longitudinal stress may be rewritten as

$$\sigma_{11} = K\varepsilon_{11} + \frac{2}{3}(\sigma_{11} - \sigma_{22}) = K\varepsilon_{11} + \frac{2}{3}\sigma_Y. \quad (2.13)$$

There will therefore be a difference of $2/3 \sigma_Y$ between the hydrostatic curve $\sigma_{11} = K\varepsilon_{11}$ (i.e. no material strength) and σ_{11} of the elastic-perfectly plastic material.

2.2.3 Shock waves

For the material described by Fig. 2.4, plastic waves will transit behind the elastic precursor. As the material strain hardens, the strain eventually reaches a point where the plastic waves become fast enough to overtake the elastic precursor. In this fully plastic region, there is a single wave with a sharp front, known as a *shock wave*. There is no longer a smooth transition in matter, but rather a sharp discontinuity between the shocked and unshocked regions in terms of density, pressure, and temperature. A shock wave of finite width is followed by a rarefaction, or release wave. Because this wave is moving into the shocked region with higher density, the material Lagrangian sound speeds will additionally be larger. The shock will thus gradually be attenuated until the pressure pulse is elastic and propagates at c_L .

For large pressures associated with shock wave loading, material strength plays a minor role, and it becomes appropriate to think of the material around the shockwave as behaving like a compressible fluid. The Rankine–Hugoniot jump conditions describe the balance of mass, momentum, and energy across a shock wave as

$$\left\{ \begin{array}{ll} \rho_0 U_s = \rho_1 (U_s - u_p) & \text{Conservation of mass} \\ P_1 - P_0 = \rho_0 U_s u_p & \text{Conservation of momentum} \\ P_1 u_p = \frac{1}{2} \rho_0 U_s u_p^2 + \rho_0 (E_1 - E_0) & \text{Conservation of energy} \end{array} \right. \quad (2.14)$$

where ρ is the mass density, U_s is the shock speed, u_p is the particle velocity, P is the pressure, E is the internal energy, and the subscripts 0 and 1 refer to material in the unshocked and shocked regions, respectively. A schematic of a shock wave moving through a material is shown in Fig. 2.5.

As described in (Cooper, 2018), the three conservation equations contain 8 variables. Assuming the initial state is known, there are then three equations with five unknowns. If two of the particle velocity, pressure, density, and shock speed are measured in the

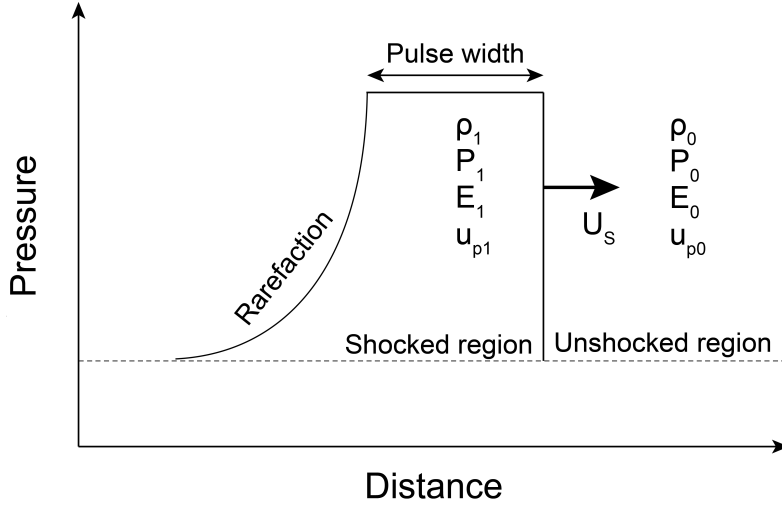


Figure 2.5: A time snapshot of a shockwave moving through a material at a shock speed U_s , creating a jump discontinuity in density ρ , pressure P , internal energy E , and particle velocity u_p . Rarefaction follows, relieving the pressure behind the shockwave.

laboratory, the complete thermodynamic state in a shock experiment is known. The locus of all possible thermodynamic states in a shock-loaded material is the *Hugoniot*. In other words, any point on a material's Hugoniot curve represents a thermodynamic end state in a single shock experiment. Out of the four variables in Eq. 2.14, the Hugoniot equation can relate any two:

$$P - U_s, P - u_p, P - \rho, U_s - \rho, \text{ and } u_p - \rho \quad (2.15)$$

The most common of these are the $U_s - u_p$ plane, $P - \rho$ plane, and the $P - u_p$ plane. For a variety of materials U_s varies linearly with u_p :

$$U_s = C_0 + s u_p \quad (2.16)$$

where C_0 is zero pressure intercept and s is the slope. C_0 for many materials approximates the bulk sound speed:

$$C_0^2 \approx C_B^2 = C_L^2 - \frac{4}{3} C_s^2. \quad (2.17)$$

In the wake of the Manhattan Project and the need for equations of state at higher pressures, the shock Hugoniot for a great many materials were compiled into the Los Alamos Scientific Laboratory (LASL) Shock Compendium, which continues to be a useful reference in the laboratory when conducting shock experiments (Mader et al., n.d.). Experimental measurements of the $U_s - u_p$ relation for many materials, including PMMA, have been found to obey an approximately linear fit, as shown in Fig. 2.6. Rewriting Eq. 2.16 for the $P - v$ relation in terms of the linear $U_s - u_p$ relation, where v is the specific volume $v/v_0 = \rho_0/\rho$, we obtain

$$P = C_0^2(v_0 - v)(v_0 - s(v_0 - v))^{-2} \quad (2.18)$$

Recall that this is not the loading curve, but rather a collection of points that represent individual shock states (or experiments). Compressive shock loading will cause the material to “jump” between states on this curve; the line connecting these two points is called the Rayleigh line, as shown in Fig. 2.7. When the material is unloaded, it continuously follows the path of the isentrope—it does not jump across states as with the Hugoniot. As an aside, the compressive states generated by the isentrope and Hugoniot can be shown to be the same to second order. As a result, the release path is often approximated as being along the Hugoniot. The slope of the Rayleigh line can be shown to be related to the shock speed as $U_s = -(slope)^{-1/2}/\rho_0$. Note that below the HEL, P_1 , the Rayleigh line is simply the linear elastic stress-strain relation. Above P_1 but below P_2 results in an elastic-plastic two-wave structure, i.e., an elastic precursor followed by a plastic shock. Loading to a pressure higher than P_2 in Fig. 2.7 will result in a Rayleigh line with slope greater than in the linear elastic region, and therefore a shock wave propagates with $U_s > C_L$.

A more general description of the material is provided by an *equation of state* (EOS). The EOS describes all possible equilibrium states of the material, that is, not only shock states, but any physically possible thermodynamic state. A thermodynamically consistent equation of state is required to produce qualitatively meaningful results, although

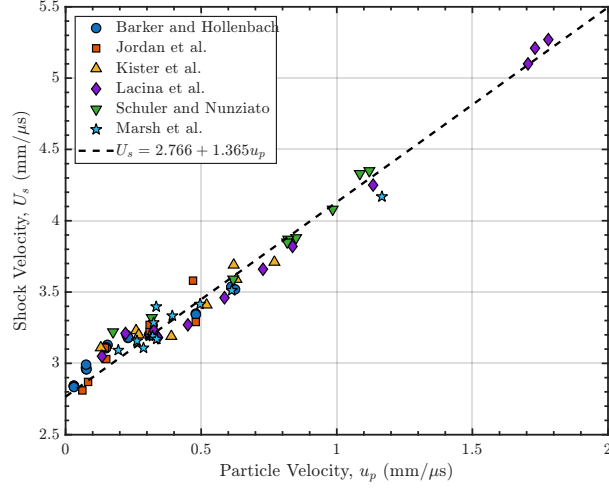


Figure 2.6: The Hugoniot of PMMA in the $U_s - u_p$ plane with a linear fit of $U_s = C_0 + s u_p$

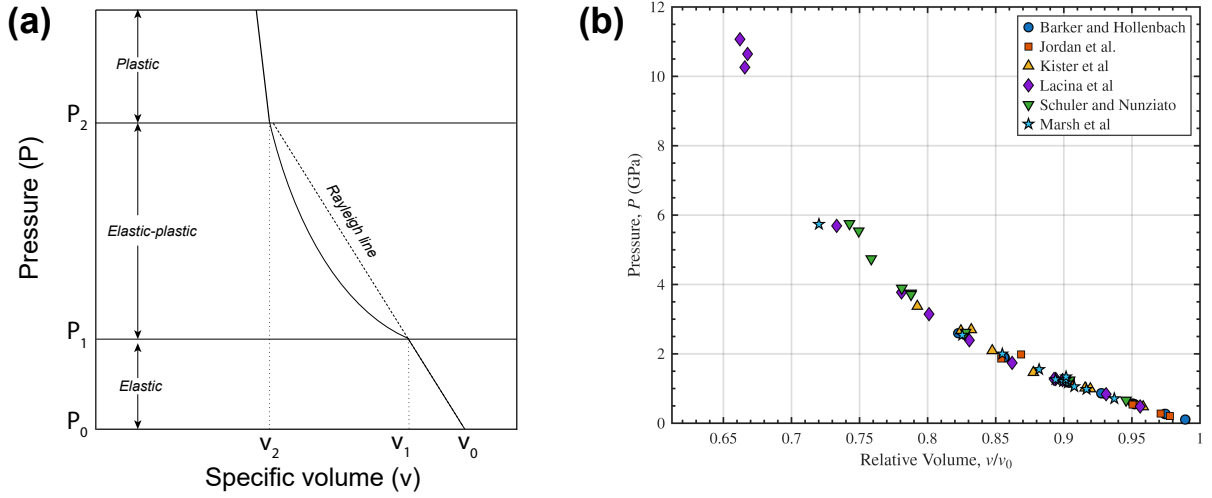


Figure 2.7: (a) The experimental Hugoniot of PMMA in the $P - v/v_0$ plane. (b) A generic material Hugoniot in the $P - v$ plane.

incomplete equations of state may be used as long as their limitations are understood. Equations of state typically aim to connect theoretical analyses of material microstructure or molecular dynamics with observations in shock experiments. There is no general theory that produces a single equation of state for all solids. Instead, an EOS must be chosen based on the material and the conditions of the problem. Discussion of the choice of an EOS for PMMA is continued in Chapters 6 and 8. In a shock experiment, an EOS represents a non-linear elastic response (i.e., it is reversible), since the material state is generally not path dependent and depends only on state variables such as density and internal energy, for example. Pressure-shear experiments often produce substantial temperature increases compared to standard plate impact experiments, and are described in the next section.

2.3 The pressure-shear plate impact experiment

While normal plate impact experiments are well-suited for understanding the pressure-volume relations under shock-loading, the possible states of stress that can be imposed are limited. Plastic strains in plate impact experiments are typically limited to a few percent, limiting test conditions for plasticity models under these extreme conditions. Moreover, shear stress cannot be directly imposed, further limiting investigations of material strength and failure.

Oblique plate impact (OPI) experiments make use of an angled flyer-target arrangement such that longitudinal and shear waves are generated upon impact. A special case of OPI is the constant pressure shear plate impact (PSPI) experiment, which allows for shearing to take place under uniform pressure. PSPI may produce strain rates of $10^5 - 10^7/\text{s}$ and plastic strains in excess of 100%, far beyond the incipient plasticity typically seen in normal plate impact (NPI) experiments, i.e., where the flyer plate normal is parallel to the impact velocity. PSPI has been conducted on many materials over the past half-century:

- Metals:
 - Iron: (Klopp et al., 1985), (Gandhi et al., 2022)
 - Steel: (Klopp et al., 1985), (Zhou, 1993), (C. R. Johnson et al., 2024)
 - Aluminum: (Klopp et al., 1985),(Gilat & Clifton, 1985), (Grunschel & Clifton, 2007), (Zuanetti et al., 2020)
 - Titanium: (Gilat & Clifton, 1985)
 - Tantalum: (Duprey, 2000)
 - Tungsten and tungsten heavy alloy: (Zhou, 1993)
 - Magnesium: (Ravindran et al., 2020)
- Lubricants (Ramesh & Clifton, 1987)
- Ceramics:
 - Soda-lime glass: (Sundaram & Clifton, 1998), (Kettenbeil, 2019)
 - Sandstone: (Helminiak & Borg, 2023)
 - Boron carbide: (Sun et al., 2020)
 - Aluminum oxide: (Espinosa et al., 1992)
- Powders: Alumina (Sundaram, 1998)
- Polymers:
 - Polyurea (Clifton et al., 2016)
 - Hydroxyl-terminated polybutadiene (HTPB): (Malhotra et al., 2022)
 - Polycarbonate: (Prakash & Mehta, 2012)
 - Polyurethane: (Sable, 2019)
 - Epoxy: (Sable, 2019)

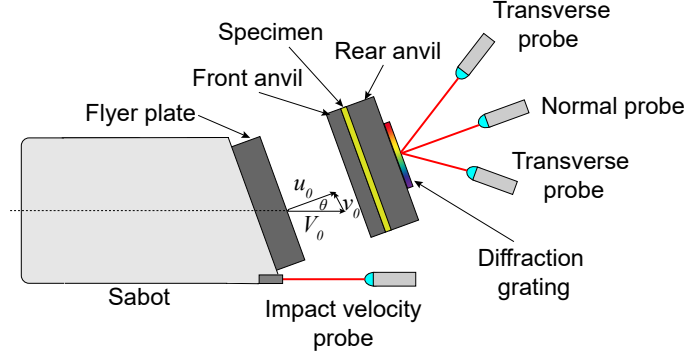


Figure 2.8: The pressure-shear plate impact experiment with a common sandwich target configuration and fiber-optic diagnostics at the rear of the target and near the projectile.

- Energetic material simulants:
 - Sucrose: (Malhotra et al., 2022)
 - Sucrose/HTPB composite: (Malhotra et al., 2021)

A typical configuration for PSPI is shown in Fig. 2.8. A flyer plate oriented at an angle θ to the impact velocity V_0 strikes a target parallel to it. Because the flyer is oriented at an angle, the velocity may be decomposed into initial normal and transverse components:

$$u_0 = V_0 \cos \theta \quad \text{Normal} \quad (2.19)$$

$$v_0 = V_0 \sin \theta \quad \text{Transverse} \quad (2.20)$$

The target assembly consists of two anvil plates and a sample sandwiched between them. For low impact angles (typically fewer than 20°), friction between the flyer plate and the front anvil plate is high enough to ensure shear tractions are transmitted. Consequently, compression and shear waves will be simultaneously transmitted into the sample. To resolve the internal stress-state, free-surface velocities are measured in directions normal and transverse to the rear anvil. Here, infrared laser light is emitted and collected via fiber-optic probes, which are used to measure the normal and transverse surface velocities,

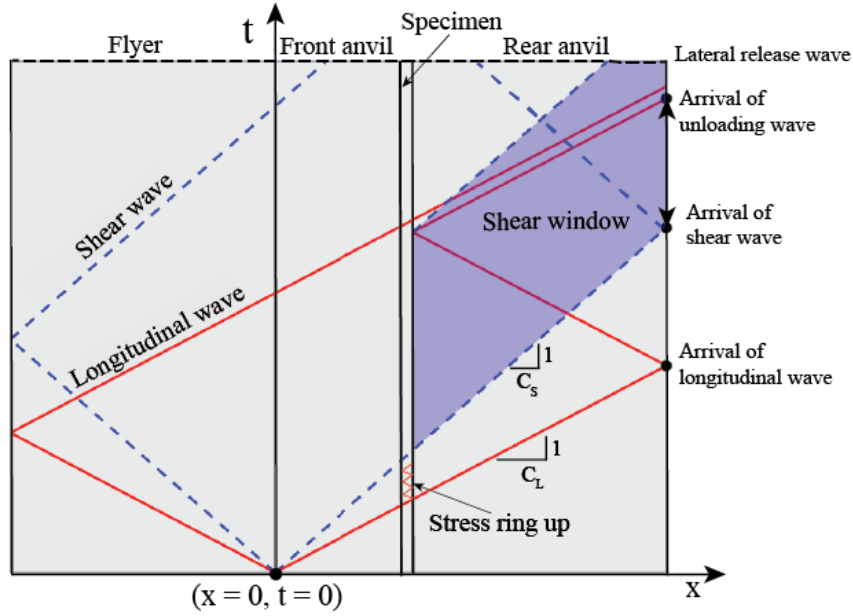


Figure 2.9: an x - t diagram for a PSPI experiment. At $(x=0, t=0)$, longitudinal and shear stress waves propagate into the flyer and target plates. The longitudinal and shear stress waves are represented by characteristic lines, with slopes c_L and c_S , respectively.

as well as the impact velocity. More information on the velocimetry system is provided in Chapter 4.

The characteristic solutions described in Section 2.1 may be applied graphically to PSPI experiments. This is the so-called $x - t$ diagram, as shown in Fig. 2.9 for a typical impact geometry. Impact occurs at $t = 0, x = 0$. Longitudinal waves move through the flyer and front target plate at c_L . Shear waves follow at the slower speed c_S . The longitudinal wave arrives first at the sample and reverberates several times, ringing the stress in the sample up to a uniform level before the shear wave arrives. The sample is therefore under a uniform normal stress before shearing begins. Stress waves continue to move through the rear anvil, reach the traction-free surface, and reflect, thereby reducing the stress. As soon as impact begins, release waves propagate from the boundary of the flyer and target. The thicknesses of the plates are chosen such that the experimental time window is maximized with respect to the arrival of boundary waves at the measurement point, approximately in the center of the target. At this time, the experiment is over, and

meaningful observations about the mechanical response of the specimen can no longer be made.

The stress state may be directly inferred by measuring surface velocities. To see this, consider four points: F in the front anvil, S in the specimen, R in the rear anvil, and fs on the free-surface of the rear anvil. Using Eqs. 2.5 and 2.8, the characteristic solutions across between $F - S$ are

$$\sigma_F - \rho_0 c_L u_F = -\rho_0 c_L u_0 \quad (2.21)$$

$$\tau_F - \rho_0 c_S V_F = -\rho_0 c_S v_0 \quad (2.22)$$

Similarly, for $S - R$ we have

$$\sigma_R + \rho_0 c_L u_R = 0 \quad (2.23)$$

$$\tau_R + \rho_0 c_S v_R = 0 \quad (2.24)$$

Lastly, for $R - fs$:

$$\sigma_R - \rho_0 c_L u_R = -\rho_0 c_L u_{fs} \quad (2.25)$$

$$\tau_R - \rho_0 c_S v_R = -\rho_0 c_S v_{fs} \quad (2.26)$$

Once the state of stress has reached a uniform value, the stress is the same everywhere: $\sigma_F = \sigma_S = \sigma_R$ and $\tau_F = \tau_S = \tau_R$. Now combining Eqs. 2.21-2.25 we obtain the equilibrium relations:

$$\sigma_S = -\frac{\rho_0 c_L u_{fs}}{2} \quad (2.27)$$

$$\tau_S = -\frac{\rho_0 c_S v_{fs}}{2} \quad (2.28)$$

$$2u_s = u_{fs} = u_0. \quad (2.29)$$

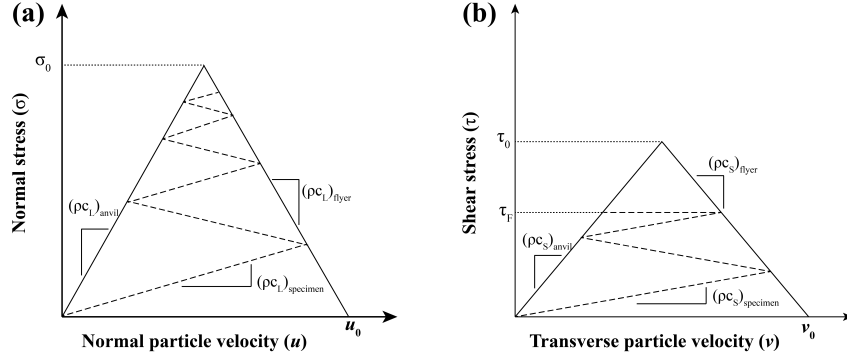


Figure 2.10: Reverberations inside the sandwiched material, with impedances marked for the anvil, specimen, and flyer for (a) normal stress (b) shear stress. In this case, the material reaches a saturation shear stress τ_F , begins to yield, and does not reach the maximum possible shear stress τ_0 .

As the normal stress wave reflects between the two anvil plates, σ follows lines with slope $\mp \rho_0 c_L$ in stress-velocity space, as shown by Fig. 2.10(a). After each reflection, σ will increase until eventually it reaches a uniform value. A parallel analysis can be repeated for τ , as shown in Fig. 2.10(b). However, if the system begins to plastically deform before reaching the maximum possible shear stress $\tau_0 = -\frac{\rho_0 c_s v_0}{2}$, then the transverse velocity will not be constant across the sample. The nominal strain rate is given by

$$\dot{\gamma} = \frac{v_F - v_R}{h} = \frac{v_0 - v_{fs}}{h} \quad (2.30)$$

where h is the thickness of the specimen. This is only valid after the sample has reached a state of uniform shear stress, i.e., when it begins to plastically deform. Integrating the shear strain rate gives the shear strain:

$$\int_0^t \dot{\gamma}(\tau) d\tau \quad (2.31)$$

With this, dynamic stress-strain curves may be obtained under extreme loading conditions inherent to PSPI experiments.

An important caveat to the above analysis is that it is predicated on an assumption of linear elastic wave propagation. Above the HEL of the anvils, plastic waves are

generated, and the mapping between transverse velocity and sample shear stress is no longer straightforward. The shear stress in the sample may be made by use of a constitutive model for the anvil, as in the two following methods. Clifton first approached this issue by means of solving the so-called inverse problem of solving the set of hyperbolic partial differential equations relating the measured free surface velocities and the transmitted stresses (Clifton et al., 2020). The analysis consists of numerically solving for the stresses via the method of characteristics. Around the same time, Kettenbeil developed a scheme that fits simulated free surface velocity profiles to experiments citep Kettenbeil2020. This calibration results in parameters for a material’s strength model. This latter method has been adopted by several research labs for a variety of materials and for normal stress up to nearly 50 GPa, well beyond the HELs of typical anvils (2-3 GPa for tool steel; 5-7 GPa for tungsten carbide) (Gandhi et al., 2022; C. R. Johnson et al., 2024; Ravindran et al., 2021).

Before moving on, a few notes about strength measurements in plate impact experiments are warranted, as PSPI is not the only experimental method that attempts to measure material strength under dynamic loading. By assuming a Tresca or von Mises yield criterion, the yield strength Y is given by

$$\sigma - \sigma_t = \pm Y \quad (2.32)$$

where σ is the longitudinal stress and σ_t is the lateral stress. Piezoresistive manganin alloy gauges are commonly used to measure these lateral stresses; this was first implemented in steel by Bernstein (Bernstein et al., 1968). A schematic of a typical stress gauge experiment is shown in Fig. 2.11. The analysis is quite straightforward; however, there are several drawbacks. First, these gauges may only be used for certain types of loading conditions (typically below 20 GPa). The material behavior of the gauges and the specimen must also be considered; these gauges typically perform poorly with ductile materials, and the long signal rise times may be problematic when dynamic failure occurs. There has also been a

recent discussion on the long accepted calibration data where Rosenberg (Rosenberg & Moshel, 2014) found the original strength of the gauges to be roughly half of the original values (Rosenberg, 1984), resulting in high values for strength in the literature. In any event, stress gauges have been used to generate strength measurements over the past few decades in a variety of materials. A few of these have been directly compared to PSPI. Millett, Gray, and Bourne (Millett et al., 2007) found reasonable agreement for tungsten and tungsten heavy alloy as measured by Zhou and Clifton (Zhou, 1993). Jordan (Jordan et al., 2022) also found reasonable agreement on polyurea when compared to PSPI experiments by Jiao and Clifton (Clifton et al., 2016). Ravindran (Ravindran et al., 2021) found that for higher pressures, there was a severe mismatch between the two measurements.

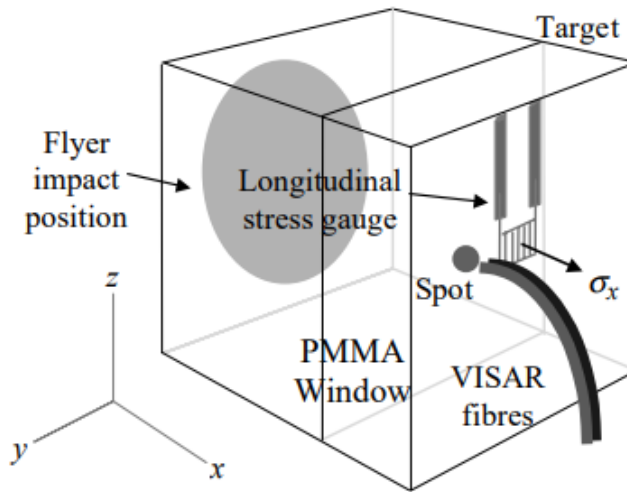


Figure 2.11: “Experimental arrangement for simultaneous gauge/VISAR experiments.”¹

Another widely used method of determining strength at high pressures was introduced by Asay and Lipkin (Asay & Lipkin, 1978): this is known as the self-consistent method. Here, the material’s yield stress is determined via particle velocity measurements during shock and release experiments. Recalling Eq. 2.13, the stress when the material is yielding

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under compression is

$$\sigma_{max} = P + \frac{4}{3}\tau. \quad (2.33)$$

When the material is released, it will eventually enter a state of tension. The stress is then at the opposite end of the yield surface:

$$\sigma_{min} = P - \frac{4}{3}\tau \quad (2.34)$$

The strength is therefore given by

$$Y = \frac{3}{4}(\sigma_{max} - \sigma_{min}) \quad (2.35)$$

Like PSPI, the self-consistent method is a non-invasive technique and is not limited by the calibration or material response of gauges. However, the technique is comparatively insensitive. Moreover, both the self-consistent method and stress gauges measure strength for limited states of stress. An attractive alternative, like PSPI, would be one where significant plastic deformation may be generated.

The advent of 3rd and 4th generation synchrotron light sources has allowed for high-speed x-ray imaging in impact experiments, thereby generating a plethora of rich, dynamic phenomena. Heterogeneous deformation may be captured by such imaging systems, and when combined with other shock diagnostics (such as PDV), complex states of loading become experimentally accessible. Several authors have leveraged this idea into parameterizing constitutive models under these finite deformation scenarios. Lind (Lind et al., 2021) examined cylindrical hole closure in copper and was able to parameterize a strength model by matching 3D simulations to the x-ray images. Jensen(Jensen et al., 2015) interrogated material strength in cerium by shock loading samples with grooved rear surfaces, as shown in Fig. 2.12. When a shock wave arrives at the rear surface, the perturbations rapidly invert, forming hydrodynamic jets with extreme local deformation, where phase changes are believed to occur. This phenomenon is a case of the Richtmeyer-

Meshkov Instability (RMI). Material strength measurements are generated from knowing ambient material properties, initial geometry, as well as the jet's dynamic shape and velocity. While highly complex material states such as this can be useful test cases for material models, there are serious drawbacks. In the vicinity of such an event, the local strain rate, plastic strain, and even stress can be impossible to know. It is for this reason that the strength model for PMMA in this work will first be calibrated to PSPI data before being tested in a case of heterogeneous deformation, that is, pore collapse.

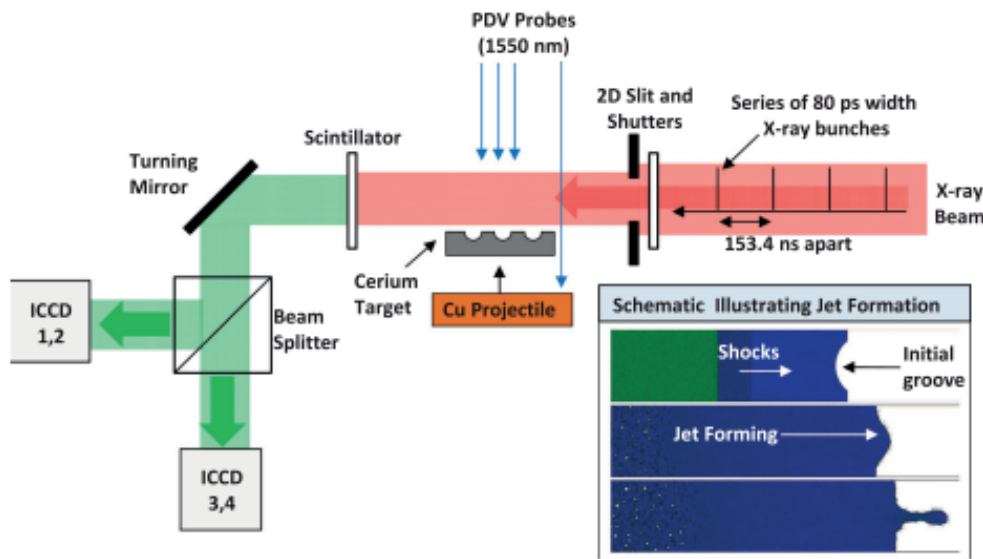


Figure 2.12: “Schematic of the experimental configuration for shock wave experiments using X-ray imaging to observe jet formation in Ce. A Cu plate (2.5 mm thick) impacted the Ce sample located in an evacuated target chamber, generating a shock wave that interacted with the machined perturbations at the back surface. The x-ray beam passed through multiple slits and shutters, interacted with the sample and evolving jets, and was incident upon a scintillator. The scintillator light was imaged onto four optically multiplexed intensified detectors, which were synchronized to the impact event and the X-rays. (Inset) Schematic showing the process of jet formation as the shock wave interacted with the groove in the sample.”²

²Reprinted from (Jensen et al., 2015), with the permission of AIP Publishing.

Construction and characterization of a plate impact facility

3.1 Overview

To measure the dynamic flow stress of PMMA, experiments were carried out on a newly constructed single-stage gas gun facility at Brown University. This chapter highlights the design process and the resulting experimental workflow in conducting PSPI experiments at Brown. Experimental PSPI data for PMMA are presented in Chapter 4.

3.2 Facility and equipment

A photo of the plate impact facility is shown in Fig. 3.1(a). A single-stage gas gun was manufactured by Applied Physics Inc. The launcher consists of a 24 foot long, 2 inch bore barrel and an approximately 12 in. $\times$ 12 in. $\times$ 12 in. target chamber, shown in Fig. 3.1 (b). The barrel has a broached rectangular keyway, shown in Fig. 2.2 (c), which is essential for projectiles to maintain their orientation after target alignment and before the experiment is over. After alignment, the projectile is pulled down the length of the barrel with a rope tied to its rear eye-bolt. The system is sealed and vacuumed to approximately 100 mTorr. When ready to fire, the pressure reservoir, or breech, is pressurized to a firing pressure P_0 . Rapidly releasing the gas then accelerates the projectile down the length of the gun barrel towards the target chamber. The experiments in this thesis used pressurized nitrogen in a breech that uses a fast-acting solenoid valve to fire. It is crucial to have a sense of a “velocity curve” which relates impact velocities as a function of projectile mass and loading pressure. Typically, the velocity v is related to P_0 and the projectile mass m by $v \sim \sqrt{P_0/m}$, since the energy balance is $\frac{1}{2}mv^2 = P_0AL$, where A is the cross section of the barrel and L is its length. Clearly, the pressure will not be constant as the projectile moves down the barrel, but this general functional form

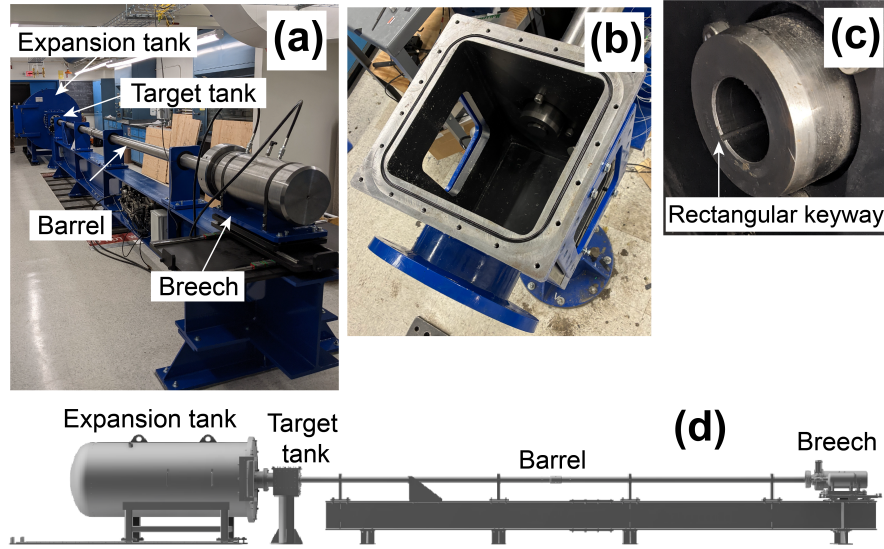


Figure 3.1: Layout of the plate impact facility (a) A view of the lab, looking downrange (b) A close-up view of the target chamber (c) The keyway (d) A profile view of a CAD assembly of the facility.

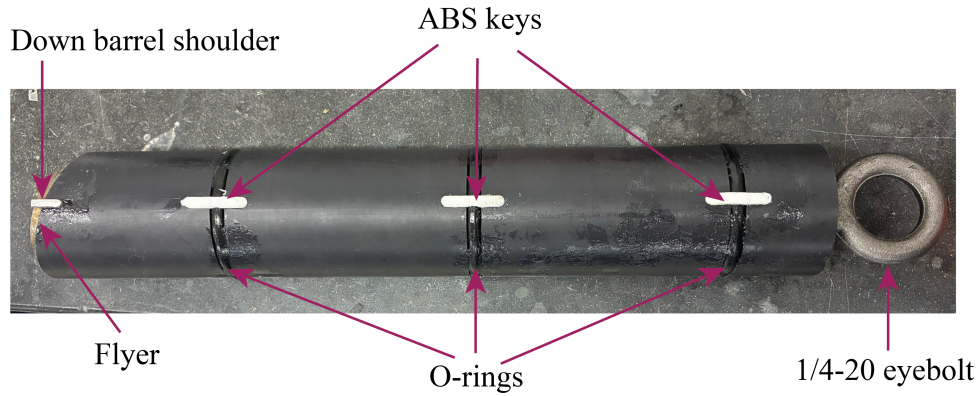


Figure 3.2: An assembled projectile.

correlates well with empirical shot data.

3.2.1 Projectile assembly

An image of an assembled projectile is shown in Fig. 3.2. The projectiles in this thesis were made of molybdenum disulfide-infused nylon, obtained from McMaster-Carr®. Due to initial problems with high tilt, longer projectiles were used, with approximate lengths of 11 inches and masses around 900 grams. Three durometer 90A Buna-N o-rings were placed in grooves along the length. Three keys made from ABS plastic were placed over

the o-rings to maintain the orientation of the projectile when firing. A small aluminum shoulder, with roughly a 5 mm X 5 mm square cross section, was attached at the front end of the projectile, which provides a reflective face from which the impact velocity may be measured. The flyer plate is epoxied into a pocket at the front end of the projectile, which itself is oriented at the impact angle θ . Finally, a 1/4"-20 eye-bolt is attached to the end of the projectile so that the projectile can be pulled the length of the barrel to the breech when firing by means of a rope. The projectile must be pulled from the front end to the rear end, as alignment must occur with the flyer and target plates as close as possible in the target chamber.

3.2.2 Target assembly

Targets in PSPI experiments in this work are in a sandwich configuration, where a thin specimen layer is sandwiched between two anvils of high acoustic impedance materials, such as tungsten carbide or D2 tool steel. Several types of experiments, including symmetric oblique impact, normal impact on quartz, and PSPI were conducted, and any nuances in sample preparation are noted in the respective sections. In any plate impact experiment, anvils are lapped on both sides with a 15 μm boron-carbide slurry in a lapping machine (Lapmaster) to a flatness of at most 2-3 helium light bands (0.58-0.87 μm). Flatness is verified by polishing a surface with successively smaller monocrystalline diamond pastes of 6 μm , 3 μm , and 1 μm . Once the plates have a mirror finish, their flatness may be verified by use of an optical flat with a monochromatic light source. It is imperative to re-lap the smooth, polished impacting surfaces for a few minutes, so as to reduce the risk of dynamic slip during impact. If slip occurs, shear tractions will correspond to the interfacial frictional strength rather than the shear strength of the material.

A completed target placed in a target holder is shown in Fig. 3.3. When polishing, rear surfaces are left with a mirror polish, as these are the free surfaces that will be monitored by the velocity diagnostics. A diffraction grating is prepared on the rear plates by means

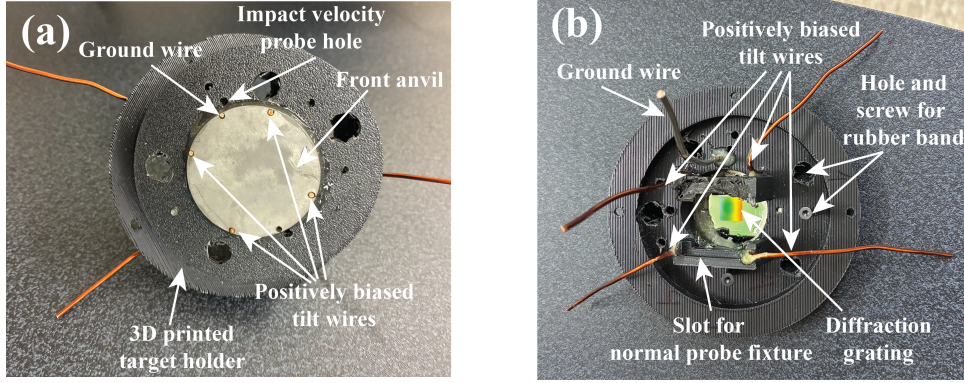


Figure 3.3: Images of a target used in a PMMA PSPI experiment (a) The impact face of the target (b) The rear surface of the target.

of photolithography. As described in (Kettenbeil et al., 2018b) and (Walze, 2005) the intensity of the 0th and 1st order diffraction spots are given by

$$I_0 = 2\beta^2 \left(1 + \cos \left(\frac{4\pi n_0 h}{\lambda} \right) \right) \quad (3.1)$$

$$(3.2)$$

$$I_1 = \frac{1}{\pi^2} \left(1 - \cos \left(\frac{4\pi n_0 h}{\lambda} \right) \right) (1 - \cos(2\pi\beta)) \quad (3.3)$$

$$(3.4)$$

where I_0 is the 0th order intensity, β is the duty cycle of the grating, n_0 is the index of

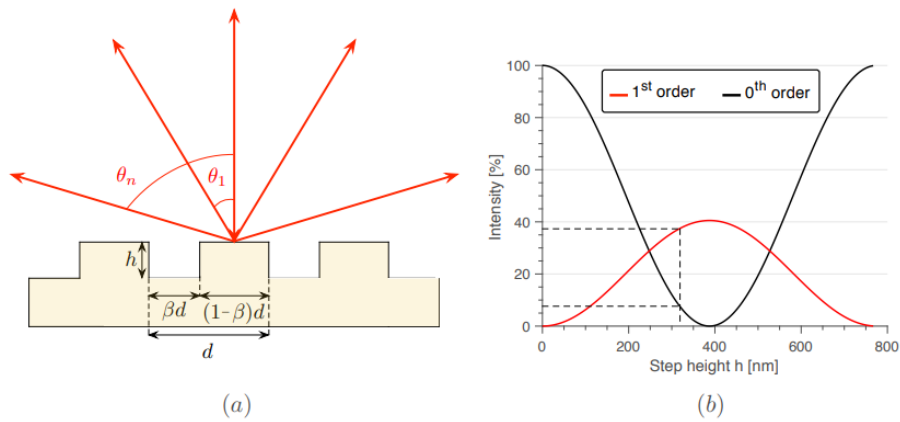


Figure 3.4: “Structure of a reflective binary diffraction grating and (b) its diffracted beam intensity as a function of step height h for a duty cycle β of 0.5.”¹

refraction, h is the step height, and λ is the wavelength of the incident light. A plot of an example grating as well as the intensities are shown in Fig. 3.4. The diffraction grating in this thesis is similar to that in (Kettenbeil et al., 2018b), with a pitch of $2.5 \mu\text{m}$, a duty cycle of $\beta = 0.5$ and a grating height of $h = 500 \text{ nm}$. In reality, the step height of the grating will be slightly different than 500 nm due to variations in the photolithography process. As a result, the intensities observed during probe alignment will vary from shot to shot.

Diffraction gratings are fabricated in Brown University’s class 1000 cleanroom, as part of the Instrumentation for Molecular and Nanoscale Innovation (IMNI) research facilities. After lapping and polishing, rear anvils are sonicated in acetone, then isopropanol, then DI water, and dried with nitrogen. The surfaces are then further cleaned by means of plasma etching with an O_2 plasma for 3 minutes. The clean plates are then spin-coated with the photoresist AZ1505 at 4000 RPM for 45 seconds; the AZ1505 spin curve lists a thickness of roughly 500 nm at 4000 RPM. This photoresist is UV sensitive, so care must be taken not to expose it to incandescent lights or sunlight. Samples are then placed on a hot-plate for a soft-bake at 110°C for 90s. Samples are then exposed with a dose of $150 \text{ mJ}/\text{cm}^2$ and a wavelength of 375 nm to produce a diffraction grating pattern with a pitch of $2.5 \mu\text{m}$, or a feature size of $1.25 \mu\text{m}$ rectangles. The exposure is performed with Heidelberg Instruments’ MLA150, a maskless aligner. The camera on the MLA is used to precisely align the diffraction grating with respect to the tilt pin holes. After exposing, the sample is developed in a beaker with 50%AZ MIF 312 (metal ion-free) developer and 50% DI water for 20 seconds. The sample is then hard-baked at 110°C for 120 seconds.

After preparing the diffraction grating, the target (whether a single plate, a quartz target, or a PSPI sandwich) is ready to be mounted. Targets are pressed and epoxied into the 3D printed target holders shown in Fig. 3.3. Tilt wires are used to align the target tilt pin holes with those of the target holder, the latter being reamed to create a slip fit. Tilt

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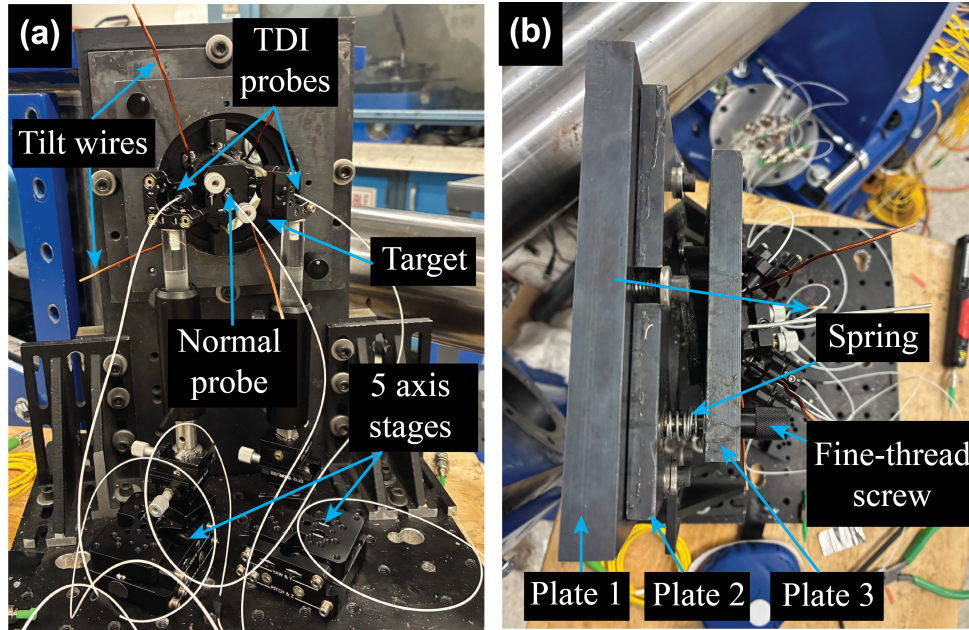


Figure 3.5: Views of the alignment fixture, depicting the location of the target, the probes, alignment stages, and the 3-plate structure of the gimbed alignment fixture itself. (a) Rear view (b) Top down view.

wires consist of 14-gauge motor winding wire. The exterior of these wires is coated with an insulating enamel, which prevents electrical connections between the wires and the target. Upon impact, the target, tilt wire faces, and flyer are all connected to a common ground; this ground is the common ground for the entire tilt system, and is connected to the target chamber, the alignment fixture, and BNC cable grounds, among other things. If the tilt wires' insulation is accidentally removed prior to shooting, there will be a short, and the tilt may not be measured. Ground wires, consisting of 14-gauge fully conductive wire, are also placed in a target hole. BNC cables are later directly grounded to this wire via butt-splice connectors. After inserting the tilt and ground wires, the front surface must be briefly re-lapped so that the wires are flush with the impact face. After cleaning the target once more, the target may be placed into the alignment fixture in the target chamber and aligned to the flyer.

3.2.3 Plate alignment and tilt

Impact occurs in the target chamber, ideally as close to the barrel as possible, so as to mitigate tilt and to ensure planar impact. The target sits in an alignment fixture as shown in Fig. 3.5. The fixture consists of three primary plates

- The first plate, which is mounted to the target chamber's breadboard via angle brackets
- The middle plate, which allows for X and Y translation of the target with respect to the barrel
- The rear plate, into which the target assembly is placed.

The rear plate has two fine threads (1/4"-100) screws placed through springs and a ball joint, which allows for precise angular alignment of the target. An autocollimator is placed outside the target chamber, as shown in Fig. 3.6(a) to conduct plate alignment. Mirrors are placed on the front anvil and flyer, and are held in place by rubber bands, as shown in Fig. 3.6(c). An autocollimator was constructed so as to simplify the alignment process by means of a digital camera. The design of the autocollimator is shown in Fig. 3.6 (d) and a simple description is as follows, and all parts were acquired from ThorLabs. A laser source (M625L4) emits laser light through a ground glass diffuser (DG1022MD) and a cross reticle (R1DS3N). This light is then transmitted through a beamsplitter (BS013) and lens doublet with focal distance 200m (AC508-200-A). After reflection, the light is re-transmitted through the doublet, through the beam splitter, and into a CMOS sensor (DCC1545M). Images of reflections are produced in the ThorCam software package. The relation describing the distance between two reflected beams as seen by an observer at the camera end of the autocollimator may be derived as

$$d = 2\theta f \tag{3.5}$$

where d is the displacement of the beams, θ is the angular displacement between the mirrors, and f is the focal length of the lens. Since the pixel pitch of the sensor is given as $5.2\mu\text{m}/\text{px}$, a pixel in the software corresponds to approximately 13μ radians of misalignment per pixel. The limit on alignment is realistically around 0.1 milliradians. The fine pitch screws on the alignment fixture are turned until crosses from the flyer and front anvil mirrors are aligned as well as possible.

After plate alignment, the probe used for measuring impact velocity is inserted into its hole in the target holder, and light return from the sabot shoulder is verified. The normal and transverse free surface velocity probes are then aligned such that the light return is maximized. The normal probe is aligned by using the tip/tilt mirror mount stage and monitoring the power meter. Aligning the TDI probes is more involved. First, IR detector cards are used to locate the XY position of the diffraction spots. Hollow aluminum tubes are then placed into mirror mounts (ThorLabs SM05PT) placed on optical posts. These optical posts are in turn fastened to kinematic stages (ThorLabs PY005), as depicted in Fig. 3.5. The tubes are maneuvered until infrared diffraction spots are visible on the outgoing side of the tube. At this point, probes (ThorLabs 50-1550A-APC) are inserted and the stages manipulated until the desired light return is achieved. For 25 dBm ($\sim 300\text{mW}$) of input power, light returns of +10 dBm and +0 dBm are desired for each of the normal and TDI probes. However, care must be taken to avoid overloading the detectors. Attenuators are therefore used to limit the power to no more than a total of -5 dBm per detector.

Finally, connections are made between tilt wires and BNC cables, and the tilt circuit is verified, which completes the alignment and target setup process. The four tilt pins have a positive voltage bias with respect to common ground; the ground is shared by the electrical equipment, BNC cables, target chamber, alignment fixture, target plate, and, once impact occurs, the flyer plate. The tilt wires, which are lapped flush to within $1\mu\text{m}$, are connected to a logic circuit. When the grounded flyer plate makes contact with the

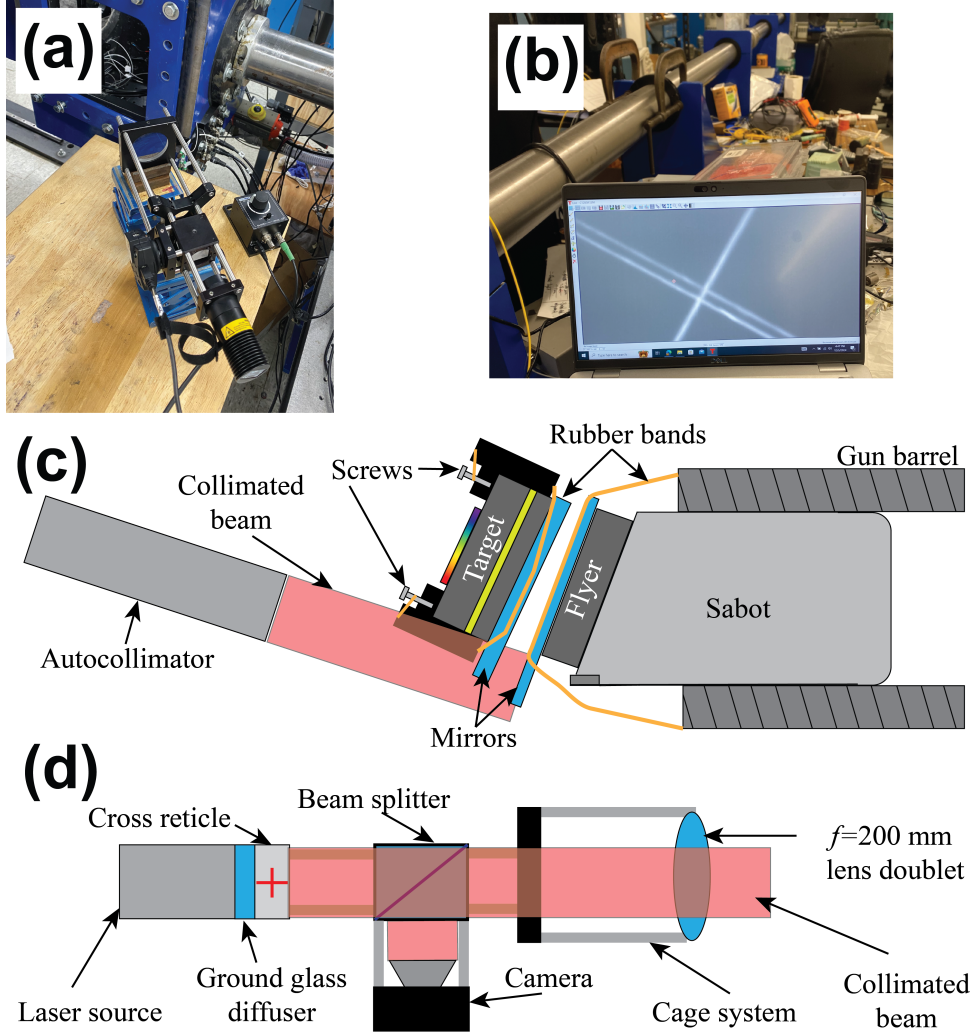


Figure 3.6: Images depicting the plate alignment procedure (a) The autocollimator oriented at the target chamber (b) The resultant image of two crosses, one from the flyer and one from the front anvil (c) A schematic of the alignment process, highlighting mirrors attached to the flyer and front anvil via rubber bands (d) A schematic of the auto-collimator, consisting of various opto-mechanical components from ThorLabs.

wires, a unique voltage drop will be detected. The system starts at nominally +5V. When any of the wires are shorted, flip-flop logic gates switch from ON to OFF. These signals are summed together, and the oscilloscope is triggered after the first pin has been shorted.

3.3 Velocimetry

An important result derived in Chapter 2 is that the internal stress state (i.e., $\sigma = \sigma_{11}$ and $\tau = \tau_{12}$) is directly related to the free surface velocities. Recall Eq. 2.27:

$$\sigma_S = -\frac{\rho_0 c_L u_{fs}}{2} \quad (3.6)$$

$$\sigma_S = -\frac{\rho_0 c_S v_{fs}}{2} \quad (3.7)$$

$$2u_s = u_{fs} = u_0. \quad (3.8)$$

We therefore seek a velocimetry system capable of resolving surface velocities with high time resolution in the normal and transverse directions via a reflective target face and diffraction grating. The following section details the construction of such a system. Various design choices will be elucidated in the following subsections.

3.3.1 Normal velocimetry

During the past half-century, a variety of diagnostic techniques have been applied to plate impact experiments. Early shock experiments, during and after World War II, made use of shorting (voltage-biased), manganin, and piezoelectric pins to capture transit times for shock waves. These methods were instrumental in constructing the previously mentioned LASL Shock Compendium, for example. Optical methods have largely supplanted such measurements. Starting in the 1950s, streak cameras were commonly used to determine EOS parameters by examining 1D motion in a shocked material. In 1968 at Lawrence Livermore National Laboratory (LLNL), Johnson and Burgess developed Fabry-Perot velocimetry, which could resolve multiple velocities using

a streak camera (P. M. Johnson & Burgess, 1968). Despite the high resolution, these systems proved difficult and expensive to build. In 1972, the Velocity Interferometer System for Any Reflector (VISAR) was developed by Barker and Hollenbach at Sandia National Laboratory (Barker & Hollenbach, 1972). Here, a time-shifted copy of Doppler-shifted light is combined with itself and allowed for continuous, nanosecond resolution measurements of velocity histories. Other interferometers, such as in Professor Rodney Clifton's Plate Impact Lab at Brown (Kim et al., 1977), were developed in the ensuing decades. By the early 2000s, the vast telecommunications industry had rendered fiber-optic equipment cheap and readily accessible. A compact, fiber-based velocimetry technique known as photonic Doppler velocimetry (PDV) was introduced at Lawrence Livermore National Laboratory in the early 2000s by Oliver Strand (Strand et al., 2006). PDV is a displacement interferometer of the Michelson variety, which relies on Doppler-shifted light from a reflective target. Telecommunications devices use near-infrared and infrared light, with typical frequencies in the 100 THz range. This is far higher than the bandwidth limit of modern oscilloscopes (usually <10 GHz), this Doppler shifted light is mixed with a reference line to produce a beat frequency in a detectable range. The recorded beat frequency history may then be converted to a velocity history, as detailed below. An excellent description of the history, working principles, and limitations of PDV may be found in (D. H. Dolan, 2020).

A schematic of a free-space equivalent of PDV is shown in Fig. 3.7 (a). Instead of mirrors and beam splitters, fiber-optic components such as circulators and couplers are used in PDV systems. Laser light moves through fiber-optic components and is transmitted by probes into free space. Light reflects off a moving target, and the Doppler-shifted light is collected by the same probe. This Doppler-shifted light moves through the bi-directional fiber-optic fibers and through a device known as a circulator. The circulator acts as a directional diode between ports and prevents input (unshifted) light from mixing with Doppler-shifted light. The circulator also prevents any reflected beams in the experiment

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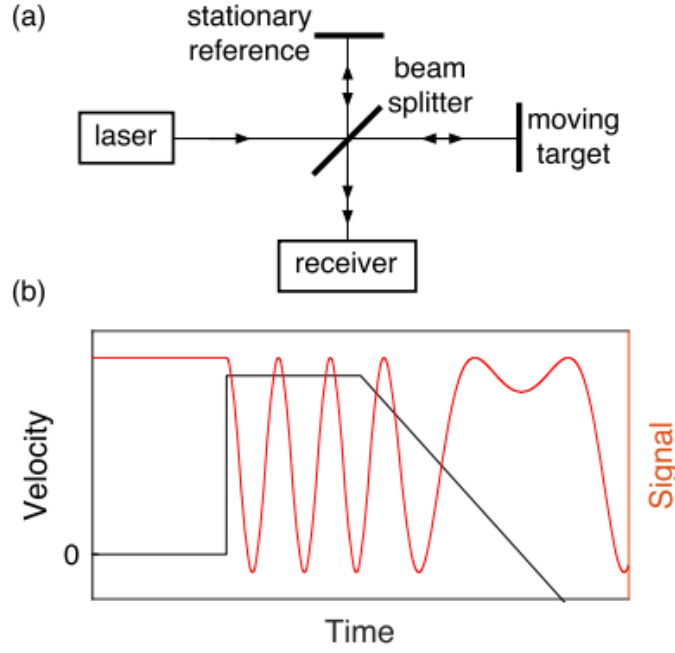


Figure 3.7: “(a) Free-space interferometer equivalent for PDV. (b) Characteristic PDV signal for a dynamic velocity.”²

from back-propagating and potentially damaging the source laser. After the circulator, the Doppler-shifted light is combined with the reference laser light of constant wavelength. This combined wave has a characteristic beat frequency B . The resultant light waves are picked up by a photodetector, which converts the optical signal into an electrical one. The latter is then sent to an oscilloscope, which saves the voltage-time record. As the target begins to move, a non-zero beat frequency is generated. As the target decelerates, the beat frequency of the resulting sinusoid also changes. The primary governing equation in PDV relates the observed velocity to the frequency of the signal:

$$f = \frac{2}{\lambda}|v| \quad (3.9)$$

For a thorough explanation of the derivation of Eq. 3.9, see (Rauls, 2015). Once the voltage history from an experiment has been stored, its frequency must be extracted as a function of time so as to obtain velocity (and therefore stress). Short-time Fourier transform (STFT) analysis is used to extract these frequencies. In this work, analysis

of PDV data was conducted with the MATLAB program Sandia InfraRed HEterodyne aNalysis (SIRHEN), which is detailed in (D. Dolan & Ao, 2010). Oscilloscopes will typically capture signals much longer than the time period of interest in an experiment, and consequently, the first step is to crop a signal to a more manageable size. Window functions are applied to overlapping subsets of the cropped data. Because discrete Fourier transforms express signals as a sum of sinusoids, sharp discontinuities at boundaries introduce artificial high-frequency components. Consequently, the signal energy may leak across adjacent frequency bins. Window functions are introduced to mitigate this spectral leakage associated with finite observation intervals. Window functions are zero-valued at and outside the boundary and ideally narrow in the region of interest. The window functions available in SIRHEN are Boxcar, Hann, Hamming, and Blackman. Almost all of the analyses in this work utilize a Hamming window, as it works well for tightly packed spectral peaks associated with impact motion.

After the window function has been applied, a power spectrum is generated, that is, a three-dimensional surface of the signal’s relative intensity versus time and frequency. To determine the correct frequencies over time from this surface, a peak-finding algorithm is applied. Frequency limits can also be set to reduce the non-physical artifacts, such as back-reflections in the PDV circuitry, which show up in the frequency spectrogram. Simply taking frequencies corresponding to intensity maxima often results in noisy data, and smooth peak-finding methods, such as a Gaussian spectral peak-finding scheme, are generally preferred. The frequency history is then related to the velocity history by Eq. 3.9.

A schematic of the PDV system used in this work is shown in Fig. 3.8, and the components are detailed in Table 3.1. There are two conventional PDV channels for the normal free surface and impact velocities. The former is time multiplexed by sending the signal through a 4 km long fiber, thereby producing a $19.83 \mu s$ time delay. Since the duration of events in plate impact experiments is generally less than $10 \mu s$, there

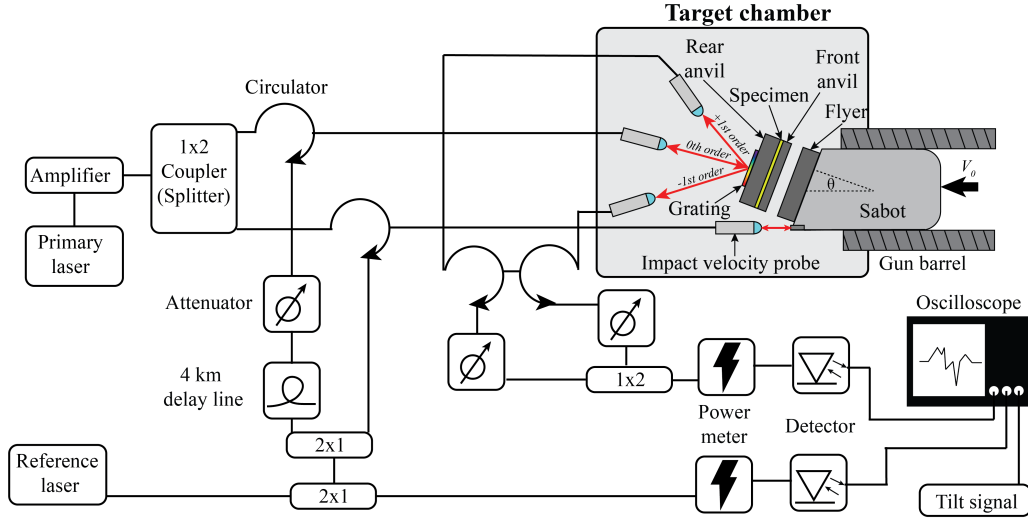


Figure 3.8: Schematic of the fiber-optic circuitry used to measure the normal and transverse velocities at the rear surface of the target, as well as the impact velocity of the sabot.

Component	Make & Model	Details
Seed Laser	NKT Photonics Basik	1550 nm CW, 40 mW
Laser Amplifier	NKT Photonics Boostik	1550 nm CW, 2W
50/50 Coupler	Agiltron FC-15119133	900 μm loose tube
90/10 Coupler	Agiltron FC-15114133	900 μm loose tube
Circulator	Agiltron OCPI-302111333	3-Port
Attenuator	OZ Optics 1550-9/125-S-50-3A3A-3-1	Variable
Delay Line	AFW Technologies SM1-22-B-4/4/0/0KM-1RU	4000m length
Power Meter	OZ Optics OPM-200-1550-9/125-S-3A3A-3-1-A	RS232 Output
Normal Probe	OZ Optics LPF-01-1550-9/125-S-9-12-1.1AS-50-3A-3-3-SP	$f = 1.1$ mm focuser
Impact Probe	AC Photonics 1CL15P070LCD01	70 mm collimator
Transverse Probe	AC Photonics 1CL15P070LCD01	70 mm collimator
Photodetector	Optilab PR-10-H-M	10 GHz
Oscilloscope	Tektronix DSOS404A	20 GS/s, 4 GHz

Table 3.1: Summary of fiber optic and data acquisition components in the Photonic Doppler Velocimetry (PDV) and transverse displacement interferometer (TDI) architecture. All connections are made with Molex FC/APC connectors, and all fibers are single-mode.

are minimal issues in analyzing the two signals in a single channel. Signals are time-multiplexed so as to leverage the higher bandwidth available when only two oscilloscope channels are used. If more than two channels are used, then the scope memory will be split in half. Finally, the PDV system used here makes use of two separate lasers in a “heterodyne” configuration. That is, light from a secondary reference laser is combined with the Doppler-shifted light originating in the primary laser. This heterodyne system allows for non-zero frequencies to be generated prior to impact. This downshifting is often done in situations where the beat frequency of interest lies beyond the available bandwidth limit.

3.3.2 Transverse velocimetry

Transverse displacement interferometry (TDI) was first implemented in oblique plate impact experiments by Kim and Clifton at Brown University (Kim et al., 1977). Here, a holographic diffraction grating was fabricated on the rear surface of a target plate. Symmetrically diffracted beams were combined with free-space optical components (i.e., beam splitters and mirrors) such that the resultant light wave collected on a photodiode was sensitive only to the transverse motion of the diffraction grating. The fundamental TDI equation is given as

$$\frac{\text{displacement}}{\text{fringe}} = \frac{d}{2} \quad (3.10)$$

where “fringe” refers to one complete segment of the sinusoidal signal (i.e., peak to peak) and d is the diffraction grating pitch, or period.

(Zuanetti et al., 2017) and (Kettenbeil et al., 2018b) later extended the TDI technique to be used with fiber-optic components. In the case of (Zuanetti et al., 2017), the system is a direct analog to that of Kim, and the relation between fringes and displacement in Eq. 3.10 is the same. Kettenbeil made use of two separate circuitry designs. The first, known as diffracted beam photon doppler velocimetry (DPDV), combines the ± 1 st order diffraction beams with a reference leg, thus generating a typical PDV wave profile

for each beam. Transverse velocity histories are then extracted by subtracting the two resultant signals. The second technique, denoted as “heterodyne transverse velocimetry” (HTV) is more directly comparable to that of Zuanetti. Here, the normal component of the motion is removed by fiber-optically combining the two diffracted waves. The primary difference between this HTV and the fiber TDI of Zuanetti is that HTV also uses an acousto-optic modulator to generate a non-zero initial frequency, which allows for the use of conventional STFT schemes. In this work, we have effectively emulated the system presented in (Zuanetti et al., 2017). TDI signals therefore have no initial frequency and are analyzed with a phase differentiation algorithm, as depicted in Fig. 3.9 and described as follows.

As with PDV analysis, typically signals are recorded far beyond the time interval of interest and must be cropped. Because shear waves move more slowly than longitudinal waves, any tilted normal motion will be observed first, followed by transverse motion corresponding to the shear wave. If impact tilt is high, disentangling these two phenomena can become intractable. Once the region of interest is obtained, high frequencies must be removed. A low-pass filter is applied to the signal, with a typical threshold being in the 10s of MHz; 10 MHz corresponds to a velocity of 12.5 m/s for a 400 lines/mm diffraction grating. This low-pass filter may affect the interpretation of results if the TDI fringe SNR is poor. The resulting sinusoid is then amplitude corrected so that the entire signal falls on the interval $[-1,1]$. Amplitude correction requires a peak finding algorithm, which must be checked by eye to reduce spurious fringes not removed by the low-pass filter. Transverse displacement is then calculated from phase information in the amplitude-corrected sinusoid. The phase is given by

$$\Delta\psi = 2\pi m + \pi(\alpha + 1) - \alpha \arccos[y(i)] - \psi_0 \quad (3.11)$$

where $\Delta\psi$ is the change in phase at the i^{th} data point, m is an integer that is updated after every fringe, $y(i)$ is the value of the signal at the i^{th} data point, and ψ_0 is the initial

phase. The transverse displacement is then given by

$$v(i) = \frac{\frac{\lambda}{2}}{2\pi} \Delta\psi(i). \quad (3.12)$$

The transverse displacement may then be numerically differentiated to obtain velocity. Of course, point-wise numerical differentiation may lead to high frequencies and noisy velocity profiles. Hence, a rolling smoothing window is typically applied. The final transverse velocity must be compared with the normal PDV signal and expected timing from the $x - t$ diagram. It is exceedingly rare that a shear wave arrives when it is not supposed to if experimental documentation is consistent.

3.4 Verification of plate impact technique

After a set of table-top experiments confirmed the basic workings of the diagnostics, two sets of experiments were conducted on quartz and tungsten carbide. These experiments aimed to (1) validate the operation of the different subsystems of the facility working in concert and (2) to provide insights into the material response of anvils used in PSPI experiments.

3.4.1 Normal plate impact of quartz

To validate the free-space velocimetry system implemented by (Kim et al., 1977) and the fiber-optic system in (Kettenbeil et al., 2018b), normal plate impact experiments were conducted on large single crystals of α -quartz which generated elastic waves in multiple directions, which were impacted by glass flyers. Experiments were similarly conducted to validate the entire plate impact system at Brown, especially the normal and transverse velocimetry equipment. Moreover, a primary experimental objective of this new facility is to study the propagation of waves in large single crystals, such as energetic materials (e.g., HMX, RDX, and PETN) and their simulants (e.g., acetaminophen and

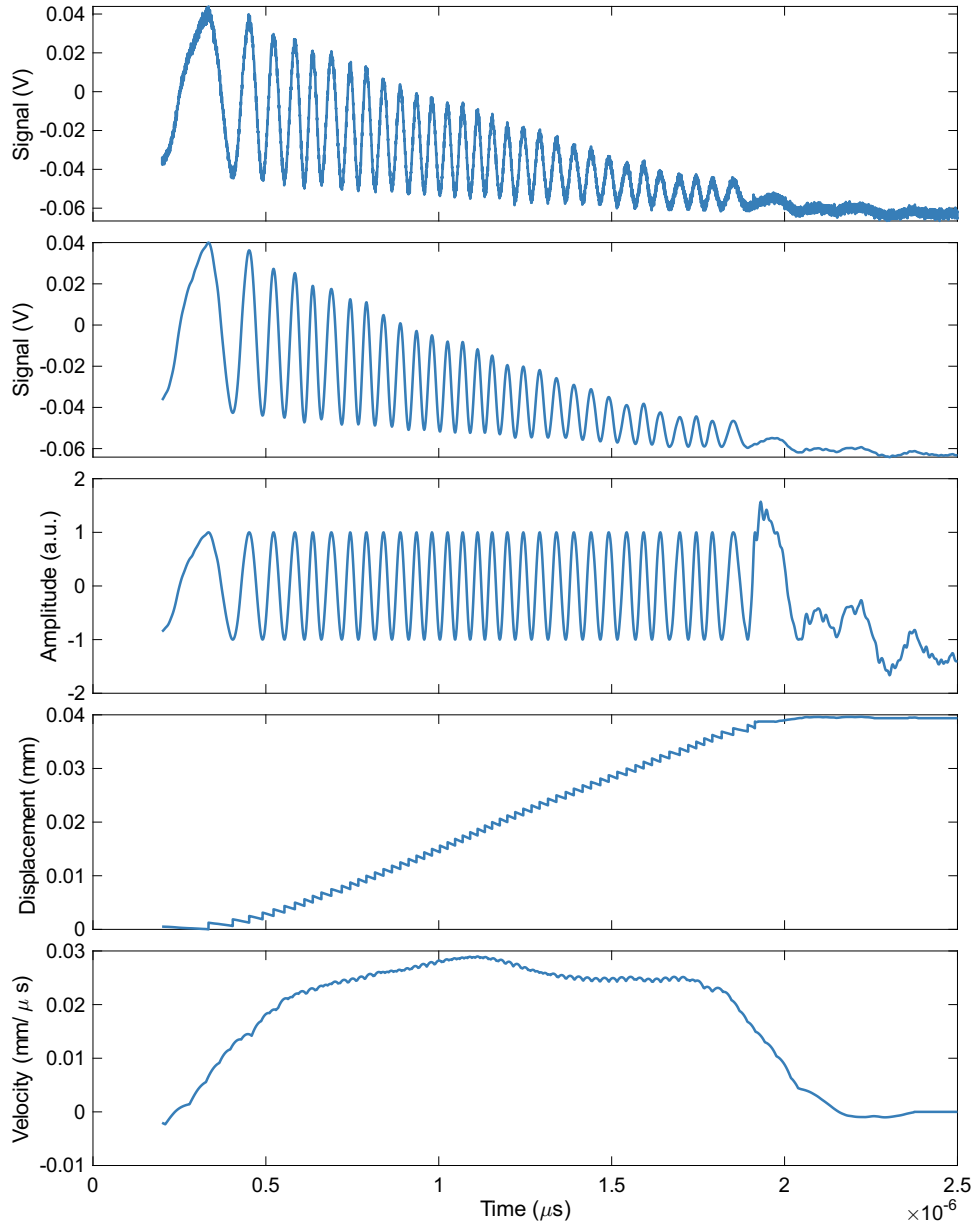


Figure 3.9: Workflow of TDI analysis, as described in the text. From top to bottom: Raw data trace, low pass filtering, amplitude correction, displacement, and velocity. The data are for shot SNTF090925, elaborated on in Section 3.4.2.

sucrose). Accordingly, the rest of this subsection discusses the methodology of plate impact experiments on single crystals as well as how these results validate the facility’s general capabilities.

The rest of the discussion deals with α -quartz and not any other polymorph, and thus simply “quartz” will be used to describe the material in the remainder of this subsection.

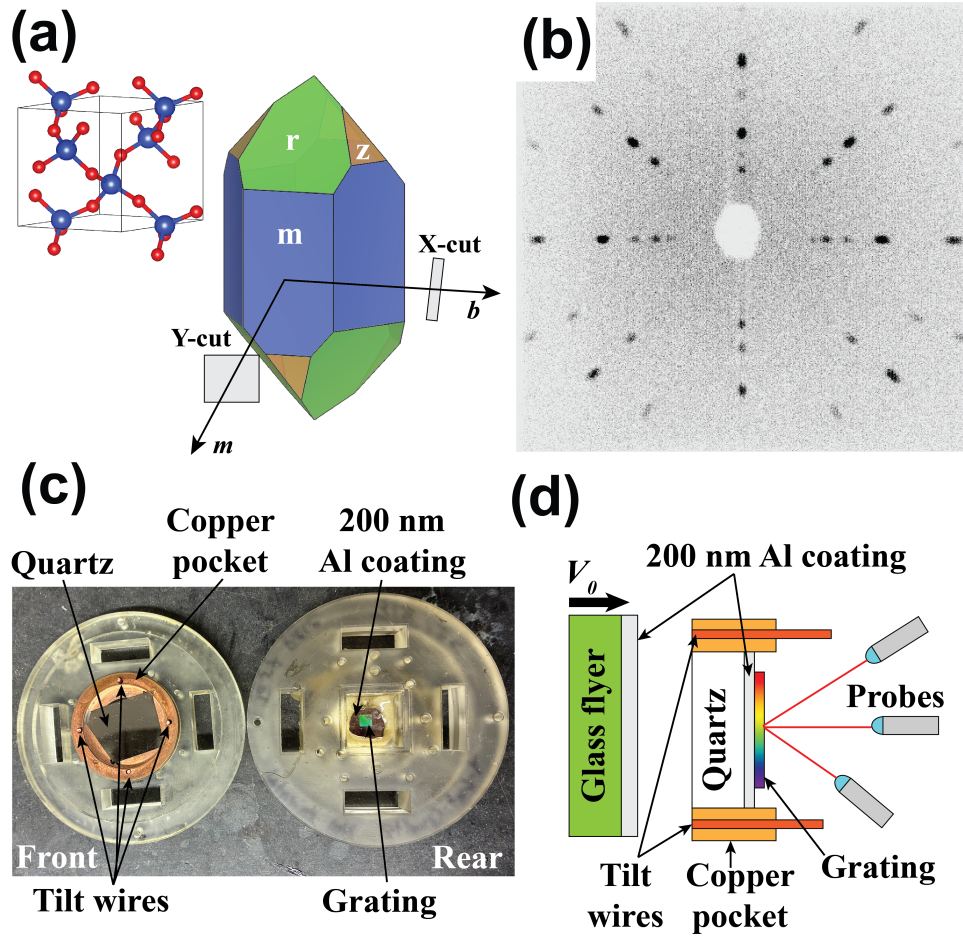


Figure 3.10: Fabrication of plate impact experiments on α -quartz. (a) A unit cell of α -quartz and an example of crystal morphology generated in Vesta. (b) A Laue diffraction pattern of Y-cut quartz along the $(1\bar{1}00)$ plane (c) Front and rear sides of the target, where quartz samples have been epoxied into and lapped flush with a copper pocket. (d) Schematic of normal plate impact on Y-cut quartz with the familiar TDI probe arrangement.

A unit cell of quartz and example morphology produced in the crystal viewing software Vesta are shown in Fig. 3.10 (a). Quartz is enantiomorphic, with the left-handed variant in the $P3_221$ space group and the right-handed variant in $P3_121$. As discussed at length first by (Donnay & Le Page, 1978) and later (Glazer, 2018), the literature is rife with confusion about the structure of quartz and resultant properties, such as piezoelectricity, which often see sign errors. In our case, the handedness of quartz will not affect elastic wave propagation, but some discussion on the morphology of quartz is still warranted. Thus far, all of the materials in this work have been assumed to be isotropic. That is, only one longitudinal and one shear wave will be generated in plate impact, with only longitudinal waves being generated in the case of normal impact. Single crystals of quartz, however, are anisotropic and will support the propagation of multiple quasi-longitudinal and quasi-transverse waves when being impacted. When impacting quartz, the velocity jumps in the normal and transverse directions may be solved directly by analyzing the wave equation, as in Eq. 2.1. A thorough discussion of this problem can be found in (J. N. Johnson, 1971). Both enantiomorphs of quartz are in the trigonal point group, which has 6 independent elastic constants. The elastic tensor C_{IJ} is given in Voigt notation by

$$[C_{IJ}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{11} & C_{13} & -C_{14} & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ C_{14} & -C_{14} & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & C_{14} \\ 0 & 0 & 0 & 0 & C_{14} & \frac{C_{11}-C_{12}}{2} \end{bmatrix} \quad (3.13)$$

There are two primary conventions used when describing the elastic tensor of quartz: the IRE 1949 Standard and the IEEE 1978 Standard. The IRE 1949 convention, which is more common and will be used here, has C_{14} negatively valued for both enantiomorphs. In accordance with this convention, the density, from (Haynes, 2016), and elastic constants, from (Bechmann, 1958), used in the rest of the analysis are given in Table 3.2. Now we

ρ (g/cc)	C_{11} (GPa)	C_{33}	C_{12}	C_{13}	C_{14}	C_{44}
2.648	86.74	107.2	6.99	11.91	-17.91	57.94

Table 3.2: Elastic material properties of α -quartz.

aim to solve for the wave velocities and polarizations for a given impulse and crystal orientation. As described by (Bower, 2009), a plane wave that travels in direction $\mathbf{p}$ with speed c has a displacement field

$$u_i = a_i f(ct - x_k p_k) \quad (3.14)$$

where a_i is the particle velocity. If $\mathbf{p} = \alpha \mathbf{a}$ then the wave velocity and particle velocity are in the same direction. If $\mathbf{p} \cdot \mathbf{a} = 0$ the wave velocity and particle velocity are perpendicular. These two cases correspond to the longitudinal and shear waves discussed earlier. Substituting the plane wave solution into the wave equation 2.1 yields

$$\Gamma_{ik} a_k f(ct - x_j p_j) = \rho a_i c^2 f(ct - x_j p_j) \quad (3.15)$$

where

$$\Gamma_{jk} = \mathbb{C}_{ijkl} p_i p_l \quad (3.16)$$

is the *acoustic tensor*. The plane wave solution satisfies

$$(\Gamma_{jk} - \rho_0 c^2 \delta_{ik}) a_k = 0 \quad (3.17)$$

$$\det(\Gamma_{jk} - \rho_0 c^2 \delta_{ik}) = 0 \quad (3.18)$$

$$(3.19)$$

This is also known as the *Christoffel equation*. The three eigenvectors and eigenvalues of this equation correspond to three wave polarization directions and speeds, respectively. For an isotropic solid, these would be the one longitudinal and two shear waves. In the case of our quartz problem, the polarization direction for the Y-cut direction is $\mathbf{p} = [0, 1, 0]$

and therefore

$$\mathbb{C}_{ijkl}p_i p_l = \Gamma_{jk} = \begin{bmatrix} 39.9 & 0 & 0 \\ 0 & 86.7 & 17.9 \\ 0 & 17.9 & 57.9 \end{bmatrix} \quad (3.20)$$

$$\Gamma_{jk}/\rho_0 = \begin{bmatrix} 15.0 & 0 & 0 \\ 0 & 32.7 & 6.8 \\ 0 & 6.8 & 21.9 \end{bmatrix} \quad (3.21)$$

The normalized eigenvectors and eigenvalues of the latter are the resultant wave polarization directions and velocities:

$$(\lambda_1, \lambda_2, \lambda_3) = (3.9, 4.3, 6.0) \text{ mm}/\mu\text{s} \quad (3.22)$$

$$V = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 0.43 & -0.9 \\ 0 & -0.9 & -0.43 \end{bmatrix} \quad (3.23)$$

The waves will therefore be, in order of speed:

- a quasi-longitudinal wave with polarization $(0, -0.9, -0.43)$ and speed $6.0 \text{ mm}/\mu\text{s}$
- a quasi-transverse wave with polarization $(0, -0.43, -0.9)$ and speed $4.3 \text{ mm}/\mu\text{s}$
- a transverse wave with polarization $(0, -0.43, -0.9)$ and speed $3.9 \text{ mm}/\mu\text{s}$

We can use these solutions along with the impedance of the flyer to predict the velocity jumps that our diagnostics will measure.

To prepare the quartz samples, the crystals first had to be correctly oriented. Disks 5 mm in thickness, 35 mm in diameter were procured from University Wafer. Glass flyers were 6 mm in thickness, 40 mm in diameter (Borofloat33, University Wafer). To rotationally orient the samples, Laue diffraction was performed on a Multiwire Labs Back-

Reflection Laue diffractometer at the MIT.nano characterization lab. Laue diffraction uses polychromatic X-rays to obtain single-crystal orientations. Crystals can be rotated on a goniometer to obtain the desired diffraction pattern, corresponding to the plane (hkl) , or $(hkil)$ in four-index notation, of interest. Y-cut quartz corresponds to the $(1\bar{1}00)$ plane. The built-in Multi-Wire software is capable of simulating diffraction patterns for known crystal structures (i.e., inputs of space groups and lattice parameters). In this way, the Z direction was carefully inscribed at the edge of the wafer, since $\pm Z$ corresponds to the transverse motion components of the quasi-longitudinal and transverse waves. A typical Laue pattern is shown in Fig. 3.10 (b). It should be noted that the Laue symmetry produced does not unambiguously define the orientation of the crystals. The crystals' orientation could in fact be 180° different. As noted in (Zecevic et al., 2026), the ambiguity in orientation can lead to difficulties in interpreting data for other impact scenarios.

Diffraction gratings were then prepared in the usual manner, except for vapor coating the surfaces. Because quartz is translucent, the wafers were then coated with a 20 nm Titanium adhesion layer followed by 300 nm of Aluminum. Vapor deposition was performed in a Lesker Lab-18 thin film deposition system. When exposing, the inscription was marked by the MLA150's microscopic camera. The glass flyers were also coated with aluminum, since the tilt system relies on an electrical short between the flyer and pins. Targets and projectiles were then assembled in the usual way. Images of targets are shown in Fig. 3.10(c). A schematic of the plate impact experiment is depicted in Fig. 3.10 (d).

Shot no.	V_0 (m/s)	Tilt (mRad)	Vacuum (mTorr)	Flyer thickness (mm)	Target thickness (mm)
SNTP032025	213.8	1.31	98	6.00	5.01
SNTP072825	147.1	0.8	140	5.97	5.00

Table 3.3: Shot matrix of the plate impact experiments on quartz.

A shot matrix describing the relevant shot parameters is depicted in Table 3.3. The resultant $t - X$ diagram is shown in Fig. 3.11. Given the thicknesses and material

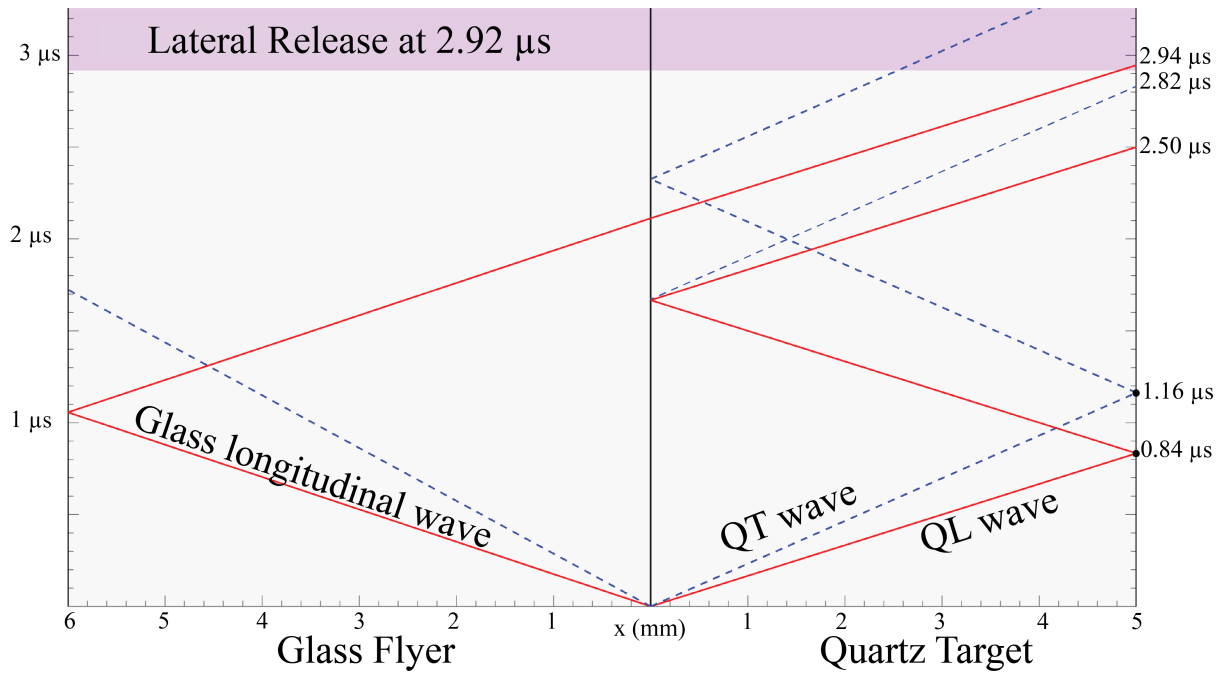


Figure 3.11: $t - X$ diagram for a 6 mm glass flyer impacting a 5 mm Y-cut quartz target. There are several times of interest at the free-surface of the target, where velocity measurements are made. At $t = 0.84 \mu s$ the QL wave arrives. At $t = 1.16 \mu s$ the QT wave arrives. At $t_1 = 2.5 \mu s$ the reflection from the target's free-surface arrives. The unloading wave from the boundary and the reflection from the free-surface of the flyer arrive almost simultaneously: $t_1 = 0.292 \mu s$ and $t_1 = 2.94 \mu s$, respectively.

properties of the glass flyer ($E = 64\text{GPa}$, $\nu = 0.2$) and quartz target, we expect the quasi-longitudinal wave to arrive at $0.84\ \mu\text{s}$ and the quasi-transverse wave to arrive at $1.16\ \mu\text{s}$, and for the signal to persist for at least $1\ \mu\text{s}$ before the arrival of any unloading waves. In the end, two plate impact experiments were conducted on quartz. The first shot gave rise to several challenges, which illuminated necessary modifications to the experiment. The tilt and down barrel velocity data are shown in Fig. 3.12. The tilt was within the standard rule of thumb of $2\ \text{mRad}$ of tilt. The raw free-surface data are shown in Fig. 3.13 and the resultant free-surface velocity profiles are shown in Fig. 3.14. Though both signals have a poor signal-to-noise ratio, a normal velocity profile produces the expected velocity jumps, and the transverse velocity nearly seems correct for the first few hundred nanoseconds, but after the arrival of the QT wave, the transverse signal is completely lost. Moreover, there is an incredibly high spike shown in the transverse velocity that appears non-physical. This spike cannot correspond to tilt, since the magnitude of this spike is even higher than the normal velocity.

Several ideas were put forth. First, gas moving past the projectile was known to be an issue at this time. However, blow-by seemed unlikely to be the source of the transverse spike for several reasons. If blow-by were affecting probes at the rear surface of the target, then this effect would be seen prior to the arrival of the longitudinal waves. The fact that the transverse spike corresponds exactly with the arrival of the quasi-longitudinal wave indicates that an event associated with the impact itself is involved. Typically, blow-by manifests as a rounding of velocity profiles prior to impact. Since rounding of both normal and transverse velocity profiles is minimal, blow-by is unlikely to be the culprit.

A second hypothesis was that there was spurious motion of the diffraction grating itself. If there were an unknown dynamic equilibration process in the grating, then that could cause the short-lived spike associated with the arrival of the longitudinal wave. This, too, seemed implausible, for such gratings have been used in myriad PSPI experiments going back decades. The grating recipe is nearly the same as in (Kettenbeil et al., 2018b) and

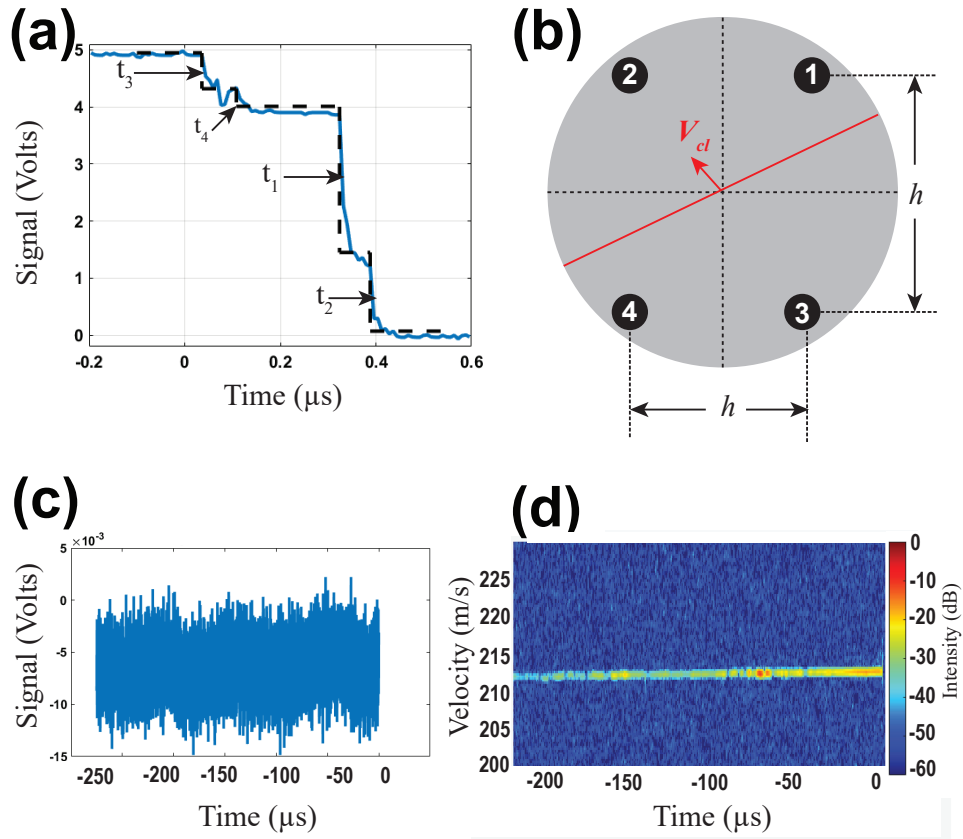


Figure 3.12: Measurements of plate tilt at impact and projectile velocity. (a) Voltage drops corresponding to the biased tilt wires being shorted. (b) Schematic of the closure velocity between the glass flyer and the quartz target, as seen looking uprange. (c) time period of interest for the experiment. (d) Spectrogram of the projectile velocity seen down the barrel.

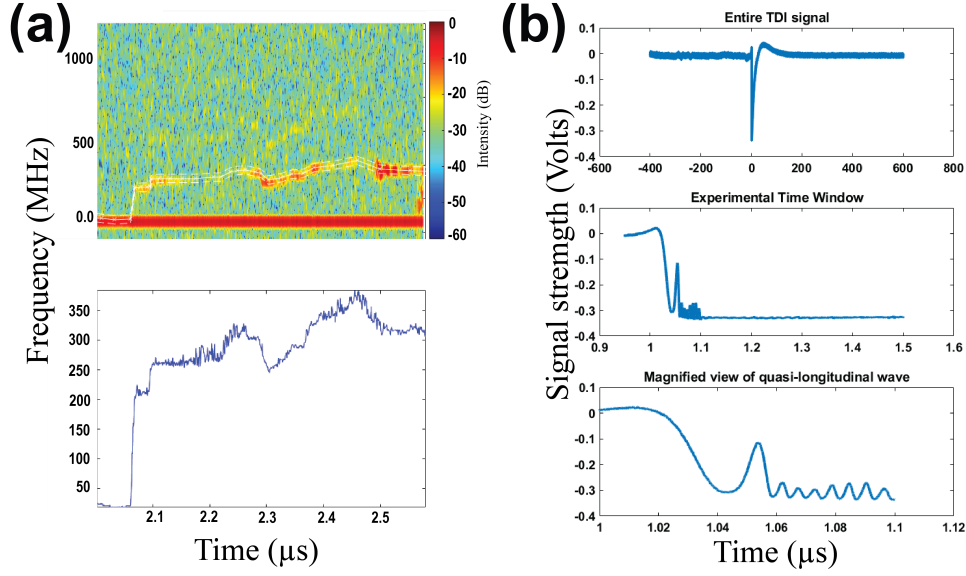


Figure 3.13: (a) Top: Spectrogram of normal free surface PDV signal. Bottom: Extracted frequency profile of the PDV signal. (b) Top: The entirety of the TDI signal as recorded by the oscilloscope. Middle: The time region of interest, based on the $t - X$ diagram. Bottom: A magnified region where the quasi-longitudinal wave induces transverse motion.

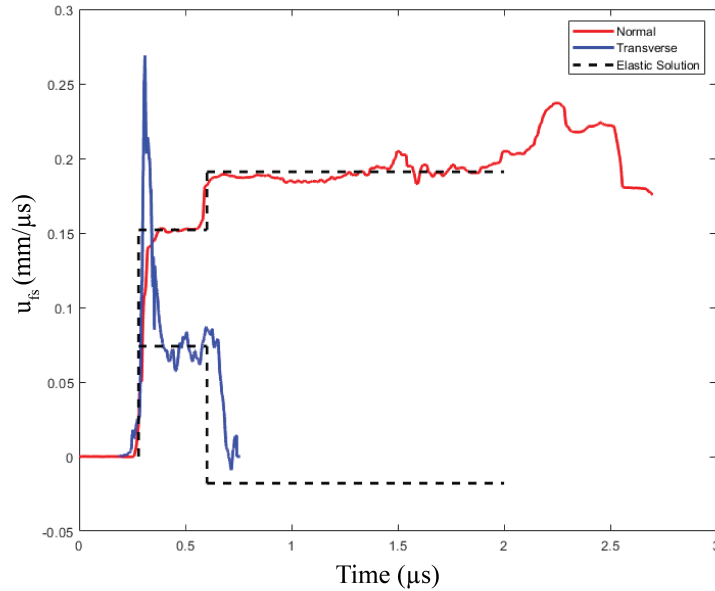


Figure 3.14: Measured normal (red) and transverse (blue) free-surface velocities, along with the elastic solution (black dashed line). The QL arrives first, followed by the QT wave roughly 300 ns later.

(Malhotra et al., 2021). Furthermore, turbulent motion on the free surface, such as in an RMI experiment, manifests not as a short-lived singular spike, but rather as a collection of smeared frequencies (Jensen et al., 2015).

A third hypothesis is that the parallelism of the plates was poor, leading to spurious motion. These wafers are commonly used in semiconductor applications and therefore have extremely fine flatness. The flatness and parallelism of the wafers were measured and found to be considerably less than $1\mu\text{m}$ and less than 0.1 mRad , respectively. However, it is possible that when the tilt pins were lapped flush with the copper pocket and quartz, an excessive amount of material was removed from the sample, rendering the parallelism poor.

A fourth hypothesis is that the probe construction or its alignment was faulty. A key strength of diffraction gratings in transverse velocimetry is that they ensure fine alignment between the probe and the rear surface of the plate. One explanation for the signal anomaly is that the construction of the probes allowed for an improper alignment to occur, resulting in a spurious signal. This explanation is at least partly unsatisfactory, as it again cannot properly explain why the anomaly is short-lived. The probe choice, however, can explain why the signal strength itself diminishes rapidly. (Kettenbeil et al., 2018b) makes the recommendation of a collimating probe for normal motion, as Zemax simulations indicated collimating probes would be less sensitive to tilt. (Zuanetti et al., 2017), however, made use of a fiber-optic focuser (PM Optics, $70\text{ }\mu\text{m}$ beam size, 9 mm working distance). Focusing probes (OZ Optics: LPF-01-1550-9/125-S-9-12-1.1AS-50-3A-3-3-SP) were implemented in the second shot on quartz, and care was taken in lapping the tilt pins flush with the sample.

The raw velocimetry data traces for the second quartz are shown in Fig. 3.15 and the velocity profiles are shown in Fig. 3.16. This shot was markedly more successful, and both normal and transverse data traces both have far better SNR. The transverse velocity is observable for several microseconds, until the arrival of the boundary waves, around $2.5\text{ }\mu\text{s}$

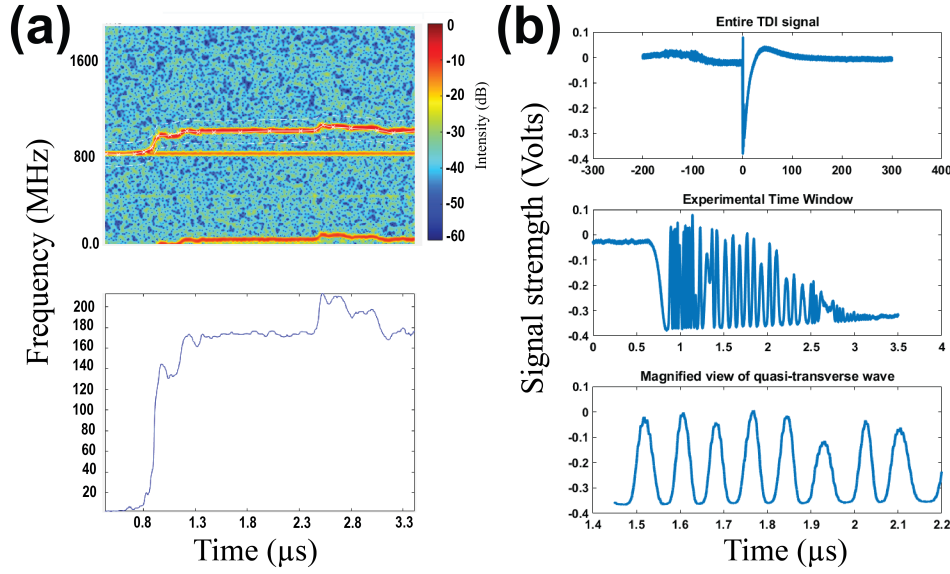


Figure 3.15: (a) Top: Spectrogram of normal free surface PDV signal. Bottom: Extracted frequency profile of the PDV signal. (b) Top: The entirety of the TDI signal as recorded by the oscilloscope. Middle: The time region of interest. Bottom: A magnified region where the quasi-transverse wave induces transverse motion.

after impact. Still, the profiles indicate suboptimal impact conditions. First, the rise times in both the normal and transverse profiles are not as sharp as in SNTF032025. There are also small velocity bumps present where constant stress levels should be present. These artifacts together indicate there was likely an air shock prior to impact. The vacuum level in SNTF072825 was higher than in SNTF032025, but this had not been an issue in other, previous normal impact experiments for similar vacuum levels. Instead, there most likely was gas blow-by that created the air cushion.

Overall, the experiments on quartz were successful in demonstrating various components of the plate impact facility (impact and rear surface velocity measurement, tilt, and vacuum), notwithstanding air cushion effects.

3.4.2 Oblique plate impact of tungsten carbide

Before conducting PSPI experiments, a calibration shot was conducted on a WC anvil. The purpose of this shot is to ascertain the HEL and to provide a basis for calibrating

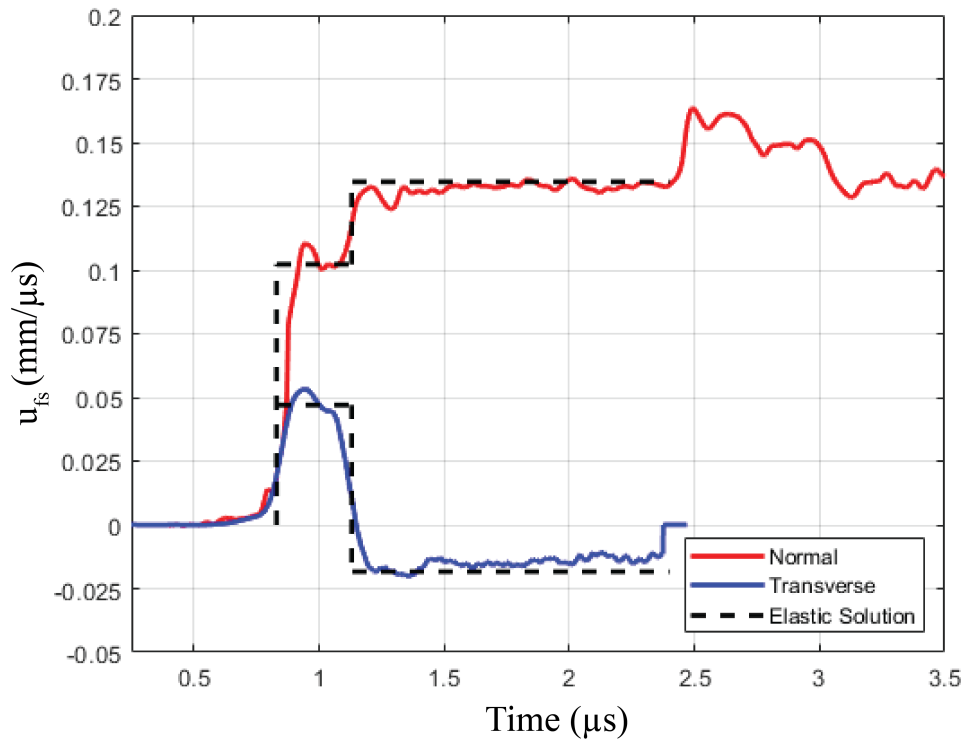


Figure 3.16: Measured normal (red) and transverse (blue) free-surface velocities, along with the elastic solution (black dashed line). The QL arrives first, followed by the QT wave roughly 300 ns later.

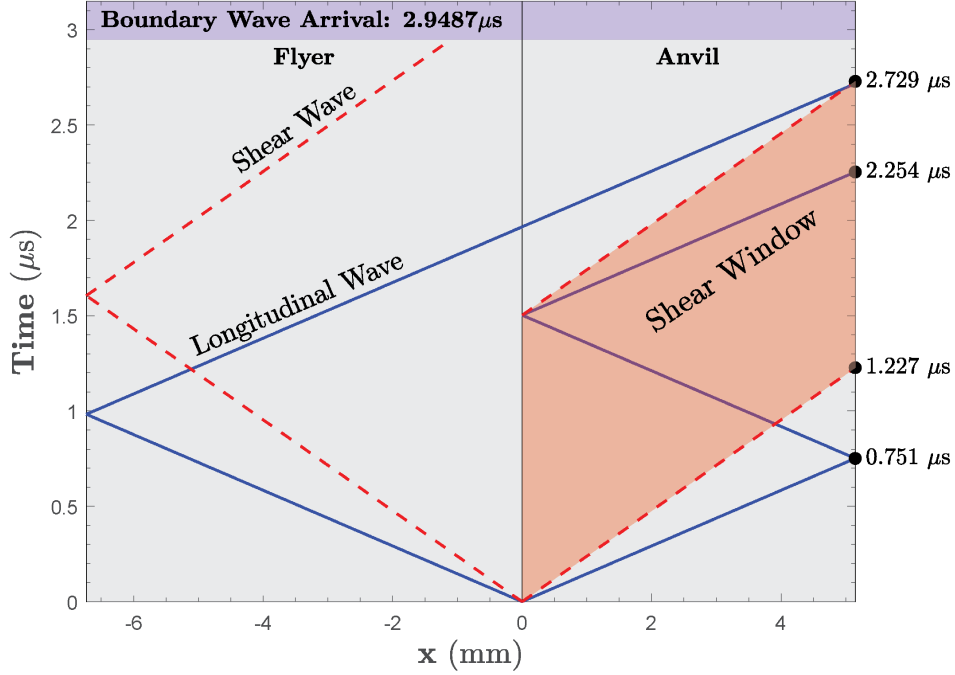


Figure 3.17: $x - t$ diagram of shot SNTTP090925.

future anvil plasticity models. In (Kettenbeil et al., 2020), a hybrid numerical-experimental approach was utilized to extend the range of PSPI experiments. The reason for this is that the PSPI equations in Chapter 2 are predicated on a linear elastic wave analysis. However, above the HEL of anvil materials, the velocity profiles at the rear surface of a PSPI target sandwich will not only reflect the propagation of plastic waves in the specimen, but also in the anvil materials. Close to the HEL (i.e., within a few GPa of normal stress), the effects are small and an elastic analysis can still be used (Kettenbeil et al., 2022). We wish to ascertain whether PSPI experiments on PMMA in the neighborhood of 10 GPa would require a hybrid or inverse analysis that incorporates plastic deformation of the anvils.

An $x - t$ diagram is shown in Fig 3.17. Shot parameters are given in Table 3.4. Longitudinal waves and shear waves transmit simultaneously into the flyer and target. Since the longitudinal wave speed C_L is higher than the shear wave speed C_S , the longitudinal wave arrives first at the rear surface of the target at $0.751 \mu\text{s}$, and the shear wave arrives at $1.227 \mu\text{s}$. At $2.95 \mu\text{s}$ the boundary waves arrive, ending the experiment.

The measured velocity profiles are given in Fig. 3.18. The transverse velocity nearly

Shot no.	V_0 (m/s)	Angle (deg.)	Tilt (mRad)	Vacuum (mTorr)	Flyer thickness (mm)	Target thickness (mm)
SNTP090925	188.3	9	1.5	168	6.73	5.14

Table 3.4: Shot matrix of the plate impact experiments on WC.

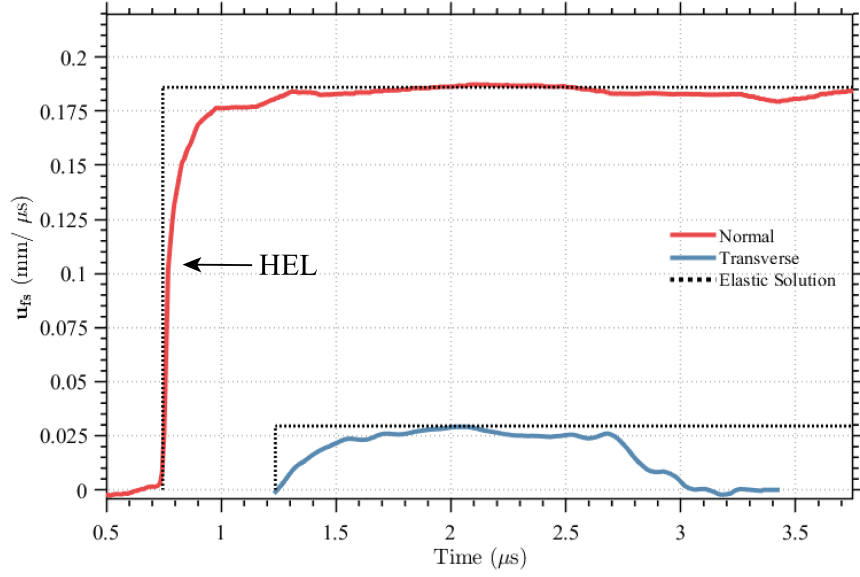


Figure 3.18: Experimental normal and transverse velocity profiles, as well as the elastic prediction based on the impact velocity.

maintains a stress plateau slightly below the elastic prediction. Since the normal stress is well above the HEL (4.9GPa), some analysis of plastic deformation in the anvils is needed. A computational framework was therefore developed to capture plasticity in the anvils, and will later be extended to sandwich target configurations.

3.5 Simulating plate impact

Just as the construction of the plate impact facility required test cases to verify the operation of each experimental subsystem, it is now appropriate to validate the computational capabilities regarding plate impact. In this section, the general computational framework for plate impact is briefly outlined and applied to the plate impact experiments on quartz and WC. All simulations in this work are carried out in the commercial finite element software ABAQUS/Explicit. The concepts here will be used and expanded upon in Chapter 5 (Numerical Modeling of PSPI) and Chapter 7 (Numerical Modeling of Pore Collapse).

3.5.1 Finite element setup

In a plate impact experiment, inertial confinement considerably simplifies the governing equations used in the analysis. So too in our simulations do we desire a finite element scheme that reduces computational complexity. To this end, the material response is assumed to vary only in the impact direction, which is a safe assumption before the arrival of release waves from the boundary. The simulation considers a small region, whose material response is representative of the entire problem geometry, as depicted in Fig. 3.19 (a). Since the material is homogeneous, a line of material may be identified in the longitudinal direction (the wave propagation direction). The other two orthogonal directions may be tessellated infinitely. Since material in these directions must fit together without gaps or overlaps, transverse displacements at these points must be the same, and this is enforced through the equation constraint. The net effect is that the finite element geometry can then be simplified as a line of C3D8R elements (8-node, linear, reduced-integration, 3D solid brick) which have periodic boundary conditions (PBCs) applied orthogonal to the impact direction, as shown in Fig. 3.19 (b). In ABAQUS, PBCs are applied by modifying the input file with the proper kinematic constraints.

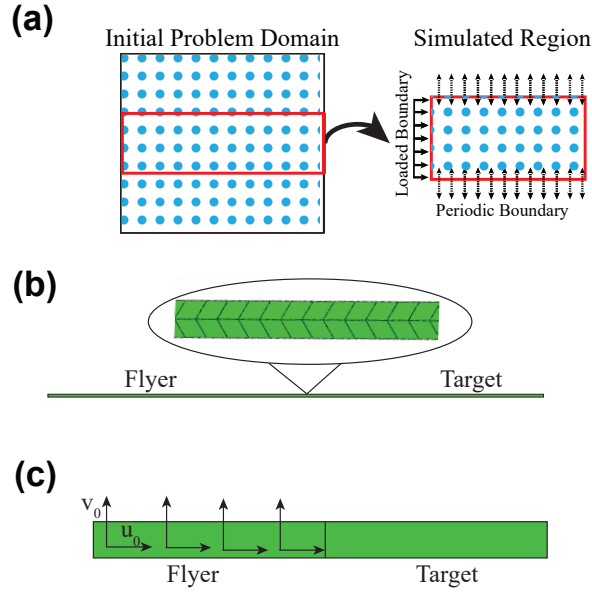


Figure 3.19: Finite element construction for plate impact problems (a) The simulation domain being generated from a larger initial problem geometry (b) Finite element mesh with 3D brick elements used in plate impact problems. (c) Schematic of the material partition and initial conditions.

A good introductory description may be found in (W. Wu et al., n.d.). ABAQUS has multiple options for constraints, such as equations, kinematic coupling, and multi-point constraints, among others (Abaqus User Manual Section 35). In the following simulations, the multi-point constraints take the form of a linear homogeneous equation

$$A_1 u_1 + A_2 u_2 + \dots + A_N u_n = 0 \quad (3.24)$$

where A_N is a constant coefficient describing the motion of nodes and u is the degree of freedom of a node. As an example, consider that the displacement at the “top” (node 1) and “bottom” (node 2) of the mesh must be the same: $u_{2,y} = u_{1,y}$. This would be implemented in the input file as

```
*equation ← type of constraint
2 ← number of terms in equation
2, 2, 1.0, 1, 2, -1. 0 ← 1.0 $u_{2,y}$  - 1.0 $u_{1,y}$  = 0
```

Setting up PBCs is the trickiest part of the simulation; the setup is straightforward. The impact velocity is specified as a velocity initial condition. In the case of normal impact, the impact velocity is the only component of this initial condition. In the case of OPI, the initial velocity is resolved into the normal and transverse components of the impact velocity (i.e. Eq. 2.19). The geometry is a line of brick elements representing the entire geometry. This geometry will then be partitioned into regions where material sections are assigned, as shown in Fig. 3.19 (c). For example, a 5 mm flyer impacting a 5 mm target will require an initial geometry of a continuous 10 mm rectangular prism. partitioned into two 5 mm sections. The element size must be small compared to the overall length, typically $5\text{-}10\mu\text{m}$ to ensure sharp stress wave profiles. Default values of linear ($b_1 = 0.06$) and quadratic viscosity ($b_2 = 1.2$) are also used to ensure smooth wave profiles are generated. There are thermodynamic consequences of artificial viscosity, which are discussed in Chapter 4, but for now, we consider only isothermal mechanical deformation, and we need not concern ourselves with this here. Finally, the element size places a constraint on the time step, since these explicit simulations are subject to the Courant-Levy-Friedrichs (CFL) condition that the stress waves must not transit an element's width in a single time step. Time steps are thus limited to $\Delta t \leq \Delta x/c$, where Δx is the element width and c is the fastest wave speed; c would be C_L for elastic problems or U_s for shock problems. This completes a basic description of the finite element setup used for most plate impact simulations in this work.

3.5.2 Normal plate impact of quartz

Quartz is modeled as an anisotropic elastic material, where the values used in the simulation are given in Table 3.2 and the properties of the glass flyer are $\rho = 2.2\text{g/cc}$, $E = 64\text{GPa}$, $\nu=0.2$. One approach for modeling the impact of Y-cut quartz is to maintain the same global orientation (“Orientation” in the Section options) and to rotate $\mathbb{C}_{ijkl}$ to

obtain the desired crystal orientation. Note that

$$\mathbb{C}_{ijkl}^{Y-cut} = Q_{im}Q_{jn}Q_{ko}Q_{lp}\mathbb{C}_{mnop} \quad (3.25)$$

where Q_{ij} is the rotation tensor. To orient the quartz crystal such that the crystal Y axis aligns with the global X axis and the crystal Z axis aligns with the global Y axis, the rotation is

$$[Q_{ij}] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (3.26)$$

For more details on simulating anisotropic linear elastic materials, refer to the Abaqus User Manual Section 22.2.1 .

Simulated velocity profiles overlaid onto the aforementioned experimental profiles are presented in Fig. 3.20. The element size in this simulation was $5 \mu m$. The simulation is able to capture the propagation of both the QL and QT waves in normal and transverse directions, and the magnitudes match the experimental results. With the basic simulation and experimental plate impact technique validated, we progress to simulations of the oblique impact experiments on tungsten carbide.

3.5.3 Oblique plate impact of tungsten carbide

As previously mentioned, in (Kettenbeil et al., 2020) a method was put forth to extract a PSPI sample's flow stress above the anvil's HEL by means of first characterizing the anvil and subsequently calibrating a material model for the sample. This calibration ensures the normal and transverse velocity profiles are reproduced before meaningful statements about the state of stress are made. The authors used a material model with two primary ingredients, separated in terms of the dilatational and deviatoric responses. This approach was repeated in subsequent works by other researchers: (Ravindran et al., 2021), (Gandhi et al., 2022). First, the common $U_s - u_p$ form of the Gruneisen equation

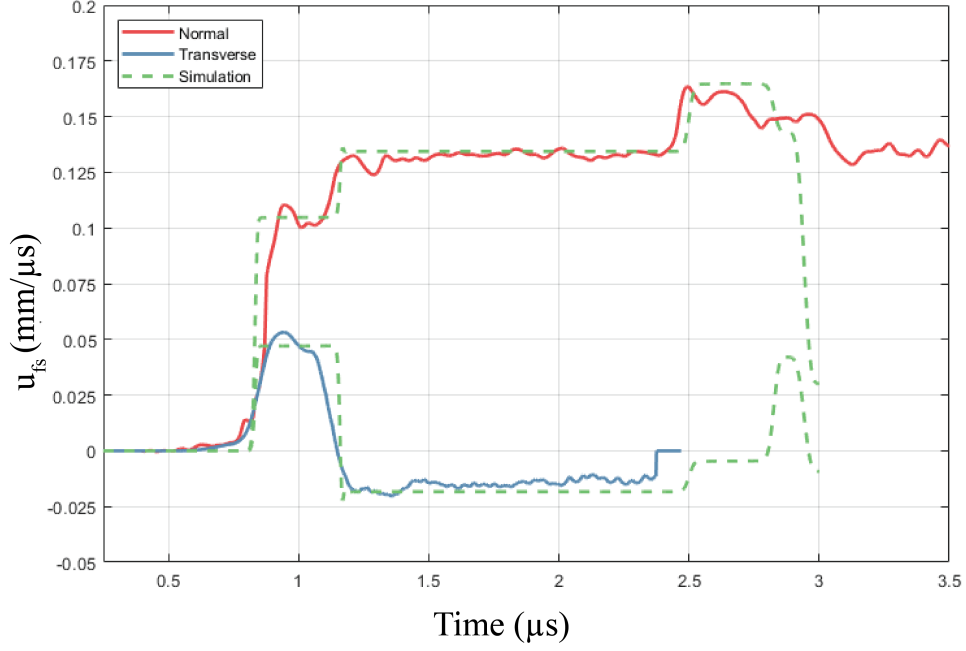


Figure 3.20: Experimental and simulated normal and transverse velocity profiles for shot SNTTP072825.

of state gives the pressure as:

$$P = \frac{\rho_0 C_0 [1 + (1 - \gamma_0/2) \mu]}{(1 - (s - 1)\mu)^2} \quad (3.27)$$

where

- C_0 and s are the linear coefficients of the ubiquitous $U_s - u_p$ fit
- μ is the relative change in density $\rho/\rho_0 - 1$
- γ_0 is the ambient Gruneisen parameter.

The values for these properties are given in Table 3.5.

ρ_0 (g/cc)	G (GPa)	C_0 (mm/ μ s)	s	γ_0
2.648	86.74	107.2	6.99	11.91

Table 3.5: EOS parameters for symmetric plate impact simulations on WC.

Along with the EOS, a shear modulus G is used to fully capture the (non-linear) elastic response of the material. The inelastic response is captured by a strain hardening power

law

$$Y(\bar{\varepsilon}^p) = A + B (\bar{\varepsilon}^p)^n \quad (3.28)$$

where $\bar{\varepsilon}^p$ is the equivalent plastic strain, A represents the initial yield strength of the material, B is the strain-hardening coefficient, and n is the strain-hardening exponent. Rate-independence is introduced via a Cowper-Symonds law of the form:

$$Y(\dot{\bar{\varepsilon}}^{pl}) = 1 + \left(\frac{\dot{\bar{\varepsilon}}^{pl}}{C\dot{\varepsilon}_0} \right)^{1/P} \quad (3.29)$$

where $\dot{\bar{\varepsilon}}^p$ is the effective plastic strain rate, $\dot{\varepsilon}_0$ is the reference strain rate, C is a coefficient, and $1/P$ is the strain-rate hardening power. The entire strength model is then

$$Y(\bar{\varepsilon}^{pl}, \dot{\bar{\varepsilon}}^{pl}) = (A + B (\bar{\varepsilon}^{pl})^n) \left(1 + \left(\frac{\dot{\bar{\varepsilon}}^{pl}}{C\dot{\varepsilon}_0} \right)^{1/P} \right) \quad (3.30)$$

This form is similar to the Johnson-Cook model with rate-dependence available in Abaqus, which takes the form

$$Y(\bar{\varepsilon}^{pl}, \dot{\bar{\varepsilon}}^{pl}) = (A + B (\bar{\varepsilon}^{pl})^n) \left(1 + C \ln \left[\frac{\dot{\bar{\varepsilon}}^{pl}}{\dot{\varepsilon}_0} \right] \right) (1 - \hat{\theta}^m) \quad (3.31)$$

where the homologous temperature $\hat{T}^m$ is given in terms of the current temperature T , the reference temperature T_{ref} , and the melt temperature T_m by

$$\hat{T}^m = \frac{T - T_{ref}}{T_m - T_{ref}} \quad (3.32)$$

. If no value is specified for m , temperature dependence is ignored. This is taken to be the case in the present simulations, for the melt temperature of WC is in excess of 2000 K, and thermal softening effects for the weak shocks at hand are therefore minimal. The strength parameters, which are based on the parameters provided in (Ravindran et al., 2021), are given in Table 3.6. The results of the simulation, overlaid onto the previously discussed experimental velocity profiles, are shown in in Fig. 3.21. The normal profile

A (GPa)	B (GPa)	n	C	$\dot{\epsilon}_0$ (s ⁻¹)
3.0	11	0.25	0.044	1

Table 3.6: Johnson-Cook model parameters for plate impact simulations on WC.

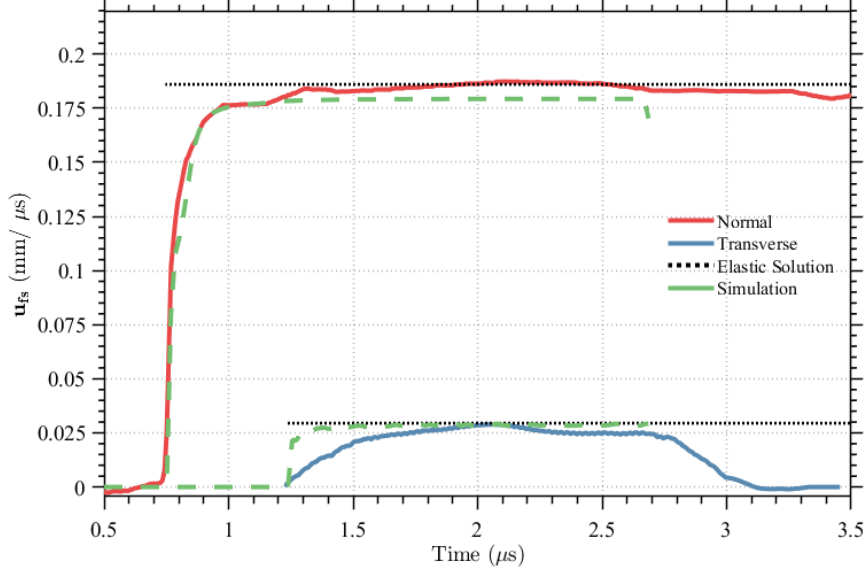


Figure 3.21: Experimental and simulated (Johnson-Cook) normal and transverse velocity profiles, as well as the elastic prediction based on the impact velocity.

matches quite well, and the plateau and HEL (~ 110 m/s) are reproduced. However, the rise time in the transverse velocity is a poor match. Moreover, the increase in the experimental normal velocity above the simulated plateau should be noted at this time. In (Zuanetti et al., 2019), OPI experiments were conducted on pure WC at varying angles of obliquity. Experiments for small angles resulted in wave profiles that increased to and were maintained at the elastic plateau. For higher levels of obliquity (i.e., 22°), the normal velocities saw anomalous increases well before the arrival of any release waves. The authors remarked that such increases were reminiscent of similar features in works on K8 glass by (Bless et al., 1992), in sodalime glass by (Dandekar & Beaulieu, 1995), and by (Dandekar, 2004) in SiC-B. In (Kettenbeil et al., 2022), another model was introduced to incorporate aspects of damage in WC, the Johnson-Holmquist (JH2) model, which will now be implemented for the problem at hand.

The JH2 model was introduced by (G. R. Johnson & Holmquist, 1994) in an effort to model brittle materials subjected to high strains, large pressures, and high strain-rates. The material strength incorporates two models: one for the intact material and one for the failed material. Both are described by a power-law pressure hardening function and a strain-hardening function. Much of what is presented below follows the ABAQUS Example Problems Guide “2.1.18 High-velocity impact of a ceramic target,” which makes use of a Johnson-Holmquist-Beissel, JH2, and Drucker-Prager plasticity model for gold impacting a silicon carbide (ceramic) target. The JH2 material model is implemented in ABAQUS by means of a built-in user subroutine. The built-in JH2 subroutine may be invoked by adding a suffix of “_jh2” to material names in the input file.

The strength in the JH2 model is expressed in terms of the normalized von Mises stress as

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*) \quad (3.33)$$

where σ_i^* is the normalized intact equivalent stress, σ_f^* is the normalized fractured equivalent stress, and D is the damage variable. $0 \leq D \leq 1$, where $D = 1$ represents fully damaged material. The normalized equivalent stresses have the general form $\sigma^* = \bar{\sigma}/\sigma_{HEL}$ where $\bar{\sigma}$ is the von Mises equivalent stress. The normalized intact and fractured stresses are written as

$$\sigma_i^* = A(P^* + T^*)^N(1 + C \ln \dot{\epsilon}^*) \leq \sigma_i^{max} \quad (3.34)$$

$$\sigma_f^* = B(P^*)^M(1 + C \ln \dot{\epsilon}^*) \leq \sigma_f^{max} \quad (3.35)$$

$$(3.36)$$

$A, B, C, M,$ and N are material parameters and σ_i^{max} and σ_f^{max} are the limits for the strength. The normalized pressure is $P^* = P/P_{HEL}$ (the pressure at the HEL). $T^* = T/P_{HEL}$ is the normalized maximum tensile hydrostatic pressure, which is the maximum tensile pressure that the material can tolerate. $\dot{\epsilon}^* = \dot{\epsilon}^{pl}/\dot{\epsilon}_0$ is the dimensionless strain rate.

Finally, the damage initiation parameter ω accumulates with plastic strain according to

$$\omega = \sum \frac{\Delta \bar{\varepsilon}^{pl}}{\bar{\varepsilon}_f^{pl}(P)} \quad (3.37)$$

where $\Delta \bar{\varepsilon}^{pl}$ is the increment in plastic strain and $\bar{\varepsilon}_f^{pl}(P)$ is equivalent plastic strain to fracture defined as

$$\bar{\varepsilon}_f^{pl} = D_1(P^* + T^*)^{D_2} \quad (3.38)$$

$$\bar{\varepsilon}_{f,min}^{pl} \leq \bar{\varepsilon}^{pl} \leq \bar{\varepsilon}_{f,max}^{pl} \quad (3.39)$$

where D_1 and D_2 are material parameters. The damage variable D increases with plastic deformation as $D = \omega$.

The dilatational part has two forms, for compression and tension:

$$P = K_1\mu + K_2\mu^2 + K_3\mu^3 \quad \text{if } \mu \geq 0 \text{ (compression)} \quad (3.40)$$

$$P = K_1\mu \quad \text{if } \mu \leq 0 \text{ (tension)} \quad (3.41)$$

$$(3.42)$$

where μ is again the compression factor $\mu = \rho/\rho_0 - 1$. Though this is the general form, in the current parameterization $K_2 = K_3 = 0.0$ and thus the dilatational behavior is the same in compression and tension. An additional pressure increment ΔP is included to capture additional dilatational effects, or bulking, when brittle materials fail

$$P = K_1\mu + K_2\mu^2 + K_3\mu^3 + \Delta P \quad (3.43)$$

$$\Delta P_{t+\Delta t} = -K_1\mu_{t+\Delta t} + \sqrt{(K_1\mu_{t+\Delta t} + \Delta P_t)^2 + 2\beta K_1\Delta U} \quad (3.44)$$

where ΔU is the elastic energy increment and β is the elastic energy loss converted to potential hydrostatic energy, with $0 \leq \beta \leq 1$.

As noted by (Kettenbeil et al., 2022), the implementation here focuses on the use of pressure-dependent strength modeling rather than the use of damage. As such, a large value for σ_i^{max} is used so as to by-pass the evolution of the damage function with strain. This now completes the description of the JH2 model. Material parameters, which are unchanged from (Kettenbeil et al., 2022), are given in Table 3.7.

ρ_0 (g/cc)	G (GPa)	A (GPa)	N	B (GPa)	M	C	$\dot{\epsilon}_0$ (s ⁻¹)
15,600	273	0.8	0.5	0.8	0.5	0.03	1
T^*	σ_i^{max}	σ_f^{max}	$\sigma_{HEL}(GPa)$	$P_{HEL}(GPa)$	β	D_1	D_2
0.4	1e+20	0.1	6.0	3.0	0.0	10.0	0.0
ϵ_f^{max}	ϵ_f^{min}	FS	lDamage	K_1 (GPa)	K_2 (GPa)	$K_3(GPa)$	
1.0	0.0	0.0	0.0	379	0.0	0.0	

Table 3.7: WC material parameters for the JH2 model.

The resultant simulation for the JH2 model is overlaid onto the previous experimental results in Fig. 3.22. The simulated transverse velocity now matches the experimental profile remarkably well. The HEL behavior of the normal profile is similar, although the bulking in the model is not able to reproduce the rise in normal velocity. It may be that the model requires further calibration near the HEL. The microstructure of the plates was not analyzed prior to the experiment, but this question should be revisited for experiments well above the HEL. The long rise time is reproduced due to the reduction in shear wave speeds, which are a consequence of imposing inelasticity in the JH2 model. Lastly, a few comments about the state of stress are warranted. For the shot SNTTP090925, the predicted experimental stress plateau is close to the elastic limit for both the normal and shear stress profiles. But even below the HEL, inelasticity and material bulking have been observed (Zuanetti et al., 2019). Moreover, (Kettenbeil et al., 2022) simulated a variety of WC OPI experiments and found excellent matches between the model and these experiments over a variety of impact scenarios, i.e., from 6 GPa (near the HEL) up to nearly 40 GPa and for varying angles of obliquity. In our case, clearly, the long rise time is not an elastic response of the material, which suggests the linear elastic PSPI equations

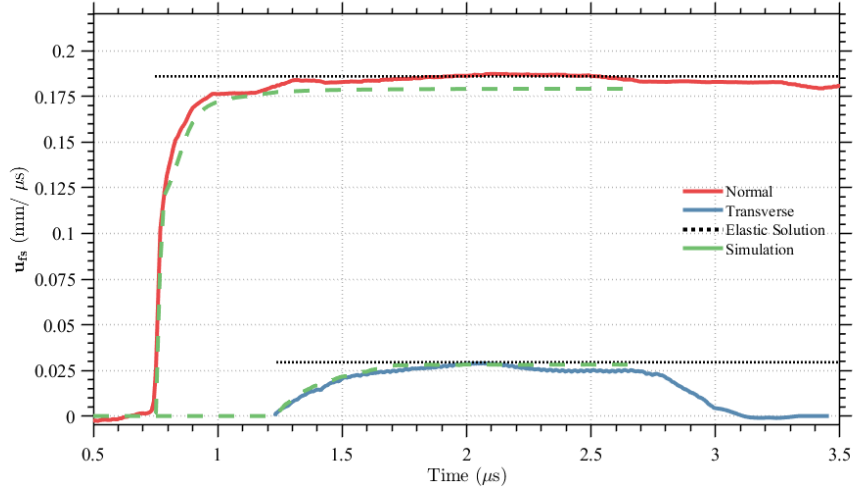


Figure 3.22: Experimental and simulated (JH2) normal and transverse velocity profiles, as well as the elastic prediction based on the impact velocity.

are not applicable in interpreting the initial response. However, as (Gandhi et al., 2022) remarked, oftentimes the primary measurement of interest in these experiments is simply the shear stress plateau, often reported as a single value. The difference between the elastic and simulated plastic normal and transverse stress is less than 3%, so in this instance, an assumption of linear elasticity is not inappropriate. In short, the necessity of either an inverse wave analysis or a hybrid numerical-experimental scheme to interpret PSPI results will depend on the objectives of the experiment and the impact conditions. With these closing comments, the analysis of the anvil material is complete, and we will now advance to the discussion of PSPI experiments on PMMA.

Pressure Shear Plate Impact Experiments on PMMA

4.1 Overview

PSPI experiments were conducted on thin films of PMMA at four impact velocities, generating normal stress of up to nearly 10 GPa. Dynamic failure is believed to have occurred in the samples, as there exists a competition between pressure confinement, thermal softening, and damage accumulation in the material, although it is difficult to ascertain when failure occurs. Dynamic shear strengths are shown to match existing data taken with manganin stress gauges, and failure is discussed in the context of other experiments with different loading configurations.

4.2 Target fabrication

As mentioned in Chapter 2, there has been a good deal of discussion on the importance of polymer pedigree. Since PolyCast UVA Type II has emerged as the top candidate to replace the legacy Rohm& Haas material, sheets of PolyCast II were purchased from Spartech. Material properties for PolyCast PMMA are given in Table 4.1¹.

¹Elastic parameters for D2 are from (Malhotra et al., 2022). Parameters for WC are from (Kettenbeil et al., 2022). Moduli for PMMA are taken from but not explicitly measured in (Lacina et al., 2018); instead, parameters are generated here by means of standard elastic formulae from the measured wave speeds C_L and C_S . The HEL of PMMA is from (Rosenberg & Partom, 1994)

Material	WC	D2	PMMA
Density (g/cc)	15.600	7.787	1.188 (Lacina et al., 2018)
Young's Modulus (GPa)	650	210	6.08
Poisson's ratio	0.2	0.29	0.325
Longitudinal wave speed ($mm/\mu s$)	6.848	5.945	2.73
Shear wave speed ($mm/\mu s$)	4.193	3.233	1.38
HEL (GPa)	6.1	2.6	0.7

Table 4.1: Elastic properties of anvil materials used in PMMA PSPI shots.

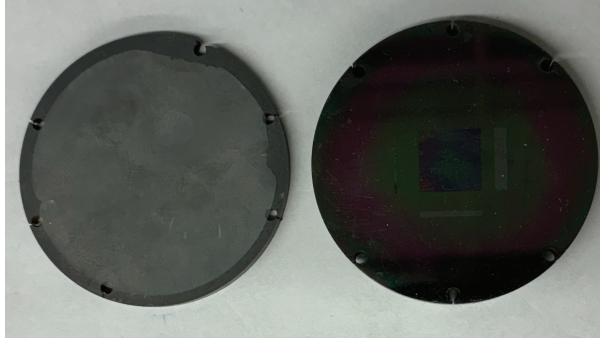


Figure 4.1: A $100\mu\text{m}$ PMMA sample on a WC front anvil (left) next to a WC rear anvil with diffraction grating (right) prior to final assembly.

For sandwich targets in PSPI experiments, normal stress waves reverberate between the two anvil plates, resulting in a constant normal stress in the sample prior to the arrival of the shear wave. This reverberation time places a limit on the maximum sample thickness. A target thickness of $100\ \mu\text{m}$ was chosen to meet this constraint. The minimum sheet thickness provided by Spartech is 1.6 mm, which must be lapped down to this thickness. Sections of these sheets were laser cut into large disks, roughly 100 mm in diameter. The coarse material removal portion occurred on the standard lapping machine with details provided in Chapter 3. Because a significant amount of material was removed, the process typically took 6-8 hours in total per disk. The disks were locally flat, but in order to ensure the strict requirements of global parallelism, samples were then hand-lapped. 40 mm disks—the size of a target—were cut from the already lapped 100 disks. Hand-lapping occurred with silicon carbide lapping films on a granite block in a manner similar to that of (Malhotra et al., 2021), except that, rather than a vacuum chuck, a crystal wafer mounting adhesive (CrystalBond 555) was used to affix samples to the flat lapping chuck. This process again lasted for several hours, before resulting in sample films ranging from 50-300 μm . The final samples were globally parallel to within 0.1 mRad. A picture of the sample and anvil plates prior to final assembly is shown in Fig. 4.1.

Two anvil materials were used: WC BC-3, which has already been discussed at length, and D2 steel—a high-carbon tool steel. The D2 plates were hardened in accordance with the experiments conducted by Malhotra (Malhotra et al., 2021), which resulted in a

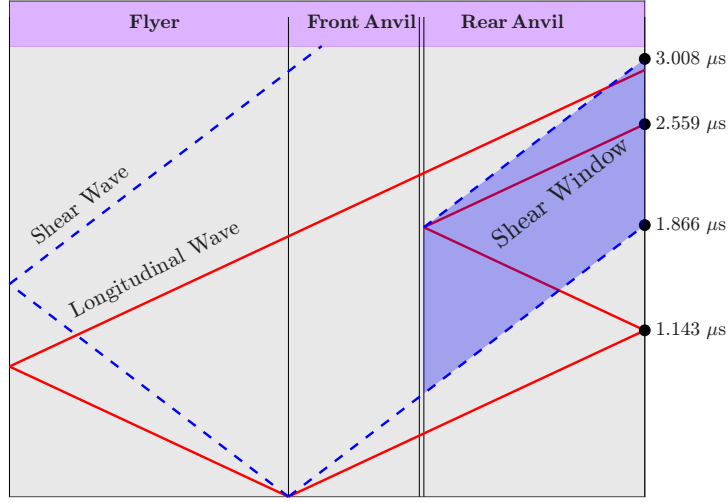


Figure 4.2: an $x - t$ diagram of PSPI experiment TP111125.

hardness of HRC 62. The material properties for both anvils are given in Table 4.1. An example $x - t$ diagram for the shot TP112525 is shown in Fig. 4.2. Due to the brittle nature of PMMA at high strain-rates, it was hypothesized that shear measurements would not be possible after the arrival of the unloading wave from the rear surface of the sample due to the accumulation of damage. This issue had been seen before in experiments on HTPB (Malhotra et al., 2021), where no shear measurements were reported after this time. Thus, an effort was made to maximize the time window prior to the arrival of the rear surface unloading wave by selecting the appropriate anvil thicknesses, and this is reflected in the $x - t$ diagram in Fig. 4.2. The remainder of the assembly process has largely been outlined in Chapter 2. Samples were lightly clamped between prepared anvils (i.e., both lapped and a diffraction grating on the rear anvil). A two-part epoxy (HARDMAN DOUBLE/BUBBLE©) was applied to the periphery of the sandwich before letting it cure for several days. The sandwich was then placed in the target holder, and the ± 1 st order diffraction direction was aligned with the horizontal axis of the target, meaning the direction of shear.

Shot no.	Target material	h_s (μm)	h_F (mm)	h_{FA} (mm)	h_{RA} (mm)	V_0 (m/s)	Tilt (mRad)	σ_{max} (GPa)	τ_{max} (GPa)	$\dot{\gamma}$ (μs^{-1})
TP110425	WC	98	5.867	2.845	5.910	101.9	0.98	5.12	0.861	0.049
TP111125	D2	92	7.560	2.923	5.576	113.1	0.62	2.19	0.309	0.11
TP111825	WC	96	6.021	3.885	4.977	194.5	1.46	9.83	n/a	n/a
TP112525	WC	67	6.126	2.874	4.851	201.5	n/a	9.72	0.404	0.75

Table 4.2: Shot matrix of the PSPI experiments on PMMA. All shots were conducted at angle of obliquity of 18° . h_s : sample thickness, h_F : flyer thickness, h_{FA} : front target plate thickness, h_{RA} : rear target plate thickness, V_0 : impact velocity, σ_{max} : peak normal stress, τ_{max} : peak shear stress, $\dot{\gamma}$: shear strain-rate at max shear stress

4.3 Experimental results

A shot matrix summarizing the experimental conditions of the four PSPI shots on PMMA is depicted in Table 4.2. All shots were successful, except for TP111825, where the transverse velocity was not measured. Prior to the shot, the PDV photodetector was swapped with the TDI photodetector, as the former seemed to have a better sensitivity. However, after the experiment, it was observed that this detector was unable to generate any TDI fringes on a tabletop test. With the detector configuration changed to the original setup, TDI was again produced normally in shot TP112525. Additionally, tilt was not successfully measured in shot TP112525.

With an assumption of linear elastic anvil behavior, normal and shear stress plots are given in Fig. 4.3 and Fig. 4.4, respectively. The wave profiles are indicative of complex behavior in the samples. First, some normal stress reverberations are observed for lower-pressure shots (especially in TP110425). This phenomenon has previously been used to measure Lagrangian wave speeds in polymers under isentropic loading, such as polyurea (Clifton et al., 2016) and HTPB (Malhotra et al., 2021), but is not feasible in the present case due to the thinness of the samples. The wave profiles are otherwise somewhat unique in the context of PSPI experiments. Whereas the symmetric shot on WC, SNTF090925, demonstrated a constant plateau of normal stress, here the normal stress in the sample tends to fluctuate with the arrival of the shear wave. This effect becomes more significant at higher impact velocities, with shots TP111825 and TP112525

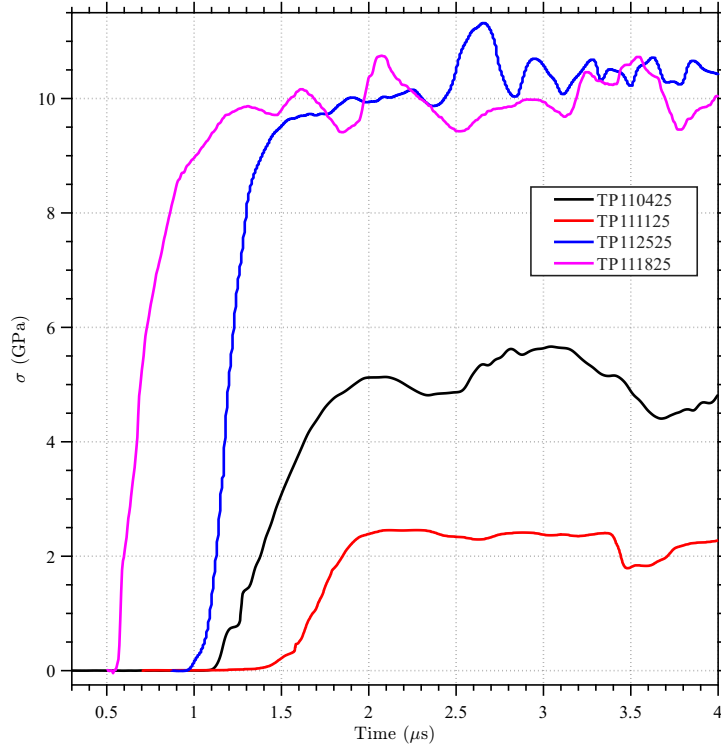


Figure 4.3: Normal stress profiles for PSPI experiments on PMMA, offset in time for clarity.

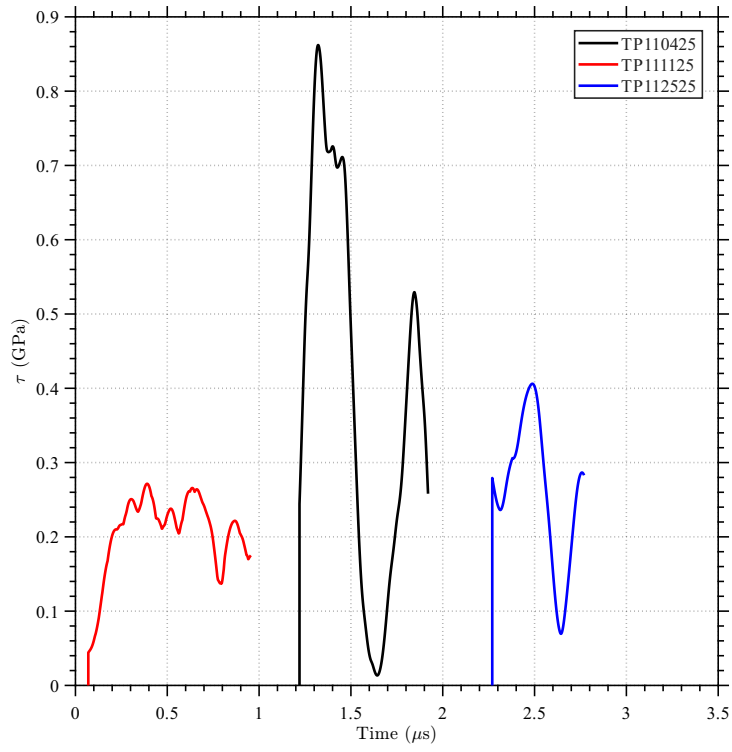


Figure 4.4: Shear stress profiles for PSPI experiments on PMMA, offset in time for clarity.

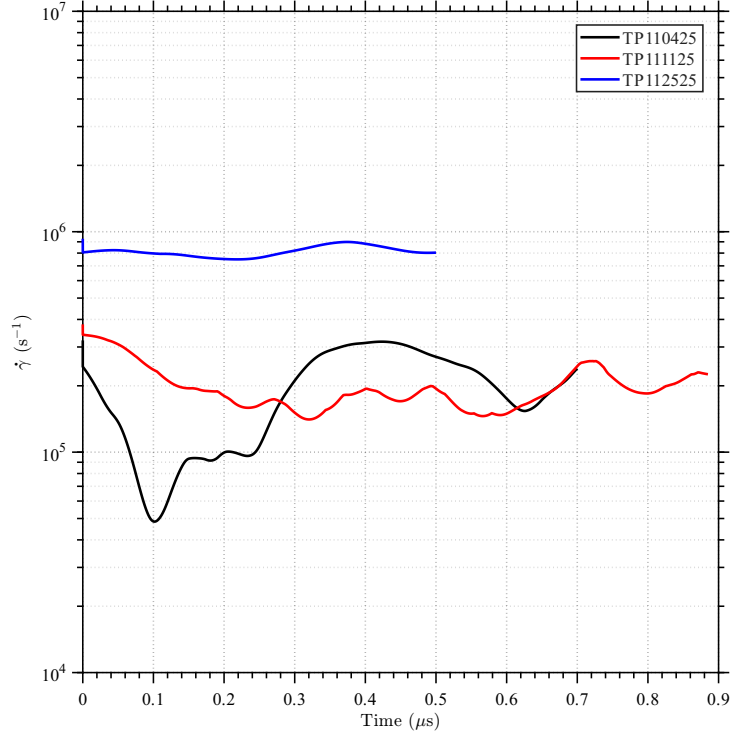


Figure 4.5: Shear strain-rates over time in PSPI experiments on PMMA.

seeing considerable fluctuations until the arrival of the unloading waves. The shear stress profiles also present complex behavior, where only a small amount of shear strain is imposed before the shear stress rapidly decreases. In all of the experiments, there is a stress “rebound” where the shear stress reaches some small value before rebounding to a higher value. These structures largely indicate that failure has occurred, but it is difficult to say when and if the samples have failed. In the case of TP110425, the shear stress does not decrease as much as in the other two shots, but the presence of this “rebound” likely indicates material failure. To obtain stress-strain curves, recall that by Eq. 4.5 we can obtain the shear strain rate, which in turn may be integrated to obtain the total shear strain. The shear strain rates are provided in Fig. 4.5. Numerical integration of these gives way to shear stress-shear strain curves, as shown in Fig. 4.6. The two experiments at lower pressures, TP110425 and TP111125, show markedly low levels of shear strain prior to failure. TP112525 shows a larger amount of strain to failure, but because of the complex wave profile, it is difficult to say when exactly the samples have failed.

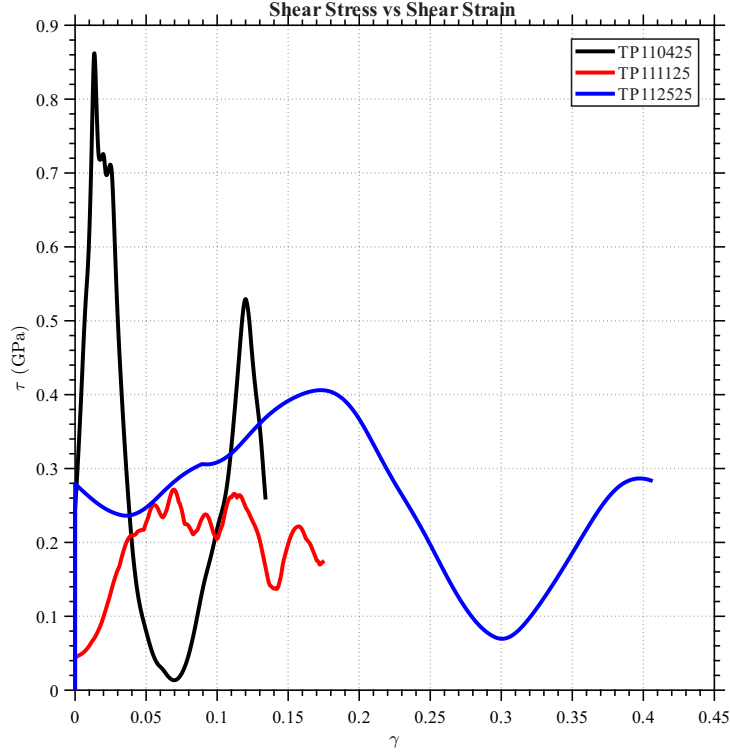


Figure 4.6: Shear stress-strain curves for PSPI experiments on PMMA..

4.4 Discussion

As mentioned in Chapter 2, several experiments have previously been conducted that measured strength in PMMA via manganin stress gauges. The results of these experiments, as well as the maximum shear stress obtained in the current experiments, are shown together in Fig. 4.7. Though the strain-rates are quite different across these experiments, the PSPI results presented here follow the trend of two predominant regions: (1) an extremely pressure-dependent shear strength until roughly 6 GPa, following which (2) a region of strength softening occurs. It has been argued that these regions reflect a runaway effect of thermal softening (Bat'kov et al., 1996), as thermocouple measurements in shock-loaded PMMA revealed temperature rises of up to 200 K for shock pressures around 3 GPa (Bloomquist & Sheffield, 1981). On the other hand, PMMA, like many polymers, is an extremely strain-rate-dependent material. In this case, there is a high amount of strain-rate hardening in the plastic response, which is in direct competition

with thermal softening. Malhotra observed somewhat similar behavior in sucrose, where shear bands were believed to have been present in all PSPI experiments. Shear bands have previously been directly observed in PSPI experiments by Zhou (Zhou, 1993), where scanning electron microscope (SEM) images were taken of soft-recovered samples. The images revealed the high amounts of shear strain in individual crystal grains, as well as shear band thickness. In the case of sucrose and PMMA, the amount of damage that occurs in the samples is simply too great to conduct a meaningful post-mortem analysis of shear banding or any other damage mechanisms. However, the experiments by Malhotra (Malhotra et al., 2022) demonstrated very large shear strains of up to $\gamma = 2.5$, which are prototypical of shear bands. In our case, the low levels of strain to failure do not indicate the presence of thermal softening, ductile flow, and shear banding.

This brittle response is somewhat surprising, given the high degree of confinement, which in other materials might be sufficient to suppress damage. As in Hopkinson bar experiments with mechanically confined samples by Rittel (Rittel & Brill, 2008), the right combination of boundary conditions and sample geometry, brittle and ductile failure modes are certainly possible in PMMA over a variety of strain-rates. As discussed in Chapter 2, the modern consensus is that plasticity below the glass transition temperature in glassy polymers is predominantly mediated by the activation of microscopic shear bands (P. D. Wu & van der Giessen, 1995). The presence and interaction of these shear bands here is not disputed, and likely has a role in governing the observed failure response, but localization via shear banding cannot explain features seen in the wave profiles. In our case, these PSPI experiments seem to confirm that PMMA can only tolerate relatively low amounts of plastic strain before failure under high strain rates.

Future experiments are warranted on PMMA to fully disentangle the complex failure landscape believed to be present in the material. Though the literature is rife with discussion on the competition between thermal softening and strain-rate hardening, as well as on the consequent emergence of localization in the form of shear bands, *in-situ* or

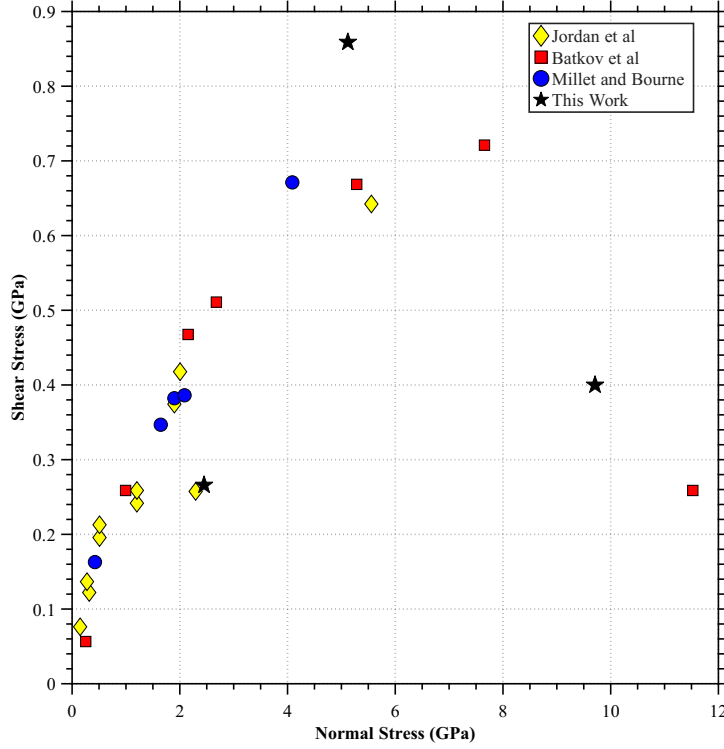


Figure 4.7: Strength measurements on PMMA.

even post-mortem observations of such events are, at the time of writing, scant. Designing experiments that leverage a combination of soft-recovery and high-speed infrared imaging would be invaluable in better understanding the processes governing dynamic failure in PMMA. Experiments on other glassy polymers may also shed light on the problem. For example, in PSPI experiments on polycarbonate (PC), (Prakash & Mehta, 2012) observed shear strains of up to roughly $\gamma = 0.25$ for normal stresses below 1 GPa. Compared to PMMA, PC is known to behave in a more ductile fashion for higher strain rates (Fleck et al., 1990). Conducting experiments on these materials and others may lead to new discoveries on damage mechanisms native to thermoplastics subjected to high-strain-rate loading.

Numerical Modeling of Pressure Shear Plate Impact
Experiments on PMMA

5.1 Overview

This chapter details constitutive modeling of PMMA for high-strain-rate problems and direct numerical simulations of PSPI, which were carried out in the commercial finite element software ABAQUS. Constitutive modeling centers on the comparison between two primary models, which have been appropriately specified for the material at hand, PMMA. The first model employs an equation of state (EOS), which separates the dilatational and distortional parts of the non-linear thermoelastic response, and is adapted from (Malhotra et al., 2022). This model also captures material strength via the Johnson-Cook thermoviscoplastic model. The Johnson-Cook parameters are taken from (Holmquist et al., 2016). The Mie-Grüneisen and Johnson-Cook model will hence be referred to as the CMGJC model. The second constitutive formulation is based on the Glassy Amorphous Polymer (GAP) model by Clements (B. E. Clements, 2012). This thermomechanical model incorporates an equation of state based on a judicious choice of the Helmholtz Free Energy. The model incorporates both bulk viscoelasticity and deviatoric viscoelasticity, as well as a viscoplastic flow rule. While the CMGJC model is calibrated only to high-strain rate behavior, the GAP model is parameterized to a variety of experiments across many decades of strain rates. The end result is that while the Johnson-Cook model tends to lead to localization via thermal softening, the GAP model predicts a broader failure landscape, in which thermal softening is neither necessary nor sufficient for high-strain rates and plastic strains seen in PSPI experiments.

5.2 Complete Mie-Grüneisen equation of state with a Johnson-Cook strength model

The following sections are an abridged summary of the model presented in (Malhotra et al., 2022), where a more detailed derivation of the model, its assumptions, and limitations may be found. A more general description of the complete Mie-Grüneisen equation of state may be found in (Menikoff, 2012). The CMGJC model makes use of the following variables:

$\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t),$	motion;
$\mathbf{F} = \nabla \boldsymbol{\chi}, \quad J = \det \mathbf{F} > 0,$	deformation gradient;
$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p,$	elastic-plastic decomposition of $\mathbf{F}$;
$\mathbf{F}^e, \quad J^e = \det \mathbf{F}^e > 0,$	elastic distortion;
$\mathbf{F}^p, \quad J^p = \det \mathbf{F}^p = 1,$	inelastic distortion (assuming plastic incompressibility);
$\mathbf{F}^e = \mathbf{R}^e \mathbf{U}^e,$	polar decomposition of $\mathbf{F}^e$;
$\mathbf{C}^e = \mathbf{F}^{e\top} \mathbf{F}^e,$	elastic right Cauchy-Green deformation tensor;
$\mathbf{B}^e = \mathbf{F}^e \mathbf{F}^{e\top},$	elastic left Cauchy-Green deformation tensor;
$\mathbf{E}^e = \ln \mathbf{U}^e = \frac{1}{2} \ln \mathbf{C}^e,$	logarithmic strain tensor;
$\bar{\mathbf{C}}^e = J^{-2/3} \mathbf{C}^e,$	elastic distortional right Cauchy-Green deformation tensor;
$\bar{\mathbf{B}}^e = J^{-2/3} \mathbf{B}^e,$	elastic distortional left Cauchy-Green deformation tensor;
$\bar{I}_1^e = \text{tr} \bar{\mathbf{C}}^e = \text{tr} \bar{\mathbf{B}}^e,$	elastic distortional first principal stretch invariant;
$\mathbf{L}^p = \dot{\mathbf{F}}^p \mathbf{F}^{p-1}, \quad \text{tr} \mathbf{L}^p = 0,$	plastic distortion rate tensor;
$\mathbf{D}^p = \text{sym}(\mathbf{L}^p),$	plastic stretching tensor;
$\mathbf{W}^p = \text{skw}(\mathbf{L}^p),$	plastic spin tensor;
$\mathbf{T} = \mathbf{T}^\top,$	Cauchy stress;
$\mathbf{M}^e = \mathbf{C}^e \mathbf{T}^e,$	Mandel stress in the intermediate configuration;

ψ_R ,	Helmholtz free energy density per unit volume of intermediate structural space;
$T > 0$,	absolute temperature.

Before continuing with the CMGJC model, it must be noted that the Mie-Grüneisen EOS comes in a variety of flavors, with perhaps the most common being an *incomplete* variant that is fit to Hugoniot data. Here, the following pressure is not associated with any thermodynamic potential and therefore does not allow access to temperature and entropy:

$$p = \frac{\rho_0 c_0^2 \eta}{(1 - s\eta)^2} \quad (5.1)$$

where $\eta = 1 - \rho_0/\rho$ is the nominal volumetric strain, and c_0 and s are the linear Hugoniot fitting parameters. This is the $p - v$ relation available as a built-in option in Abaqus, as well as a multitude of hydrocodes.

5.2.1 Free energy

The free energy density is taken to be a function of the three invariants of logarithmic strain (K_1, K_2, K_3) , as originally proposed by (Criscione et al., 2000). These invariants capture specific aspects of the deformation process in an isotropic body. $K_1 (= \text{tr} \mathbf{E}^e = \ln J^e)$, represents the volume change. The second invariant $K_2 (= |\mathbf{E}_0^e|)$, represents the magnitude of the deviatoric response of the material. The third invariant $K_3 (= 3\sqrt{6} \det \mathbf{N})$ represents the mode of distortion, where $\mathbf{N} = \frac{\mathbf{E}_0^e}{|\mathbf{E}_0^e|}$. K_3 is confined to the range $-1 \leq K_3 \leq 1$, and $K_3 = -1$ in compression, $K_3 = 0$ in simple shear, and $K_3 = 1$ in simple tension. The Helmholtz free energy ψ_R here is taken to depend on (K_1, K_2, T) such that the distortional contribution is uncoupled from the contribution due to volume changes:

$$\tilde{\psi}_R(K_1, K_2, T) = f_1(K_1, T) + G(K_1, T)K_2^2 \quad (5.2)$$

where $G(K_1, T)$ is the temperature and volumetric strain dependent shear modulus. Drawing our attention to the first term, we can rewrite this in terms of the specific volume v as $\psi(v, T)$ since $v = Jv_0 = v_0 \exp(K_1)$. We consider a complete Mie-Grüneisen equation of state, in which $\psi(v, T)$ is given by (Menikoff, 2012):

$$\begin{aligned} \psi(v, T) = & - \int_{v_0}^v P_{T_0}(v') dv' \\ & - \int_0^{T/\phi(v)} \frac{(T - \phi(v)T')}{T'} c_{v_0}(T') dT' \\ & + \int_0^{T_0/\phi(v)} \frac{(T_0 - \phi(v)T')}{T'} c_{v_0}(T') dT' \end{aligned} \quad (5.3)$$

with

$$\phi(v) = \exp \left(- \int_{v_0}^v \frac{\Gamma(v')}{v'} dv' \right).$$

The complete Mie-Grüneisen equation of state (5.3) is constructed from three functions of a single variable: (1) $P_{T_0}(v)$, the isothermal equation of state at the ambient temperature T_0 , (2) $\Gamma(v)$, the temperature-independent Grüneisen parameter, and (3) $c_{v_0}(T)$, the specific heat capacity at constant volume as a function of T along the isochore v_0 . Specific forms for each function are given below:

- The isothermal equation of state $P_{T_0}(v)$ is taken to be given by a third-order logarithmic equation of state (Poirier & Tarantola, 1998):

$$P_{T_0}(v) = K_{T_0} \left(\frac{v_0}{v} \right) \ln \left(\frac{v_0}{v} \right) \left[1 + \frac{K'_{T_0} - 2}{2} \ln \left(\frac{v_0}{v} \right) \right], \quad (5.4)$$

where K_{T_0} and K'_{T_0} are the isothermal bulk modulus and the first derivative of the isothermal bulk modulus with respect to pressure, respectively, in the ambient state (v_0, T_0) .

- The temperature-independent Grüneisen parameter $\Gamma(v)$ is taken to be

$$\Gamma(v) = \left(\frac{\Gamma_0}{v_0} \right) v, \quad (5.5)$$

where Γ_0 is the Grüneisen parameter at the ambient specific volume v_0 , so that $\phi(v)$ is

$$\phi(v) = \exp \left(- \int_{v_0}^v \frac{\Gamma(v')}{v'} dv' \right) = \exp \left[\Gamma_0 \left(1 - \frac{v}{v_0} \right) \right]. \quad (5.6)$$

- Finally, the specific heat capacity along the isochore v_0 , $c_{v_0}(T)$, is taken to be given through the Einstein model for the specific heat of a solid (Kittel, 2018; Kittell & Yarrington, 2016; Winey et al., 2002):

$$c_{v_0}(T) = A \left(\frac{T_E}{T} \right)^2 \frac{\exp(T_E/T)}{(\exp(T_E/T) - 1)^2}, \quad (5.7)$$

where $A = \lim_{T \rightarrow \infty} c_{v_0}(T)$ is the Dulong-Petit limit of the specific heat with units of J/kg-K and T_E is the Einstein characteristic temperature.

5.2.2 Stress

The Cauchy stress is given by

$$\mathbf{T}(K_1, K_2, T) = -P(K_1, T) \mathbf{I} + \frac{2}{J} G(K_1, T) K_2 \mathbf{N}' \quad (5.8)$$

where $\mathbf{N}' = \mathbf{R}^e \mathbf{N} \mathbf{R}^{eT}$ The pressure is

$$P = -\frac{1}{3} \text{Tr} \mathbf{T} = -\frac{\partial \psi}{\partial v} = P_{T_0}(v) + \frac{\Gamma(v)\phi(v)}{v} \int_{T_0/\phi(v)}^{T/\phi(v)} c_{v_0}(T') dT'. \quad (5.9)$$

Using (5.4), (5.5), and (5.7) in (5.9), we have

$$P = K_{T_0} \left(\frac{v_0}{v} \right) \ln \left(\frac{v_0}{v} \right) \left[1 + \frac{K'_{T_0} - 2}{2} \ln \left(\frac{v_0}{v} \right) \right] + \frac{\Gamma_0 \phi(v)}{v_0} A T_E \left[\frac{1}{\exp(\phi(v) T_E / T) - 1} - \frac{1}{\exp(\phi(v) T_E / T_0) - 1} \right], \quad (5.10)$$

with $\phi(v)$ given through (5.6). The Mandel stress is given by

$$\mathbf{M}^e(K_1, K_2, T) = -P(K_1, T)J\mathbf{I} + 2G(K_1, T)K_2\mathbf{N} = -P(v, T)J\mathbf{I} + 2G(v, T)\mathbf{E}_0 \quad (5.11)$$

and the corresponding equivalent tensile stress is

$$\bar{\sigma} = \sqrt{\frac{3}{2}}|\text{dev}\mathbf{M}^e| \quad (5.12)$$

5.2.3 Flow rule

The deformation gradient is multiplicatively decomposed as $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$. The evolution equation for $\mathbf{F}^p$ is

$$\left. \begin{aligned} \dot{\mathbf{F}}^p &= \mathbf{D}^p\mathbf{F}^p, & \mathbf{F}^p(\mathbf{X}, 0) &= \mathbf{1}, \\ \mathbf{D}^p &= \frac{3}{2}\dot{\bar{\epsilon}}^p \frac{\mathbf{M}_0^e}{\bar{\sigma}}, \end{aligned} \right\} \quad (5.13)$$

where $\dot{\bar{\epsilon}}^p = \sqrt{2/3}|\mathbf{D}^p| \geq 0$ is the equivalent tensile plastic strain rate. In (5.13), we have assumed plastic irrotationality, $\mathbf{W}^p = \mathbf{0}$, and codirectionality of the plastic stretching tensor $\mathbf{D}^p$ and the Mandel stress deviator $\text{dev}\mathbf{M}^e$. The equivalent tensile accumulated plastic strain $\bar{\epsilon}^p$ is given by

$$\bar{\epsilon}^p = \int_0^t \dot{\bar{\epsilon}}^p(\xi) d\xi \quad (5.14)$$

plays the role of a non-negative isotropic hardening internal variable that does not decrease in time and accounts for the past history of plastic deformation. As in (Holmquist et al., 2016), the equivalent tensile stress $\bar{\sigma}$, pressure P , equivalent tensile plastic strain rate $\dot{\bar{\epsilon}}^p$, isotropic hardening variable $\bar{\epsilon}^p$, and temperature T are then taken to be related through the modified Johnson-Cook model by (G. R. Johnson, 1983):

$$\bar{\sigma} = [A + B(\bar{\epsilon}^p)^n] \left[1 + \left(\frac{\dot{\bar{\epsilon}}^p}{\dot{\bar{\epsilon}}^0} \right)^{C_3} \right] (1 - \hat{T}^M) + C_4 P, \quad (5.15)$$

where $\{A, B, n, C_3, \dot{\varepsilon}_0, M, C_4\}$ are material parameters, the homologous temperature

$$\hat{T} = \begin{cases} 0 & \text{for } T < T_{\text{ref}}, \\ \frac{T - T_{\text{ref}}}{T_{\text{m}}(P) - T_{\text{ref}}} & \text{for } T_{\text{ref}} \leq T \leq T_{\text{m}}(P), \\ 1 & \text{for } T > T_{\text{m}}(P) \end{cases} \quad (5.16)$$

is a function of temperature and pressure, T_{ref} is the reference temperature below which there is no temperature dependence of the flow stress, and $T_{\text{m}}(P)$ is the pressure-dependent melt temperature. The parameter $\dot{\varepsilon}_0$ is the reference strain rate, and A is the yield strength at the reference strain-rate $\dot{\varepsilon}_0$ and reference temperature T_{ref} . The parameters B and n describe isotropic strain hardening, in which B is the strain-hardening modulus and n is the strain-hardening exponent. The parameter C_3 is the strain-rate-sensitivity exponent, M is the thermal softening exponent, and C_4 is the pressure sensitivity coefficient. Equation (5.15) may be inverted to obtain an explicit expression for $\dot{\varepsilon}^p$ as follows:

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \left[\left(\frac{\bar{\sigma} - C_4 P}{[A + B(\bar{\varepsilon}^p)^n](1 - \hat{T}^m)} - 1 \right)^{1/C_3} \right], \quad (5.17)$$

with $\hat{T}$ given through (5.16). A conditional is applied such that if $\bar{\sigma} - C_4 P \leq 0$ then $\dot{\varepsilon}^p = 0$. In practice, to ensure that $\bar{\varepsilon}^p$ remains bounded when the melt temperature is exceeded, $T > T_{\text{m}}(P)$, the equivalent tensile plastic strain rate is taken to be

$$\dot{\varepsilon}^p = \left(\frac{1}{\dot{\varepsilon}_{\text{JC}}^p} + \frac{1}{\dot{\varepsilon}_{\text{lim}}^p} \right)^{-1}, \quad (5.18)$$

where $\dot{\varepsilon}_{\text{JC}}^p$ is given by (5.17), and $\dot{\varepsilon}_{\text{lim}}^p$ is an upper limiting value of $\dot{\varepsilon}^p$.

(Holmquist et al., 2016) also makes use of a damage model, originally reported by (G. R. Johnson & Cook, 1985), where the plastic strain to failure is given by

$$\varepsilon_p^f = [D_1 + D_2 \exp(D_3 \sigma^*)] [1 + D_4 \ln \dot{\varepsilon}^*] [1 + D_5 T^*] \quad (5.19)$$

where $\sigma^* = P/\bar{\sigma}$ is the ratio of the pressure P to von Mises equivalent stress $\bar{\sigma}$, $\dot{\epsilon}^*$ is the non-dimensionalized strain rate ($\dot{\epsilon}/\dot{\epsilon}_0$), T^* is the homologous temperature $(T - T_0)/(T_m - T_0)$, T_0 is the ambient temperature, and T_m is the melt temperature, which is taken to be the glass transition temperature $T_g = 100^\circ\text{C}$. Because Boolean damage (e.g. with element deletion once the failure strain has been exceeded) is often problematic in modeling shock waves, a scalar damage variable D is used to degrade the deviatoric stresses in an element:

$$D = \frac{\bar{\epsilon}^p}{\epsilon_p^f} \quad (5.20)$$

$$\boldsymbol{\sigma}'_0 = \boldsymbol{\sigma}_0(1 - D) \quad (5.21)$$

where D is constrained such that $D \in (0, 1)$. There are two works of relevance to this thesis that have adapted the parameterization from Holmquist, which applied the Johnson-Cook strength model to pore collapse in PMMA: first (Rai et al., 2020) and more recently (Lawlor et al., 2024). These latter models use nearly the same expression for capturing plastic flow:

$$\bar{\sigma} = [A + B(\bar{\epsilon}^p)^n] \left[1 + C \ln \left(\frac{\dot{\bar{\epsilon}}^p}{\dot{\bar{\epsilon}}_0} \right) \right] (1 - \hat{T}^M), \quad (5.22)$$

This, however, is a different formulation from Holmquist. First, these models use logarithmic strain-rate dependence, rather than a power-law dependence, which is likely of no consequence, provided the model is well-fit to experimental data. Second, and more importantly, these models neglect the pressure dependence of the shear strength of PMMA. PMMA, as is true for many polymers, has widely been characterized as having a pressure-dependent yield surface (P. B. Bowden & Jukes, 1972; Rabinowitz et al., 1970). It will later be shown that this term leads to stark differences in simulated results, both for the PSPI experiments in this chapter, as well as in the pore collapse simulations in Chapter 7. The absence of the pressure term will, in general, more readily lead to thermal softening, runaway, and shear banding. It should also be noted that these studies do not incorporate damage, which will later be shown to reduce plastic work and heating. The

runaway effect of thermal softening and intense plastic deformation is exacerbated further still by the fact that these studies do not incorporate a pressure-dependent shear modulus, as was done by Holmquist. In (Rai et al., 2020) $G = 5.9$ GPa and in (Lawlor et al., 2024) $G = 2.19$ GPa, which is the ambient value. As done by Holmquist, a pressure-dependent shear modulus is fit to measurements coming from (Gupta, 1980): $G(\eta) = G_a + G_b\eta$, where G_a and G_b are constants. This completes the description of the plasticity model.

5.2.4 Complete Mie-Grüneisen EOS parameters

Before fits to experimental data can be obtained, a few useful thermodynamic relations are needed. The ambient Grüneisen parameter Γ_0 is not directly measurable and is instead written in terms of other ambient material properties:

$$\Gamma_0 = \frac{1}{2} \left[\sqrt{\frac{1}{(\alpha_{v0}T_0)^2} + \frac{4C_{\eta0}^2}{T_0c_{v0}}} - \frac{1}{(\alpha_{v0}T_0)} \right] = 1.09 \quad (5.23)$$

where c_{v0} is the ambient specific heat at constant volume, α_{v0} is the Ambient coefficient of volume expansion, and $C_{\eta0}$ is the ambient isentropic wave speed. Next, we may write the ambient isothermal bulk modulus as

$$K_{T0} = \frac{\Gamma_0 \rho_0 c_{v0}}{\alpha_{v0}} = 8.17 \text{ GPa} \quad (5.24)$$

The bulk modulus here is several times higher than the ambient value, which was observed experimentally by (Gupta, 1980), where large instantaneous jumps in the bulk modulus were observed for high strain-rate loading.

We now fit the specific heat to data from (Brandrup et al., 1999) using Eq. 5.7. The experimental specific heat and Einstein model are shown in Fig. 5.1.

As in (Malhotra et al., 2022), numerical integration of this function is also approximated

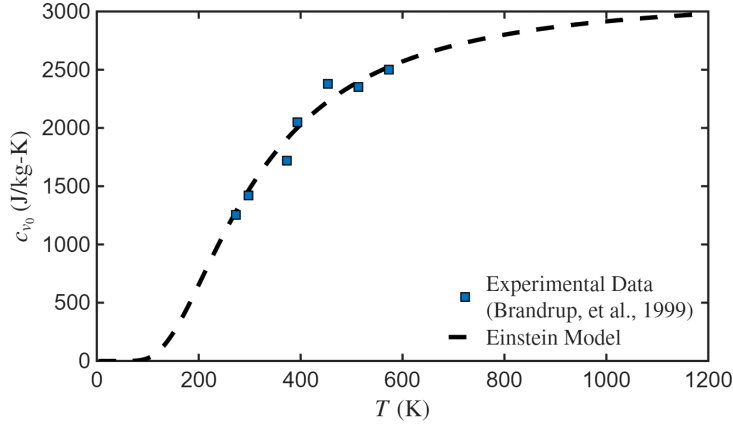


Figure 5.1: Specific heat varied with temperature for the Einstein model.

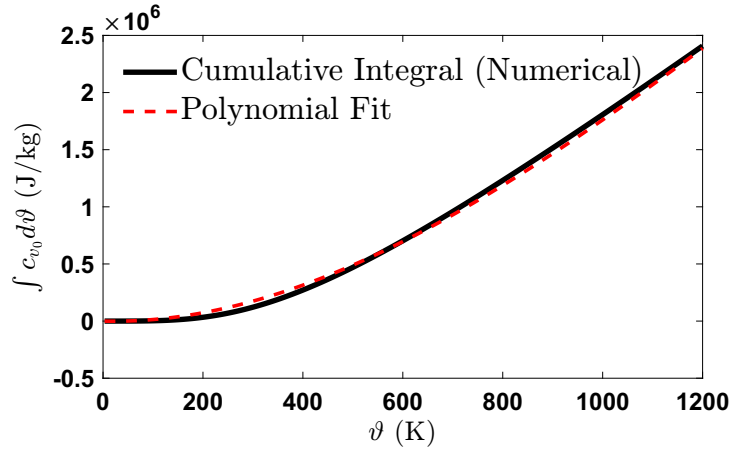


Figure 5.2: Numerical integral and approximate fit of the integral of specific heat with respect to temperature.

by a polynomial:

$$\int_0^T c_v(T') dT' \approx a_1 + a_2 T + a_3 T^2 \quad (5.25)$$

This polynomial fit and the integrated empirical specific heat are depicted in Fig. 5.2. Note that this polynomial is used directly to calculate the thermal pressure in Eq. 5.9.

The shock speed U_S as a function of particle velocity u_p is shown in Fig. 5.3. The model, as well as the fit to the LASL Shock Compendium (Mader et al., n.d.) are approximately linear in the region of interest. Other researchers, such as (Jordan et al., 2016), have generated piecewise polynomial fits to account for the highly non-linear response for low shock pressures, typically < 0.4 GPa. Our PSPI experiments were all conducted at normal

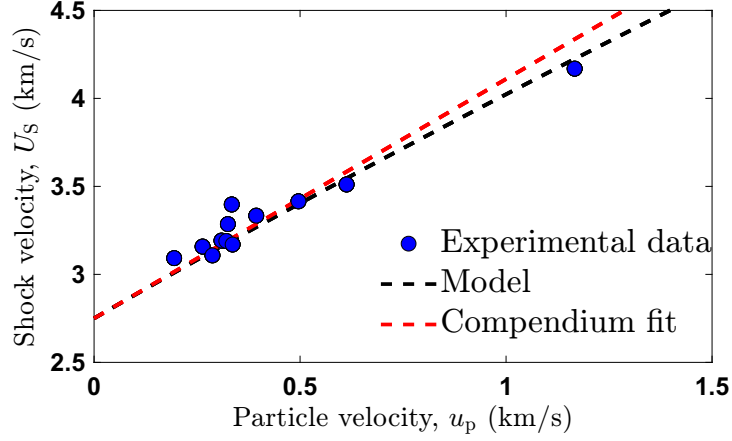


Figure 5.3: $U_s - u_p$ Hugoniot plane for PMMA. The experimental data are taken from the LASL shock compendium (Mader et al., n.d.).

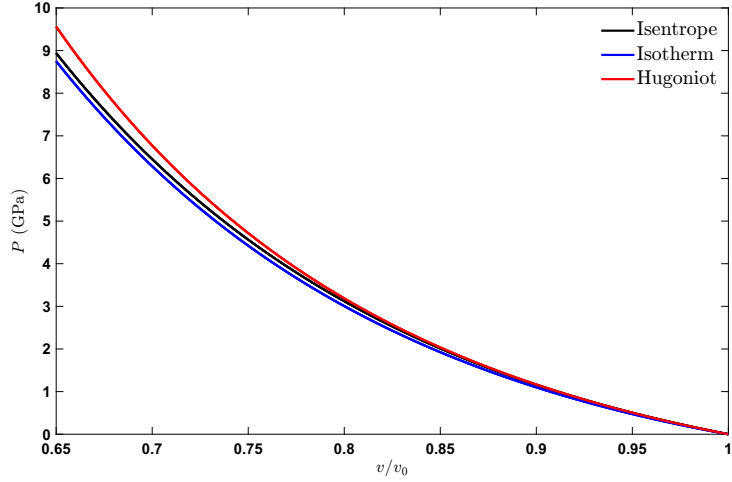


Figure 5.4: Pressure varied with specific volume for isothermal, isentropic, and shock (Hugoniot) loading.

stresses above 2.5 GPa, and therefore, the model's response is adequate for our current purposes. A linear least-squares regression can be performed with respect to experimental data in the range of interest to obtain K'_{T0} such that the linear Hugoniot behavior is captured.

The pressure generated from loading along the isotherm, isentrope, and Hugoniot is shown in Fig. 5.4. Note that for low levels of compression, the Hugoniot and isentrope are similarly valued. The absolute temperature T is generated for these same loading profiles in Fig. 5.5.

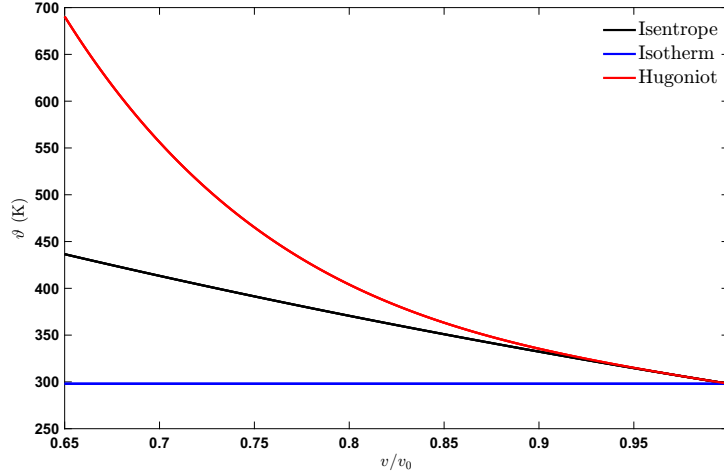


Figure 5.5: Temperature varied with specific volume for isothermal, isentropic, and shock (Hugoniot) loading.

A summary of the material properties for PMMA used in the Complete Mie-Grüneisen equation of state model, as well as all other material properties, is presented in Table 5.3.

5.2.5 Johnson-Cook plasticity parameters

The fits below are all to experimental data that was collected by (Holmquist et al., 2016). Here, strength model parameters were not provided, and thus the approach outlined by (G. R. Johnson, 1983) is followed. Most of the parameters provided by (Rai et al., 2020) generated excellent fits to the data and are largely used here, with the exception of the reference temperature and the pressure hardening terms. The parameters used in the Johnson-Cook strength model are presented in Table 5.3.

The thermal softening parameters were fit to experimental data from (Nasraoui et al., 2012) and (Quinson et al., 1997). As noted above, the values from (Rai et al., 2020) provide excellent fits, though the reference temperature here and in (Holmquist et al., 2016) is $T_0 = 213\text{K}$, whereas $T = 298\text{K}$ in (Rai et al., 2020). The resultant flow stress σ_y as a function of homologous temperature is depicted in Fig. 5.6.

For the strain-rate hardening data, the parameters from (Rai et al., 2020) are checked against experiments by (Nasraoui et al., 2012; Richeton et al., 2006; Rittel & Brill, 2008).

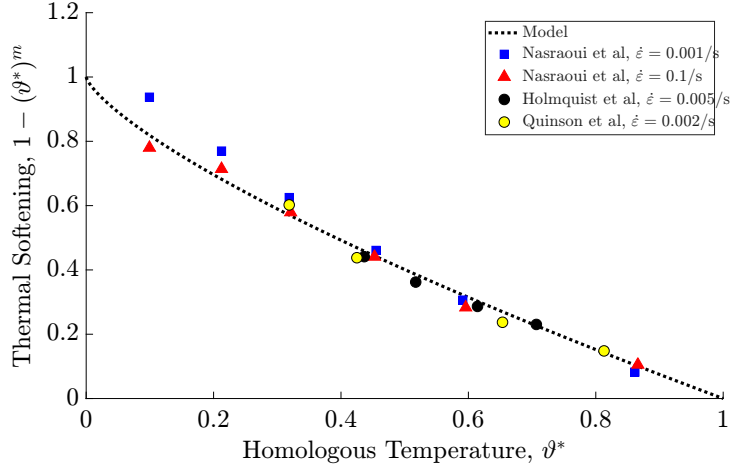


Figure 5.6: Compressive yield stress varied with homologous temperature T^* .

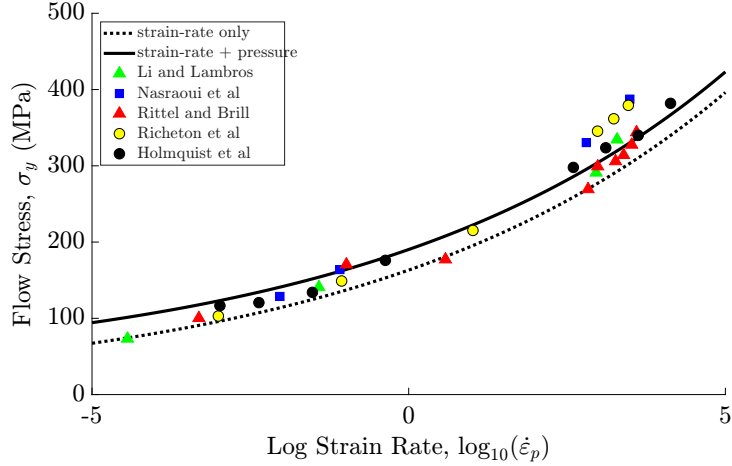


Figure 5.7: Compressive yield stress varied with strain-rate.

The resultant flow stress σ_y as a function of strain-rate is depicted in Fig. 5.7. The pressure-fits are compared with the experimental data from several sources, as presented in (Holmquist et al., 2016), and are shown in Fig. 5.8. The most problematic set of parameters was the strain hardening parameters n and B . These are somewhat arbitrarily defined, in that the Johnson-Cook model cannot replicate the complex post-yield softening and hardening behaviors in PMMA with just one strain hardening term. This inability to capture post-yield behavior leads to an ambiguous parameterization. The plastic strain level reported in (Holmquist et al., 2016) does not correspond to the actual onset of plasticity; rather, this plastic strain level only corresponds to regions in the stress-strain

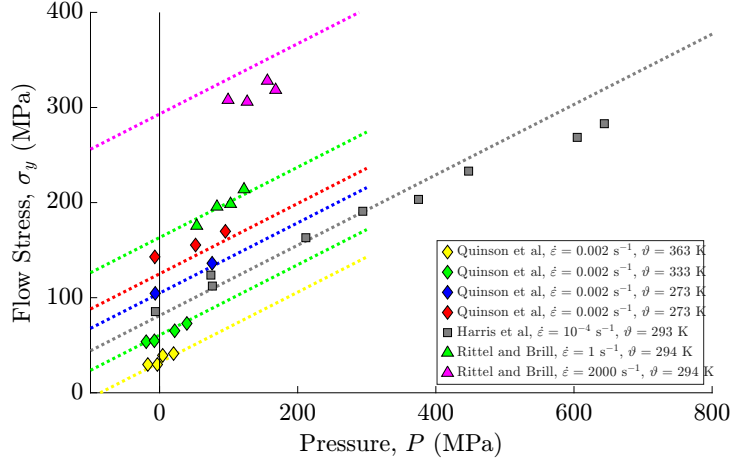


Figure 5.8: Yield stress varied with pressure. The model fits from this work are given as dashed lines.

curves where hardening begins to dominate. Prior to this, PMMA softens considerably under quasi-static compression. Though it is outside the scope of this present work, the Johnson-Cook model must be modified to capture this softening and hardening behavior. The original strain hardening parameters by (Rai et al., 2020) are used to generate the curves in Fig. 5.9 (a) and (c). Though fits for all other parameters were generally excellent, here there is a gross mismatch between experiment and model. If the value of B is increased by a factor of 10 to be $B = 900$ MPa, the fits are much better, as shown in Fig. 5.9 (b) and (d). Given the fidelity of fits for other parameters by (Rai et al., 2020), this behavior is indeed strange. On the other hand, the apparent ambiguity in defining the onset of plastic deformation with only a strain-hardening term may be to blame for this discrepancy. As previously noted, this sole strain-hardening term ought to be modified in future work. With this caveat in mind, since fits provided by the rest of the parameter set in (Rai et al., 2020) are generally of good quality, the strain-hardening parameters will also be used, so that results between this current work and that of Rai may be more directly compared when discussing void collapse simulations in Chapter 7.

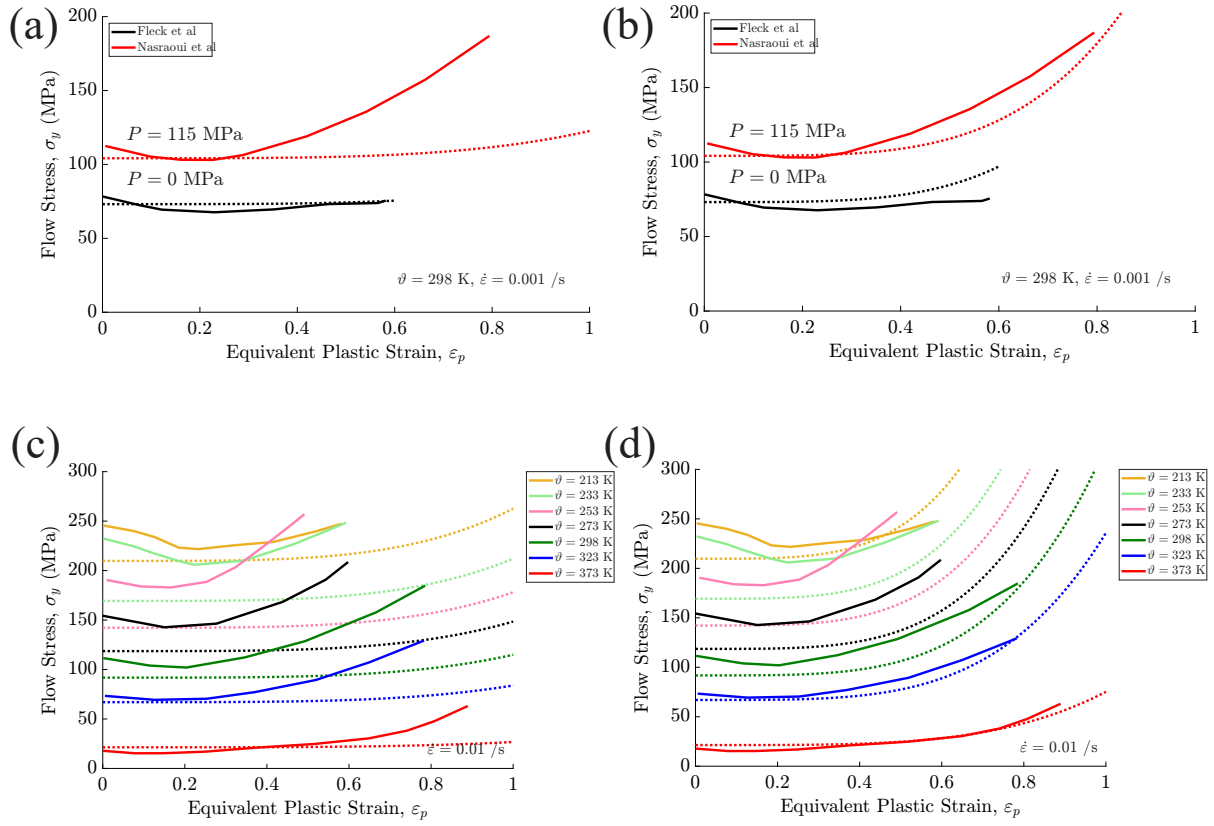


Figure 5.9: Yield stress varied with plastic strain (a) for varying pressure with $B = 90$ MPa (b) for varying pressure with $B = 900$ MPa (c) for varying temperature with $B = 90$ MPa (d) for varying temperature with $B = 900$ MPa

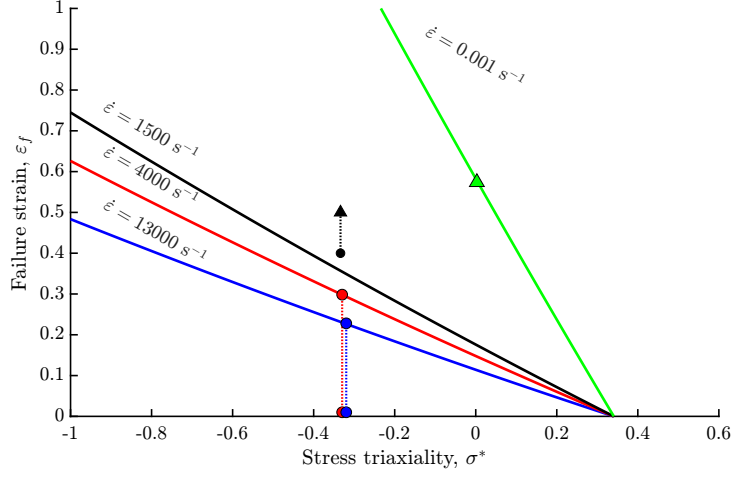


Figure 5.10: Plastic failure strain varied with stress triaxiality.

5.2.6 Johnson-Cook damage parameters

The data presented in (Holmquist et al., 2016) make for a straightforward parameterization of the Johnson-Cook damage model, with experimental data coming from (Fleck et al., 1990), (Nasraoui et al., 2012), (Holmquist et al., 2016), (P. D. Wu & van der Giessen, 1995), and (Barker & Hollenbach, 1970)

The plastic failure strain ε_f^p varied with pressure is presented in Fig. 5.10. When failure is believed to have occurred, a single data point is used. When the test was stopped before failure, a single point with a vertical arrow is used. When failure occurred, but it is not clear at what strain level failure transpired, two points are connected by a vertical dotted line. Several strain rates are used to generate predictions from the model. Here, one may observe the competition between strain-rate, which decreases failure strain, and triaxiality, which increases failure strain, that is, under higher levels of compression.

The plastic failure strain ε_f^p varied with strain-rate is presented in Fig. 5.11. The data from the PSPI experiments are also shown here as a reference, as well as several color-coded lines corresponding to different amounts of stress triaxiality. As an example, the tensile experiments by Wu et al are depicted in red, as is the model prediction, for which $\sigma^* = 0.33$.

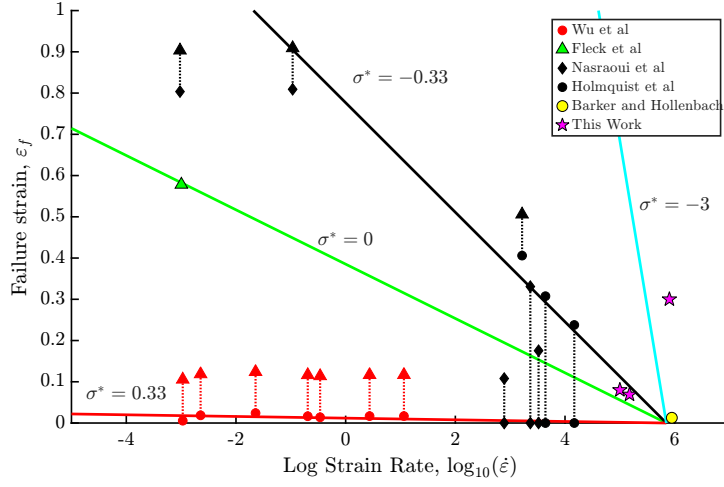


Figure 5.11: Plastic failure strain varied with strain-rate.

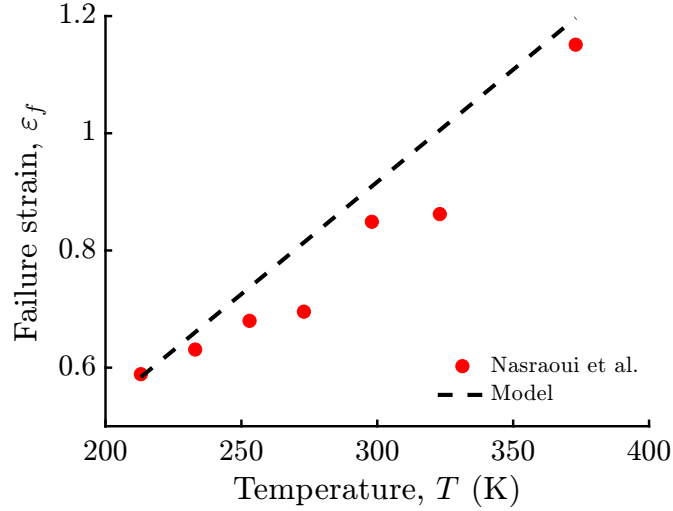


Figure 5.12: Plastic failure strain varied with temperature.

Finally, the plastic failure strain ε_f^p varied with temperature is presented in Fig. 5.12. A higher degree of thermal softening increases the ductility of the material; for higher temperatures, larger failure strains are observed. This softening behavior is observed well below the glass transition temperature as well. As a corollary, the material fails in a highly brittle manner for cold temperature tests.

The end result of the parameterization for the CMGJC model for PMMA is summarized by Table 5.3 in the chapter appendix.

5.3 The Glassy Amorphous Polymer (GAP) Model

The Glassy Amorphous Polymer (GAP) model was first described in its entirety by (B. E. Clements, 2012). The GAP model incorporates theoretical work by Clements and others at Los Alamos National Laboratory, primarily for polymers used as binders in polymer-bonded explosives (PBX). The viscoelastic part of the model has since been incorporated into the VISCOSCRAM model (Bennett et al., 1998; Buechler & Luscher, 2012) and more recently, the Mechanically Activated Thermal CHemistry (MATCH) material model at Los Alamos (B. Clements & Ma, 2018) for a variety of polymers, including PMMA, polycarbonate, Kel-F, HDPE, UHMWPE, epoxy, and others. VISCOSCRAM itself was a marriage between viscoelasticity and the brittle damage model ISOSCRAM (Addessio & Johnson, 1990). The name “ISOSCRAM” refers to an **isotropic** implementation of the statistical **crack** modeling introduced by J. Dienes in the 1980s (Dienes & Kershner, 1998; Dienes & Margolin, 1980). SCRAM is a physically based, micromechanical description for the large deformations of brittle materials. The continuum-level response is determined by combining strains associated with many penny-shaped cracks. This work grew out of a damage framework applied to geological materials and was later applied to PBXs, as damage at the binder-explosive interface plays a key role in the sensitivity and performance of these materials.

The GAP model and its modeling relatives (e.g., VISCOSCRAM) have also been implemented at other DOE and DOD facilities such as Sandia National Labs (Cearley et al., 2024) and the Air Force Research Laboratory (B. E. Clements, 2011). Many such projects are pursued as part of the DOE/DoD Joint Munitions Program. In an L2 report from Los Alamos, two variants of polyethylene, high-density polyethylene (HDPE) and UHMWPE (Ultra-high-molecular-weight polyethylene), were characterized for a variety of experiments, most notably for high deformation experiments like Taylor cylinder impact and dynamic tensile failure (spall) experiments (Coe et al., 2017). This report was recently

expanded upon by (Dattelbaum et al., 2026).

The GAP material subroutine in this work, which was implemented in ABAQUS/Explicit, was developed independently of any existing codes for MATCH, VISCOSCRAM, or any sibling code databases. The codes here were written *ab initio* at Brown University solely based on publicly available materials. Parameters for PMMA were provided in the original publication (B. E. Clements, 2012), except where new modifications were introduced. The primary adjustments to the model come in (i) the equilibrium equation of state and (ii) the hierarchical flow stress (HFS) plasticity model.

The GAP model makes use of the following variables:

ψ ,	Helmholtz free energy density per unit volume;
ρ ,	density;
$v = 1/\rho$,	specific volume;
$m = 1, \dots M$,	m^{th} volumetric viscoelastic Maxwell element;
	$m = 0$ signifies equilibrium element;
$\bar{\tau}^{(m)}$	bulk relaxation time;
$n = 1, \dots N$,	n^{th} viscoelastic deviatoric Maxwell element;
	$n = 0$ signifies equilibrium element;
$\tau^{(n)}$	deviatoric relaxation time;
$p^{(m)}$,	pressure (positive in compression);
$s_{ij}^{(n)}$,	Cauchy stress deviator;
$\varepsilon_v^{(m)}$,	infinitesimal volumetric strain (positive in compression);
$e_{ij}^{(n)}$,	infinitesimal strain deviator;
$e_{ij}^{P(n)}$,	infinitesimal plastic strain deviator;
T ,	absolute temperature;
$\eta^{(m)}$,	entropy;

$$\begin{aligned}
c_v^{(m)} &\approx c_v^{(0)}, & \text{specific heat at constant volume;} \\
c_p^{(m)} &\approx c_p^{(0)}, & \text{specific heat at constant pressure;} \\
\gamma^{(m)} &\approx \gamma^{(0)}, & \text{Grüneisen parameter;} \\
\beta^{(m)} &\approx \beta^{(0)}, & \text{volumetric expansion coefficient;}
\end{aligned}$$

5.3.1 Model overview

Amorphous polymers consist of long chains of molecules that form complex networks throughout the material. These networks will have a variety of responses to a given mechanical load, resulting in a macroscopic non-equilibrium response. In viscoelastic models, polymers are modeled as having a variety of relaxation modes, which result in a non-linear macroscopic stress-strain response. Because viscoelastic models are often applied to scenarios with relatively small volumetric deformation, e.g., quasi-static uniaxial tension, viscoelasticity is typically accounted for solely in the deviatoric response. Volumetric, or bulk, viscoelasticity is typically not needed and therefore neglected. In the case of plate impact experiments, the volumetric strain is large, and bulk viscoelastic effects can no longer be neglected.

In addition to both bulk and deviatoric viscoelasticity, the model incorporates a novel plasticity model termed the “Hierarchical Flow Stress” (HFS) model. A non-associated, modified Prandtl-Reuss flow rule is taken to depend on plastic work, strain-rate, stress-level, and temperature. This, in combination with viscoelasticity, reproduces the complex rate and temperature-dependent material response of PMMA outlined in Chapter 1.

The model also incorporates a thermodynamically consistent equation of state, which forms the backbone of the model. In the original formulation, this equation of state was written in terms of a semi-empirical form of the Gibbs free energy, which reproduces (1) an empirical fit to specific heat data and (2) the Tait equation of state, which itself is fit to experimental dilatometry data. In this work, the equation of state is modified to

include the Birch-Murnaghan equation of state in lieu of the Tait equation of state, in order to better capture high-pressure behavior.

5.3.2 Continuum framework

In the GAP model, a small deformation framework is used, where the stress measure is the Cauchy stress tensor $\boldsymbol{\sigma}$ and the strain measure is the infinitesimal strain tensor $\boldsymbol{\varepsilon}$. This choice of framework is particularly advantageous for the applied strain-rate and temperature-dependent plasticity and damage models, which are based on the assumption of additive decomposition of strain rates. It should be noted that the time derivative of the Cauchy stress by itself is not objective, and its use will lead to spurious results under rigid body rotation. The small deformation framework here is applied in a hypoelastic time-stepping procedure. That is, at each time step, small stress increments are determined from strain increments. These calculations are performed in the co-rotational frame before being updated via the Green-Naghdi objective stress rate.

For small deformations, the rate of deformation tensor D_{ij} is approximated by the time derivative of ε_{ij} from which the trace is

$$D_{kk} = \frac{\partial v_k}{\partial x_k} \approx \frac{d}{dt} \left(\frac{\partial u_k}{\partial X_k} \right) = \dot{\varepsilon}_{kk}. \quad (5.26)$$

where V_k is the velocity, u_k is the displacement, X_k is the material coordinate, and x_k is the spatial coordinate. Thus, in this framework, the stress power can then be approximated as $\sigma_{ij}\dot{\varepsilon}_{ij}$. The balance laws of mass, momentum, and energy are given by

$$\dot{\rho} - \rho\dot{\varepsilon}_v = 0 \quad (5.27)$$

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho\dot{v}_i \quad (5.28)$$

$$\rho\dot{u} = \sigma_{ij}\dot{\varepsilon}_{ij} + \rho r - \frac{\partial q_k}{\partial x_k} \quad (5.29)$$

where u is the internal energy, r is the internal heat production, and $\frac{\partial q_k}{\partial x_k}$ is the heat flux leaving the system.

σ_{ij} and ε_{ij} may be additively decomposed into volumetric and deviatoric components:

$$\sigma_{ij} = s_{ij} - p\delta_{ij} \quad (5.30)$$

$$\varepsilon_{ij} = e_{ij} - \varepsilon_v \delta_{ij} + \quad (5.31)$$

$$(5.32)$$

(note: dilatational signs are positive in compression).

The balance of energy now becomes

$$\rho \dot{u} = s_{ij} \dot{e}_{ij} - \rho p \dot{v} + \rho r - \frac{\partial q_k}{\partial x_k}. \quad (5.33)$$

The Clausius-Duhem form of the second law of thermodynamics states that the internal entropy production $= \eta_D$ evolves according to

$$\rho T \eta_D = \rho T \dot{\eta} - \rho r + \frac{\partial q_k}{\partial x_k} - \frac{1}{T} q_k \frac{\partial T}{\partial x_k} \geq 0. \quad (5.34)$$

Given the dynamic nature of the problems at hand, the current work considers only adiabatic processes and thus neglects thermal diffusion. Recalling that the Helmholtz free energy may be written as $\psi = u - T\eta$ and combining the expressions of the first and second laws, therefore yields

$$\rho T \eta_D = \rho T \dot{\eta} = -\rho \dot{\psi} - \rho \eta \dot{T} + s_{ij} \dot{e}_{ij} - \rho p \dot{v} \geq 0. \quad (5.35)$$

With the general thermodynamic description of the model framework complete, a final few remarks on the model design are needed. Recall that in this framework, the total strain ε_{ij} may be decomposed into volumetric and deviatoric components. The volumetric

strain ε_v itself may be additively decomposed into elastic, thermoelastic, and viscoelastic components as

$$\dot{\varepsilon}_v = \dot{\varepsilon}_v^{E(m)} + \beta \dot{T} + \dot{\varepsilon}_v^{VE(m)} \quad (5.36)$$

where m refers to the m^{th} volumetric generalized Maxwell branch. The first branch is the equilibrium branch where no viscoelastic strains are present, while the remainder have viscoelastic components. The volumetric viscoelastic strain-rate for the m^{th} element is given by $\dot{\varepsilon}_v^{VE(m)} = p^{(m)} / (B_T^{(m)} \bar{\tau}^{(m)})$. The volumetric rheological model is depicted in Fig. 5.13 (left).

In the deviatoric case, since it is further assumed that plasticity is isochoric and thus is captured by the deviatoric response of the material only. Therefore, the total strain deviator rate $\dot{e}_{ij}$ is written as

$$\dot{e}_{ij} = \dot{e}_{ij}^{E(n)} + \dot{e}_{ij}^{P(n)} + \dot{e}_{ij}^{VE(n)} \quad (5.37)$$

where where m refers to the m^{th} deviatoric generalized Maxwell branch. The deviatoric rheological model is depicted in Fig. 5.13 (right).

5.3.3 Specialization of free energy

The net effect on the model is that an *ansatz* for the free energy must include equilibrium and non-equilibrium components. The specific Helmholtz free energy is given as the sum of these:

$$\rho\psi(\varepsilon_v^E, e_{ij}^E, T; \{\dot{\varepsilon}_v^E, \dot{e}_{ij}^E, \dot{T}\}) = \rho\psi^{(0)}(\varepsilon_v^E, e_{ij}^E, T) + \frac{1}{2} \left(\sum_{n=1}^N \frac{s_{ij}^{(n)} s_{ij}^{(n)}}{2\mu^{(n)}} + \sum_{m=1}^M \frac{p^{(m)} p^{(m)}}{B_T^{(m)}} \right) \quad (5.38)$$

and the variables are described in detail at the beginning of this section. A few comments, however, are warranted. $\psi^{(0)}$ is the equilibrium specific Helmholtz free energy, and the second term represents the non-equilibrium contributions for both deviatoric and volumetric

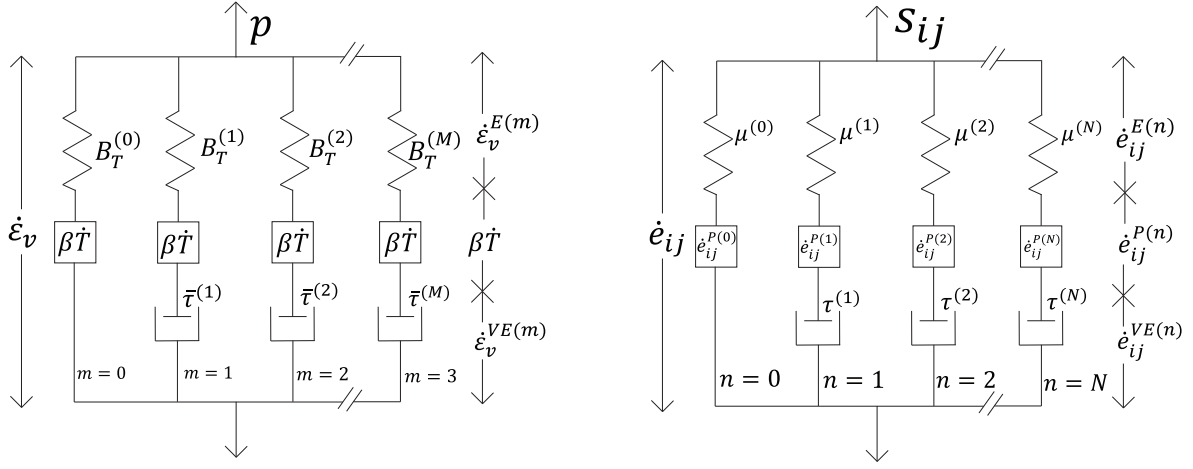


Figure 5.13: Generalized Maxwell models for bulk visco-thermoelasticity (left) and deviatoric viscoelasticity and plasticity (right).

parts of the free energy. The overall response of the volumetric and deviatoric components is represented by generalized Maxwell elements, which have equilibrium (or steady-state) responses as well as non-equilibrium branches, which give rise to viscoelasticity. The former are depicted as having mechanical springs, and the latter are represented by springs and viscous dashpots in series, as shown in Fig. 5.13. There are additionally thermoelastic and viscoplastic contributions to each of these, which will be discussed shortly. The $n = 1, \dots, N$ deviatoric terms and $m = 1, \dots, M$ volumetric terms in Eq. 5.38 represent the elements in our physical analog of the model. The curly brackets on the left indicate that the Helmholtz free energy is a functional of the strain and temperature history. The independent variables enter through non-equilibrium contributions to the pressure $p^{(m)}$ and deviatoric stress $s_{ij}^{(n)}$ as

$$p^{(m)}(t) = \int_0^t dt_1 e^{\bar{\phi}^{(m)}(t_1) - \bar{\phi}^{(m)}(t)} B_T^{(m)} \dot{\epsilon}_v(t_1) + \int_0^t dt_1 e^{\bar{\phi}^{(m)}(t_1) - \bar{\phi}^{(m)}(t)} \beta^{(m)} B_T^{(m)} \dot{T}(t_1) \quad (5.39)$$

and

$$s_{ij}^{(n)}(t) = \int_0^t dt_1 e^{\phi^{(n)}(t_1) - \phi^{(n)}(t)} 2\mu^{(n)} \dot{\epsilon}_{ij}(t_1) \quad (5.40)$$

The factor $\bar{\phi}$ is defined as

$$\phi^{(n)}(t) \equiv 1/\tau^{(n)}(t) = \int_0^t \frac{dt'}{\tau^{(n)}(t')} \quad (5.41)$$

in the deviatoric case and

$$\bar{\phi}^{(m)}(t) \equiv 1/\bar{\tau}^{(m)}(t) = \int_0^t \frac{dt'}{\bar{\tau}^{(m)}(t')} \quad (5.42)$$

in the volumetric case. $\tau^{(n)}$ and $\tau^{(m)}$ are the deviatoric and volumetric relaxation times for a given element. These relaxation times depend on temperature via the shear and volumetric shift factors a_T and $\bar{a}_T$, respectively, such that

$$\tau^{(n)}(T) = a_T(T)\tau^{(n)} \quad (5.43)$$

$$\tau^{(m)}(T) = \bar{a}_T(T)\tau^{(m)} \quad (5.44)$$

$$(5.45)$$

Here, as in (B. E. Clements, 2012), polynomial forms are used for the shift functions:

$$\log_{10} a_T(T) = p_0 + p_1 T + p_2 T^2 \quad (5.46)$$

$$\bar{a}_T(T) = \bar{p}_0 + \bar{p}_1 T \quad (5.47)$$

$$(5.48)$$

The shift functions are presented in Fig. 5.14.

As mentioned in the general model overview, the hypoelastic formulation involves writing stress increments at the end of each time step. To this end, differential forms for Eq. 5.39 and Eq. 5.40 are given by

$$dp^{(m)} = B_T^{(m)} d\varepsilon_v + \beta^{(m)} B_T^{(m)} dT - \frac{p^{(m)}}{\bar{\tau}^{(m)}} dt \quad (5.49)$$

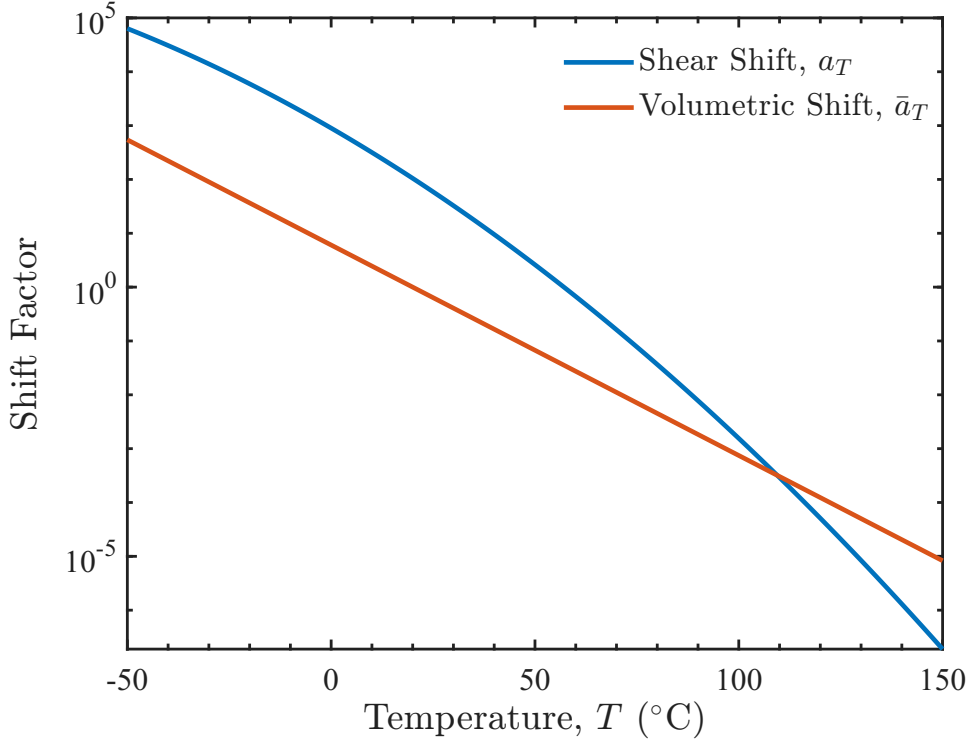


Figure 5.14: Volumetric and shear shift factors varied with temperature.

$$ds_{ij}^{(n)} = 2\mu^{(n)}de_{ij}^{(n)} - \frac{s_{ij}^{(n)}}{\tau^{(n)}}dt = 2\mu^{(n)}(de_{ij} - de_{ij}^{P(n)}) - \frac{s_{ij}^{(n)}}{\tau^{(n)}}dt \quad (5.50)$$

where $de_{ij}^{P(n)}$ is the plastic strain deviator increment for the n^{th} element. Subtracting this from the total strain deviator gives the elastic strain deviator increment. In Eq. 5.49 and 5.50, as well as for the rest of the analysis, it is assumed that all elements are at the same temperature $T(t)$. This is known as the *quasi-equilibrium hypothesis*, derived from the notion that heat may rapidly be transported between elements, which seems plausible in the absence of purely thermal agitations. The equilibrium analogs of the above differential forms are nearly identical, sans the relaxation terms:

$$dp^{(0)} = B_T^{(0)} d\varepsilon_v + \beta^{(0)} B_T^{(0)} dT \quad (5.51)$$

$$ds_{ij}^{(n)} = 2\mu^{(0)}(de_{ij} - de_{ij}^{P(0)}) \quad (5.52)$$

where “0” denotes the equilibrium elements where no viscoelastic relaxation occurs.

The differential of the free energy is

$$\rho d\psi = \rho d\psi^{(0)} + \sum_{m=1}^M \frac{p^{(m)}}{B_T^{(m)}} dp^{(m)} + \sum_{n=1}^N \frac{s_{ij}^{(n)} ds_{ij}^{(n)}}{2\mu^{(n)}} \quad (5.53)$$

. Substituting Eq. 5.49 and Eq. 5.50 into Eq. 5.53 gives

$$\begin{aligned} \rho d\psi = & \rho d\psi^{(0)} - \sum_{m=1}^M \frac{p^{(m)}}{v} dv + \sum_{m=1}^M \beta^{(m)} p^{(m)} dT + \sum_{n=1}^N s_{ij}^{(n)} de_{ij}^{(n)} \\ & - \sum_{m=1}^M \frac{p^{(m)} p^{(m)}}{B_T^{(m)} \bar{\tau}^{(m)}} dt - \sum_{n=1}^N \frac{s_{ij}^{(n)} s_{ij}^{(n)}}{2\mu^{(n)} \tau^{(n)}} dt \end{aligned} \quad (5.54)$$

The equilibrium contributions to the pressure, deviatoric stress, and specific entropy are

$$p^{(0)} = - \left(\frac{\partial \psi^{(0)}}{\partial v} \right)_{e_{ij}^{E(0)}, T}, \quad s_{ij}^{(0)} = \rho \left(\frac{\partial \psi^{(0)}}{\partial e_{ij}^{E(0)}} \right)_{\varepsilon_v, T}, \quad \eta^{(0)} = - \left(\frac{\partial \psi^{(0)}}{\partial T} \right)_{v, e_{ij}^{E(0)}} \quad (5.55)$$

The dissipation inequality is given as

$$Td\eta = -d\psi - \eta dT + \frac{1}{\rho} s_{ij} de_{ij} - p dv \geq 0 \quad (5.56)$$

Substituting Eq.5.53 into Eq. 5.56 and following the Coleman-Noll procedure (whereby the prefactors of dv , de_{ij} , and T are set to zero) results in expressions for pressure

$$p = p^{(0)} + \sum_{m=1}^M p^{(m)} \quad (5.57)$$

the stress deviator

$$s_{ij} = s_{ij}^{(0)} + \sum_{n=1}^N s_{ij}^{(n)} \quad (5.58)$$

and the specific entropy

$$\eta = \eta^{(0)} + \sum_{m=1}^M \eta^{(m)}. \quad (5.59)$$

The remaining terms in the dissipation inequality are the internal dissipation:

$$\rho T \frac{d\eta}{dt} = \sum_{m=1}^M \frac{p^{(m)} p^{(m)}}{B_T^{(m)} \bar{\tau}^{(m)}} + \sum_{n=1}^N \frac{s_{ij}^{(n)} s_{ij}^{(n)}}{2\mu^{(n)} \tau^{(n)}} + \sum_{n=1}^N s_{ij}^{(n)} \dot{e}_{ij}^{P(n)} \geq 0 \quad (5.60)$$

The first two terms here arise due to bulk and deviatoric viscoelastic dissipation, and the third term represents plastic dissipation.

We now wish to find the temperature evolution equation. First, from the above inequality, we generate a variety of standard thermodynamic relations:

$$T \left(\frac{\partial \eta^{(0)}}{\partial T} \right)_v = c_v^{(0)} \quad T \left(\frac{\partial \eta^{(0)}}{\partial T} \right)_p = c_p^{(0)} \quad (5.61)$$

$$v \left(\frac{\partial \eta^{(0)}}{\partial v} \right)_T = \gamma^{(0)} c_v^{(0)} \quad v \left(\frac{\partial T}{\partial v} \right)_\eta = -\gamma^{(0)} T \quad (5.62)$$

$$v \left(\frac{\partial p^{(0)}}{\partial v} \right)_T = -B_T^{(0)} \quad \left(\frac{\partial p^{(0)}}{\partial T} \right)_v = \beta^{(0)} B_T^{(0)} \quad (5.63)$$

$$\frac{1}{v} \left(\frac{\partial v}{\partial t} \right)_p = \beta^{(0)} \quad (5.64)$$

From these and the Maxwell relation

$$T \left(\frac{\partial \eta^{(0)}}{\partial v} \right)_T = T \left(\frac{\partial p^{(0)}}{\partial T} \right)_v \quad (5.65)$$

we have

$$\gamma^{(0)} c_v^{(0)} = v \beta^{(0)} B_T^{(0)} \quad (5.66)$$

and also

$$c_p^{(0)} - c_v^{(0)} = T \beta^{(0)} \gamma^{(0)} c_v^{(0)}. \quad (5.67)$$

We may rewrite Eq. 5.54 as

$$\rho d\psi = \rho d\psi^{(0)} + \sum_{m=1}^M p^{(m)} d\varepsilon_v^{E(m)} + \sum_{n=1}^N s_{ij}^{(m)} de_{ij}^{E(n)} - \rho \sum_{m=1}^M \eta^{(m)} dT \quad (5.68)$$

From here, another Maxwell relation is generated

$$-\left.\frac{\partial\eta^{(m)}}{\partial\varepsilon_v}\right)_{T,e_{ij}^{E(0)}}=v\left.\frac{\partial p^{(m)}}{\partial T}\right)_{\varepsilon_v^{E(m)},e_{ij}^{E(n)}}=\gamma^{(m)}c_v^{(m)} \quad (5.69)$$

Multiplying this by $Td\varepsilon_v^{E(m)}$, the above definition for $c_v^{(m)}$ by dT , and adding them together yields:

$$-T\gamma^{(m)}c_v^{(m)}d\varepsilon_v^{(m)}+c_v^{(m)}dT=Td\eta^{(m)} \quad (5.70)$$

Summing over all of the elements results in the volumetric temperature increment:

$$dT^{vol}=\frac{1}{c_v^{eff}}\sum_{m=0}^MTd\eta^{(m)}-\rho T\gamma_{eff}dv-\frac{Tv}{c_v^{eff}}\sum_{m=0}^M\beta^{(m)}\frac{p^{(m)}}{\bar{\tau}^{(m)}}dt \quad (5.71)$$

where $c_v^{eff}\approx c_v^{(0)}$, $\beta^{(m)}\approx\beta^{(0)}$, and $\gamma^{eff}\approx\gamma^{(0)}$, which are consequences of the quasi-equilibrium hypothesis. The temperature increment for the deviatoric branches is similarly given by

$$dT^{dev}=\frac{1}{\rho c_v^{eff}}\left[\sum_{n=1}^N\frac{s_{ij}^{(n)}s_{ij}^{(n)}}{2\mu^{(n)}\tau^{(n)}}+\sum_{n=0}^Ns_{ij}^{(n)}\dot{e}_{ij}^{P(n)}\right] \quad (5.72)$$

The total temperature increment is the sum of these,

5.3.4 Volumetric response

Equilibrium is difficult to define for many polymers. Here, it refers to part of the equation of state that represents mechanical responses on very long time-scales (e.g., hours) compared to the typical problem time in plate impact problems (e.g., microseconds). The equilibrium EOS presented in (B. E. Clements, 2012) is a semi-empirical form of the Gibbs free energy:

$$g^{(0)}(p^{(0)}, T)=\sum_{m=0}c_mT^m+c_LT\ln T+v_0\left[p^{(0)}-R\left\{\left(B_T^{(0)}+p^{(0)}\right)\ln\left(1+p^{(0)}/B_T^{(0)}\right)-p^{(0)}\right\}\right]. \quad (5.73)$$

where R is a common parameter for polymer modeling, 0.0894. This EOS is artificially constructed to reproduce two features: (i) a particular empirical fit to the specific heat and (ii) the Tait EOS. The Tait EOS was originally used to model seawater (Tait, 1965), and has since found remarkable success in modeling polymer behavior. In the context of polymers, the Tait EOS is most frequently applied to static dilatometry experiments, where the specific volume is tabulated as a function of temperature and pressure (Quach et al., 1974). The Tait EOS gives v as

$$v(p, T) = \left(\frac{\partial g}{\partial P} \right)_T = v_0 \left\{ 1 - R \ln \left[1 + \frac{p}{B_T} \right] \right\}. \quad (5.74)$$

v_0 and B_T have the parametric forms

$$B_T(T) = B_0 \exp(B_1 T + B_2 T^2) \quad (5.75)$$

$$v_0(T) = a_0 + a_1 T. \quad (5.76)$$

Note that the temperature dependence here will manifest when taking the appropriate derivatives to obtain thermodynamic quantities of interest. Derivatives of $g(p, T)$ give rise to thermodynamic quantities required by the model:

$$c_p(p, T) = -T \left(\frac{\partial^2 g}{\partial T^2} \right)_p \quad (5.77)$$

$$B_T^{-1} = -T \left(\frac{\partial^2 g}{\partial T^2} \right)_p \quad (5.78)$$

$$\beta(p, T) = -\frac{1}{v} \left(\frac{\partial^2 g}{\partial p \partial T} \right)_p \quad (5.79)$$

$$c_v = c_p - v B_T \beta^2 T \quad (5.80)$$

$$\gamma = \frac{v \beta B_T}{c_v} \quad (5.81)$$

In the original GAP formulation, a severe mismatch was found between its Hugoniot and experimental data, such as by (Barker & Hollenbach, 1970). This discrepancy motivated

the introduction of bulk viscoelasticity, which results in a much higher instantaneous bulk modulus under high-strain-rate loading. This was previously discussed in the CMGJC model, where the “ambient” bulk modulus is several times that of the actual ambient bulk modulus. The incorporation of bulk viscoelasticity was largely successful; however, when implementing the GAP model in this work, it was observed that the Hugoniot was only well matched for low shock pressures, ~ 1 GPa. A re-parameterization for the model was attempted, but in the end, the Tait EOS was deemed unable to capture the $U_s - u_p$ experimental data. A separate issue was found with the Tait EOS, where thermodynamic quantities, such as the volumetric expansion β , quickly became non-physical for shock pressures around 2-4 GPa. As these quantities become negative, so do temperature increments for increasing pressure. The volumetric expansion as a function of temperature is presented in Fig. 5.15. At higher pressures, deformation quickly becomes unbounded, and impact simulations crash.

To deal with the shortcomings of the Tait EOS, another equilibrium EOS was introduced. Parametric sweeps were attempted for the Tait, Vinet, and 2^{nd} and 3^{rd} order Birch-Murnaghan equations of state. In the end, the 3^{rd} order Birch-Murnaghan EOS provided the best fit to dilatometry and shock data, with the added constraint that derived thermodynamic quantities remain bounded for the regime of interest, up to 10 GPa, which is the maximum normal stress imposed in the PSPI experiments in Chapter 4.

As discussed in (Katsura & Tange, 2019), the Helmholtz free energy should increase with compression and may be expressed as a series of our strain measure:

$$\Delta\psi = a_0 + a_1 f_\varepsilon + a_2 f_\varepsilon^2 + a_3 f_\varepsilon^3 + \dots \quad (5.82)$$

The absolute value is arbitrary, and hence we take a_0 to be zero. The second term represents pressure, which will be taken to be zero at an initial state. Finally, note that

the strain measure in our case is given as

$$f_\varepsilon = 1 - \left(\frac{v}{v_0} \right)^{(-1/3)}; \quad (5.83)$$

Truncating Eq. 5.82 gives

$$\Delta\psi = a_2 f_\varepsilon^2 + a_3 f_\varepsilon^3 \quad (5.84)$$

where it can be shown (e.g. (Katsura & Tange, 2019)) that

$$a_2 = \frac{9}{2} * B_T[T] v_0[T] \quad (5.85)$$

$$a_3 = -4a_2 \quad (5.86)$$

where B_{T_0} and v_0 are the parametric forms defined above. Similar to the semi-empirical form of the Gibbs free energy in GAP, the semi-empirical GAPHP equation of state is given in its entirety as

$$\psi = a_2 f_\varepsilon^2 + a_3 f_\varepsilon^3 + c_0 + c_1 T + c_2 T^2 + c_3 T^3 + c_4 T^4 + c_L T \ln T \quad (5.87)$$

In general, the full expressions for derived quantities are considerable in length. These expressions are readily generated by means of the symbolic toolbox Mathematica. The results of the fits are shown in Fig. 5.16. Here, the experimental dilatometry data for specific volume are from (Theobald et al., 2001) and the specific heat data are from the ATHAS database (Wunderlich, 1995). Note that the glass transition temperature T_g is taken to be a function of pressure $T_g = T_{g0} + \alpha p$, where $\alpha = 1.6 \times 10^{-7} \text{ Pa}^{-1}$, the same value as in the original GAP formulation. Two sets of parameters are provided above and below this temperature. That is, there are two unique parameter sets for the (pressure-dependent) glassy and rubbery regimes. At the start of each time increment in a simulation, the current temperature is compared to the current value of T_g . If the temperature is below this, then the glassy values are used. Above T_g , the rubbery

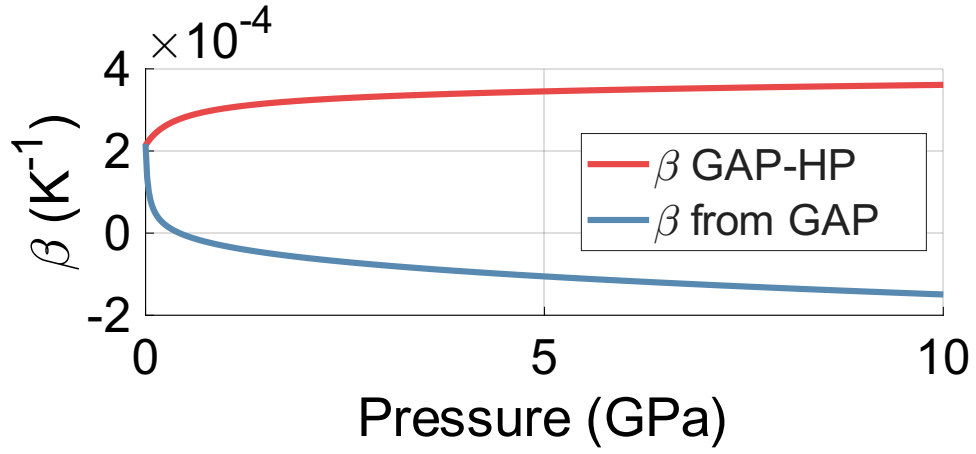


Figure 5.15: $\beta^{(0)}$ for the old GAP parameterization and the new high-pressure parameterization, which will be referred to as GAP-HP hence. Negative values result in negative thermal pressure for positive temperature increments, which are non-physical, and result in simulations crashing.

parameters are used.

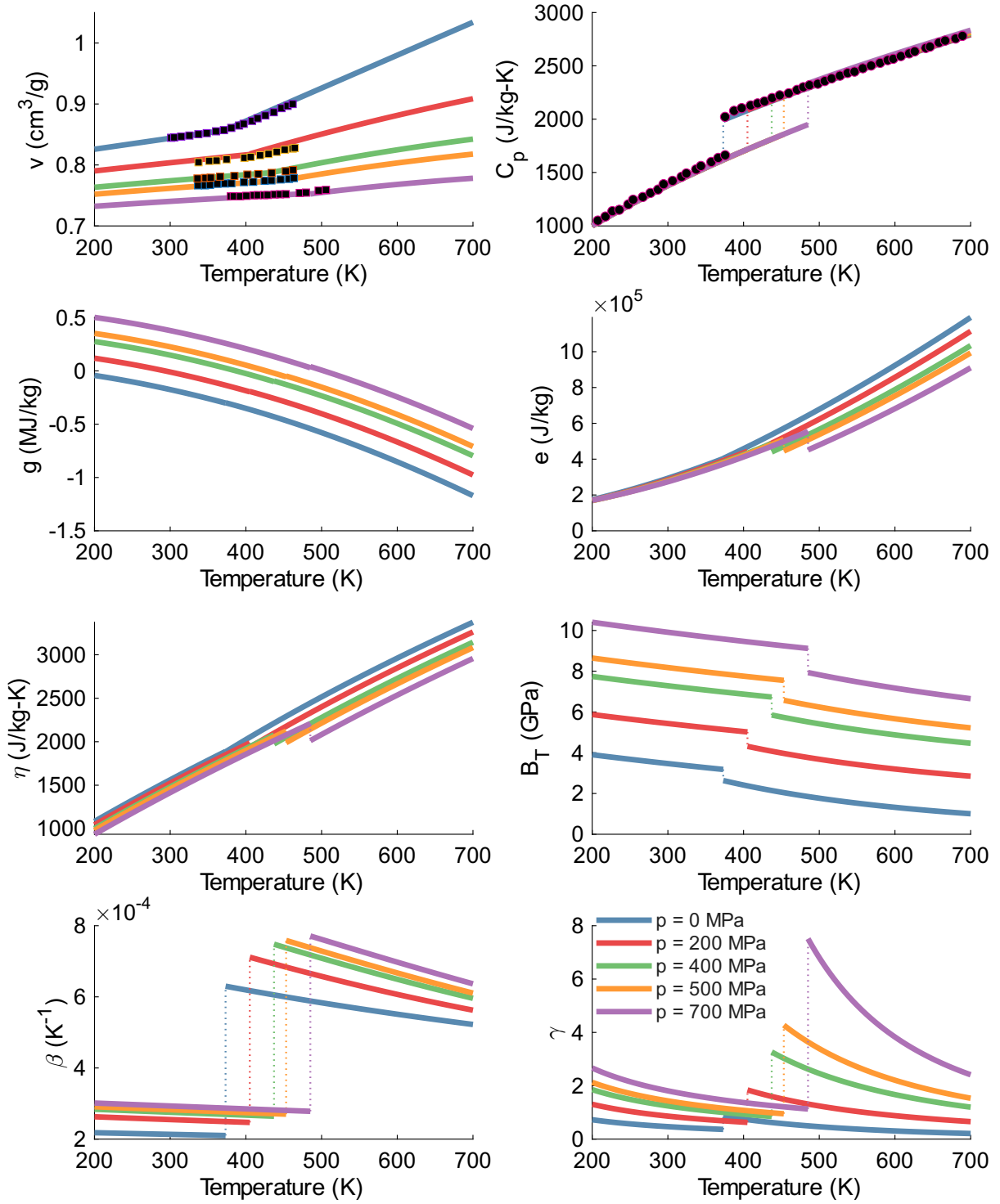


Figure 5.16: Thermodynamic state variables for the GAP-HP model. From left to right, top to bottom: specific volume, specific heat at constant pressure, Gibbs free energy, internal energy, entropy, isothermal bulk modulus, volumetric expansion, and Grüneisen parameter.

5.3.5 Plasticity

The plasticity in the original GAP formulation is known as the *hierarchical flow stress model*. Plastic flow follows a Prandtl-Reuss viscoplastic flow rule (Hill, 1998):

$$de_{ij}^{P(n)} = \Lambda_p^{(n)} s_{ij}^{(n)} dt \quad (5.88)$$

with a flow rule similar to the Bodner-Partom model (Bodner & Partom, 1975) and the version applied to polycarbonate by (Frank & Brockman, 2001):

$$\Lambda_p^{(n)}(t) = \frac{D_0}{\sqrt{J_2^{(n)}(t)}} \left(\frac{\dot{\epsilon}_{eff}}{\dot{\epsilon}_0} \right)^{n_z} \left(\frac{3J_2}{[f(t)Z^{(n)}(t)]^2} \right)^{n_p}. \quad (5.89)$$

This flow rule is not associated with any particular yield surface; in fact, the material is yielding for all possible loading paths, although plastic strains will, of course, be small for small deformations. Each rheological element, both equilibrium and non-equilibrium, has its own flow function.

$Z^{(n)}(t)$ is a load-history dependent variable related to microstructural arrangement during straining. Practically, its purpose is to capture the initial yielding, strain softening, and hardening seen in uniaxial stress-strain curves for PMMA. D_0 is the limiting strain rate. The factor $2J_2^{(n)} \equiv s_{ij}^{(n)} s_{ij}^{(n)}$ is the second invariant of the stress deviator. n_z and n_p are strain rate sensitivity parameters. The effective strain rate is given by

$$\dot{\epsilon}_{eff} = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} \quad (5.90)$$

With $\bar{W}_p^{(n)}$ denoted as the plastic work per unit volume for each Maxwell element, the plastic work increment is given by

$$d\bar{W}_p^{(n)} = s_{ij}^{(n)} de_{ij}^{P(n)} \quad (5.91)$$

$Z^{(n)}(t)$ depends on the plastic work as

$$Z^{(n)}(t) = Z_0^{(n)}(t) + \alpha_W \bar{W}_p^{(n)}(t) \quad (5.92)$$

where α_W is a unitless material parameter.

To incorporate hardening and softening behaviors, $Z_0^{(n)}(t)$ evolves according to a set of hierarchical ordinary differential equations:

$$dZ_j^{(n)}(t) = m_j \left(1 - \frac{Z_j^{(n)}(t)}{Z_{j+1}^{(n)}(t)} \right) d\bar{W}_p^{(n)}(t), \quad j = 0, \dots, J \quad (5.93)$$

with initial conditions

$$Z_j^{(n)}(t=0) = K_j Z_0 \left(\frac{\mu^{(n)}}{\sum_{k=0} \mu^{(k)}} \right) d\bar{W}_p^{(n)}(t), \quad j = 0, \dots, J+1 \quad (5.94)$$

Thus, for the n^{th} element, $Z_j^{(n)}(t=0)$ scales with the distribution of shear relaxation strengths.

The scale factor $f(t)$ enables further temperature and strain-rate sensitivity:

$$f = 1 + f_T \log(a_T) + f_\epsilon \log(\dot{\epsilon}_{eff}/\dot{\epsilon}_0) \quad (5.95)$$

This completes the description of the HFS model. The parameters for the GAPHP model in this work made use of the same parameters as in the original GAP formulation. There, parameters were obtained by matching stress-strain curves in uniaxial compression experiments from $\dot{\epsilon}_{eff} = 10^{-3}$ /s (quasi-static compression via hydraulic machine) to $\dot{\epsilon}_{eff} = 10^4$ /s (dynamic compression via split Hopkinson bar). Simulations were compared over a variety of strain-rates and temperatures, as shown in Fig. 5.17. Both sets of data are taken from (Richeton et al., 2006) and the simulated profiles from (B. E. Clements, 2012) are included as well as dashed lines. All simulations consisted of a single CPE4R

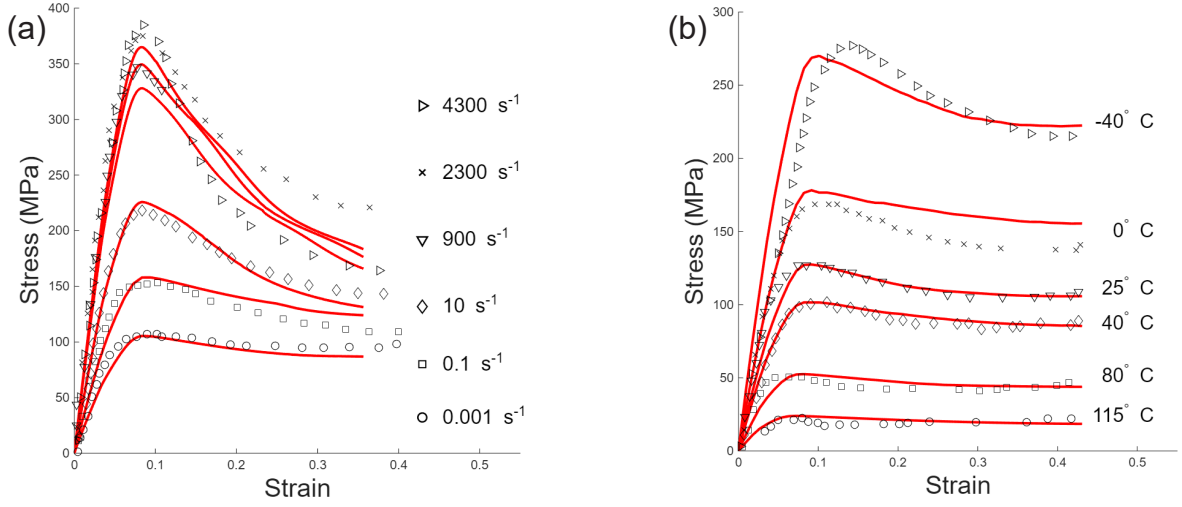


Figure 5.17: Comparison of experimental and simulated stress-strain profiles. The red curves are from this work, and the symbols are from experiments by (Richeton et al., 2006).

element, which had three adjacent faces with roller-type boundary conditions and a fourth face with a velocity boundary condition. The result is a state of uniaxial stress.

Additional plate impact simulations (with the methodology described in the previous two chapters) were conducted to verify GAP-HP for high strain-rate problems. The result is shown in Fig. 5.18. Each line represents 50 simulations conducted for a variety of impact velocities, resulting in temperature, pressure, and volumetric strains associated with a single Hugoniot state.

5.4 Simulations of PSPI experiments on PMMA

The simulated normal stress profiles for shots TP110425, TP111125, and TP112525 with the CMGJC and GAP-HP models are depicted in Fig. 5.19 and Fig. 5.20, respectively. Due to the strong dependence on strain-rate and, to a greater extent, for these experiments, pressure, the CMGJC predicts shear stresses much higher than the experimentally measured profiles. Stress continues to slowly build until the arrival of the release waves. The GAP-HP model, on the other hand, predicts a relatively low, constant shear strength. After reaching its maximum, the shear deformation results in

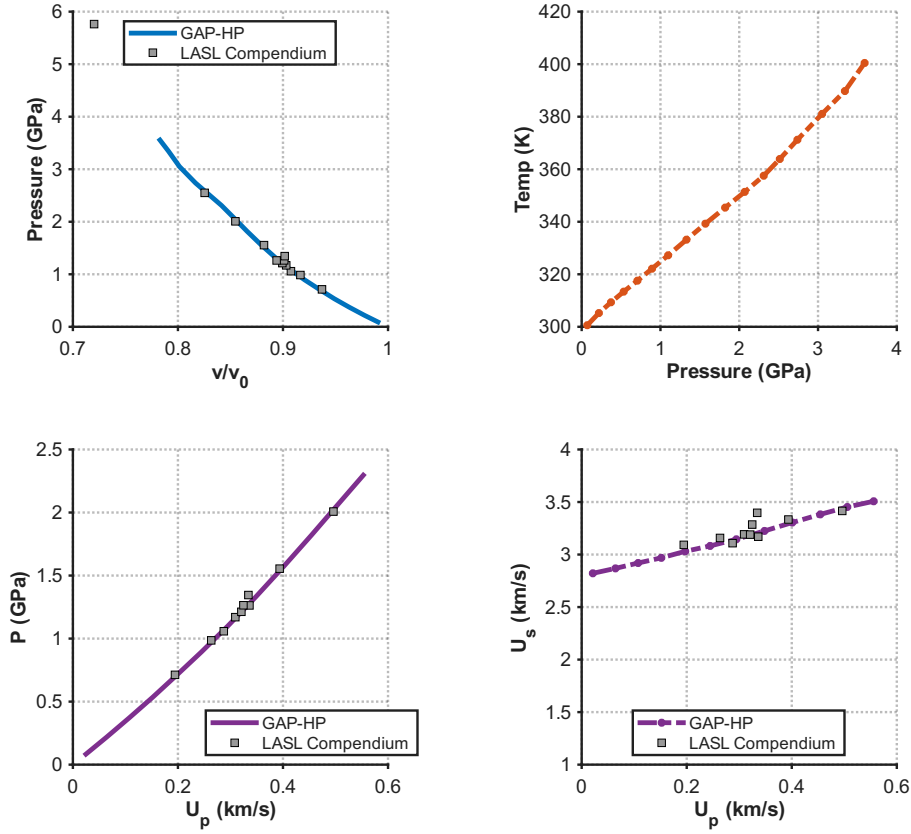


Figure 5.18: Hugoniot states from the GAP model. Experimental data are from the LASL shock compendium. (Mader et al., n.d.).

large amounts of softening. Softening occurs due to both the inherent strain softening in the HFS plasticity model as well as elastic thermal softening.

As has been discussed at length, the pressure dependence of shear strength is a known effect in PMMA as well as other polymers. A model which bears great resemblance to the HFS plasticity model and which is also based on the plastic flow function introduced by (Bodner & Partom, 1975) is the model introduced by (Frank & Brockman, 2001). In this work, the phenomenological flow parameter $Z(t)$ is written to depend explicitly on pressure:

$$Z_{(k)} = Z'_{(k)} \exp \left(-p_{(k)}/P_{0k} \right) \quad (5.96)$$

where k refers to the k^{th} Maxwell element. The pressure sensitivity factor P_0 is assumed

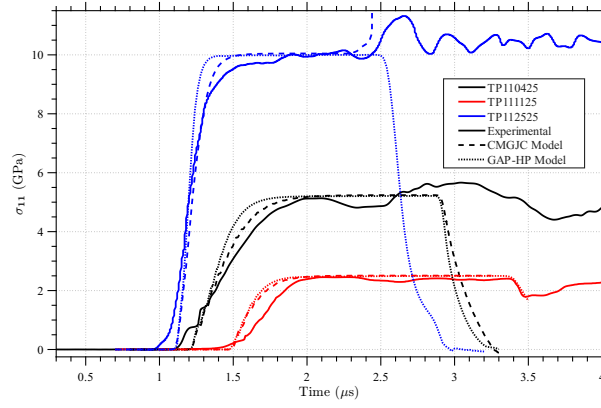


Figure 5.19: Comparison of experimental and simulated normal stress profiles, shifted arbitrarily in time for clarity.

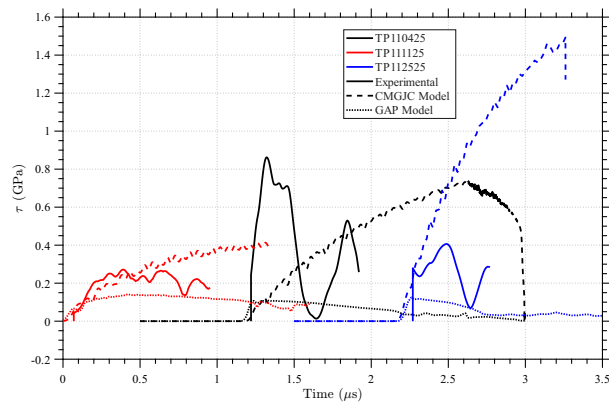


Figure 5.20: Comparison of experimental and simulated normal stress profiles, shifted arbitrarily in time for clarity.

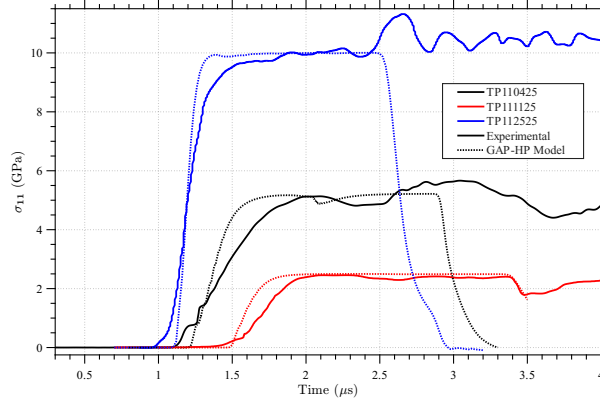


Figure 5.21: Comparison of experimental and pressure-adjusted GAP-HP normal stress profiles, shifted arbitrarily in time for clarity.

to have a distribution time similar to that of the bulk modulus:

$$P_{0(k)} = P_0 \frac{K_{(k)}}{\sum_{i=0}^N K_i} \quad (5.97)$$

In the present state, the HFS model depends only on strain rate, temperature, and plastic work. Thus, a natural step is to employ the apparatus of (Frank & Brockman, 2001) and to set

$$Z^{(n)}(t) = Z_0^{(n)}(t) \left(1 + \left(\frac{p}{p_0} \right)^{p_z} \right) + \alpha_W \bar{W}_p^{(n)}(t) \quad (5.98)$$

where p_0 and p_z are the pressure sensitivity coefficient and pressure sensitivity exponent. Having the yield function depend explicitly on the pressure in this way leads to strikingly different results for the shear stress, as shown in Fig. 5.22. The normal stress profiles are also given in Fig. 5.21. There is a much stronger competition between softening and (pressure) hardening, which results in far better agreement between experiment and simulation. Moreover, for the two shots with higher pressures, TP110425 and TP112525, the failure “rebound” is observed in these profiles, where, after reaching maximum, the shear stress drops precipitously and recovers to some intermediate value.

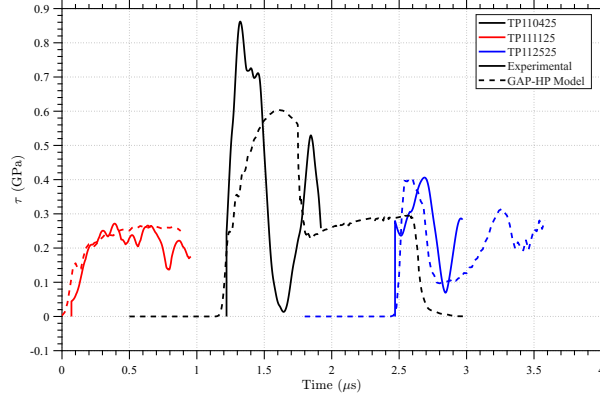


Figure 5.22: Comparison of experimental and pressure-adjusted GAP-HP shear stress profiles, shifted arbitrarily in time for clarity.

5.5 Discussion

Though there is generally good agreement between the simulated and experimental profiles, the normal stress profiles' rise times are in poor agreement. The model here is overestimating the longitudinal wave speed in the weak shock regime. To better account for this, the bulk viscoelastic parameters should be better calibrated to the isentropic wave speed. Reducing the longitudinal wave speed in this region would likely also give rise to the ring-up steps seen in TP110425 and TP111125. This correction could be accomplished by future experiments on thick ($>200 \mu m$) sandwiched samples in normal plate impact, in a manner similar to (Malhotra et al., 2021) for the rubbery polymer HTPB.

The stress rebound in the higher-pressure shots is a result of profound thermal softening, resulting in high levels of shear strain that temporarily stabilize the deformation, again leading to high temperatures, followed eventually by complete collapse of the material at later times. The temperature rise will affect both the bulk and shear shift functions and therefore the corresponding element moduli as well as the plastic flow function $\Lambda^{(n)}$. It is natural to wonder whether the material can actually support such harsh behavior, or whether this would give rise to damage and fracture. Further, if failure does occur, does it proceed in a brittle or ductile manner? Though the temperature increases are extreme, recall that there is a temperature dependence of the glass transition temperature in the

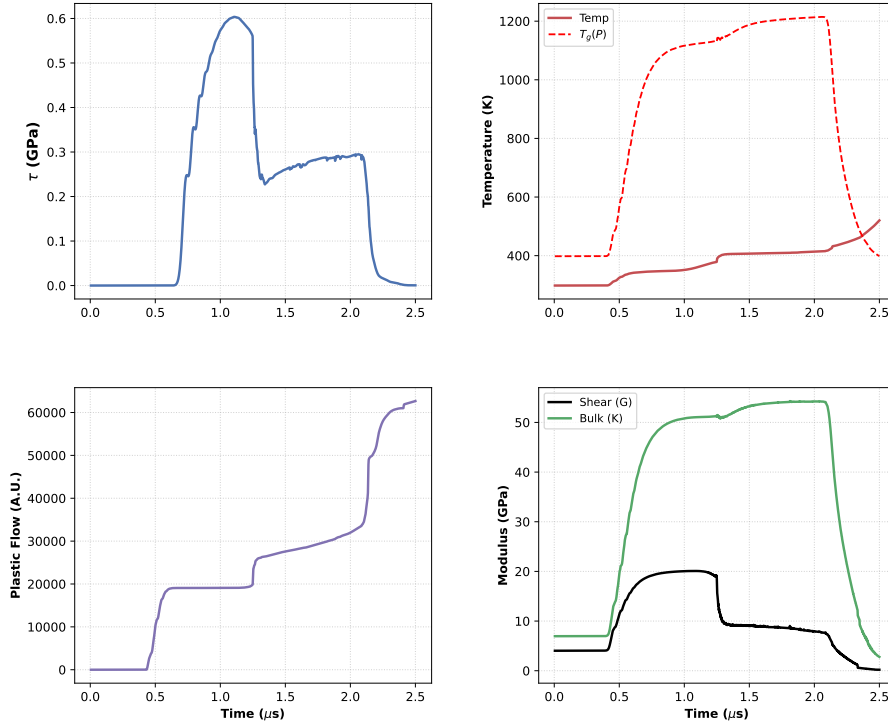


Figure 5.23: Derived simulated quantities over time, from shot TP112525 (a) Shear stress (b) Temperature and $T_g(P)$ (c) Plastic flow (d) Effective bulk and shear moduli.

GAP-HP model. Take shot TP112525 as an example. Here, the pressure is $p = 5.2\text{GPa}$ at the plateau and thus $T_g = 1200\text{ K}$. The temperature plateaus at around 500 K when the shear stress begins to initially decrease. The material is therefore in its glassy state and is not described by the model as being rubbery or having any other secondary relaxation. The implication of this is that though significant thermal softening has occurred, it is not, as in the case of the Johnson-Cook model, simply melting and resulting in a zero-valued stress deviator. The material response at hand is more complex, and there are a variety of mechanisms working in concert to result in the observed response. Looking at the shear stress for TP112525 Fig. 5.23(a) next to the temperature in Fig. 5.23(b), we see there the most significant rise in temperature is initially due to thermoelastic heating. When the shear wave arrives, plastic flow occurs, as seen in Fig. 5.23(c). Eventually, the thermally softened plastic flow function reaches a level of plastic strain that results in structural softening via the hierarchical ODEs in the HFS function until the shear stress reaches a plateau and deforms in an elastic-perfectly plastic manner until the arrival of the release

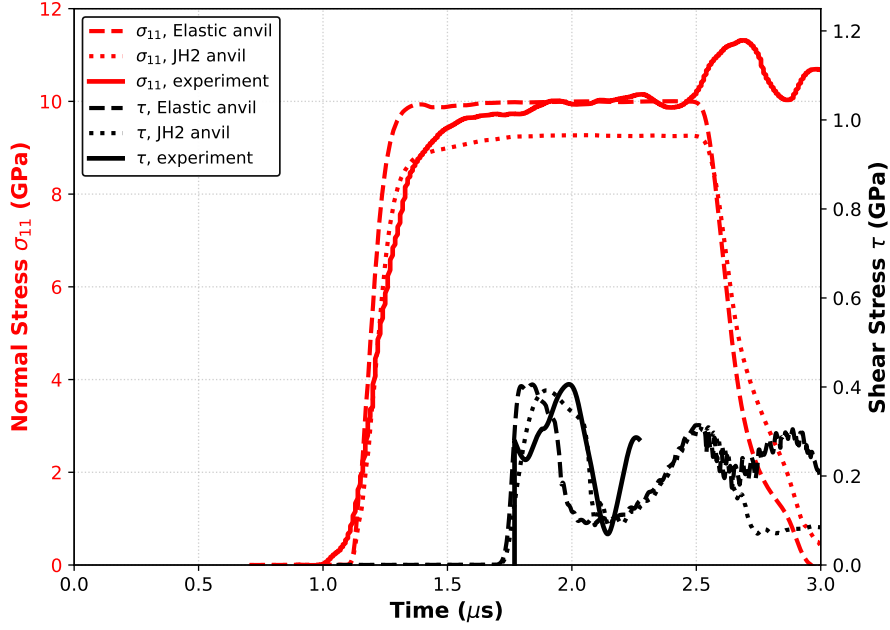


Figure 5.24: Experimental and simulated profiles for shot TP112525 with the GAP-HP model for PMMA, and elastic and JH2 models for the WC anvils.

waves. It should be noted that during this time the effective shear modulus μ_{eff} only negligibly decreases due to the effect of thermal softening (which occurs via the shear shift function a_T), as shown in Fig. 5.23 (d).

Before moving on, anvil plasticity should be discussed. In Chapter 3, it was shown that an elastic assumption was unable to reproduce the long rise time in the experiment. This long rise time relates to the shear wave speed, which may be reduced in the presence of inelastic deformation. The normal stresses for shots TP111125 and TP110425 are at or below the HEL, and thus, inelastic effects are minimal. Shot TP112525, however, witnessed a normal stress of nearly twice the HEL and should therefore be addressed in the model. An additional simulation with the JH2 model for WC (see Chapter 3) was carried out and is presented in Fig. 5.24. The JH2 model variant produces fairly similar shear stress profiles as in the elastic case. Though the anvils' shear wave speeds have been reduced by inelasticity, these speeds and the effective shear moduli are still far greater than that of PMMA. Thus, while the rise time is slightly longer for the JH2 case, the

overall wave profiles are highly similar. More than this, the shear strength is virtually identical. The normal stress, on the other hand, is markedly different in the elastic and inelastic simulations. In particular, the JH2 model gives rise to a normal stress plateau nearly 800 MPa less than the elastic case. The experimental profile lies between these two extremes, where initially the plateau is approximately 100 MPa less than the elastic prediction, but at later times this limit is reached. There is likely an element of inelasticity occurring in the plates, and further calibration of the plasticity model for this range of normal stresses is merited.

In conclusion, the GAP-HP model, while broadly successful in representing plastic flow and the pressure dependence of the flow stress in PMMA, has a level of complexity that could be better understood with novel loading conditions. To gain a better sense of the limitations of the model, as well as what other constitutive ingredients are necessary to capture the dynamic behavior of PMMA subjected to large shear strain rates, another experimental platform is required. The next chapter discusses such an approach in the form of shock-loaded void collapse.

5.A Material parameters

Table 5.3: Summary of the material parameters for the CMGJC model with damage.

Variable	Description	Value
ρ_0	Initial density	1188 kg/m ³
Elastic Properties		
G_0	Shear modulus at ambient	2.19 GPa
G_η	Pressure coefficient of shear modulus	10.69
K_{T0}	Isothermal bulk modulus	8.17 GPa
K'_{T0}	Derivative of bulk modulus with respect to pressure	4.86
$C_{\eta 0}$	Isentropic wave speed	2750 m/s
Γ_0	Grüneisen parameter at ambient	1.09
Thermal Properties		
T_0	Ambient temperature	298 K
c_p^*	Specific heat capacity at ambient	1466 J/(kg·K)
α_{v0}	Volume expansion	227×10^{-6}
k	Thermal Conductivity	0.190 W/(m·K)
c_0		8.22×10^3 (kg·K ⁴)/J
c_1	Empirical specific heat:	7.06×10^{-2} (kg·K ³)/J
c_2	$T^3/(c_0 + c_1T + c_2T^2 + c_3T^3)$	1.16×10^{-2} (kg·K ²)/J
c_3		3.34×10^{-4} J/(kg·K)
a_1	Fit to integral of specific heat:	-320.9 (kg·K)/J
a_2	$\int_0^T c_v dT \approx a_3T + a_1T^2 + a_2T^3$	2.81 J/(kg·K ²)
a_3		0.0 J/(kg·K ³)

Continued on next page

Table 5.3 – continued from previous page

Variable	Description	Value
Johnson-Cook Plasticity		
A	Static yield strength	357 MPa
B	Strain-hardening coefficient	90 MPa
n	Strain-hardening exponent	4
C_3	Strain-rate hardening coefficient	0.077
$\dot{\epsilon}_0$	Reference effective plastic strain-rate	1 s ⁻¹
$\dot{\epsilon}_p^{lim}$	Limiting strain-rate	10 ⁷ s ⁻¹
T_{ref}	Transition temperature	213
T_M	Melt temperature	398
m	Thermal softening exponent	0.74
C_4	Pressure coefficient	0.27
Johnson-Cook Damage		
D_1	Static coefficient	-7.71
D_2	Stress triaxility multiplier	8.1
D_3	Stress triaxility coefficient	-0.143
D_4	Strain-rate coefficient	-0.171
D_5	Thermal softening coefficient	0.316

Table 5.4: Summary of the material parameters for the GAP-HP model.

Variable	Description	Value
General & Physical Properties		

Continued on next page

Table 5.4 – continued from previous page

Variable	Description	Value
ρ	Density	1188 kg/m ³
R	Tait EOS constant	0.0894
T_g	Glass transition temperature	373.15 K
α_{Tg}	Pressure dependence of T_g	$1.6 \times 10^{-7} \text{ }^\circ\text{C}/Pa$
T_0	Reference ambient temperature	298 K
K'_T	Bulk modulus slope in Birch-Murnaghan EOS	0.0
Viscoelastic Relaxation (<i>m</i>-series / Volumetric)		
τ_{m1}	Volumetric relaxation time 1	$1.0 \times 10^4 \text{ s}$
τ_{m2}	Volumetric relaxation time 2	$1.0 \times 10^{-3} \text{ s}$
τ_{m3}	Volumetric relaxation time 3	$1.0 \times 10^{-4} \text{ s}$
τ_{m4}	Volumetric relaxation time 4	$1.0 \times 10^{-5} \text{ s}$
τ_{m5}	Volumetric relaxation time 5	$1.0 \times 10^{-6} \text{ s}$
τ_{m6}	Volumetric relaxation time 6	$1.0 \times 10^{-7} \text{ s}$
$B_{T,m1}$	Viscoelastic bulk modulus 1	$1.5 \times 10^9 \text{ s}$
$B_{T,m2}$	Viscoelastic bulk modulus 2	$1.2 \times 10^9 \text{ s}$
$B_{T,m3}$	Viscoelastic bulk modulus 3	$0.3 \times 10^9 \text{ s}$
$B_{T,m4}$	Viscoelastic bulk modulus 4	$0.3 \times 10^9 \text{ s}$
$B_{T,m5}$	Viscoelastic bulk modulus 5	$0.3 \times 10^9 \text{ s}$
$B_{T,m6}$	Viscoelastic bulk modulus 6	$0.3 \times 10^9 \text{ s}$
Viscoelastic Relaxation (<i>n</i>-series / Deviatoric)		
τ_{n1}	Deviatoric relaxation time 1	$1.0 \times 10^4 \text{ s}$
τ_{n2}	Deviatoric relaxation time 2	$1.0 \times 10^3 \text{ s}$
τ_{n3}	Deviatoric relaxation time 3	$1.0 \times 10^2 \text{ s}$

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Table 5.4 – continued from previous page

Variable	Description	Value
τ_{n4}	Deviatoric relaxation time 4	1.0×10^1 s
τ_{n5}	Deviatoric relaxation time 5	1.0 s
τ_{n6}	Deviatoric relaxation time 6	1.0×10^{-1} s
τ_{n7}	Deviatoric relaxation time 7	1.0×10^{-2} s
τ_{n8}	Deviatoric relaxation time 8	1.0×10^{-3} s
τ_{n9}	Deviatoric relaxation time 9	1.0×10^{-4} s
τ_{n10}	Deviatoric relaxation time 10	1.0×10^{-5} s
τ_{n11}	Deviatoric relaxation time 11	1.0×10^{-6} s
τ_{n12}	Deviatoric relaxation time 12	1.0×10^{-7} s
μ_{n0}	Viscoelastic shear modulus 0	200.0×10^6 s
μ_{n1}	Viscoelastic shear modulus 1	80.0×10^6 s
μ_{n2}	Viscoelastic shear modulus 2	60.0×10^6 s
μ_{n3}	Viscoelastic shear modulus 3	70.0×10^6 s
μ_{n4}	Viscoelastic shear modulus 4	80.0×10^6 s
μ_{n5}	Viscoelastic shear modulus 5	90.0×10^6 s
μ_{n6}	Viscoelastic shear modulus 6	130.0×10^6 s
μ_{n7}	Viscoelastic shear modulus 7	160.0×10^6 s
μ_{n8}	Viscoelastic shear modulus 8	200.0×10^6 s
μ_{n9}	Viscoelastic shear modulus 9	260.0×10^6 s
μ_{n10}	Viscoelastic shear modulus 10	300.0×10^6 s
μ_{n11}	Viscoelastic shear modulus 11	340.0×10^6 s
μ_{n12}	Viscoelastic shear modulus 12	380.0×10^6 s
Equation of State (Glassy Regime)		
$B_{0,lt}$	Bulk modulus scaling parameter	3.2×10^8 Pa

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Table 5.4 – continued from previous page

Variable	Description	Value
$B_{1,lt}$	Bulk modulus temperature dependence 1	$-1.18 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$
$B_{2,lt}$	Bulk modulus temperature dependence 2	$0.00 \text{ }^{\circ}\text{C}^{-2}$
$a_{0,lt}$	Specific volume intercept	$8.38 \times 10^{-4} \text{ m}^3/\text{kg}$
$a_{1,lt}$	Specific volume slope	$1.8 \times 10^{-7} \text{ m}^3/\text{kg}\cdot^{\circ}\text{C}$
$c_{0,lt}$	Specific heat expansion coefficient 0	$6.8 \times 10^4 \text{ J/kg}$
$c_{1,lt}$	Specific heat expansion coefficient 1	$11.0 \text{ J/kg}\cdot\text{K}$
$c_{2,lt}$	Specific heat expansion coefficient 2	$-3.0 \text{ J/kg}\cdot\text{K}^2$
$c_{3,lt}$	Specific heat expansion coefficient 3	$9.5 \times 10^{-4} \text{ J/kg}\cdot\text{K}^3$
$c_{4,lt}$	Specific heat expansion coefficient 4	$-2.75 \times 10^{-7} \text{ J/kg}\cdot\text{K}^4$
$c_{L,lt}$	Logarithmic specific heat parameter	$0.0 \text{ J/kg}\cdot\text{K}$
Equation of State (Rubbery Regime)		
$B_{0,gt}$	Bulk modulus scaling parameter	$3.3 \times 10^8 \text{ Pa}$
$B_{1,gt}$	Bulk modulus temperature dependence 1	$-3.47 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$
$B_{2,gt}$	Bulk modulus temperature dependence 2	$1.00 \times 10^{-6} \text{ }^{\circ}\text{C}^{-2}$
$a_{0,gt}$	Specific volume intercept	8.035×10^{-4}
$a_{1,gt}$	Specific volume slope	5.4×10^{-7}
$c_{0,gt}$	Specific heat expansion coefficient 0	$-8.7 \times 10^4 \text{ J/kg}$
$c_{1,gt}$	Specific heat expansion coefficient 1	$3.2 \times 10^3 \text{ J/kg}\cdot\text{K}$
$c_{2,gt}$	Specific heat expansion coefficient 2	$-2.75 \text{ J/kg}\cdot\text{K}^2$
$c_{3,gt}$	Specific heat expansion coefficient 3	$7.917 \times 10^{-4} \text{ J/kg}\cdot\text{K}^3$
$c_{4,gt}$	Specific heat expansion coefficient 4	$-1.917 \times 10^{-7} \text{ J/kg}\cdot\text{K}^4$
$c_{L,gt}$	Logarithmic specific heat parameter	$-480.0 \text{ J/kg}\cdot\text{K}$
Shift Factors & Kinematics		

Continued on next page

Table 5.4 – continued from previous page

Variable	Description	Value
$p_{1,s}$	Shear shift parameter 1	$-3.169 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$
$p_{2,s}$	Shear shift parameter 2	$-0.101 \text{ }^\circ\text{C}^{-2}$
$p_{3,s}$	Shear shift parameter 3	$6.8 \text{ }^\circ\text{C}^{-1}$
$p_{1,v}$	Volumetric shift parameter 1	$1.8 \text{ }^\circ\text{C}^{-1}$
$p_{2,v}$	Volumetric shift parameter 2	$-0.09 \text{ }^\circ\text{C}^{-2}$
$\dot{\epsilon}_0$	Reference effective strain-rate	$1.0 \times 10^{-4} \text{ /s}$
Stiffening of Moduli		
χ_1	Non-equilibrium bulk modulus coefficient 1	7.0
χ_2	Non-equilibrium bulk modulus coefficient 2	24.0
χ_3	Shear modulus coefficient	0.6
Yield, Flow & Plasticity/Damage Parameters		
f_t	Flow parameter f_t	0.05
f_ϵ	Flow parameter parameter f_ϵ	1.0×10^{-4}
n_z	Hardening/Flow parameter n_z	1.0
n_p	Hardening/Flow parameter n_p	10.0
D_0	Flow function reference strain-rate D_0	$1.0 \times 10^4 \text{ /s}$
Z_0	Yield parameter Z_0	$5.5 \times 10^8 \text{ Pa}$
K_0	Flow function ODE parameter K_0	1.0
K_1	Flow function ODE parameter K_1	1.7
K_2	Flow function ODE parameter K_2	1.6
K_3	Flow function ODE parameter K_3	1.3
K_4	Flow function ODE parameter K_4	0.8
K_5	Flow function ODE parameter K_5	0.9

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Table 5.4 – continued from previous page

Variable	Description	Value
m_0	Flow function ODE parameter m_0	540.0
m_1	Flow function ODE parameter m_1	70.2
m_2	Flow function ODE parameter m_2	16.2
m_3	Flow function ODE parameter m_3	8.1
m_4	Flow function ODE parameter m_4	6.7
α_w	Plastic work coefficient	1.1
p_0	Plastic flow pressure sensitivity coefficient	2.1×10^9 Pa
p_z	Plastic flow pressure sensitivity exponent	3.1

Dynamic void collapse

6.1 Overview

In this chapter, the interaction between shock waves and cylindrical voids in PMMA is presented. Impact experiments are combined with ultra-high-speed x-ray imaging at synchrotron facilities. Shock waves were generated via flyer plate impact, and shock pressures ranging from 0.5 to 2.5 GPa were imposed. Digital Image Correlation (DIC) was used to extract the transient displacement and strain fields under plane strain conditions, allowing for a quantitative 2D analysis of the deformation processes. The resulting kinematic fields were in good accord with predictions from the known material shock Hugoniot curve. The extracted quantitative kinematic fields around collapsing voids suggest that extreme viscoplastic flow occurs before fracture for lower shock pressures, and for higher shock pressures, only viscoplastic flow occurs. In combination with other published results on void collapse in PMMA, these findings hint at a highly complex failure landscape in PMMA.

6.2 Background

Void collapse in solid media plays a key role in energy dissipation and localization, and has seen considerable interest in recent years in the shock physics community. Bowden and Yoffe were among the first to demonstrate that void collapse could result in the formation of “hot-spots” in energetic materials (F. P. Bowden & Yoffe, 1948). These hot-spots are regions of localized high temperature where mechanical energy is dissipated as heat through a variety of mechanisms. Some of these hot spots could become critical, initiating self-sustaining reaction fronts emanating from them, leading to ignition. It is estimated that for ignition, hot-spots have to be of dimensions 0.1-10 μm , occur on time-scales of

$10^{-5} - 10^{-3}$, and temperatures of greater than ~ 700 K (Field, 1991). A variety of hot-spot mechanisms have been considered in the literature, such as: adiabatic compression of gas during void collapse, viscous or plastic heating of surrounding material during void collapse, friction between sliding surfaces or particles in solid matrix, adiabatic shear bands, heating at crack tips, and viscous heating of material extruded at the impacting surface, among others (Field, 1991). The specific mechanisms that produce a hot-spot depend on loading conditions, meso-structure, and material properties, and the identification of these regimes continues to be a topic of study into the present-day (Barua et al., 2013).

Void collapse, as one type of hot-spot mechanism, has seen considerable research interest in recent decades for a variety of materials, geometries, and loading scenarios. In general, for strong impacts (initial pressures on the order of several GPa), hydrodynamic collapse occurs (Meyers, 1994), resulting in the formation of a jet and a vortex that dissipate energy and reach high temperatures (Ranjan et al., 2011). On the other hand, at low to moderate shock loading, localization mechanisms such as adiabatic shear bands and friction become dominant, in which material strength plays an important role (Escauriza et al., 2020; Rai et al., 2020). A variety of computational simulations of void collapse have predicted that the region surrounding a shock-loaded void undergoes shear localization in the form of families of shear bands, which is an important source of hot-spots even in the absence of jet formation and thus may potentially lead to ignition. However, the computational simulations vary a great deal in their predictions with respect to the shear band width, spacing, and energy dissipation (Austin et al., 2015; Hardin & Mares, 2018; Rai et al., 2020; Springer et al., 2018).

Answering these questions would be greatly facilitated by measuring the dynamic temperature fields at such fine scales. Real-time thermal measurements of shear bands and crack propagation, for instance, would prove invaluable in uncovering the nature of extreme deformation and failure in these materials. Pyrometry and Stokes-anti-Stokes thermometry are two contemporary techniques that result in high temporal resolution

data of temperature (B. P. Johnson et al., 2024; Roy et al., 2022). Unfortunately, such techniques are limited to an averaging over one dimension for fine time-scales, and thus, while these techniques provide unique insights into the thermal evolution for shock-loaded materials, they cannot capture the full temperature field around a collapsing void. High-speed thermal imaging cameras are also unable to provide the spatial resolution obtained in this work. Taking a black body at an elevated shock temperature of 500 K results in a peak wavelength of $5.8 \mu\text{m}$ of infrared light. By the Rayleigh criterion of $\lambda/2\text{NA}$, this wavelength would be diffraction-limited to approximately $12 \mu\text{m}$ for a typical 10x objective with a numerical aperture of 0.25. This is the best-case scenario and does not take into account the deficiency in temporal resolution. Shear bands, for example, may have a width on the order of a few microns and a spacing of less than a hundred microns, depending on the material and loading conditions. On the other hand, the diffraction limit of visible light, which is what is emitted by the scintillator and collected by high-speed cameras, is approximately $1\mu\text{m}$ for the same 10x objective with x-ray sources.

Alternatively, thermomechanical models may be parameterized to full-field measurements of transient kinematic fields as obtained in this study. Reproducing the experimentally measured kinematic fields in direct numerical simulations raises confidence in predicting the evolution in thermodynamics surrounding defects in a material, an evolution that in turn may drive the rapid chemistry seen in an ignition process. Thus, in addition to measuring the full deformation field associated with a collapsing void, it is essential to resolve any localization that forms. However, imaging strain fields in the interior of a solid is a challenging task due to (i) the absence of internal markers in the material, (ii) optical aberrations due to the presence of a solid-air interface in the optical path between the solid and the imaging system, and (iii) the need for imaging at high spatial and temporal resolutions simultaneously.

The highly coherent and brilliant X-rays produced by 3rd generation synchrotron facilities are uniquely suited to capture internal motion in shock-loaded materials. The

pulse width of synchrotron pulses is typically 100 ps or less, resulting in minimal intraframe motion and consequent blurring. The combination of high-speed x-ray imaging and plate impact facilities, therefore, creates a rich experimental platform for quantitative measurements during shock-loaded void collapse. Correlating fine features between successive images to obtain the deformation fields is possible via digital image correlation (DIC). Several researchers have made use of this platform in recent years. Lu et al were the first to report full-field quantitative deformation measurements around dynamically loaded voids, and quantified shear band propagation in aluminum at the Advanced Photon Source (APS) (Lu et al., 2014). Wagner et al applied DIC to an additively manufactured simulant and verified the results with photon doppler velocimetry (PDV) profiles at the rear surface of the target (Wagner et al., 2020). Lawlor et al reported on DIC with visible light imaging applied to spherical pore collapse in PMMA (Lawlor et al., 2024) for a pore diameter of 0.8 mm and pressures between 0.4 and 1.0 GPa. Shear localization was observed and explained through an instability induced by thermal softening, with a transition between ductile damage and brittle failure regimes within the pressure range. This work also builds upon the earlier efforts of Escauriza et al, who obtained high-speed x-ray images of spherical pore collapse in PMMA for pore diameters between 3 and 6 mm and pressure between 0.49 GPa and 16.6 GPa (Escauriza et al., 2018). The lower end of shock pressures corresponds to the so-called “strength” regime where plasticity dictates much of the pore collapse process, while the higher end corresponds to the “hydrodynamic” regime where inertial forces dominate. That is, for large pressures and strain-rates, material strength becomes insignificant and solid media begin to deform in a manner akin to fluids. Experiments in the strength regime (for pressures between 0.49 GPa and less than 2 GPa) by Escauriza et al showed damage accumulation, fracture, and perturbations around the pore, possibly indicative of shear banding. As the shock pressure increases, the spacing of the perturbations decreases, and the failure transitions from a brittle mode to an inertially dominated one (for pressures between roughly 2 GPa and 3 GPa). For much higher pressures in the hydrodynamic regime, the pores collapse

completely, and the rapid collapse gives rise to jetting, vortical flow, and phase changes.

In this chapter, the full displacement and strain fields are characterized around a collapsing cylindrical void in PMMA. Given the existing questions on the nature of plasticity, damage, and failure in PMMA under dynamic loading, this characterization is performed in the spirit of developing an experimental methodology that, in the future, can be applied to more complex problems, such as hot-spot mechanisms in energetic materials.

6.3 Methods

Experiments were conducted at two synchrotron facilities: the Dynamic Compression Sector (DCS) at the Advanced Photon Source (APS), located at Argonne National Laboratory, and at the European Synchrotron Research Facility (ESRF) beamline ID-19, located in Grenoble, France. Synchrotron facilities are particle accelerators at their core, and produce highly coherent, brilliant X-rays by means of accelerating electrons to nearly the speed of light. As electrons accelerate and bend at sections of a large ring, typically 1000 m in diameter, high-energy photons are produced. Hard x-rays of energies in the tens of kilo electron-volt range (keV) are common for synchrotrons. Both APS and ESRF are user facilities, whereby researchers apply for “beam-time”, which is typically a period of 1-3 days when experiments are conducted at a beamline. Beamlines are experimental stations along the synchrotron ring, of which there are several dozen. Though many synchrotron facilities exist, there are relatively few that (1) are open to external university researchers on a rolling basis and (2) have impact experiment capabilities. Impact experiments at DCS and ESRF make use of one or two common setups: x-ray diffraction and phase contrast imaging (PCI), which is described in more detail in Section 6.3.1. Both facilities have single-stage and two-stage gas guns, and DCS has a variety of other impact experiments available, such as laser shock and split-Hopkinson bars. The combination of these experimental apparatuses, the full-time scientists and researchers who operate them, and the attractive imaging capabilities made possible by synchrotron x-rays allows for

significant amounts of rich data to be generated in only a few days' time.

6.3.1 Imaging systems

Table 6.1: Comparison of high-speed phase-contrast imaging specifications between the Dynamic Compression Sector (DCS) and the European Synchrotron Radiation Facility (ESRF).

Parameter	DCS (APS)	ESRF
Camera	Pi-Max 4	Shimadzu HPV-X2
Number of Frames	8	384
Sample Size (mm ³)	10.3 × 10.3 × 10.3	28 × 28 × 28
Effective Field of View (mm ²)	2.2 × 2.2	12.8 × 8
Pixel Size (μm)	2.45	32
Mean X-ray energy (keV)	26	30
Sample-detector distance (m)	~ 1	~7
X-ray Pulse Width (ps)	80	100
Interframe Time (ns)	153.4 (24-bunch mode)	176 (16-bunch mode)*
X-ray Attenuation (PMMA)	~60% (10.3 mm thick)	~63% (28 mm thick)
X-ray Attenuation (Gold)	~47% (10 μm thick)	~74% (25 μm thick)

* Note: 176 ns is the time between pulses, and 176.67 is the interframe time.

Technical specifications for the respective imaging systems are given in Table 6.1. Imaging was conducted via X-ray phase contrast imaging (PCI), with the high-energy X-rays coming from the synchrotron beams, as shown in Fig. 6.1. The experiment occurs several hundred meters downstream from the X-ray source. When X-ray beams propagate through a sample, light waves undergo two changes: attenuation (traditional absorption contrast) and a phase shift. If the detector is placed sufficiently far from the sample, the phase-shifted components of the wave will interfere with the unshifted components. In the Fresnel diffraction regime, this phase difference produces discernible intensity variations, resulting in the characteristic “edge enhancement” that allows PCI to resolve fine features such as cracks, interfaces, and shock waves in low-density materials. The detector, therefore, is placed several meters downstream from the sample. After being transmitted through and attenuated by the target chamber windows, PMMA block, and

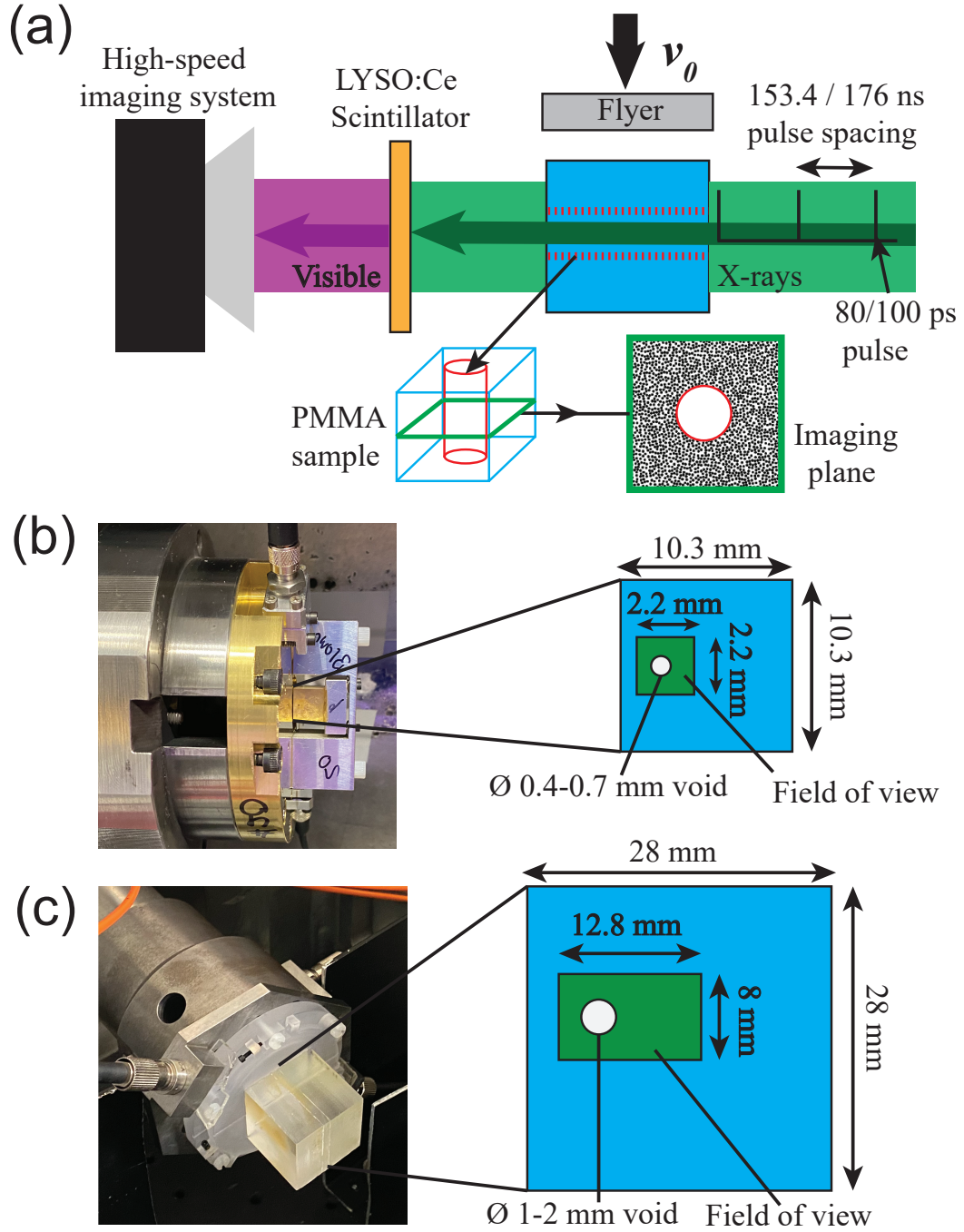


Figure 6.1: Configuration of the impact experiment (a) A schematic of the imaging setup (b) Target assembly and field of view at DCS (c) A schematic of the imaging setup (b) Target assembly and field of view at ESRF.

gold speckles, X-ray pulses are collected by a Lutetium Yttrium Orthosilicate: Cerium (LYSO:Ce) scintillator, which converts the incoming X-rays into visible light. The visible light image is then recorded by high-speed cameras.

At DCS, X-rays were emitted in pulses in 24 bunch mode, which has a temporal spacing of 153.4 ns. Four multiple-gated intensified-charge-coupled-device (ICCD) PI-MAX4 cameras (Princeton Instruments) are used to store 8 image frames. These ICCDs have an inter-frame time of 153.4 ns, which is specifically set to capture images in 24 bunch mode. The x-ray pulse width (80 ps) allows for sharp images with minimal blurring. For particle velocities of up to 1 km/s and a pixel pitch of $2.5 \mu\text{m}/\text{pixel}$, the time for a material point to transit a given pixel on an image is $2.5/1000=2.5$ ns. Since the decay time (~ 40 ns) of the scintillator is also far less than the inter-frame time, blurring is therefore negligible in the obtained images.

The imaging system is triggered using a custom-built laser-photodiode system. Just before impact, the projectile physically blocks the laser, which sends the trigger signal, corresponding to a 50 % voltage drop from the photodiode. For the 5x objective that was used, the ICCD field of view is approximately 2.5 mm by 2.5 mm, or 1024 px by 1024 px. However, the smaller size of the X-ray beam limits the field of view to 2.2 mm by 2.2 mm, or roughly 900 px by 900 px.

At ESRF, the x-ray beam was set in 16 bunch mode, yielding x-ray pulses spaced 176 ns apart. Two HPV-X2 Shimadzu cameras were used for shots 1-4, and three were used for the remainder. 128 images were collected per camera, resulting in a rather wide time window of $67.8 \mu\text{s}$. However, the boundary waves would arrive after approximately 2-10 μs , depending on the impact velocity and type of flyer used. The inter-frame time on the HPV-X2 may only be set to 10 ns increments, and thus, for the case of three cameras, each camera's inter-frame time was set to 530 ns. This resulted in a phase drift throughout the image sequence. In other words, the difference between x-ray pulses and inter-frame time ($520-528=2$ ns) resulted in a smearing between adjacent pulses that

ranged from 0 % in the first frame to 50 % in the worst case (i.e., after approximately 88 images). Nonetheless, the window of interest was less than 10 μs , and so this did not significantly affect the DIC analysis. Images correspond to a 12.8 mm by 8 mm field of view, as shown in 6.1 (c). However, due to the arrangement of the pellicle, there was significant vignetting, which limited the usable portion of the images in the analysis. Each image was 400 by 250 pixels, and the pixel size was 32 μm .

Before using the raw images in DIC, they must be pre-processed. First, the images must be flat-field corrected, whereby non-uniform illumination and sensor artifacts are mitigated. In addition to images taken during an experiment, dark field images are taken, where the image shutters are closed. Whitefield or flat-field images are taken, ideally as close to the experiment as possible. Synchrotron beams typically receive reinjection bunches every 20 minutes to every hour; if flat-field images are taken far before or far after an experiment, the intensity will likely not be representative of what was used in the experiment. To produce a corrected image C from a raw image R , the flat-field equation, which makes use of the dark-field image D and the flat-field image F , is

$$C = m \frac{R - D}{F - D} \quad (6.1)$$

where m represents the average value of $F - D$ (Willmott, 2011). Once the flat-field correction has been applied, images must be registered. Because image sequences consist of inter-woven records from multiple cameras, different camera positions will result in apparent motion in the image record. Prior to the experiment, a calibration image must be taken with each camera. Ideally, this is a grid of known size, e.g., 5 μm dots spaced 10 μm apart. One camera is chosen as a reference, and the image sequences from the remaining cameras are affinely transformed by means of the Image Registration Toolbox in MATLAB. Finally, cropping the images may be required depending on the imaging setup used, thereby reducing the field of view. Once X-ray images have been flat-field corrected and registered, they may be used in DIC.

6.3.2 X-ray Digital Image Correlation

In recent years, DIC has proliferated in use to become one of the most prevalent techniques in experimental mechanics. Here, kinematic fields are extracted during the deformation of a test sample by applying a speckle pattern to the sample surface. Undeformed and deformed images are then compared to obtain displacement fields, which in turn may be differentiated to obtain strain fields. A variety of studies have optimized DIC patterns that produce strain fields that best match analytical results (for example, the stress concentration around a hole in tension or stress intensity factors near a crack tip). The most important features of DIC patterns are as follows:

1. The ideal speckle size is at least 3-by-3 pixels to avoid aliasing, but small enough to ensure a high-density pattern. (Reu, 2014).
2. The ideal DIC pattern has a fill (speckle area divided by total area) of roughly 50 % (Reu, 2014).
3. There is good grayscale contrast. It can be shown that the variance in the measurement of motion is given by $\text{Var}(t)=2 \sigma^2/\sigma_i \nabla F^2(x_i)$ where t is the difference between the exact and measured displacements, σ is the Gaussian white noise standard deviation, and $\nabla F^2(x_i)$ represents the spatial intensity gradients in the image F over pixels x_i (Schreier et al., 2009).
4. The DIC pattern moves and deforms with the material but does not exert significant mechanical stresses on the sample (Barranger et al., 2012).

More detailed descriptions are available elsewhere, but a simple explanation of a prototypical technique is as follows. Images are taken at each time step: $t_0, t_0 + \Delta t, \dots, t_0, +n\Delta t$, resulting in one reference image and n current images. For each current image, the current image and reference images are sectioned into subsets. Each subset in the reference image is subjected to a trial displacement. The reference and current subsets are correlated with

one another by means of some metric, such as least squares. Once the minima of the metric are found by means of an iterative scheme such as the Newton-Raphson method, the calculated displacement is obtained to move from the reference to the current configuration. One should note that this description corresponds to what is known as the Lagrangian form of local DIC, wherein images are divided into subsets and correlated with one another across time steps. Another approach is the global DIC method, which makes use of the mathematical machinery developed for finite element methods. This latter approach is particularly advantageous for finite deformation problems where kinematic compatibility is not ensured for local DIC, since displacements are calculated only for individual subsets. Other hybrid methods, such as the Augmented Lagrangian DIC (AL-DIC) method (Yang & Bhattacharya, 2019), harmonize the computational performance of local DIC with the kinematically compatible fields produced with the global method. Developed at Sandia National Laboratory, the Digital Correlation Engine (DICE) allows for the use of both global and local approaches (Turner, 2017) depending on the problem of interest. However, despite the apparent arguments in favor of applying a global method, the local algorithms Ncorr and VIC2D were found to be highly successful in extracting kinematic fields for comparatively low computational cost.

When preparing speckles on the mid-plane, films as thin as possible were desired, in order not to affect the phenomenon of interest. On the other hand, the film should be thick enough to attenuate the X-rays strongly enough to provide sufficient contrast for propagation-based PCI. Gold was therefore chosen due to its high linear attenuation coefficient μ , allowing for thin but strongly attenuating films. The intensity of light I produced by propagation-based attenuation is captured by the Beer-Lambert law:

$$I = I_0 e^{-\mu d} \quad (6.2)$$

where I_0 is the incident intensity, μ is the linear attenuation coefficient, and d is the propagation distance. Using mass attenuation values for the x-ray energies at hand, we

may estimate the ratio of x-ray attenuation in areas of the sample where gold is present and where it is not. The attenuation levels for these respective regions are provided in Table 6.1. These parameters were able to generate quality DIC results for relatively thin gold films.

The size of speckles must be determined for the imaging system at hand. In the case of the imaging system at DCS, the pixel size was $2.45\text{ }\mu\text{m}$, and therefore a mean speckle size of $10\text{ }\mu\text{m}$ was chosen. For the imaging system at ESRF, the pixel size was $32\text{ }\mu\text{m}$, and a speckle size of $75\text{ }\mu\text{m}$ was targeted.

6.3.3 Sample preparation

Samples were machined from cast PMMA sheets (PLEXIGLAS®). The first set of samples, studied at DCS, was approximately 10.3 by 10.3 by 10.3 mm cubes and consisted of two halves that were bonded together by a cyanoacrylate adhesive. The second set of samples, studied at ESRF, was 28mm by 28 mm by 28 mm cubes after bonding. Samples were machined to size and subsequently lapped with $30\text{ }\mu\text{m}$ lapping disks on a granite block to ensure proper flatness.

In both sets of samples, a gold speckle pattern was applied so that after bonding, the mid-plane contained this pattern, as shown in 6.1(a). Initially, patterns were attempted by vapor depositing gold with the Lesker Lab 18 physical vapor deposition machine. This process proved to be exorbitant, however, as approximately 1 gram of gold produced 100 nm of thickness on the samples. During experimental beamtime, it was observed that in order to generate sufficient speckle contrast, a minimum of $5\text{ }\mu\text{m}$ was needed for each sample.

A far more economical means of preparing patterns involves the use of thin foils. After being machined to size, samples were sonicated with isopropanol and de-ionized water and dried with nitrogen. $7\text{ }\mu\text{m}$ foils at DCS and $25\text{ }\mu\text{m}$ foils at ESRF were carefully bonded to one side of the half-blocks via cyanoacrylate. The photolithography recipe outlined

in Chapter 4 was repeated for the blocks, except with specific DIC speckle patterns for each geometry. These patterns were generated by means of a Python macro implemented in the photolithography design software KLayout. The two primary variables were the speckle size and the in-fill, which was set at 50%.

After exposing, developing, and cleaning in de-ionized water, samples were placed in a bath of potassium iodide. Potassium iodide is a common gold etchant with etchant rates typically around $28\text{\AA}/\text{s}$. Following this, photoresist was cleaned from the samples by means of oxygen ashing (PlasmaTherm©). Samples were then bonded with cyanoacrylate and lightly clamped to reduce the presence of trapped air. To ensure flat impact surfaces, samples were again lapped along the bonded edges. Flyer plates, due to their small size, were additionally lapped by hand in this way. Finally, cylindrical holes were introduced by drilling and reaming holes to the desired size. Water was used as a coolant to reduce the effect of thermal stress.

After samples were constructed, two target fixtures were used. At DCS, an aluminum tip/tilt stage was used. Impact surfaces were aligned to the front side of brass mounting plates by means of an autocollimator (see Chapter 3). These brass plates additionally carried visible laser light, which was used as the time-of-arrival trigger for the imaging experiment. The assembly is depicted in Fig. 6.1. At ESRF, samples were simply bonded to plastic target holders that were mechanically fastened to the muzzle end of the gas gun.

6.3.4 Impedance Matching

Depending on the objective of the particular shot, a target shock pressure was selected prior to shot execution. Based on the desired shock pressure, impact velocities may be generated from a technique known as impedance matching. By applying a momentum balance at the impact surface, the pressures and particle velocities must match at the moment of impact. Additionally, the particle velocity of the flyer prior to impact is simply the impact velocity V_0 . These conditions define the resultant shock pressure and

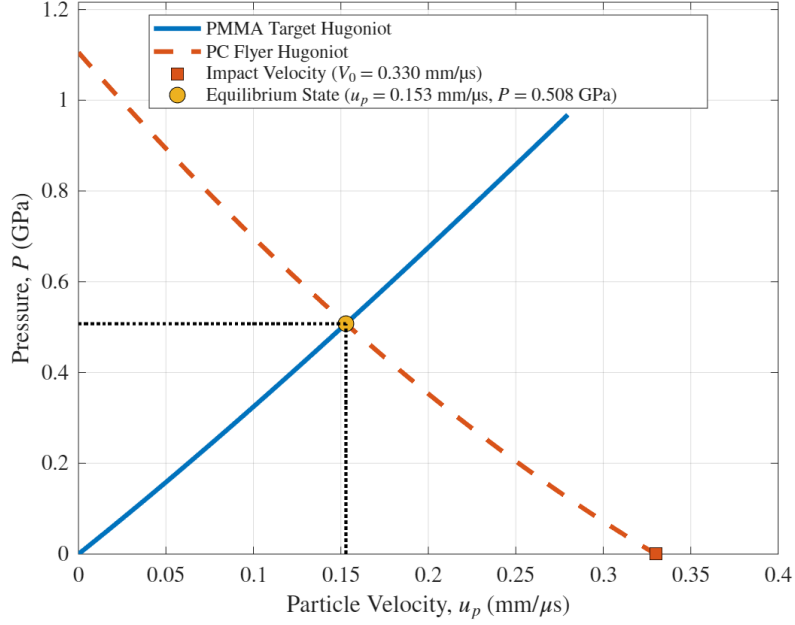


Figure 6.2: Impedance matching diagram for shot ESRF-52318.

particle velocity, which may be determined by Hugoniot data available in the literature. An example impedance match is shown in Fig. 6.2. Such an analysis results in impact velocities for a desired shock pressure list, and these velocities may be determined from a given gas gun's velocity curve (impact velocity versus breech pressure).

6.4 Results

A shot matrix detailing the shots conducted at the two facilities is given in Table 6.2. Impact occurs from the left-hand side in all images. One may observe the shock entering the frame, moving along in, and beginning to interact with the void in Fig. 6.3. At DCS, camera trigger times were chosen so that, even with the uncertainty of the timing, at least one dynamic frame would be captured before the arrival of the shock wave at the hole interface.

At ESRF, 128 images were captured for each camera for a total of 256 for the two-camera arrangement and 384 for the three-camera arrangement. The experimental time window of interest is before the arrival of the boundary waves, after a few (~ 3 -5) microseconds.

Table 6.2: Experimental shot matrix organized by impact conditions, target geometry, calculated shock states, and predicted arrival times.

Shot	Flyer	Impact	Geometry (mm)				Shock State				Timing (μ s)		
		V_0 (mm/ μ s)	D_0	x_{hole}	l_f	l_s	P (GPa)	U_p (mm/ μ s)	U_s (mm/ μ s)	$U_{s,f}$ (mm/ μ s)	t_{front}	t_{bound}	Δt_{shock}
DCS-23-2-026	PMMA	0.696	0.475	1.74	4.00	10.3	1.30	0.348	3.15	3.15	1.43	2.11	2.54
DCS-23-2-027	PMMA	0.400	0.466	1.73	4.00	10.3	0.681	0.202	2.85	2.90	1.58	2.33	2.76
DCS-23-2-028	PMMA	0.494	0.488	1.74	4.00	10.3	0.859	0.250	2.89	2.97	1.56	2.30	2.69
DCS-23-2-029	PMMA	0.582	0.647	1.82	4.00	10.3	1.03	0.297	2.93	3.04	1.54	2.27	2.63
DCS-23-2-030	PMMA	1.03	0.475	1.74	4.00	10.3	2.08	0.513	3.43	3.43	1.31	1.94	2.34
DCS-23-2-031	PMMA	0.703	0.353	1.68	4.00	10.3	1.31	0.352	3.15	3.15	1.43	2.11	2.54
DCS-23-2-032	PMMA	0.704	0.348	1.67	4.00	10.3	1.32	0.352	3.15	3.15	1.43	2.11	2.54
DCS-23-2-035	PMMA	1.02	0.666	1.83	4.00	10.3	2.07	0.510	3.42	3.42	1.32	1.94	2.34
DCS-23-2-036	PMMA	0.700	0.490	1.75	4.00	10.3	1.31	0.350	3.15	3.15	1.43	2.11	2.54
ESRF-52302	PC	0.479	1.15	5.58	6.00	28.0	0.778	0.228	2.87	2.60	5.22	6.61	4.62
ESRF-52303	PC	0.333	1.06	5.53	6.00	28.0	0.513	0.154	2.80	2.41	5.36	6.79	4.99
ESRF-52304	Al	0.390	1.06	5.53	5.00	28.0	1.10	0.315	2.94	5.45	5.10	6.46	1.83
ESRF-52305	Al	0.635	1.55	5.78	5.00	28.0	2.02	0.500	3.40	5.53	4.41	5.58	1.81
ESRF-52306	PC	0.329	1.60	5.80	6.00	28.0	0.506	0.152	2.80	2.40	5.36	6.79	5.00
ESRF-52317	Cu	0.656	1.03	5.51	3.00	28.0	2.48	0.587	3.55	4.04	4.22	5.35	1.48
ESRF-52318	PC	0.330	2.17	6.09	6.00	28.0	0.508	0.153	2.80	2.40	5.36	6.79	5.00

Variables: V_0 : impact velocity; D_0 : void diameter; x_{hole} : hole center coordinate; l_f : flyer thickness; l_s : sample thickness; P : peak shock pressure; U_p : particle velocity; U_s : target shock velocity; $U_{s,f}$: flyer shock velocity; t_{front} : shock front arrival time at the boundary lateral coordinate; t_{bound} : lateral boundary release arrival time at the center of the sample; Δt_{shock} : total shock transit time through the sample thickness.

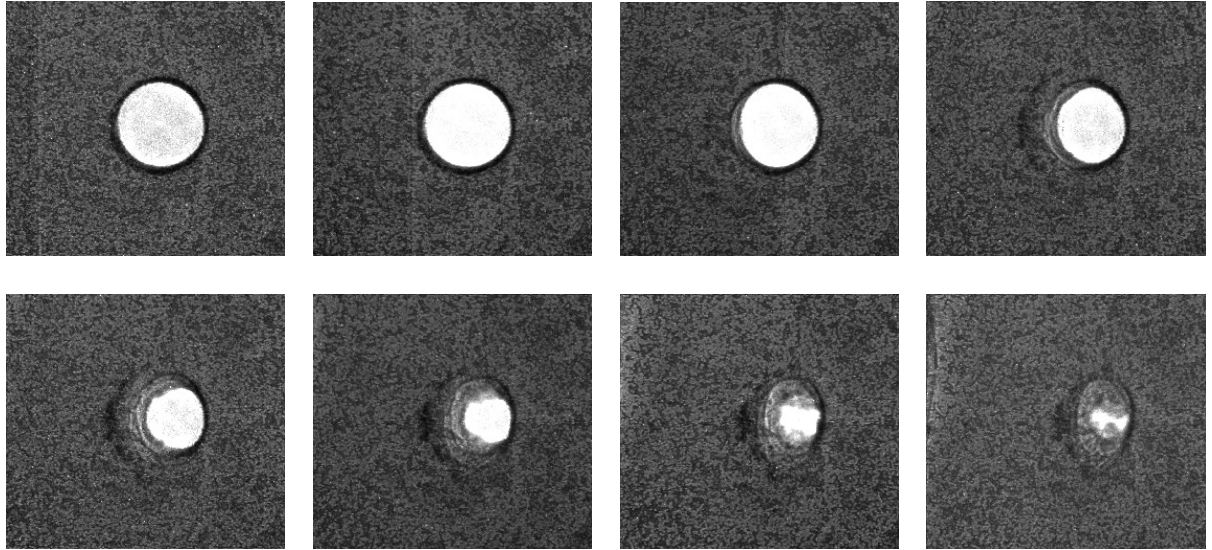


Figure 6.3: Raw images from DCS shot DCS-23-2-029 increasing in time by 153.4 ns from left to right, top to bottom.

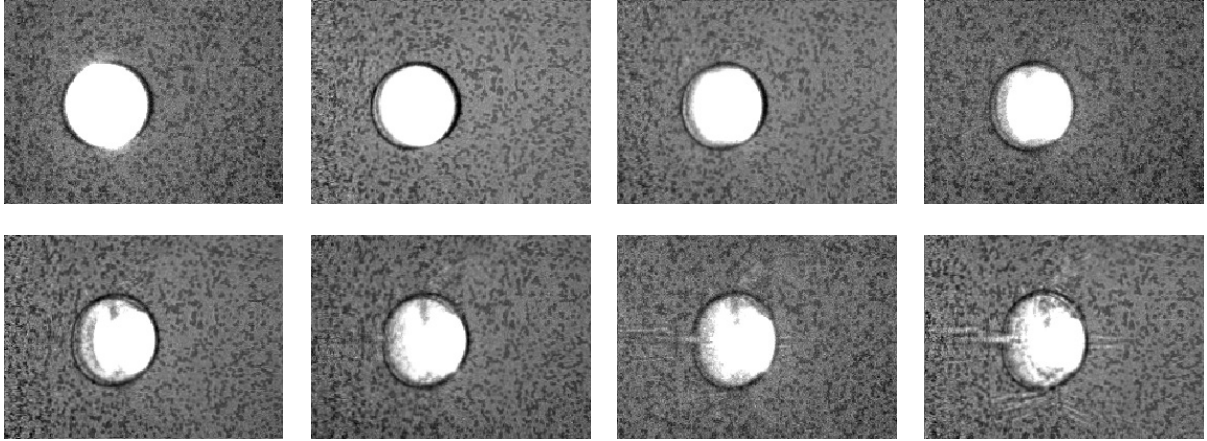


Figure 6.4: Raw images from ESRF shot ESRF-52318 increasing in time by $1.76 \mu\text{s}$ from left to right, top to bottom.

Again, one may observe the first interaction of the shock wave with the void and the subsequent collapse of the void in Fig. 6.4.

In general, the overall contrast from these X-ray images is amenable to available DIC algorithms, and the patterns showed high fidelity to the samples with minimal amounts of distortion prior to the arrival of the release waves. After the arrival of release waves, damaged regions quickly opened up into mixed-mode fracture, in particular for the lower-pressure shots. Laterally, Mode I cracks opened up on either side of the pore. These cracks propagate at close to the ambient Rayleigh speed, approximately 800-900 m/s for all lower-pressure shots at ESRF. A quintessential example of this post-release fracture is given by ESRF-51238 in Fig. 6.4. Such features were generally only observed at ESRF, as the constraint on the number of frames at DCS limited the experimental time window. However, small whisker-like features were observed near the pore for DCS-23-2-027 and DCS-23-2-029 and are believed to be micro-cracks only a few pixels ($< 10\mu\text{m}$) wide. A clear crack was observed for DCS-23-2-028, which is discussed further in Section 6.4.3.

A “phase diagram” of the collapse modes is depicted in Fig. 6.5. These frames were selected just before complete collapse occurs, or just after the arrival of the release waves for the void that did not completely collapse. (Escauriza et al., 2020) noted that for spherical void collapse in PMMA, complete collapse was observed above roughly 1.25 GPa,

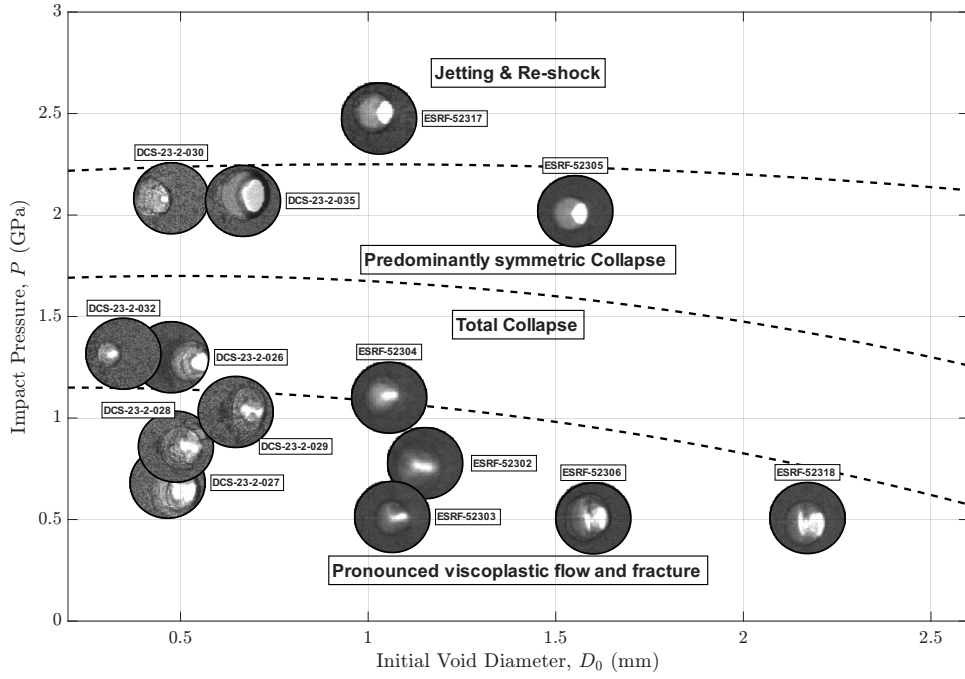


Figure 6.5: A “phase diagram” representing the modes of void collapse observed in the current work.

which is similar to what is observed for this experimental campaign. Parenthetically, the collapse pressure threshold is somewhat difficult to discern in practice. When shock-loaded cylindrical voids begin to completely collapse, collapse first occurs in the central region of the hole (that is, central length-wise). When the sample is shocked, release waves propagate inward from the lateral traction-free surfaces, thereby reducing the applied stress. The overall result is an hourglass shape, where the mid-plane distorts to a greater extent than the outer regions. This can be observed in shot DCS-23-2-030 in Fig. 6.5, for example. In this grayscale image, the bulk is dark gray, while the left-hand side of the pore appears as light gray, which indicates that only part of the cylindrical void has completely collapsed. To the right of this, the void remains bright, indicating x-rays are unobstructed by this yet uncollapsed region of the void.

Below the collapse pressure threshold, pronounced viscoplastic flow occurs above and below the void, pinching in towards the center. In the regime where the collapse pressure threshold has just been reached, as in ESRF-52304, there is still substantial viscoplastic

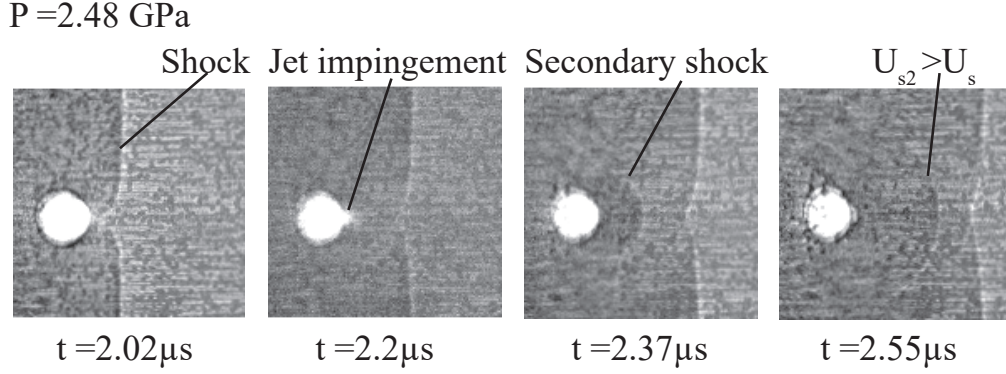


Figure 6.6: ESRF-52317, which shows the emergence of a secondary shock after hydrodynamic jetting.

flow, resulting in relatively elongated voids at the moment of collapse. As the shock pressure increases, the collapse morphology tends towards symmetry. At the highest pressure, in ESRF-52317, jetting appears to occur, resulting in a secondary shock that eventually catches up with the initial shock wave, as shown in Fig. 6.6. Unfortunately, at this high end of the induced shock pressure range, speckle patterns become extremely distorted, thereby limiting DIC analysis after the shock wave reaches the void.

6.4.1 Kinematic analysis with DIC

The overall contrast from these X-ray images is amenable to the available DIC algorithms. The uncertainty in displacement may be assessed by means of a program written by (Estrada & Franck, 2015), which computationally distorts a static image, thereby simulating simple forms of deformation such as tension and simple shear. This analysis yielded an uncertainty of ± 1 pixel in displacement for both the DCS data and the ESRF data. The noise floor can also be estimated by correlating two static images and recording the variance in displacement. The noise floor was determined to be approximately 0.1 px for the DCS data and 0.5 px for the ESRF data, depending on the sample. Subpixel displacements are possible due to the use of interpolation splines. For ESRF, DIC was primarily performed with a subset size of 27 px and a spacing of 0 px. For DCS, a larger subset size of 65 px and a spacing of 2 px was used.

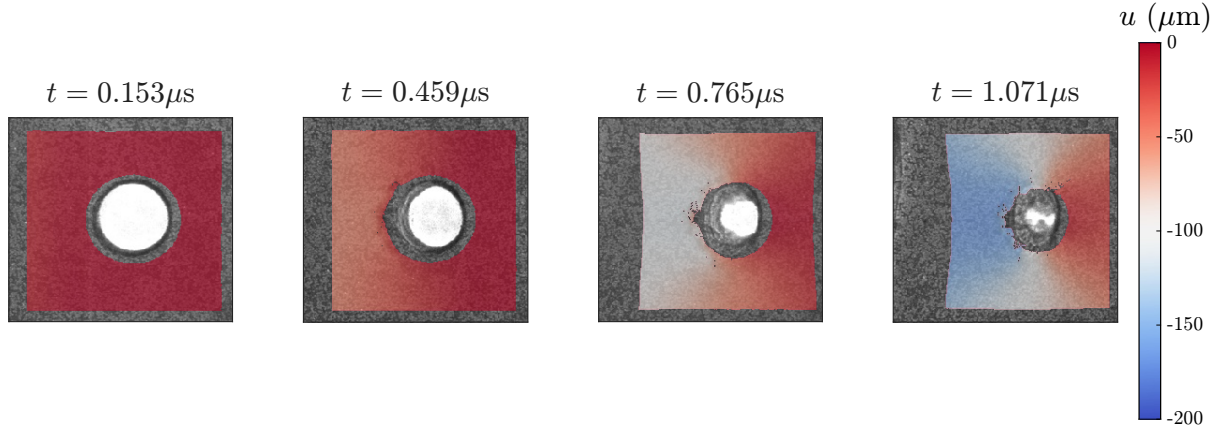


Figure 6.7: Horizontal displacement fields from shot DCS-23-2-029.

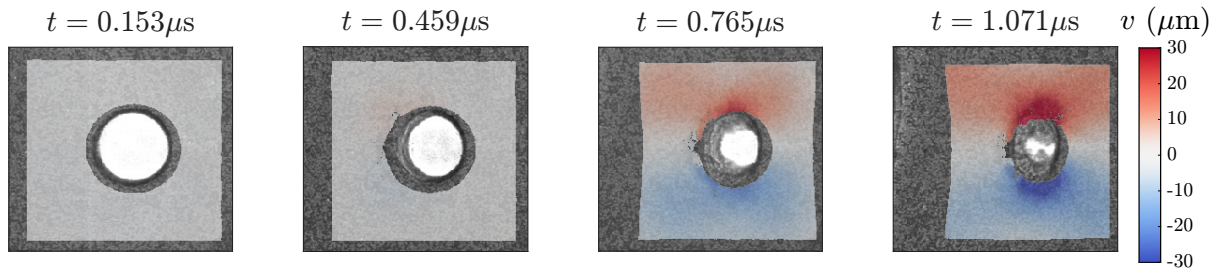


Figure 6.8: Vertical displacement fields from shot DCS-23-2-029.

Lagrangian horizontal (U) and vertical displacement fields (V) for a typical shot at DCS are shown in Fig. 6.7 and 6.8, respectively. Deformation fields may be resolved to within a fraction of the void diameter, but in the vicinity of collapse, the speckle pattern distorts, rendering analysis impossible for the current configuration. An attempt was made to coat the interior of the pore with a gold based paint to provide a better qualitative picture of the collapse evolution, but this resulted in little success.

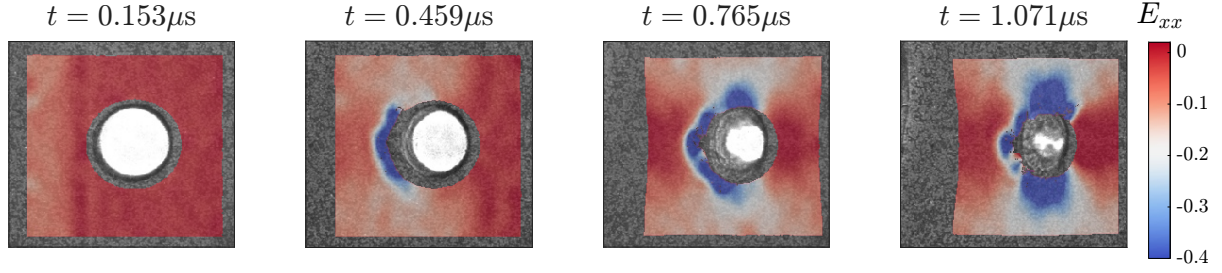


Figure 6.9: XX component of strain fields from shot DCS-23-2-029.

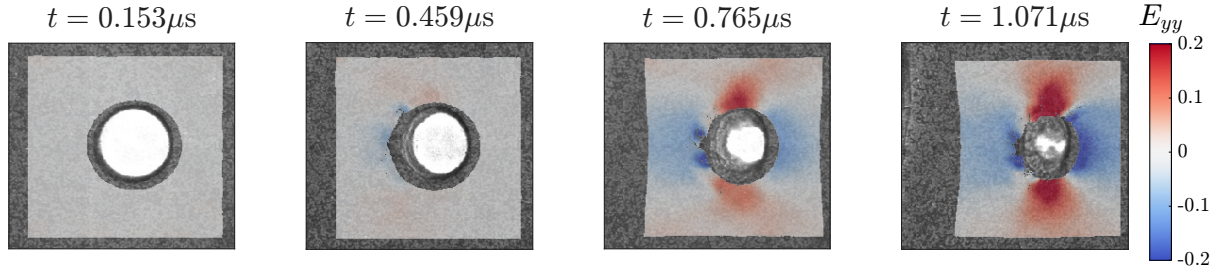


Figure 6.10: YY component of strain fields from shot DCS-23-2-029.

These displacement fields may be differentiated to obtain (Lagrangian) strain fields:

$$E_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} + \frac{\partial U_k}{\partial X_i} \frac{\partial U_k}{\partial X_j} \right) \quad (6.3)$$

$$(6.4)$$

$$E_{XX} = \frac{\partial U}{\partial X} + \frac{1}{2} \left(\left(\frac{\partial U}{\partial X} \right)^2 + \left(\frac{\partial V}{\partial X} \right)^2 \right) \quad (6.5)$$

$$(6.6)$$

$$E_{YY} = \frac{\partial V}{\partial Y} + \frac{1}{2} \left(\left(\frac{\partial U}{\partial Y} \right)^2 + \left(\frac{\partial V}{\partial Y} \right)^2 \right) \quad (6.7)$$

$$(6.8)$$

$$E_{XY} = \frac{1}{2} \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} + \frac{\partial U}{\partial X} \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \frac{\partial V}{\partial Y} \right) \quad (6.9)$$

$$(6.10)$$

The E_{XX} strain field shows the presence of the shock wave quite well. The bulk initially experiences uniform compressive strain due to the passage of the shock wave. After the shock interacts with the traction-free surface of the void, pressure is relieved, resulting in

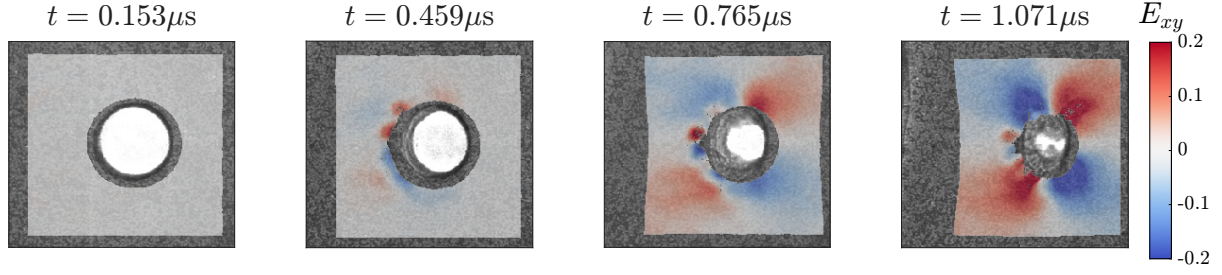


Figure 6.11: XY component of strain fields from shot DCS-23-2-029.

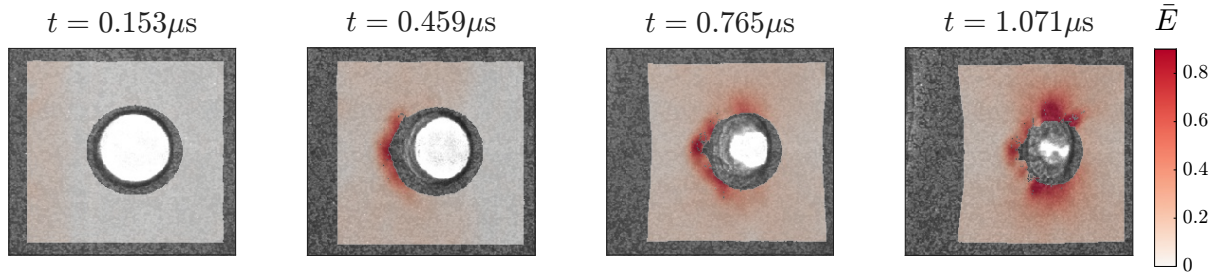


Figure 6.12: Mises strain fields from shot DCS-23-2-029.

a state of near-zero strain on the left side of the void. No pressure is transmitted through the void, as shots are done under vacuum, and thus the right side is also minimally compressed. The greatest areas of longitudinal strain develop above and below the void. E_{YY} demonstrates the opposite effect, where maxima are observed above and below the void where material flows towards the center of the void (at least in the strength-dominated regime), and minima form to either side of the void. Minima and maxima of the shear strain field E_{XY} are observed at approximately 45° to the impact direction. Intuitively, this stands to reason, since the shear stress on a plane angled at θ relative to the impact axis is given by

$$\tau = \frac{\sigma_1 - \sigma_2}{2} \sin(2\theta). \quad (6.11)$$

Since the principal axis is aligned with the impact axis with principal stress σ_1 , the material therefore experiences the maximum driving force for shear deformation at $\theta = \pm 45^\circ$. Finally, we plot the Mises equivalent strain $\bar{E}$ given by

$$\bar{E} = \frac{2}{3} \sqrt{E_{11}^2 - E_{11}E_{22} + E_{22}^2 + 3E_{12}^2}. \quad (6.12)$$

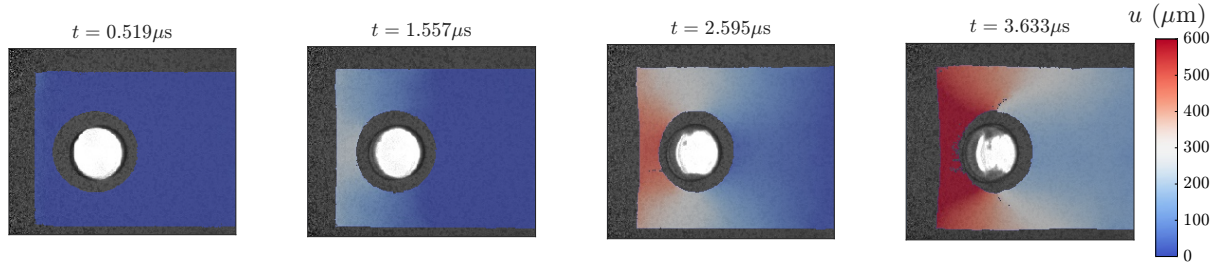


Figure 6.13: Horizontal displacement field from shot ESRF-52318.

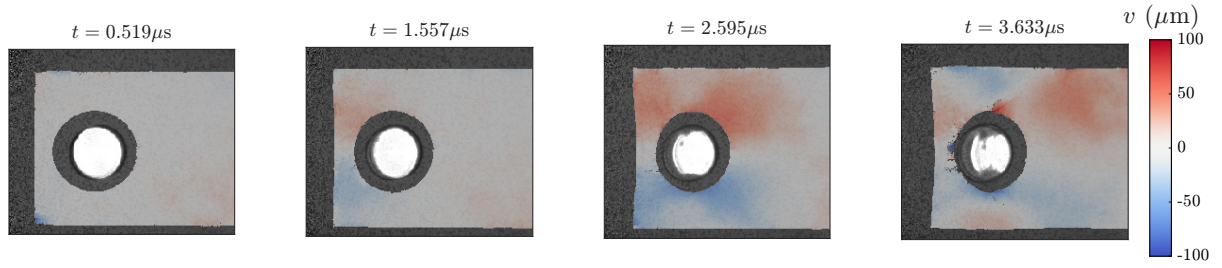


Figure 6.14: Vertical displacement field from shot ESRF-52318 at ESRF.

The Mises strain for DCS-23-2-029 is given in Fig. 6.12. Mises equivalent strain helps to characterize regions of intense inelastic deformation, which occur above and below the pore, where viscoplastic flow is the most extreme.

Finally, this procedure is repeated for another typical experiment, ESRF-52318: displacements U and V are given in Fig. 6.13 and Fig. 6.14, respectively, while the strain field E_{XX} is given in Fig. 6.15, E_{YY} in Fig. 6.16, E_{XY} in Fig. 6.17, and the Mises strain $\bar{E}$ in 6.18.

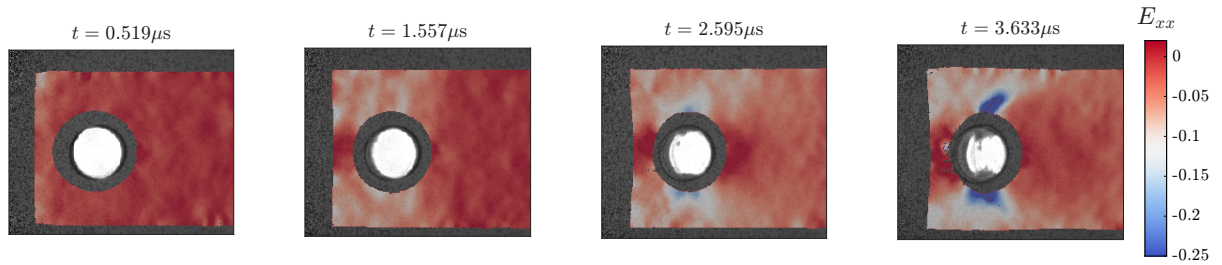


Figure 6.15: XX component of strain from shot ESRF-52318 at ESRF.

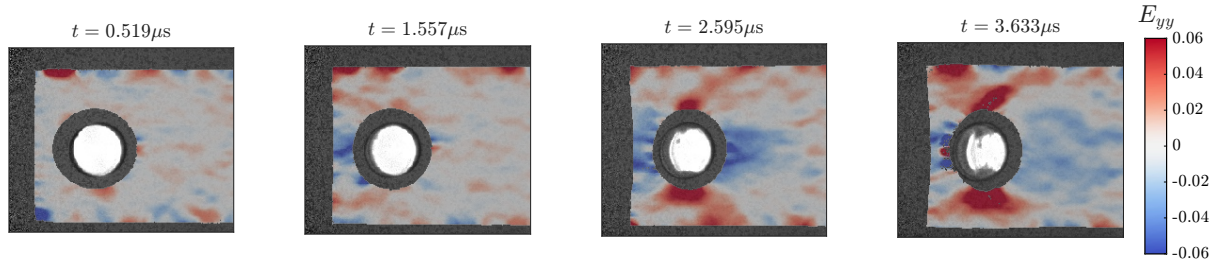


Figure 6.16: YY component of strain from shot ESRF-52318 at ESRF at increasing time steps of 153 ns, from left to right, top to bottom.

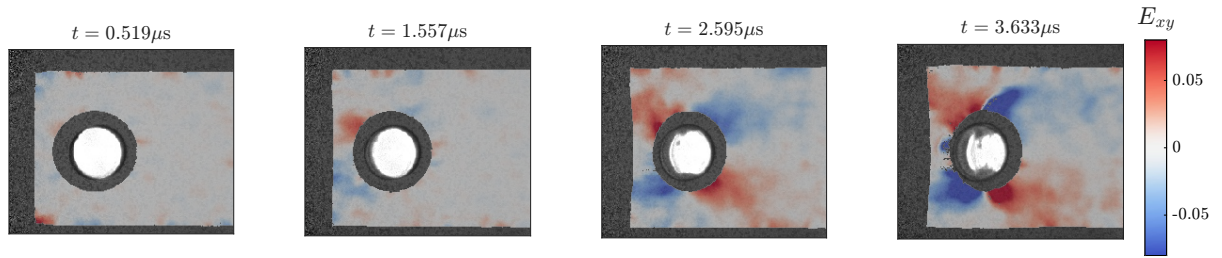


Figure 6.17: XY component of strain from shot ESRF-52318 at ESRF.

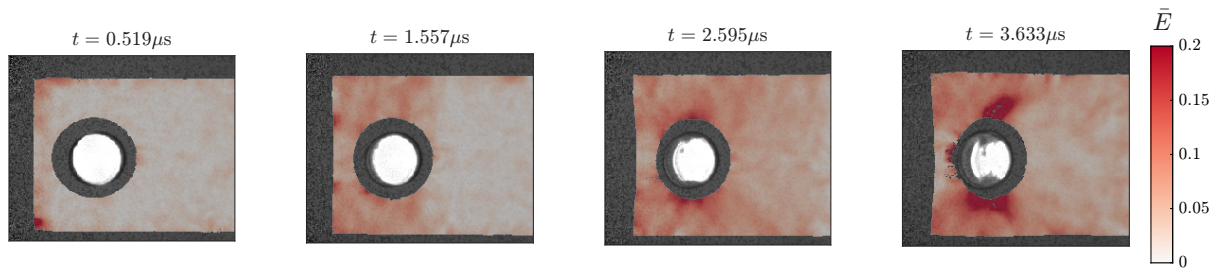


Figure 6.18: Mises strain from shot ESRF-52318 at ESRF.

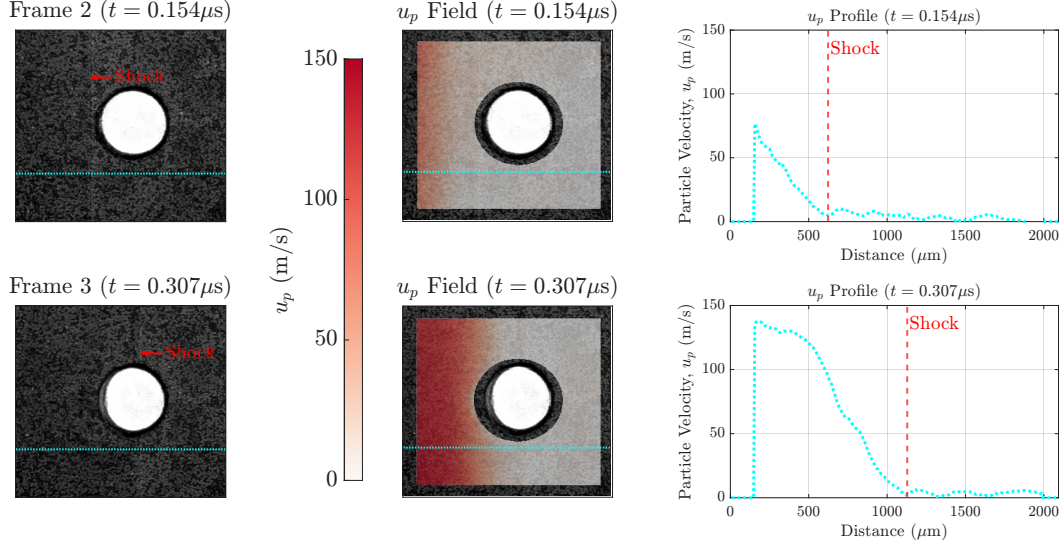


Figure 6.19: Extraction of particle and shock velocities.

6.4.2 Hugoniot Calculations

To verify the magnitudes of the extracted quantities, the shock velocity U_s and the particle velocity u_p are extracted when possible in the set of experiments. Recall that the Hugoniot defines the locus of possible thermodynamic states in a shock-loaded material. That is, each point on the Hugoniot represents a single shock state, typically written in terms of the variables U_s, u_p, p , and v/v_0 , where U_s is the shock speed, u_p is the particle velocity, p is the shock pressure, and v/v_0 is the nominal volumetric compression. The most common of these relations is $U_s - u_p$ which is serendipitously linear for most materials. Thus, we may commonly write $U_s = C_0 + s u_p$. C_0 , the linear intercept, is the bulk sound speed and s is the slope of the Hugoniot in the $U_s - u_p$ plane. Typically, these parameters, which are of great interest to shock experimentalists and modelers alike, are obtained electromechanically by means of piezoelectric transducers or stress gauges, which are placed in a sample at precisely known distances. The transit time and therefore the shock speed U_s may then be determined by examining the arrival time of the shock wave between these gauges.

With DIC, however, one can discern shock speeds and particle velocities optically. One

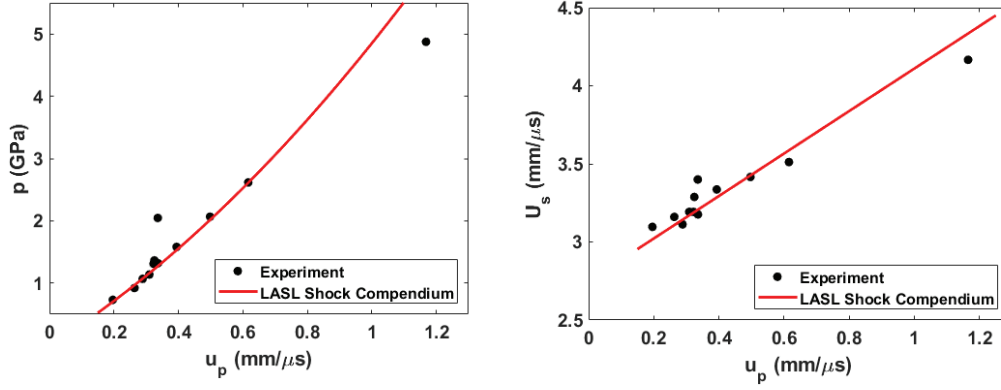


Figure 6.20: Extracted Hugoniot of PMMA via DIC analysis.

may obtain particle velocities by taking the inter-frame change in displacement field in the x direction, u_x , and taking its quotient with the interframe time Δt , $u_p = u_x / \Delta t$. After the shock wave reaches the hole interface, the displacement field becomes highly complex, rendering this 1D Hugoniot analysis invalid. To circumvent this issue, values of u_x are taken in the region downstream from the hole before the arrival of the shock wave at the hole. u_x is then averaged over a line segment in this region. Values of U_s may be determined in a similar manner, by examining the propagation speed of the leading edge of the displacement field, prior to the first interaction with the hole, as shown in Fig. 6.19. Another common technique for measuring shock velocities (that is, in experiments in the literature where DIC is not used) consists of subtracting the n th image from the $(n - 1)^{th}$ image. The pixel values of the shock front may then be tracked across frames to obtain U_s values. In reality, measurements made with X-ray DIC are not an especially economical way of characterizing a Hugoniot, as stress gauge data from a plate impact experiment is more readily generated than in our PCI data collected at synchrotron facilities. Here, the primary motivation for performing this check is to strengthen confidence in the fidelity of the measured displacement and strain fields. Fig. 6.20 shows points on the PMMA Hugoniot for experiments where displacement fields were obtainable prior to the arrival of the shock for multiple frames; the curves are again from the LASL shock compendium.

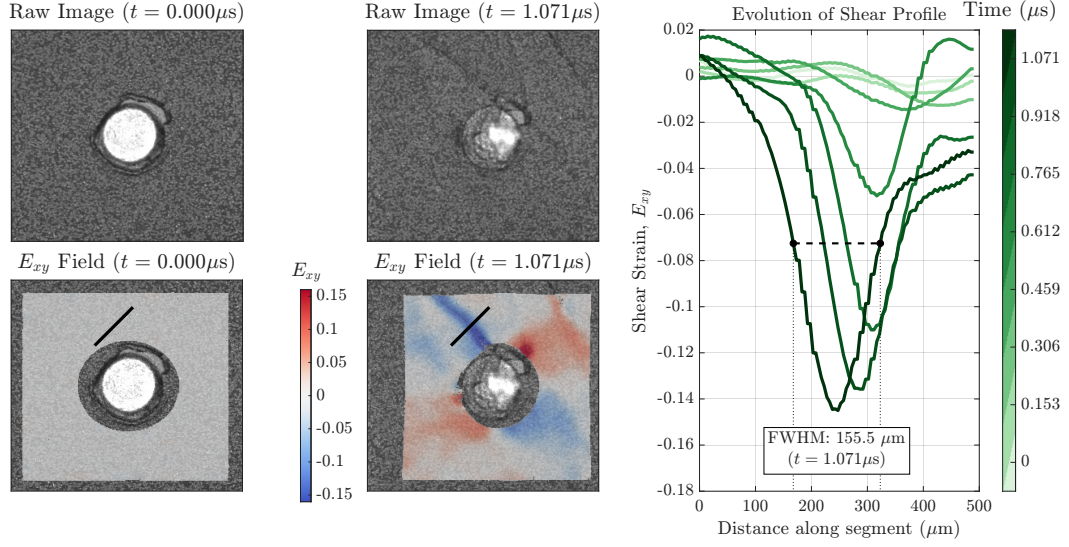


Figure 6.21: Shear strains over a line profile over a crack for shot DCS-23-2-028.

6.4.3 Damage and failure

While DIC patterns tend to become highly distorted when voids completely collapse (i.e., in the 1-2 GPa range and beyond), direct visualization of material failure is possible for lower shock pressures. In shot DCS-23-2-028, an apparent crack was observed in the vicinity of the pore. The crack seemingly grew at 45 degrees from the impact axis, in the direction of a maximal shear. Though growth of this feature is observed, the position of its tip over time could not be confidently determined. Shear strains along a line placed over this feature are presented in Fig. 6.21. Here, we observe that a strain discontinuity emerges over time. However, remains difficult to properly characterize the feature. While cracks typically manifest as a breakdown in the correlation coefficient in DIC (Helm, 2008), and this is not observed here, the region where the feature emerges is under an enormous amount of pressure, and newly formed surfaces cannot separate. Mode II cracking is possible under pressure, and the enormous amounts of pressure would affect the surrounding strain field. It should also be noted that shear strains of around ~ 0.14 are measured in the final frame, indicating that at the very least, there is an accumulation of ductile damage. The full width at half maximum (FWHM) is given as $292 \mu\text{m}$. This

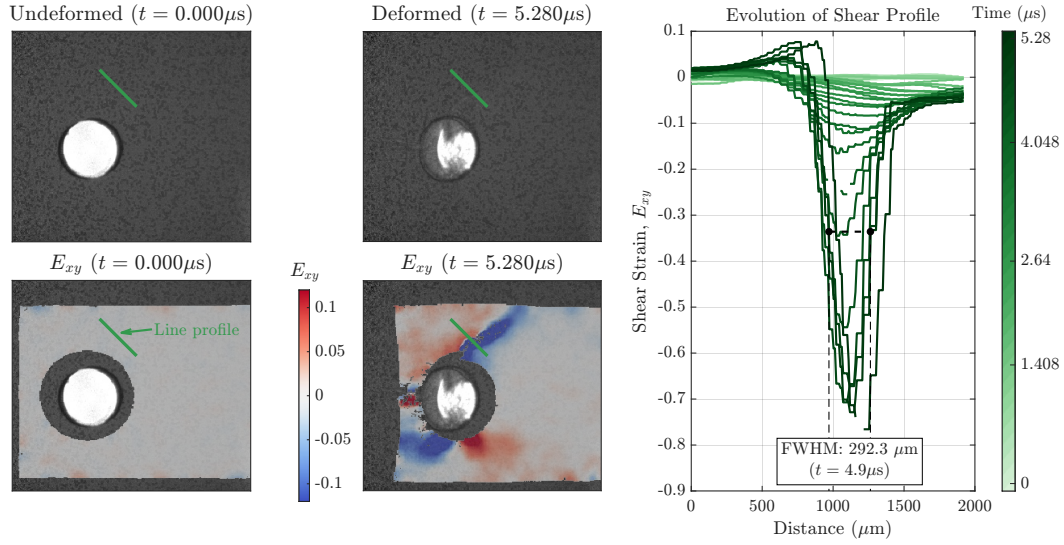


Figure 6.22: Shear strains over a line profile over a crack for shot ESRF-52318.

does not indicate the actual width of a shear band, but rather the apparent continuum strain-localization zone, since the DIC subset size acts as a low-pass spatial filter. An estimate for a shear band width under these conditions is in the range of $10 - 20\mu\text{m}$ (Lawlor et al., 2024). Instead of in the width of the band, a key hint may be in the void itself, where a small amount of material appears to propagate down and to the left-side boundary of the void, and at the same time leaving a small white hole at the top of the void, as seen in Fig. 6.21. It is therefore at this point that the material has at least partially fragmented due to the accumulation of damage. However, this occurs in the final frame, precluding any further evidence of damage from presenting itself.

The ESRF dataset is comparatively advantageous in this regard. The longer time record allows for a clearer characterization of the shear damage feature that emerges at the top of the void, as in shot ESRF-52318. The resulting shear strain profile is indicated in Fig. 6.22, along with the raw images and shear strain plots at $t = 0$ and at $t = 5.28\mu\text{s}$ when the release waves arrive. Until this time, shear strains are measurable around the localization zone. Only after the pressure has been relieved does the correlation coefficient begin to break down, as cracks completely open up in this area. It is for this reason that points are missing on these plots, as the opening of cracks renders correlation intractable.

Prior to the arrival of the release waves, two surfaces on either side of this band appear to be sliding past one another.

6.5 Discussion

In the preceding experiments, dynamic fracture was observed in the vicinity of shock-loaded cylindrical voids in a predominantly Mode II envelope. Seen most clearly in shot ESRF-52318, high levels of plastic strain precede fracture. These bands are likely initiated by defects that form locally intense stress concentrations, thereby leading to localization. The general regions of propagation, however, are not random and always seem to reside in the lobes of shear maxima and minima, oriented at approximately 45 degrees to the impact axis. For higher shock pressures, damage is suppressed as the material gives way to inertially dominated collapse.

A natural question now arises as to why such large strains are tolerated for comparatively similar strain rates and pressures as in the PSPI experiments in Chapter 4. The simulations in Chapter 5 indicated that there were two primary competing effects of pressure hardening and thermal softening, which resulted in an initially strong pressure dependence of shear strength, followed by a drop at higher pressures where thermal softening began to dominate. In the next chapter, we will again employ the CMGJC and GAP models to explore this paradox, that is, why for PSPI experiments on PMMA, the failure strains are much lower than the measured strain fields for void collapse experiments.

Numerical Modeling of Dynamic Void Collapse Experiments

7.1 Overview

The constitutive models CMGJC and GAP-HP, which were described in Chapter 5, are implemented here to capture the void collapse experiments of Chapter 6. The models predict extreme temperatures and large amounts of plastic work in the vicinity of the voids, which readily give rise to localization in the form of shear bands. Experimentally, deformation is not observed to this extreme extent, and thus a correction is needed. Following the work of (Coe et al., 2017), the GAP-HP model is updated with a damage model based on statistical crack mechanics. This damage model is calibrated to DIC data by directly updating a regression on material parameters, where the loss function is tabulated in terms of the Mises strain from the simulation and the DIC data. Once calibrated, the damage model reduces the intense plastic work and resultant heating during the collapse process, and produces collapse morphologies that reduce error between simulation and experiment, but do not completely agree, hinting at the need for ductile damage mechanisms in the model.

7.2 Simulation framework

2D finite element simulations were performed in ABAQUS under plane strain and adiabatic conditions, that is, without thermal diffusion. Each mesh consists of CPE4R elements. A convergence study was conducted and found that below an approximate mesh size of $d/100$, where d is the diameter of the void, the collapse morphology and shear band size were relatively constant. Small shear band branches emerged for smaller mesh sizes; however, the overall effect on temperature, plastic work, and localization was minimal.

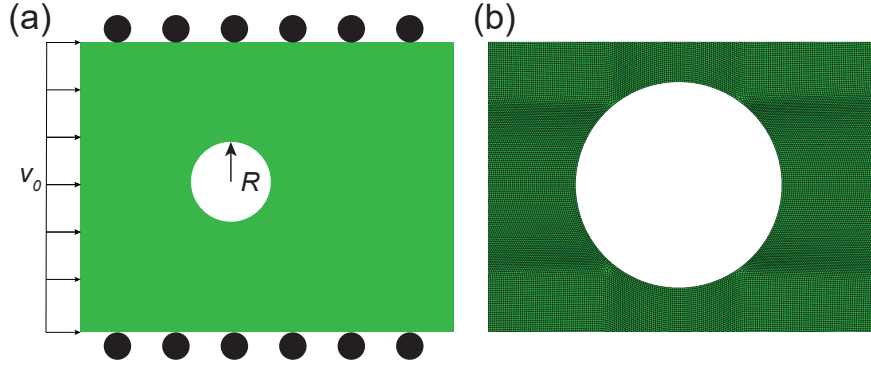


Figure 7.1: The geometry used in pore collapse simulations (a) Symmetry boundary conditions are applied to the top and bottom surfaces, and a velocity pulse is applied to the impact face (b) Magnified region around the pore with mesh size $d/100$.

7.3 Void collapse via CMGJC and GAP-HP models

Before discussing the models' performances, the number of decades of strain-rates generated in these problems is first highlighted by Fig. 7.2. Here, it can be observed that the problem at hand gives rise to several decades of strain rates over time and over a certain profile length at a fixed moment in time. This, in combination with the known material history of PMMA, highlights the need for highly rate-dependent material models that capture the transient evolution of the stress-strain response of the material. Shot ESRF-52318 is uniquely suited as a test case for the model due to its well-resolved DIC pattern (thereby enabling quality measurements of deformation fields) and presence of substantial crack formation prior to the arrival of the release waves. Simulations were performed with the same geometry as the experiment, i.e., a 2.17 mm void with the center 6 mm from the left boundary. Loading was applied by applying a velocity pulse to the left-hand side of the sample, and the width of this shock pulse is dictated by the width of the flyer used.

Overall, the GAP-HP matches the final contour quite well. This is due to the GAP-HP's ability to capture viscoelastic and viscoplastic deformation over a variety of strain rates. The pressure is given in Fig. 7.3. Both simulations give rise to shock waves that

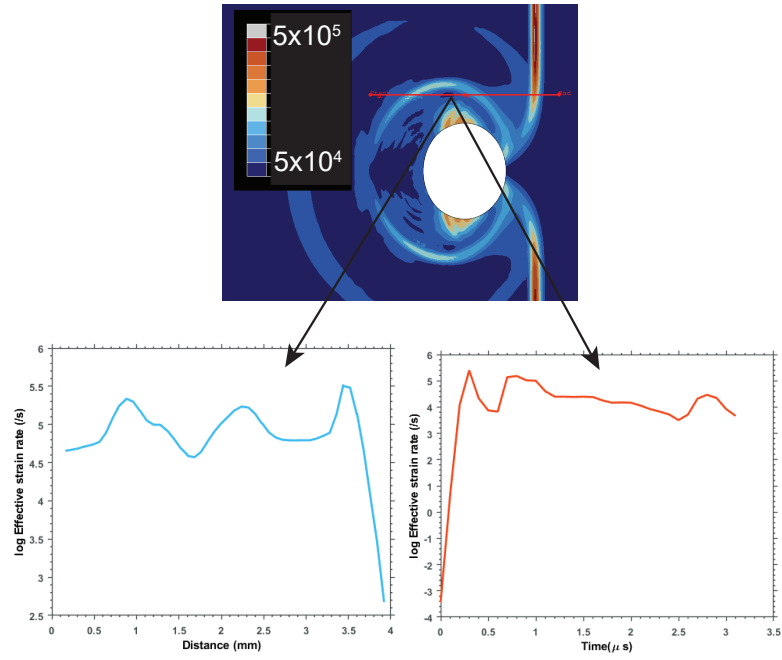


Figure 7.2: Top: A time snapshot just after the shock wave has transited the void with a line profile drawn in red at the top of the void. Bottom left: effective strain rate over the line profile. Bottom right: Effective strain rate over time from the center of the line profile.

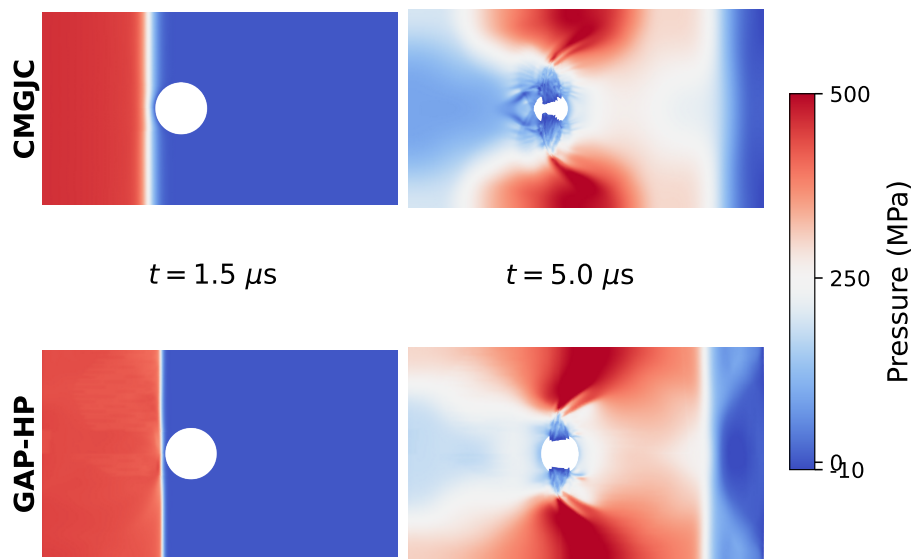


Figure 7.3: Contours of pressure for the CMGJC and GAP-HP models.

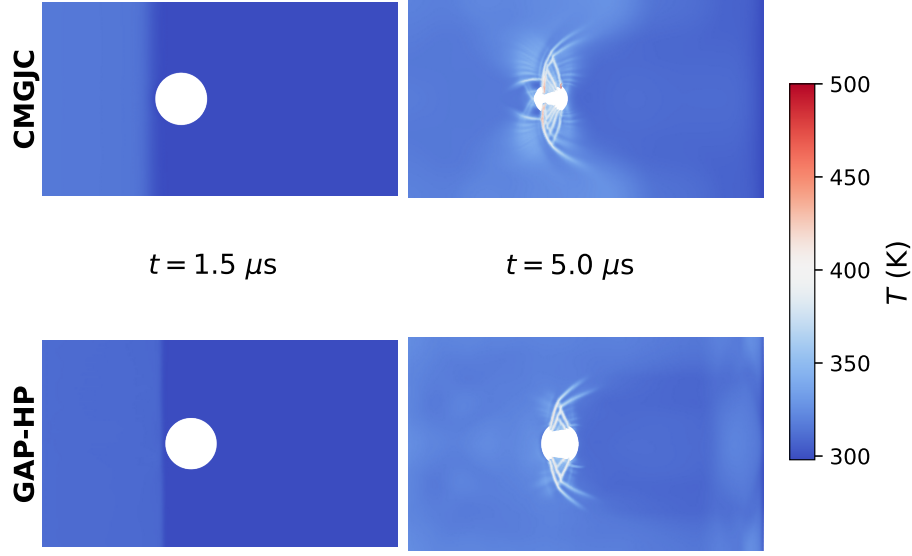


Figure 7.4: Contours of temperature for the CMGJC and GAP-HP models.

transit at the shock speed U_S and which reach the void at approximately the same time.

Temperature fields are given in Fig. 7.4. Thermoelastic shock heating initially raises the temperature by approximately 12 K. As the void collapses and substantial plastic deformation occurs, shear bands form in the vicinity of the void. The shear bands have widths of approximately $100 \mu\text{m}$ and see temperature increases of well over 150 K. In the case of the CMGJC model, this is in excess of the T_g and at this point the stress deviator is set to zero as the material is assumed to have melted. For the GAP-HP model, T_g is pressure dependent (see Chapter 5) and is therefore not exceeded. However, substantial softening occurs. The shear bands that develop in the GAP-HP model are consequences of structural softening in the HFS plasticity model and thermal softening in both the plastic flow function and the effective moduli.

Since deformation fields were obtained for these experiments, strain fields may be directly compared between experiment and simulation. The XX component of Lagrangian strain is given in Fig. 7.5. Just as in the experimental DIC field, strain maxima form on the horizontal axis of the void and minima form on the vertical axis. In regions outside of the immediate zone of collapse, agreement between experiment and simulation is acceptable, with minima in the range of -0.25 to -0.35 appearing outside of the shear

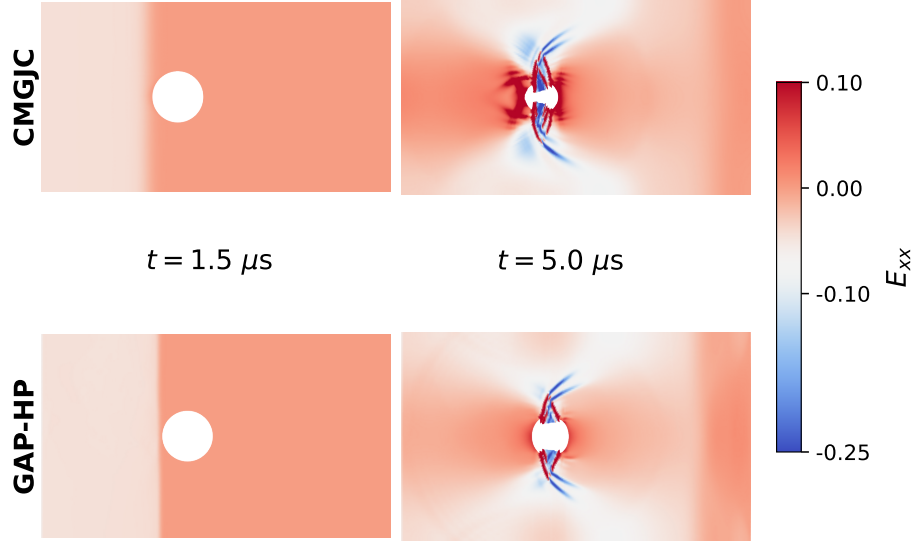


Figure 7.5: Contours of E_{xx} for the CMGJC and GAP-HP models.

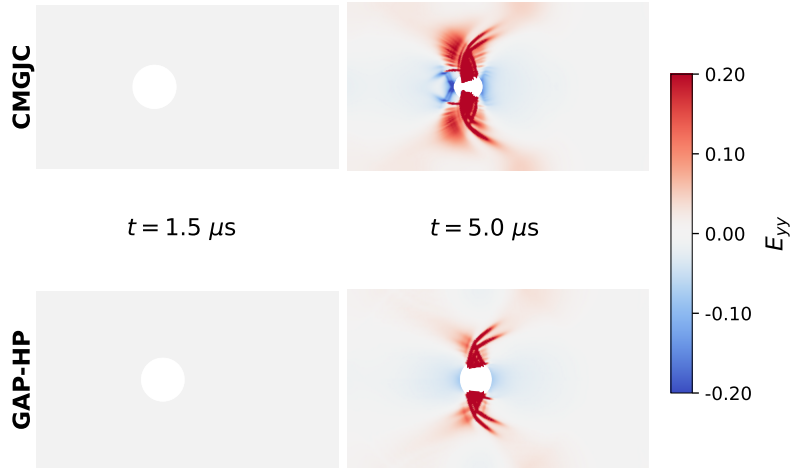


Figure 7.6: Contours of E_{yy} for the CMGJC and GAP-HP models.

bands, similar to the experimental profiles.

The results for the YY and XY components of strain follow a similar story and are shown in Fig. 7.6 and 7.7. Outside the immediate vicinity of shear bands, general agreement is reached, but the enormous strains created in the shear bands prevent 1:1 agreement with the experiment, which only shows one dominant localization zone. Since DIC acts as a low-pass spatial filter, it is possible that higher levels of strain are simply not able to be captured due to limitations in the high-speed imaging system and post-processing methods. To assist in creating a model that better captures the experiment at

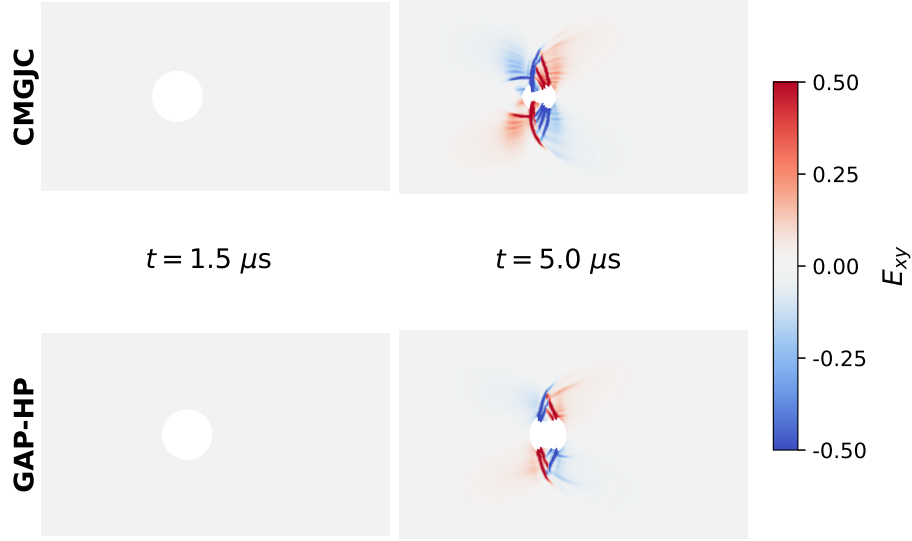


Figure 7.7: Contours of E_{XY} for the CMGJC and GAP-HP models.

hand and consequently uncovers new aspects of the damage and failure process, a damage model based on statistical crack mechanics was introduced.

7.4 Damage in GAP-HP

The damage model employed in this work is derived from the implementation by (Coe et al., 2017), which was parameterized for polyethylene. In this model, polymers are damaged by brittle-like crack mechanisms. It remains to be seen to what extent dynamic failure in PMMA is truly brittle. To this point, the PSPI and void collapse experiments indicate that plasticity is likely playing a large role in the damage process. Nevertheless, the current computational results are unable to fully capture what appears to be a shear crack emerging in the vicinity of the void. It is for this reason that we pursue the brittle crack network model that follows.

Much of the damage model is based on the work of J. Dienes (Dienes & Kerschner, 1998). In linear elastic fracture mechanics, it is not the crack length c nor the applied stress σ that governs crack growth in a brittle material, but rather a combination of these: the stress intensity factor K , which was first developed by George Irwin (Irwin,

1968). The stress intensity factor is one of the most fundamental and useful parameters in fracture mechanics and is used to establish failure criteria due to brittle fracture. Rather than resolving the propagation of individual cracks, the damage model here considers micro-crack formation and ascribes scalar damage to the material response as a result. For a given strain increment, these micro-cracks attenuate the resultant stress. Micro-cracks may form due to imposed tensile (Mode I) and shear (Mode II) deformation. Since this work deals with experiments where materials are subjected only to compressive dilatational states (which suppress Mode I cracks), the parameterization will center on Mode II crack propagation.

In Mode I, the stress intensity factor $\bar{K}$ is taken to be

$$\bar{K} = \sqrt{\frac{9}{2}\pi\bar{c}p^2} \quad (7.1)$$

where $\bar{c}$ is the average Mode I crack length. The Mode II stress intensity factor is given by

$$K = \sqrt{\frac{3}{2}\pi cs_{ij}s_{ij}} \quad (7.2)$$

where c is the average Mode II crack length.

In modeling fracture processes, Dienes used the experimental observation that crack growth $\dot{c}$ is divided into slow and rapid growth regimes. The value of the stress intensity factor at the transition point between the two regimes is the fracture toughness, written as $\bar{K}$ for mode I and $K_{0\mu}$ for Mode II cracks.

Up to a constant factor f_1 , the transition between slow and rapid Mode II crack growth is governed by $K_{0\mu}$. These cracks can never exceed the shear wave speed of the material, and thus an upper limit is applied: $\dot{c} < \nu\sqrt{\mu/\rho} \equiv v_{max}$ where $0 < \nu < 1$ is a scale factor. Again, the same concepts here apply to Mode I cracks.

In the slow growth regime, the following expression for crack growth is used

$$\dot{c} = v_{max} \frac{1}{f_2^m} \left(\frac{K}{K_{0\mu}} \right)^m, \quad K < K_{0\mu} f_1 \quad (7.3)$$

where f_1 and f_2 are given by

$$f_1 = \sqrt{1 + \frac{2}{m}} \quad (7.4)$$

$$f_2 = f_1 \left(1 + \frac{m}{2} \right)^{1/m} \quad (7.5)$$

In the rapid growth regime, the crack growth is given by

$$\dot{c} = v_{max} \left[1 - \left(\frac{K_{0\mu}}{K} \right)^2 \right], \quad K > K_{0\mu} f_1 \quad (7.6)$$

The shear fracture toughness $K_{0\mu}$ is pressure, strain-rate, and temperature dependent:

$$K_{0\mu} = K_0(\dot{\varepsilon}, T) \left[1 + \frac{\pi \mu' \Theta(p) \sqrt{c}}{K_0(\dot{\varepsilon}, T)} \left(1 + \frac{\mu' \Theta(p) \sqrt{c}}{K_0(\dot{\varepsilon}, T)} \right) \right]^{1/2} \quad (7.7)$$

where μ_s is the coefficient of friction and

$$\mu' \equiv \mu_s \sqrt{\frac{45}{2(3 - 2\mu_s)^2}}. \quad (7.8)$$

Θ is defined to be $\Theta(p) = p$ in compression and zero otherwise; the net effect is that frictional forces are generated when pressure is applied to two surfaces of a Mode II crack.

$K_0(\dot{\varepsilon}, T)$ is given by

$$\log_{10} K_0(\dot{\varepsilon}, T) = k_0 + k_T \log_{10} a_T(T) - k_E \log_{10} \left(\frac{\dot{\varepsilon}_{eff}}{\dot{\varepsilon}_0} \right) \quad (7.9)$$

where $\dot{\varepsilon}_0$ is a material parameter and $\dot{\varepsilon}_{eff}$ is the effective strain rate, as defined earlier in the GAP-HP model.

Before deformation occurs, it is assumed that there are a large number of initial cracks that act as nucleation sites for crack growth. The average initial size of such cracks is c_0 . The density of these cracks per unit volume is given as a_0 , which, unlike c , is a material parameter that does not evolve due to deformation.

The Mode II crack evolution equations are given by

$$\mu_{eff} \dot{e}_{ij}^c = \frac{d}{dt} \left[\left(\frac{c}{a_0} \right)^3 \sum_{n=0}^N s_{ij}^{(n)} \right] \quad (7.10)$$

$$\mu_{eff} = \sum_{n=0}^N \mu^{(n)} \frac{1 - e^{-\Delta\phi^{(n)}}}{\Delta\phi^{(n)}} \quad (7.11)$$

$$\Delta\phi^{(n)}(t) = \int_{t-\Delta t}^t \frac{dt'}{\tau^{(n)}(t')} \quad (7.12)$$

$$c_{v0} = A \left(\frac{\theta_E}{\tilde{\theta}_0} \right)^2 \frac{e^{\frac{\theta_E}{\tilde{\theta}_0}}}{\left(e^{\frac{\theta_E}{\tilde{\theta}_0}} - 1 \right)^2} \quad (7.13)$$

As before, these crack strains are easily integrated into the additive decomposition of strain:

$$\dot{e}_{ij} = \dot{e}_{ij}^{E(n)} + \dot{e}_{ij}^{P(n)} + \dot{e}_{ij}^{VE(n)} + \dot{e}_{ij}^{c(n)} \quad (7.14)$$

This completes the description of the model.

To obtain material parameters for the model, the values used for HDPE in (Coe et al., 2017) were used as a starting point. Since Mises strain is chosen as the scalar field that best represents highly damaged regions in the material. Mises strain fields from the experiment Shot ESRF-52318 are directly compared with values generated by the GAP-HP model. A parametric study was performed whereby several hundred simulations were run for a variety of values for a_0 , c_0 , and k_0 ; otherwise all parameters are the same as the original values for polyethylene.

As shown in Fig. 7.8, simulations with GAP-HP that included damage showed mixed

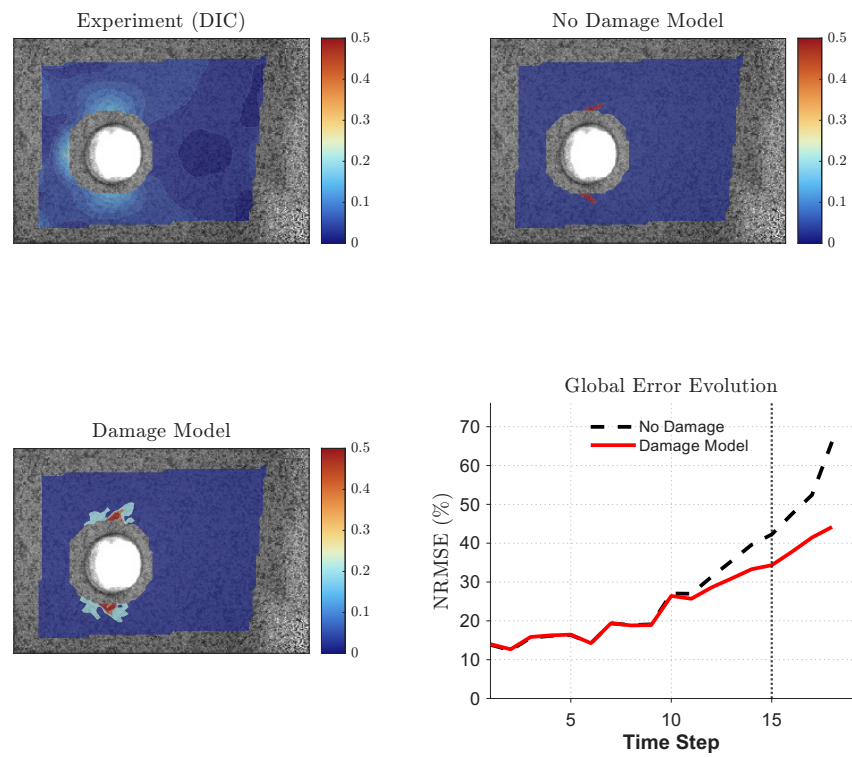


Figure 7.8: Comparison of Mises strain between DIC patterns and Abaqus simulations to parameterize the damage model. The dashed line on the lower left indicates the current frame.

success in reducing the degree of localization. When damage is enabled, shear banding gives rise to cracks and subsequent stress deterioration. Since the level of Mises strain is large to begin with, the model already resides near a local minimum. The inclusion of the damage model, however, does overall reduce the intensity of the shear bands, and the total error is reduced. Still, the inclusion of the damage model is unable to recreate the failure envelope of the void, which, it should be noted, is not unique to this particular test. Similar failure modes were observed in shots ESRF-52306 and ESRF-52303, for example. While localization and failure are strongly dependent on the exact loading condition and initial defects in a sample, the general collapse morphology is approximately the same between this set of experiments.

7.5 Discussion

Though the damage model was unable to capture the exact evolution of damage and failure in the material, the general collapse envelope is well captured by the GAP-HP model as well as the CMGJC model. Fig. 7.9 presents the normalized pore area over time, for the experiment ESRF-52318, CMGJC model, GAP-HP model, GAP-HP-DMG model, and the GAP-HP model run under isothermal conditions. The primary deficiency is that none of the models are able to capture experimental observation of the detachment between the two crack surfaces and subsequent rapid movement into the void interior, but otherwise the CMGJC and GAP-HP models perform admirably.

While the isothermal GAP-HP model did not produce the high degree of plastic flow observed for the other simulations, it nonetheless did show high levels of strain softening and the emergence of shear bands, as shown in Fig. 7.10. The magnitude of the plastic strain here is certainly less than in the normal (adiabatic) case, but the softening here highlights one of a number of components in the GAP-HP model that lead to a loss of strength.

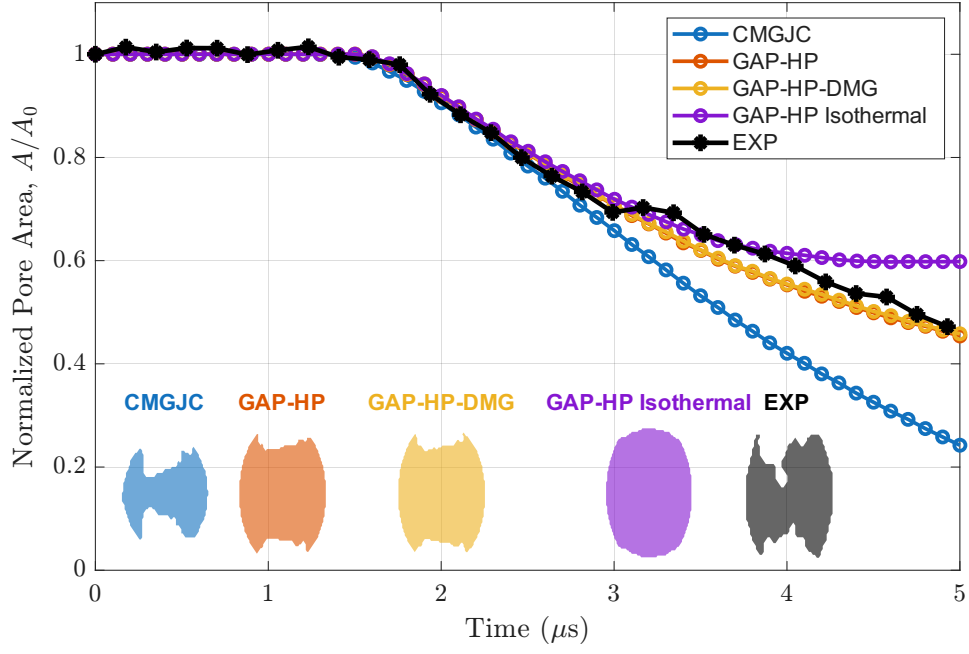


Figure 7.9: Normalized void area over time for shot ESRF-52318 and the simulation models used in this work.

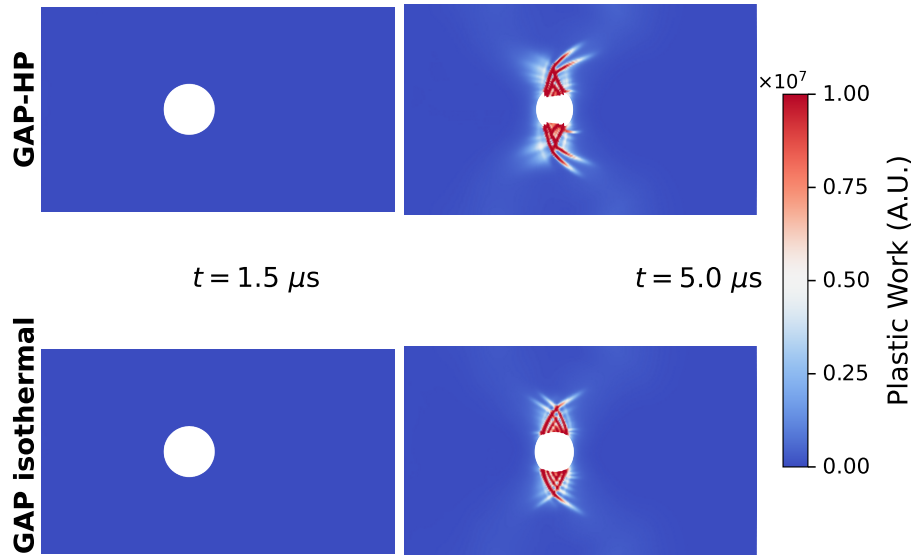


Figure 7.10: Plastic work for the GAP-HP model under adiabatic and isothermal conditions.

These results highlight the importance of thermal softening in the loss of strength, as was seen in the PSPI experiments. Examining the contributions to the evolution of temperature at a point in a shear band in Fig. 7.11, shock heating initially raises the temperature by roughly 10 K. As viscoplastic flow occurs, these extreme amounts of plastic strain result in greater temperature increases, resulting in localization. As was the case with PSPI, thermal softening plays a significant role in facilitating plastic flow. In contrast to the PSPI results, the effects of softening are also extended to the effective moduli, as shown in Fig. 7.12. Two integration points are selected here: Point 1, which is within a shear band, and Point 2, which is far from localization effects. Thermoelastic heating occurs after the arrival of the shock. Point 1 is substantially heated by plastic work in the evolving shear band. Around 3 μs , there is a precipitous drop in the effective shear modulus. However, because significant plastic heating and thermal softening have occurred up to this point, this is simply a consequence, and not a cause of, localization. Recall that in PSPI, there was no such drop in the effective shear modulus; rather, the modulus was constant, and softening occurred primarily via the plastic flow function. In this case, the free surface of the pore reduces pressure in its vicinity, which has two primary effects. Pressure hardening of the flow stress will be greatly diminished, allowing for runaway plastic deformation to occur. For these reasons, the model gives rise to shear bands at the surface of the void, both in isothermal and adiabatic conditions. In the adiabatic case, thermal softening greatly amplifies the growth of these shear bands, resulting in the runaway localization process. Future work on PMMA must address the shortcomings of the damage model. A potential avenue is the inclusion of small-scale ductile damage, which leads to predominantly Mode II fracture. Though no shear bands were observed at the mesoscale and failure appeared to be predominantly brittle, it is possible that micro shear bands exist in the wake of intense distortional deformation. These sites of locally intense plastic strain may weaken the material, allowing cracks to develop. In sum, there is likely a complex interplay between plasticity and fracture that is not clearly revealed by examining the collapse process. After extracting quantitative deformation

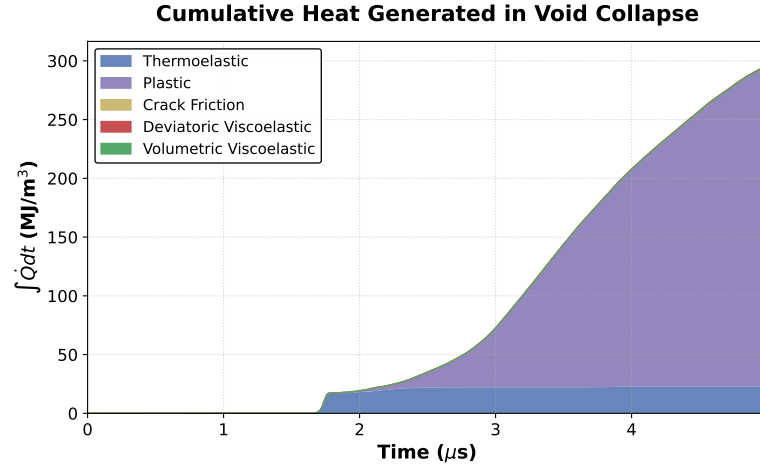


Figure 7.11: Cumulative heating contributions over time from thermoelastic, viscoelastic, and inelastic sources.

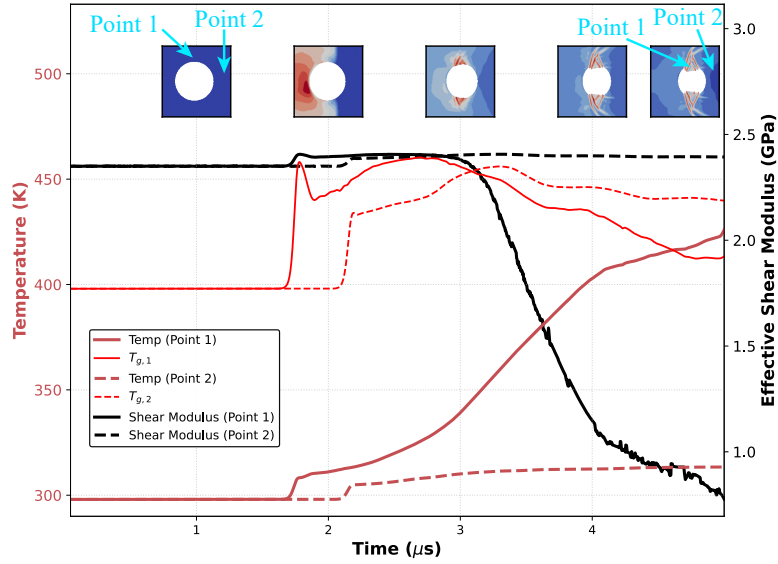


Figure 7.12: Temperature profiles over time for a point within a shear band and a point outside the localization zone, as well as T_g and the effective moduli at each integration point.

fields, it appears that the origin of failure likely originates below the resolution limit. Future experiments that subject PMMA to such extreme dynamic conditions and are able to examine micron or sub-micron surface features would likely be highly illuminating. As an example, soft-recovery experiments that are able to preserve samples in as intact a state as possible after shock loading may offer hints on the nature of the failure landscape of glassy polymers.

7.1.

7.A Damage Parameters

Parameters for the damage model are provided in Table 7.1.

c_0 (μm)	a_0 (μm)	k_0 ($\log \text{Pa}\sqrt{m}$)	κ_T	κ_E	$\dot{\epsilon}_0 (s^{-1})$	ν	μ_s
6.0	68.0	7.8	0.0	0.2	0.2	1.0	0.4

Table 7.1: Damage model properties, for Mode II cracking only.

Concluding Remarks

8.1 Summary of results

The chief aim of this thesis was to shed light on the complex mechanisms governing material failure in PMMA. PMMA is among the most abundantly studied materials in shock physics, which provides a unique platform for material analysis and modeling. Despite this, the failure landscape of PMMA remains poorly understood under simultaneous levels of high pressure and high strain rates.

In the introduction, background information was provided on PMMA. As one of the most studied materials in shock physics, a great many dynamic compression studies have been conducted. Under uniaxial compression, PMMA is ductile for low strain-rates and brittle for high strain-rates. A paradigm has emerged in modeling glassy polymers like PMMA whereby plasticity is mediated by the presence of densely packed micro shear bands. The shear strength and plastic strain to failure generally increase with pressure and are in competition with softening effects, both structural and thermal in origin.

Chapter 2 presented an overview of the theoretical basis for plate impact experiments. These experimental frameworks are introduced to simplify material geometry and boundary conditions to provide clearer characterizations of material behavior under dynamic conditions.

Chapter 3 described the characterization of a newly constructed plate impact facility at Brown. Characterization experiments were conducted on Y-cut quartz as well as tungsten carbide. These experiments illuminated several challenges and limitations of plate impact.

Chapter 4 presented pressure-shear experiments on PMMA at pressures ranging from 2.5 to 10 GPa and total strain-rates of $10^5 - 10^6 \text{ s}^{-1}$. Measurements of shear strength match

existing measurements made by several researchers with lateral stress gauges. Plastic failure strains ranging from approximately $\bar{\varepsilon}^p = 0.1 - 0.3$ were observed. The overall response suggests a brittle-like failure mode under such high-strain rates.

Chapter 5 covered two constitutive models for PMMA under high-strain-rate loading. The first comprises the complete Mie-Grüneisen equation of state, adapted from (Malhotra et al., 2022), and a Johnson-Cook strength model, adapted from (Holmquist et al., 2016), and is overall referred to as the CMGJC model. The second model is a modification of the viscoelastic and viscoplastic Glassy Amorphous Polymer (GAP) model as presented in (B. E. Clements, 2012). The GAP model was modified such that material behavior was better captured at high pressures. Without this modification, thermodynamic quantities like the volumetric expansion, Grüneisen coefficient, and specific heat would attain negative, non-physical values for shock pressures above a few GPa. To this end, the free energy was modified to incorporate a modified Birch-Murnaghan equation of state. The high-pressure GAP-HP model was shown to capture quasi-static, high-strain-rate, and plate impact experiments. The GAP-HP and CMGJC models were compared to PSPI experiments from Chapter 4, and initial disagreement was found in the shear stress profiles. This discrepancy was accounted for by introducing an explicit pressure dependence into the GAP-HP flow function.

Chapter 6 detailed shock-loaded cylindrical pore collapse experiments conducted at two synchrotron light facilities: (1) the Dynamic Compression Sector, at the Advanced Photon Source as part of Argonne National Laboratory and (2) the European Synchrotron Radiation Facility (ESRF) in Grenoble, France. Quantitative measurements of kinematic fields in the vicinity of collapsing pores, giving key insights into material localization and failure.

Chapter 7 presented finite element simulations of shock-loaded pore collapse for the two primary constitutive models outlined in Chapter 5. These models again give rise to different interpretations on the nature of failure mechanisms inherent in the collapse

process in the regime where material strength dominates. Interestingly, the GAP-HP model predicts shear bands even under isothermal conditions. The introduction of damage reduces the extent to which localization in the form of shear banding occurs, thereby providing better agreement between experiment and simulation. Despite this, the damage model was only partially successful, and further damage modeling is needed.

8.2 Directions for Future Work

8.2.1 PSPI experiments for a variety of loading conditions

The PSPI experiments in this thesis were conducted at three pressures and for varying strain rates. Though the response appeared highly brittle, the interplay between stress triaxiality, pressure, and strain-rate requires further investigation. First, experiments at pressures beyond 10 GPa would reveal whether there is a total loss of shear strength or whether there is some saturation in the response. Non-monotonic growth in the shear strength at higher pressures is possible due to the complex interplay of the hardening and softening mechanisms present in the material. The PSPI simulations in this thesis suggested that for higher pressures, significant thermal softening occurs, which leads to abrupt and intense plastic flow. This plastic flow is eventually arrested, resulting in the shear stress rebounding to some intermediate value below the initial shear strength. It remains to be seen whether this phenomenon would occur for higher pressures. At the same time, there is a strain-rate hardening effect that may also contribute to a stabilization in the pressure dependence of shear strength. An experimental campaign that consists of several PSPI experiments at constant strain-rates for varying pressure, constant pressure for varying strain-rate, as well as varying triaxiality (i.e., angle of obliquity) would likely elucidate the nature of these competing mechanisms.

8.2.2 Soft-recovery experiments

Both PSPI and void collapse experiments, which are able to recover intact samples, would be invaluable in deciphering the micromechanical damage process. The extent to which shear bands propagating in the vicinity of collapsing voids in the strength dominated regime could be aided by post-mortem fractography, along the lines of experiments by (Rittel & Brill, 2008).

8.2.3 Model synthesis

The phenomenological CMGJC does a remarkable job at capturing the primary features of the experiments, and with some additional parameterization, this model could better reproduce features seen in the experiments. For example, the immense pressure dependence in the Johnson-Cook model was shown to taper for higher pressures in PSPI experiments. It would not be unreasonable to introduce a functional form for this simple linear term, as is done for the strain hardening, strain-rate hardening, and thermal softening terms. In a similar vein, the ambiguous parameterization of the strain hardening parameters could be ameliorated by a non-monotonic function here. That is, additional terms which capture (structural) strain softening as well as hardening would result in producing features seen in simulations with the GAP-HP model, such as shear bands near collapsing voids, even under isothermal conditions.

The GAP-HP model has the opposite issue, where the complexity of the model is likely not needed for the relevant time scales and loading conditions. The viscoelastic aspects of the model could likely be greatly reduced in complexity; it is certainly possible that a great many of the 12 deviatoric Maxwell elements could be removed, since the time-scales here are many orders of magnitude longer than the few microseconds typical of plate impact experiments.

It seems possible that these models may be synthesized to capture the most salient

features of the material response. Simplicity is not desired for its own sake; rather, the enormous number of material parameters and state variables often creates difficulties in properly interpreting the implications of the model. The evolution of the flow stress in the GAP-HP model, for example, is quite difficult to dissect into different elements. Some simplification of this model would allow for greater understanding of the heart of already complex damage and failure mechanisms present in PMMA.

8.2.4 Ductile damage modeling

Though the damage model introduced in Chapter 7 was able to reduce the total root-mean-square-error between experiment and simulation, it was unable to generate simulations where the collapse morphology was in line with experiment to a few percent. Additional experiments, such as soft-recovery, may illuminate the extent to which ductility plays a role in damage and failure. At the very least, there are likely locally intense regions of plastic flow. The extent to which these weaken the surrounding material is currently unknown. A ductile damage model that supplements the existing one may provide additional insights into how the material fails, even in the absence of new experimental data. Resolving the *in – situ* deformation processes at the micro scale will likely prove to be quite challenging, and a modeling effort that considers micromechanical ductile damage should be pursued in parallel.

8.2.5 Neural network modeling

The recent meteoric success of machine learning in mechanics presents a unique opportunity to gain further insights into dynamic phenomena. Physically Inspired Neural Networks (PINNs) can embed the knowledge of physical laws inherent to a dataset, as described by the systems’ governing partial differential equations (Karniadakis et al., 2021). However, many of these techniques typically rely on large amounts of data that neural networks can be trained on. Plate impact experiments, as with many experiments in

mechanics and materials science, are highly throughput-limited. Experimental campaigns typically take months of preparation, and the individual experiments can take several weeks to conduct. Even after all of this effort, the limited access to diagnostics to probe such extreme environments often results in ambiguity in analyzing the material response. PSPI experiments, for example, rely on velocity profiles which may have features that could be explained not only by complex properties in the material of interest, but also anvil inelasticity. The classical picture of these experiments does not seem immediately applicable to machine learning techniques.

The rich, full-field data from DIC present an interesting opportunity to develop new insights into deformation and failure. In future work, DIC data from the shock-loaded void collapse experiments could be combined with DIC data of loaded voids for a variety of strain rates, such as by split-Hopkinson bars or even for quasi-static rates. Such experiments and computational analysis may be able to provide further insights into the nature of transitions in failure behavior. Whether such a dataset could be combined with burgeoning techniques (e.g. (Ferreira & Bessa, 2026)) in machine learning requires further study.

Bibliography

- Addessio, F. L., & Johnson, J. N. (1990). A constitutive model for the dynamic response of brittle materials. *Journal of Applied Physics*, 67(7), 3275–3286.
- Ames, N. M., Srivastava, V., Chester, S. A., & Anand, L. (2009). A thermo-mechanically coupled theory for large deformations of amorphous polymers. part II: Applications. *International Journal of Plasticity*, 25(8), 1495–1539.
- Anand, L., Ames, N. M., Srivastava, V., & Chester, S. A. (2009). A thermo-mechanically coupled theory for large deformations of amorphous polymers. part i: Formulation. *International Journal of Plasticity*, 25(8), 1474–1494.
- Anand, L., & Gurtin, M. E. (2003). A theory of amorphous solids undergoing large deformations, with application to polymeric glasses. *International Journal of Solids and Structures*, 40(6), 1465–1487.
- Archer, J. S., & Lesser, A. J. (2011). Shear band formation and mode II fracture of polymeric glasses [eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1002/polb.22159>]. *Journal of Polymer Science Part B: Polymer Physics*, 49(2), 103–114.
- Argon, A. S. (1973). A theory for the low-temperature plastic deformation of glassy polymers [eprint: <https://doi.org/10.1080/14786437308220987>]. *The Philosophical Magazine: A Journal of Theoretical Experimental and Applied Physics*, 28(4), 839–865.
- Arruda, E. M., Boyce, M. C., & Jayachandran, R. (1995). Effects of strain rate, temperature and thermomechanical coupling on the finite strain deformation of glassy polymers. *Mechanics of Materials*, 19(2), 193–212.
- Asay, J. R., & Lipkin, J. (1978). A self-consistent technique for estimating the dynamic yield strength of a shock-loaded material. *Journal of Applied Physics*, 49(7), 4242–4247.
- Austin, R. A., Barton, N. R., Reaugh, J. E., & Fried, L. E. (2015). Direct numerical simulation of shear localization and decomposition reactions in shock-loaded HMX crystal. *Journal of Applied Physics*, 117(18).

- Barker, L. M., & Hollenbach, R. E. (1970). Shock-wave studies of PMMA, fused silica, and sapphire. *Journal of Applied Physics*, *41*(10), 4208–4226.
- Barker, L. M., & Hollenbach, R. E. (1972). Laser interferometer for measuring high velocities of any reflecting surface. *Journal of Applied Physics*, *43*(11), 4669–4675.
- Barranger, Y., Doumalin, P., Dupré, J. C., & Germaneau, A. (2012). Strain measurement by digital image correlation: Influence of two types of speckle patterns made from rigid or deformable marks. *Strain*, *48*(5), 357–365.
- Barua, A., Kim, S., Horie, Y., & Zhou, M. (2013). Ignition criterion for heterogeneous energetic materials based on hotspot size-temperature threshold. *Journal of Applied Physics*, *113*(6).
- Bat'kov, Y. V., Novikov, S. A., & Fishman, N. D. (1996). Shear stresses in polymers under shock compression. *AIP Conference Proceedings*, *370*, 577–580.
- Bechmann, R. (1958). Elastic and piezoelectric constants of alpha-quartz. *Physical Review*, *110*(5), 1060–1061.
- Bennett, J. G., Haberman, K. S., Johnson, J. N., & Asay, B. W. (1998). A constitutive model for the non-shock ignition and mechanical response of high explosives. *Journal of the Mechanics and Physics of Solids*, *46*(12), 2303–2322.
- Bernstein, D., Godfrey, C., Klein, A., & Shimmin, W. (1968). Research on manganin pressure transducers. *Behaviour of Dense Media under High Dynamic Pressures*. N.-Y, 461.
- Bless, S. J., Brar, N. S., Kanel, G., & Rosenberg, Z. (1992). Failure waves in glass [eprint: <https://ceramics.onlinelibrary.wiley.com/doi/pdf/10.1111/j.1151-2916.1992.tb04174.x>]. *Journal of the American Ceramic Society*, *75*(4), 1002–1004.
- Bloomquist, D. D., & Sheffield, S. A. (1981). Shock-compression temperature rise in polymethyl methacrylate determined from resistivity of embedded copper foils. *Applied Physics Letters*, *38*(3), 185–187.
- Bodner, S. R., & Partom, Y. (1975). Constitutive equations for elastic-viscoplastic strain-hardening materials. *Journal of Applied Mechanics*, *42*(2), 385–389.

- Bowden, F. P., & Yoffe, A. (1948). Hot spots and the initiation of explosion. *Symposium on Combustion and Flame, and Explosion Phenomena*, 3(1), 551–560.
- Bowden, P. B., & Jukes, J. A. (1972). The plastic flow of isotropic polymers. *Journal of Materials Science*, 7(1), 52–63.
- Bower, A. F. (2009). *Applied mechanics of solids*. CRC press.
- Boyce, M. C., Parks, D. M., & Argon, A. S. (1988). Large inelastic deformation of glassy polymers. part i: Rate dependent constitutive model. *Mechanics of Materials*, 7(1), 15–33.
- Brandrup, J., Immergut, E. H., Grulke, E. A., Abe, A., & Bloch, D. R. (1999). *Polymer handbook* (Vol. 89). Wiley New York.
- Buechler, M. A., & Luscher, D. J. (2012). *A semi-implicit integration scheme for the stress update of viscoSCRAM*. Los Alamos National Laboratory (LANL).
- Cearley, G., Bradley, A., O’Brien, C., Fuller, T., Sanchez, J., & Niederhaus, J. (2024, October 1). *Viscoelastic modeling of polymers in ALEGRA with the GAP model* (SAND–2024-14744, 2480093).
- Chen, W., & Ravichandran, G. (1996). Static and dynamic compressive behavior of aluminum nitride under moderate confinement. *Journal of the American Ceramic Society*, 79(3), 579–584.
- Clayton, J. D. (2026, February 7). Constitutive theory for mechanics of amorphous thermoplastic polymers under extreme dynamic loading.
- Clements, B., & Ma, X. (2018). Mechanically activated thermal chemistry (MATCH) model for PAGOSA users. *LANL report*.
- Clements, B. E. (2012). A continuum glassy polymer model applicable to dynamic loading. *Journal of applied physics*, 112(8).
- Clements, B. E. (2011). *Temperature calculation using viscoscrum with discussion of supporting experiments*. Los Alamos National Laboratory (LANL).
- Clifton, R. J., Song, S., & Jiao, T. (2020). Inverse problem for PSPI experiments, 070008.

- Clifton, R. J., Wang, X., & Jiao, T. (2016). A physically-based, quasilinear viscoelasticity model for the dynamic response of polyurea. *Journal of the Mechanics and Physics of Solids*, 93, 8–15.
- Coe, J. D., Brown, E., Cady, C. M., Carlson, C. A., Clements, B. E., Dattelbaum, D. M., Fezzaa, K., Gustavsen, R. L., Hooks, D. E., Iverson, A. J., Jensen, B. J., Jordan, J. L., Jones, D. R., Junghans, S. A., Lang, J. M. J., LeBrun, T. J., Lewis, M. W., Maerzke, K. A., Pierce, T. H., ... Welch, P. M. J. (2017, October 10). *Equation of state and damage in polyethylene* (LA-UR-17-29234). Los Alamos National Laboratory (LANL).
- Cooper, P. W. (2018). *Explosives engineering*. John Wiley & Sons.
- Criscione, J. C., Humphrey, J. D., Douglas, A. S., & Hunter, W. C. (2000). An invariant basis for natural strain which yields orthogonal stress response terms in isotropic hyperelasticity. *Journal of the Mechanics and Physics of Solids*, 48(12), 2445–2465.
- Dandekar, D. P., & Beaulieu, P. A. (1995). Failure wave under shock wave compression in soda lime glass [Pages: 211–218 Publication Title: Metallurgical and Material Applications of Shock-Wave and High-Strain-Rate Phenomena].
- Dandekar, D. P. (2004). Spall strength of silicon carbide under normal and simultaneous compression-shear shock wave loading [_eprint: <https://ceramics.onlinelibrary.wiley.com/doi/pdf/10.1002/7402.2004.tb00178.x>]. *International Journal of Applied Ceramic Technology*, 1(3), 261–268.
- Dattelbaum, D. M., Schilling, B. F., Clements, B. E., Jordan, J. L., Welch, C. F., & Stull, J. A. (2026). Shock response and dynamic failure of high density-(HDPE) and ultra-high molecular weight polyethylene (UHMWPE). *Journal of Dynamic Behavior of Materials*, 12(1), 37–48.
- Dienes, J. K., & Kershner, J. D. (1998, December 31). *Multiple-shock initiation via statistical crack mechanics* (LA-UR-98-3046; CONF-980803–). Los Alamos National Lab., NM (United States).

- Dienes, J. K., & Margolin, L. G. (1980). A computational approach to rock fragmentation. *ARMA US Rock Mechanics/Geomechanics Symposium*, ARMA-80.
- Dolan, D. H. (2020). Extreme measurements with photonic doppler velocimetry (PDV). *Review of Scientific Instruments*, *91*(5), 051501.
- Dolan, D., Iii, & Ao, T. (2010, June 1). *SIRHEN : A data reduction program for photonic doppler velocimetry measurements*. (SAND2010-3628, 989357).
- Donnay, J. D. H., & Le Page, Y. (1978). The vicissitudes of the low-quartz crystal setting or the pitfalls of enantiomorphism. *Acta Crystallographica Section A: Crystal Physics, Diffraction, Theoretical and General Crystallography*, *34*(4), 584–594.
- Duprey, K. E. (2000). Pressure-shear response of thin tantalum foils. *AIP Conference Proceedings*, *505*, 447–450.
- Escauriza, E. M., Duarte, J. P., Chapman, D. J., Rutherford, M. E., Farbaniec, L., Jonsson, J. C., Smith, L. C., Olbinado, M. P., Skidmore, J., Foster, P., Ringrose, T., Rack, A., & Eakins, D. E. (2020). Collapse dynamics of spherical cavities in a solid under shock loading [ISBN: 4159802064669]. *Scientific Reports*, *10*(1), 1–16.
- Escauriza, E. M., Olbinado, M. P., Rutherford, M. E., Chapman, D. J., Jonsson, J. C. Z., Rack, A., & Eakins, D. E. (2018). Ultra-high-speed indirect x-ray imaging system with versatile spatiotemporal sampling capabilities. *Applied Optics*, *57*(18), 5004.
- Eshelby, J. D. (1957). The determination of the elastic field of an ellipsoidal inclusion, and related problems. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, *241*(1226), 376–396.
- Espinosa, H. D., Raiser, G., Clifton, R. J., & Ortiz, M. (1992). Experimental observations and numerical modeling of inelasticity in dynamically loaded ceramics. *J. Hard Mater*, *3*(3), 285–313.
- Estrada, J. B., & Franck, C. (2015). Intuitive interface for the quantitative evaluation of speckle patterns for use in digital image and volume correlation techniques. *Journal of Applied Mechanics*, *82*(95001).

- Eyring, H. (1936). Viscosity, plasticity, and diffusion as examples of absolute reaction rates. *The Journal of Chemical Physics*, *4*(4), 283–291.
- Ferreira, B. P., & Bessa, M. A. (2026). Automatically differentiable model updating (ADiMU): Conventional, hybrid, and neural network material model discovery including history-dependency. *Journal of the Mechanics and Physics of Solids*, *206*, 106408.
- Field, J. E. (1991). The physics of liquid impact, shock wave interactions with cavities, and the implications to shock wave lithotripsy. *Physics in Medicine and Biology*, *36*(11), 1475–1484.
- Fleck, N. A., Stronge, W. J., & Liu, J. H. (1990). High strain-rate shear response of polycarbonate and polymethyl methacrylate. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, *429*(1877), 459–479.
- Frank, G. J., & Brockman, R. A. (2001). A viscoelastic–viscoplastic constitutive model for glassy polymers. *International Journal of Solids and Structures*, *38*(30), 5149–5164.
- Fredenburg, D. A., & Chisolm, E. D. (2014). *(u) equation of state and compaction modeling for CeO₂*. Los Alamos National Laboratory (LANL), Los Alamos, NM (United States).
- Gandhi, V., Ravindran, S., & Ravichandran, G. (2022). Dynamic strength of iron at high pressures and strain rates. *Physical Review Letters*, *128*(1), 015705.
- Gearing, B. P., & Anand, L. (2004). On modeling the deformation and fracture response of glassy polymers due to shear-yielding and crazing. *International Journal of Solids and Structures*, *41*(11), 3125–3150.
- Gilat, A., & Clifton, R. J. (1985). Pressure-shear waves in 6061-t6 aluminum and alpha-titanium. *Journal of the Mechanics and Physics of Solids*, *33*(3), 263–284.
- Glazer, A. M. (2018). Confusion over the description of the quartz structure yet again. *Journal of Applied Crystallography*, *51*(3), 915–918.

- Grunschel, S. E., & Clifton, R. J. (2007). Dynamic plastic response of aluminum at temperatures approaching melt. *Metallurgical and Materials Transactions A*, 38(12), 2885–2890.
- Guduru, P. R. (2001). *An investigation of dynamic failure events in steels using full field high-speed infrared thermography and high -speed photography* [Doctoral dissertation, California Institute of Technology].
- Gupta, Y. M. (1980). Determination of the impact response of PMMA using combined compression and shear loading. *Journal of Applied Physics*, 51(10), 5352–5361.
- Hardin, D. B., & Mares, J. O. (2018). Temperature rise from collapsing pores. *AIP Conference Proceedings*, 1979.
- Haynes, W. M. (2016). *CRC handbook of chemistry and physics*. CRC press.
- Helm, J. D. (2008). Digital image correlation for specimens with multiple growing cracks. *Experimental Mechanics*, 48(6), 753–762.
- Helminiak, N. S., & Borg, J. P. (2023). The equation of state and strength properties of copper mountain sandstone. *AIP Conference Proceedings*, 2844(1).
- Hill, R. (1998). *The mathematical theory of plasticity* (Vol. 11). Oxford university press.
- Holmquist, T., Bradley, J., Dwivedi, A., & Casem, D. (2016). The response of polymethyl methacrylate (PMMA) subjected to large strains, high strain rates, high pressures, a range in temperatures, and variations in the intermediate principal stress. *The European Physical Journal Special Topics*, 225(2), 343–354.
- Hopkinson, B. (1914). X. a method of measuring the pressure produced in the detonation of high, explosives or by the impact of bullets. *Philosophical Transactions of the Royal Society of London, Series A: Containing Papers of a Mathematical or Physical Character*, 213(497), 437–456.
- Irwin, G. R. (1968). Linear fracture mechanics, fracture transition, and fracture control. *Engineering Fracture Mechanics*, 1(2), 241–257.
- Jensen, B. J., Cherne, F. J., Prime, M. B., Fezzaa, K., Iverson, A. J., Carlson, C. A., Yeager, J. D., Ramos, K. J., Hooks, D. E., Cooley, J. C., & Dimonte, G. (2015).

- Jet formation in cerium metal to examine material strength. *Journal of Applied Physics*, 118(19), 195903.
- Johnson, B. P., McGrane, S. D., Lang, J. M., Ramos, K. J., Bolme, C. A., & Aslam, T. (2024). Temperature measurements in shocked PBX 9501 using raman thermometry. *AIP Conference Proceedings*, 3066(1), 570004.
- Johnson, C. R., Borg, J. P., Alexander, C. S., LaJeunesse, J. W., Helminiak, N. S., & Specht, P. E. (2024). Flow strength measurements of wrought and AM SS304l via pressure shear plate impact experiments. *Journal of Dynamic Behavior of Materials*, 10(1), 2–19.
- Johnson, G. R. (1983). A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures. *Proceedings of the 7th International Symposium on Ballistics, Am. Def. Prep. Org.(ADPA), Netherlands, 1983*, 541–547.
- Johnson, G. R., & Cook, W. H. (1985). Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures. *Engineering Fracture Mechanics*, 21(1), 31–48.
- Johnson, G. R., & Holmquist, T. J. (1994). An improved computational constitutive model for brittle materials. *AIP Conference Proceedings*, 309, 981–984.
- Johnson, J. N. (1971). Shock propagation produced by planar impact in linearly elastic anisotropic media. *Journal of Applied Physics*, 42(13), 5522–5530.
- Johnson, P. M., & Burgess, T. J. (1968). Free surface velocity measurement of an impacted projectile by optical doppler shift. *Review of Scientific Instruments*, 39(8), 1100–1103.
- Jordan, J. L., Casem, D., & Zellner, M. (2016). Shock response of polymethylmethacrylate. *Journal of Dynamic Behavior of Materials*, 2(3), 372–378.
- Jordan, J. L., Casem, D. T., & Robinette, J. (2022). Hugoniot and dynamic strength in polyurea. *Journal of Applied Physics*, 131(16), 165903.

- Jordan, J. L., Casem, D. T., Sturtevant, B. T., & Sutherland, G. (2020). Dynamic strength in polymethylmethacrylate, 040006.
- Karniadakis, G. E., Kevrekidis, I. G., Lu, L., Perdikaris, P., Wang, S., & Yang, L. (2021). Physics-informed machine learning. *Nature Reviews Physics*, 3(6), 422–440.
- Katsura, T., & Tange, Y. (2019). A simple derivation of the birch–murnaghan equations of state (EOSs) and comparison with EOSs derived from other definitions of finite strain. *Minerals*, 9(12), 745.
- Kettenbeil, C., Lovinger, Z., Jiao, T., Mello, M., Clifton, R., & Ravichandran, G. (2022). Inelastic behavior of tungsten carbide at high pressures. *Journal of the Mechanics and Physics of Solids*, 159, 104762.
- Kettenbeil, C., Lovinger, Z., Ravindran, S., Mello, M., & Ravichandran, G. (2020). Pressure-shear plate impact experiments at high pressures. *Journal of Dynamic Behavior of Materials*, 6(4), 489–501.
- Kettenbeil, C. (2019). *Dynamic strength of silica glasses at high pressures and strain rates*.
- Kettenbeil, C., Mello, M., Bischann, M., & Ravichandran, G. (2018a). Heterodyne transverse velocimetry for pressure-shear plate impact experiments. *Journal of Applied Physics*, 123(12).
- Kettenbeil, C., Mello, M., Bischann, M., & Ravichandran, G. (2018b). Heterodyne transverse velocimetry for pressure-shear plate impact experiments. *Journal of Applied Physics*, 123(12), 125902.
- Kim, K.-S., Clifton, R. J., & Kumar, P. (1977). A combined normal- and transverse-displacement interferometer with an application to impact of y-cut quartz. *Journal of Applied Physics*, 48(10), 4132–4139.
- Kittel, C. (2018, June 6). *Kittel’s introduction to solid state physics, global edition* [Google-Books-ID: nNpVEAAQBAJ]. John Wiley & Sons.
- Kittell, D. E., & Yarrington, C. D. (2016). A physically-based mie–grüneisen equation of state to determine hot spot temperature distributions [eprint: <https://doi.org/10.1080/13647830.2016.125902>]. *Combustion Theory and Modelling*, 20(5), 941–957.

- Klopp, R. W., Clifton, R. J., & Shawki, T. G. (1985). Pressure-shear impact and the dynamic viscoplastic response of metals. *Mechanics of Materials*, 4(3), 375–385.
- Kolsky, H. (1949). An investigation of the mechanical properties of materials at very high rates of loading. *Proceedings of the Physical Society. Section B*, 62(11), 676.
- Lacina, D., Neel, C., & Dattelbaum, D. (2018). Shock response of poly[methyl methacrylate] (PMMA) measured with embedded electromagnetic gauges. *Journal of Applied Physics*, 123(18), 185901.
- Lawlor, B. P., Gandhi, V., & Ravichandran, G. (2024). Full-field quantitative visualization of shock-driven pore collapse and failure modes in PMMA. *Journal of Applied Physics*, 136(22), 225901.
- Lemke, R. W., Knudson, M. D., & Davis, J. -. (2011). Magnetically driven hyper-velocity launch capability at the sandia z accelerator. *International Journal of Impact Engineering*, 38(6), 480–485.
- Lind, J., Nelms, M. D., Robinson, A. K., Kumar, M., & Barton, N. R. (2021). Examining material constitutive response under dynamic compression and large plastic strains using in situ imaging of hole closure. *Acta Materialia*, 206, 116584.
- Lu, L., Fan, D., Bie, B. X., Ran, X. X., Qi, M. L., Parab, N., Sun, J. Z., Liao, H. J., Hudspeth, M. C., Claus, B., Fezzaa, K., Sun, T., Chen, W., Gong, X. L., & Luo, S. N. (2014). Note: Dynamic strain field mapping with synchrotron x-ray digital image correlation. *Review of Scientific Instruments*, 85(7), 076101.
- Mader, C. L., Gibbs, T. R., Hopson, J. W., Marsh, S. P., Popolato, A., Hoyt, M. S., Thayer, K. V., Barnes, J. F., Craig, B. G., Deal, W. E., Dick, R. D., Johnson, J. N., Marshall, E., Morris, C. E., Neal, T. R., Peterson, S. W., Rogers, R. N., Thieme, M. T., Wackerle, J. D., & Walsh, J. M. (n.d.). LOS ALAMOS SERIES ON DYNAMIC MATERIAL PROPERTIES.
- Malhotra, P., & Guduru, P. (2020). A technique for high-speed microscopic imaging of dynamic failure events. *Submitted to Experimental Mechanics*.

- Malhotra, P., Jiao, T., Clifton, R. J., & Guduru, P. R. (2021). Dynamic shearing resistance of a polymer-bonded energetic simulant: Composite of sucrose and hydroxyl-terminated polybutadiene (HTPB). *Journal of Applied Physics*, 130(18), 185902.
- Malhotra, P., Jiao, T., Henann, D. L., Clifton, R. J., & Guduru, P. R. (2022). Dynamic shearing resistance of an energetic material simulant: Sucrose. *Journal of the Mechanics and Physics of Solids*, 159.
- Menikoff, R. (2012, June 28). *Complete mie-gruneisen equation of state* (LA-UR-12-22592). Los Alamos National Laboratory (LANL), Los Alamos, NM (United States).
- Meyers, M. (1994). *Dynamic behavior of materials*. John Wiley & sons, 1994.
- Millett, J. C. F., & Bourne, N. K. (2000). The deviatoric response of polymethylmethacrylate to one-dimensional shock loading. *Journal of Applied Physics*, 88(12), 7037–7040.
- Millett, J. C. F., Gray, G. T., & Bourne, N. K. (2007). Measurement of the shear strength of pure tungsten during one-dimensional shock loading. *Journal of Applied Physics*, 101(3), 033520.
- Nasraoui, M., Forquin, P., Siad, L., & Rusinek, A. (2012). Influence of strain rate, temperature and adiabatic heating on the mechanical behaviour of poly-methylmethacrylate: Experimental and modelling analyses. *Materials & Design*, 37, 500–509.
- Oleinik, E. F., Rudnev, S. N., & Salamatina, O. B. (2007). Evolution in concepts concerning the mechanism of plasticity in solid polymers after the 1950s. *Polymer Science Series A*, 49(12), 1302–1327.
- Poirier, J. .-, & Tarantola, A. (1998). A logarithmic equation of state. *Physics of the Earth and Planetary Interiors*, 109(1), 1–8.
- Prakash, V., & Mehta, N. (2012). Uniaxial compression and combined compression-and-shear response of amorphous polycarbonate at high loading rates [eprint: <https://4spepublications.onlinelibrary.wiley.com/doi/pdf/10.1002/pen.22190>]. *Polymer Engineering & Science*, 52(6), 1217–1231.

- Quach, A., Wilson, P. S., & Simha, R. (1974). The effects of stereoregularity on the pressure-volume-temperature and related properties of poly(methyl methacrylate) [eprint: <https://doi.org/10.1080/00222347408204553>]. *Journal of Macromolecular Science, Part B*, 9(3), 533–550.
- Quinson, R., PEREZ, J., RINK, M., & PAVAN, A. (1997). Yield criteria for amorphous glassy polymers. *Journal of Materials Science*, 32(5), 1371–1379.
- Rabinowitz, S., Ward, I. M., & Parry, J. S. C. (1970). The effect of hydrostatic pressure on the shear yield behaviour of polymers. *Journal of Materials Science*, 5(1), 29–39.
- Rai, N. K., Escauriza, E. M., Eakins, D. E., & Udaykumar, H. S. (2020). Mechanics of shock induced pore collapse in poly(methyl methacrylate) (PMMA): Comparison of simulations and experiments. *Journal of the Mechanics and Physics of Solids*, 143.
- Ramesh, K. T., & Clifton, R. J. (1987). A pressure-shear plate impact experiment for studying the rheology of lubricants at high pressures and high shearing rates. *Journal of Tribology*, 109(2), 215–222.
- Ranjan, D., Oakley, J., & Bonazza, R. (2011). Shock-bubble interactions. *Annual Review of Fluid Mechanics*, 43, 117–140.
- Rauls, M. B. (2015). Shock wave behavior of particulate composites [ISBN: 9781369368376]. *ProQuest Dissertations and Theses*, 2015, 180.
- Ravindran, S., Gandhi, V., Lovinger, Z., Mello, M., & Ravichandran, G. (2021). Dynamic strength of copper at high pressures using pressure shear plate experiments. *Journal of Dynamic Behavior of Materials*, 7(2), 248–261.
- Ravindran, S., Lovinger, Z., Kettenbeil, C., Mello, M., & Ravichandran, G. (2020). Pressure-shear plate impact experiments of magnesium at high pressures. *AIP Conference Proceedings*, 2272(1), 120022.
- Reu, P. (2014). *All about speckles: Speckle size measurement*.
- Richeton, J., Ahzi, S., Vecchio, K. S., Jiang, F. C., & Adharapurapu, R. R. (2006). Influence of temperature and strain rate on the mechanical behavior of three

- amorphous polymers: Characterization and modeling of the compressive yield stress. *International Journal of Solids and Structures*, 43(7), 2318–2335.
- Rittel, D., & Brill, A. (2008). Dynamic flow and failure of confined polymethylmethacrylate. *Journal of the Mechanics and Physics of Solids*, 56(4), 1401–1416.
- Robertson, R. E. (1966). Theory for the plasticity of glassy polymers. *The Journal of Chemical Physics*, 44(10), 3950–3956.
- Rosenberg, Z. (1984). Determination of the dynamic phase transitions in bismuth with in-material manganin gauges. *Journal of Applied Physics*, 56(11), 3328–3329.
- Rosenberg, Z., & Moshel, G. (2014). Revisiting the calibration of manganin gauges for lateral stress measurements in shock-loaded solids. *Journal of Applied Physics*, 115(10), 103511.
- Rosenberg, Z., & Partom, Y. (1994). Accounting for the hugoniot elastic limits of polymers by using pressure-dependent yield criterion. *Journal of Applied Physics*, 76(3), 1935–1936.
- Roy, S., Johnson, B. P., Zhou, X., Nguyen, Y. T., Dlott, D. D., & Udaykumar, H. S. (2022). Hot spot ignition and growth from tandem micro-scale simulations and experiments on plastic-bonded explosives. *Journal of Applied Physics*, 131(20), 205901.
- Sable, P. A. (2019). Multi-scale traction dynamics in obliquely impacted polymer-metal targets. *Dissertations*.
- Schreier, H., Orteu, J. J., & Sutton, M. A. (2009). *Image correlation for shape, motion and deformation measurements: Basic concepts, theory and applications* [Publication Title: Image Correlation for Shape, Motion and Deformation Measurements: Basic Concepts, Theory and Applications]. Springer US.
- Schuler, K. W., & Nunziato, J. W. (1974). The dynamic mechanical behavior of polymethyl methacrylate. *Rheologica Acta*, 13(2), 265–273.

- Springer, H. K., Bastea, S., Nichols, A. L., Tarver, C. M., & Reaugh, J. E. (2018). Modeling the effects of shock pressure and pore morphology on hot spot mechanisms in HMX. *Propellants, Explosives, Pyrotechnics*, 43(8), 805–817.
- Strand, O. T., Goosman, D. R., Martinez, C., Whitworth, T. L., & Kuhlow, W. W. (2006). Compact system for high-speed velocimetry using heterodyne techniques. *Review of Scientific Instruments*, 77(8), 083108.
- Sun, X., Chauhan, A., Mallick, D. D., Tonge, A. L., McCauley, J. W., Hemker, K. J., LaSalvia, J. C., & Ramesh, K. T. (2020). Granular flow of an advanced ceramic under ultra-high strain rates and high pressures. *Journal of the Mechanics and Physics of Solids*, 143, 104031.
- Sundaram, S., & Clifton, R. J. (1998). Flow behavior of soda-lime glass at high pressures and high shear rates. *AIP Conference Proceedings*, 517–520.
- Sundaram, S. (1998). *Pressure-shear plate impact studies of alumina ceramics and the influence of an intergranular glassy phase* [Doctoral dissertation, Brown University].
- Tait, P. G. (1965). *Report on some of the physical properties of fresh water and of sea water*. Johnson Reprint Corporation.
- Theobald, S., Pechhold, W., & Stoll, B. (2001). The pressure and temperature dependence of the relaxation processes in poly(methylmethacrylate). *Polymer*, 42(1), 289–295.
- Turner, D. (2017). *Digital image correlation engine*.
- Wagner, K. B., Keyhani, A., Boddorff, A. K., Kennedy, G., Montaigne, D., Jensen, B. J., Beason, M., Zhou, M., & Thadhani, N. N. (2020). High-speed x-ray phase contrast imaging and digital image correlation analysis of microscale shock response of an additively manufactured energetic material simulant. *Journal of Applied Physics*, 127(23).
- Walze, G. (2005). *Mikrostrukturierte oberflächen in kombination mit optischen schaltungsmechanismen zum tageslichtmanagement* [Doctoral dissertation, Freiburg (Breisgau), Univ., Diss., 2006].

- Willmott, P. (2011). An introduction to synchrotron radiation. an introd. to synchrotron radiat. tech. appl.
- Winey, J. M., Gruzdkov, Y. A., Dreger, Z. A., Jensen, B. J., & Gupta, Y. M. (2002). Thermomechanical model and temperature measurements for shocked ammonium perchlorate single crystals. *Journal of Applied Physics*, *91*(9), 5650–5656.
- Wu, P. D., & van der Giessen, E. (1995). On neck propagation in amorphous glassy polymers under plane strain tension. *International Journal of Plasticity*, *11*(3), 211–235.
- Wu, W., Owino, J., Al-Ostaz, A., & Cai, L. (n.d.). Applying periodic boundary conditions in finite element analysis.
- Wunderlich, B. (1995). The ATHAS database on heat capacities of polymers. *Pure and Applied Chemistry*, *67*(6), 1019–1026.
- Yang, J., & Bhattacharya, K. (2019). Augmented lagrangian digital image correlation. *Experimental Mechanics*, *59*(2), 187–205.
- Zecevic, M., Zuanetti, B., Leong, A. F. T., Meredith, C. S., Bolme, C. A., Ramos, K. J., & Cawkwell, M. J. (2026). Phase-field modeling and experiments of dynamic fracture in single crystal quartz. *Acta Materialia*, *305*, 121819.
- Zhou, M. (1993). *Dynamic shear localization in a tungsten heavy alloy and ductile rupture in a spheroidized 1045 steel*. Brown University.
- Zuanetti, B., Wang, T., & Prakash, V. (2017). A compact fiber optics-based heterodyne combined normal and transverse displacement interferometer. *Review of Scientific Instruments*, *88*(3), 033108.
- Zuanetti, B., Wang, T., & Prakash, V. (2019). Plate impact investigation of the dynamic response of commercial tungsten carbide under shock-induced compression and combined compression-and-shear loading. *International Journal of Impact Engineering*, *131*, 200–208.

Zuanetti, B., Wang, T., & Prakash, V. (2020). Dynamic behavior of polycrystalline metals under combined pressure and shear loading at elevated temperatures. *AIP Conference Proceedings*, 2272(1), 100024.