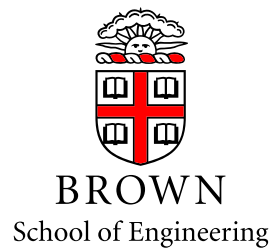


Bubble Dynamics and Coalescence: bubble-droplet filtration and microgravity
bubble emulsions

Madeline E. Federle



A dissertation submitted in partial fulfillment
of the requirements of the degree of Doctor of Philosophy
in the School of Engineering at Brown University

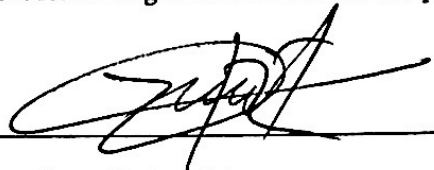
Providence, Rhode Island

April 2026

Copyright © 2026 Madeline E. Federle

This dissertation by Madeline E. Federle is accepted in its present form
by the School of Engineering as satisfying the
dissertation requirement for the degree of Doctor of Philosophy.

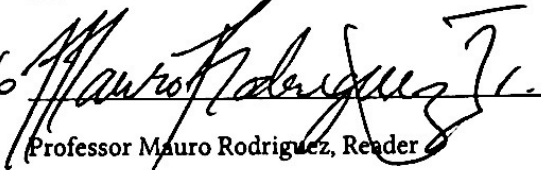
Date April 8, 2026



Professor Roberto Zenit, Advisor

Recommended to the Graduate Council

Date April 14, 2026



Professor Mauro Rodriguez, Reader

Date April 08, 2026



Professor Dominique Legendre, Reader

Date April 8, 2026



Professor Ralph Milliken, Reader

Approved by the Graduate Council

Date _____

David P. Lindstrom, Dean of the Graduate School

ABSTRACT

Multiphase flows, systems in which more than one phase is present, are inherently complex yet ubiquitous across a wide range of industrial and natural phenomena. A subset of multiphase flows, bubbly flows, serve as the central focus of this dissertation. Despite their apparent simplicity, bubbly flows support critical technologies ranging from industrial fluid filtration to liquid propellant management in space. This dissertation aims to explore the fundamental physics governing bubbly flows and extend this understanding to microgravity environments.

The first area of investigation examines bubbly flows as a method of fluid filtration. Specifically, the process of aeration, in which bubbles capture and transport impurities toward the fluid surface, was studied as a mechanism for separating oil droplets from produced water, a byproduct of oil extraction. Contrary to classical assumptions, experimental results demonstrated that coalescence via flotation is unlikely. Instead, a velocity enhancement of the oil droplet, driven by the bubble's surrounding flow field, was both experimentally observed and theoretically modeled. A mathematical framework was developed to quantify this hydrodynamic interaction and identify the governing parameters that dictate the conditions under which this velocity enhancement occurs.

Building on this foundation, the work was extended to study bubbly flows in reduced gravity environments. These experiments were conducted using an in-house designed homogeneous isotropic turbulence chamber, with microgravity conditions achieved through NASA drop tower facilities and Zero-G parabolic flight campaigns. The primary objectives of this work were to quantify bubble breakup, size distributions, and bubble dispersion under varying turbulent conditions in microgravity. Additionally, the speed of sound through the bubbly liquid was measured. It was found that bubble size distributions in microgravity differ significantly from those observed under Earth's gravity, and a critical bubble diameter was established as a function of turbulence intensity. Furthermore, the reduction in speed of sound through a bubbly liquid relative to the continuous phase was experimentally verified.

CURRICULUM VITÆ

Madeline Federle is a mechanical engineering graduate from The George Washington University (GWU). Her passion for engineering developed due to her inherit success in math from a young age but also her desire to pursue her dream of becoming an astronaut. During her time at GWU, she joined the rocket team which further developed her interests in rocketry, becoming level 1 certified in rocketry, as well as space science. During her years at GWU, she interned at the NAVSEA, a department of the Navy, working on submarine ballistic launched missiles. Madeline graduated from GWU in 2019 with a B.S in Mechanical Engineering and a minor in philosophy.

Driven by a desire to pursue a more academically focused career, Madeline left her position at Raytheon in 2020, where she worked on CAD models for sonar devices, to begin her doctoral studies in Fluids and Thermal Sciences at Brown University. At Brown, she worked under the guidance of Professor Roberto Zenit, investigating the complexities of bubbly flows across a range of applications.

Her research focused on the hydrodynamics governing coalescence between an oil droplet and an air bubble. Through extensive experimentation, she quantified the probability of coalescence during flotation processes. Her findings revealed that coalescence between these phases during flotation is highly unlikely; instead, she observed a droplet velocity enhancement resulting from interactions with the surrounding bubble-induced flow field.

Expanding her research interests into space science, Madeline also examined the effects of microgravity on bubble emulsions. In this work, she studied the formation and stability of bubble emulsions in microgravity environments and quantified their dispersion and breakup under turbulent conditions. She has also began investigating the speed of sound in bubble emulsions, aiming to further evaluate and refine the well-established theoretical models used to predict acoustic wave propagation in bubbly liquids.

Through this body of work, Madeline has developed deep expertise in multiphase flow physics, experimental fluid mechanics, and the coupling between hydrodynamics and acoustics. Her research advances fundamental understanding of complex two-phase systems while contributing insights relevant to energy, environmental, and space applications.

Publications

1. **Federle, M. E.**, A. Das, V. Mathai, and R. Zenit. The interaction of an oil droplet and a gas bubble rising in a quiescent fluid. *International Journal of Multiphase Flow*, 171:104682, 2024.
2. **Federle, M. E.** and R. Zenit. Physical conditions for droplet–bubble coalescence in flotation. *International Journal of Multiphase Flow*, 179:104933, 2024.
3. M. Ravisankar, **Federle, M. E.**, M. Bahadori, A. Das, H. Kesari, and R. Zenit. The mechanics of the squash nick shot. *Proceedings of the National Academy of Sciences*, 122(20):e2505715122, 2025.
4. A. Trevino, J.-W. Barotta, **Federle, M. E.**, and R. Zenit. Blending fluid mechanics and art part i: Making a splash with high speed photography. *The Physics Teacher*, 64:348–352, 2026.
5. **Federle, M. E.**, B. Lyons, M. Ravisankar, and R. Zenit. Design of an accessible turbulence chamber for laboratory experiments. *HardwareX*, page e00745, 2026.
6. J.-W. Barotta, **Federle, M. E.**, A. Trevino, and R. Zenit. Blending fluid mechanics and art part ii: Flow visualization. *The Physics Teacher*, NA:In press, 2026.
7. **Federle, M. E.**, J.-W. Barotta, A. Trevino, and R. Zenit. Blending fluid mechanics and art part iii: Fluids art installation. *The Physics Teacher*, NA:In press, 2026.
8. **Federle, M. E.** and R. Zenit. Formation of bubble emulsions in microgravity. (*in prep*)., 2026.

Conferences

- | | |
|---|---------------|
| 1. 12th International Conference on Multiphase Flow, Toulouse, France | April 2025 |
| 2. American Physical Society - DFD 77th Annual Meeting | November 2024 |
| 3. American Physical Society - DFD 76th Annual Meeting | November 2023 |
| 4. 11th International Conference on Multiphase Flow, Kobe, Japan | April 2023 |
| 5. 2023 NASA RI Space Grant Annual Spring Symposium | April 2023 |
| 6. American Physical Society - DFD 75th Annual Meeting | November 2022 |

- | | |
|--|---------------|
| 7. 2022 NASA RI Space Grant Annual Spring Symposium | April 2022 |
| 8. American Physical Society - DFD 74th Annual Meeting | November 2021 |
| 9. 27th Annual Meeting of the Fluids Division of the Mexican Physics Society | November 2021 |

Teaching experience

1. Graduate Teaching Assistant - Fluid Mechanics (Fall 21), Fluid Mechanics of Aerospace and Energy Systems (Spring 23), Art Fluid Engineering (Spring 24&25), and Engineering Statics and Dynamics (Spring 2026).

News & Press

- *Aspiring Astronaut, PhD student studies bubbles—in space*, Brown Alumni Magazine, December 2025.
- *The ingredients for an unbeatable squash shot*, Physics Today Back Scatter, September 2025.
- *Madeline Federle: This aspiring astronaut is studying bubble behavior in zero gravity*, News from Brown, August 2025.
- *Student Spotlight: Bubble emulsions in microgravity*, Rhode Island Space Grant Social Media, July 2025.
- *No player can return this killer shot. Physics explains how it works.*, ScienceNews, June 2025.
- *With potential implications for mechanical systems, study reveals physics of ‘nick shot’ in squash*, News from Brown, June 2025.
- *The Nick, explained by Science!*, US SquashSite News, USA, June 2025.

PREFACE AND ACKNOWLEDGMENTS

This dissertation consists of original research by Madeline E. Federle between September 2020 and April 2026 under the supervision of Professor Roberto Zenit at the Brown University School of Engineering. Contributions from others are an integral part of this work and are noted in the text where appropriate.

The completion of my doctorate would not have been possible without the support and dedication of those who surrounded me during my time at Brown. I thoroughly believe that doing 'hard things' is a team effort, and if it weren't for the collective group of people rooting me on, discussing scientific topics, and helping me prepare for talks, exams, and more, I would not have made it to the finish line.

With that, I would like to express my sincerest gratitude to my advisor, *Prof. Roberto Zenit*. I will forever be grateful for the researcher you have cultivated me into. You did not only give me a greater understanding of fluid dynamics, but also showed me what it is to love this work, to be excited about the research we are conducting, to love bubbles, and to do all that we do well. I feel extremely lucky to have someone so wise pave the path for my success. You have challenged me, supported me, and most importantly, trusted me. With this trust, you allowed me to reach for the stars, and hey, we got pretty close! Thank you for showing me the beauty of scientific research and for sharing the balance between fun and rigor. Because of you, I will always 'press on'.

I also want to thank my committee members, *Prof. Ralph Milliken*, *Prof. Mauro Rodriguez*, and *Prof. Dominique Legendre*. Each of you has supported me in many ways. *Prof. Milliken*, your support has allowed me to thrive, and I am so grateful for all that you and the Rhode Island Space Grant Consortium have made possible. You gave me access to research opportunities I only dreamed of. Thank you! *Prof. Rodriguez*, thank you for your support and guidance, from asking thoughtful questions about my research to discussing future career paths and offering expert insight into the journey ahead. I have, and always will, appreciate your willingness to show up not only for me, but for all students at Brown. *Prof. Legendre*, the boss, thank you for always believing in me and serving as our secondary bubble boss. Your expertise has consistently helped improve my work, and your unwavering confidence in my ability to succeed has given me so much confidence.

A special thanks to the Fluids faculty, *Prof. Thomas Powers*, *Prof. Kenneth Breuer*, *Prof. Daniel Harris*, *Prof. Monica Martinez Wilhelmus*, and *Prof. Jesse Ault*, for your support and knowledge. It has been an

honor to learn from each and every one of you. Your diverse expertise has given me insight into the many exciting avenues of research worth exploring. I have truly felt that each of you were cheering me on and genuinely excited about my work. Thank you for your moral support, research guidance, and arguably one too many letters of recommendation!

Jamila Wilson, a thank you does not cut it. Your administrative support, help processing orders, and warm smile were all crucial to my success. I am beyond grateful for all the last-minute purchases you made on my behalf, even when they seemed like a daily occurrence. Thank you for always prioritizing this for me! *Kathleen DiOrio*, thank you as well for all your administrative help. Your guidance and encouragement never went unnoticed. Thank you both!

The BDW and JEPIS staff, thank you! I genuinely could not have done this without you, *Ben Lyons*. Your willingness to jump on any design idea I had truly instilled confidence in me and made it all possible. You are one of a kind and an incredibly skilled machinist. I have valued your opinion and support throughout these years more than you may know. Beyond that, your friendship has made my PhD and my life so much more enjoyable. You always had a smile on your face, whether we were working late in the BDW or nervously floating in microgravity. I don't know how to thank you enough. *John Shilko*, thank you for your encouragement and machinist expertise! I'm grateful that for my awesome nickname, 'Sally', and for truly believing in my ability to become an astronaut. Your kindness was unparalleled.

To the Brown student community, if I wrote about all of you, I'd need to add another hundred pages to this thesis. Each and every one of you has made my time at Brown truly great. From badminton matches to scientific discussions, you have all shaped my journey. I am thankful to the WiFTS (Women in FTS) for reminding me of the importance of my place at the table. I am thankful to my badminton teammates for always providing a space to decompress, and for reminding me just how terrible my serve was. I am thankful to each lab group for teaching me new techniques, lending equipment, or simply saying hello in the hall.

I am, of course, extremely grateful for the Zenit Lab, who supported me throughout this entire journey. To the original crew, *Mithun Ravisankar*, *Asimanshu Das*, *Andres Zambrano*, and *Yunxing Su*, thank you for supporting me as I took my first steps in the lab. You got me on my feet, and I am so grateful. *Mithun* and *Asimanshu*, I don't know where to start. You both taught me what it means to be a researcher, from the small things, like making plots more aesthetically pleasing and knowing who's who at conferences, to the big

things, like sitting down to derive a model that took far too many hours, or running PIV measurements until they were just right. You showed me how to conduct research and how to do it well, all while supporting each other and having fun along the way. You supported me academically and personally, and I will forever cherish our friendship. To the newer members of the Zenit Lab, *Pf Guadio*, *Jensine*, *Anxu*, and *Orr*, it has been an absolute privilege to get to know you all. Thank you for keeping my spirits high and for always offering your support.

Maria Tatulea-Codrean, thank you for being an amazing role model. I always appreciated our ability to go from talking science to talking wedding planning in the same breath. You reminded me how wonderful it is to be a woman in STEM, thank you!

To all the undergraduate students I have had the pleasure of working with over the years, you each deserve a special shoutout. I am extremely grateful for your project support over the semesters. Your hard work and effort were always valued, and I feel so lucky to have met so many brilliant students.

Finally, I have to thank the true support system behind all of this, my family and friends. To my friends *Grace King*, *Sara Wilton*, *Alissa Assad* and my family of PVD neighbors: you have all supported this journey, encouraged me when I needed it, and let me rant when I needed that too. Thank you for being so invested in me. I love you all for it.

Most importantly, I am grateful for the support of my *parents*. You have shown me what it means to work hard, and to succeed or fail with grace. *Mom*, I am forever grateful for your support throughout this journey. You have always reminded me how proud you are of me, whether I had quit on day one or pushed through to day two thousand of my PhD. Thank you for always believing in me, for reminding me how proud you are of the life I live and the grace with which I live it, and for listening on the days it doesn't feel graceful at all. To my *in-laws*, honestly, thank you for being so wonderful. Not everyone gets so lucky. I am grateful for all the ways you have supported me over the years, from scientific conversations to care packages. Your support and generosity mean the world to me.

To my sisters, I am so grateful to have you. *Izzy*, thank you for always assuming I'm a genius based solely on my thinking face. Your little compliments go a long way and have given me so much confidence over the years. *Cheylsea*, thank you for being my ride or die. You have supported me through my PhD, but you have also given me so much more. I am grateful for the community you helped me build in Providence by sharing your friends, for being my neighbor so we could share so many meals, dog walks, and emergency

crying sessions, and for always believing that I can do anything I put my mind to. You have supported me in so many ways and helped me become someone I am truly proud of.

And finally, to the one who had to handle the most of this, who always had their arms open whenever I walked through the door, on the good days and especially the hard ones, with *Comet* faithfully by your side ready to lick me till I smiled. To my husband, *Colin Sullivan*, you are my MVP. You cultivated an environment of care that allowed me to succeed. Thank you for making dinners when I couldn't, for listening to my practice talks so many times you could recite them yourself, for never forgetting about 'our guy' Reynolds, for your photography and musical expertise that proved incredibly helpful in my work, and for your unwavering belief in my ability to succeed. You truly deserve an honorary doctorate from the Zenit Lab. Thank you for being you, and for supporting me in becoming me. I love you!

CONTENTS

Curriculum vitae	v
List of Tables	xvii
List of Figures	xviii
1 Introduction	1
1.1 Bubbly flows in applications	3
1.1.1 Bubble addition as an engineering tool	4
1.1.2 Bubble mitigation and control without buoyant forces	6
1.2 Outline of the dissertation	8
2 The interaction of an oil droplet and a gas bubble rising in a quiescent fluid	10
2.1 Introduction	11
2.2 Experimental Arrangement	13
2.3 Results and Discussions	18
2.4 Interaction Model	21
2.4.1 Conclusions	25
3 Physical conditions for droplet–bubble coalescence in flotation	27
3.1 Introduction	27
3.2 Conditions for coalescence	29
3.2.1 Surface energy conditions	30
3.2.2 Interaction time conditions	30
3.3 Comparison with experiments	34
3.4 Conclusions and discussion	35
4 Experiments of bubbles in microgravity: an introduction	37
4.1 NASA Drop Tower Experiments	38
4.2 Zero G Parabolic Flights	40

4.3	Conclusions	43
5	Design of an accessible turbulence chamber for laboratory experiments	46
5.1	Introduction	46
5.2	Hardware description	48
5.3	Design files summary	48
5.4	Bill of materials summary	49
5.5	Build instructions	51
5.5.1	Tank Body	51
5.5.2	Welded Frame	51
5.5.3	Acrylic Windows	52
5.6	Motor Arms	55
5.7	Impellers	56
5.8	Mounting Experiment	57
5.9	Operation instructions	58
5.10	Validation and characterization	59
5.11	Conclusions	63
6	Speed of sound in bubbly liquids	65
6.1	Introduction	65
6.2	Experimental Arrangement	68
6.3	Data Processing	71
6.4	Results	74
6.5	Conclusions	78
7	Bubble emulsions in microgravity	83
7.1	Introduction	83
7.2	Experimental setup	85
7.3	Analysis Methodology	88
7.4	Preliminary Results	90

7.4.1	Flow Analysis	91
7.4.2	Bubble Breakup Formation	93
7.5	Conclusions	97
8	Conclusions	99
8.1	Summary	99
8.2	Future directions	101
9	Bibliography	103
	Appendices	115
A	The Mechanics of the Squash Nick Shot	116
A.1	Introduction	116
A.2	Experimental Setup	117
A.2.1	Measurement of the ball's Young's Modulus	118
A.2.2	Materials and Methods	119
A.3	Condition for a nick shot	120
A.3.1	Characteristic times for wall collision	122
A.4	Results and discussion	123
A.5	The mechanical model of the ball contact	125
A.5.1	Stage 1: Contact with the wall, $t = t_1$	125
A.5.2	Stage 2: contact with the floor, while already in contact with the wall, $t = t_2$	126
A.5.3	Stage 3: restitution	127
A.6	Detailed description of experiments	127
A.7	Effect of initial spin	129
A.8	Collision videos	130
B	Blending Fluid Mechanics and Art Part I: Making a Splash with High-Speed Photography	131
B.1	Introduction - Overview of the Idea	131
B.1.1	Worthington-Edgerton crowns	131

B.1.2	The process of a drop impact on a free surface	132
B.1.3	Arduino microprocessor	133
B.2	Design and Setup	134
B.2.1	Circuit	134
B.2.2	Operating Procedure	134
B.3	Results	136
B.3.1	Examples	136
B.3.2	Analysis	138
B.4	Teaching Outcomes	138
B.5	Extensions and Future Directions	139
B.6	Conclusions	140
B.7	Bill of Materials	141
B.8	Attribution Table	142
C	Blending Fluid Mechanics and Art Part II: Flow Visualization	143
C.1	Introduction - Overview of the Idea	143
C.2	Design and Setup	144
C.3	Results	147
C.4	Conclusions	151
C.5	Bill of Materials	153
C.6	Attribution Table	154
D	Blending Fluid Mechanics and Art Part III: Fluids Art Project	155
D.1	Introduction - Overview of the Idea	155
D.2	Design and Setup	156
D.3	Results	159
D.3.1	Spacewalker: An Artistic Display of Rayleigh-Bénard Convection Cells	159
D.3.2	Clouds on the Loose: Creating indoor clouds	161
D.3.3	Love is in the Air: Wingtip Vortices Visualization	162
D.3.4	The Flow of Color: Vortex-Shedding-Driven Paper Marbling	164

D.4	Teaching Outcomes	165
D.5	Conclusions	166
D.6	Tips, Disclaimers, Room for Future Directions	166
D.7	Attribution Table	167

LIST OF TABLES

2.1	Physical properties of test fluids. For the water-glycerin mixture the interfacial tension represents the liquid-air value.	14
2.2	Bubble and droplet experimental characteristics, including average diameters (d_D and d_B), terminal velocities (U_∞^D and U_∞^B), ratios of both velocities and diameters between the bubble and droplet, Reynolds number, and Weber number.	16
4.1	Experimental system components, dimensions, and weights.	41
5.1	An overview of some current HIT chamber designs highlighting the geometry, method of agitation, and Re_λ	47
5.2	All files included in open source for design and fabrication of the Turbulence Chamber . .	49
5.3	Bill of materials for HIT chamber, including part name, assembly item, and cost in U.S dollars	50
6.1	Measured delay times and corresponding computed speeds of sound for individual bursts for two gas volume fractions, $\alpha = 1$ and 2, and in bubble column under Earth's gravity. . .	76
6.2	Measured delay times and corresponding computed speeds of sound compared to theoretical predictions from drop tower tests for bubbles with diameters of approximately 0.55 mm. .	76
6.3	Measured speed of sound values for individual bursts corresponding to experiments conducted on parabolic flights where, $\alpha = 0.2\%$ and the expected speed of sound value is equal to 222.39 m/s.	77
7.1	Various turbulent Reynolds numbers and their corresponding dissipation rates	92
7.2	Critical Weber numbers for set bubble diameter in HIT with turbulent Reynolds number equal to 703.	96
7.3	Various turbulent Reynolds numbers and their corresponding dissipation rates.	97
A.1	Range of conditions and ball properties for the nick squash shot experiments	120

LIST OF FIGURES

1.1	(a) A flow regime map for liquid-gas flows in vertical flows and (b) illustrations of each regime. The figure has been adapted from C. Brennen [9].	1
1.2	Examples of bubbles in different settings: (a) 18th century oil painting titled 'Soap Bubbles' by Jean-Baptiste-Siméon Chardin , (b) methane bubbles frozen in lake in Russia (credits: Kristina Makeeva), (c) bubbles used for filtration in aeration tank (credits: Reserve Control Solutions), (d) bubble chains in a glass of sparkling wine (credits: Colin F. Sullivan).	3
1.3	(a) Droplet bubble coalescence seen at head of capillary [10], (b) a representation of bubble plumes used for spill oil containment [11], (c) a diagram of an aeration tank which uses droplet bubble coalescence for filtration [12].	5
1.4	Examples of bubbly flows in microgravity over time: (top) pool boiling experiments conducted on sounding rockets to study formation and coalescence of vapor bubbles in microgravity [13], and (bottom) electrolysis experiments conducted on parabolic flights to study the effect of electromagnetic fields on driving bubble motion [14].	7
2.1	Experimental setup where U_B is the bubble velocity, and U_D is the droplet velocity, d_B , and d_D are the bubble and droplet diameters respectively and ϕ is the collision angle defined as the vertical angle between the bubble and droplet centers at a vertical distance of d_B	13
2.2	A time series of images showing a bubble-droplet interaction	15
2.3	Drag Coefficient vs Reynolds Number Verification	17
2.4	Velocity Jump	19
2.5	Experimental Results	20
2.6	Flow Field Diagram	21
2.7	Model fit for experimental data	24
3.1	A visual representation showing the sliding time of a droplet as it advances over the bubble surface.	31

3.2	The coalescence conditions for a variety of different air bubble and oil droplet pairs as a function of E^* and T^* . The solid lines show the prediction for potential flow, whereas, the dashed lines show the prediction for Stokes flow, see Eq. 3.5, 3.16, 3.17. Experimental values, from Federle et al. [1], are seen in the black points.	35
4.1	Image from NASA of NASA Glenn 2.2 drop tower detailing the 8 floor drop with release at floor 8, preparation at floor 5, and an airbag for capture at floor 1 (photo credits: NASA). .	38
4.2	Educational drop tower rig from NASA with camera (top left), HIT chamber (top right), and controls/computer (bottom center) mounted in place.	39
4.3	Images showing the details of ZeroG parabolic flight, including (a) the airplane flown (credits: ZeroG), (b) a schematic (credits: Wiki-MikeRun) of the trajectory of the airplane highlight intervals of 1.8 g and microgravity, (c) accelerometer data collected aboard parabolic flight showing regions of hypergravity and microgravity.	42
4.4	Components of parabolic flight experiment with (a) showing the entire mounted system including the base plate, double containment, HIT chamber, and camera, (b) detailing the structural load analysis on the HIT chamber, and (c) is a force diagram of the necessary 9 g's of force required for structural testing by ZeroG.	43
4.5	Photograph of team working aboard the flight in microgravity. Those featured in the image include, Roberto Zenit (back), Madeline Federle (left), and Ben Lyons (front).	44
4.6	Two examples of bubble emulsions in (a) NASA drop tower test, and (b) ZeroG parabolic flight.	45
5.1	Components of turbulence chamber frame(a) angled steel ligaments, inset shows the front view of the ligament,(b) the inside of one assembled face , (c) the outside view of all faces assembled and welded.	52
5.2	Faces for truncated cube including five octagons, one octagon with a large center hole, and one cap for the center hole.	53
5.3	Schematic for epoxy and silicone sealant. Yellow dotted lines indicate where the epoxy should be first applied, followed by the sealant.	54
5.4	(a) Full arm assembly, (b) Separated arm assembly, highlighting the connection points. . .	55

5.5	The corner mount horizontally sliced in half, where (a) is the isometric view highlighting the locations of helicoils and screws, and (b) is the top view showing the threads and geometry of the connection points.	56
5.6	The entirety of the arm assembly from an (a) isometric view, including the: (1) impeller, (2) shaft, (3) mechanical seal, (4) ball bearing, (5) rigid coupler, and (6) DC motor, and (b) from the top view.	57
5.7	(Left) A photo of the HIT chamber in its final, fully assembled state. The experiment is filled with water. (Right) A CAD rendering of fully assembled experiment.	58
5.8	A simple schematic to show the control system, including a laptop computer, an Arduino Mega, eight ESC controllers, eight DC motors, and a 12 V power supply.	59
5.9	Additional colormaps of the isotropic ratio in the horizontal and vertical directions normalized by the length of the tank. Bounding boxes show the average isotropic ratio in that region. Reynolds ranging from $220 < Re_\lambda < 820$, increasing from left to right.	61
5.10	A colormap of the isotropic ratio in the horizontal and vertical directions normalized by the length of the tank. Bounding boxes show the average isotropic ratio in that region. . .	61
5.11	PDFs of the (top) horizontal and (bottom) vertical fluid velocity fluctuations in the HIT chamber at an Reynolds ranging from $220 < Re_\lambda < 820$, increasing from left to right . The dashed black line represents the Gaussian profile. Each line corresponds to each time instance throughout the measurement (2 ms).	62
5.12	The (a) horizontal and (b) vertical spectra of liquid velocity fluctuations for various Reynolds numbers, $220 < Re_\lambda < 820$. The x-axis represents the wave number, k , normalized by the wavenumber corresponding to the propeller diameter, k_d , and the y-axis is the turbulent kinetic energy, $E(k)$ normalized by the variance $d\langle u'^2 \rangle$. The blue and red lines are the largest and smallest scales for the inertial subrange.	63
6.1	Speed of sound measurements using Woods equation and experimental values from Wilson and Roy [15]	67
6.2	A schematic of the experimental setup including bubble emulsion, 8 impellers, high speed camera, and hydrophone speaker arrangement.	69

6.3	A snapshot image at one time instance that shows bubble breakup and dispersion due to mechanical agitation, $\alpha = 0.2\%$, $Re \approx 425$. Bubble sizes range from 0.5 to 4 mm, majority being approximately 1 mm.	70
6.4	(left) A preview of the hydrophone signals from hydrophone 1, red, and hydrophone 2, blue. Hydrophone 1 signal is very clear and located near the underwater speaker, whereas hydrophone 2 is located roughly 20 cm from the speaker and contains noise from signal interaction with bubbles. (right) A similar preview of hydrophone signals zoomed in from 0.4 to 0.5 seconds.	70
6.5	A plot of the gas volume fraction, α , versus the speed of sound, c . Using Eq. 6.1, an estimated speed of sound based on the fluid properties can be estimated. The black points represent the estimated experimental values, with gas volume fractions ranging from 0.1% to 10%. The inset figure shows the zoomed in portion of these results to highlight the steep decay in the speed of sound at small gas volume fractions.	72
6.6	(a) Signal recorded from hydrophones where the red is the entire signal captured including the mixing of the fluids, shown by the large amplitude noise. The blue is the designated burst of interest recorded by hydrophone 1 and filtered as shown by the above blue signal. The black is the designated burst of interested recorded by hydrophone 2, the signal below is the original signal filtered an amplified. (b) A zoomed in representation of the burst of interested showing the first hydrophone in blue and the second in red with their respective filtered and amplified versions in the the dashed lines.	73
6.7	A spectrogram of hydrophone 1, top, and hydrophone 2, bottom, signals. The red lines show the targeted frequencies and the black dotted lines are the cut off bounds.	74
6.8	Cross correlation plot between hydrophone 1 and hydrophone 2 for the sound signal sweeping from 1700 to 2100 Hz, wavelength between 0.87 m and 0.70 m. The peaks shown are the maximum alignment between the two signals, with the overall maximum being the second peak at 0.000786 seconds.	75

6.9	Experimental data plotted in comparison with the theoretical prediction (black line) of the speed of sound in a bubble emulsion. The various shapes represent the experimental environment; drop tower, ground-based, or parabolic flights, whereas, the colors designate the gas volume fraction ranging from 0.1 to 10 %.	78
7.1	Images of experimental setup including (a) truncated cube shaped tank and (b) impellers used for agitation.	86
7.2	Bubble breakup over time in HIT chamber.	91
7.3	Two side by side images that show bubble identification. The left image is the image without any labeling, as the right image showed the identical image with bubbles labeled in various colors.	92
7.4	Color maps of the turbulence intensity in the entire tank (left), far from impellers, and near one impeller (right) plotted over the horizontal and vertical directions normalized by the impeller radius. The white region in the right figure is the masked region due to the shadow of the impeller. Both horizontal and vertical axes are normalized by the impeller radius.	93
7.5	Plots from preliminary tracking model showing bubble breakup over time where (a) shows the overall distribution of the bubble sizes, (b) shows the change in bubble size over time, and (c) counts the bubbles over time. Turbulent Reynolds number increases from top to bottom.	94
7.6	Plots showing the turbulent Weber number for both (left) the center of the HIT chamber, and (right) near the impeller for the critical estimated bubble diameter of 0.09 mm. A bubble of this size will on breakup if it is larger than $We_{crit} = 0.0340$.	95
7.7	Critical bubble diameters plotted for a critical Weber number equal to 3.8 for (left) the center of the tank in the region of HIT, and (right) the region of high intensity turbulence near the impeller.	97
A.1	Mechanics of squash nick shot	121
A.2	Map of conditions for nick occurrence	124
B.1	Worthington-Edgerton crown created by a droplet impacting the free surface of a stagnant liquid.	132

B.2	Image sequence of the collision of a water droplet against the free surface of a stagnant water. A droplet falls from above onto a quiescent liquid bath. At impact, the droplet deforms the free surface downwards causing a crown of droplets to form around the cavity. As the cavity springs back up, a jet emerges from the free surface eventually destabilizing into a line of droplets. Experiment shown here are with a droplet of radius $R = 5$ mm at an impact speed of $U = 3.75$ m/s with corresponding Reynolds and Weber numbers of $Re = 1025$ and $We = 2675$, respectively. The image sequence was obtained with a Photron SA3 model 120K-M3 at 2000 frames per second, though a similar photo series can readily be reproduced with a standard DSLR or smartphone camera.	133
B.3	Arduino and breadboard circuit used to control experimental setup. Output pins send pulses to each element of the setup as dictated by user defined time delays within the Arduino code.	135
B.4	Experimental setup The setup consists of a peristaltic pump which draws fluid from a fluid reservoir, producing the droplet at the syringe needle. A camera and flash controlled by an Arduino (not shown here) capture the moment of impact of the droplet with the stagnant fluid bath.	136
B.5	Artistic representation of droplet splashes. The objective of this lab is for students to produce original and artistic images following the theme of droplet collisions. Shown here are a subset of the final results produced in the 2023 and 2024 iterations of the course. See Appendix III for attribution [16].	139
B.6	Splash regime map. This map, proposed by [17], locates the expected splash behavior for given values of the Reynolds and Weber numbers. Examples from students experiments are shown, as discussed in the manuscript.	140
C.1	(a) Sketch of flow in a vortex ring. (b) Experimental setup for generating vortex rings. A box with a cut-out orifice is filled with smoke from a fog/smoke generator. The back side of the box is made of flexible material, plastic wrap. By shining a laser, a plane of interest is illuminated. Tapping the flexible side of the box produces an impulse producing the vortex ring out of the orifice which can be recorded with a phone camera set up close by.	144

C.2	Plane visualization of turbulence. (a) Flow in the room visualized via the lasers placed a distance away from the vortex ring generator box after waving one's arms around. (b) A vortex ring can be visualized in an equivalent way. All photographs are reproduced with permission from the students. See Appendix III for attribution.	146
C.3	Generation of laminar and turbulent vortex rings. By changing the intensity of the impulse on the back of the container, both a laminar ($Re = 3400$) (a) and turbulent ($Re = 8300$) (b) vortex rings are generated. While the laminar vortex ring is characterized by visible streamlines in the ring itself, the turbulent vortex ring exhibits an unsteady blob of smoke with a pronounced wake behind the traveling ring. All photographs are reproduced with permission from the students. See Appendix III for attribution.	149
C.4	Visual additions to the vortex ring experiment. A large array of behavior was visualized as the students were given the freedom to adapt the experimental setup.(a) A laminar vortex ring is placed within a group of stick figures in the work to mimic Henri Matisse's 1905 Joy of Life. (b) A toy car is accelerated on the table leading to a large disturbance in the ambient flow fields of the air. All photographs are reproduced with permission from the students. See Appendix III for attribution.	150
C.5	Artistic representation of vortex rings. At the end of the lab, students presented their favorite images from their exploration of which a small subset of the final results produced in the 2023 and 2024 iterations of the course. All photographs are reproduced with permission from the students. See Appendix III for attribution.	152
D.1	(a) Rayleigh-Bénard convection cell experimental setup. (b) Displayed installation of Rayleigh-Bénard convection with a diameter of about 75 cm. (c) Zoomed in view of convection cells. Attributions are listed in Appendix II.	160
D.2	Liquid nitrogen clouds in various locations on campus, including our performing arts center, greenhouse, quiet green, and cafe on campus. Attributions are listed in Appendix II. . . .	162
D.3	(a) Final configuration of foam airplane mounted below rail system with fog machine on the floor. (b) Wingtip vortices are visualized with fog and blue lighting. Attributions are listed in Appendix II.	163

D.4	(a)Experimental schematic for vortex shedding. (b-e) Examples of the final prints, which used subtle, gradual transitions in colors to capture vortices.	164
-----	--	-----

1 INTRODUCTION

Multiphase flows, systems in which more than one phase is present, including the specific case of two-phase flows involving only two interacting phases, are a cornerstone of fluid mechanics and appear across many disciplines and exhibit a wide range of behaviors that can differ substantially from one problem to another. Due to their importance in industrial and natural applications, the research on this subject is vast, however, open questions still remain as the topic is so diverse. Multiphase flow problems have relevance in engineering, physics, chemistry, geophysics, applied math, and simply, many of our day to day activities or functions. Due to the vast nature of multiphase flow problems, subcategories are created and researched to hone in on the details of the problem such as liquid-solid, gas-solid, or liquid-gas flows. Liquid-solid flows can consist of topics such as sedimentation or the flow of liquids through a porous media [18]. Whereas, gas-solid flows consists of research related to fluid structure interaction, such as the wind interacting with wind turbines, or studying designs of cars or airplanes to make them more aerodynamic [19]. Another division of multiphase flows, is liquid-gas flows. Liquid-gas flows in themselves offer a diverse set of research problems, ranging from combustion to droplets and bubbles [20]. The focus of this thesis will be on gas-liquid two-phase flows, specifically, bubbly flows.

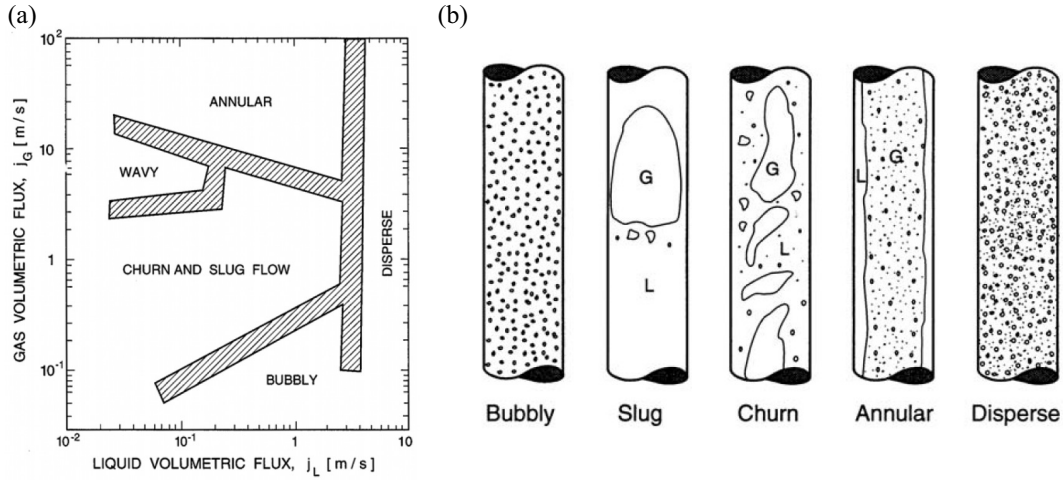


Figure 1.1: (a) A flow regime map for liquid-gas flows in vertical flows and (b) illustrations of each regime. The figure has been adapted from C. Brennen [9].

Bubbly flows, a subset of two-phase flows, encompass several subcategories of their own. The gas volume fraction, α , plays a central role in determining the flow characteristics and regime. For this reason,

flow regime maps have been developed to classify liquid–gas flows according to parameters such as gas volume fraction and flow morphology. Within these maps, bubbly flow corresponds to relatively low gas volume fractions in which discrete bubbles are dispersed throughout a continuous liquid phase. Figure 1.1(a) presents a representative liquid–gas flow regime map, identifying the relevant regions, while Fig. 1.1(b) illustrates the corresponding flow structures.

Beyond classification, bubbly flows introduce a distinct set of complex and highly specific challenges. Even at a fundamental level, the governing equations for two-phase flow differ from those of single-phase flow. The momentum equations for two-phase flow are commonly derived using a volume-averaging approach, which results in separate momentum balances for each phase. These equations are then coupled through interfacial momentum exchange terms that account for the interactions between the phases [21]. Owing to these complexities, bubbly flows have been studied extensively over the past several decades, motivated both by their prevalence in industrial and natural systems and by the rich physical phenomena they exhibit. Investigating such a niche yet diverse topic is both challenging and rewarding. Fluid mechanics has a remarkable way of manifesting in our everyday lives, and bubbles are no exception. The curiosity and admiration surrounding bubbles have long captured human interest, from artistic accounts of soap bubbles symbolizing the impermanence of life in the 17th and 18th century to Leonardo da Vinci’s depiction of the zig-zag rise of bubbles [22]. These early observations highlight how frequently bubbles appear in our daily experiences. Bubbles are often associated with enjoyment, appearing in many food products ranging from champagne to the incorporation of air in ice cream [23, 24]. Beyond this, bubbles play an important role in numerous industrial applications and are commonly found in a variety of filtration and mixing processes [25]. See Fig. 1.2 for examples.

Despite their seemingly simple appearance, bubbly flows are highly complex and can vary significantly depending on a range of governing factors. At a fundamental level, considerations such as bubble size, quantity, or distribution play a critical role. These parameters help distinguish between the different types of bubbly flows that may be encountered. Bubbly flows may involve the study of a single bubble rising through a fluid or the collective behavior of many bubbles within a fluid, which is often quantified using the gas volume fraction and the homogeneity of the mixture.

Several physical effects must be considered when solving bubbly flow problems, including buoyancy, surface and interfacial tension, bubble-bubble interactions, and interactions between the bubble and the

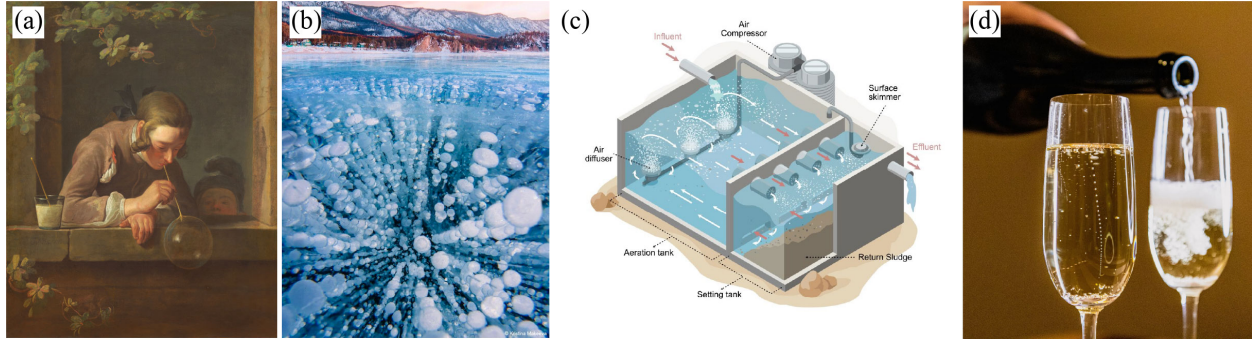


Figure 1.2: Examples of bubbles in different settings: (a) 18th century oil painting titled 'Soap Bubbles' by Jean-Baptiste-Siméon Chardin , (b) methane bubbles frozen in lake in Russia (credits: Kristina Makeeva), (c) bubbles used for filtration in aeration tank (credits: Reserve Control Solutions), (d) bubble chains in a glass of sparkling wine (credits: Colin F. Sullivan).

surrounding fluid. An air bubble will rise through a liquid at a characteristic speed due to buoyancy. The terminal velocity, at which drag and buoyancy forces balance, depends on the bubble diameter, the density difference between the bubble and the surrounding fluid, and the bubble shape. This terminal velocity in turn determines how the bubble influences the surrounding fluid. As the bubble rises, the surrounding fluid will be impacted by the added mass of the bubble and wake formation. The region associated with added mass makes the bubble appear more massive than it truly is, but it provides important insight into the near field effects of bubble motion. Farther from the bubble, the wake becomes the dominant influence on the flow. Both effects influence the bubble rise trajectory and its interactions with the surrounding fluid, as well as with other bubbles, droplets, or particles.

Overall, much of the research on bubbly flows is aimed at controlling bubbles, either by suppressing their formation or by intentionally producing them. By understanding how bubbles disperse in a fluid medium, how their shape may change in response to external factors, and how interaction effects arise, researchers can begin to leverage this knowledge to motivate new technological advancements.

1.1 Bubbly flows in applications

Bubbles play a critical role not only in many day to day activities but also in a wide range of engineering processes. Depending on the application, bubbles may be either beneficial or detrimental. In some engineering systems, bubbles are intentionally introduced into a fluid to enhance processes such as impurity filtration or noise reduction. In other cases, bubbles can lead to performance degradation or failure if

they become trapped within devices and cannot be effectively removed. For this reason, it is essential to understand the bubble dynamics associated with specific operating conditions in order to design processes involving bubbles that are both effective and efficient.

1.1.1 Bubble addition as an engineering tool

As noted above, bubbles are often desirable in many engineering applications. One prominent example is the use of bubbles for filtration. Many conventional filtration processes rely on density differences that allow impurities to rise to the surface; however, these processes are often extremely slow, particularly when the impurities are small. To enhance removal efficiency, bubbles can be introduced into the fluid to collide with, capture, or attach to impurities such as droplets or particles.

Across much of the literature on bubble-assisted filtration, a common underlying assumption is that bubbles possess an effective “stickiness” that allows contaminants to adhere to their interfaces. Understanding this behavior requires consideration of several key factors, including interfacial properties, flow conditions, and fundamental parameters such as the size, number, and spatial distribution of both bubbles and impurities. These factors collectively determine the likelihood of bubble–impurity contact and the nature of their interaction. Consequently, much of the existing work frames filtration performance in terms of collision rates and coalescence efficiency, implicitly suggesting that successful filtration depends on frequent interactions and favorable interaction outcomes.

One major class of applications where this framework is employed is oil containment and removal. As represented in Fig. 1.3, bubbles have been widely used as an engineering tool for the containment and filtration of oil, both in produced water and in large-scale oil spill scenarios [25–28]. Numerous studies have investigated the use of bubble plumes or bubble nets as containment mechanisms for oil spills, [11, 27–29]. In particular, Hazar et al. conducted numerical and experimental studies examining how bubble or aperture size, air discharge, and pollutant type influence the effectiveness of bubbles as containment devices, depicted in Fig. 1.3 (b). Overall, the study found that bubbles can successfully be used for oil containment, with aperture size identified as one of the most influential parameters. Specifically, larger apertures were found to increase the effective containment distance and overall efficiency.

Similarly, significant research has focused on the filtration, or removal of impurities from produced water, the oil–water mixture generated during oil extraction, through the use of bubbles, schematically

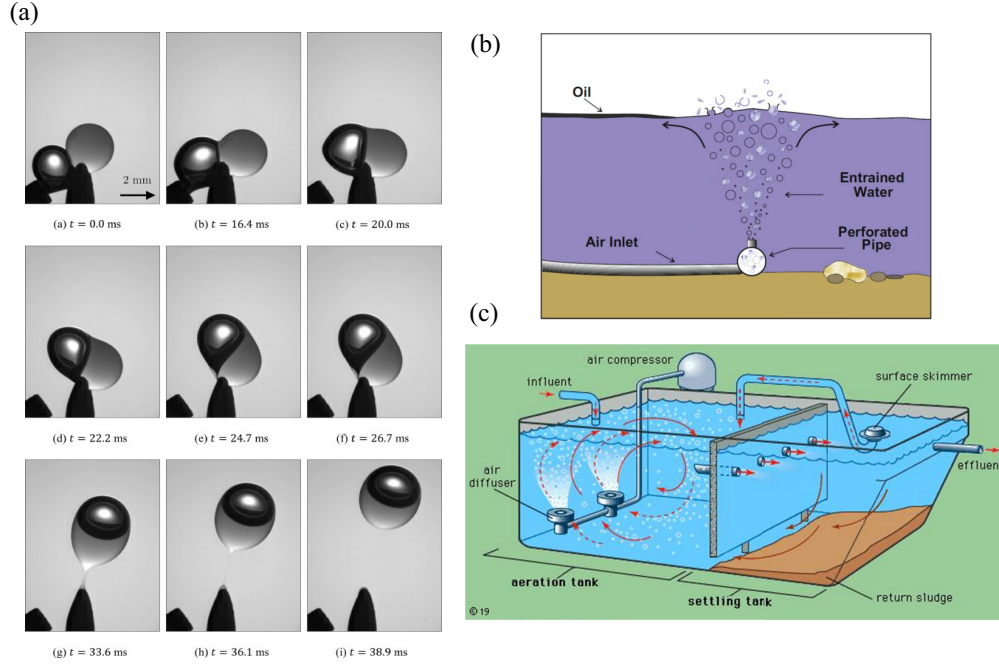


Figure 1.3: (a) Droplet bubble coalescence seen at head of capillary [10], (b) a representation of bubble plumes used for spill oil containment [11], (c) a diagram of an aeration tank which uses droplet bubble coalescence for filtration [12].

shown in Fig. 1.3 (c). The body of work on this topic consistently shows that bubble addition improves the rate at which impurities can be captured and removed from these systems. Some studies attribute this improvement to coalescence between oil droplets and bubbles [25, 26, 30, 31], shown in detail in Fig 1.3 (a), while others suggest that the enhanced filtration rates are driven primarily by the flow field induced by rising bubbles [1]. Although there is ongoing disagreement regarding the dominant mechanism, there is broad consensus that bubbles play a beneficial role in facilitating oil removal.

These concepts extend beyond oil-related systems and into surface-cleaning and solid-contaminant removal applications. Beyond oil filtration, bubbles have also been employed to remove solid contaminants from surfaces and fluids. For example, Guan et al. investigated the effects of galloping bubbles interacting with solid surfaces [32]. Their work demonstrated that the unsteady motion of a galloping bubble can generate flow structures capable of overcoming the adhesive forces that bind microplastics to surfaces, thereby enabling surface cleaning. In a related context, Kaur et al. examined the use of bubbles for microplastics filtration during flotation processes. They found that introducing small bubbles increased filtration efficiency by promoting adhesion between the microplastics and the bubbles [33].

Finally, both J. Bird and Lee et al. investigate the use of bubbles for removing impurities from distinct and unconventional surfaces, including underwater pilings and even tomatoes. J. Bird explores the use of underwater bubbles to remove biofilms from submerged pilings [34], whereas Lee et al. examine how bubbles can be utilized to eliminate *Salmonella* contamination from tomato surfaces [35].

While these studies collectively demonstrate the versatility of bubble-assisted filtration, the detailed mechanisms governing bubble–impurity interactions remain an active area of investigation. As noted earlier, disagreement persists regarding the dominant processes responsible for enhanced filtration in oil flotation systems. This uncertainty is largely due to the limited availability of scaled experiments capable of resolving the interaction physics. In many filtration applications, optical access is severely restricted by high impurity concentrations and turbid flow conditions, making it challenging to directly observe processes such as coalescence. As a result, fully characterizing the microscale mechanisms underlying bubble-assisted filtration continues to present significant experimental challenges.

1.1.2 Bubble mitigation and control without buoyant forces

While the previous section highlights the many ways in which bubbles can be leveraged as an engineering tool, their presence is not universally beneficial. The same interfacial properties that enable bubbles to capture and transport impurities can, in other contexts, disrupt flow behavior, reduce efficiency, or even lead to system failure. Common terrestrial examples of failure due to unwanted bubbles include inkjet printing [36], combustion systems [37], and other fluid-handling technologies where bubbles disrupt performance [38].

In many Earth-based systems, however, the effects of unwanted bubbles are often minimized or neglected because buoyancy rapidly removes them from the fluid. When buoyancy is no longer a driving force, such as in microgravity environments, this natural removal mechanism disappears, leaving a more complex problem that requires active control of bubble behavior.

Bubbles in microgravity, defined as a state of continuous free fall or a space-like environment, behave fundamentally differently from bubbles under Earth’s gravity. In the absence of buoyant forces, bubbles do not rise and instead remain suspended within the fluid unless acted upon by an external force. As a result, bubbles may persist, accumulate on surfaces, or remain dispersed in regions where they are not desired. These altered dynamics have motivated extensive research into the impact of stationary bubbles

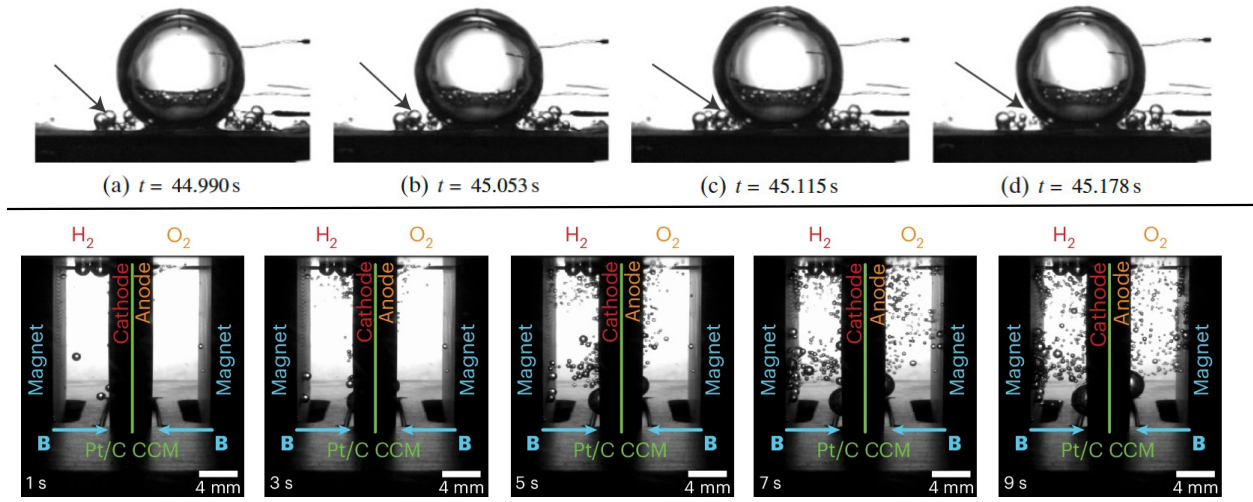


Figure 1.4: Examples of bubbly flows in microgravity over time: (top) pool boiling experiments conducted on sounding rockets to study formation and coalescence of vapor bubbles in microgravity [13], and (bottom) electrolysis experiments conducted on parabolic flights to study the effect of electromagnetic fields on driving bubble motion [14].

on common physical processes. One notable example is boiling in microgravity, see Fig. 1.4 (top) [13]. Under terrestrial conditions, vapor bubbles formed during boiling detach from the heated surface and rise, promoting heat transport throughout the fluid. In contrast, during boiling in microgravity, vapor bubbles tend to remain attached to the heated surface, significantly reducing heat transfer efficiency.

As a result, dedicated efforts have focused on understanding bubble dynamics in microgravity and developing methods to influence or control bubble motion. One common approach involves introducing shear flows to induce bubble movement. This strategy is frequently employed in microgravity boiling experiments, where a mean flow is used to remove bubbles from heated surfaces and thereby enhance heat transfer [39]. For example, Sielaff et al. generated individual vapor bubbles using localized heating on a flat plate and then applied a shear flow to control bubble motion along the surface [40]. In related studies, the role of bubble coalescence has also been examined. When bubbles coalesce on a surface, the resulting interaction can cause the merged bubble to jump away from the wall, increasing detachment and improving boiling performance [41].

Similarly, there has been recent research conducted to address the use of magnetic fields for bubble migration in electrolysis. Electrolysis in microgravity provided a novel approach for the beginnings of understanding how one might create rocket propellant in space from available resources. Similar to boiling,

electrolysis will not work efficiently if bubbles can not be removed from the electrode. The recent work on this subject, Fig. 1.4 (bottom), uses magnetic fields to induce convection and drive bubbles away from the electrode [14, 42].

Overall, bubbles can cause significant disruptions in space-based applications; however, research in this area remains relatively niche and limited due to the restricted accessibility of microgravity experiments. While microgravity conditions can be achieved experimentally through drop towers, parabolic flights, or space-based platforms such as the International Space Station or small satellite missions, these approaches are associated with substantial costs that limit widespread access. In addition to these experimental constraints, there remains a gap in fundamental understanding of the physical mechanisms governing bubble dynamics in microgravity. In particular, key processes such as bubble formation, dispersion, coalescence, and long-term stability are not yet fully understood.

1.2 Outline of the dissertation

In this thesis, I investigate two problems: first, the role of bubbles in enhancing filtration processes, and second, the fundamental behavior of bubble suspensions in microgravity. The first part of this work presents detailed experiments, along with a model, examining the interaction between an oil droplet and an air bubble in a quiescent fluid. The second part introduces a new experimental design, describes the microgravity experiments conducted, and outlines the analysis methods used to study bubble suspensions.

Chapter 2 examines the interaction between an oil droplet and an air bubble rising in a quiescent fluid. Using high-speed video, the droplet and bubble are tracked during flotation in order to quantify their coupled motion. This analysis is used to understand the flow fields of both phases and how they interact. The bubble-induced flow field causes the droplet to experience a measurable velocity jump. An interaction model is developed using Oseen flow theory to describe the droplet's equation of motion under the influence of the bubble's flow field.

Chapter 3 builds upon this interaction framework to explore the governing parameters that determine coalescence efficiency. This work introduces two scaling parameters, based on time and energy considerations, to characterize the likelihood of droplet–bubble coalescence. A map of coalescence probability is presented.

Chapter 4 marks the transition to the portion of this thesis focused on bubbles in microgravity. This chapter explains the experiments that were conducted in microgravity specifically focusing on how microgravity conditions were achieved. The bubble emulsion experiment was conducted in both NASA drop towers and parabolic flights. The details of these tests are explained in this chapter.

Chapter 5 chapter describes the microgravity experimental setup in detail, which consists of a homogeneous isotropic turbulence chamber. The design, construction, and validation of this chamber are presented and discussed.

Chapter 6 uses data collected from NASA drop tower experiments and parabolic flights to investigate the speed of sound in bubble emulsions. Existing theoretical descriptions of the speed of sound in bubbly liquids are typically formulated without explicitly accounting for bubble diameter [43]. The goal of this chapter is to explore the influence of bubble size on acoustic propagation. Experiments were conducted over a range of bubble sizes and gas volume fractions in order to quantify the resulting speed of sound.

Chapter 7 investigates the properties of bubble emulsions in microgravity. Using high-speed imaging, experiments were conducted aboard parabolic flights and in drop tower facilities to generate bubble emulsions through mechanical agitation. A fixed gas volume fraction was selected, while the impeller rotation rate was varied to control the turbulence intensity. The influence of turbulence intensity on the resulting bubble size distribution was examined.

Finally, Chapter 8 provides a conclusion of the results and summarizes the primary contributions of this thesis.

The appendices present additional research conducted throughout my doctoral degree. Appendix A presents the experimental results of “The Mechanics of the Squash Nick Shot.” The nick shot, an unreturnable shot, occurs when the ball hits the bottom corner of the wall and the floor just right, to bounce back with only horizontal velocity. The mechanics of this were investigated in detail focusing on angle of attack, ball height from the floor, and contact time. Appendix B, C, and D present a compilation of three works that show how fluid mechanics can be used as an artistic media. This three-part series examines high-speed photography of droplet impacts, flow visualization of vortices, and an open-ended student art-fluids project.

2 THE INTERACTION OF AN OIL DROPLET AND A GAS BUBBLE RISING IN A QUIESCENT FLUID

Preface The first two chapters of this thesis are dedicated to the research conducted on the interaction between a oil droplet and an air bubble. The goal was to improve the understanding of aeration, a process of filtration using air bubbles. Much of the previous research was dedicated to the efficiency of filtration of oil from water by coalescence of air bubbles and oil droplets – however, as stated in the following two works the process of filtration occurs via a velocity enhancement of the oil droplet due to the bubbles flow field rather than coalescence. The first work focused on the experimental setup, designed and built within my first year, the images acquired during these experiences, and finally a model using Oseen’s flow to model the velocity enhancement of the droplet (Chapter 2). The second work focuses on the efficiency of coalescence by defining two necessary time scales. The time scales look at the time it will take fluid to drain between a bubble and droplet, as well as the sliding time, the time the bubble and droplet will be in close contact (Chapter 3).

The following research, including experiments, modeling, and publishing papers was conducted throughout years 1 through 3 of my doctoral degree.

Abstract Gas flotation for oily-water mixtures is common practice in many industrial cleanup applications. To better understand the process experiments were conducted considering interacting pairs of different bubble and droplet sizes, as well as changing the viscosity of the surrounding liquid. The experiments demonstrated, that in most cases, the air bubble and oil drop did not coalesce but rather bounced off one another, resulting in an oil droplet velocity enhancement. A mathematical model was developed to capture the droplet dynamics by incorporating the effect of the bubble-induced field, which is in good agreement with the experimental measurements. A force balance of buoyancy, weight, drag, added mass, and a bubble velocity-induced force was used to predict the interaction process prior to and after the collision. The present work offers new insights into understanding droplet-bubble interactions that can help design novel applications for the removal of oil droplets suspended in water.

Information This chapter is a modified version of the work published in *International Journal of Multiphase Flow* [1].

2.1 Introduction

Bubbly flows, or two-phase flows, have been studied extensively in recent decades due to their various applications in industry and nature, as well as the novel findings of their physical phenomena [22, 44–48]. There have been many efforts to understand bubbly flows, bubble formation [49, 50], naturally occurring bubbles such as methane bubbles in the ocean [51, 52], interfacial phenomena of bubbles, bubble stability, bubble–bubble pairs, and their interactions [44–46, 53, 54]. Researchers have conducted two types of experiments, one type is aimed to understand industrial-scale phenomena, such as using bubbles for oil filtration or oil spills in the ocean. Specifically, research has been conducted on how bubble foams or bubble rafts can be an effective way to contain oil due to the hydrodynamic interactions of the bubble–bubble pairs as well as the bubble–oil pair [55, 56]. The other types of experiments are aimed at answering fundamental questions about bubbles and droplets, such as understanding the details of their interactions. The present study attempts to bridge these types of experiments by looking at an application in which bubbles are used to capture oil from a mixed fluid but emphasizing the details of the interaction between one bubble and one droplet.

The current study is aimed to understand some aspects of aeration tanks. The purpose of aeration tanks is to separate out oil from produced water, a byproduct of oil extraction, via bubble injection. The literature notes that the use of bubbles can be an efficient way to speed up the oil separation process [25, 26, 30, 31]. For instance, Igunnu et al. [30] and Saththasivam et al. [25] give overviews of different filtration methods commonly used in industry, focusing on aeration tanks but also including different methods of filtration. Moosai et al. [26] explore the surface science and the stages of attachment of the oil and gas phases in flotation. In addition, Lee et al. [31] detailed an experimental setup to remove all impurities from a produced water system. These researchers argue that bubbles are able to capture oil droplets and decrease the time it will take them to rise to the surface. When the bubble is engulfed by the oil droplet, it allows for the droplet’s velocity to be comparable to the bubble’s velocity. The focus of these studies is on the compound

drop or coalesced drop dynamics rather than the interaction process of the droplet and bubble during flotation. We argue that the details of how the interaction happens in this application are a fundamental step to understanding how aeration tanks can be used effectively and efficiently.

Overall, there are numerous studies that have been conducted to understand, bubble–bubble, bubble–particle, and particle–particle interactions in addition to other flow field questions such as particles in turbulent flow fields [45, 47, 57]. Again, some of the literature focuses on the application side of bubble or droplet interactions, however, the fundamental understanding side of the bubble and droplet interaction is not complete. For instance, many authors have studied the phenomena that occur when two bubbles rise. They found that two bubbles rising in line in a Newtonian fluid will typically experience the so-called drafting-kissing-tumbling phenomena (DKT). DKT is due to wake interaction and instability. First, the trailing bubble will speed up towards the leading bubble. The leading bubble’s low-pressure region in the wake will pull the trailing bubble to rise faster, the drafting phase. The trailing bubble will then touch the leading bubble, the kissing phase, and be in an unstable configuration where the bubbles are vertically aligned. This will then lead to the tumbling phase, in which the two bubbles will tumble to be relatively horizontal with one another and finally, they will rise separately without further interaction [45, 53, 58]. This research has also been extended to understand how the DKT phenomena change in a non-Newtonian fluid, where bubbles experience extended oscillating interaction due to the viscoelasticity of the liquid [46].

The key mechanism that drives the interaction in pair interactions is the wake of the leading bubble. For example, the computational work of Hallez and Legendre [53] investigates viscous wake effects of bubble–bubble pairs concluding that the interaction is a combination of three effects: the potential effect, a viscous correction, and a wake effect which impacts the drag and transverse forces on the trailing bubble. In general, there has been extensive fundamental experimental research conducted to understand the interaction process of bubbles. Our work aims to extend this understanding by studying the interaction processes of a pair of objects of different fluids, specifically air bubbles and oil droplets.

We conduct experiments of a one-to-one interaction of an air bubble and an oil droplet. In this paper, we will present how the droplet velocity increases due to the interaction with the bubble and its flow field. We report that the contact time, collision angle, and diameter ratio between the two phases of the interaction can change the outcome of the interaction by affecting the velocity of the droplet. Interactions with smaller collision angles and smaller diameter ratios tend to have larger impacts on the droplet’s overall velocity. It

is experimentally shown that the droplet velocity can increase significantly due to the interaction with an oncoming bubble. A theoretical model for the droplet velocity enhancement induced by the bubble flow field, considering Oseen's flow, is proposed and compared to our experimental results.

2.2 Experimental Arrangement

The experimental setup consisted of a rectangular acrylic tank, two syringe pumps [59], capillary needles, a high-speed camera, a backing light source, the bulk fluid, and of course an oil droplet and air bubble. A schematic of the setup is shown in Fig. 2.1. The details of the components of the experimental setup are explained below.

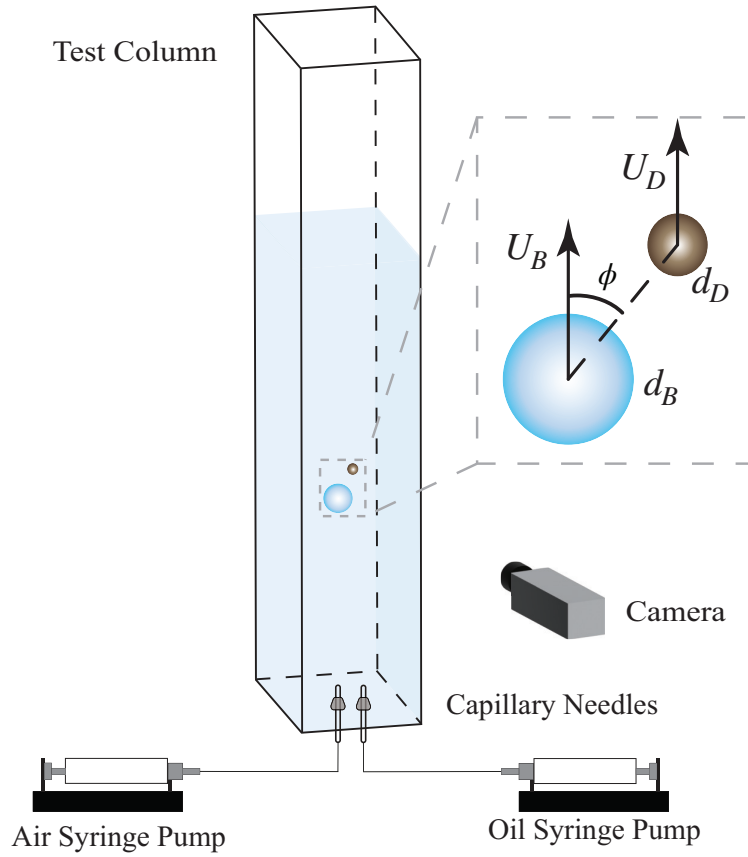


Figure 2.1: Experimental setup where U_B is the bubble velocity, and U_D is the droplet velocity, d_B , and d_D are the bubble and droplet diameters respectively and ϕ is the collision angle defined as the vertical angle between the bubble and droplet centers at a vertical distance of d_B .

The air bubble and oil droplet were both created by Arduino-run syringe pumps [59]. The syringe pumps resemble a linear rail system that allows the user to control the formation of droplets and bubbles

Fluid	Density, ρ , kg/m ³	Viscosity, μ , Pa · s	Interfacial tension, σ , N/m
70% Glycerin 30% Water	1181	0.026	0.067
10W-30 Motor Oil	875	0.13	0.020

Table 2.1: Physical properties of test fluids. For the water-glycerin mixture the interfacial tension represents the liquid-air value.

independently at a precise rate. Using such pumps allowed us to create one droplet or one bubble within a certain time period to make a one-to-one interaction to be observed between the droplet and the bubble.

An oil (10W-30 motor oil) was used for experimentation; its properties are shown in Table 2.1. The oil was dyed blue with a soluble oil-based food coloring (LorAnn) in order to visualize the droplet better. The oil was dyed by mixing about 2 grams of powdered coloring into the oil for 10 hours. The oil was then filtered through a paper filter to remove any of the dye that was not completely dissolved. The oil's interfacial tension within the glycerin mixture was then measured in order to verify that the dye had no impact on the properties of the oil.

The test column shown in Fig. 2.1, was filled with a bulk fluid composed of 70% glycerin and 30% water, with properties reported on Table 2.1. This mixture gave the best results for producing spheroidal bubbles and droplets as well as slowing rise velocities to have a straight path in order to control the visualization of experiments. The test column was constructed from laser-cut acrylic sheets with a cross-section measuring 21 cm × 6.2 cm × 6.4 cm, this is roughly 20 times the size of the bubble, ensuring that wall effects are negligible. The height of the tank allowed for both phases to reach terminal velocity before and after an interaction.

The bottom of the tank had two holes spaced about 2 cm apart. The holes were plugged with rubber stoppers to prevent leakage but allow the capillary needles to puncture through to the inside of the tank. Standard capillary needles were used to conduct the experiments. The gauge size of the capillary needles limited the diameter ratios that were examined. The needles were inserted at different angles to vary the collision angle between the bubble and the drop.

A high-speed camera (*Photron FASTCAM SA5*) was placed on one side of the acrylic tank parallel to the capillary needles and a LED light panel was placed on the other. Experiments were conducted and filmed at 500 frames per second (fps). An example of a typical experiment can be seen in Fig. 2.2. The recorded images were processed using a *Canny* edge detection algorithm in Matlab to identify the individual free-rising droplet and bubble [60]. A checkerboard pattern, with a grid size of 10 mm was used for calibration. The

resolution of the image was 20 pixels/mm. The diameter of both the droplet, d_D , and bubble, d_B , as well as bubble velocity, U_B , and droplet velocity, U_D , were found through image analysis techniques. The diameter was found by subtracting the background from the images and focusing on either the droplet or the bubble. Then, using, $d = (d_{max}^2 d_{min})^{\frac{1}{3}}$, where d is the equivalent bubble or droplet diameter, d_{max} is the longest bubble or droplet diameter and d_{min} is the shortest bubble or droplet diameter. The temporal variation in the droplet and bubble diameter was negligible and is solely attributed to experimental uncertainty. The mean absolute deviation is approximately 0.01 mm, as both the droplet and bubble reached terminal values. An average was then obtained over all the frames to give a mean diameter.

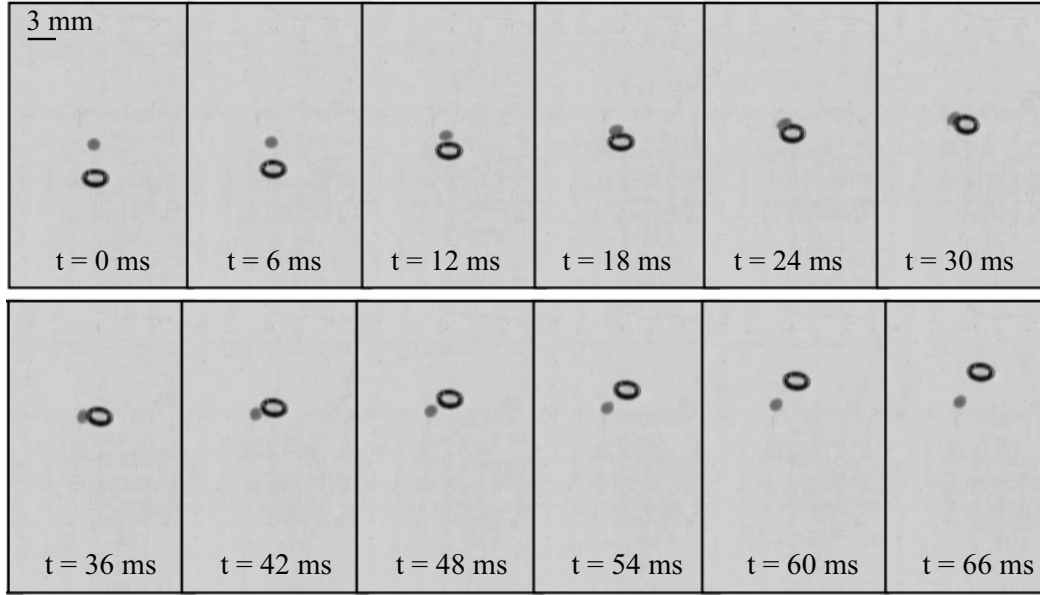


Figure 2.2: Time series of images showing bubble-droplet interaction with bubble diameter of 3 mm, $Re_B \approx 30$, and droplet diameter of 1.5 mm, $Re_D \approx 3$. The interval between the images is 6 ms.

In using the known value for the time between each frame, based on the fps, the velocity could be calculated by tracking how much the bubble or droplet moved over one frame. This process was repeated for the entirety of the image sequence. A central difference scheme was used in order to reduce the uncertainty. The range of diameter ratios, d_D/d_B , based on the corresponding capillary gauge size, and the terminal velocities of the bubble, U_{∞}^B , and the drop, U_{∞}^D , throughout experimentation can be seen in Table 2.2. The

d_D [mm]	d_B [mm]	Diameter Ratio [d_D/d_B]	U_∞^D , mm/s	U_∞^B , mm/s	Velocity Ratio [U_∞^D/U_∞^B]	Re_D	Re_B	We_D	We_B
1.47	3.08	0.48	18.97	185.36	0.10	1.31	25.90	0.03	1.85
1.83	3.53	0.52	25.65	202.71	0.13	2.14	34.09	0.07	2.81
1.97	3.00	0.66	27.38	188.01	0.15	2.46	27.13	0.09	1.96
2.06	2.96	0.70	29.14	177.28	0.16	2.72	23.80	0.11	1.63
2.26	2.01	1.12	35.59	98.74	0.36	3.66	9.04	0.17	0.34
2.46	2.01	1.22	35.01	98.74	0.35	3.92	9.04	0.18	0.34
2.83	2.01	1.41	44.48	98.74	0.45	5.73	9.04	0.34	0.34

Table 2.2: Bubble and droplet experimental characteristics, including average diameters (d_D and d_B), terminal velocities (U_∞^D and U_∞^B), ratios of both velocities and diameters between the bubble and droplet, Reynolds number, and Weber number.

terminal Reynolds number, drag coefficient, and Weber number are also calculated according to:

$$Re_B = \frac{U_\infty^B d_B \rho}{\mu}, \quad Re_D = \frac{U_\infty^D d_D \rho}{\mu} \quad (2.1)$$

$$Cd_B = \frac{g V_B d_B (\rho_f - \rho_B)}{\rho_f U_\infty^{B^2}}, \quad Cd_D = \frac{g V_D d_D (\rho_f - \rho_D)}{\rho_f U_\infty^{D^2}} \quad (2.2)$$

$$We_B = \frac{U_\infty^{B^2} d_B \rho}{\sigma_B}, \quad We_D = \frac{U_\infty^{D^2} d_D \rho}{\sigma_D}. \quad (2.3)$$

where ρ, ρ_B, ρ_D are the densities corresponding to the fluid, bubble and droplet, and V_B and V_D are the droplet and bubble volumes calculated as $\frac{1}{6}\pi d^3$. Their values are shown in Table 2.2. Note that the terminal droplet Reynolds number, Re_D , is always less than 6, whereas, the terminal bubble Reynolds, Re_B , the number is always less than 35. Also, note that both the droplet Weber number, We_D , and bubble Weber number, We_B , are of order 1, meaning both droplet and bubble are mostly spheroidal. Specifically, the aspect ratio range for the bubble spans approximately from 1.47 to 1.00, while the aspect ratio for the droplet varies within the range of 1.04 to 0.93, therefore, the bubble may undergo a subtle degree of deformation.

We verified our experimental measurements by comparing the free-rising of a single bubble or droplet to theoretical predictions proposed by Mei et al. [61] and Rachih et al. [62]. The predicted relationship between the Reynolds number ranging from 0 to 100, Re , and the drag coefficient, C_d , for a bubble rising in a stagnant fluid, is given by

$$C_d^B = \frac{16}{Re_B} \left(\frac{16 + 3.315 Re_B^{\frac{1}{2}} + 3 Re_B}{16 + 3.315 Re_B^{\frac{1}{2}} + Re_B} \right) \quad (2.4)$$

where $Re_B = \rho_f U_B d_B / \mu_f$, ρ_f , and μ_f are the bulk fluid's density and viscosity respectively, U_B is the velocity of the corresponding bubble or drop, and d_B is the corresponding diameter of the bubble or drop [61]. For a solid sphere the relationship between C_d and Re , we consider the Schiller & Naumann [63] relation given by

$$C_d^s = \frac{24}{Re} (1 + 0.15 Re^{0.687}). \quad (2.5)$$

Lastly, the relationship between C_d and Re for the droplet is given by

$$C_d^D = \frac{C_d^B + \mu^* C_d^s}{1 + \mu^*} \quad (2.6)$$

where μ^* is the viscosity ratio of the internal to external fluid [62].

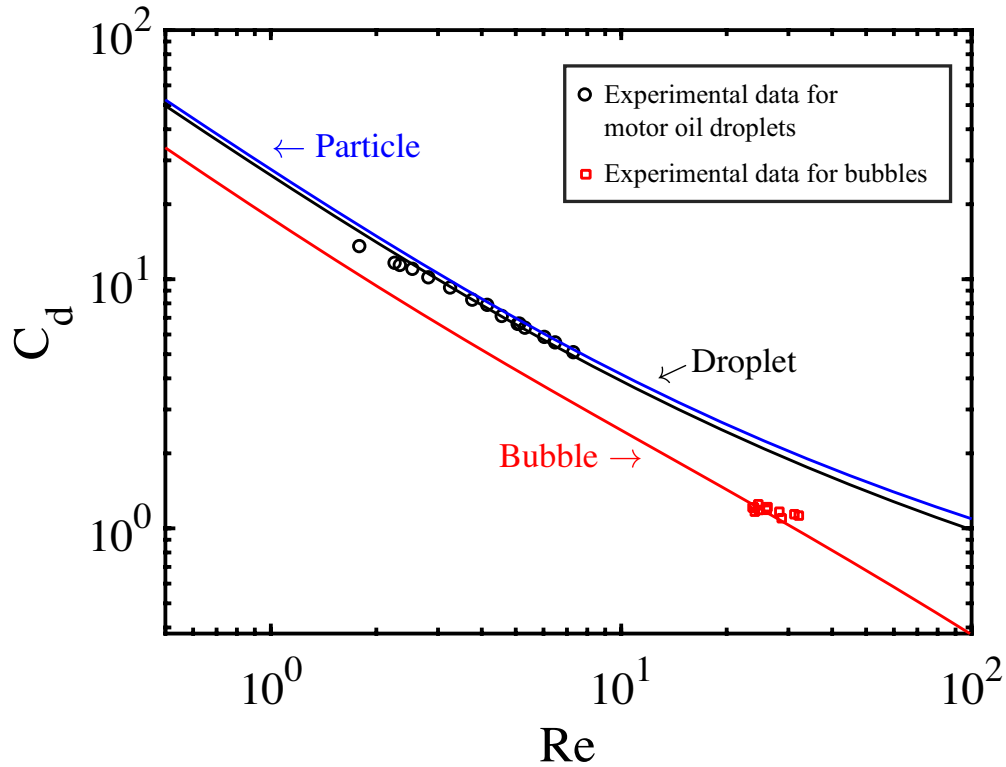


Figure 2.3: The drag coefficient as a function of the Reynolds number for all experimental measurements of an oil droplet or air bubble rising in a quiescent fluid, empty circles, and squares, respectively. Solid lines represent the prediction for a bubble (Eq. 2.4)[61], droplet (Eq. 2.6) [62], and particle (Eq. 2.5)[63].

A plot representing these relationships is shown in Fig. 2.3, where the solid lines indicate the model and the squares or circles represent the experimental data for an individual bubble or droplet. Good agreement

between experimental data and the prediction validated our processing technique. The free-rising bubble measurements are aligned with the prediction from Eq. 2.4. Whereas, the experimental measurements for the droplet lay in between the prediction for the bubble (Eq. 2.4) and the particle (Eq. 2.5), and on top of the prediction for the droplet (Eq. 2.6), as expected.

2.3 Results and Discussions

An analysis of the experiments conducted, capturing the interaction of a free-rising bubble and droplet, was performed. A typical experimental interaction is shown in Fig. 2.4(a), where the terminal velocities of a single droplet and bubble are plotted as a function of time. The points on the plot show one experimental case of the increase in droplet velocity due to the interaction of the bubble. The bubble, on the other hand, experiences very small changes in its speed. The experimental data is supported by the images in Fig. 2.4(b) and the position plot in Fig. 2.4(c) to gain an intuition of the orientation and position of the droplet relative to the bubble. Before the interaction, $t^* < 0$, the droplet travels at its terminal velocity, as shown by the relatively horizontal region in the experimental data from time -2 to -0.5. When the bubble is approximately one diameter away, the velocity of the droplet starts to increase, as seen in images 1 through 3. After some time, $t^* > 0$, the orientation of the droplet and bubble change due to the bubble surpassing the droplet. At this point, shown in images 3 through 6, the droplet is in the bubble's wake. In the wake, the droplet speed decreases but at a smaller rate than its viscous relaxation. The viscous relaxation of the droplet was calculated using Eq. 2.7, where the droplet started at its maximum velocity, which can be seen in the prediction in Fig. 2.7 (grey dashed-dashed line). The collision angle, ϕ , shown in Fig. 2.1, is the vertical angle between the droplet and bubble centers when they are at a vertical distance of one bubble diameter apart and was aimed to be as small as possible, thus having the two phases nearly inline. The collision angle varied in each experiment, as it was not controlled but rather a product of the experimental setup. The experiments showed a droplet velocity enhancement due to the interaction with the bubble and its flow field. The droplet velocity enhancement was caused by the interaction with the bubble and its flow field. The velocity of the droplet typically increased about 2 to 3 times its terminal velocity, U_∞^D , seen in Fig. 2.4. Many experiments were conducted and shared similar phenomena, yet coalescence was never observed. Overall, the experimental observations indicate a classical transfer of momentum, as would be expected

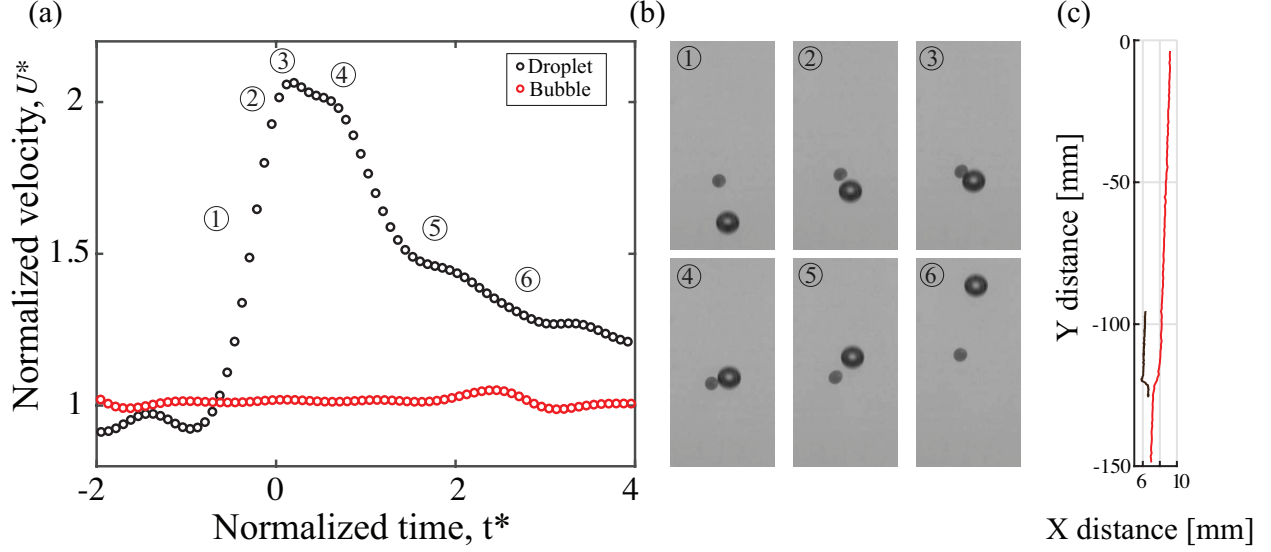


Figure 2.4: (a) A typical experimental plot of the droplet velocity jump as a function of normalized velocity, $U^* = U/U_\infty$, and normalized time, $t^* = tU_\infty/d$, (b) images that support the bubble-droplet orientation and (c) a plot of the temporal evolution of the relative positions between the droplet and bubble during the interaction where the red line is the bubble position, and black is the droplet position. Time $t^* = 0$ is set to be the time where the bubble and droplet interact ($\phi \approx 90^\circ$).

in the case of two moving bodies interacting in air, however, rather than experiencing a direct transfer of momentum, the interaction of the bubble and droplet within the viscous fluid demonstrates a non-trivial interaction of a deformable object in a velocity-induced flow field.

In the experimental results, it can be seen that a smaller offset between the bubble and drop, or a smaller collision angle, leads to a higher increase in the droplet velocity, as a more direct interaction causes more momentum to be transferred from the bubble to the droplet. This is highlighted in Fig. 2.5(a), where two experiments with the same diameter ratio but different collision angles are compared. The case with the smaller collision angle (blue) has a higher increase in droplet terminal velocity than that of the larger collision angle (black). Also, a smaller diameter ratio, in which the diameter ratio is defined as the droplet diameter, d_D , over the bubble diameter, d_B , also leads to a higher increase in the droplet velocity. Two experiments are highlighted in Fig. 2.5(b) to highlight this. The two experiments shown have the same collision angle, yet different diameter ratios. The one with the smaller diameter ratio, defined by having a small droplet and large bubble, has a larger increase than the case with the larger diameter ratio – a droplet and bubble more equal in size. Fig. 2.5(c) shows the collision angle and diameter ratio compared simultaneously to the velocity ratio, U_{max}^D/U_∞^D . This shows that both the size ratio and collision angle

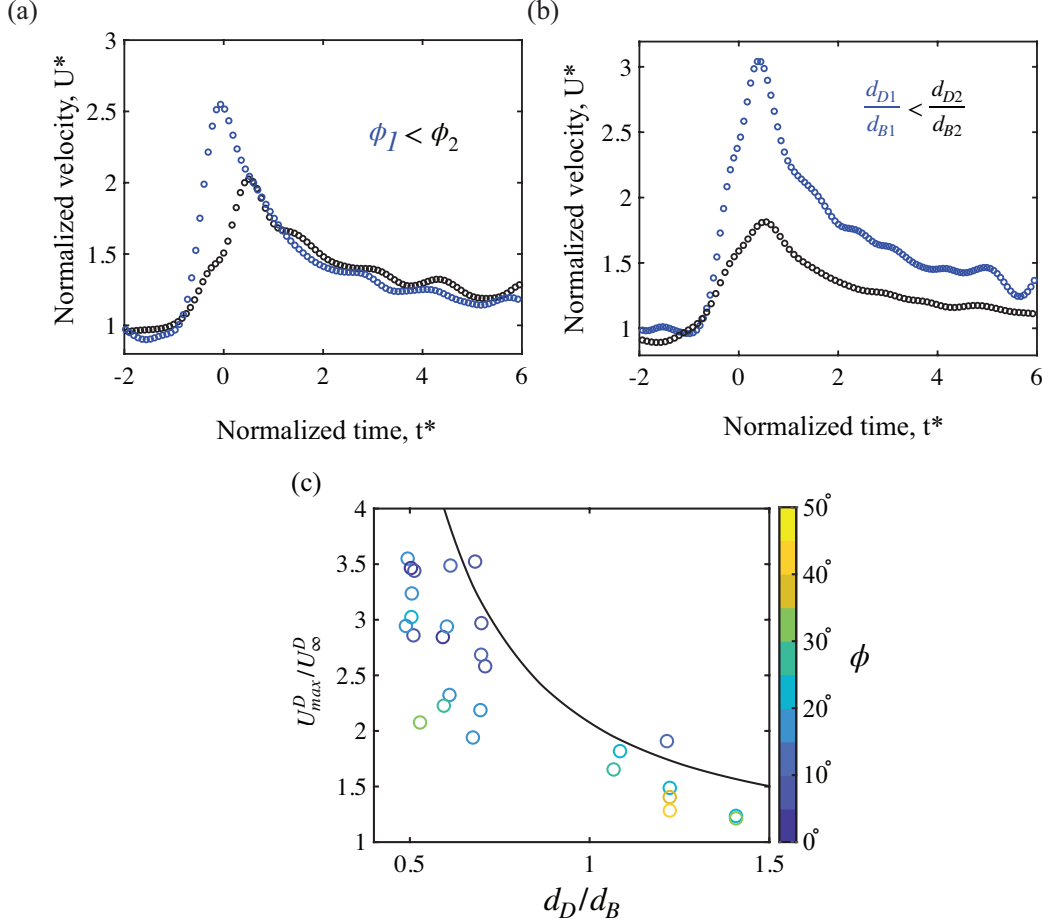


Figure 2.5: The experimental results showing a comparison of (a) collision angles ($\phi_1 = 7.3^\circ$, $\phi_2 = 28.1^\circ$, and $d_{D1}/d_{B1} \approx d_{D2}/d_{B2} \approx 0.64$) and (b) diameter ratio ($d_{D1}/d_{B1} = 0.5$, $d_{D2}/d_{B2} = 0.7$, and $\phi_1 \approx \phi_2 \approx 22.9^\circ$) effects on the max droplet velocity, and (c) a plot illustrating the relationship between collision angle ϕ , diameter ratio, d_D/d_B , and velocity ratio of the maximum droplet velocity, U_{max}^D , and the droplet terminal velocity, U_∞^D with a prediction (seen in black solid curve) of the velocity ratio from Eq. 2.9, U_{max}^D/U_∞^D , as a function of the diameter ratio, d_D/d_B .

collectively influence the enhancement of speed, where the smallest collision angle and diameter ratio result in the largest increase in droplet velocity.

In order to rationalize our experimental results and the physical phenomena that occur, we analyzed the parameters impacting droplet velocity enhancement and developed a model to explore the advection of the droplet due to the bubble-induced flow field. The following section outlines the model of the bubble flow field with respect to the droplet.

2.4 Interaction Model

The following section describes a model for the oil droplet's velocity increase due to the bubble's interaction. A force balance was used to represent the buoyancy, drag, and weight acting on the droplet. The force balance on a droplet, irrespective of the bubble is

$$(\rho_f - \rho_d) \frac{4}{3} \pi r_D^3 g - 6\pi \mu_f r_D U_D = (m + m_{ad}) \frac{dU_D}{dt} \quad (2.7)$$

where μ_f and ρ_f are the bulk fluid viscosity and density, ρ_d is the density of the oil drop, r_D is the radius of the drop, g is Earth's gravitational acceleration, $m = \frac{4}{3} \rho_d \pi r_D^3$ and $m_{ad} = \frac{2}{3} \rho_f \pi r_D^3$ which are the mass and the added mass of the droplet respectively. The first term on the left-hand side is the buoyant force minus the body force and the second term is the viscous drag force. The right-hand side is the change of momentum of the droplet.

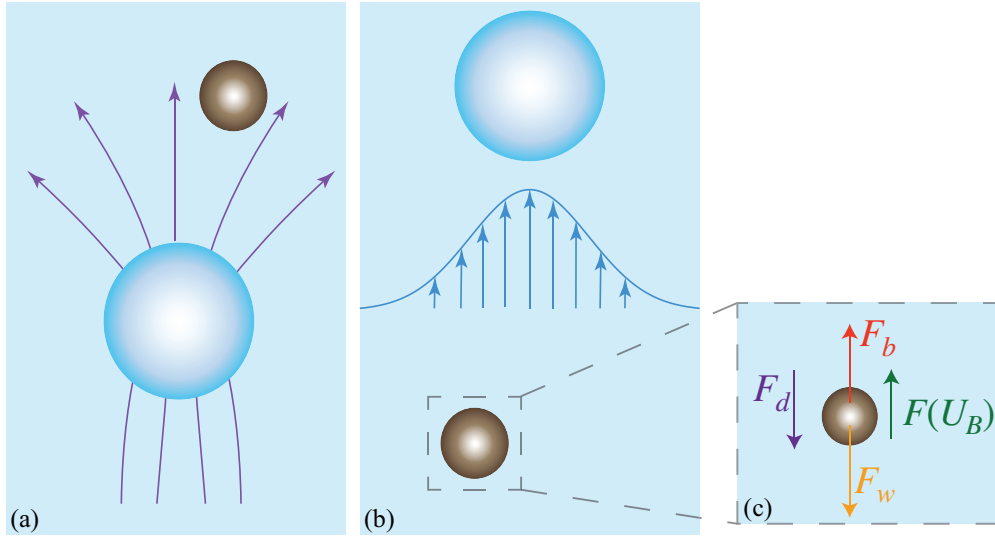


Figure 2.6: A diagram representing (a) the flow field in front of the bubble, (b) the flow field behind the bubble, and (c) a simple force diagram representing the various forces acting on the droplet; drag, buoyancy, weight, and bubble induced force.

An additional term, shown schematically in Fig. 2.6, was used to represent the force acting on the droplet due to flow field changes induced by the bubble velocity, $F(U_B)$. The velocity disturbance due to the bubble was found by examining how the flow field around the droplet changed due to the interaction with the bubble. As a first approximation, we can obtain the velocity distribution in front of the bubble using

potential flow, with which the pressure field can be determined using a standard flow around a sphere solution. Assuming that the pressure field is not affected by the droplet, a force can be calculated by solving for the change in pressure as the bubble approaches the droplet. This was achieved by solving for the flow field using potential flow and subsequently using Bernoulli's equation to determine the pressure difference in the gap. The force was then quantified as the product of the change in pressure within the gap between the droplet and bubble, ΔP , and the cross-sectional area of the bubble, A . This term is added to Eq. 2.7 and its prediction is shown in Fig. 2.7 (grey dashed-dotted line). Clearly, the prediction shows a rapid increase in the velocities of the droplet, much faster than that observed experimentally. Since potential flow neglects viscous effect, the interaction is shorter range. After contact, we can consider a wake model for the velocity disturbance induced by the bubble. A simple wake expression such as $U_{wake} = \frac{A}{t} e^{\frac{-By^2}{t}}$ was used to describe the flow behind the bubble which led to a reasonable prediction of the droplet velocity decay after contact; however, this model depends on empirical fitting parameters [53, 64]. Therefore, we considered Oseen's flow which is valid for a moderate Reynolds number viscous flow and avoids the use of fitting parameters [64, 65].

Oseen's flow describes the flow field around a sphere in viscous, incompressible fluids at low to moderate Reynolds numbers [65], which was applied with the underlying assumption that a no-slip boundary condition exists at the surface of the bubble, therefore, resembling a solid sphere with clearly defined flow conditions. Using Oseen's flow the entire interaction process can be modeled due to the inherent vertical asymmetric nature of this flow, given by

$$\begin{aligned} \mathbf{u} = & U \left[-\frac{a^3}{2r^3} \cos \theta + \frac{3a^2}{\text{Re } r^2} - \frac{3a^2}{\text{Re}} \left(\frac{1}{r^2} + \frac{k}{r} [1 - \cos \theta] \right) e^{-kr(1+\cos \theta)} \right] \hat{r} \\ & - U \left[\frac{a^3}{4r^3} \sin \theta + \frac{3a^2}{\text{Re } r} k \sin \theta e^{-kr(1+\cos \theta)} \right] \hat{\theta} \end{aligned} \quad (2.8)$$

where U is the free-stream velocity (in the case of these experiments the bubble velocity), a is the radius, θ represents the polar coordinate (180° represents the flow in front of the bubble and 0° represents the flow following the bubble or the wake), r is the distance from the bubble, k is a constant defined as $\text{Re}/4a$, and Re is the Reynolds number define as $\text{Re} = 2\rho Ua/\mu$. This equation gives us the disturbance induced by the bubble to affect the velocity field surrounding the droplet.

In assuming the droplet can be modeled as a small sphere compared to the relative flow field, the effect of the flow field on the droplet can then take a similar form to the Maxey-Riley equation [66]. The equation for particle (or drop) motion considers how a relatively small drop is affected by a surrounding flow field. The assumption for the equation of motion is that the drop must be sufficiently small in order to follow the path of the flow field. To characterize the drops as sufficiently small, the Stokes number is used. The Stokes number is defined as $St = \rho_D U_D d_D / 18\mu$, where ρ_D is the density of the drop, U_D is the velocity of the droplet, d_D is the diameter of the drop and μ is the viscosity of the bulk fluid. For $St \ll 1$ the drop will closely follow the flow streamlines [67]. For the case of the experimental data presented here, the Stokes number is approximately 0.2, therefore, justifying the use of the Maxey-Riley equation of particle motion.

Finally, modifying Eq. 2.7 to account for the influence of the bubble via Eq. 2.8, the droplet equation of motion can be expressed as

$$(\rho_f - \rho_d) \frac{4}{3} \pi r_D^3 g + 3m_{ad} \frac{DU_v^B}{Dt} - 6\pi\mu f r_D (U_D - U_v^B) = (m + m_{ad}) \frac{dU_D}{dt}. \quad (2.9)$$

The bubble velocity disturbance is U_v^B , which is the vertical component of $\mathbf{u}$ from Eq. 2.8. The left-hand side of this equation represents the forces acting on the droplet and the induced flow field of the bubble. The right-hand side is the resulting change of momentum of the droplet. Overall, Eq. 2.9 can be used to solve for the increase in the droplet velocity due to the interaction with the bubble, $U_D(t)$.

Solving Eq. 2.9, we find a good agreement between experiments and theory. It is assumed that both the droplet and bubble start at a far enough distance or time in which they both are at their terminal velocities. It is also assumed that the interaction occurs when the droplet is one bubble radius away from the bubble and the collision angle is equal to zero. This result is plotted in Fig.7. For time $t^* < 0$, the purple solid line represents the droplet equation of motion solved for the flow in front of the bubble (or before the interaction) whereas, for $t^* > 0$, the blue line is representative of the flow behind the bubble (or after the interaction). The model aligns with the experimental observations and shows that the interaction with the bubble results in an enhanced droplet velocity. For the cases where the collision angle was much greater than zero, the model struggled to predict whether the droplet was located in the region in front or behind the bubble. Nevertheless, the model overall was able to predict the maximum droplet velocity. Although it is beyond the scope of this paper, it would be advantageous to include a horizontal velocity component of

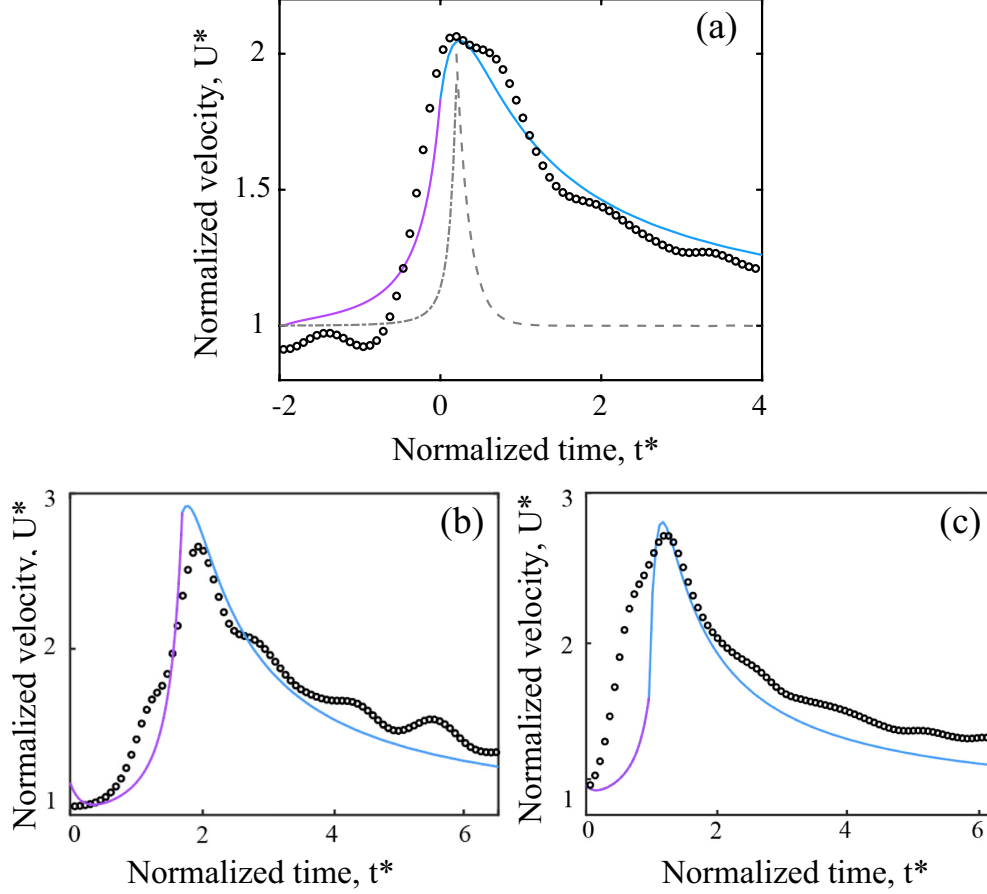


Figure 2.7: (a) A plot that represents the model fit, for $t^* < 0$ (purple solid line) and for $t^* > 0$ (blue solid line), for an experimental data set of one bubble droplet interaction case ($\circ$). Where the normalized velocity is $U^* = U/U_\infty$, and normalized time is $t^* = tU_\infty/d$. The grey lines show the prediction from a potential flow model ($-\cdot-$) and the relaxation of the droplet calculated from Eq. 2.7 ($- -$). Two other model fits are shown in (b) and (c) with different contact angles and rise velocities.

the droplet to gain a deeper understanding of how non-zero collision angles affect the underlying physics of this problem. With that said, the current model detailed the following (1) the difference in the droplet rise and decay time scales, (2) how the collision angle could change the droplet velocity outcome, and (3) the parameters that influence this impact phenomena.

The time scales of the ascent and decay emphasize the influence the bulk fluid has on the interaction. In air, there would be an instantaneous increase and decrease in the velocity of a body being impacted by another high-velocity moving body, however, in the case where there is a viscous fluid, the ascent to maximum velocity and the relaxation to terminal velocity are slowed. This is due to both the surrounding fluid being pushed by the bubble ($t^* < 0$) and the wake of the bubble or the surrounding fluid being pulled by the bubble ($t^* > 0$). In the case of these experiments, the ascent and decay time scales are not symmetric.

The model also shows how the velocity of the droplet starts increasing around $t^* \approx -1$, rather than when the droplet and bubble are actually in contact, $t^* \approx 0$. This is a result of the bubble's flow field extending beyond the bubble diameter [68, 69]. A similar difference in time scales is also seen in the second portion, $t^* > 0$. The effect of a trailing flow field extends much further than the flow field in front of the bubble, this is again due to the inherent asymmetry in Oseen's flow approximation [64]. Therefore, the time it takes for the droplet to decay from the maximum velocity is greater than that of the time for the droplet to reach the maximum velocity.

Furthermore, the model was also able to predict the dependence of the diameter ratio on the velocity ratio. Fig. 2.5 (c), shows the upper bound of these results, where the velocity ratio, diameter ratio, and contact ratio are shown. The line in black, found using Eq. 2.9, shows the predicted relationship between velocity ratio and diameter ratio at a zero collision angle or the ideal case. From this relationship, it can be verified that the most advantageous case, where the droplet velocity is maximized, is when the bubble diameter is much greater than the droplet diameter. As they get closer in size, or the droplet is larger than the bubble, the jump in velocity decreases.

2.4.1 Conclusions

We have experimentally studied the interaction of an air bubble and an oil droplet within a quiescent fluid. Experiments were conducted in order to better understand the interaction of an oil droplet and an air bubble inspired by industrial applications. The experiments showed a droplet velocity enhancement due to the interaction with the bubble and its flow field. A velocity jump of 2 to 3 times its terminal velocity was shown in a typical case. Parameters such as diameter ratio and collision angle impacted how much the droplet velocity jumped. We also presented a physical model to support the experimental results. The model approximates the flow around the bubble to be Oseen's, then we solve the equation of the motion for the droplet incorporating the effect of this bubble-induced flow field. The model was compared to experimental results and showed good agreement. The model captured a faster ascend of the droplet velocity when the bubble was behind the droplet and then a slower decay as the droplet was in the bubble's wake, which was also seen experimentally.

This study can be used to further explore the importance of interaction in two-phase flows specifically their significance in industrial applications. As mentioned, bubbly flows are used in the oil industry in

aeration tanks, in applying the research from this study we can relate the decrease in the rise time of oil droplets to the droplet velocity enhancement due to the bubble and its flow field. In practice, the droplet will interact with the bubble and flow field a multitude of times as both rise to the surface of the tank which will make the process of filtration more efficient. We believe that this research can be extended to further investigate the role of the contact time between the two phases. In these experiments, the contact time remained very short, however, if we were to create a system with neutrally buoyant bubbles and droplets, such as in microgravity, we may increase the contact time, therefore, producing a different novel result. The theoretical formulation introduced for a single bubble–droplet interaction can be extended to many body-interacting systems especially to study bubble-induced turbulence. This work provides a framework to study bubble–particle interaction systems with potential applications to mitigate oil spills and build efficient aeration tanks.

3 PHYSICAL CONDITIONS FOR DROPLET–BUBBLE COALESCENCE IN FLOTATION

Abstract Gas flotation for oily-water mixtures is common practice in many industrial cleanup applications. Gas bubbles, typically air, are expected to come into contact with dispersed oil drops leading to the formation of compound drops that rise faster. Although this process is widely used, the details of the interaction and the capture conditions are not well understood. The present analysis discusses the necessary conditions for coalescence considering the fluid properties of the three phases. A critical time ratio is formulated considering the drainage and sliding times of the interaction; an engulfing parameter is proposed considering the surface tension of the three phases. Considering our experiments and the conditions typically found in applications, we argue that coalescence due to free interaction is unlikely to occur.

Information This chapter is a modified version of the work published in *Journal of Fluid Mechanics* [2].

3.1 Introduction

The hydrodynamic interaction between bubbles and oil droplets has been studied due to their prominence in industrial applications. Many researchers have studied how bubbly flows, specifically aeration tanks, can be used in the oil industry to aid the filtration of oils or other toxins [1, 25, 26, 30, 31]. Aeration tanks work by using air bubbles to capture oil droplets, thus increasing their rise velocity, and in turn, making the oil separation process faster. There have been many efforts to try to understand oil filtration by comparing different methods [31, 70, 71], exploring the process of bubble–droplet coalescence [25, 72–74], and investigating the efficiency of bubbly flows as a means of filtration [26, 30]. Iggunu et al. [30] provided an overview of different filtration methods commonly used in industry. Other researchers, such as Saththasivam et al. [25] and Moosai et al. [26], explore the surface science of the two phases, specifically looking at the interfacial tension. Moosai et al. focused on the stages of attachment, emphasizing the process of the oil droplet spreading around the bubble to achieve coalescence [26]. Whereas Saththasivam

et al. looked at the size and shape of the bubbles and how pre-treating the produced wastewater could alter the interfacial tension therefore increasing your probability of bubble–droplet coalescence[25].

The research conducted on this subject typically considers the so-called spreading parameter, S . The spreading parameter gives the condition for coalescence based on interfacial tensions [48]. Many researchers have used this as a stand-alone condition to dictate if coalescence will occur. Additional studies have demonstrated the importance of other factors such as drainage time and sliding time in influencing the likelihood of coalescence [72, 75, 76]. The size and velocity of both the droplet and bubble can change the drainage and sliding times hence affecting the outcome of the interaction. Chesters’ work explores the drainage time by modeling the interaction of the droplet and bubble as a lubrication problem between two parallel plates under a given force [72] which is equivalent to the proposed drainage time given by Jeffreys and Davies, and Davis et al. [75, 77, 78]. In changing the driving force of the interaction, different time scales can be obtained [72, 75]. Other researchers have considered the induction time (or attachment time) of the interaction which is the summation of the drainage time, rupture time, and spreading time. Again, the drainage time, the primary, and most prominent phase of attachment, is the time the film takes to drain. The secondary phase is the time it takes for the thin film to rupture and finally, the final phase, the spreading time is the time it will take for the oil to spread over the bubble surface. It is not possible to calculate the time of attachment analytically; this time is determined empirically [79].

The present study considers both surface energy and time argument conditions for coalescence during flotation. The ideas are verified experimentally. In a previous investigation, we studied the bubble–droplet interaction in detail [1]. The experiments looked at a one-to-one interaction of an oil droplet and an air bubble in a quiescent fluid. We use some of those experiments to test our hypothesis. In all cases, over a range of diameter ratios, coalescence was not observed, rather, the droplet experienced a velocity enhancement due to the interaction with the bubble and its flow field. The velocity enhancement was modeled using an Oseen flow approximation of the bubble flow field and its influence on the droplet [1]. We found that for free rising droplet–bubble pairs, in the absence of any attractive forces, coalescence is not possible. For the case when the energy landscape is favorable for coalescence, the size of the droplet is too small for the film in between the bubble and droplet to drain during the interaction time. These findings are in agreement with Niewiadomski et al. [80], who argued that attachment by collision is impossible.

3.2 Conditions for coalescence

As stated, a majority of the work done on this subject is restricted to a set of defined conditions that are related to the spreading parameter, S . It has been argued that coalescence is expected if the so-called spreading parameter [48, 81] is larger than zero. The spreading parameter S is defined as

$$S = \sigma_{wa} - (\sigma_{ow} + \sigma_{oa}) \quad (3.1)$$

where σ_{ij} is the interfacial tension between substances i and j , which in this case can be water, w , oil, o , or air, a . This assumption is derived from the well-known wetting phenomenon that looks at how a droplet will spread on a flat surface [48]. This parameter has been used by some studies to predict bubble–droplet coalescence [25, 26, 70]. Considering typical values for surface tension values for air, oil, and water combination, $S > 0$; hence, coalescence should always occur. Although the energy minimization reasoning behind the concept of the spreading parameter is correct, it has several drawbacks. First, the spreading parameter does not consider the finite area of the droplet or the bubble. Some researchers have noted the importance of the ratio between droplet and bubble diameter yet have failed to consider it for coalescence conditions [25, 26, 82].

By considering only the surface energies, the dynamic process needed for the bubble and droplet to merge is disregarded. For a compound droplet to form, first, the droplet and bubble need to make physical contact. This can only occur if the droplet inertia is sufficiently large [83, 84]. Once in contact, the droplet and bubble need to remain in contact for long enough for the liquid in the film between them to drain. Details of the relationship between the time scales of these processes and the surface energy condition have not been discussed simultaneously. It is important to note that although there is a vast literature on the subject of flotation, due to its wide industrial use, the condition for free rise coalescence has not been clearly identified. In contrast, when bubbles and droplets are pushed onto one another, compound drops can be readily formed [10, 82, 85, 86].

3.2.1 Surface energy conditions

To modify the concept of the spreading parameter for finite-size surfaces, we can first consider the surface energy of an air bubble immersed in water is

$$S = \frac{\pi}{4} d_B^2 \sigma_{aw} \quad (3.2)$$

where d_B is the bubble diameter. If the bubble is completely engulfed by a layer of oil, the new configuration will have a surface energy of

$$S' = \frac{\pi}{4} d_B^2 \sigma_{ao} + \frac{\pi}{4} d_D'^2 \sigma_{ow} \quad (3.3)$$

where $d_D' = (d_B^3 + d_D^3)^{1/3}$ and d_D is the diameter of the oil droplet.

Therefore, by extending the concept of the spreading parameter, Eq. 3.1, to finite surfaces, we propose an engulfing parameter as

$$E = \frac{\pi}{4} d_B^2 \sigma_{aw} - \left(\frac{\pi}{4} d_B^2 \sigma_{ao} + \frac{\pi}{4} d_D'^2 \sigma_{ow} \right). \quad (3.4)$$

If the surface energy of the compound droplet is smaller than the bubble, then $E > 0$. Otherwise, if $E < 0$, only partial converge of the bubble will occur for which the coalesced droplet may not form or will not be stable [74, 87].

The engulfing parameter can be normalized to be conveniently written in dimensionless form as:

$$E^* = \frac{E}{\frac{\pi}{4} d_B^2 \sigma_{aw}} = 1 - \left(\frac{\sigma_{ao}}{\sigma_{wa}} + \frac{\sigma_{ow}}{\sigma_{wa}} (1 + d^{*3})^{2/3} \right) \quad (3.5)$$

where $d^* = d_D/d_B$ is the diameter ratio. For $E^* > 0$, considering typical values of surface tension for oil, air, and water $d^* < 1$, meaning that only droplets smaller than bubbles would have favorable conditions for coalescence.

3.2.2 Interaction time conditions

Relying solely on the surface energy condition is insufficient. To fully address the dynamics of the compound droplet, it's essential to also incorporate the characteristic time for interaction. For coalescence to occur, the droplet and bubble must remain in contact for a time longer than the time it will take for the liquid film

to drain between them. For the bubble and droplet to come into contact a critical Stokes number needs to be met [83, 84]. The Stokes number is defined as $St = (1/9)U_D d_D \rho_d / \mu$, where m is the mass of the droplet, ρ_D is the density of the oil droplet, U_D is the velocity of the droplet, d_D is the diameter of the droplet, and μ is the viscosity of the fluid [84]. The Stokes number compares the inertial effects of the droplet with the viscous dissipation by the surrounding fluid. If the Stokes number is small, then the droplet will follow the streamlines of the flow, therefore never making contact with the bubble itself. If the Stokes number is greater than 10, then the droplet will have too much inertia and possibly bounce off the bubble. Therefore, for coalescence to occur, the Stokes number needs to be slightly larger than zero such that the droplet is inertial enough to cross streamlines but smaller than 10 to prevent bouncing off the bubble [83, 84].

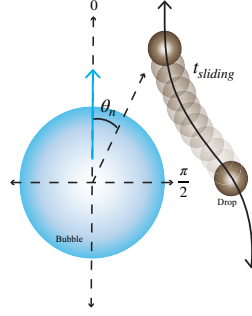


Figure 3.1: A visual representation showing the sliding time of a droplet as it advances over the bubble surface.

Once in contact, another condition must be met for coalescence to occur. Additionally, the time of contact and the time of drainage need to be considered. The time the droplet and bubble are in contact can be calculated by the sliding time, or the time it takes for the droplet to slide over the bubble surface. The sliding time is calculated analytically for two conditions; potential flow and Stokes flow. The sliding time, seen in Fig. 3.1, is defined as the time it takes for the droplet to go from the collision angle, θ_n , to the point in which the droplet begins to trail away from the bubble, $\theta_n \approx \pi/2$, as a function of the summation of the bubble diameter, d_B , and the droplet diameter, d_D , and the tangential droplet velocity, $U_{D\theta}$. The collision angle is defined as the vertical angle between the bubble and droplet centers at a vertical distance of d_B . Considering the flow solution around a bubble at either potential flow or creeping flow approximations the sliding time can be calculated.

In potential flow the tangential liquid velocity is equivalent to [88]

$$u_{\theta}^{pot} = U_B \sin \theta (1 + 0.5(1/d')^3) \quad (3.6)$$

where d' is an effective diameter equivalent to $d_B/(d_B + d_D)$, U_B is the bubble velocity and θ is the necessary radial coordinate.

Similarly, the Stokes flow tangential liquid velocity is

$$u_{\theta}^{st} = U_B \sin \theta (1 - \frac{3}{4}d' - (1/d')^3). \quad (3.7)$$

Therefore, the tangential droplet velocity for both potential flow and Stokes flow is the following:

$$u_{D\theta} = u_{\theta}^i + U_D \sin \theta \quad (3.8)$$

where u_{θ}^i is the potential or Stokes flow tangential liquid velocity and U_D is the droplet velocity.

The sliding time, t_s , for both potential flow and Stokes flow is calculated by [88]:

$$t_s = \int_{\theta_n}^{\pi/2} \frac{(d_D + d_B)d\theta}{2U_{D\theta}} \quad (3.9)$$

Therefore, the sliding time for potential flow is

$$t_s^p = \frac{(d_D + d_B)}{2(U_B + U_D) + U_B(1 + d^*)^3} \ln \left(\cot \frac{\theta_n}{2} \right) \quad (3.10)$$

whereas the sliding time for Stokes flow is

$$t_s^s = \frac{(d_D + d_B)}{\frac{1}{2}(-1 + (1 + d^*)^3 + 3d^*)U_B + U_D} \ln \left(\cot \frac{\theta_n}{2} \right) \quad (3.11)$$

The sliding time will be at its maximum when the bubble and droplet interact head-on, conversely, the sliding time will be at a minimum, or tend to zero, if the two phases interact at a 90-degree angle.

We can compare the sliding time with the drainage time to determine if coalescence is likely. The drainage time between two deformable bodies has been studied by many researchers. There have been

attempts to solve for the drainage time with experimental results, simplified mathematical models, and computational models [72, 76, 89–92]. An expression that is often used was originally proposed by Ye and Miller (1988), who studied the contact time between bubble/particle collisions. They hypothesized that the film thickness did not vary much throughout the sliding process, and therefore made attachment and rupture unlikely to occur during sliding [90]. They concluded that the induction time had a linear relationship with particle diameter independent of the particle type [89, 90]. They proposed $t_{induction} = Ad_D^B$, where A and B are empirical parameters to be determined experimentally [90, 93]. Although this equation can offer some insight, it does not have physical significance.

Considering that the time of attachment is only the drainage time, the primary step of coalescence, we can gain a more physical insight to predict attachment or coalescence. The drainage time is derived considering the flow between the bubble and droplet as a parallel squeezed flow [72, 94]. First, the flow between the approaching droplet and bubble, the squeezed flow, is modeled as the change in the height, h , or the distance between the two, over time, as a function of the terminal velocities, which is expressed as:

$$\frac{dh}{dt} = U_B - U_D \quad (3.12)$$

The squeezed flow between the droplet and bubble can then be equated to the flow that is displaced outward to allow the droplet and bubble to come closer in contact. The flow that is displaced outward is found using the Poiseuille relation for flow between quasi-parallel surfaces and is expressed as the force, F , given by [72, 75]:

$$F = \frac{3\mu\pi d_{eq}^2}{8h} \frac{dh}{dt} \quad (3.13)$$

Integrating Eq. 3.13 and solving for the time, we obtain:

$$t_d = \frac{3\pi\mu}{2F} d_{eq}^2 \ln\left(\frac{h_i}{h_f}\right) \quad (3.14)$$

Solving this equation for flotation, where gravity is supplying the interaction force F , and substituting in the buoyancy force, $F_b = \frac{1}{6}\pi d_D^3 \Delta\rho g$, we acquire Eq. 3.15.

$$t_d = \frac{3\pi\mu}{2(\frac{1}{6}\pi d_D^3 \Delta\rho g)} d_{eq}^2 \ln\left(\frac{h_i}{h_f}\right) \quad (3.15)$$

where the equivalent diameter is $d_{eq} = d_D(1/d^*) + d_B d^*$, $\Delta\rho = \rho - \rho_D$, and h_i/h_f is the ratio of the initial distance of the interaction to the rupture distance. The initial distance, h_i , represents the displacement between the bubble and the droplet at the onset of their interaction, or when the droplet first experiences the bubble's effects. Experimentally, this has been determined to be a vertical distance of d_B . A typical value of h_f is 10 nm, for which thermal fluctuations initiate the film to rupture [95].

To determine if coalescence can occur, we can compare the sliding time to the drainage time. To do so, we propose the ratio, T^* , defined as $T^* = t_s/t_d$. If $T^* < 1$, the duration of the interaction will be shorter than the drainage time, leading to no coalescence.

Considering the two expressions obtained for t_d and t_s , for potential flow and Stokes flow we obtain

$$T_{potential}^* = \frac{t_s}{t_d} = \frac{St}{2Fr^2\rho^*} \left[\frac{(1+d^*)^3}{d^*[1+U^*(1+(1+d^*)^3)]} \frac{\ln \cot(\theta/2)}{\ln(h_i/h_f)} \right] \quad (3.16)$$

and

$$T_{Stokes}^* = \frac{t_s}{t_d} = \frac{St}{2Fr^2\rho^*} \left[\frac{4(1+d^*)^3}{d^*[4U^{*-1} + U^{*-1}(1+d^*)^3 + 3(1+d^*)U^{*-1} + 1]} \frac{\ln \cot(\theta/2)}{\ln(h_i/h_f)} \right] \quad (3.17)$$

where St is the droplet Stokes number, $\rho^* = (\rho - \rho_D)/\rho_D$, Fr is the droplet Froude number ($Fr = U_D/\sqrt{gd_D}$), d^* is the diameter ratio (d_D/d_B), and U^* is the velocity ratio (U_D/U_B). These expressions can be readily compared with experimental results. In particular, we use data from Federle et al. [1], where the isolated interaction of the droplets and bubbles are considered.

As stated, for coalescence to occur, the sliding time needs to be greater than the drainage time, meaning that the bubble and droplet are in contact for longer than it would take for the thin film to drain. Therefore, coalescence is probable when $T^* > 1$.

3.3 Comparison with experiments

We argue that for coalescence to occur two conditions should be met: $E^* > 0$ and $T^* > 1$. The first condition ensures that the surface energy landscape is favorable for a bubble to remain captured inside a droplet. The second condition limits interactions to last sufficiently long for the fluid film to drain. These conditions are shown in Fig. 3.2, where the upper right-hand quadrant shows the region of coalescence when the limits for T^* and E^* are met. Fig. 3.2 presents experimental data from previous studies [1] alongside predictions

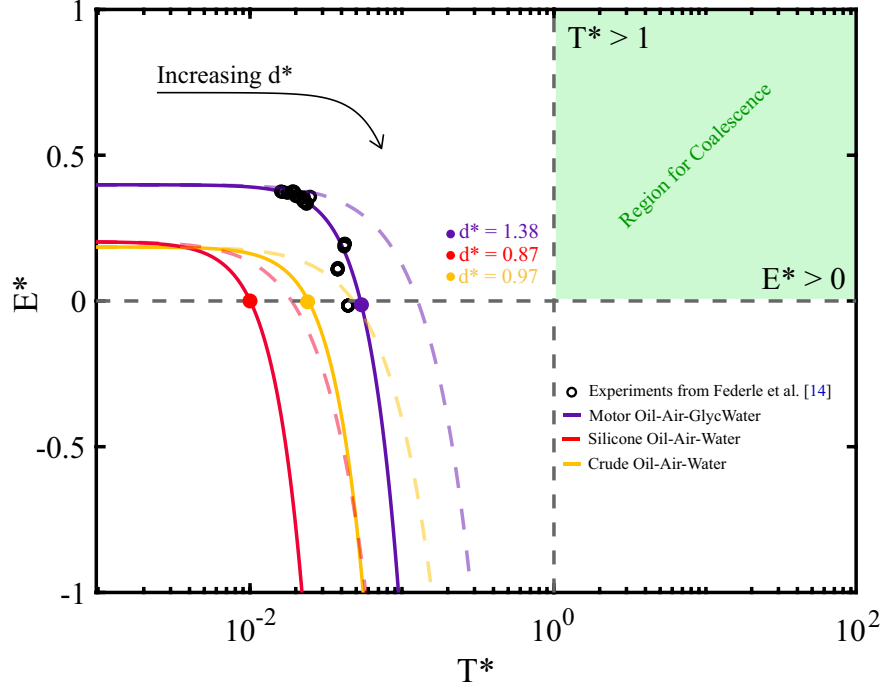


Figure 3.2: The coalescence conditions for a variety of different air bubble and oil droplet pairs as a function of E^* and T^* . The solid lines show the prediction for potential flow, whereas, the dashed lines show the prediction for Stokes flow, see Eq. 3.5, 3.16, 3.17. Experimental values, from Federle et al. [1], are seen in the black points.

for various diameter ratios, spanning magnitudes from 10^{-3} to 10^3 , with corresponding velocities. Fig. 3.2 also includes predictions for a few density ratios for different specified oils. The lines show the predictions from Eq. 3.16 and Eq. 3.5 for a variety of different oil, air, and water combinations. Clearly, the conditions for coalescence are not met. The critical value for d^* is shown for each case in which $E^* > 0$. The lack of coalescence is a result of the conflicting required diameter ratios for E^* to be positive and T^* to be greater than 1. For E^* to be positive, the diameter ratio must be less than 1, meaning the droplet must be smaller than the bubble. Conversely, for T^* to be greater than 1 the diameter ratio needs to be greater than 1.

3.4 Conclusions and discussion

The likelihood of coalescence occurring during free interaction is highly improbable, given that T^* and E^* limits will not be met. We show that T^* and E^* encompass the required conditions for coalescence that include surface energy and interaction effects. Interestingly, aeration is still used in water remediation. We envision the following possible scenarios. First, the droplet velocity is enhanced due to the interaction

with the bubble and its flow field therefore decreasing the rise time of oil droplets thus allowing for more efficient filtration, as demonstrated by Federle et al. [1]. Also, in practical applications the surrounding fluid is not stagnant. Some studies [96, 97] have shown that bubbles would tend to aggregate in vortex cores in turbulent fields, while droplets would remain dispersed. This process, not considered here, may lead to an increased interaction in between bubbles and drops that could lead to capture. Second, there could be attractive forces driving the bubble and droplet to remain together, therefore increasing contact time. The drainage time is affected by the viscosity and density of the bulk fluid. In principle, reducing the fluid viscosity would shorten the drainage time. However, viscosity reduction also increases the velocity of the bubble and droplet, affecting sliding time. Consequently, altering the fluid viscosity does not ensure a higher probability of coalescence. The bulk fluid can also contain other additives, such as salt. The surface forces of the air bubble and oil droplet, although not considered in the scope of this paper, impact the probability of coalescence. At very small scales, where the bubble and droplet get close enough to interact, surface forces, such as van der Waals forces, need to be considered. However, in the case of an oil droplet and an air bubble, they are both negatively charged therefore they would repel one another [98–101]. It has been found that this interfacial repulsive force can be reduced by the addition of salt, therefore causing coalescence to happen more rapidly [102]. Contrarily, it has also been stated that the addition of salt can inhibit coalescence, therefore, the addition of salt does not guarantee an increase in coalescence probability [103, 104]. It should also be noted that T^* would need to be roughly 100 times higher to meet the condition for the coalescence limit. The addition of salt is unlikely to have this large of an effect on T^* . Third, the introduction of surfactants could alter surface energies or interaction times, promoting coalescence. Surfactants such as SLS (anionic), HTAB (cationic), Tergitol, and Toluene have demonstrated the ability to significantly reduce water-oil interfacial tension [105]. Nonetheless, the practicality of adding such surfactants is limited due to the potential further contamination of produced water. In summary, the possibility of coalescence occurring during free interaction is highly unlikely due to unmet T^* and E^* limits, even considering the potential scenarios involving surfactant or attractive forces between bubbles and droplets. Conducting visualizations of bubble–droplet interactions in industrial settings to verify the occurrence of coalescence would be of significant interest. However, such experiments pose considerable challenges due to the complexity of the environment.

4 EXPERIMENTS OF BUBBLES IN MICROGRAVITY: AN INTRODUCTION

The following chapters (4 - 7) of this thesis are focused on the study of bubbles in microgravity. Conducting experiments in microgravity is inherently challenging and requires a substantial amount of preparation and effort, much of which is not fully captured within the chapters that discuss results. Experimental campaigns were carried out at NASA Glenn Research Center in Cleveland, Ohio, using the 2.2-second drop tower, as well as aboard Zero-G parabolic flights. Both campaigns were resource-intensive and required extensive experimental planning, coordination, and logistics. Funding to support these efforts was secured through the Rhode Island Space Grant Consortium and NASA Rapid Research Response (R3) proposals, which were written by the myself and Prof. Roberto Zenit.

The details of the experimental apparatus are presented in Chapter 5, including a construction guide for the homogeneous isotropic turbulence (HIT) chamber and its validation through PIV measurements of the turbulent flow within the tank. The design and validation of this experiment were inherently challenging, as the fabrication of HIT chambers is not a trivial task [106–110].

In addition to the standard design considerations required for Earth-based testing, several additional constraints had to be addressed to enable operation in microgravity, most notably size limitations. The chamber dimensions were restricted by the volume constraints imposed by NASA for testing in the drop tower facility. The specific size requirements are detailed in the following section. Beyond geometric limitations, additional constraints included power supply, data acquisition, and accessibility.

Power availability was a particular concern for the drop tower experiments, as no onboard power source was provided. Consequently, all powered components had to operate on batteries. Compact Li-Po batteries were used for the experimental systems, while cameras and laptops were operated on their internal batteries until recharging was required. In contrast, during parabolic flight campaigns, aircraft power was available, allowing direct power supply to the experimental equipment.

Finally, the experiment had to be designed with minimal manual intervention during operation. This requirement necessitated a high degree of automation, enabling the system to function reliably under the limited access conditions inherent to both drop tower and parabolic flight testing. Further details regarding the automation strategy are provided in this chapter.

4.1 NASA Drop Tower Experiments

The first experimental campaign was conducted using the 2.2-second drop tower at NASA Glenn, schematic shown in Fig. 4.1. The drop tower is in an eight story (24 m) building, resembling a large elevator shaft. The experiment is placed in a drag shield on the 5th floor, the preparation stage, where it is hoisted up by a crane pulley system. At the top level, the release stage, the experiment is attached to singular 15 mm pin which will hold it until release is triggered. The drag shield with the experiment inside is then dropped and remains in free fall for 2.2 seconds before impacted a large airbag at the capture stage. The experiment is isolated from the aerodynamic drag because it is not attached to the drag shield and has 19 cm to fall within the drag shield itself during drop.

One of the most challenging aspects of both experimental campaigns was the necessity of operating outside of the traditional laboratory environment. The experiments were required to function reliably not only at Brown University but also under an entirely different set of operational conditions.

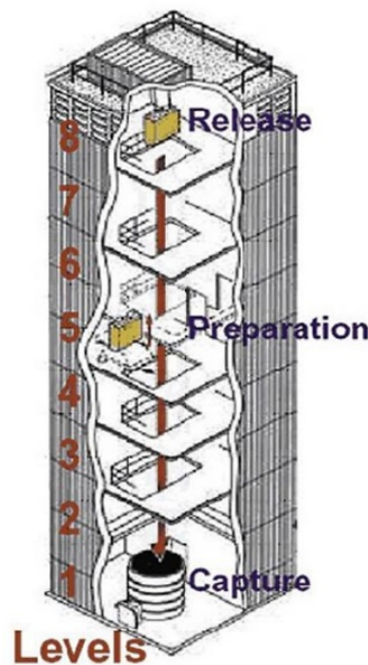


Figure 4.1: Image from NASA of NASA Glenn 2.2 drop tower detailing the 8 floor drop with release at floor 8, preparation at floor 5, and an airbag for capture at floor 1 (photo credits: NASA).

The primary constraints on the experiment's size, weight, and power consumption were dictated by the dimensions of the NASA-provided drop tower rig. The entire experimental system, including data acquisition and power components, had to be mounted to this structure. The rig, approximately 1 m tall, 1

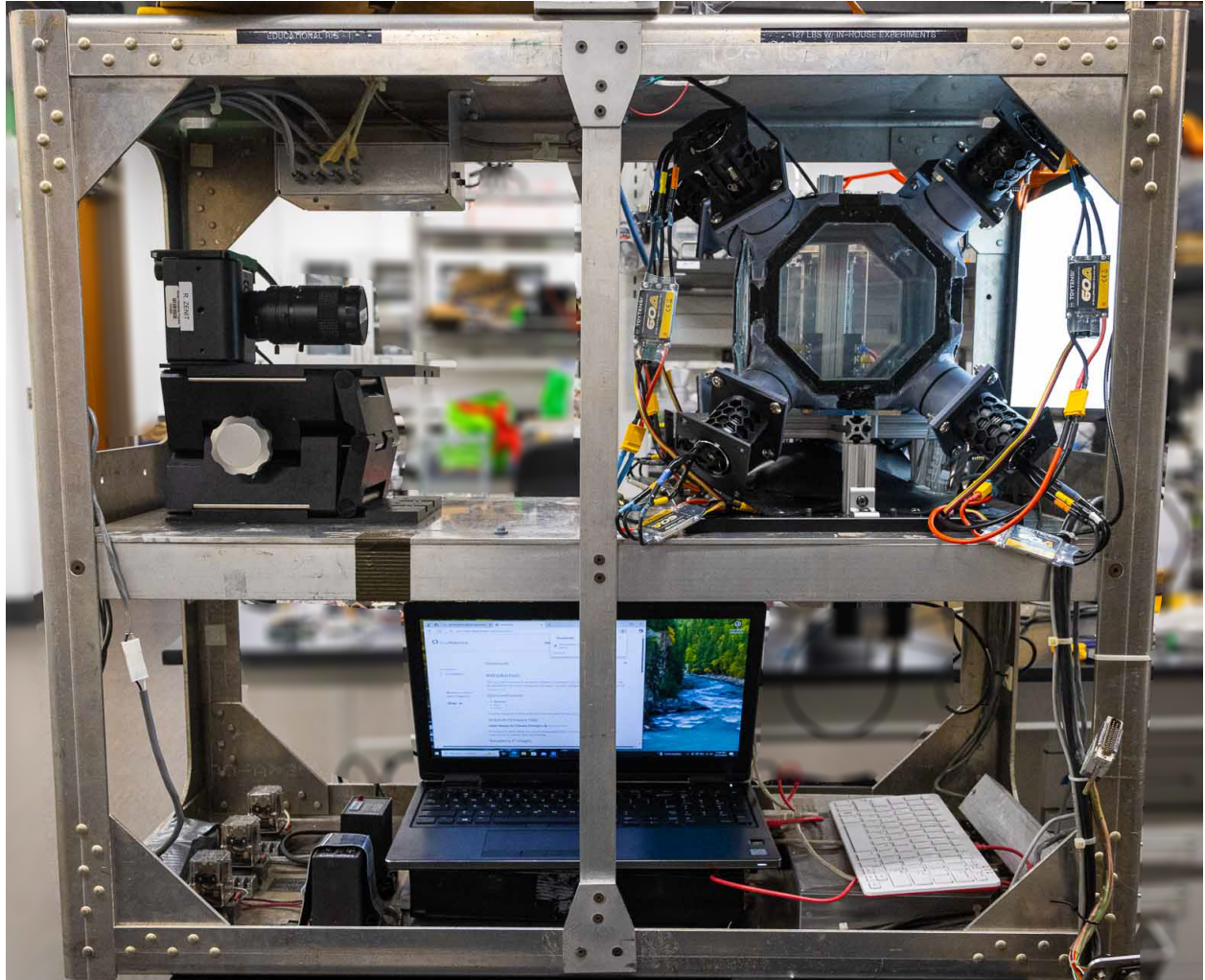


Figure 4.2: Educational drop tower rig from NASA with camera (top left), HIT chamber (top right), and controls/computer (bottom center) mounted in place.

m wide, and 0.5 m deep, was shipped to Brown University to allow full integration of the setup. It consisted of two levels: the upper level supported the turbulence chamber and high-speed camera, while the lower level housed the laptop used to control the experiment and store data. This configuration is shown in Fig. 4.2. After integration and testing, the system was disassembled; the rig was returned to NASA, and all experimental components were packed in a hard case and transported to NASA Glenn for testing.

In addition to geometric limitations, the system was constrained by power availability. During each drop, no external power source was accessible, requiring the experiment to operate entirely on onboard batteries. Small Li-Po batteries powered the mixing motors, while a larger NASA-supplied battery pack

maintained power to the camera and laptop. If any device, including the main battery pack, lost charge, testing was paused to allow for recharging.

The experiment also could not be manually adjusted during a drop. Access to the experiments was available before and after drop with the assistance of the NASA engineers onsite. NASA engineers were in charge of making sure the rig was prepared for drop including putting the drag shield on, checking the center of mass, pulling the rig up after drop, and releasing the rig to drop. Their support, specifically, Vittorio Valletta, was crucial to the success of the drop campaign. All critical actions during the drop had to be automated. Motor shutoff and camera recording were therefore triggered automatically using an impulse-based system developed with support from undergraduate students. The entire rig was connected to a voltage input prior to release; once the drop initiated, the voltage signal was removed, triggering the camera to begin recording and stopping the motors approximately 0.25 seconds later. Because the high-speed camera has an internal memory for only 3 seconds precise trigger timing was essential.

Finally, the system had to withstand decelerations of up to 30 g at the end of each drop. This phase imposed the greatest structural stress, yet ground-based testing could not fully replicate these loading conditions. To mitigate risk, additional structural reinforcements were incorporated to ensure the apparatus could endure the high deceleration forces.

Over the course of two days, the experiment was successfully dropped 14 times, with data collected using high-speed imaging and hydrophone measurements.

4.2 Zero G Parabolic Flights

The second experimental campaign was conducted aboard Zero Gravity Corporation parabolic flights (ZeroG), see Fig. 4.3(a). The parabolic flight achieves microgravity in a similar way as the drop tower by attaining a state of free fall. However, in this case, when the airplane is on the peak of each parabola the pilot reduces thrust and lift to match the atmospheric drag, therefore allowing everything inside the plane to float or be in microgravity. Each flight consisted of 30 parabolas, including two lunar-gravity parabolas and twenty-eight microgravity parabolas. Each parabola involved approximately 30–40 seconds of ascent at 1.8 g, followed by approximately 20 seconds of microgravity, and concluding with 30–40 seconds of descent at 1.8 g (see Fig. 4.3(b) for a schematic). Fig. 4.3(c) shows true accelerometer data for the parabolic

flights with g-jitter noise captured. After every five parabolas, a pause of approximately 2–3 minutes was provided to permit minor experimental adjustments.

Preparation for the flight campaign required an extensive safety and engineering review of the experimental payload. This review included detailed documentation of power consumption, mass properties, mounting structures, structural load analyses, and hazardous materials assessments. A formal technical report was submitted, followed by an in-person payload walkthrough with the flight operations team prior to final approval. Some key details of that report are summarized in this section.

The experiment consisted of the following components:

Description	Dimensions (in)	Weight (lbs)	Quantity	Purpose
Experimental payload with fluid	–	42	1	Experimental payload
Zoom recorder	$1.88 \times 2.95 \times 3.04$	0.53	1	Speed of sound measurements
Camera and mount	–	–	–	–
Floor testing stand	$18 \times 30 \times 0.5$	27.86	1	Mounting
Power strip	$12 \times 2 \times 0.75$	0.81	2	Powering components
Power supply	$4.5 \times 8.5 \times 1.97$	1.6	2	Powering components
Top plate	$16 \times 35 \times 0.25$	13.64	1	Closing second containment
Laptops	$15 \times 10 \times 0.5$	5.5	2	Data collection
Total	$18 \times 30 \times 20$	107.8	n/a	Experiment

Table 4.1: Experimental system components, dimensions, and weights.

While the core experimental apparatus remained consistent with the drop tower configuration, the arrangement was modified to accommodate the aircraft environment and its distinct spatial constraints. In addition to the homogeneous isotropic turbulence (HIT) chamber, high-speed camera, and laptop computer, the system was required to meet double-containment standards. To satisfy this requirement, a custom-fabricated Plexiglas enclosure was constructed to surround the entire apparatus (see Fig. 4.4(a) for schematic representation). Comprehensive load analyses were performed for both the integrated system (Fig. 4.4(b)) and the internal experimental components (Fig. 4.4(c)).

The experiment was mounted to an aluminum optical breadboard, which was subsequently secured to a structural metal baseplate containing designated mounting holes compatible with the aircraft floor interface. The fully assembled system was then rigidly fastened to the aircraft floor to ensure structural integrity under varying acceleration conditions.

During the campaign, the turbulence intensity within the experimental tank was systematically varied every three parabolas. In the final five parabolas, a controlled quantity of oil was introduced into the system.

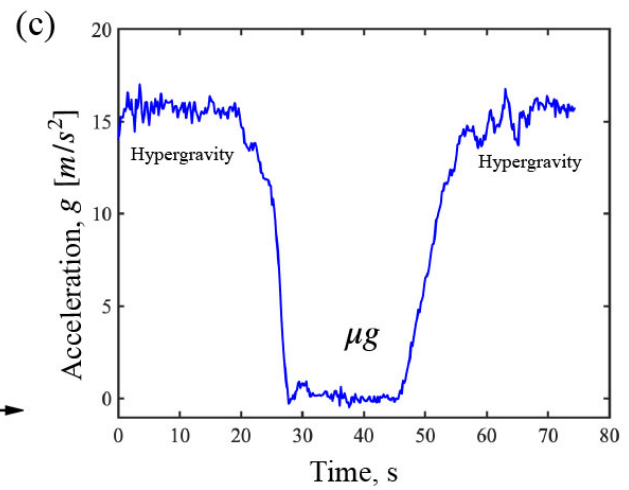
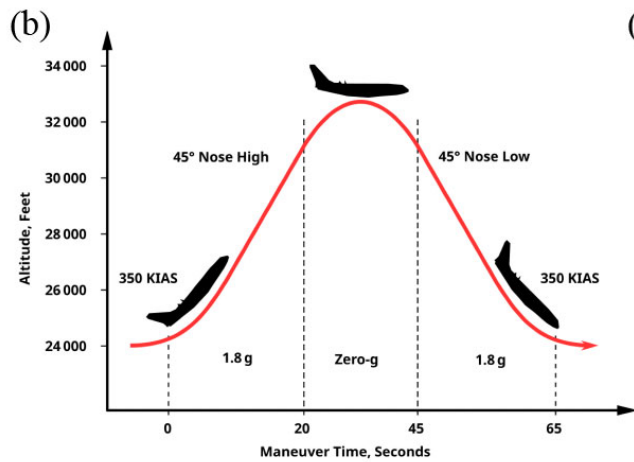


Figure 4.3: Images showing the details of ZeroG parabolic flight, including (a) the airplane flown (credits: ZeroG), (b) a schematic (credits: Wiki-MikeRun) of the trajectory of the airplane highlight intervals of 1.8 g and microgravity, (c) accelerometer data collected aboard parabolic flight showing regions of hypergravity and microgravity.

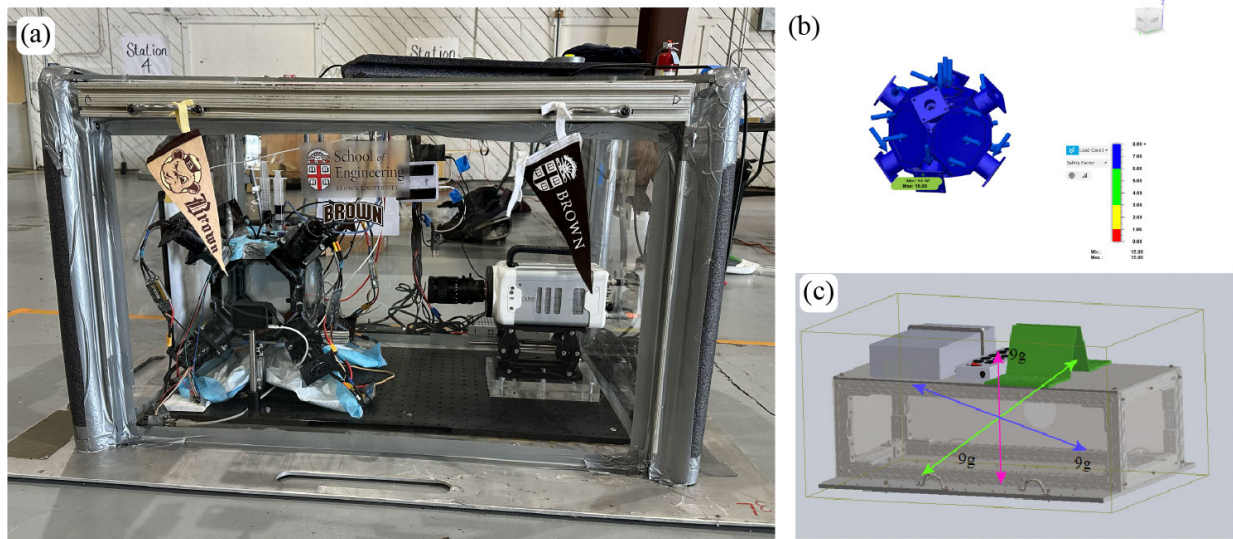


Figure 4.4: Components of parabolic flight experiment with (a) showing the entire mounted system including the base plate, double containment, HIT chamber, and camera, (b) detailing the structural load analysis on the HIT chamber, and (c) is a force diagram of the necessary 9 g's of force required for structural testing by ZeroG.

As in the drop tower experiments, the setup was designed to operate with a high degree of autonomy. The experimental team consisted of three members: myself, Prof. Roberto Zenit, and Benjamin Lyons (see Fig. 4.5). Responsibilities were distributed such that one team member triggered the high-speed camera at predetermined intervals, another controlled the mixing system to achieve the prescribed turbulence intensity, and the third operated the hydrophone data acquisition system. These roles were intentionally structured to be minimal and interchangeable to maintain operational continuity if necessary.

The primary operational challenge encountered during the campaign was data transfer, specifically the time required to download high-speed image data to the onboard computer between parabolas. This limitation was successfully mitigated through the implementation of an overlapping triggering protocol, ensuring that all experimental data were reliably captured and stored.

4.3 Conclusions

The remaining chapters discuss the output of the drop tower and parabolic flight tests including a short study on the speed of sound in bubbly liquids (Chapter 6) and the results of bubble breakup in microgravity (Chapter 7). Bubbles do not rise in microgravity and, as a result, form a distinct class of bubbly flows. The following two chapters focus on what are referred to here as bubble *emulsions*, see Fig. 4.6 for an example



Figure 4.5: Photograph of team working aboard the flight in microgravity. Those featured in the image include, Roberto Zenit (back), Madeline Federle (left), and Ben Lyons (front).

of such a flow. Additionally, see attached videos for comparison of bubble emulsion formation in (1) the lab at Brown University under [Earth's gravity](#), (2) [NASA drop tower tests](#), and (3) [ZeroG parabolic flights](#). Each of these emulsions consists of similar gas volume fractions, $\alpha \approx 0.2\%$, and filmed at similar frame rates. Further details on the breakup process will be discussed in Chapter 7.

Within the scope of this thesis, a bubble emulsion is defined as a two-phase mixture of air bubbles and water. Although the term emulsion is traditionally used to describe mixtures of two immiscible liquids, it is adopted in this work to describe a gas–liquid mixture. A similar system could alternatively be described as a bubble *suspension*; however, the term suspension is typically reserved for solid–liquid mixtures. Accordingly, these chapters seek to investigate the dynamics of bubble emulsions in order to better understand this unique two-phase flow regime. This work was conducted during years 3-6 of my doctoral degree.

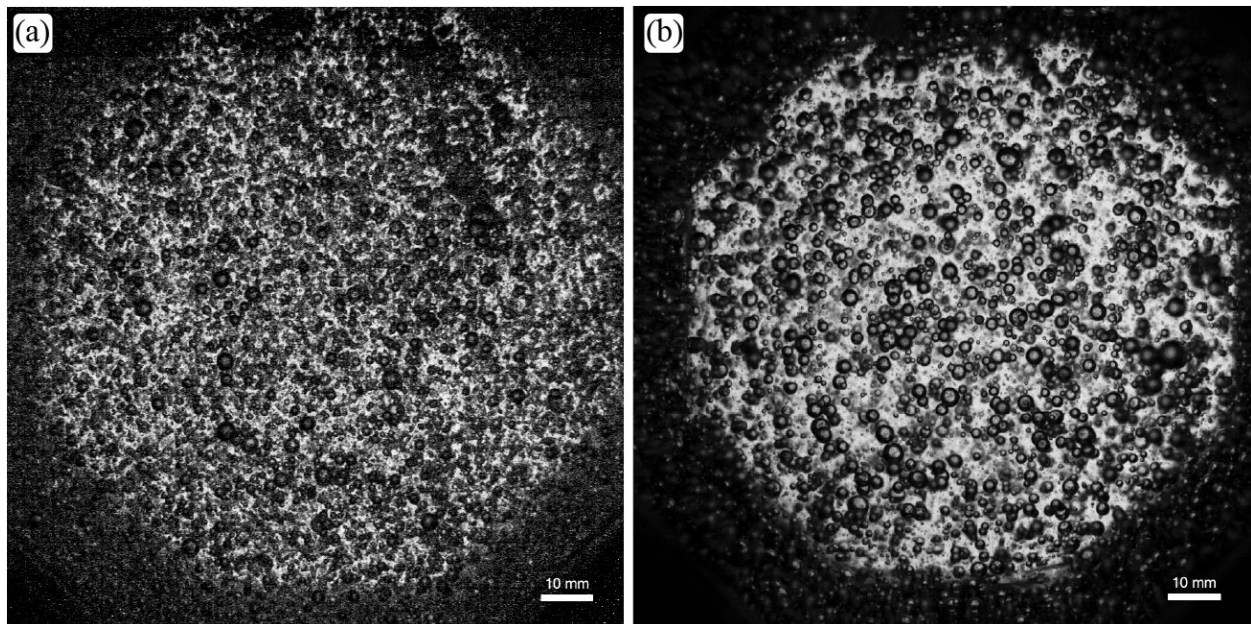


Figure 4.6: Two examples of bubble emulsions in (a) NASA drop tower test, and (b) ZeroG parabolic flight.

5 DESIGN OF AN ACCESSIBLE TURBULENCE CHAMBER FOR LABORATORY EXPERIMENTS

Abstract Homogeneous isotropic turbulence (HIT) chambers provide a valuable platform for experimentally studying turbulence across a wide range of systems. Existing designs are often characterized by high cost, complex fabrication requirements, and reliance on outsourced components. To address these challenges, we present a low-cost, accessible, and open-source HIT chamber that can be fabricated entirely in-house. The chamber consists of a truncated cube-shaped steel frame with acrylic windows, with a volume of 5.5 L. Turbulence is generated using eight propellers driven by DC motors, providing controlled agitation of the flow. To assess the performance of our design, particle image velocimetry (PIV) was used to quantify the statistical properties of the turbulence. The design leverages simple machining processes and readily available parts, making it practical for laboratories regardless of resources. This open-source design aims to broaden access to HIT experimentation and provide a cost-effective platform for turbulence research and education.

Information This chapter is a modified version of the work published in *HardwareX* [5].

5.1 Introduction

Homogeneous isotropic turbulence (HIT) is characterized by statistical uniformity in both space and direction, such that, at any given time, the velocity fluctuations about the mean are evenly distributed and have a mean value of approximately zero [111]. Studying homogeneous isotropic turbulence presents significant challenges but offers substantial value for both industrial and natural systems. Given its complexity and importance in scientific problems, HIT has been the focus of extensive experimental investigation [106, 107, 109]. Homogeneous isotropic turbulence has been used to study a variety of topics, such as ice melting [112], cloud drop tracking [113], ocean mixing [114], or even in combustion chambers [108]. All these studies rely on being able to create reliable and repeatable HIT conditions. While turbulence is often regarded as a chaotic phenomenon, numerous theories, models, and experimental approaches have been developed to quantify and investigate this flow regime.

Given the complexity of homogeneous isotropic turbulence (HIT), many researchers have developed HIT chambers in both air and water environments. These chambers are typically characterized by statistical averages of turbulence intensity and designed to meet the scaling and uniformity requirements of HIT. While various approaches exist, the overarching goal remains the same: to mix the fluid uniformly. One common method employs jet-stirred tanks [106, 107, 115]. For example, McCutchan et al. designed a tank with four custom nozzles arranged uniformly within a cubic tank [112]. By pumping fluid through this nozzle array, they generated turbulence, which they characterized using particle image velocimetry (PIV) and acoustic Doppler velocimetry (ADV). Another approach uses impellers—similar to the method adopted in the present study [109, 110]. By employing multiple, dispersed impellers within a tank, it is possible to achieve isotropy and homogeneity with minimal mean flow. Although these two methods differ in how turbulence is generated, the resulting flows exhibit similar characteristics and share common design features, including tank geometries that are often nearly spherical. While the objectives and specific outcomes vary between chambers, the turbulent Reynolds number typically serves as a common metric for quantifying turbulence intensity. Table 5.1 provides an overview of several HIT chambers employing different mixing techniques, sizes, and corresponding turbulent Reynolds numbers, Re_λ .

Table 5.1: An overview of some current HIT chamber designs highlighting the geometry, method of agitation, and Re_λ .

Authors	Tank Geometry	Fluids type	Agitation Type	Agitation device quantity	Size	Re_λ
Current Design	Truncated cube chamber	Water	Impellers	8	36 cm by 36 cm by 36 cm;	260 – 860
McCutchan et al. (2023) [112]	Cube chamber with jets located at corners and midpoints	Water	Synthetic Jets	80	44.6 cm by 44.6 cm wide and 50.4 cm high	68 – 176
Variano et al. (2008) [107]	Cube chamber with synthetic jet array at base of tank	Water	Synthetic Jets	64	80 cm by 80 cm wide and 100 cm high	280 – 386
Mannaa et al. (2021) [108]	Spherical chamber	Air/ Gas	Fan stirred	Four 8 bladed fans	440 mm inner diameter	Does not explicitly state Re_λ
Wang et al. (2025) [109]	20-faced polyhedron	Water	Impellers	12	400 mm edge length	205 – 602
Zimmerman et al. (2010) [110]	20 faced icosahedron	Water	Propellers	12	40 cm edge length	≈ 350

Although numerous HIT chambers have been fabricated by many researchers, they are often mechanically complex, requiring specialized components, custom fabrication, and outsourced manufacturing. Such large-scale and costly designs can limit accessibility, particularly for smaller laboratories or teaching institutions. Notably, through personal communication, it was reported that the 20-faced polyhedral tank

[109] alone cost upwards of 50 times the price of the system presented here. In response, the present work introduces a low-cost, ≈ 900 USD, do-it-yourself HIT chamber, designed to be built in-house using readily available materials and basic machining capabilities. By providing an open-source framework, our aim is to make HIT experimentation more accessible and affordable for universities and research groups worldwide.

5.2 Hardware description

The turbulence chamber is a compact, benchtop-scale apparatus developed to enable a wide range of experiments on turbulent flows. The chamber features a truncated cube-shaped tank with transparent acrylic windows, providing clear optical access for flow visualization and measurements. Eight impellers are mounted at the interior corners of the tank and driven by DC motors to generate mechanical agitation, producing homogeneous and isotropic turbulence within the working fluid. The impeller rotation rates—and consequently the turbulence intensity—can be easily adjusted through Arduino-based motor control. All components of the chamber, including the tank and mechanical assemblies, were customized and fabricated in-house using 3D printing and standard machining techniques.

While many previous turbulence chamber designs have relied on costly components or specialized fabrication processes, the present design prioritizes accessibility and reproducibility. All materials can be sourced from standard suppliers or local hardware stores, and the fabrication process can be completed in most academic or research laboratories equipped with basic tools and a 3D printer. By sharing this design as an open-source project, the goal is to broaden access to experimental turbulence research, enabling students, educators, and researchers to construct their own chambers and contribute to a growing community of experimentalists advancing the study of turbulent flows.

The following sections detail the design and fabrication steps. Table 5.3 is a bill of materials with associated costs. For more details on CAD models and codes, see the project repository on Zenodo [116].

5.3 Design files summary

All design files required to build, customize, and operate the turbulence chamber are open-source and available on the Zenodo page: <https://doi.org/10.5281/zenodo.17288660>. The repository includes the Arduino code for motor control, as well as CAD files for the corner mounts, motor housings, frame, and

Table 5.2: All files included in open source for design and fabrication of the Turbulence Chamber

Design filename	File type	Open source license	Location of the file
<i>Arduino Code</i>	<i>Arduino File (.ino)</i>	<i>CC BY-SA 4.0</i>	<i>digital_repository</i>
<i>Corner Mount</i>	<i>CAD file (.step)</i>	<i>CC BY-SA 4.0</i>	<i>digital_repository</i>
<i>Motor Housing</i>	<i>CAD file (.step)</i>	<i>CC BY-SA 4.0</i>	<i>digital_repository</i>
<i>Impeller</i>	<i>CAD file (.step)</i>	<i>CC BY-SA 4.0</i>	<i>digital_repository</i>
<i>Tank Body</i>	<i>Figures</i>	<i>included in text</i>	<i>included in text</i>

propellers. Details of the remaining components—those not 3D-printed or coded—are provided in the main text of this paper.

The design files listed in Table 5.2 are the following:

1. Arduino Code: The code written and implemented to control the speed of the motors. The user can input a value that will dictate the RPM of the motors collectively. This can be adjusted based on the users needs.
2. CAD files for corner mount, motor housing, and impeller. Each of these CAD files are included in the open source Zenodo page. These items are explained in detail in the build instructions portion of the article.
3. Tank Body: The details of the tank body are included in the article in the form of figures and instructions, this includes the welded frame and the acrylic windows, as well as mounting instructions. Note that all components can be scaled the desired size. The chamber must not be scaled down to where the near-field flow regions of the impellers interact, nor should it be scaled too large that the motors can not provide sufficient power to effectively mix the entire tank.

5.4 Bill of materials summary

See Table 5.3 for a complete bill of materials. If identical parts or materials are not readily available, suitable alternatives may be used, provided they meet the same functional specifications.

Table 5.3: Bill of materials for HIT chamber, including part name, assembly item, and cost in U.S dollars

Designator	Component	Number	Cost per unit (USD)	Total cost (USD)	Source of materials	Material type
tank frame	Thick Plain Steel Angle	3	15.93	47.79	Home Depot	Metal
tank windows	Acrylic sheets	8	13.90	111.20	Amazon	Polymer
Epoxy windows	Six10 West System epoxy	1	32.99	32.99	West Marine	Polymer
silicone windows	Aqueon Aquarium Silicone Sealant	1	12.99	12.99	Amazon	Polymer
coating of frame	Rust-Oleum spray coating	1	7.48	7.48	Home Depot	Polymer
shaft seals	Mechanical Shaft Seal Replacement 103-12	3	11.99	35.97	Amazon	Composite
shaft ball bearing	Ball Bearings 608-RS	1	11.99	11.99	Amazon	Metal
shaft	Shaft	1	12.99	12.99	Amazon	Metal
impellers/corner mounts	Elegoo Resin (for 3D printing)	1	29.99	29.99	Amazon	Polymer
controls	Arduino Mega	1	49.90	49.90	Amazon	Semiconductor
DC motor	KingVal Replacement 3660 2150KV Brushless Motor	8	45.68	365.44	Amazon	Metal
ESC × 8	Electronic Speed Controllers	8	12.99	103.92	Amazon	Semiconductor
Power supply	DC 12V 50A 600W Power Supply Adapter Transformer Switch	1	31.99	31.99	Amazon	Semiconductor
Total				854.64		

5.5 Build instructions

The setup is a truncated cube-shaped tank, filled with a fluid, commonly water, however, other non-corrosive fluids can also be used. The agitation is produced by eight impellers, located on each truncated corner, and rotated by DC motors. The use of such an impeller arrangement produces homogeneous, isotropic turbulence throughout the fluid. For our research aims, this chamber was built to examine how bubbles break up within turbulent fields. If the turbulence is sufficiently strong, the gas bubbles fragment and disperse [117]. The impellers are rotated at a range of speeds to induce different levels of turbulent intensities.

5.5.1 Tank Body

5.5.2 Welded Frame

The chamber is a truncated cube; a cube with the corners cut off. This geometry provided eight exposed triangular faces that are used for the impeller arms, as seen in Fig. 5.1. The faces of the chamber are octagonal and provide ample access for visualization. This geometry was chosen as it allowed for symmetry among the motors. The dimensions of the tank body are adaptable to meet the needs of the user's experiment, however the measurements of this tank are 36 cm by 36 cm by 36 cm;. Although we selected a configuration with eight impellers, other turbulence chambers have employed a wide range of geometries, using anywhere from four propellers [107] to as many as twenty impellers [109]. While increasing the number of impellers leads to a finer control over turbulence characteristics, it also introduces substantial design complexity. The eight-impeller configuration offers a balance: geometric symmetry is maintained but the design and operation remain tractable.

The turbulence chamber's external structure is constructed from steel angle stock, chosen to provide a flat surface for mounting the tank faces. Steel was selected for this experimental setup to ensure sufficient structural integrity during high-force experiments. However, the chamber body can also be fabricated from alternative material, or even 3D printed, if the intended application does not subject the structure to significant mechanical loads. The machining and assembly process was carried out using the following steps:

1. Cut each ligament (48 total) of the octagon at a 67.5-degree angle, using a Computer Numerical Control machine to the desired length, Fig. 5.1(a).

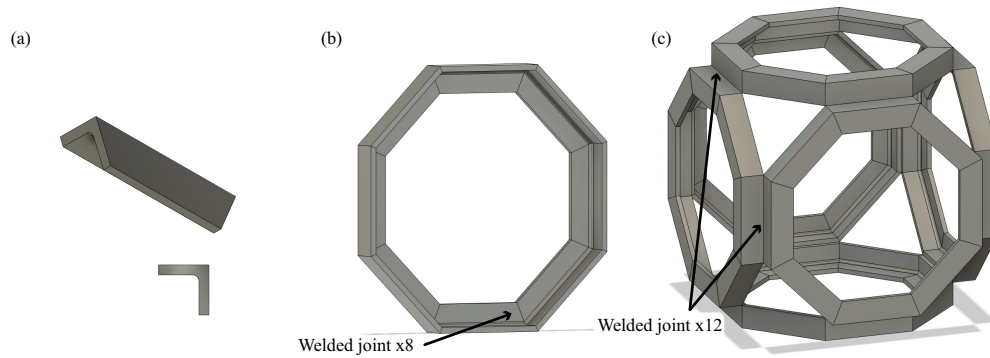


Figure 5.1: Components of turbulence chamber frame(a) angled steel ligaments, inset shows the front view of the ligament,(b) the inside of one assembled face , (c) the outside view of all faces assembled and welded.

2. Weld 8 pieces of the cut steel to one another, creating an octagon. Tungsten Inert Gas (TIG) welding was used for this step [118]. It is recommended to start by tack welding each piece together and then, once the geometry is set, finalizing the octagon by applying a full weld to each joint. This geometry can be seen, looking from the inside of the tank, in Fig. 5.1(b).
3. Weld the following 5 octagons together in the same way, using the remaining ligaments.
4. Weld each octagon to one another (6 in total) from the inside of the angle steel. Again, start by tack welding each octagon to the others. This will allow for some wiggle room to perfect the truncated cube geometry. Use a carpenter's square to ensure each corner is 90 degrees. Once the octagons are welded to one another, fully weld each joint on the interior and exterior. The final frame should resemble Fig. 5.1(c).
5. Coat the entire frame in a rust-resistant coating, such as a spray paint, Flex Seal, or powder coating. This will also seal any small gaps in the weld. If degradation causes the coating to chip or wear away, it should be reapplied to those areas to minimize the risk of rust contamination.

5.5.3 Acrylic Windows

The windows are attached from the inside of the tank to reduce leaks as the pressure from the inside of the tank, when full, pushes outward. The window's dimensions depend on the overall dimensions of the octagon. The windows need to be small enough to fit inside the tank frame but large enough to fit the inner edge of the angle steel, the dimensions for this design is a 15.5 cm wide octagon. The laser-cut octagon

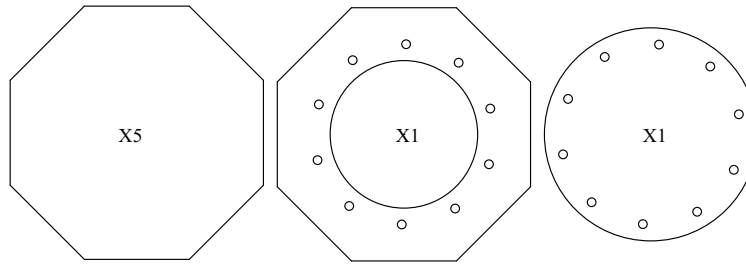


Figure 5.2: Faces for truncated cube including five octagons, one octagon with a large center hole, and one cap for the center hole.

windows were made from 1/8 inch thick acrylic sheets, as seen in Fig. 5.2. The thickness of the acrylic sheet can be adjusted based on the overall tank size. Larger tanks will require thicker acrylic. A large circular hole was cut into the top lid. It was sealed using a laser-cut gasket, with the same hole pattern and geometry as the top lid, and a circular piece of acrylic to allow access inside the tank. The top hole in the lid can be used for drainage via a siphon as well as for access to clean the experimental setup.

The windows were attached with marine-grade epoxy and sealed with marine-grade silicone. It is crucial to secure the bottom window, the one directly across from the window with the circular hole, last. This will allow for the final base window to be epoxied through the hole from the lid. It is recommended to secure a maximum of two windows at a time, such that there is ample room to weigh down or clamp each window during the epoxy's set and cure times. To securely fasten each window onto the tank frame, the following steps should be taken:

1. Select Initial Windows: Choose a pair of windows to start with, such as two side panels.
2. Apply Epoxy: Spread a thin layer of epoxy along the inside edge of the angle steel, see Fig. 5.3 for reference.
3. Position the Window: Place the acrylic window, sanded side down, onto the epoxy-coated edge, ensuring the sanded edge aligns with the epoxy for optimal contact with both the window and the steel frame.
4. Secure the Window: Use weights or clamps to hold the window in place for at least the epoxy's curing time (typically 5–6 hours).

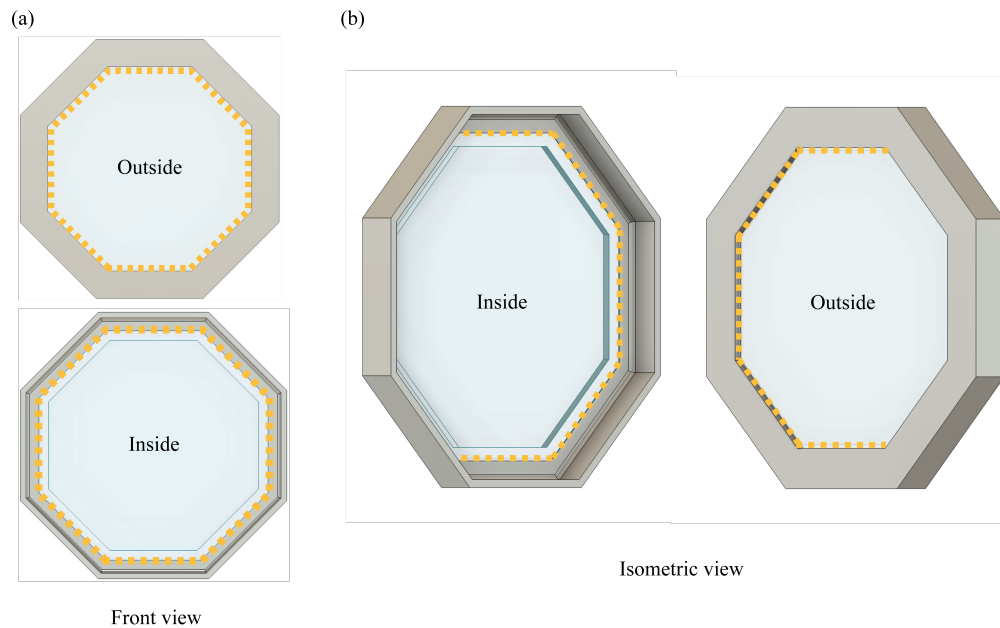


Figure 5.3: Schematic for epoxy and silicone sealant. Yellow dotted lines indicate where the epoxy should be first applied, followed by the sealant.

5. Repeat for Opposite Face: Follow the same process for the opposite window. If working with pairs is challenging, install one window at a time.
6. Seal Inner Edges: Remove the clamps to apply a thin layer of silicone. Using a caulk gun, apply a minimal amount of silicone along the inside edges. Wearing gloves, smooth the silicone with your finger or a tool to fill any gaps evenly.
7. Continue with Remaining Windows: Repeat these steps for the remaining window pairs, completing the bottom face last. This will allow for access to the interior of the tank throughout the entire process. Using the hole in the lid, the bottom face can be epoxied and sealed with silicone.
8. Seal Exterior Seams: After all windows are sealed from the inside, apply a thin layer of silicone to the exterior seams of the tank frame and windows.
9. Curing Time: Allow the silicone to dry for at least 24 hours before filling the tank with water.

As noted, it is best to do this entire process in pairs or one by one to ensure that access to the inside of the tank is least obstructed. Once the tank is completely sealed, accessing the interior of the tank is much more difficult, as the hole in the lip of the tank is the only access point. Following these steps ensures that

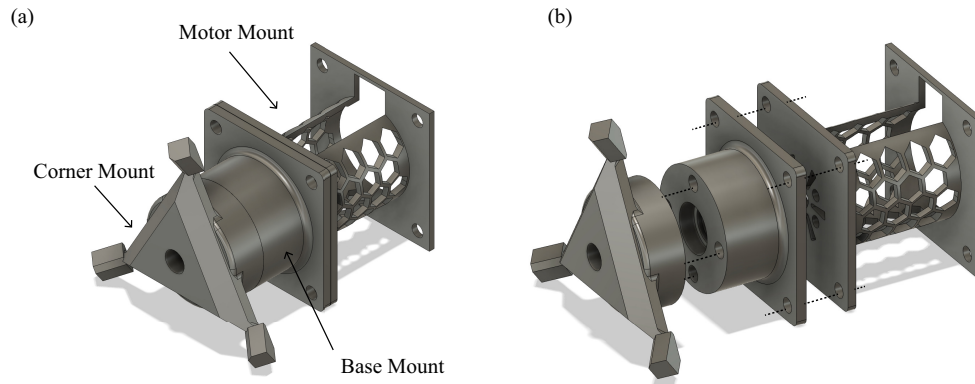


Figure 5.4: (a) Full arm assembly, (b) Separated arm assembly, highlighting the connection points.

the tank is leak-proof. If leaks develop over time due to silicone degradation, the recommended procedure is to remove the existing silicone using a sharp straight-edged tool (e.g., a razor blade), allow the surfaces to dry completely, and then reapply silicone to the seams. The windows remain securely fixed by the epoxy, making reapplication of the silicone straightforward.

5.6 Motor Arms

The arms, constructed of three 3D printed parts (corner mount, base mount, and motor mount), hold together the driving motor, a ball bearing, a shaft, a mechanical seal, and the propeller, highlighted in Fig. 5.4. The base of the arm, the corner mount, is sealed onto the tank; this component has a triangular face that, similarly to the windows, is attached to the steel frame using epoxy. The geometry of the face of this component is designed to fill the missing corner of the steel frame, which should also be scaled as needed. In the center of this component, there is a hole for the shaft.

The opposite side of the triangular corner mount features a hole of the same diameter to allow the shaft to pass through, along with a larger recessed opening designed to accommodate a mechanical seal. Mechanical seals are commonly used hardware components that allow a rotating shaft to pass through a surface while preventing fluid leakage. The force applied to the mechanical seal spring is critical, excessive compression or insufficient compression can lead to leakage or prevent proper rotation. The 3D printed housing for the mechanical seal is to ensure the appropriate spring compression and minimize these issues. The diameter of this hole matches that of the mechanical seal, such that the base of the mechanical seal sits snugly to prevent leakage.

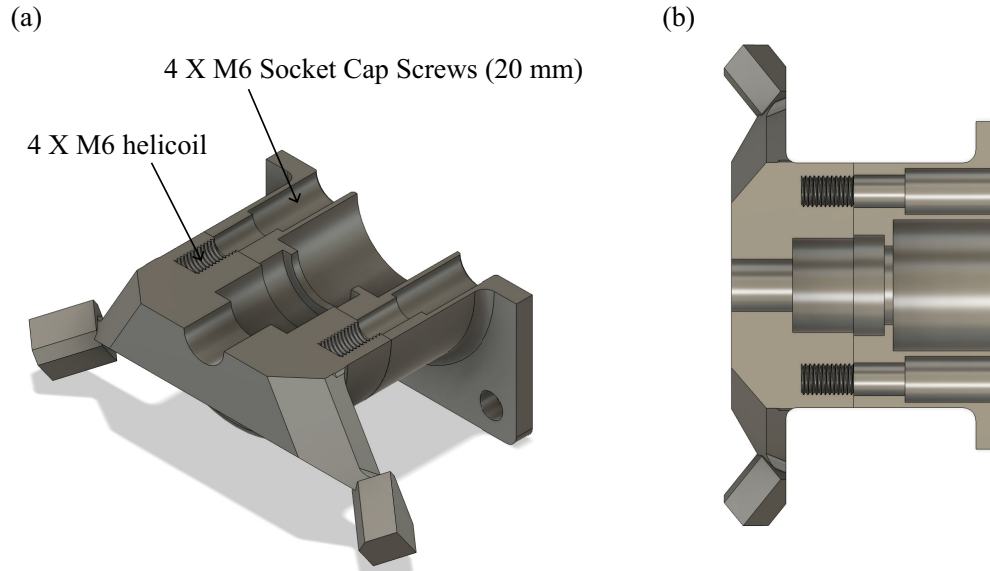


Figure 5.5: The corner mount horizontally sliced in half, where (a) is the isometric view highlighting the locations of helicoils and screws, and (b) is the top view showing the threads and geometry of the connection points.

The next component of each arm is the base mount, a 3D-printed part designed to sit flush against the corner mount, securing the mechanical seal via a pressure fit and providing support for a ball bearing and shaft. The base mount is fastened using four screws that align with helicoils embedded in the corner mount. To allow for the installation and compression of the mechanical seal's spring, the base mount is split into two pieces. Fig. 5.5 illustrates the assembly and the interlocking geometry of these components.

The shaft is long enough to go from the inside of the tank through the corner mount and the base mount to reach the motor, which is on the other side of the base mount. Finally, the assembly of the arms is completed by attaching the motor to the shaft. The shaft is mounted to the motor using a rigid coupler, and the motor is supported by another 3D-printed part attached to the base mount using four screws and nuts through the holes shown at the corners of both of these prints, as seen in Fig. 5.6.

5.7 Impellers

The impellers, mounted to the shaft that runs from the inside of the tank to the motors, are used to induce mixing in the tank. The shape and size of these impellers can be modified based on your tank's requirements. In the case of our tank, we used small toroidal impellers, as seen in Fig. 5.6. This design, modified from [109]

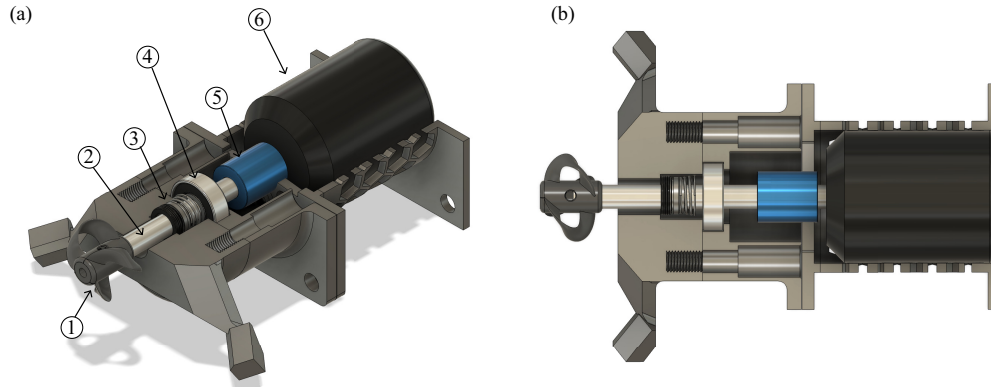


Figure 5.6: The entirety of the arm assembly from an (a) isometric view, including the: (1) impeller, (2) shaft, (3) mechanical seal, (4) ball bearing, (5) rigid coupler, and (6) DC motor, and (b) from the top view.

and studied by others [119, 120], has been proven to provide intense mixing. However, other impellers can be used, such as a typical boat impeller or fan design.

5.8 Mounting Experiment

Finally, for our design, the experiment was mounted on an 18 in. x 18 in. aluminum breadboard using aluminum T-slotted rail hardware. The experiment needs to be mounted at a certain height to allow space for the bottom arms to fit. As seen in Fig. 5.7, T-slotted rail hardware was used to make an X-shaped frame to support the weight of the chamber. This offered enough support for the tank to sit stationary in the lab. If the experiment is being tested under extreme conditions, such as vibrations or any x-y-z directional changes, additional supports are added. For example, adding two screw pins to the welded frame that can then be secured to the T-slotted rail frame, or adding additional supports to secure the tank from the top. The design for this can be modified as seen fit for individual experimental setups.

In our experiments, the tank was mounted to T-slotted rails, which were affixed to a square aluminum optical breadboard. This mounting configuration provided sufficient structural support while conforming to the size constraints imposed by the NASA drop tower facility. As stated previously, the primary objective of the experiment was to investigate bubble breakup in turbulence; however, we also examined the influence of microgravity on bubble breakup and dispersion [8].

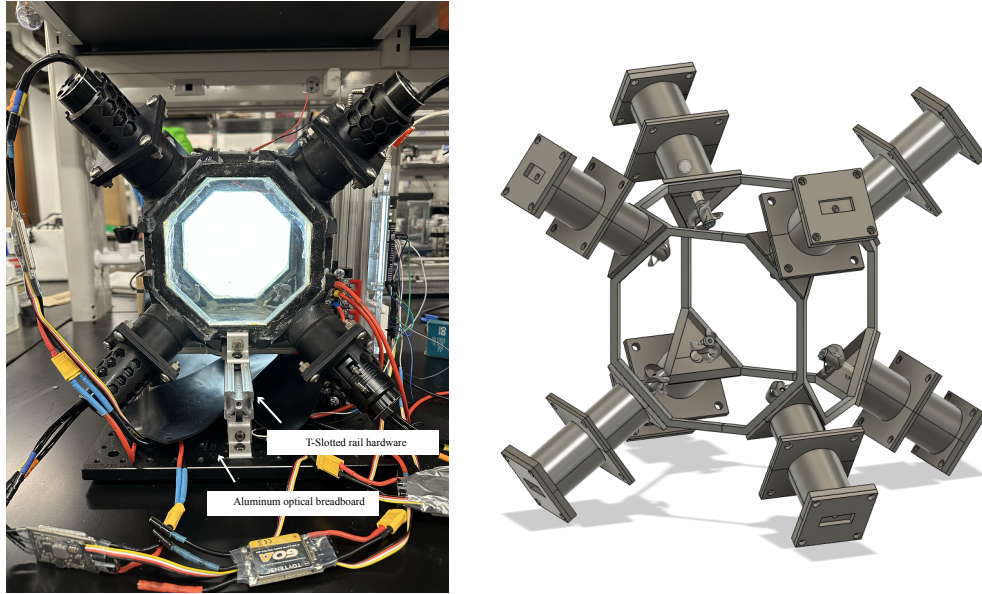


Figure 5.7: (Left) A photo of the HIT chamber in its final, fully assembled state. The experiment is filled with water. (Right) A CAD rendering of fully assembled experiment.

5.9 Operation instructions

The final step for the homogeneous isotropic turbulence chamber is the motor control. The motors must generate sufficient torque to overcome friction from the mechanical seal and shaft coupling while also achieving a rotational speed that is high enough to produce the desired turbulence intensity. For this, the KingVal brushless motors worked for our application. These motors are easily controlled using a microcontroller such as an electronic speed controller (ESC) and an Arduino Mega, see Fig. 5.8 for a schematic. The electronic speed controller, 40A 5V brushless 2-4s Lipo, acts as the driver interface between the Arduino Mega and the motor, and also controls the direction. Based on which order the pins from the ESC are plugged into each motor, the direction of rotation will change. Simply switching the order of two of the pins will switch the direction.

An Arduino Mega is used because it has sufficient PWM (Pulse Width Modulation) pins, pin 2 through 13, to run eight motors. To regulate the rotational speed of each brushless motor, an Arduino sends a PWM signal to the ESC. The ESC interprets the PWM signal and adjusts the motor speed accordingly, with each PWM value corresponding to a specific motor RPM (rotations per minute). This approach enables precise and repeatable control of the impeller rotation rate during experiments. A simple code, written in Arduino can be found on the Zenodo repository, which is used to input an RPM value to run all motors

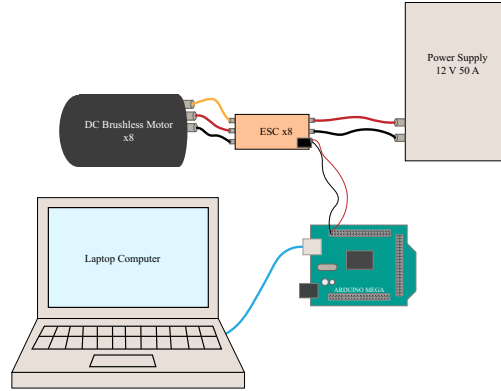


Figure 5.8: A simple schematic to show the control system, including a laptop computer, an Arduino Mega, eight ESC controllers, eight DC motors, and a 12 V power supply.

simultaneously or individually, or adjust a timing scheme for run time intervals. Motor calibration should be performed prior to experimentation. Because these motors are relatively low-cost, variations in rotational speed may occur even when identical input signals are supplied. To address this, a series of calibration sweeps was carried out, during which the output speed of each motor was optically measured and adjusted to ensure consistent performance across all units. In addition, although not implemented in the current design, encoders can be used to create an active feedback loop for precise RPM control. Each ESC is connected to a terminal block, allowing all units to be powered simultaneously. A 12 V DC power supply capable of delivering 50 amps is used to drive all eight motors. Sufficient current capacity is essential to ensure stable motor operation. While lithium-ion polymer batteries can also be used, a power supply is preferable for long-duration experiments, as batteries tend to drain quickly, leading to fluctuations in motor rotational speed.

5.10 Validation and characterization

In order to characterize the turbulence intensity and uniformity within the turbulence chamber, we conducted Particle Image Velocimetry (PIV) measurements. PIV is an optical measurement technique in which the fluid is seeded with tracer particles. By cross-correlating successive images, PIV determines the average particle displacement within each interrogation window, providing an estimate of the instantaneous velocity field. From this velocity field, quantities such as vorticity and basic turbulence statistics can be calculated.

We conducted a sweep of 2D PIV experiments in which we tested each experiment, at a set Reynolds number, 3 times. The field of view for our measurements was $75 \text{ mm} \times 75 \text{ mm}$ (1024×1024 pixels), resulting in a spatial resolution of 0.675 mm between vectors. Each snapshot consisted of 111×111 vectors. We varied the rotational rate of the motors to induce a range of turbulence intensities, $260 < Re_\lambda < 860$, a typical range observed for highly turbulent fields [121]. The turbulent Reynolds number, based on the Taylor microscale, λ , the root-mean-square of the velocity fluctuations (turbulent kinetic energy), u' , and the kinematic viscosity of the fluid, ν , is defined as $Re_\lambda = u'\lambda/\nu$ [121]. The Taylor microscale is defined as $\lambda = \sqrt{15\nu u'/\epsilon}$, where ϵ is the turbulent kinetic energy dissipation rate calculated from the typical way using experimental PIV data [121]. The bulk fluid that was used for testing was pure water.

For the experiments, the turbulence chamber was completely filled with water to eliminate any bubbles that could interfere with the post-processing. Additionally, degassing the bulk fluid is considered good practice and is recommended during experiments. A green laser beam (532 nm) was used to illuminate the $55 \text{ }\mu\text{m}$ diameter tracer particles in the bulk fluid. High-speed images were taken at 5000 fps using a Photron FASTCAM SA5. The recorded images were then analyzed using PIVLab in Matlab [122]. The mean and fluctuating velocity components were calculated considering 1000 frames for each case recorded at $5000 \text{ frames per second}$.

Statistical turbulence metrics were extracted from the PIV data to evaluate the flow characteristics within the tank. In particular, we examined the isotropic ratio, defined as u_{rms}/v_{rms} , which provides a measure of flow isotropy by comparing the root-mean-square fluctuations of the horizontal and vertical velocity components. In our experiments, the center of the domain corresponds to $u_{rms}/v_{rms} \approx 1$, suggesting near-isotropic behavior. Fig. 5.10 shows an ideal case of the color map of the isotropic ratio, whereas 5.9 show more tested cases for a range of Reynolds numbers. The average isotropic ratio for various areas; 8 cm^2 , 4 cm^2 , and 2.5 cm^2 , is shown in the corresponding bounding box. The regions of higher fluctuations, when $u_{rms}/v_{rms} \neq 1$, are seen closer to the boundaries where near wall and impeller effects are having larger impact.

Furthermore, we analyzed the distribution of horizontal and vertical velocity fluctuations by evaluating their fit to a Gaussian profile. Fig. 5.11 presents the probability distribution functions (PDFs) for an intermediate Reynolds number, $Re_\lambda = 517$. The PDF represents the likelihood of a given velocity fluctuation relative to the mean, providing insight into the statistical nature of the flow field. In the case of homogeneous

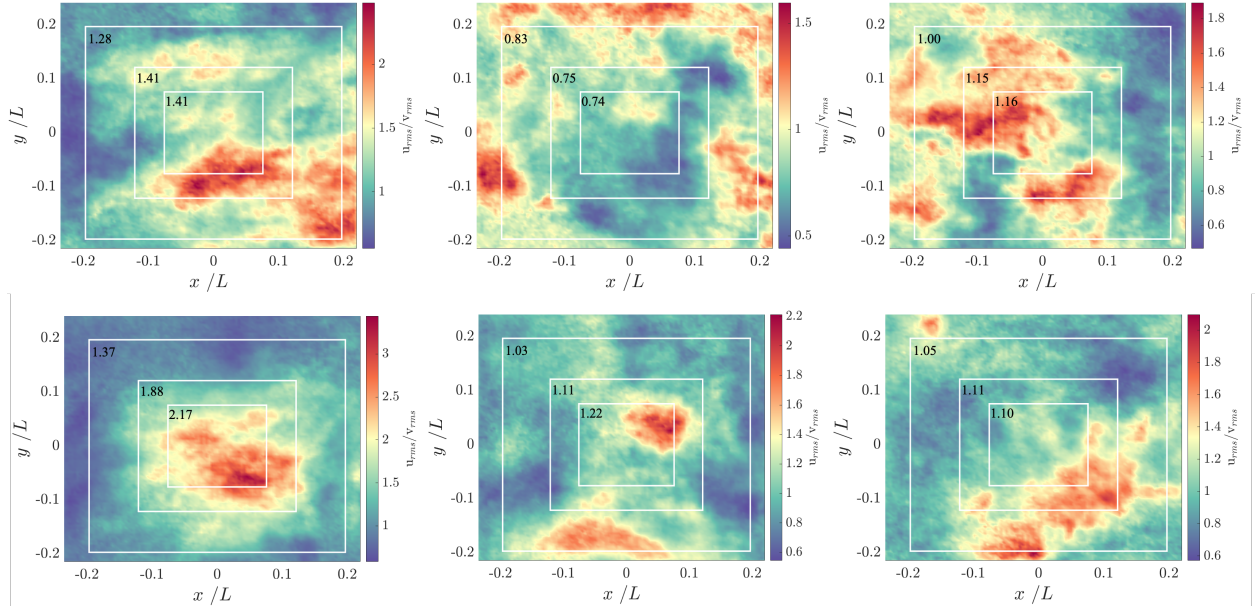


Figure 5.9: Additional colormaps of the isotropic ratio in the horizontal and vertical directions normalized by the length of the tank. Bounding boxes show the average isotropic ratio in that region. Reynolds ranging from $220 < Re_\lambda < 820$, increasing from left to right.

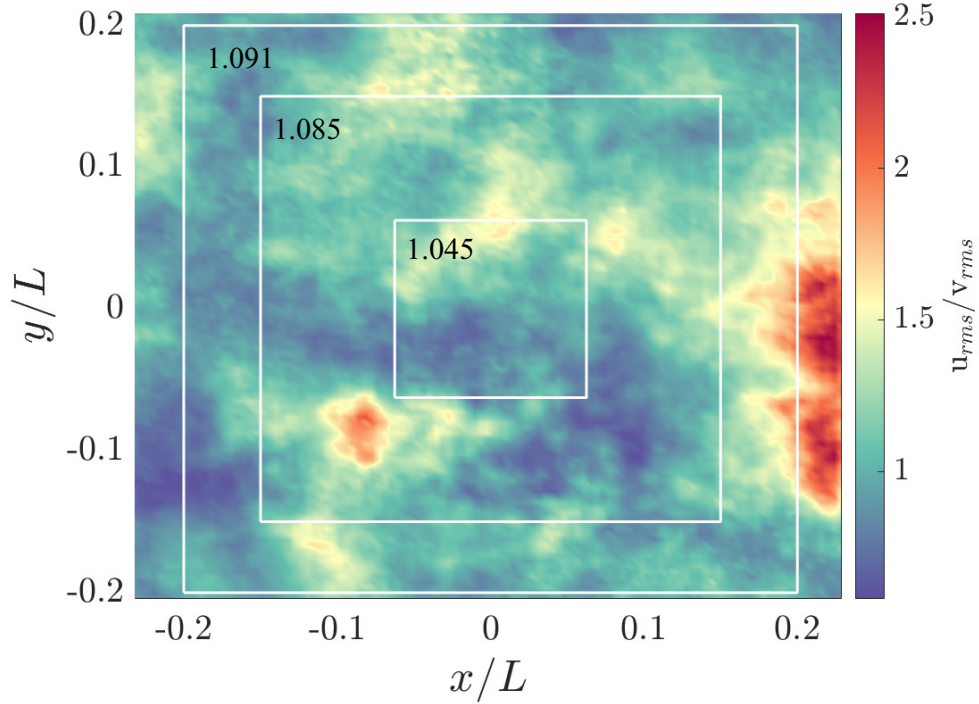


Figure 5.10: A colormap of the isotropic ratio in the horizontal and vertical directions normalized by the length of the tank. Bounding boxes show the average isotropic ratio in that region.

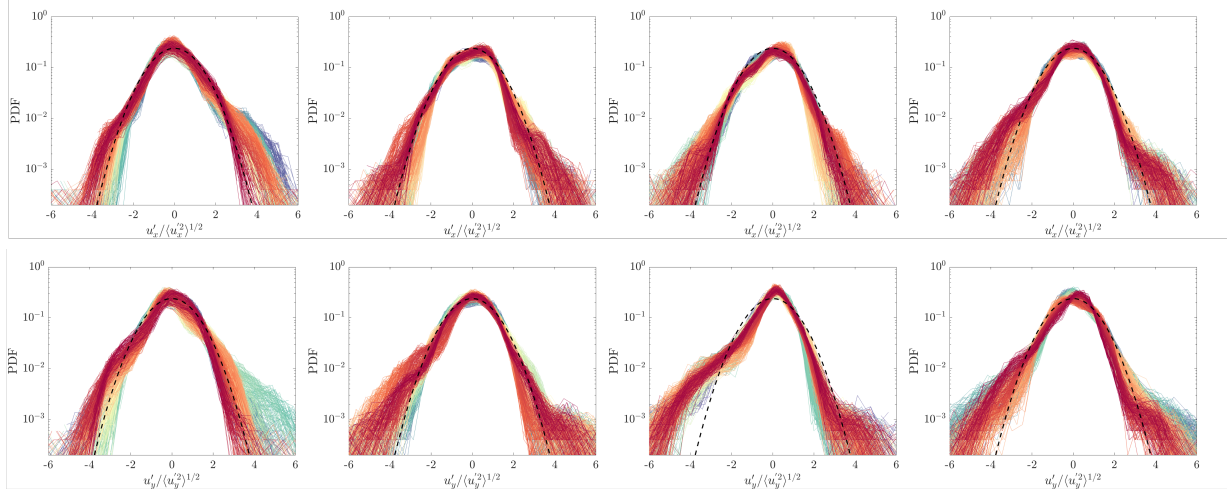


Figure 5.11: PDFs of the (top) horizontal and (bottom) vertical fluid velocity fluctuations in the HIT chamber at an Reynolds ranging from $220 < Re_\lambda < 820$, increasing from left to right . The dashed black line represents the Gaussian profile. Each line corresponds to each time instance throughout the measurement (2 ms).

isotropic turbulence, the velocity fluctuations in both directions are expected to follow a Gaussian distribution, with the highest probability centered around 0, where the velocity fluctuations in either direction are equivalent to the mean i.e., $u' / \langle u'^2 \rangle^{1/2}$ [123]. Across the range of Reynolds numbers considered, the experimental data demonstrated good agreement with the Gaussian model, consistent with the characteristics of isotropic turbulence [123].

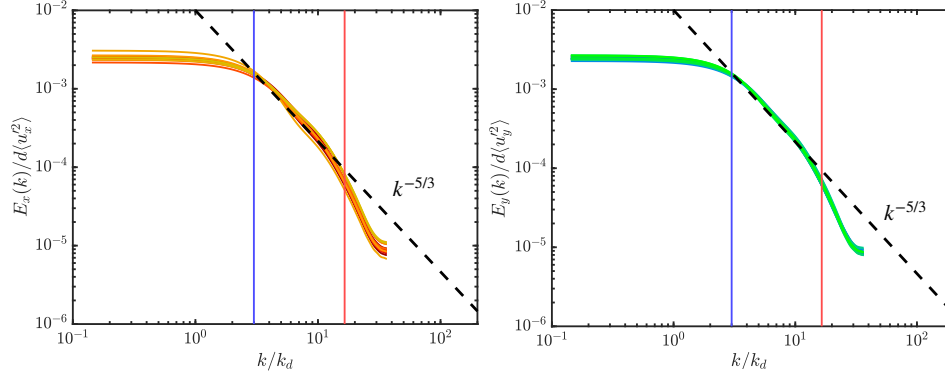


Figure 5.12: The (a) horizontal and (b) vertical spectra of liquid velocity fluctuations for various Reynolds numbers, $220 < Re_\lambda < 820$. The x-axis represents the wave number, k , normalized by the wavenumber corresponding to the propeller diameter, k_d , and the y-axis is the turbulent kinetic energy, $E(k)$ normalized by the variance $d\langle u'^2 \rangle$. The blue and red lines are the largest and smallest scales for the inertial subrange.

Finally, we evaluated the energy spectra of the turbulence in our tank. Fig. 5.12 shows the kinetic energy normalized by the variance, versus the wave number, normalized by the impeller diameter, k_d . Using a Fast Fourier Transform, the experimental results show classical Kolmogorov's power-law decay, $E(k) \sim k^{-5/3}$, the typical scaling for HIT in Newtonian fluids [121, 123], is observed for the inertial subrange, highlighted between the blue and red lines. The data was compared to the John Hopkins turbulence database and showed very good agreement for the scales of interest in this experiment [124]. Over the range of Reynolds numbers tested, this scaling is met in both the horizontal and vertical directions. Based on these measurements, we can conclude that the flow produced by the proposed device is homogeneous isotropic turbulence.

5.11 Conclusions

We have designed, fabricated, and tested a homogeneous isotropic turbulence (HIT) chamber. While the concept of HIT chambers is not new, our work presents a novel approach by creating an open-source, affordable, and accessible design. The chamber can be built entirely in-house by universities with limited resources using simple machining, 3D printing, and low-cost components, with a total cost of approximately 900 USD.

Experimental testing confirmed that the apparatus is capable of generating turbulence with a high degree of homogeneity and isotropy. Minor deviations from ideal HIT were observed, primarily due to

slight motor speed variations. These could be mitigated by integrating encoders to provide real-time RPM measurements and enable a feedback control loop, ensuring all motors operate at identical speeds.

Compared to existing HIT chamber designs, this work offers a practical and cost-effective alternative that enables a broader range of researchers to study phenomena influenced by HIT.

6 SPEED OF SOUND IN BUBBLY LIQUIDS

Abstract The speed of sound in bubbly media, or bubble emulsions, decreases rapidly with increasing gas volume fraction. Existing theoretical approaches have proved that even with small gas volume fractions the speed of sound is much less than that in pure water or air; however, experimental validation remains limited due to the difficulty of maintaining stable bubble suspensions under terrestrial gravity. Experiments were conducted in microgravity to generate stable and homogeneous bubble emulsions. Bubble emulsions were produced through controlled mechanical agitation to promote bubble breakup and regulate bubble size distributions. Acoustic waves were propagated through the mixture and recorded using two hydrophones. The recorded signals were analyzed to determine the speed of sound across a range of gas volume fractions and bubble size distributions. Both experimental measurements and theoretical predictions demonstrate a decay in sound speed with increasing gas volume fraction.

Information This work is currently under development and has not yet been submitted for publication.

6.1 Introduction

It is well established that the speed of sound in a bubbly liquid is significantly lower than that in either of its pure components; water or air. The foundational work by Wood in the 1930s demonstrated that the sound speed in such mixtures is primarily governed by the gas volume fraction, α [43], as seen in Eq. 6.1

$$\frac{1}{c_m^2} = \left[(1 - \alpha)\rho_l + \alpha\rho_g \right] \left[(1 - \alpha)\kappa_l + \alpha\kappa_g \right] \quad (6.1)$$

where c_m is the speed of sound in the medium, α is the gas volume fraction, defined as the ratio between the volume of gas, V_g to the volume of the mixture, V_m .

$$\alpha = V_g/V_m \quad (6.2)$$

The volume of the mixture is defined as $V_m = V_g + V_l$, where V_l is the liquid volume and V_g is the gas volume. The liquid and gas densities are ρ_l and ρ_g respectively, and κ_l and κ_g are the compressibility of the liquid and gas phase. Eq. 6.1 can be derived from the general equation for the speed of sound, c , with the modification due to the effective density and compressibility of the mixture.

$$c^2 = \frac{1}{\rho\kappa} \quad (6.3)$$

To expand the density to that of the mixture an effective density can be defined by the density of the gas and liquid. The effective density, or mixture density, ρ_m , is

$$\rho_m = (1 - \alpha)\rho_l + \alpha\rho_g \quad (6.4)$$

Additionally, the compressibility of the mixture has a large contribution to the speed of sound. The compressibility of the mixture, κ_m , is defined as

$$\kappa_m = (1 - \alpha)\kappa_l + \alpha\kappa_g \quad (6.5)$$

where κ_l and κ_g are the compressibilities of the liquid and gas phases respectively. Finally, Eq. 6.1 can be found by substituting Eq. 6.5 and Eq. 6.4 into Eq. 6.3.

As shown by Wilson and Roy in Fig. 6.1, even a small gas volume fraction, on the order of $\alpha = 0.5\%$, can reduce the speed of sound in water by a factor of 100 [15]. Similarly, McWilliam et al. quantified the speed of sound in air-water mixtures and found trends that are in good agreement with Woods equation [125]. However, as noted in this work, minimal experimental data was used for validation. To our understanding, majority of the work that has been done to quantify the speed of sound in bubbly liquids was conducted over a decade ago and/or is theoretical [126–129].

When a sound wave propagates through a bubble curtain, it is partially reflected, scattered, and absorbed due to the compressibility of the bubbles. This significant reduction in acoustic transmission has motivated a variety of underwater applications, particularly in studies examining both reductions in the speed of sound and acoustic attenuation in bubbly media. Much of the research on sound attenuation has focused on mitigating underwater noise pollution generated during marine construction activities, with the goal of

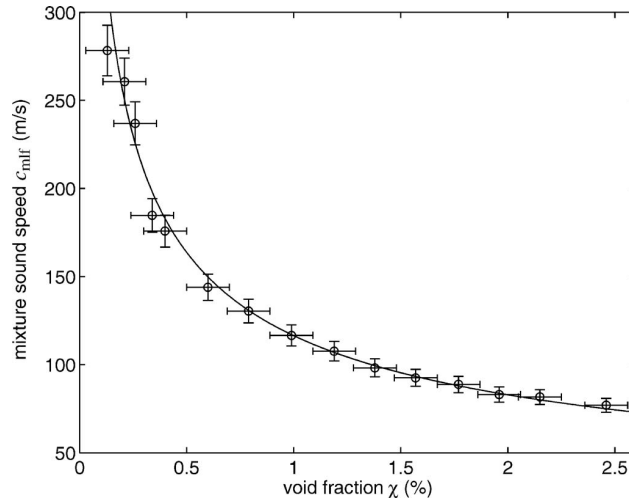


Figure 6.1: Speed of sound measurements using Woods equation and experimental values from Wilson and Roy [15]

protecting marine life. Many fish species communicate using specific frequency ranges, and reducing noise within these frequencies during underwater operations could help minimize disturbances to these species.

Research on this phenomenon is extensive yet open questions still remain. Numerous studies have investigated sound propagation and absorption in bubbly media in different contexts. For instance, bubble curtains, streams of bubbles released into the water to form an acoustic barrier, are used to dampen underwater noise during industrial processes such as pile driving for offshore wind turbines, thereby protecting marine life [129–131]. Specifically, Tsouvalas et al. conducted a parametric study that demonstrated that the effectiveness of noise reduction depends strongly on the diameter of the piles used during offshore construction. Specifically, their results showed that for large-diameter piles, the dominant mechanism for noise reduction is the acoustic impedance mismatch between seawater and the bubble curtain, which leads to increased reflection of the sound waves. In contrast, for smaller piles, the resonant frequency of the bubbles plays a more significant role in the attenuation process. In this case, the bubble curtain primarily governs the acoustic absorption of the sound waves [129].

Similarly, certain whale species generate bubble nets around schools of fish, disorienting their prey and facilitating capture [132]. More recently, there has been growing interest in the potential use of bubbles as acoustic dampeners in underwater defense systems, offering a means of acoustic stealth [133]. The study of sound propagation and attenuation in bubble liquids has broad implications: environmentally, by improving acoustic protection for marine ecosystems [134]; technologically and commercially, by enhancing

underwater manufacturing and offshore energy operations [131]; and socially, by contributing to national defense applications where acoustic control is critical.

Despite these advances, the influence of bubble diameter, flow structure and behavior, and the properties of the surrounding fluid have received surprisingly little attention. Additionally, many of these studies rely on analytical and theoretical models with little experimental validation data. It is commonly stated that when the resonant frequency of a bubble differs from the frequency of the generated sound, the bubble's diameter can be considered negligible [15, 125, 127]. However, when the bubble's resonant frequency closely matches the frequency of the sound wave, the behavior becomes much more complex. Similarly, the flow dynamics within bubble curtains or nets, including bubble wake interactions, which themselves depend on the bulk fluid properties, are typically overlooked when studying sound attenuation in these systems.

The present study examines the speed of sound in a bubble emulsion, a unique two-phase flow consisting of a small gas volume fraction of air dispersed as bubbles in a water-like fluid. Experiments were conducted in microgravity, creating a stable bubble emulsion, to further quantify the speed of sound in bubbly liquids. Over a variety of tests, the distribution and size of bubbles were changed to measure the change in the speed of sound. The current results of this study show a change in the speed of sound however it is not developed enough to fully quantify the true effect of size and distribution.

6.2 Experimental Arrangement

A homogeneous isotropic turbulence (HIT) chamber was used to create a bubble emulsion in microgravity. The details of the HIT chamber can be found in our previous work seen in Chapter 5[116]. In the case of these experiments, the HIT chamber was used to introduce bubbles into the bulk fluid. The truncated cube shaped tank, as seen in Fig. 6.2, roughly 5.5 L, was filled with a majority of water and a small gas volume fraction. The gas volume fractions tested ranged from 0.1% to 10%. The gas volume fraction was controlled by removing a set amount of fluid from the tank before completely sealing the tank. Once the tank was set to the desired air water ratio, a set of eight impellers, located on the corner of the truncated cube, were used to induce agitation to the fluid and introduced bubbles into the flow, see Fig. 6.3. The intensity of agitation was changed by inputting a range of rotational rates to the impellers. The rotational rates correspond to

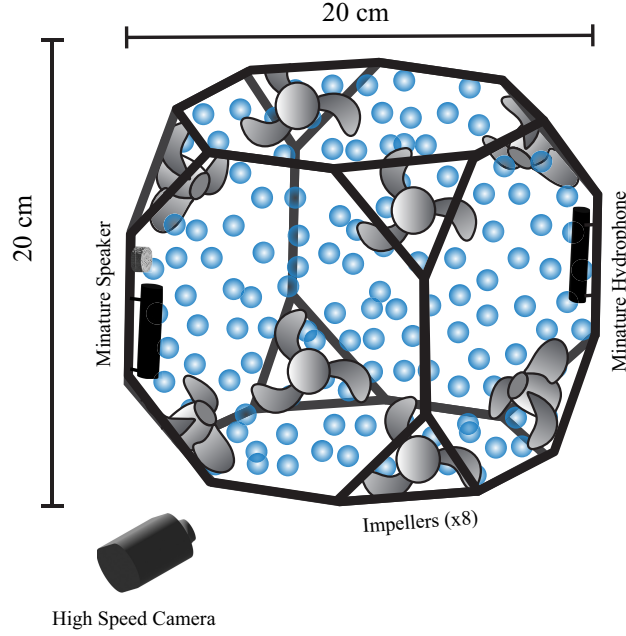


Figure 6.2: A schematic of the experimental setup including bubble emulsion, 8 impellers, high speed camera, and hydrophone speaker arrangement.

turbulent Reynolds numbers ranging from $260 < Re_\lambda < 860$. These intensities lead to a range of bubble diameters, however the mean bubble diameter typically is between 0.4 to 0.7 mm, resulting in a natural bubble frequency of approximately 6kHz. The natural frequency is defined as [135];

$$f_0 = \frac{1}{2\pi R} \sqrt{\frac{3\gamma P}{\rho}} \quad (6.6)$$

where R is the bubble radius, γ is the polytropic coefficient which is 1.4 for air, P is the ambient pressure, and ρ is the density of water.

The hydrophone-speaker arrangement was used to measure the speed of sound in a bubbly liquid. A waterproofed speaker was placed at one end of the tank. The speaker created an audible noise that swept over range of frequencies from approximately 1700 to 2100 Hz. An Aquarian Hydrophones AS-1 hydrophone was used to record these frequencies and was placed near the speaker. A second hydrophone was placed on the opposite side of the tank. The audio data collected at each hydrophone was then compared to one another. Fig. 6.4 shows a typical recorded signal.

These experiments were conducted at NASA Glenn drop tower and aboard a parabolic flight campaign, see Chapter 4 for details. The drop tower, which is an 8 story high shaft allowed for 2.2 seconds of

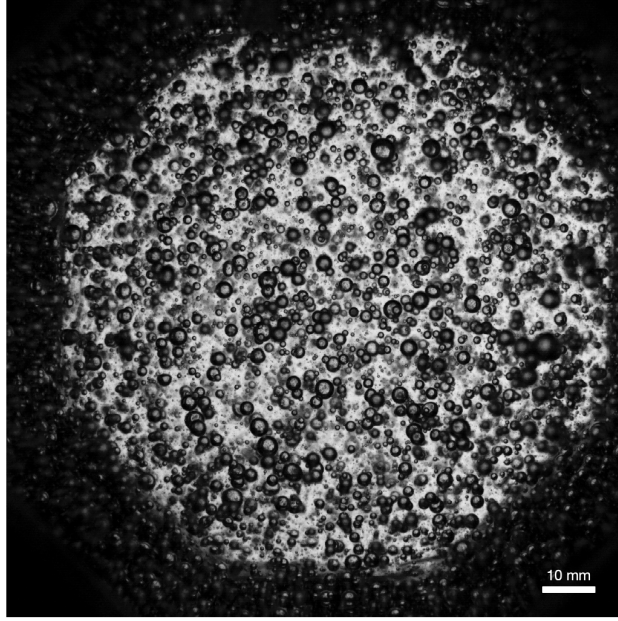


Figure 6.3: A snapshot image at one time instance that shows bubble breakup and dispersion due to mechanical agitation, $\alpha = 0.2\%$, $Re \approx 425$. Bubble sizes range from 0.5 to 4 mm, majority being approximately 1 mm.

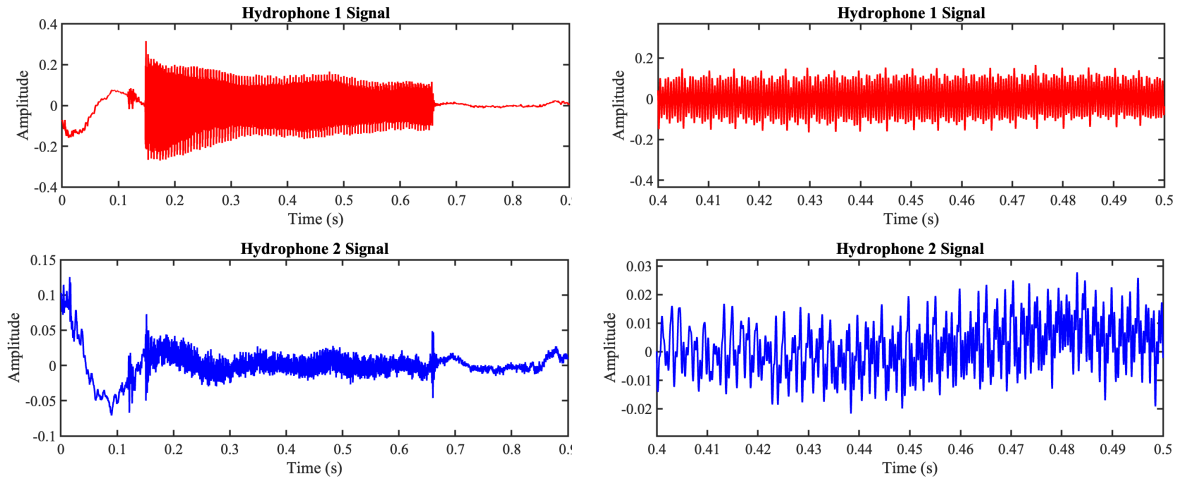


Figure 6.4: (left) A preview of the hydrophone signals from hydrophone 1, red, and hydrophone 2, blue. Hydrophone 1 signal is very clear and located near the underwater speaker, whereas hydrophone 2 is located roughly 20 cm from the speaker and contains noise from signal interaction with bubbles. (right) A similar preview of hydrophone signals zoomed in from 0.4 to 0.5 seconds.

microgravity to be achieved as the experiment fell. The parabolic flights allowed for roughly 20 seconds of microgravity; however, majority of the experimental results will focus on the drop tower as a wider range

of gas volume fractions was achievable under these conditions. The microgravity environment provides a unique environment in which the bubbles, after dispersed in the bulk liquid, will not rise and remain dispersed in the liquid, creating a stable bubble emulsion. This unique environment therefore provided access to longer duration experimentation of the speed of sound in a bubbly liquid. We hypothesize that this environment leads to a better way to accurately determine the speed of sound.

A set of 15 drop tower experiments were conducted. Each experiment consisted of testing a different gas volume fraction a minimum of 3 times. The experiment began with roughly 3 seconds of mixing prior to drop, and 0.25 seconds of mixing into the drop. Once the mixing was complete, the hydrophones and a high speed camera would collect audio and visual data of the sound wave and dispersion of the bubbles receptively. Using the high speed images and the hydrophone data in parallel provided an understanding of the dispersion of the bubbles on the speed of sound.

6.3 Data Processing

The results presented in this section are preliminary and aim to establish a methodology for estimating the speed of sound in a bubble emulsion. First, to evaluate the experimental measurements, the estimated speed of sound was compared with classical theoretical predictions for acoustic propagation in two-phase flows. One of the earliest and most widely used models was proposed by Wood (1930) [43], which relates the speed of sound in a mixture to its effective compressibility, Eq. 6.1.

Using this formulation, we can achieve an estimate for the speed of sound based on a range of gas volume fractions tested during experiments, as seen in Fig. 6.5. The major limitation to this theoretical approach, and the purpose of these experiments, was to understand how the size and distribution of the bubble emulsion, factors that are not accounted for in Eq. 6.1, effect the speed of sound.

To evaluate the speed of sound directly from the experimental data the cross correlation between the pair of hydrophone signals was computed. The cross correlation function is defined as

$$CCF(\tau) = \int_{-\infty}^{+\infty} f(t)g(t - \tau)dt. \quad (6.7)$$

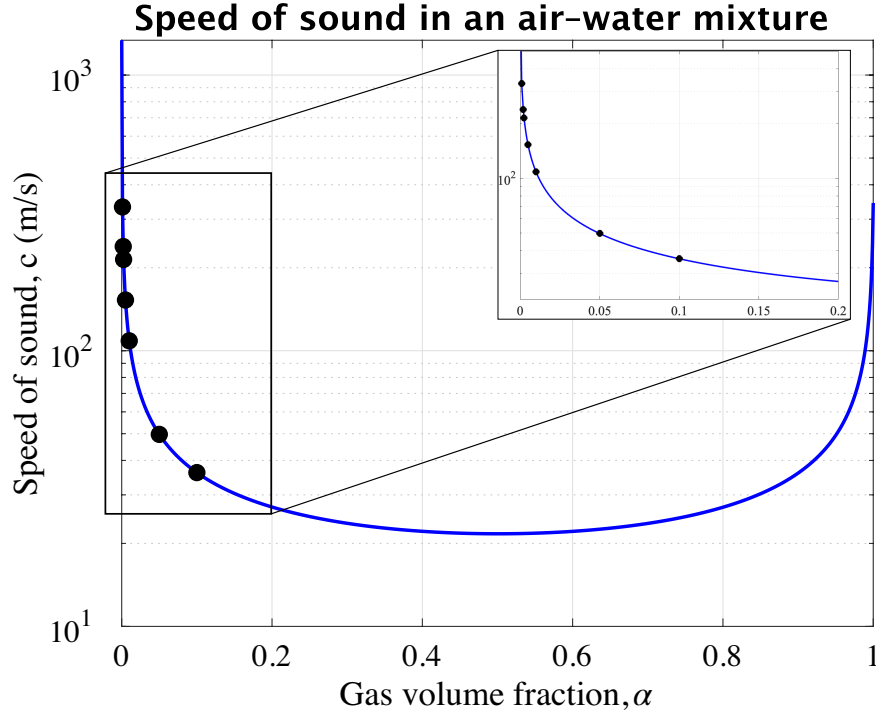


Figure 6.5: A plot of the gas volume fraction, α , versus the speed of sound, c . Using Eq. 6.1, an estimated speed of sound based on the fluid properties can be estimated. The black points represent the estimated experimental values, with gas volume fractions ranging from 0.1% to 10%. The inset figure shows the zoomed in portion of these results to highlight the steep decay in the speed of sound at small gas volume fractions.

where f and g are the recorded audio signals, and τ is the lag between the two signals. The cross correlation function reaches a maximum at the lag $\tau_{optimal}$ corresponding to the time delay between the two signals when they are similar. The speed of sound can then be calculated:

$$C = L/\tau_{optimal} \quad (6.8)$$

where L is the known distance between the two hydrophones.

Although this expression appears straightforward, defining the lag time is nontrivial. Both hydrophones recorded acoustic signals; however, hydrophone 2, located a distance $L \approx 20$ cm from the speaker, exhibited substantially higher noise levels due to scattering and attenuation caused by bubbles, as seen in Fig. 6.4.

To improve the measurement, the signals were filtered and amplified. The aim is to isolate the frequencies in the range of 1700 to 2100 Hz. Each experiment was divided into sound 'bursts', where each burst was the input acoustic sweep lasting for approximately 1 second. Isolating individual bursts allowed the temporal

and spectral characteristics of the signal to be examined independently. The experiments conducted in the drop tower had only 1 burst in microgravity, as seen in Fig. 6.4, whereas in the parabolic flight experiments 4 to 5 bursts were recorded as seen in Fig. 6.6(a).

Fig. 6.6 (a) shows the second isolated burst for this set experiment. During this interval, microgravity conditions were achieved, with an average gravitational acceleration of approximately 0.087 m/s^2 . Due to small variation in gravitational acceleration, also known as g-jitter, the average acceleration and photographic observations were used to characterize the bubble distribution during the burst.

Both hydrophone signals were band-pass filtered to remove any noise above 7500 Hz and below than 1000 Hz. The signal recorded from hydrophone 2 was amplified as this signal was substantially damped due to the fluid. Fig. 6.6 (b) illustrates this filtration and amplification.

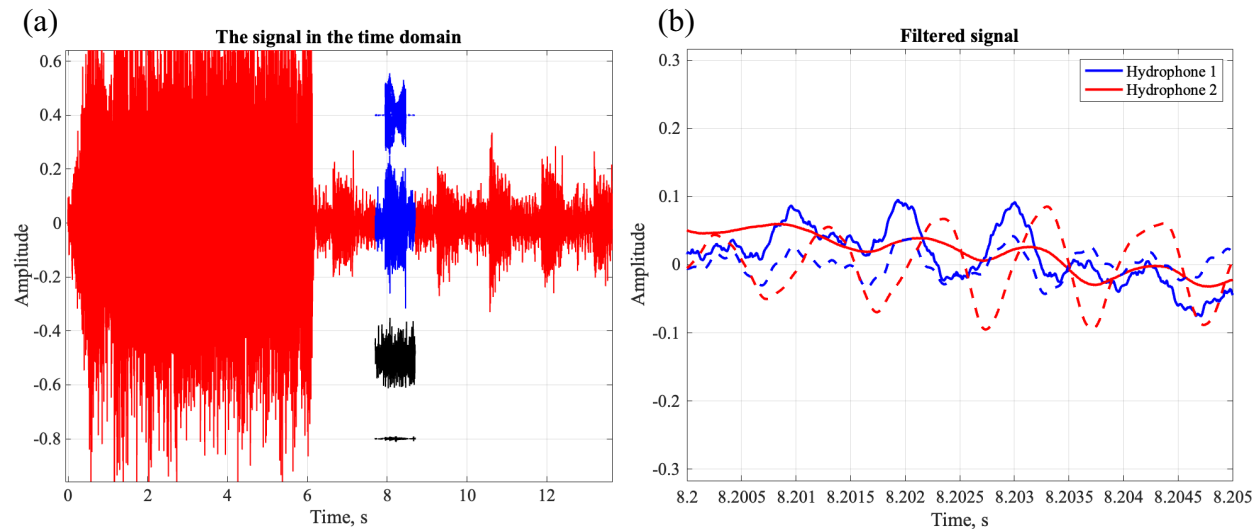


Figure 6.6: (a) Signal recorded from hydrophones where the red is the entire signal captured including the mixing of the fluids, shown by the large amplitude noise. The blue is the designated burst of interest recorded by hydrophone 1 and filtered as shown by the above blue signal. The black is the designated burst of interest recorded by hydrophone 2, the signal below is the original signal filtered and amplified. (b) A zoomed-in representation of the burst of interest showing the first hydrophone in blue and the second in red with their respective filtered and amplified versions in the dashed lines.

A spectral analysis was conducted to verify the dominant frequency content of the bursts and identify additional frequency components generated during the experiment. As shown in Fig. 6.7, the dominant peaks appear within the targeted frequency range from 1700 and 2100 Hz for both hydrophones. Additional peaks can be seen between 4000 to 6000 Hz. These higher-frequency components are likely associated with acoustic reflections within the tank and scattering caused by bubble interactions. Due to noise contributions

from both the experimental environment and reflections within the tank, isolating the direct acoustic signal proved challenging. For this reason, room acoustic processing methods were considered.

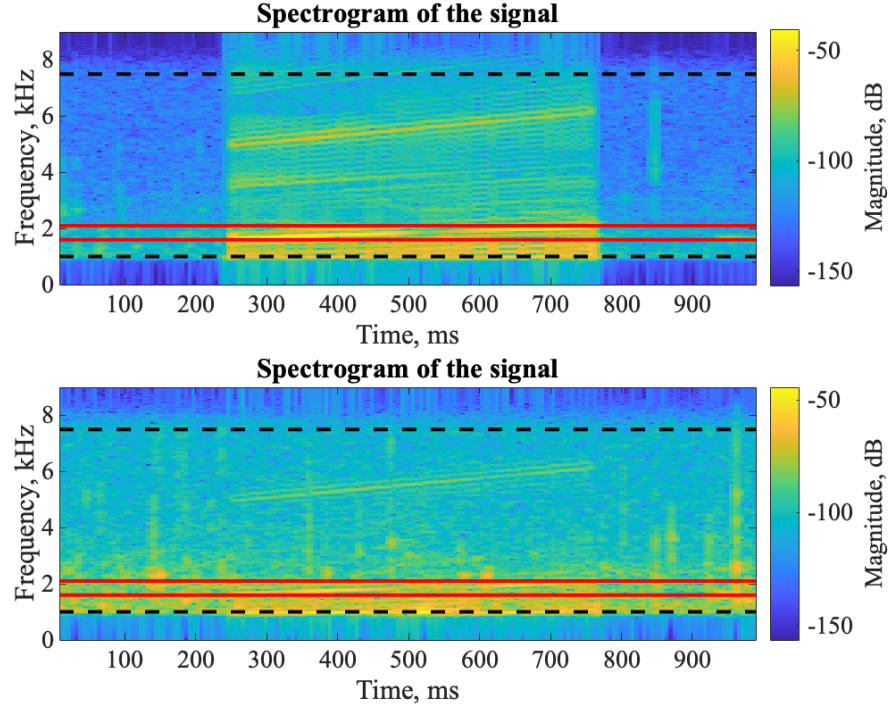


Figure 6.7: A spectrogram of hydrophone 1, top, and hydrophone 2, bottom, signals. The red lines show the targeted frequencies and the black dotted lines are the cut off bounds.

6.4 Results

Following the signal processing described above, the cross correlation between the two hydrophone signals was computed. An example of the cross-correlation result is shown in Fig. 6.8. A cross-correlation coefficient of 0.26 was obtained, corresponding to a delay time of approximately 7.86×10^{-4} s and an estimated speed of sound of approximately 266 m/s. This result indicates a moderate linear relationship between the two signals. In this particular experiment, the second correlation peak coincides with the maximum peak, providing additional confidence in the measurement.

The first set of experiments were conducted under Earth's gravity in a bubble column. Although the experimental configuration varied slightly, see details in the work done by Ravisankar and Zenit [136], the hydrophones were arranged in a similar way, with one located near the speaker and the second placed

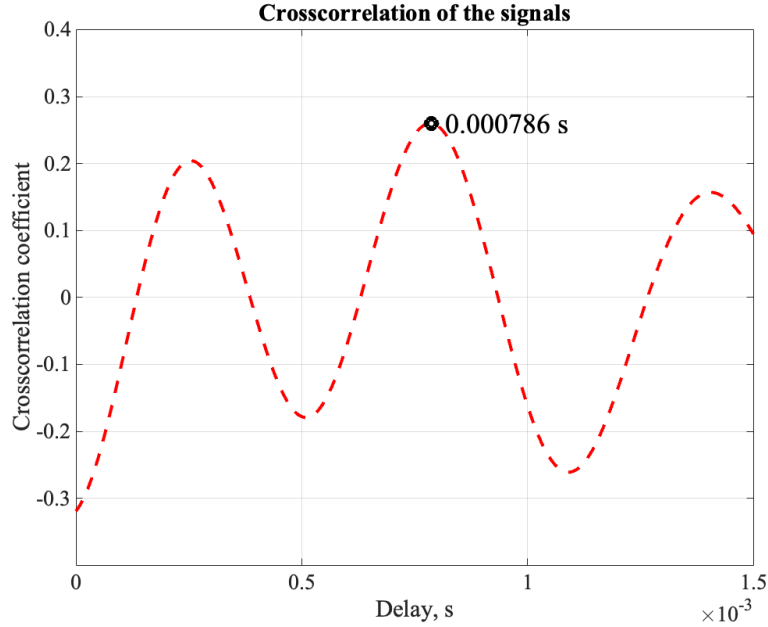


Figure 6.8: Cross correlation plot between hydrophone 1 and hydrophone 2 for the sound signal sweeping from 1700 to 2100 Hz, wavelength between 0.87 m and 0.70 m. The peaks shown are the maximum alignment between the two signals, with the overall maximum being the second peak at 0.000786 seconds.

at the opposite end of the tank. These experiments aimed to serve as a baseline for comparison with the microgravity measurements.

The results of the Earth gravity experiments are summarized in Table 6.1. Each experiment consisted of multiple acoustic bursts, and the delay times corresponding to the first three peaks as well as the maximum cross-correlation coefficient were used to compute the corresponding speeds of sound. The expected speed of sound based on the 1% and 2% gas volume fraction tests were 100.8 and 71.7 m/s respectively. The calculated speeds of sound vary significantly from the expected value predicted by Eq. 6.1. This discrepancy could be due to bubbles interacting with the hydrophones as they rise, causing misreadings in the hydrophone data, or due to acoustic scattering from the tank itself.

Following the baseline Earth experiments, the same signal processing methodology was applied to the microgravity experiments conducted in both the drop tower and parabolic flight campaigns. Despite the agreement with the expected and measured speed of sound, the estimated sound of speed should be interpreted as an approximation. In many of the experiments, the cross-correlation coefficient at the maximum peak did not yield physically reasonable values for the speed of sound. For this reason, multiple peaks within each burst were examined rather than relying solely on the global maximum. As shown in

Trial - Burst	α	Delay 1 [s]	Delay 2 [s]	Delay 3 [s]	Delay (Max) [s]	SOS 1 [m/s]	SOS 2 [m/s]	SOS 3 [m/s]	SOS (MaxCoeff) [m/s]
1-1	1.00	4.32×10^{-4}	8.28×10^{-4}	1.44×10^{-3}	2.11×10^{-2}	1758.07	917.74	528.70	36.02
1-2	1.00	3.85×10^{-4}	1.03×10^{-3}	1.93×10^{-3}	7.08×10^{-3}	1971.89	740.71	394.38	107.29
1-3	1.00	4.69×10^{-5}	6.93×10^{-4}	1.25×10^{-3}	4.69×10^{-5}	16213.33	1097.14	608.00	16213.33
1-4	1.00	2.60×10^{-5}	4.69×10^{-4}	1.01×10^{-3}	2.35×10^{-2}	29184.00	1621.33	752.16	32.36
1-5	1.00	8.85×10^{-5}	6.98×10^{-4}	1.32×10^{-3}	6.98×10^{-4}	8583.53	1088.96	576.76	1088.96
2-1	1.00	4.38×10^{-4}	9.38×10^{-4}	1.46×10^{-3}	3.52×10^{-3}	1737.14	810.67	521.14	215.86
2-2	1.00	1.25×10^{-4}	7.08×10^{-4}	1.29×10^{-3}	4.74×10^{-1}	6080.00	1072.94	588.39	1.60
2-3	1.00	3.54×10^{-4}	9.79×10^{-4}	1.58×10^{-3}	1.58×10^{-3}	2145.88	776.17	480.00	480.00
2-4	1.00	2.92×10^{-4}	8.75×10^{-4}	1.50×10^{-3}	2.92×10^{-4}	2605.71	868.57	506.67	2605.71
2-5	1.00	4.17×10^{-4}	9.79×10^{-4}	1.56×10^{-3}	1.27×10^{-2}	1824.00	776.17	486.40	59.90
3-1	1.00	1.30×10^{-4}	8.18×10^{-4}	9.27×10^{-4}	6.81×10^{-3}	3379.20	538.09	474.61	64.64
3-2	1.00	8.33×10^{-5}	6.04×10^{-4}	1.17×10^{-3}	5.96×10^{-3}	5280.00	728.28	377.14	73.85
3-3	1.00	1.15×10^{-4}	6.77×10^{-4}	1.23×10^{-3}	8.80×10^{-3}	3840.00	649.85	357.97	50.02
3-4	1.00	3.28×10^{-4}	1.08×10^{-3}	1.99×10^{-3}	1.99×10^{-3}	1340.95	406.15	221.15	221.15
3-5	1.00	1.77×10^{-4}	8.49×10^{-4}	1.34×10^{-3}	3.71×10^{-3}	2484.71	518.28	328.72	118.65
4-1	1.00	3.13×10^{-4}	9.58×10^{-4}	1.54×10^{-3}	5.54×10^{-3}	1408.00	459.13	285.41	79.40
4-2	1.00	2.71×10^{-4}	8.33×10^{-4}	1.40×10^{-3}	2.52×10^{-2}	1624.62	528.00	315.22	17.48
4-3	1.00	1.25×10^{-4}	7.29×10^{-4}	1.31×10^{-3}	1.25×10^{-4}	3520.00	603.43	335.24	3520.00
4-4	1.00	4.38×10^{-4}	1.04×10^{-3}	1.67×10^{-3}	1.67×10^{-3}	1005.71	422.40	264.00	264.00
4-5	1.00	6.25×10^{-4}	1.15×10^{-3}	1.81×10^{-3}	4.88×10^{-3}	704.00	384.00	242.76	90.26
5-1	2.00	4.95×10^{-4}	1.15×10^{-3}	1.67×10^{-3}	2.96×10^{-1}	1536.00	663.27	456.00	2.57
5-2	2.00	1.04×10^{-5}	5.42×10^{-4}	1.16×10^{-3}	3.28×10^{-2}	72960.00	1403.08	654.35	23.16
5-3	2.00	1.04×10^{-4}	6.67×10^{-4}	1.30×10^{-3}	1.01×10^{-2}	7296.00	1140.00	586.02	75.37
5-4	2.00	1.82×10^{-4}	7.19×10^{-4}	1.39×10^{-3}	2.61×10^{-3}	4169.14	1057.39	546.52	290.68
5-5	2.00	4.79×10^{-4}	1.07×10^{-3}	1.67×10^{-3}	8.77×10^{-3}	1586.09	708.35	456.00	86.70
6-1	2.00	4.79×10^{-4}	1.33×10^{-3}	2.27×10^{-3}	4.82×10^{-2}	1586.09	570.00	334.68	15.76
6-2	2.00	6.67×10^{-4}	1.56×10^{-3}	2.27×10^{-3}	2.21×10^{-2}	1140.00	486.40	334.68	34.42
6-3	2.00	5.63×10^{-4}	1.44×10^{-3}	2.33×10^{-3}	1.27×10^{-2}	1351.11	528.70	325.71	60.00
6-4	2.00	4.17×10^{-5}	9.17×10^{-4}	1.75×10^{-3}	2.35×10^{-2}	18240.00	829.09	434.29	32.37
6-5	2.00	6.25×10^{-5}	7.29×10^{-4}	1.27×10^{-3}	5.38×10^{-2}	12160.00	1042.29	598.03	14.12
7-1	2.00	7.29×10^{-4}	1.60×10^{-3}	2.53×10^{-3}	1.99×10^{-2}	603.43	275.18	173.83	22.13
7-2	2.00	2.50×10^{-4}	1.46×10^{-3}	2.45×10^{-3}	8.40×10^{-3}	1760.00	301.71	179.36	52.41
7-3	2.00	4.22×10^{-4}	1.38×10^{-3}	2.23×10^{-3}	1.04×10^{-2}	1042.96	320.00	197.38	42.49
7-4	2.00	2.08×10^{-4}	1.33×10^{-3}	2.36×10^{-3}	6.23×10^{-3}	2112.00	331.29	186.49	70.58
7-5	2.00	4.38×10^{-4}	1.46×10^{-3}	2.34×10^{-3}	1.83×10^{-1}	1005.71	300.64	187.73	2.40
8-1	2.00	3.75×10^{-4}	1.35×10^{-3}	2.31×10^{-3}	5.23×10^{-3}	1173.33	324.92	190.27	84.14
8-2	2.00	3.33×10^{-4}	1.29×10^{-3}	2.29×10^{-3}	4.27×10^{-3}	1320.00	340.65	192.00	103.02
8-3	2.00	3.13×10^{-4}	1.19×10^{-3}	2.23×10^{-3}	5.17×10^{-3}	1408.00	370.53	197.38	85.16
8-4	2.00	8.75×10^{-4}	1.71×10^{-3}	2.46×10^{-3}	4.92×10^{-1}	502.86	257.56	178.98	0.89
8-5	2.00	1.04×10^{-4}	1.04×10^{-3}	2.02×10^{-3}	7.17×10^{-3}	4224.00	422.40	217.73	61.40

Table 6.1: Measured delay times and corresponding computed speeds of sound for individual bursts for two gas volume fractions, $\alpha = 1$ and 2, and in bubble column under Earth's gravity.

Fig. 6.8, the first three peaks, including the maximum peak, were analyzed. These peak locations were used to compute four corresponding estimates of the speed of sound, which were then compared to the theoretical prediction based on the prescribed gas volume fraction using Eq. 6.1. The results are summarized in Table 6.2.

Test	α	Delay 1 [s]	Delay 2 [s]	Delay 3 [s]	Delay (Max) [s]	SOS 1 [m/s]	SOS 2 [m/s]	SOS 3 [m/s]	SOS (Max) [m/s]	Expected [m/s]
1	0.10	5.00×10^{-4}	1.08×10^{-3}	1.60×10^{-3}	5.94×10^{-3}	418.50	193.15	130.44	35.24	311.25
2	0.10	1.67×10^{-4}	6.25×10^{-4}	1.15×10^{-3}	1.15×10^{-3}	1255.50	334.80	182.62	182.62	311.25
3	0.25	1.67×10^{-4}	5.42×10^{-4}	9.17×10^{-4}	2.92×10^{-2}	1255.50	386.31	228.27	7.16	198.53
4	0.25	4.79×10^{-4}	1.08×10^{-3}	1.67×10^{-3}	3.38×10^{-3}	436.70	193.15	125.55	62.00	198.53
5	0.25	4.58×10^{-4}	1.00×10^{-3}	1.50×10^{-3}	2.60×10^{-3}	456.55	209.25	139.50	80.35	198.53
6	0.50	3.33×10^{-4}	1.00×10^{-3}	1.48×10^{-3}	2.15×10^{-3}	627.75	209.25	141.46	97.51	140.93
7	1.00	6.25×10^{-5}	6.04×10^{-4}	1.17×10^{-3}	6.04×10^{-4}	3348.00	346.34	179.36	346.34	100.03
8	1.00	5.83×10^{-4}	1.19×10^{-3}	1.73×10^{-3}	2.85×10^{-3}	358.71	176.21	121.01	73.31	100.03
9	5.00	4.17×10^{-5}	5.63×10^{-4}	1.08×10^{-3}	5.63×10^{-4}	5022.00	372.00	193.15	372.00	45.71
10	5.00	5.63×10^{-4}	1.10×10^{-3}	1.63×10^{-3}	3.25×10^{-3}	372.00	189.51	128.77	64.38	45.71
11	10.00	6.04×10^{-4}	1.81×10^{-3}	2.75×10^{-3}	5.54×10^{-3}	346.34	115.45	76.09	37.76	33.21

Table 6.2: Measured delay times and corresponding computed speeds of sound compared to theoretical predictions from drop tower tests for bubbles with diameters of approximately 0.55 mm.

Similarly, the first peak and maximum peak were identified for the parabolic flights, see Table 6.3. These results were also compared to the theoretical predication. The main difference between the parabolic flight and drop tower experiments was the change in the gas volume fraction. For the experiments conducted on the drop tower the gas volume fraction was varied, with less access the experiment aboard the parabolic flights, the gas volume fraction remained constant. However the rotational rate of the mixing was varied, increasing from test 4 to 21. No conclusions have been made on the impact of the rotational rate on the speed of sound.

Test – Burst No.	d_b (Mean) [mm]	SOS_{max} [m/s]	SOS_1 [m/s]
4 – 2	0.63	204.97	484.05
4 – 5	0.63	2363.30	304.36
5 – 1	0.63	232.23	232.23
5 – 5	0.63	140.96	233.58
6 – 2	0.59	3348.00	313.87
6 – 4	0.59	1385.40	297.60
7 – 2	0.59	4017.60	332.03
7 – 5	0.59	4464.00	346.34
8 – 2	0.59	208.17	208.17
8 – 4	0.59	∞	181.79
9 – 2	0.61	2008.80	289.03
9 – 4	0.61	186.86	349.35
10 – 2	0.61	542.91	221.96
10 – 4	0.61	139.50	213.70
11 – 2	0.61	3652.00	340.47
11 – 4	0.61	139.50	217.16
12 – 2	0.58	913.09	273.30
12 – 4	0.58	221.96	221.96

Test – Burst No.	d_b (Mean) [mm]	SOS_{max} [m/s]	SOS_1 [m/s]
13 – 2	0.58	161.34	286.97
13 – 4	0.58	∞	163.98
14 – 2	0.58	203.90	203.90
14 – 4	0.58	∞	219.54
15 – 2	0.56	∞	168.10
15 – 3	0.56	167.40	313.87
16 – 2	0.56	1488.00	299.80
16 – 4	0.56	1913.10	311.40
17 – 2	0.56	1296.00	309.04
17 – 4	0.56	2678.40	355.50
18 – 2	0.54	954.57	262.60
18 – 3	0.54	165.30	291.10
19 – 2	0.54	2363.30	329.30
19 – 4	0.54	168.10	295.40
20 – 2	0.54	446.40	199.88
20 – 4	0.54	267.84	267.84
21 – 2	0.55	1004.40	324.00
21 – 4	0.55	2114.50	318.00

Table 6.3: Measured speed of sound values for individual bursts corresponding to experiments conducted on parabolic flights where, $\alpha = 0.2\%$ and the expected speed of sound value is equal to 222.39 m/s.

Although several of the calculated values were in reasonable agreement with the expected prediction, the measurements did not consistently align with theory. Fig. 6.9 plots the most reasonable data for all experimental campaigns compared to the theoretical prediction. At present, we are unable to conclusively explain why certain estimates fall within the anticipated range while others deviate significantly. Bubble-induced scattering and attenuation significantly distort the acoustic signal, making identification of a distinct cross-correlation peak difficult. Additional refinement of signal processing methods will be required to improve measurement accuracy.

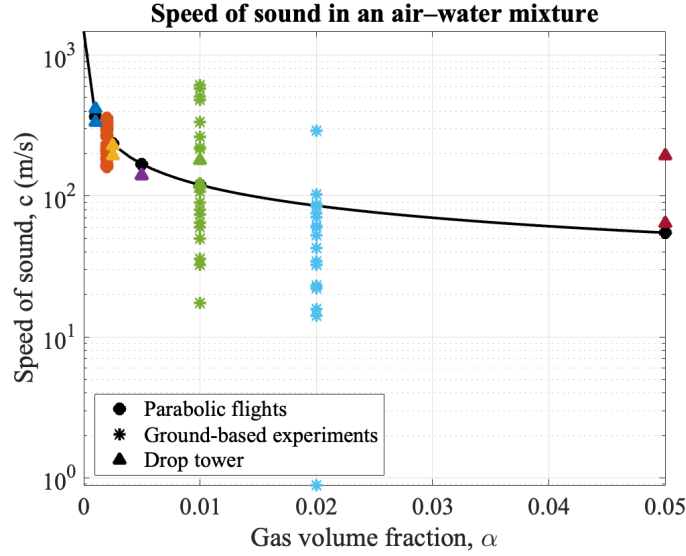


Figure 6.9: Experimental data plotted in comparison with the theoretical prediction (black line) of the speed of sound in a bubble emulsion. The various shapes represent the experimental environment; drop tower, ground-based, or parabolic flights, whereas, the colors designate the gas volume fraction ranging from 0.1 to 10 %.

6.5 Conclusions

The speed of sound in a bubble emulsion decreases significantly relative to pure water as gas volume fraction increases. Both the microgravity experimental measurements and theoretical predictions demonstrate that the speed of sound ranges from approximately 300 to 30 m/s for gas volume fractions between 0.1% and 10%. Experiments were conducted under Earth's gravity as a baseline and then in microgravity conditions using both drop tower testing and parabolic flight campaigns to create a stable bubble emulsion. The speed of sound was estimated by calculating the cross-correlation between two hydrophone signals generated by an acoustic wave propagating through the bubbly medium. While the experimental data confirm the expected reduction in sound speed, measurement uncertainty was introduced by noise caused by scattering from tank walls and bubbles, making precise identification of cross-correlation peaks challenging.

Experimental results were compared with Wood's (1930) theoretical model and showed some deviation. These differences are likely attributable to bubble size distribution and dispersion effects, which are not accounted for in the theoretical formulation.

This study represents an initial investigation into acoustic wave propagation in bubble emulsions and remains ongoing. Future work will focus on developing a more comprehensive understanding of sound

propagation within the experimental tank. This includes incorporating room acoustics analysis methods and investigating the influence of different acoustic frequencies on the measurements. In the present study, the sound frequency was selected based on the optimal operating range of the hydrophones rather than the natural frequency of the bubbles. Future experiments should therefore examine a wider range of acoustic frequencies, ideally those significantly different from the natural frequency of the bubbles.

Supplementary Material

Below is the Matlab script that was used for the analysis of the hydrophone signals.

```

1 [caption={MATLAB implementation used for acoustic analysis of bubbly liquid experiments},
2 label={lst:soundanalysis}]
3
4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
5 %   Sound Analysis with MATLAB Implementation   %
6 %                                               %
7 % Author: M.Sc. Eng. Hristo Zhivomirov  04/01/14 %
8 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
9 % Modified by R. Zenit and M. Federle to analyze sound in bubbly liquids. 02/10/25
10
11 set(groot,'defaultLineLineWidth',2)
12 set(groot,'defaultAxesFontName','Times New Roman')
13 set(groot,'defaultTextFontName','Times New Roman')
14 set(groot,'defaultAxesFontSize',18)
15 set(groot,'defaultTextFontSize',18)
16
17 [x, fs] = audioread('ZeroG/250503_003_Tr1.WAV'); % load hydrophone signal 1
18 [x2, fs2] = audioread('ZeroG/250503_003_Tr2.WAV'); % load hydrophone signal 2
19
20 N = length(x);
21 t = (0:N-1)/fs;
22
23 N2 = length(x2);
24 t2 = (0:N2-1)/fs2;
25

```

```

26 % Trim signal
27 duration = 1; % seconds
28 t0 = 6.4 + 1.3*(1); % manually selected start time
29
30 x_s = x(t0*fs:(t0+duration)*fs-1);
31 t_s = t(t0*fs:(t0+duration)*fs-1);
32
33 x2_s = x2(t0*fs:(t0+duration)*fs-1);
34 t2_s = t2(t0*fs2:(t0+duration)*fs2-1);
35
36 %% Plot signal waveform
37 figure('Name','Signal')
38 plot(t, x, 'r')
39 hold on
40 plot(t_s, x_s, 'b')
41 plot(t2_s, x2_s-0.5, 'k')
42 xlabel('Time, s')
43 ylabel('Amplitude')
44 title('Signal in time domain')
45 grid on
46
47 %% Filtering parameters
48 fc_h = 7500; % high cutoff frequency
49 fc_l = 1000; % low cutoff frequency
50
51 fis_h = 2100; % input sound frequency range
52 fis_l = 1600;
53
54 % Filter design
55 LP = designfilt('lowpassiir','FilterOrder',8,...
56     'PassbandFrequency',fc_h,'PassbandRipple',0.2,...
57     'SampleRate',fs);
58
59 HP = designfilt('highpassiir','FilterOrder',8,...
60     'PassbandFrequency',fc_l,'PassbandRipple',0.2,...

```



```

61         'SampleRate',fs);
62
63 % Apply filters
64 x_s_f = filtfilt(LP,x_s);
65 x2_s_f = filtfilt(LP,x2_s);
66
67 x_s_f = filtfilt(HP,x_s_f);
68 x2_s_f = filtfilt(HP,x2_s_f);
69
70 %% Spectrogram
71 windowsize = 2^12;
72
73 figure('Name',"Spectrogram")
74
75 subplot(2,1,1)
76 spectrogram(x,windowsize,3/4*windowsize,[],fs,'yaxis')
77 title('Spectrogram Hydrophone 1')
78
79 subplot(2,1,2)
80 spectrogram(x2,windowsize,3/4*windowsize,[],fs2,'yaxis')
81 title('Spectrogram Hydrophone 2')
82
83 %% Power spectral density
84 [X,f] = pwelch(x,ones(windowsize,1),windowsize/2,windowsize,fs,'power');
85 X = 20*log10(sqrt(X)*sqrt(2));
86
87 [X_s,f_s] = pwelch(x_s,ones(windowsize,1),windowsize/2,windowsize,fs,'power');
88 X_s = 20*log10(sqrt(X_s)*sqrt(2));
89
90 %% Cross correlation
91 [Rx,lags] = xcorr(x_s,x2_s,'coeff');
92 d = lags/fs;
93
94 [Rx2,lags2] = xcorr(x_s_f,x2_s_f,'coeff');
95 d2 = lags2/fs2;

```

```

96
97 figure('Name','Crosscorrelation')
98 plot(d,Rx,'r')
99 hold on
100 plot(d2,Rx2,'r--')
101 xlabel('Delay, s')
102 ylabel('Crosscorrelation coefficient')
103 title('Crosscorrelation of signals')
104 grid on
105
106 % Peak detection
107 mask = (d2 >= 0) & (d2 <= 0.0015);
108 d2_window = d2(mask);
109 Rx2_window = Rx2(mask);
110
111 [all_peaks,all_locs] = findpeaks(Rx2_window,d2_window);
112
113 if length(all_peaks) >= 2
114     peak_val = all_peaks(2);
115     peak_time = all_locs(2);
116 end
117
118 % Speed of sound calculation
119 dist = 0.20925;
120 sos = dist / peak_time

```

7 BUBBLE EMULSIONS IN MICROGRAVITY

Abstract Bubble breakup in turbulent flows remains an open question, largely due to the limited availability of high-quality experimental datasets. The objective of this work is to contribute to the development of such a dataset by quantifying bubble breakup under controlled turbulent conditions. In this study, bubble breakup was experimentally investigated in turbulence under microgravity conditions, which eliminate buoyancy effects and allow bubbles to remain suspended within the flow. A homogeneous isotropic turbulence (HIT) chamber was used to generate sufficiently intense turbulence to induce bubble breakup. The turbulence intensity within the chamber was quantified using particle image velocimetry (PIV) measurements. Under these highly turbulent conditions, rapid bubble breakup was observed. An artificial intelligence and machine learning (AI/ML) framework was used to analyze high-speed imaging data and extract key parameters governing the breakup process. These parameters include the critical Weber number, turbulent dissipation rate, characteristic turbulence timescales, and the resulting critical bubble diameter. Distributions of bubble sizes were obtained for a range of turbulence intensities, and the time evolution of the mean bubble diameter was evaluated to characterize the decay in bubble size during turbulent mixing. Finally, both local and averaged critical Weber numbers were calculated for the various turbulent flow cases examined.

Information This work is currently under development and has not yet been submitted for publication.

7.1 Introduction

Emulsions are typically defined as mixtures of two immiscible fluids [137], most commonly oil and water. They are present in many everyday products, including cosmetics [138], as well as in pharmaceutical [139] and food formulations [140] where stability is essential. To characterize and understand emulsions, dispersion and size distribution are critical parameters. Emulsions are defined by the relationship between the bulk phase and the dispersed phase. Quantifying the volume fraction of the dispersed phase within the bulk phase, along with the droplet size distribution and characteristic droplet diameters, directly influences

the stability of the emulsion and, consequently, its performance or intended application [137]. We aim to use a similar framework for a new type of emulsion: a bubble emulsion.

Although a bubble emulsion is not a classical emulsion in the traditional liquid–liquid sense, it follows the same underlying principles. In this case, the bulk phase is a liquid, such as water, and the dispersed phase consists of air distributed throughout the liquid in the form of bubbles. This unique two-phase system would separate rapidly under normal gravitational conditions; however, in the absence of gravity a bubble emulsion can remain stable due to the lack of buoyant forces. Such a situation has been studied theoretically [141] and numerically [142], but to our knowledge no experimental studies currently exist. Many investigations of two-phase bubbly flows in microgravity have been conducted, including flow boiling [143], electrolysis [42], and bubble breakup [117], yet none have examined the properties of bubble emulsions in detail. Of particular interest for practical applications are the formation process and stability of bubble emulsions [144, 145]. Once these mechanisms are better understood, we foresee a range of potential applications in space engineering, including thermal control (a porous liquid could exhibit unique insulating properties), propellant handling (bubbles significantly influence the performance of liquid propellant systems), and water remediation (the performance of aeration tanks may be substantially altered in microgravity) [143, 145].

Understanding how such bubble emulsions form requires examining the mechanisms that generate and distribute bubbles within the liquid phase. The formation of a bubble emulsion requires bubble breakup, which in this work is achieved through turbulent agitation. Bubble breakup in turbulent flows has been studied extensively, including the review by Ni [146]. These studies identify the governing parameters that control breakup and, consequently, the resulting bubble size distribution as a function of the flow field. In general, breakup occurs when the turbulence intensity is sufficiently large; strong turbulence promotes the formation of smaller bubbles, whereas weak turbulence may be insufficient to fragment larger bubbles. Despite this understanding, accurately predicting bubble size distributions in turbulent flows remains a significant challenge.

Bubble breakup is classically defined as occurring when inertial forces overcome surface tensions forces, or reaches a critical Weber number which is the ratio of these two forces. The foundational work conducted by Hinze aimed to categorized breakup based on the cause of deformation [147]. Specifically noting turbulence as one of the cause of deformation and breakup. However, turbulence is inherently complex and thus complicates the prediction of bubble breakup. To resolve this, many researchers have

focused specific studies to quantify all these forms of break up. Specifically, the 1997 work by Fabre and Risso, focused on bubble breakup in turbulence, identifies the breakup mechanics for a single bubble in a turbulent field without gravitational effects [117]. Their study, conducted in a turbulence chamber in microgravity, found that breakup will occur when there is a critical Weber number of 5. However, the study leaves questions remaining about the bubble breakup over larger time scales and therefore the resulting size distribution of bubble breakup over time.

Recent studies by Zhong and Ni emphasized the continued need for high-quality experimental investigations of bubble breakup in turbulent fields, particularly to better quantify breakup frequency [148]. Bubble breakup is inherently sensitive to the characteristics of the turbulence. As the turbulence intensity evolves, decays over time, or varies spatially, the breakup behavior changes accordingly. In addition, interactions between neighboring bubbles can significantly alter the local flow field, thereby modifying the breakup mechanism. More importantly, to the best of our knowledge, our study represents the first experimental study investigating the formation process of a bubble emulsion for potential space applications.

By conducting experiments over approximately 20-second intervals in microgravity, we are able to isolate and examine bubble breakup in controlled turbulent conditions without buoyancy. Two conditions of particular interest are: (1) the frequency of bubble breakup over a longer duration of time, (2) a critical diameter value that bubbles reach after sustained agitation.

This work provides preliminary evidence of distinct breakup timescales in homogeneous isotropic turbulence. The data for the critical Weber number under these conditions is limited and requires further investigation. Therefore, we seek to understand how the critical Weber number evolves under sustained agitation, where repeated breakup events occur over extended periods.

7.2 Experimental setup

Using a homogeneous isotropic turbulence (HIT) chamber, a bubble emulsion was created. The details of the HIT chamber are summarized in Chapter 5. The truncated cube shaped tank, seen in Fig. 7.1(a), was filled with majority water and small gas volume fractions of air, from 0.1 % to 10% and then sealed off. This created a small air pocket at the top of our tank. Intense turbulence, $420 < Re_\lambda < 820$, was then induced via mechanical agitation of eight evenly spaced impellers located at the cut off corners of the truncated

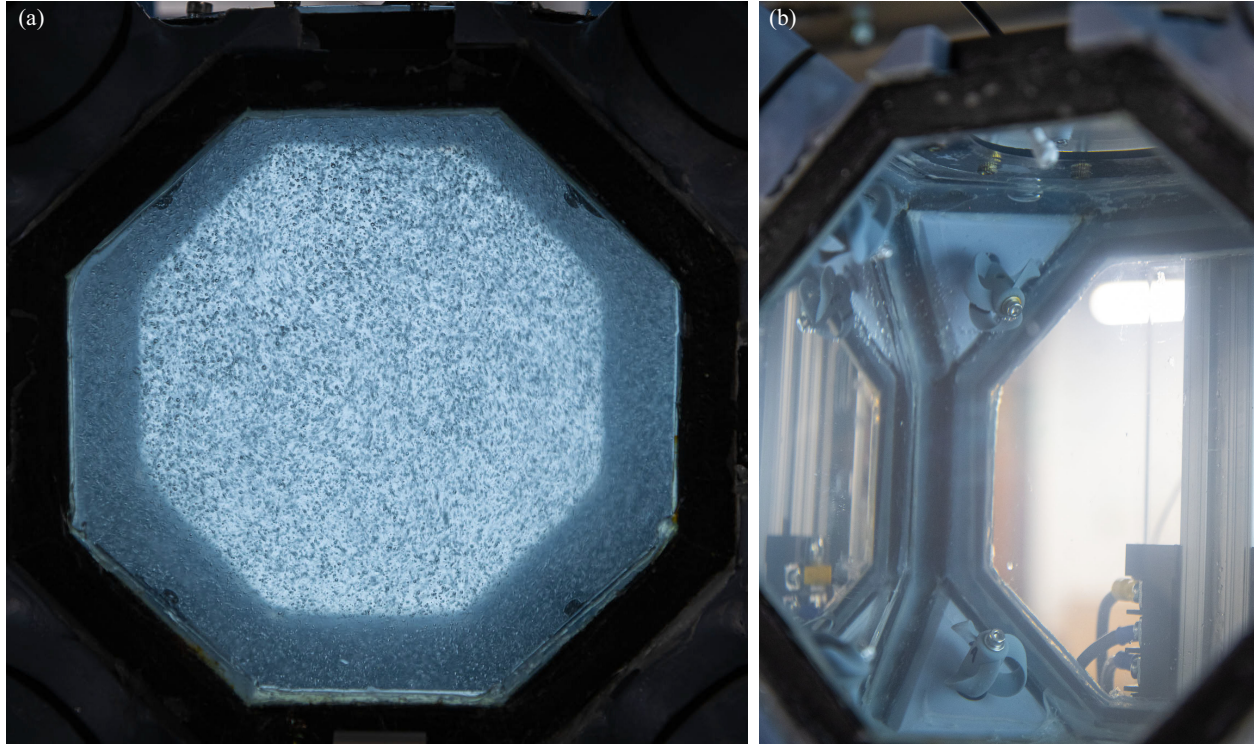


Figure 7.1: Images of experimental setup including (a) truncated cube shaped tank and (b) impellers used for agitation.

cube shaped tank, as detailed in Fig. 7.1(b). The turbulence intensity was changed by adjusting the angular velocity of the impellers. The aim was to adjust the angular velocity and in turn change the bubble breakup size.

The experiments were conducted in microgravity in order to study bubble emulsions in the absence of buoyancy-driven motion. Under normal gravity conditions, bubbles rapidly rise to the liquid surface due to buoyancy forces, which limits the time available for observing bubble breakup and dispersion. In contrast, the microgravity environment allows bubbles to remain suspended within the fluid for extended durations, enabling detailed observation of bubble breakup and subsequent dispersion within the turbulent flow.

Microgravity conditions were achieved using two experimental platforms: the NASA Glenn 2.2 second drop tower and a series of parabolic flight campaigns. While both platforms were utilized during the broader experimental program, the analysis presented in this work focuses primarily on the parabolic flight experiments, which provide significantly longer microgravity durations. The parabolic flight experiments also explore a wide range of turbulence intensities therefore offered a larger comprehensive data set.

The parabolic flight, described in Chapter 4, produces microgravity by flying an aircraft through a series of parabolic trajectories. During each parabola, the aircraft experiences approximately 20 seconds of microgravity near the peak of the trajectory, followed by 30–40 seconds of hypergravity conditions (approximately 1.8 g) during the pull-out phase.

High-speed imaging was used to capture the evolution of the bubble emulsion during the microgravity period. Images were recorded using a Photron FASTCAM Nova R5 operating at 1250 frames per second (fps). Careful timing was required to ensure that bubble breakup occurred during the microgravity phase of the flight. For each parabola, the experimental sequence was triggered immediately after the onset of microgravity. The experimental timeline consisted of the following steps: (1) 3 seconds of delay, allowing the aircraft to stabilize in microgravity conditions, (2) 6 seconds of active mixing, during which the impellers were operated to induce turbulence and initiate bubble breakup, and (3) approximately 11 seconds of mixing off, allowing observation of bubble dispersion after turbulence generation ceased. This timing sequence ensured that both bubble breakup and the subsequent dispersion dynamics were captured within the microgravity window. The high-speed camera was triggered simultaneously with the experimental system and recorded continuously for 20 seconds. The initial delay in mixing ensured that the bubble breakup process occurred entirely within the microgravity environment.

In addition to the high-speed imaging, a GoPro camera was used to record the full duration of each experimental flight. Each flight consisted of approximately 30 parabolas over a period of roughly 45 minutes. The GoPro footage provides a broader view of the experimental setup and the evolution of the bubble emulsion throughout the entire campaign. While this footage was not used for quantitative analysis, it offers a qualitative visualization of the system behavior and experimental conditions during flight.

The high-speed images were post-processed using an artificial intelligence (AI)–based machine learning model designed to detect and quantify bubbles within the image sequences. The model was used to extract key characteristics of the bubble emulsion, including bubble size and spatial distribution. The details of the machine learning methodology and image analysis pipeline are described in the following sections.

7.3 Analysis Methodology

Within the Kolmogorov–Hinze framework, bubble breakup is assumed to occur when the turbulent Weber number reaches a critical value. The turbulent Weber number, the ratio between inertial and surface tension forces, can be defined as [149]:

$$We_t = \frac{\overline{\rho \delta u^2} (d_b) d_b}{\sigma} \quad (7.1)$$

where ρ is the fluid density, u is the fluid velocity, d_b is the bubble diameter, and σ is the interfacial tension. Assuming homogeneous and isotropic turbulence, the velocity fluctuation can be approximated as $\overline{(\delta u)^2} \approx C(\epsilon d)^{2/3}$, where the Kolmogorov constant is taken to be $C = 2.3$ and ϵ is the turbulence dissipation rate. Substituting this relationship into Eq. 7.1 yields

$$We_t = \frac{2.3 \rho \epsilon^{2/3} d_b^{5/3}}{\sigma}. \quad (7.2)$$

Hinze (1955) demonstrated that bubble breakup occurs when turbulent stresses exceed the restoring surface tension forces, corresponding to a critical Weber number, $We_{crit} \sim \mathcal{O}(1 - 10)$. This criterion defines a critical bubble diameter, commonly referred to as the Hinze scale, given by

$$d_h \sim \left(\frac{We_{crit} \sigma}{2.3 \rho \epsilon^{2/3}} \right)^{3/5}. \quad (7.3)$$

Bubbles larger than d_h are expected to break, while smaller bubbles will remain stable against turbulent stresses. In the present experiments, this framework provides a theoretical reference scale for interpreting the measure bubble diameter distributions.

However, the Hinze framework alone does not fully describe the bubble breakup process. The Hinze framework only considers breakup due to the interplay of inertial and surface tension forces, but fails to consider the necessary timescales that aim to understand not only bubble breakup, but also bubble breakup frequency. Bubble breakup frequency is typically modeled using a Boltzmann-type equation to predict the size distribution of bubbles, particle or drops in turbulence. The Boltzmann-type equation, Eq. 7.4, originally proposed by Williams (1985) [150], describes the evolution of the number of bubbles or droplets, $\frac{\partial n}{\partial t}$, of a certain size, d_b .

$$\frac{\partial n}{\partial t} + \nabla \cdot (vn) = -\frac{\partial}{\partial d_b}(Rn) + \dot{Q}_b + \dot{Q}_c \quad (7.4)$$

The advection velocity of bubbles is, $v(d_b, x, t)$, $R = dd_b/dt$ is the rate of change of bubble size due to mass dissolution, and $\dot{Q}_b$ and $\dot{Q}_c$ are the rate of change of bubble number density due to breakup and convalescence respectively [148].

This population balance equation is most often implemented in computational studies, since many experimental investigations do not directly resolve the temporal evolution of breakup frequency. Applying the full formulation to experimental data is therefore challenging. To address this difficulty, simplified approaches have been proposed in which the breakup frequency is estimated using measurable quantities, such as the initial number of bubbles n_0 , the number of bubbles that remain unbroken n_1 , an average breakup time t_b , and the mean residence time, τ . In terms of these variables, the breakup frequency may be approximated as [151]:

$$g(d_b) = \frac{n_0 - n_1}{\tau n_1 + \sum_i t_{b,i}}. \quad (7.5)$$

For experimental results, it is also important to consider the characteristic timescales of the flow, for the case of turbulence, the eddy turnover time, t_e , is typically proposed as the characteristic time scale, defined as:

$$t_e = \epsilon^{-1/3} d_b^{2/3} \quad (7.6)$$

where ϵ is the turbulence dissipation rate, and d_b is the bubble size. The turbulence dissipation rate can be found experimentally, for instance, from PIV data. The dissipation rate is defined as:

$$\epsilon = 2\nu \langle s_{ij}s_{ij} \rangle \quad (7.7)$$

where S_{ij} is the fluctuating rate of strain, and ν is the kinematic viscosity. Assuming local isotropy for 2D particle image velocimetry (PIV), ϵ is defined as:

$$\epsilon = \nu \left\langle 4 \left(\frac{\partial u'_x}{\partial x} \right)^2 + 4 \left(\frac{\partial u'_y}{\partial y} \right)^2 + 3 \left(\frac{\partial u'_x}{\partial y} \right)^2 + 3 \left(\frac{\partial u'_y}{\partial x} \right)^2 + 4 \left(\frac{\partial u'_x}{\partial x} \frac{\partial u'_y}{\partial y} \right) + 6 \left(\frac{\partial u'_x}{\partial y} \frac{\partial u'_y}{\partial x} \right) \right\rangle \quad (7.8)$$

where u' and v' are the horizontal and vertical velocity fluctuations, respectively. These quantities are obtained directly from the instantaneous velocity fields measured using 2D PIV.

Similarly, the turbulence kinetic energy was calculated from the PIV measurements and is defined as

$$TI = \frac{\sqrt{\frac{1}{2}(u_{rms}^2 + v_{rms}^2)}}{U_{tip}} \quad (7.9)$$

where u_{rms} and v_{rms} are the root-mean-square velocity fluctuations in the horizontal and vertical directions, and $U_{tip} = 2\pi r\omega$ is the tip speed of the impeller with radius, r , rotating at speed, ω measured in rotations per second.

7.4 Preliminary Results

To experimentally quantify bubble size distributions high speed photography was used. During experimentation, bubble breakup was rapidly observed in microgravity as seen in Fig. 7.2. The process of breakup during turbulence, roughly 6 seconds of mixing, was filmed as well as approximately 11 seconds of dissipation. The images captured during experimentation, were used to find the size distribution of the bubbles achieved at various turbulent Reynolds numbers.

To accurately quantify the size distribution of the bubble emulsion, a machine learning framework was developed. The workflow consisted of (1) manually labeling bubbles using bounding boxes, (2) training a segmentation model, and (3) applying the Segment Anything framework to process large sets of high-speed images. At its current stage, as seen in Fig. 7.3, the model performs reasonably well in identifying bubbles. However, to improve its accuracy and robustness, it must be trained on a substantially larger dataset, ideally hundreds of images. As with most machine learning approaches, performance improves as the model is exposed to more representative data. The results presented in the following sections should therefore be viewed as a framework for understanding bubble breakup within the limitations of the present ML implementation. As the model is further refined and trained on additional data, these results will be revisited to reflect a more accurate characterization of the bubble size distribution.

Due to the image resolution of 1024×1024 pixels, the smallest bubble size reliably detectable by the algorithm corresponds to approximately 10 pixels, which is equivalent to roughly 1 mm in physical space.

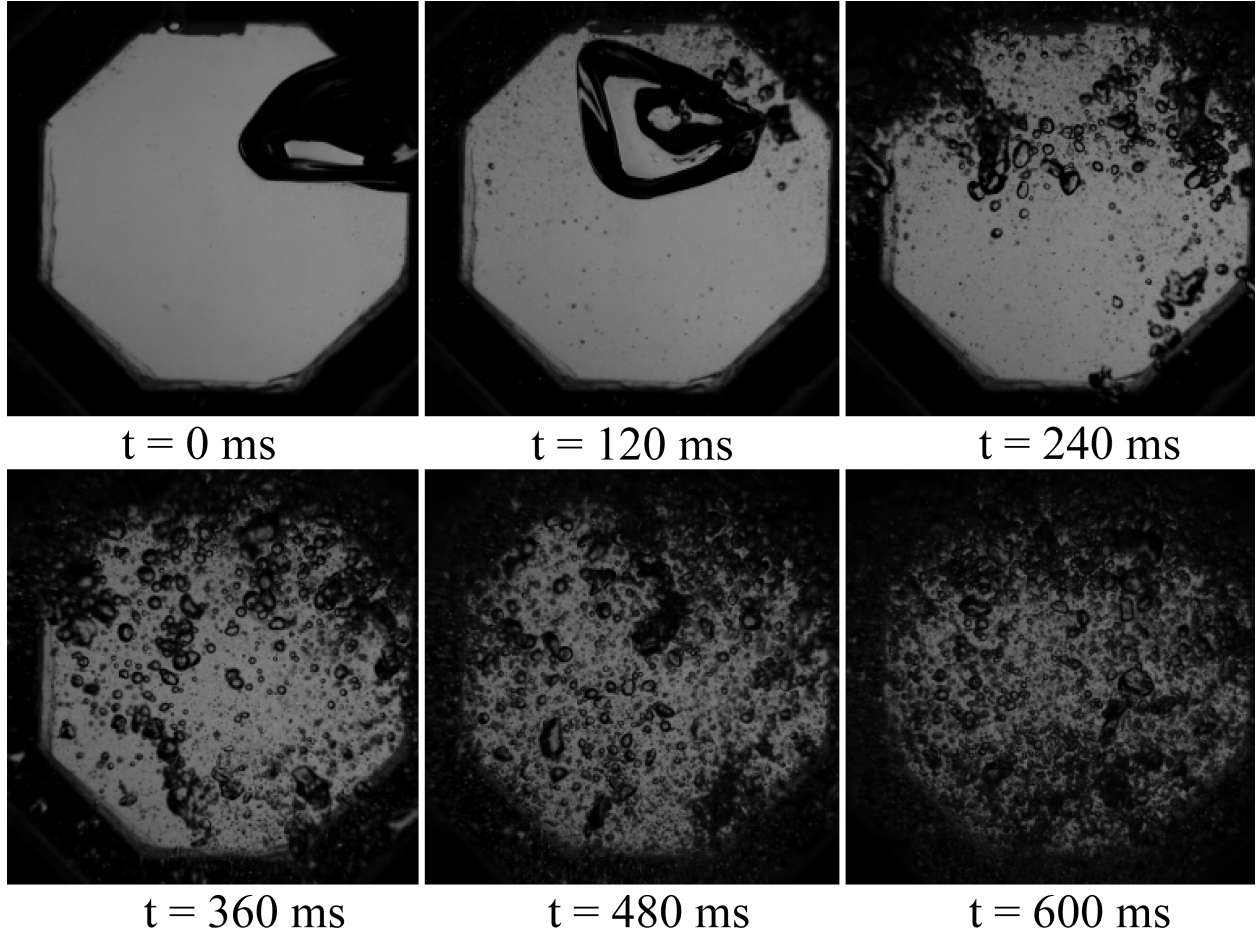


Figure 7.2: Bubble breakup over time in HIT chamber.

Although this represents a limitation of the current identification framework, an estimate of the bubble size distribution can still be obtained by extending the fitted lognormal distribution toward smaller diameters.

For each detected bubble, the area was found using a pixel to mm ratio. Using this area, A , an equivalent diameter was computed as

$$d_b = \sqrt{\frac{4}{\pi} A}. \quad (7.10)$$

7.4.1 Flow Analysis

A series of two-dimensional PIV measurements were performed across a range of operating conditions, see Chapter 5 for more details. Multiple turbulent Reynolds numbers were achieved by varying the rotational speed of the motors. The measurement window covered an area of $75 \text{ mm} \times 75 \text{ mm}$ (1024×1024 pixels),

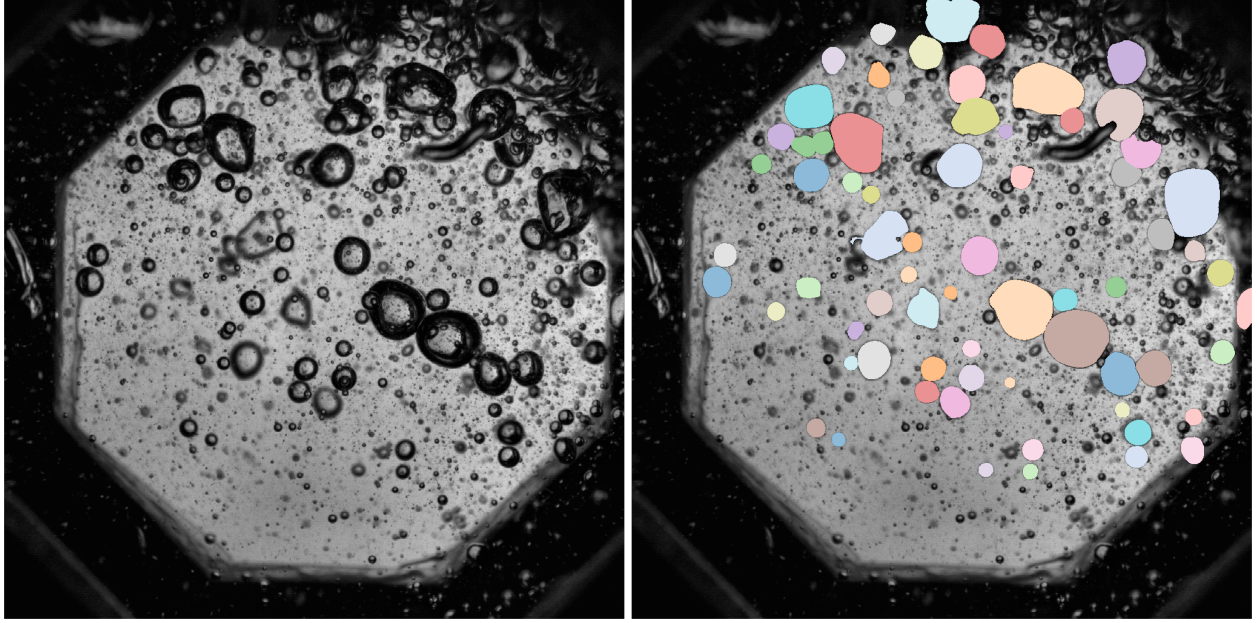


Figure 7.3: Two side by side images that show bubble identification. The left image is the image without any labeling, as the right image showed the identical image with bubbles labeled in various colors.

yielding a spatial vector resolution of 0.675 mm. Each instantaneous velocity field consisted of 111×111 vectors.

By adjusting the impeller rotation rate, turbulence intensities corresponding to $420 < Re_\lambda < 820$ were generated, spanning a range characteristic of strongly turbulent flows [121]. The turbulent Reynolds number based on the Taylor microscale is defined as $Re_\lambda = u' \lambda / \nu$, where u' is the velocity fluctuations (related to the turbulent kinetic energy), λ is the Taylor microscale, and ν is the kinematic viscosity of the fluid [121]. The Taylor microscale is given by $\lambda = \sqrt{15 \nu u' / \epsilon}$, where ϵ is calculated using Eq. 7.8. The dissipation rate for various turbulent Reynolds numbers is shown in Table 7.1.

PWM Input	RPM	Turbulent Reynolds Re_λ	Dissipation Rate ϵ
1170	14,776	422	0.761
1220	25,764	514	1.063
1245	33,493	682	1.578
1270	34,889	703	2.438
1320	44,329	821	3.127

Table 7.1: Various turbulent Reynolds numbers and their corresponding dissipation rates

Fig. 7.4 illustrates the turbulence intensity in two regions of the chamber. Fig. 7.4 (left) is the central region of the tank, located far from the impellers, roughly $1.5r$, where r is the impeller radius equal to 14

mm. Fig. 7.4 (right) is the near-field region close to an impeller. As expected, the turbulence intensity is highest near the impellers and decreases toward the center of the chamber. The high intensity flow region generated by the impellers occupies roughly 60% of the total tank volume, while the remaining 40% corresponds to the central region where the flow more closely approaches homogeneous isotropic turbulence (HIT). Because bubble breakup is driven by local turbulent stresses, bubbles near the impeller are expected to experience stronger deformation and more frequent breakup events than those located in the central region.

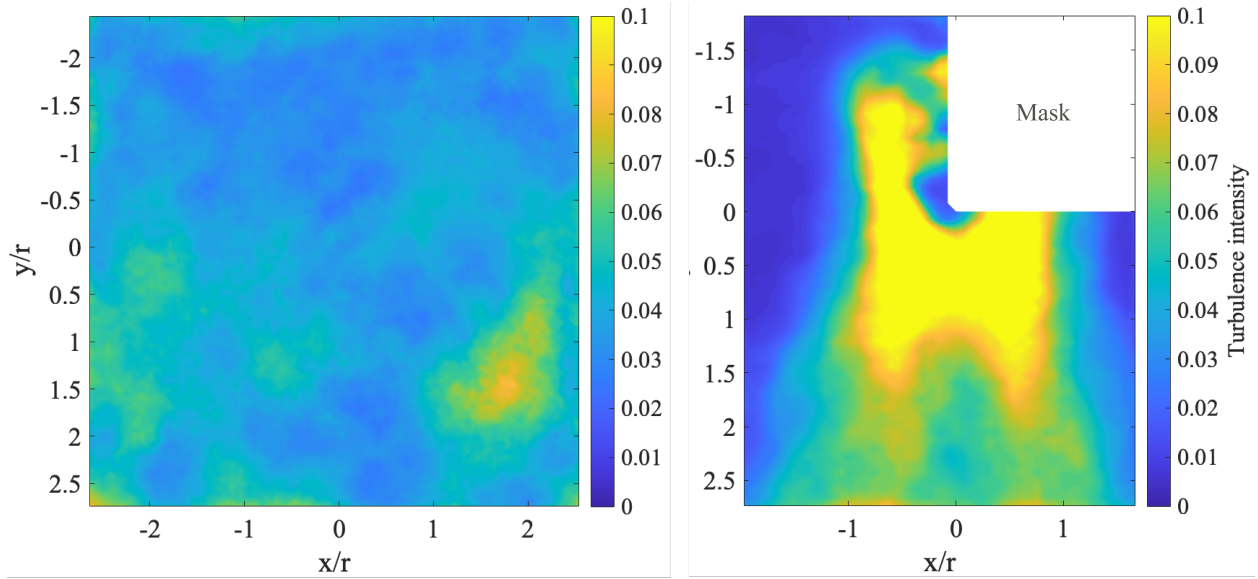


Figure 7.4: Color maps of the turbulence intensity in the entire tank (left), far from impellers, and near one impeller (right) plotted over the horizontal and vertical directions normalized by the impeller radius. The white region in the right figure is the masked region due to the shadow of the impeller. Both horizontal and vertical axes are normalized by the impeller radius.

7.4.2 Bubble Breakup Formation

During experimentation, bubble breakup was rapidly observed in microgravity, as shown in Fig. 7.2. Approximately 0.6 seconds of mixing resulted in substantial breakup of the initially large single bubble. At the initial time prior to mixing, t_i , the gas volume fraction determines the initial bubble size. In these experiments, the gas volume fraction was maintained at 0.2%, producing a single bubble with an approximate diameter of 47 mm. The evolution of this bubble was subsequently tracked in time in order to quantify changes in the bubble size distribution during turbulent mixing.

Fig. 7.5 shows how bubble breakup evolves as the turbulent Reynolds number increases. At the lower Reynolds number, shown in Fig. 7.5 (top), both the bubble diameter and bubble count take longer to reach a plateau compared to the middle case. In the highest Reynolds number case, Fig. 7.5 (bottom), the bubble count reaches a plateau; however, the bubble diameter does not exhibit a similar trend. This discrepancy is likely due to limitations in the tracking algorithm, as accurately identifying bubble sizes becomes more difficult when the images become densely populated with bubbles. Overall, there is little difference observed in the minimum or mean bubble diameters across the cases.

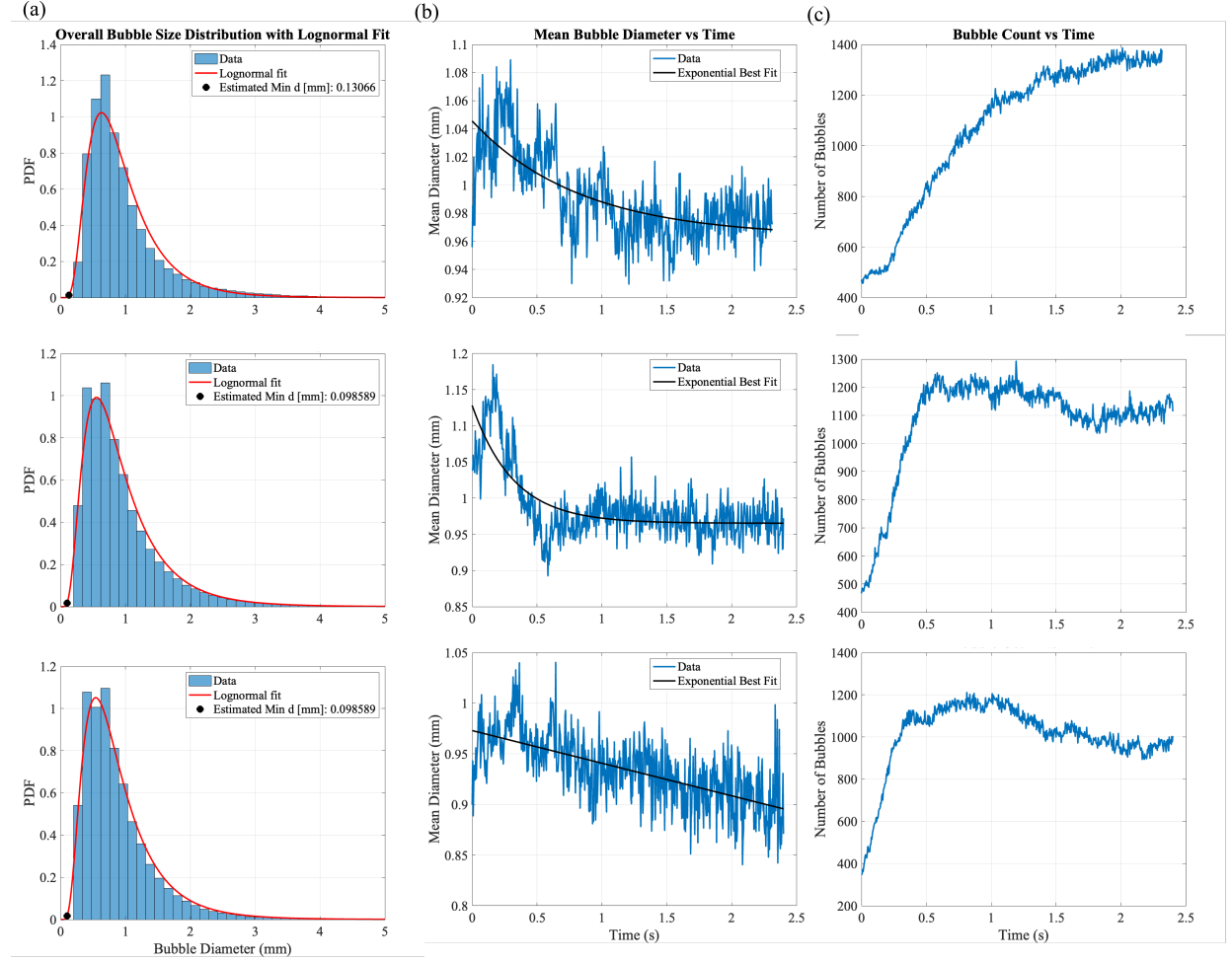


Figure 7.5: Plots from preliminary tracking model showing bubble breakup over time where (a) shows the overall distribution of the bubble sizes, (b) shows the change in bubble size over time, and (c) counts the bubbles over time. Turbulent Reynolds number increases from top to bottom.

During the mixing process, the initially large bubble fragments into smaller bubbles as it interacts with turbulent fluctuations in the flow, leading to an increase in bubble count (see Fig. 7.5 (c)). Breakup

occurs when turbulent eddies deform a bubble sufficiently to overcome surface tension forces. This process continues until the bubbles become small enough that the turbulent stresses can no longer overcome the restoring capillary forces. As a result, the bubble population evolves toward a quasi-steady distribution in which both the bubble count and characteristic bubble size approach a plateau (see Fig. 7.5 (b)). At this stage, the smallest stable bubble size can be estimated. For the present experiments, a lognormal fit to the measured distribution yields a minimum bubble diameter of approximately 0.091 mm for the intermediate Reynolds number case (see Fig. 7.5 (a)). Using this estimated diameter, the corresponding turbulent Weber number was calculated using Eq. 7.2. As shown in Fig. 7.6, a bubble diameter of 0.091 mm corresponds to $We_{crit} \approx 0.123$. This value is significantly smaller than values commonly reported in the literature, where $We_{crit} \sim \mathcal{O}(1-10)$. This discrepancy likely arises from limitations in the current machine learning framework and the image resolution used for bubble identification.

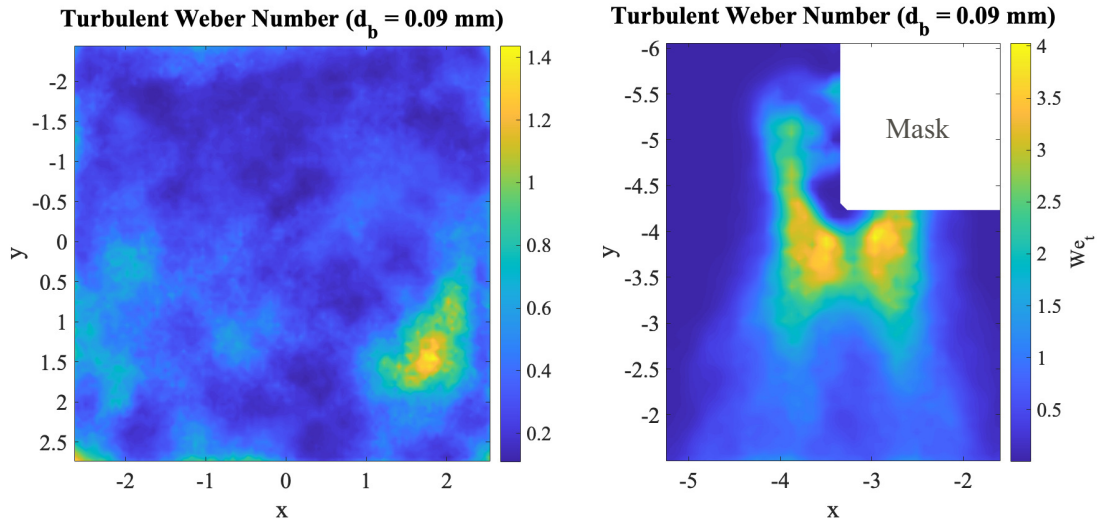


Figure 7.6: Plots showing the turbulent Weber number for both (left) the center of the HIT chamber, and (right) near the impeller for the critical estimated bubble diameter of 0.09 mm. A bubble of this size will on breakup if it is larger than $We_{crit} = 0.0340$.

To further assess this discrepancy, the Weber number was evaluated over a range of bubble diameters from 0.05 to 1.5 mm for both the far-field and near-field regions. As seen in Table 7.2, as the bubble diameter increases, the corresponding Weber number, in the region of HIT, also increases, indicating that larger bubbles require stronger turbulent fluctuations to undergo breakup. Considering that the typical critical Weber number is on the order of 1–10, bubble diameters between approximately 0.1 mm and 1 mm represent the most physically reasonable range. Using a bubble diameter of 1 mm, corresponding to the approximate

lower detection limit of the image processing algorithm results in a more physically reasonable estimate of the critical Weber number of 3.8, which lies within the expected range reported in the literature.

Bubble diameter [mm]	Critical Weber number
0.05	0.190
0.1	0.378
0.5	1.898
1.0	3.794
1.5	5.713

Table 7.2: Critical Weber numbers for set bubble diameter in HIT with turbulent Reynolds number equal to 703.

Using this critical Weber number, the range of bubble diameters under these conditions can be explored. As seen in Fig. 7.7, the average resulting bubble diameter near the center of the tank will be roughly 1 mm. In contrast, maintaining a similar Weber number of approximately 3.8 but near the impellers would result in bubbles that are roughly an order of magnitude smaller than those broken up by turbulent fluctuations in the center of the tank, as illustrated in Fig. 7.7 (right). The predicted bubble diameter near the impellers, approximately 0.1 mm, agrees well with the experimentally estimated minimum bubble diameter reported in Table 7.3.

The critical diameter for various turbulent Reynolds numbers is summarized in Table 7.3. As the turbulent Reynolds number increases, the estimated critical diameter changes slightly, and the corresponding critical Weber number also varies. Comparing the minimum turbulent Reynolds number of 422 with the maximum value of 821 shows a difference in the predicted critical diameter of approximately 0.05 mm. This variation is relatively small and can be considered negligible within the range of Reynolds numbers investigated in this study. Therefore, significant changes in the critical bubble diameter are not expected across this range of mixing intensities. However, because the critical Weber number depends on both the bubble diameter and the turbulent dissipation rate, even this small difference in diameter leads to a more noticeable change in the critical Weber number.

These preliminary results provide an initial framework for interpreting bubble breakup behavior in the HIT chamber. As the machine learning model is further refined and trained using larger datasets, these results will be revisited to obtain a more accurate characterization of the bubble size distribution and the corresponding breakup dynamics.

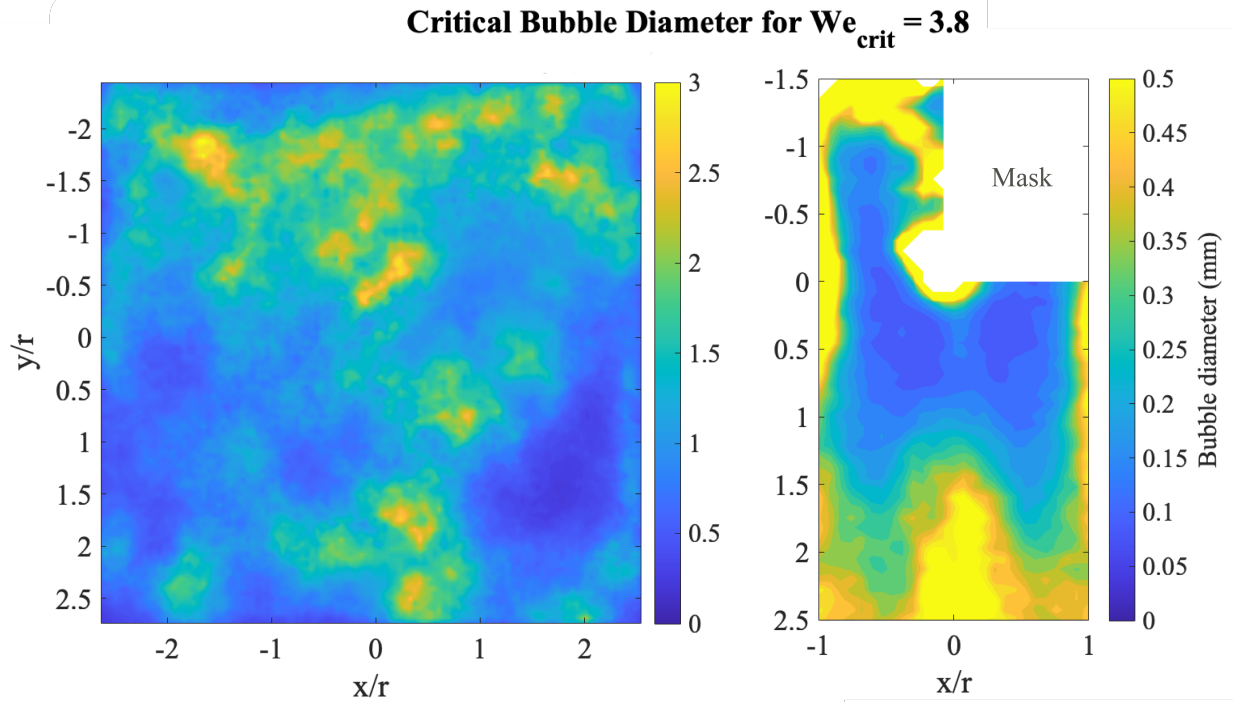


Figure 7.7: Critical bubble diameters plotted for a critical Weber number equal to 3.8 for (left) the center of the tank in the region of HIT, and (right) the region of high intensity turbulence near the impeller.

Turbulent Reynolds Re_λ	Minimum Estimated Bubble Diameter	Critical Weber Number
422	0.131 mm	9.00×10^{-3}
514	0.088 mm	5.77×10^{-3}
682	0.099 mm	9.17×10^{-3}
703	0.099 mm	1.23×10^{-1}
821	0.088 mm	1.19×10^{-2}

Table 7.3: Various turbulent Reynolds numbers and their corresponding dissipation rates.

7.5 Conclusions

The present study serves as a preliminary understanding of the experimental work conducted on bubble breakup in a turbulent field. Using high speed imaging bubble break up was recorded across a range of turbulent Reynolds numbers. An AI/ML-based tracking framework was developed to quantify the size distribution during breakup. The model crudely captured the major and minor axes of in-plane bubbles, to quantify a diameter for each detected bubble.

Analysis of the data revealed that the majority of breakup events occurred over relatively short timescales. Specifically, the largest number of breakup events occurred within approximately 0.6 seconds

after turbulence was initiated. Although breakup events continued while agitation persisted, the bubble size distribution typically approached a plateau after approximately 3–4 seconds of turbulent mixing.

At later times, the smallest observed bubble diameter was approximately 0.091 mm, corresponding to an estimated critical Weber number of 0.34. This value is smaller than those commonly reported in the literature. However, this estimate should be assumed to be an approximation, as the bubble tracking methodology is still under development and may introduce uncertainties in the measured bubble sizes.

When the smallest bubble diameters were compared across the range of Reynolds numbers investigated, only minor variations were observed. This result may reflect limitations in the current tracking approach; however, it may also indicate that the smallest attainable bubble size within the tested turbulence range is approximately 0.1 mm. If this interpretation is correct, it suggests that stronger turbulence levels than those tested here would be required to produce smaller bubbles through breakup, whereas weaker turbulence would likely be insufficient to break bubbles to this scale.

8 CONCLUSIONS

The focus of this thesis is to characterize a subset of two-phase flows; gas-liquid flows. Bubbly flows can be understood in a range of different phenomena, for this thesis, we focus on coalescence, breakup, and bubble distributions in both quiescent and turbulent flows.

8.1 Summary

In Chapter 2 and Chapter 3, the interaction between an oil droplet and an air bubble in a quiescent fluid was investigated. The primary objective of Chapter 2 was to better understand a process commonly referred to as “aeration” or the removal of oil from produced water through bubble injection. In this study, a simplified one-to-one interaction between a single air bubble and a single oil droplet rising in a water-like fluid was examined. Using high-speed photography, the interaction between the two phases was recorded and analyzed. It was initially expected that coalescence between the bubble and droplet would occur under these conditions. However, the experiments instead revealed that the droplet experienced a vertical velocity enhancement due to the hydrodynamic interaction with the bubble. To further understand this behavior, a predictive model was developed based on Oseen flow to describe the influence of the bubble flow field on the droplet. Using the Maxey–Riley equation, the resulting equation of motion for the droplet was solved to predict its trajectory and velocity.

While Chapter 2 successfully described the hydrodynamic interaction between the bubble and droplet, it did not fully explain the observed lack of coalescence. Therefore, Chapter 3 focused on examining the coalescence efficiency between bubbles and droplets. In this chapter, the conditions required for coalescence were explored through an energy balance between the bubble, the droplet, and the resulting compound droplet. By considering the characteristic contact time between the phases and the associated energy requirements for coalescence, a map defining the conditions for coalescence was developed. The analysis demonstrated that coalescence is extremely unlikely, and potentially impossible, under the conditions encountered during flotation.

Together, these chapters revisited a well-known coalescence phenomenon and revealed that coalescence is improbable in the studied configuration. Instead, the dominant effect observed during aeration is the hydrodynamic interaction between bubbles and droplets, which leads to an enhancement in droplet velocity.

Despite the absence of coalescence, this mechanism can still contribute to the filtration process, suggesting that aeration remains an effective method for filtering oil from oil–water mixtures.

The subsequent chapters, Chapter 4, Chapter 5, Chapter 6, and Chapter 7, focused on the study of a bubbly flow system referred to as a *bubble emulsion*. A bubble emulsion is defined here as a mixture of water and air in which the air is dispersed throughout the liquid in the form of bubbles. As described in Chapter 4, these experiments were conducted in microgravity environments, including NASA drop towers and ZeroG parabolic flights. Performing these experiments in microgravity provided a unique environment in which bubbles were not influenced by gravitational forces, specifically buoyancy. By removing buoyancy effects, it becomes possible to create and sustain a stable bubble emulsion in which bubbles remain suspended within the liquid rather than rising to the surface.

Creating such a bubble emulsion in microgravity, however, is not straightforward. In typical terrestrial experiments, bubbles can be introduced into a fluid through capillaries or gas injection, forming a bubbly flow as seen in Chapters 2 and 3. In microgravity, however, bubbles do not detach easily from capillaries because buoyancy is absent. To overcome this challenge, mechanical agitation was used to generate and disperse bubbles throughout the fluid.

Chapter 5 describes the experimental apparatus used to achieve this mechanical agitation: a homogeneous isotropic turbulence chamber. This chamber, developed in-house, represents a novel and cost-effective approach to generating controlled turbulent flows. The total cost of constructing the device was approximately 900 USD, making the design accessible to a broad range of researchers. One of the goals of this chapter was to present the chamber as an open-source design so that other researchers could more easily conduct turbulence experiments. In the present work, this chamber was used to generate bubble emulsions by filling the tank with a prescribed mixture of air and water and mechanically agitating the fluid. The mixing process dispersed the air uniformly throughout the tank in the form of bubbles.

In Chapter 6, experiments were conducted to investigate the speed of sound in bubbly liquids. The objective of these experiments was to understand how the speed of sound varies as a function of bubble concentration and dispersion within the liquid. To measure the speed of sound, two hydrophones were placed at opposite corners of the tank, and an acoustic signal was generated using an underwater speaker located near one of the hydrophones. The signals recorded by the two hydrophones were cross-correlated to determine the time delay between them, allowing the speed of sound within the bubbly liquid to be

calculated. These experiments were conducted in microgravity using drop tower facilities. During the 2.2-second drop, the fluid was initially agitated to disperse the bubbles. The agitation was then stopped, leaving a stable bubble emulsion while the acoustic signal was transmitted and recorded. Although this work is ongoing, the goal is to combine the acoustic measurements with high-speed imaging in order to establish a correlation between the speed of sound and the bubble size distribution and spatial dispersion.

Finally, Chapter 7 examined bubble emulsions in greater detail by quantifying bubble dispersion, breakup, and size distributions within the turbulent flow. These experiments were conducted primarily during parabolic flight campaigns. In this chapter, the turbulence intensity within the chamber was varied to determine how turbulent stresses influence bubble breakup and the resulting bubble size distribution. To quantify bubble diameters, an AI/ML framework was developed to identify and track bubbles in high-speed images. Using this approach, the smallest stable bubble diameter within the flow could be estimated. From this critical diameter, a corresponding critical Weber number was calculated, providing insight into the breakup dynamics of bubbles within the turbulent microgravity environment.

8.2 Future directions

Future work related to Chapter 6 concerns the continued investigation of the speed of sound in bubbly liquids. Although the experiments conducted in this study enabled the collection of preliminary measurements of the speed of sound, additional work is required to fully understand the relationship between acoustic propagation and bubble distributions. Several experimental parameters must be refined to obtain more accurate and reliable measurements. In particular, the wavelength of the acoustic signal used in these experiments was large relative to the dimensions of the experimental tank. As a result, accurately determining the speed of sound within the system proved challenging. Future experiments should therefore employ acoustic signals with wavelengths that are better matched to the scale of the experimental apparatus.

In addition, as discussed in Chapter 5, a room acoustics characterization of the experimental tank should be conducted. Performing such a calibration would provide a clearer understanding of how acoustic waves propagate throughout the entire tank volume. This would make it possible to distinguish between signals that result from reflections at the tank boundaries and those that arise from interactions with the dispersed bubbles. A more comprehensive acoustic characterization would significantly improve the interpretation

of the recorded hydrophone signals. Ideally, a much larger number of experiments would be conducted in order to develop a sufficiently large dataset for accurately characterizing the speed of sound in bubbly liquids. However, due to the complexity and cost associated with microgravity experimentation, such extensive data collection was not feasible within the scope of the present study.

There also remain several important directions for future research related to the work presented in Chapter 7. One area of particular interest is the continued development of the AI/ML framework used for bubble detection and tracking. In its current form, the algorithm performs well when identifying bubbles at relatively low gas volume fractions. However, as the gas volume fraction increases and bubbles become more densely packed, the identification task becomes significantly more challenging. Improving the robustness of the model under these conditions will require additional training data and further refinement of the detection algorithm.

One potential approach to addressing this challenge would be the development of an open-source database of bubbly flow images. This would allow researchers working on a wide variety of bubbly flow problems to contribute labeled datasets that can be used to train and improve machine learning algorithms for bubble detection and tracking. While many researchers have independently developed algorithms for identifying and tracking bubbles, a centralized repository for bubbly flow datasets does not currently exist to the best of our knowledge. Establishing such a collaborative framework could significantly benefit the broader research community by enabling the development of more robust and generalizable machine learning models for analyzing complex bubbly flows.

Finally, the dataset obtained during the experiments described in Chapter 7 contains a substantial amount of information that has not yet been fully explored. While the primary focus of this chapter was on bubble breakup within turbulent flow, the recorded data also include extensive observations of the bubble emulsion after mechanical agitation was stopped. During this post-agitation period, numerous coalescence events can be observed within the bubble population. Future work will aim to analyze these events in greater detail, with the goal of quantifying bubble oscillation frequencies as a function of bubble size. Such measurements could provide additional insight into the dynamics of bubble interactions and the stability of bubble emulsions in microgravity environments.

9 BIBLIOGRAPHY

- [1] **Federle, M. E.**, A. Das, V. Mathai, and R. Zenit. The interaction of an oil droplet and a gas bubble rising in a quiescent fluid. *International Journal of Multiphase Flow*, 171:104682, 2024.
- [2] **Federle, M. E.** and R. Zenit. Physical conditions for droplet–bubble coalescence in flotation. *International Journal of Multiphase Flow*, 179:104933, 2024.
- [3] M. Ravisankar, **Federle, M. E.**, M. Bahadori, A. Das, H. Kesari, and R. Zenit. The mechanics of the squash nick shot. *Proceedings of the National Academy of Sciences*, 122(20):e2505715122, 2025.
- [4] A. Trevino, J.-W. Barotta, **Federle, M. E.**, and R. Zenit. Blending fluid mechanics and art part i: Making a splash with high speed photography. *The Physics Teacher*, 64:348–352, 2026.
- [5] **Federle, M. E.**, B. Lyons, M. Ravisankar, and R. Zenit. Design of an accessible turbulence chamber for laboratory experiments. *HardwareX*, page e00745, 2026.
- [6] J.-W. Barotta, **Federle, M. E.**, A. Trevino, and R. Zenit. Blending fluid mechanics and art part ii: Flow visualization. *The Physics Teacher*, NA:In press, 2026.
- [7] **Federle, M. E.**, J.-W. Barotta, A. Trevino, and R. Zenit. Blending fluid mechanics and art part iii: Fluids art installation. *The Physics Teacher*, NA:In press, 2026.
- [8] **Federle, M. E.** and R. Zenit. Formation of bubble emulsions in microgravity. (*in prep.*), 2026.
- [9] C. E. Brennen. Fundamentals of multiphase flow. 2005.
- [10] J. Karp, E. Mancilla, F. S. da Silva, D. Legendre, R. Zenit, and R. E. Morales. The dynamics of compound drops at high reynolds numbers: Drag, shape, and trajectory. *International Journal of Multiphase Flow*, 142:103699, 2021.
- [11] C. Hazar, M. Mollaoglu, and A. C. Töz. Fuzzy-based evaluation of the air bubble barrier as an alternative oil spill response equipment. *Water, Air, & Soil Pollution*, 236(8):485, 2025.
- [12] J. A. Nathanson and A. Ambulkar. Wastewater treatment. *Encyclopedia Britannica*, February 2026. Published 13 February 2026. Accessed 23 February 2026. Available at: <https://www.britannica.com/technology/wastewater-treatment>.
- [13] C. Colin, O. Kannengieser, W. Bergez, M. Lebon, J. Sebilliau, M. Sagan, and S. Tanguy. Nucleate pool boiling in microgravity: Recent progress and future prospects. *Comptes Rendus. Mécanique*, 345(1): 21–34, 2017.
- [14] Ö. Akay, M. Monfort-Castillo, T. St Francis, J. Becker, S. Saravanabavan, Á. Romero-Calvo, and K. Brinkert. Magnetically induced convection enhances water electrolysis in microgravity. *Nature Chemistry*, 17(11):1673–1679, 2025.
- [15] P. S. Wilson and R. A. Roy. An audible demonstration of the speed of sound in bubbly liquids. *American Journal of Physics*, 76(10):975–981, 2008.
- [16] Readers can access the Appendices at TPT Online at XXX, under the Supplemental tab.
- [17] R. Deegan, P. Brunet, and J. Eggers. Complexities of splashing. *Nonlinearity*, 21(1):C1, 2007.
- [18] R. Lenormand. Liquids in porous media. *Journal of Physics: Condensed Matter*, 2(S):SA79, 1990.

- [19] R. Kamakoti and W. Shyy. Fluid–structure interaction for aeroelastic applications. *Progress in Aerospace Sciences*, 40(8):535–558, 2004.
- [20] R. Clift, J. R. Grace, and M. E. Weber. *Bubbles, Drops and Particles*. Academic Press, New York, 1978.
- [21] M. Ishii and T. Hibiki. *Thermo-fluid dynamics of two-phase flow*. Springer Science & Business Media, 2010.
- [22] A. Prosperetti. Bubbles. *Physics of Fluids*, 16(6):1852–1865, 2004.
- [23] O. Atasi*, M. Ravisankar*, D. Legendre, and R. Zenit. Presence of surfactants controls the stability of bubble chains in carbonated drinks. *Physical Review Fluids*, 8(5):053601, 2023.
- [24] B. M. Adhikari, T. Truong, S. Prakash, N. Bansal, and B. Bhandari. Impact of incorporation of CO₂ on the melting, texture and sensory attributes of soft-serve ice cream. *International Dairy Journal*, 109: 104789, 2020.
- [25] J. Saththasivam and S. Loganathan, K. and Sarp. An overview of oil–water separation using gas flotation systems. *Chemosphere*, 144:671–680, 2016.
- [26] R. Moosai and R. A. Dawe. Gas attachment of oil droplets for gas flotation for oily wastewater cleanup. *Separation and Purification Technology*, 33(3):303–314, 2003.
- [27] W. T. Jones. Air barriers as oil-spill containment devices. *Society of Petroleum Engineers Journal*, 12 (02):126–142, April 1972. ISSN 0197-7520. DOI: [10.2118/3050-PA](https://doi.org/10.2118/3050-PA).
- [28] C. Hazar and A. C. Töz. Numerical and experimental simulation on efficiency of air bubble barrier on various pollutants for preventing oil spill spread. *Marine Pollution Bulletin*, 201:116254, April 2024. ISSN 0025-326X. DOI: [10.1016/j.marpolbul.2024.116254](https://doi.org/10.1016/j.marpolbul.2024.116254).
- [29] J.-M. Lo. The effect of air-bubble barriers in containing oil-slick movement. *Ocean Engineering*, 24(7): 645–663, July 1997. ISSN 0029-8018. DOI: [10.1016/S0029-8018\(96\)00037-6](https://doi.org/10.1016/S0029-8018(96)00037-6).
- [30] E. T. Igunnu and G. Z. Chen. Produced water treatment technologies. *International Journal of Low-Carbon Technologies*, 9(3):157–177, 2014.
- [31] J. Lee, W. Cho, K. Poo, S. Choi, T. Kim, E. Son, Y. Choi, Y. Kim, and K. Chae. Refractory oil wastewater treatment by dissolved air flotation, electrochemical advanced oxidation process, and magnetic biochar integrated system. *Journal of Water Process Engineering*, 36:101358, 2020.
- [32] J. H. Guan, S. I. Tamim, C. W. Magoon, H. A. Stone, and P. J. Sáenz. Galloping bubbles. *Nature Communications*, 16(1):1572, February 2025. ISSN 2041-1723. DOI: [10.1038/s41467-025-56611-5](https://doi.org/10.1038/s41467-025-56611-5).
- [33] G. Kaur, J. Fettig, M. Oldenburg, S. Grass, and C. Marx. Enhanced removal of microplastics using microflotation. *Science of The Total Environment*, 1013:181259, January 2026. ISSN 0048-9697. DOI: [10.1016/j.scitotenv.2025.181259](https://doi.org/10.1016/j.scitotenv.2025.181259).
- [34] J. Bird. Bubbling underwater to breakup biofilms and lift early settlers (bubbles). 2020.
- [35] J. J. Lee, J. D. Eifert, S. Jung, and L. K. Strawn. Cavitation bubbles remove and inactivate listeria and salmonella on the surface of fresh roma tomatoes and cantaloupes. *Frontiers in Sustainable Food Systems*, 2:61, 2018.

- [36] D. Lohse. Fundamental fluid dynamics challenges in inkjet printing. *Annual Review of Fluid Mechanics*, 54:349–382, 2022.
- [37] L. Lamoot, B. Manescau, K. Chetehouna, and N. Gascoin. Review on the effect of the phenomenon of cavitation in combustion efficiency and the role of biofuels as a solution against cavitation. *Energies*, 14(21):7265, 2021.
- [38] C. E. Brennen. The amazing world of bubbles. *Engineering and Science*, 70(1):30–41, 2007.
- [39] C. Konishi, H. Lee, I. Mudawar, M. M. Hasan, H. K. Nahra, N. R. Hall, J. D. Wagner, R. L. May, and J. R. Mackey. Flow boiling in microgravity: Part 1–interfacial behavior and experimental heat transfer results. *International Journal of Heat and Mass Transfer*, 81:705–720, 2015.
- [40] A. Sielaff, D. Mangini, O. Kabov, M. Q. Raza, A. I. Garivalis, M. Zupančič, S. Dehaeck, S. Evgenidis, C. Jacobs, D. Van Hoof, *et al.* The multiscale boiling investigation on-board the international space station: An overview. *Applied thermal engineering*, 205:117932, 2022.
- [41] M. Q. Raza, C. Colin, and J. Sebilliau. Coalescence-induced bubble departure in microgravity: Effect of number of bubbles. In *Conference on Fluid Mechanics and Fluid Power*, pages 587–597. Springer, 2023.
- [42] Á. Romero-Calvo, H. Schaub, and G. Cano-Gómez. Diamagnetically enhanced electrolysis and phase separation in low gravity. *Journal of Spacecraft and Rockets*, 59(1):59–72, 2022.
- [43] A. B. Wood and R. Lindsay. A textbook of sound, 1930.
- [44] R. Zenit and J. J. Feng. Hydrodynamic interactions among bubbles, drops, and particles in non-newtonian liquids. *Annual Review of Fluid Mechanics*, 50:505–534, 2018.
- [45] D. Legendre, J. Magnaudet, and G. Mougin. Hydrodynamic interactions between two spherical bubbles rising side by side in a viscous liquid. *Journal of Fluid Mechanics*, 497:133–166, 2003.
- [46] M. Ravisankar, A. G. Correa, Y. Su, and R. Zenit. Hydrodynamic interaction of a bubble pair in viscoelastic shear-thinning fluids. *Journal of Non-Newtonian Fluid Mechanics*, 309:104912, 2022.
- [47] V. Mathai, S. G. Huisman, C. Sun, D. Lohse, and M. Bourgoïn. Dispersion of air bubbles in isotropic turbulence. *Physical Review Letters*, 121(5):054501, 2018.
- [48] P. G. De Gennes, F. Brochard-Wyart, D. Quéré, *et al.* *Capillarity and wetting phenomena: drops, bubbles, pearls, waves*, volume 315. Springer, 2004.
- [49] B. Krull, S. Tschisgale, and J. Fröhlich. An immersed boundary method for complex-shaped bubbles represented by spherical harmonic functions. In *Conference: 9th International Conference on Multiphase Flow (ICMF)*, 2016.
- [50] M. K. Tripathi, K. C. Sahu, and R. Govindarajan. Dynamics of an initially spherical bubble rising in quiescent liquid. *Nature Communications*, 6(1):6268, 2015.
- [51] I. Leifer and R. K. Patro. The bubble mechanism for methane transport from the shallow sea bed to the surface: A review and sensitivity study. *Continental Shelf Research*, 22(16):2409–2428, 2002.
- [52] M. G. Domingos and S. S. Cardoso. Turbulent two-phase plumes with bubble-size reduction owing to dissolution or chemical reaction. *Journal of Fluid Mechanics*, 716:120–136, 2013.

- [53] Y. Hallez and D. Legendre. Interaction between two spherical bubbles rising in a viscous liquid. *Journal of Fluid Mechanics*, 673:406–431, 2011.
- [54] D. Legendre, R. Zenit, and J. R. Velez-Cordero. On the deformation of gas bubbles in liquids. *Physics of Fluids*, 24(4):043303, 2012.
- [55] S. Pesch, A. Schlüter, M., Z. M., Malone, D. K., Krause, and C. B. Paris. Behavior of rising droplets and bubbles: impact on the physics of deep-sea blowouts and oil fate. In *Deep Oil Spills*, pages 65–82. Springer, 2020.
- [56] G. Eidnes, F. Leirvik, T. A. McClimans, G. S.H, and E. Grimaldo. Containing oil spills by use of air bubbles. In *The Twenty-third International Offshore and Polar Engineering Conference*. OnePetro, 2013.
- [57] D. D. Joseph, Y. J. Liu, M. Poletto, and J. Feng. Aggregation and dispersion of spheres falling in viscoelastic liquids. *Journal of Non-newtonian Fluid Mechanics*, 54:45–86, 1994.
- [58] T. Sanada, A. Sato, M. Shirota, and M. Watanabe. Motion and coalescence of a pair of bubbles rising side by side. *Chemical Engineering Science*, 64(11):2659–2671, 2009.
- [59] D. Harris. Instructable diy syringe pump. www.instructables.com/DIY-Syringe-Pump/, 2016.
- [60] L. Ding and A. Goshtasby. On the canny edge detector. *Pattern Recognition*, 34(3):721–725, 2001.
- [61] J. F. Mei, R. and Klausner and C. J. Lawrence. A note on the history force on a spherical bubble at finite Reynolds number. *Physics of Fluids*, 6(1):418–420, 1994.
- [62] A. Rachih, D. Legendre, E. Climent, and S. Charton. Numerical study of conjugate mass transfer from a spherical droplet at moderate reynolds number. *International Journal of Heat and Mass Transfer*, 157:119958, 2020.
- [63] L. Schiller. Über die grundlegenden Berechnungen bei der Schwerkraftaufbereitung. *Z. Vereines Deutscher Inge.*, 77:318–321, 1933.
- [64] G. K. Batchelor. *An introduction to fluid dynamics*. Cambridge University Press, 2000.
- [65] H. Schlichting and K. Gersten. *Boundary-Layer Theory*. Springer Berlin Heidelberg, 2017.
- [66] M. R. Maxey and J. J. Riley. Equation of motion for a small rigid sphere in a nonuniform flow. *Physics of Fluids*, 26(4):883–889, 1983.
- [67] V. Mathai, D. Lohse, and C. Sun. Bubbly and buoyant particle–laden turbulent flows. *Annual Review of Condensed Matter Physics*, 11:529–559, 2020.
- [68] R. Van Hout, J. Eisma, G. E. Elsinga, and J. Westerweel. Experimental study of the flow in the wake of a stationary sphere immersed in a turbulent boundary layer. *Physical Review Fluids*, 3(2):024601, 2018.
- [69] D. Legendre, A. Merle, and J. Magnaudet. Wake of a spherical bubble or a solid sphere set fixed in a turbulent environment. *Physics of Fluids*, 18(4):048102, 2006.
- [70] H. Chakibi, I. Hénaut, A. Salonen, D. Langevin, and J.-F. Argillier. Role of bubble–drop interactions and salt addition in flotation performance. *Energy & Fuels*, 32(3):4049–4056, 2018.

- [71] R. M. Jenson, M. M. Wollman, A.P. and Weislogel, L. Sharp, R. Green, P. J. Canfield, J. Klatte, and M. E. Dreyer. Passive phase separation of microgravity bubbly flows using conduit geometry. *International Journal of Multiphase Flow*, 65:68–81, 2014.
- [72] A. Chesters. Modelling of coalescence processes in fluid-liquid dispersions: a review of current understanding. *Chemical Engineering Research and Design*, 69(A4):259–270, 1991.
- [73] D. Y. Chan, E. Klaseboer, and R. Manica. Film drainage and coalescence between deformable drops and bubbles. *Soft Matter*, 7(6):2235–2264, 2011.
- [74] R. E. Johnson and S. Sadhal. Fluid mechanics of compound multiphase drops and bubbles. *Annual Review of Fluid Mechanics*, 17(1):289–320, 1985.
- [75] D. Liao, Y. and Lucas. A literature review on mechanisms and models for the coalescence process of fluid particles. *Chemical Engineering Science*, 65(10):2851–2864, 2010.
- [76] B. Albijanic, O. Ozdemir, A. Nguyen, and D. Bradshaw. A review of induction and attachment times of wetting thin films between air bubbles and particles and its relevance in the separation of particles by flotation. *Advances in Colloid and Interface Science*, 159(1):1–21, 2010.
- [77] R. H. Davis, J. A. Schonberg, and J. M. Rallison. The lubrication force between two viscous drops. *Physics of Fluids A: Fluid Dynamics*, 1(1):77–81, 1989.
- [78] G. Jeffreys and G. Davies. Coalescence of liquid droplets and liquid dispersion. In *Recent Advances in Liquid-Liquid Extraction*, pages 495–584. Elsevier, 1971.
- [79] H. Schulze. Hydrodynamics of bubble-mineral particle collisions. *Mineral Processing and Extractive Metallurgy Review*, 5(1-4):43–76, 1989.
- [80] M. Niewiadomski, A. V. Nguyen, J. Hupka, J. Nalaskowski, and J. D. Miller. Air bubble and oil droplet interactions in centrifugal fields during air-sparged hydrocyclone flotation. *International Journal of Environment and Pollution*, 30(2):313–331, 2007.
- [81] A. W. Adamson, A. P. Gast, *et al.* *Physical chemistry of surfaces*, volume 15. Interscience publishers New York, 1967.
- [82] S. Yan, X. Yang, Z. Bai, X. Xu, and H. Wang. Drop attachment behavior of oil droplet-gas bubble interactions during flotation. *Chemical Engineering Science*, 223:115740, 2020.
- [83] R. Zenit and D. Legendre. The coefficient of restitution for air bubbles colliding against solid walls in viscous liquids. *Physics of Fluids*, 21(8), 2009.
- [84] G. Joseph, R. Zenit, M. Hunt, and A. Rosenwinkel. Particle-wall collisions in a viscous fluid. *Journal of Fluid Mechanics*, 433:329–346, 2001.
- [85] S. Yan, Y. Zhang, X. Yang, Y. Huang, Z. Bai, and X. Xu. Interfacial behavior and internal microflow of an oil droplet during the process of the oil droplet covering a gas bubble: Without and with NaCl. *Industrial & Engineering Chemistry Research*, 60(16):6006–6015, 2021.
- [86] H. Ami Ahmadi, A. Ebadi, and S. M. Hosseinalipour. Experimental study of bubble-droplet interaction in water; the effect of water salinity on the drainage time. *Journal of Dispersion Science and Technology*, pages 1–14, 2021.

- [87] M. Dudek and G. Øye. Microfluidic study on the attachment of crude oil droplets to gas bubbles. *Energy & Fuels*, 32(10):10513–10521, 2018.
- [88] G. Dobby and J. Finch. A model of particle sliding time for flotation size bubbles. *Journal of Colloid and Interface Science*, 109(2):493–498, 1986.
- [89] J. Je, D. Lee, J. Kwon, and H. Cho. Simulation of bubble–particle attachment process and estimation of attachment probability using a coupled smoothed particle hydrodynamics–discrete element method model. *Minerals Engineering*, 183:107581, 2022.
- [90] Y. Ye and J. Miller. The significance of bubble/particle contact time during collision in the analysis of flotation phenomena. *International Journal of Mineral Processing*, 25(3-4):199–219, 1989.
- [91] J. Chevaillier, E. Klaseboer, O. Masbernat, and C. Gourdon. Effect of mass transfer on the film drainage between colliding drops. *Journal of Colloid and Interface Science*, 299(1):472–485, 2006.
- [92] A. Nguyen, H. Schulze, and J. Ralston. Elementary steps in particle–bubble attachment. *International Journal of Mineral Processing*, 51(1-4):183–195, 1997.
- [93] Z. Dai, D. Fornasiero, and J. Ralston. Particle–bubble attachment in mineral flotation. *Journal of Colloid and Interface Science*, 217(1):70–76, 1999.
- [94] I. Bazhlekoy, A. Chesters, and F. Van de Vosse. The effect of the dispersed to continuous-phase viscosity ratio on film drainage between interacting drops. *International Journal of Multiphase Flow*, 26(3):445–466, 2000.
- [95] H. Lhuissier and E. Villermaux. Bursting bubble aerosols. *Journal of Fluid Mechanics*, 696:5–44, 2012.
- [96] E. Calzavarini, M. Kerscher, D. Lohse, and F. Toschi. Dimensionality and morphology of particle and bubble clusters in turbulent flow. *Journal of Fluid Mechanics*, 607:13–24, 2008.
- [97] E. Calzavarini, M. Cencini, D. Lohse, and F. Toschi. Quantifying turbulence-induced segregation of inertial particles. *Physical Review Letters*, 101:084504, Aug 2008. DOI: [10.1103/PhysRevLett.101.084504](https://doi.org/10.1103/PhysRevLett.101.084504). URL <https://link.aps.org/doi/10.1103/PhysRevLett.101.084504>.
- [98] I. Vakarelski and S. Kamoliddinov, F. and Thoroddsen. Why bubbles coalesce faster than droplets: The effects of interface mobility and surface charge. *Langmuir*, 2024.
- [99] W. Weyl. Surface structure of water and some of its physical and chemical manifestations. *Journal of Colloid Science*, 6(5):389–405, 1951.
- [100] J. Beattie and A. Djerdjev. The pristine oil/water interface: Surfactant-free hydroxide-charged emulsions. *Angewandte Chemie International Edition*, 43(27):3568–3571, 2004.
- [101] K. Marinova, R. Alargova, N. Denkov, O. Velez, D. Petsev, I. Ivanov, and R. Borwankar. Charging of oil- water interfaces due to spontaneous adsorption of hydroxyl ions. *Langmuir*, 12(8):2045–2051, 1996.
- [102] K. Giribabu, M. Reddy, and P. Ghosh. Coalescence of air bubbles in surfactant solutions: Role of salts containing mono-, di-, and trivalent ions. *Chemical Engineering Communications*, 195(3):336–351, 2007.
- [103] Y. M. Yang and J. R. Maa. Bubble coalescence in dilute surfactant solutions. *Journal of Colloid and Interface Science*, 98(1):120–125, 1984.

- [104] J. D. Paulsen, R. Carmigniani, A. Kannan, J. C. Burton, and S. R. Nagel. Coalescence of bubbles and drops in an outer fluid. *Nature Communications*, 5(1):1–7, 2014.
- [105] A. Bera, A. Mandal, and B. B. Guha. Synergistic effect of surfactant and salt mixture on interfacial tension reduction between crude oil and water in enhanced oil recovery. *Journal of Chemical & Engineering Data*, 59(1):89–96, January 2014. ISSN 0021-9568. DOI: [10.1021/je400850c](https://doi.org/10.1021/je400850c). URL <https://doi.org/10.1021/je400850c>. Publisher: American Chemical Society.
- [106] A. L. McCutchan and B. A. Johnson. An experimental apparatus for generating homogeneous isotropic turbulence. *Experiments in Fluids*, 64(11):177, 2023.
- [107] E. A. Variano and E. A. Cowen. A random-jet-stirred turbulence tank. *Journal of Fluid Mechanics*, 604:1–32, 2008.
- [108] O. A. Manna, M. S. Mansour, S. H. Chung, and W. L. Roberts. Characterization of turbulence in an optically accessible fan-stirred spherical combustion chamber. *Combustion Science and Technology*, 193(7):1231–1257, 2021.
- [109] C. Wang, S. G. Huisman, T. Basset, R. Volk, and M. Bourgoïn. Lagrangian flow statistics in experimental homogeneous isotropic turbulence. *arXiv preprint arXiv:2503.03891*, 2025.
- [110] R. Zimmermann, H. Xu, Y. Gasteuil, M. Bourgoïn, R. Volk, J.-F. Pinton, E. Bodenschatz, *et al.* The lagrangian exploration module: An apparatus for the study of statistically homogeneous and isotropic turbulence. *Review of Scientific Instruments*, 81(5), 2010.
- [111] A. N. Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large reynolds numbers. *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, 434(1890):9–13, 1991.
- [112] A. L. McCutchan, C. R. Meyer, and B. A. Johnson. Enhancement of ice melting in isotropic turbulence. *Physical Review Fluids*, 9(7):074601, 2024.
- [113] M. Pinsky and A. Khain. A model of a homogeneous isotropic turbulent flow and its application for the simulation of cloud drop tracks. *Geophysical & Astrophysical Fluid Dynamics*, 81(1-2):33–55, 1995.
- [114] D. R. Caldwell and J. N. Mourn. Turbulence and mixing in the ocean. *Reviews of Geophysics*, 33(S2): 1385–1394, 1995.
- [115] H. Wang, Y. Xu, and J. Wang. Experimental study on bubble pairs and induced flow fields using tomographic particle image velocimetry. *Experiments in Fluids*, 66(5):1–27, 2025.
- [116] M. Federle. Open-source design of an accessible turbulence chamber. <https://github.com/Madeline-Federle/Open-Source-Design-of-an-Accessible-Turbulence-Chamber>, 2025. GitHub repository.
- [117] F. Risso and J. Fabre. Oscillations and breakup of a bubble immersed in a turbulent field. *Journal of Fluid Mechanics*, 372:323–355, 1998.
- [118] N. Jeyaprakash, A. Haile, and M. Arunprasath. The parameters and equipments used in tig welding: A review. *The International Journal of Engineering and Science (IJES)*, 4(2):11–20, 2015.
- [119] H. Jansen. Impact of toroidal propeller design on unmanned aerial vehicle acoustic signature and aerodynamic performance. *Int. J. Aerosp. Eng.(IJASE)*, 2:1–11, 2024.

- [120] I. Gabriel and I. Simion. Performance of 3d printed conventional and toroidal propeller for small multirotor drones. *Journal of Industrial Design and Engineering Graphics*, 18(1):27–32, 2023.
- [121] H. Tennekes and J. L. Lumley. *A first course in turbulence*. MIT press, 1972.
- [122] W. Thielicke and R. Sonntag. Particle image velocimetry for matlab: Accuracy and enhanced algorithms in pivlab. *Journal of Open Research Software*, 9(1), 2021.
- [123] P. Davidson. *Turbulence: an introduction for scientists and engineers*. Oxford University Press, 2015.
- [124] Y. Li, E. Perlman, M. Wan, Y. Yang, C. Meneveau, R. Burns, S. Chen, A. Szalay, and G. Eyink. A public turbulence database cluster and applications to study lagrangian evolution of velocity increments in turbulence. *Journal of Turbulence*, (9):N31, 2008.
- [125] D. McWilliam and R. Duggins. Speed of sound in bubbly liquids. In *Proceedings of the Institution of Mechanical Engineers, Conference Proceedings*, volume 184, pages 102–107. SAGE Publications Sage UK: London, England, 1969.
- [126] S. Domenico. Acoustic wave propagation in air-bubble curtains in water—part i: history and theory. *Geophysics*, 47(3):345–353, 1982.
- [127] K. W. Commander and A. Prosperetti. Linear pressure waves in bubbly liquids: Comparison between theory and experiments. *The Journal of the Acoustical Society of America*, 85(2):732–746, 1989.
- [128] B. Würsig, C. Greene Jr, and T. Jefferson. Development of an air bubble curtain to reduce underwater noise of percussive piling. *Marine environmental research*, 49(1):79–93, 2000.
- [129] A. Tsouvalas and A. Metrikine. Noise reduction by the application of an air-bubble curtain in offshore pile driving. *Journal of Sound and Vibration*, 371:150–170, 2016.
- [130] S. D. Jarriel, Y. Jézéquel, N. Formel, S. Cones, J. A. Stanley, and T. A. Mooney. Pile driving sound induces short-term behavioral changes in black sea bass (*centropristis striata*): A field study. *The Journal of the Acoustical Society of America*, 157(4):2350–2364, 2025.
- [131] Y. Peng and A. Tsouvalas. A three-dimensional model of an air bubble curtain in offshore pile driving. In *Journal of Physics: Conference Series*, volume 2647, page 232006. IOP Publishing, 2024.
- [132] M. R. Noatch and C. D. Suski. Non-physical barriers to deter fish movements. *Environmental Reviews*, 20(1):71–82, 2012.
- [133] C. Croëne, J. O. Vasseur, L. Roux, C. Audoly, and A.-C. Hladky. A review of acoustic metamaterials for naval and underwater defense applications: from historical concepts to new trends. *Acta Acustica*, 9:24, 2025.
- [134] M. Dähne, J. Tougaard, J. Carstensen, A. Rose, and J. Nabe-Nielsen. Bubble curtains attenuate noise from offshore wind farm construction and reduce temporary habitat loss for harbour porpoises. *Marine Ecology Progress Series*, 580:221–237, 2017.
- [135] V. Leroy, M. Devaud, and J.-C. Bacri. The air bubble: Experiments on an unusual harmonic oscillator. *American Journal of Physics*, 70(10):1012–1019, 2002.
- [136] M. Ravisankar and R. Zenit. Velocity fluctuations for bubbly flows at small Re. *Journal of Fluid Mechanics*, 1001:A34, 2024.

- [137] X. Shi, D. Qi, C. Lin, and J. Li. A technical review on characterization methods for structures and properties of emulsion. *APL Materials*, 12(11), 2024.
- [138] D. Venkataramani, A. Tsulaia, and S. Amin. Fundamentals and applications of particle stabilized emulsions in cosmetic formulations. *Advances in Colloid and Interface Science*, 283:102234, 2020.
- [139] B. A. Khan, N. Akhtar, H. M. S. Khan, K. Waseem, T. Mahmood, A. Rasul, M. Iqbal, and H. Khan. Basics of pharmaceutical emulsions: A review. *African journal of pharmacy and pharmacology*, 5(25): 2715–2725, 2011.
- [140] G. Muscholik. Multiple emulsions for food use. *Current Opinion in Colloid & Interface Science*, 12 (4-5):213–220, 2007.
- [141] S. Kang, A. S. Sangani, and D. L. Tsao, H. and Koch. Rheology of dense bubble suspensions. *Physics of Fluids*, 9(6):1540–1561, 1997.
- [142] M. E. Rosti, F. De Vita, and L. Brandt. Numerical simulations of emulsions in shear flows. *Acta Mechanica*, 230(2):667–682, 2019.
- [143] M. Narcy and C. Colin. Two-phase pipe flow in microgravity with and without phase change: recent progress and future prospects. *Interfacial Phenomena and Heat Transfer*, 3:1–17, 2015.
- [144] J. Banhart, F. García-Moreno, S. Hutzler, D. Langevin, L. Liggieri, R. Miller, A. Saint Jalmes, and D. Weaire. Foams and emulsions in space. *Europhysics News*, 39(4):26–28, 2008.
- [145] L. Liggieri, M. Ferrari, A. Passerone, F. Ravera, G. Loglio, P. Pandolfini, A. Steinchen, M. Antoni, A. Sanfeld, R. Miller, *et al.* Microgravity as a tool for studies on emulsion stability. In *ESA Special Publication*, volume 1290, pages 150–167, 2005.
- [146] R. Ni. Deformation and breakup of bubbles and drops in turbulence. *Annual Review of Fluid Mechanics*, 56(1):319–347, 2024.
- [147] J. O. Hinze. Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. *AIChE journal*, 1(3):289–295, 1955.
- [148] S. Zhong and R. Ni. On the breakup frequency of bubbles and droplets in turbulence: a compilation and evaluation of experimental data. *International Journal of Multiphase Flow*, 176:104842, 2024.
- [149] D. Legendre and R. Zenit. Gas bubble dynamics. *Reviews of Modern Physics*, 97(2):025001, 2025.
- [150] F. A. Williams. *Combustion theory*. CRC Press, 2018.
- [151] A. Håkansson. On the validity of different methods to estimate breakup frequency from single drop experiments. *Chemical Engineering Science*, 227:115908, 2020.
- [152] Professional Squash Association. Squash facts. <https://www.psasquashtour.com/>, 2024. Accessed: 2024-11-13.
- [153] R. Cross. Impact of sports balls with striking implements. *Sports Eng.*, 17:3–22, 2014.
- [154] C. Cohen and C. Clanet. Physics of ball sports. *Europhysics News*, 47(3):13–16, 2016.
- [155] R. Cross. The coefficient of restitution for collisions of happy balls, unhappy balls, and tennis balls. *American Journal of Physics*, 68(11):1025–1031, 2000.

- [156] G. J. Lewis, J. C. Arnold, and I. W. Griffiths. The dynamic behavior of squash balls. *American Journal of Physics*, 79(3):291–296, 2011.
- [157] SquashTV. Squash top 10 rolers: A collection of amazing nick shots. <https://www.youtube.com/watch?v=ELUvl26BKbk&t=2s>, 2020. Accessed: 2025-02-19.
- [158] S. Rowson, S. M. Duma, J. G. Beckwith, J. J. Chu, R. M. Greenwald, J. J. Crisco, P. G. Brolinson, A. Duhaime, T. W. McAllister, and A. C. Maerlender. Rotational head kinematics in football impacts: an injury risk function for concussion. *Annals of Biomedical Engineering*, 40:1–13, 2012.
- [159] S. Jaisee, F. Yue, and Y. H. Ooi. A state-of-the-art review on passive friction dampers and their applications. *Engineering Structures*, 235:112022, 2021.
- [160] A. N. Gent. A new constitutive relation for rubber. *Rubber Chemistry and Technology*, 69(1):59–61, 1996.
- [161] A. Das, K. S. Breuer, and V. Mathai. Nonlinear modeling and characterization of ultrasoft silicone elastomers. *Applied Physics Letters*, 116(20), 2020.
- [162] N. Maw, J. R. Barber, and J. N. Fawcett. The oblique impact of elastic spheres. *Wear*, 38(1):101–114, 1976.
- [163] K. L. Johnson. *Contact mechanics*. Cambridge University Press, 1987.
- [164] A. Theadom, N. J. Starkey, T. Dowell, P. A. Hume, M. Kahan, K. McPherson, V. Feigin, B. R. Group, et al. Sports-related brain injury in the general population: an epidemiological study. *Journal of Science and Medicine in Sport*, 17(6):591–596, 2014.
- [165] J. Misra and S. Chakravarty. A study on rotational brain injury. *Journal of Biomechanics*, 17(7): 459–466, 1984.
- [166] K. Hansen, N. Dau, F. Feist, C. Deck, R. Willinger, S. M. Madey, and M. Bottlang. Angular impact mitigation system for bicycle helmets to reduce head acceleration and risk of traumatic brain injury. *Accident Analysis and Prevention*, 59:109–117, 2013.
- [167] B. F. Berencsi and A. Kossa. Analyzing the effect of temperature on squash ball impacts using high-speed camera recordings. *Periodica Polytechnica Mechanical Engineering*, 65(4):354–362, 2021.
- [168] G. G. Joseph and M. L. Hunt. Oblique particle–wall collisions in a liquid. *Journal of Fluid Mechanics*, 510:71–93, 2004.
- [169] R. Cross. Bounce of a spinning ball near normal incidence. *American Journal of Physics*, 73(10): 914–920, 2005.
- [170] A. M. Worthington. *A study of splashes*. Longmans, Green, and Company, 1908.
- [171] H. E. Edgerton and J. R. Killian. *Flash!: Seeing the unseen by ultra high-speed photography*. CT Branford Company, 1954.
- [172] A. L. Yarin. Drop impact dynamics: splashing, spreading, receding, bouncing. *Annual Review of Fluid Mechanics*, 38:159–192, 2006.
- [173] R. Krechetnikov and G. M. Homsy. Crown-forming instability phenomena in the drop splash problem. *Journal of Colloid and Interface Science*, 331(2):555–559, 2009.

- [174] L. V. Zhang, P. Brunet, J. Eggers, and R. D. Deegan. Wavelength selection in the crown splash. *Physics of Fluids*, 22(12):122105, 2010.
- [175] P. Shah and M. M. Driscoll. Drop impact dynamics of complex fluids: A review. *Soft Matter*, 2024.
- [176] M. H. Land. Full steam ahead: The benefits of integrating the arts into stem. *Procedia computer science*, 20:547–552, 2013.
- [177] M. K. Daugherty. The prospect of an” a” in stem education. *Journal of STEM Education: Innovations and Research*, 14(2), 2013.
- [178] V. A. Segarra, B. Natalizio, C. V. Falkenberg, S. Pulford, and R. M. Holmes. Steam: Using the arts to train well-rounded and creative scientists. *Journal of microbiology & biology education*, 19(1):10–1128, 2018.
- [179] J. Day, É. Lantagne-Hurtubise, A. Sheyerman, Y.-H. T. Liao, and C. Hoyt. Quantum matter on the table: A pretty, simple hands-on activity. *The Physics Teacher*, 59(1):34–37, 2021.
- [180] D. J. Horne, L. Zheng, and B. King. A simple and cost-effective fluid dynamics apparatus to engage students in the classroom and laboratory. *The Physics Teacher*, 62(5):330–334, 2024.
- [181] D. J. Acheson. *Elementary fluid dynamics*. Oxford University Press, 1990.
- [182] T. R. Institution. The science of vortex rings. <https://www.youtube.com/watch?v=yjgACB7urOo>, 2018. YouTube, video, accessed <Access Date>.
- [183] J. Taddeucci, J. J. Peña Fernández, V. Cigala, U. Kueppers, P. Scarlato, E. Del Bello, T. Ricci, J. Sesterhenn, and S. Panunzi. Volcanic vortex rings: Axial dynamics, acoustic features, and their link to vent diameter and supersonic jet flow. *Geophys. Res. Lett.*, 48(15):e2021GL092899, 2021.
- [184] M. H. Dickinson, C. T. Farley, R. J. Full, M. A. R. Koehl, R. Kram, and S. Lehman. How animals move: an integrative view. *Science*, 288(5463):100–106, 2000.
- [185] P. G. Saffman. *Vortex dynamics*. Cambridge University Press, 2012.
- [186] S. B. Pope. *Turbulent Flows*. Cambridge University Press, 2012.
- [187] M. Van Dyke. *An album of fluid motion*, volume 176. Parabolic Press Stanford, 1982.
- [188] Kipkay. How to make vortex cannons. <https://www.instructables.com/How-to-Make-Vortex-Cannons/>, 2008. Accessed: 2025-10-06.
- [189] D. Brown. Video modeling: combining dynamic model simulations with traditional video analysis. In *American Association of Physics Teachers (AAPT) Summer Meeting*, pages 19–23, 2008.
- [190] C. E. Brennen. *Fundamentals of Fluid Mechanics*. 2005. URL <http://brennen.caltech.edu/fluidbook/Fluidbook.htm>. Online textbook, California Institute of Technology.
- [191] G. H. McKinley. Getting the (dimensionless) numbers right. *Nature Chemical Engineering*, 1(1): 116–116, 2024.
- [192] P. G. Saffman. *Vortex Dynamics*. Cambridge Monographs on Mechanics. Cambridge University Press, 1993.
- [193] S. Kida, M. Takaoka, and F. Hussain. Collision of two vortex rings. *J. Fluid Mech.*, 230:583–646, 1991.

- [194] M. R. Dhanak and B. de Bernardinis. The evolution of an elliptic vortex ring. *J. Fluid Mech.*, 109: 189–216, 1981.
- [195] G. Bangera and S. E. Brownell. Course-based undergraduate research experiences can make scientific research more inclusive. *CBE: Life Sciences Education*, 13(4):602–606, 2014.
- [196] D. Harris, K. Schlueter-Kuck, E. Austin, and K. Cohen. Course-based undergraduate research in upper-level engineering electives: A case study. *Journal of STEM Education: Innovations and Research*, 22(3), 2021.
- [197] M. C. Linn, E. Palmer, A. Baranger, E. Gerard, and E. Stone. Undergraduate research experiences: Impacts and opportunities. *Science*, 347(6222):1261757, 2015.
- [198] E. L. Dolan. Course-based undergraduate research experiences: Current knowledge and future directions. *National Research Council Committee Papers*, 1:1–34, 2016.
- [199] L. Brandimarte, A. Funk, and B. Richter. Mixing fluid mechanics with circus: how the performing arts can enhance learning in an undergraduate engineering course. *European Journal of Engineering Education*, 49(5):889–910, 2024.
- [200] J. Hertzberg and A. Sweetman. Images of fluid flow: Art and physics by students. *Journal of Visualization*, 8(2):145–152, 2005.
- [201] S. Friedlander and V. Yudovich. Instabilities in fluid motion. *Notices of the American Mathematical Society*, 46(11):1358–1367, 1999.
- [202] P. Bergé and M. Dubois. Rayleigh-bénard convection. *Contemporary Physics*, 25(6):535–582, 1984.
- [203] D. R. Mulford. Liquid nitrogen cloud demonstration for large and small groups. *Journal of Chemical Education*, 98(6):2029–2035, 2021.
- [204] R. Caballero. *Physics of the Atmosphere*. IOP Publishing, 2014.
- [205] K. Wark. *Thermodynamics 4th Ed*. McGraw-Hill, 1983.
- [206] P. K. Kundu, I. M. Cohen, and D. R. Dowling. *Fluid Mechanics*. Academic press, 2015.
- [207] M. H. Sohn and J. W. Chang. Visualization and piv study of wing-tip vortices for three different tip configurations. *Aerospace Science and Technology*, 16(1):40–46, 2012.
- [208] W. Devenport, M. C. Rife, S. I. Liapis, and G. J. Follin. The structure and development of a wing-tip vortex. *Journal of Fluid Mechanics*, 312:67–106, 1996.
- [209] M. Zdravkovich. Different modes of vortex shedding: an overview. *Journal of Fluids and Structures*, 10(5):427–437, 1996.

APPENDICES

The following appendix sections present several scholarly works undertaken during my doctoral studies that are not directly related to the central focus of this thesis. Nevertheless, these projects were instrumental in developing my research foundation and contributed significantly to my technical training and professional growth throughout the degree.

The first appendix, [A](#), describes collaborative work conducted with members of the Zenit Laboratory on The Mechanics of the Squash Nick Shot. In this project, I played a leading role in the experimental effort, which included the design and construction of a pressurized air cannon, the development of a nick-shot impact platform, and the execution of controlled experiments with squash players.

The subsequent appendix sections, [B](#), [C](#), and [D](#) comprise a three-part series titled Blending Fluid Mechanics and Art. These works were conducted and authored by myself, Prof. Roberto Zenit, Avery Trevino, and Jack-William Barrotta. The papers document the framework of a course developed by Prof. Zenit that integrates principles of fluid mechanics with the arts. These contributions were published in a teaching-focused journal with the aim of encouraging broader adoption of interdisciplinary, laboratory-based learning approaches and highlighting the aesthetic and conceptual connections between fluid mechanics and the arts.

A THE MECHANICS OF THE SQUASH NICK SHOT

Abstract Squash is a widely popular racket sport, practiced by millions of people worldwide, played inside a walled court. When played well, players can last for several minutes before the ball bounces twice on the floor. There is, however, an unanswerable shot. When the ball hits the nick between a vertical wall and the floor, under certain conditions, it rolls without any vertical bounce. We study this process experimentally. We determined that the ball must hit the vertical wall first at a narrow range of heights above the floor, but most importantly, it must touch the floor before finishing its rolling time on the vertical wall. When the rolling time is shorter than the contact time, the vertical momentum is canceled due to a mechanical frustration condition. This behavior is explained considering a contact model, which agrees with the experimental observations. In addition to its relevance to other walled ball sports, this understanding could help design rolling shock dampers with possible practical applications.

Information This work is published in *Proceedings of the National Academy of Sciences* [3]. We thank Mark Johnson and Alex Zaslavsky from the Rhode Island Squash Association and Stefano Crema from Squash Busters Providence for animated discussions. We are also grateful to Coach Arthur Gaskin, from Brown University, for his wisdom in all things squash.

A.1 Introduction

Squash is a racket sport played in a four-walled court between two players. Players take turns hitting the ball in allowable areas on the walls such that the opponent is unable to return the ball back before bouncing twice on the floor. According to the Professional Squash Association [152], squash is played by 3 million people in over 150 countries. The sport was recently approved to be included in the 2028 Summer Olympics games in Los Angeles.

The pace, speed and court size of racket sports is determined by the mechanics of the ball [153, 154]. What distinguishes squash from other racket sports is the ball and its interaction with the walls. The ball is a soft-deformable hollow rubber shell that ‘squashes’ significantly when it impacts the wall (hence the name); it is characterized by a low coefficient of restitution [155, 156]. Professional players become incredibly

skilled at retrieving the ball from all parts of the court making rallies long and physically demanding. One shot in squash is highly sought-after because it is unanswerable, the nick shot. When the ball hits the nick (the corner between a vertical wall and the floor), in some conditions, it rolls on the floor instead of bouncing [157]. This shot occurs roughly 8.7 % in professional games (see Supplemental Information, SI); hence, it plays a significant role in the outcome of a match. Similar shots are also observed in racquetball, padel, Gaelic handball, fronton (American handball) and Jai Alai (in this sport, the nick shot is called “chula”). From our observations, we determined that nick shots occur when the ball first hits one of the vertical walls (side or back) moving approximately perpendicular to it (see Fig. A.1(a)), descending at angles of incidence, θ_0 , ranging from 13 to 40 degrees, and at a range of different speeds. To our knowledge, no mechanical interpretation of this peculiar behavior exists.

Experiments were conducted to identify the set of conditions needed to observe a rolling nick shot. We rationalize our measurements by placing the results on a map of the ratio of the characteristic times (contact to rolling) and the dimensionless elasticity of the ball (the Cauchy number) for collisions near a nick. We conclude by offering an explanation of the behavior sustained on a mechanical model that predicts the conditions at which the ball ceases to bounce vertically after the interaction with the nick. Understanding the mechanics of the nick shot could be relevant beyond squash and sports mechanics. This knowledge could be useful in designing rotational dampers in sport injury prevention [158] and manufacturing control [159].

A.2 Experimental Setup

The experiments were conducted in the controlled environment of our lab rather than on the squash courts. To simulate the player hitting the squash ball, a pressurized air cannon was used to launch a squash ball towards an acrylic wall. The air cannon, constructed of PVC piping with an inner diameter close to that of the diameter of the balls, a plastic chamber, a manometer, and a valve. Once pressurized to a certain value, the valve was quickly opened to release the squash ball at a certain speed U_0 , aimed at the corner at an incident angle θ_0 . The pressure inside the air cannon was varied from 15 psi to 40 psi to produce a range of impact speeds, as seen in Table 1 in the main manuscript. The acrylic wall represents the nick, where the vertical wall and the floor meet. The corner, constructed from two pieces of 7.5 cm thick acrylic plate, was

mounted using aluminum profiles about 1 meter away from the air cannon. The angle at which the acrylic corner was mounted was then varied from 20 to 40 degrees to produce different incident angles. The impact location H was varied by displacing the acrylic corner back and forth, while keeping the position of the cannon fixed.

Three different squash balls were used, a single dot (for intermediate), a double dot (for professional players), and a hard ball (used in doubles), all manufactured by Dunlop. The diameter of the balls were $D = 40$ mm, and their masses were $m = 24$ grams.

The ball's Young's modulus was determined experimentally following the procedure described below. For the collision experiments and the compression tests, the balls were tested at room temperature, 22°C, and at 40°C, which we determined to be a typical temperature for a squash ball during a hard game. The ball was heated using a heating pad. The temperature was measured using a hand-held infrared heat gun, with an accuracy of $\pm 2^\circ\text{C}$.

The interaction between the squash ball and the wall was recorded using a high-speed camera (*Photron FASTCAM SA5*) filming at 5000 fps, with a shutter speed of 1/20000. Examples of typical experiments can be seen in movies below. The squash ball was marked up with white stripes to help image tracking.

The images collected were analyzed using image processing techniques on Matlab. Using the diameter of the squash ball for calibration, the location of the ball was tracked in time, as shown in Fig. 1(c) of the main manuscript. Hence, both vertical and horizontal velocities of the ball geometric center were determined during the interaction with the wall. The ball speed was determined in two directions, considering a central difference scheme. The occurrence of a nick shot was determined from these measurements. A collision was considered a nick if the vertical velocity after the interaction was smaller than $0.1U_0 \sin \theta_0$.

A.2.1 Measurement of the ball's Young's Modulus

To determine the Young's Modulus of the squash balls, compressive tests were conducted using an Instron machine. The machine was set to compress each ball at a controlled rate of 2 mm/s up to a predetermined deformation limit of 25 mm, well below the permanent deformation limit. The applied force as a function of deformation showed a non-linear behavior. We, therefore, used the two-parameter hyperelastic Gent model [160, 161], to determine the effective Young's modulus of the balls. The stress, σ was calculated as the ratio of the compression force over the cross sectional area of the ball. The strain, ϵ , was calculated as

the ratio of the deformation and the ball diameter. The data was fitted to

$$\sigma = G_m \left(\lambda - \frac{1}{\lambda^2} \right), \quad (\text{A.1})$$

where, $\lambda = 1 + \epsilon$ is the stretch ratio and G_m is normalized shear modulus defined as

$$G_m = G J_m / (J_m - I_1 + 3)$$

where G is the shear modulus, J_m is the locking parameter and I_1 is the first invariant of the left Cauchy-Green deformation gradient tensor. The fitting of the data to this model yielded the material constants, G and J_m . The Young's modulus was determined by

$$E = 2G(1 + \nu), \quad (\text{A.2})$$

where $\nu = 0.5$ is the Poisson ration for rubber. The values of E are shown in Table 1 of the main document.

A.2.2 Materials and Methods

Controlled collisions were produced with an air gun. It deployed squash balls (Dunlop, Professional 2-Dot, one yellow dot and elite doubles) at a range of speeds, close to those use in regular squash. The incident angle and the initial contact with the vertical wall was varied, by moving the target and its inclination. The collisions were recorded with a high speed camera. The position was determined using image processing software; the velocity was calculated using a central-difference scheme. The ball was directed to hit the nick between a vertical and a horizontal wall at different locations, H , speeds, U_0 , and incidence angles, θ_0 , see Fig. A.1(a). Experiments were conducted for three different balls, two different ball temperatures and two fixed incident angles. All experiments consider no initial rotational speed, $\omega_0 = 0$. Table A.1 summarizes all the experimental conditions considered.

We rationalize the interaction by considering the following phases: (i) contact with the wall, $t = t_1$; (ii) compression and rolling, $t_1 < t < t_2$; (iii) contact with the floor $t = t_2$; and (iv) bounce or nick process, $t > t_2$. Figures A.1(a) and A.1(b) show nomenclature and snapshots of a typical collision at $t = t_1$ and at $t = t_2$, respectively. Figures A.1(c) and A.1(d) show the trajectory and the speed time evolution, respectively,

Table A.1: Range of conditions and ball properties for all experiments. For all cases, $0.5 < H^* < 1.0$.

Case	Ball type	E (kPa)	U_o (m/s)	Ca
1	Single dot (○, ●)	99	33 to 58	0.05 to 0.12
2	Double dot (□, ■)	105	18 to 58	0.04 to 0.13
3	Doubles (△, ▲)	471	17 to 58	0.16 to 2.26
4	Single dot, at 40° C (○, ●)	96	30 to 52	0.04 to 0.15
5	Double dot, at 40° C (□, ■)	90	33 to 56	0.04 to 0.11
6	Doubles, at 40° C (△, ▲)	450	30 to 55	0.21 to 0.62

of three typical collisions (cases C1, C2 and C3, see movies in the SI). In particular, the case C1 shows the nick behavior, depicted by the red symbols in the figure. The ball first touches the vertical wall at a point A , at distance $H^* = H/D > 0.62$, at $t = t_1$. Immediately after, the ball slows down as it exchanges momentum with the wall. As the ball compresses, it also rolls without sliding at the contact point A [162] (conditions discussed in the SI). Hence, the ball's center of mass continues to move downwards (see vertical speed of the ball, U_{cm} , for $t_1 < t < t_2$), even though the point A is fixed. As the interaction evolves, the ball eventually touches the floor on point B at $t = t_2$, Fig. A.1(b). Soon after this happens, the vertical velocity of the ball becomes zero. The most impressive part of the nick behavior is displayed after this moment. As the ball decompresses, its horizontal velocity gradually increases until losing contact with the wall but, most importantly, its vertical velocity remains very close to zero. In other words, the ball leaves the corner interaction with a nearly horizontal trajectory, as seen in Fig. A.1(c). Two other cases are shown in the figure, for $H^* = 0.51$ and $H^* = 0.81$. Even though the ball collides at approximately the same speed and incident angle, the nick behavior is not observed (detailed description in SI).

A.3 Condition for a nick shot

We hypothesize that the nick will occur if the ball makes contact with the floor before it has ended its rolling motion. In other words, the contact time has to be longer than the rolling time. We can estimate the characteristic time for the compression, t_c , considering the collision of an elastic ball against a rigid wall [163]. The rolling time, t_r , can be calculated by considering the displacement of the ball's center of mass during contact and the rotational inertia of the ball (details of calculations appear in the SI).

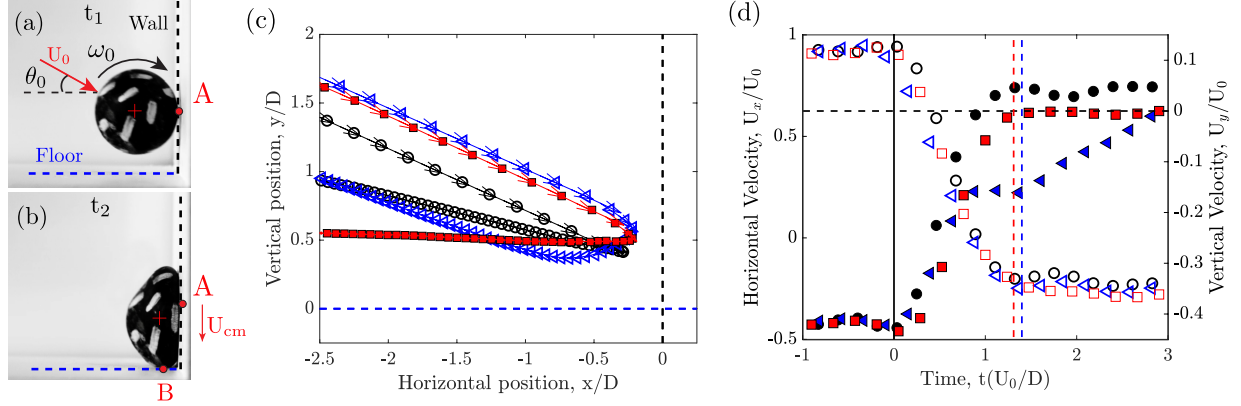


Figure A.1: (a) Nomenclature, ball first contact with vertical wall, $t = t_1$. (b) Ball first contact with floor, $t = t_2$. (c) Trajectory of the ball for three cases: Case C1, filled red squares, nick shot, $Ca = 0.25$, $H^* = 0.62$, $\theta_0 = 24.43^\circ$ and $\tau = 0.53$; Case C2, empty black circles, ball hits wall and floor simultaneously, $Ca = 0.24$, $H^* = 0.51$, $\theta_0 = 24.34^\circ$ and $\tau = 0.76$; Case C3, empty blue triangles, ball hits vertical wall at higher H , $Ca = 0.25$, $H^* = 0.81$; $\theta = 24.31^\circ$ and $\tau = 1.10$. (d) Normalized normalized ball velocity (U_x/U_0 and U_y/U_0) as function of normalized time, tU_0/D . Empty and filled symbols show horizontal and vertical velocities, respectively (left and right vertical axes, respectively). Case C1, red symbols; C2, black symbols; C3 blue symbols; vertical solid black line corresponds to time $t_1 = 0$, vertical red dashed line shows t_2 for case C1; vertical dashed blue line shows t_2 for case C3.

If we define $\tau = t_r/t_c$, the condition for a nick shot is

$$\tau < 1. \quad (\text{A.3})$$

Considering explicit expressions for t_c and t_r , we obtain

$$\tau = \beta H^* Ca^{2/5} \frac{(\cos \theta_0)^{1/5}}{\sin \theta_0 + 2\kappa\omega^*} \quad (\text{A.4})$$

where $\beta = 0.6058$ is a numerical constant, $\omega^* = \omega_0 D/U_0$, $H^* = H/D$ and Ca is the Cauchy number defined as

$$Ca = \frac{E}{\rho_{ball} U_0^2}$$

where E is the Young's modulus and $\rho_{ball} = 6m/(\pi D^3)$, is the density of the ball. Using Eqn. A.10, a map of conditions for the nick to occur can be obtained, considering the collision kinematic conditions (U_0 , ω_0 , θ_0), the location of first contact with the wall (H^*), and the ball elasticity parameter (Ca).

A.3.1 Characteristic times for wall collision

To identified the conditions for the nick shot to occur, two characteristic times are needed: the compression time and the rolling time. Here, we detail how those times are determined.

Contact time The contact time for the impact of an elastic sphere is determined by the balance of the impact force and the elastic deformation, neglecting friction and energy dissipation [163]. The impact time is give by

$$t_c = \alpha \left(\frac{m^2}{DE^2 U_0 \cos \theta_0} \right)^{1/5} \quad (\text{A.5})$$

where $\alpha = 3.29$, m is the mass of the ball, U_0 is the ball speed at contact, θ_0 is the angle at contact and E is the Young's modulus.

Rolling time Assuming that there is no slip between the ball and the wall, a condition met for contact with friction and at small incident angles [162], the ball will execute a rolling motion on the wall with in contact with it. The time that the wall will take to reach the floor, while rolling, is

$$t_r = \frac{H}{U_{cm}}$$

where H is the distance above the floor where the ball first makes contact with the wall. U_{cm} is the velocity of ball's center of mass at time t_1 (see Fig. 1(a) in the main text), which can be defined as

$$U_{cm} = \frac{D(U_0 \sin \theta_0 - D\omega_0/2)}{D_J + D} \quad (\text{A.6})$$

where $D_J = 2J/(mD)$, J being the ball's polar moment of inertia and ω_0 is the ball's angular velocity at the moment of contact. This calculation the classic rolling theory [163]. Therefore, we can rewrite t_r as:

$$t_r = \frac{H(2D_J + D)}{D(U_0 \sin \theta_0 + D_J \omega_0)} \quad (\text{A.7})$$

where $D_J = 2J/(mD)$. A squash ball is a thick spherical shell of thickness T , for which the polar moment of inertial is

$$J = \frac{1}{10} m D^2 \frac{1 - (1 - 2T/D)^5}{1 - (1 - 2T/D)^3}. \quad (\text{A.8})$$

Considering that $T/D = 0.113$ for a squash ball, $J = \kappa m D^2$ where $\kappa = 0.1346$.

Therefore, the rolling time simplifies to:

$$t_r = \frac{H(4\kappa + 1)}{U_0 \sin \theta_0 + 2\kappa D \omega_0} \quad (\text{A.9})$$

The time ratio The ratio of rolling to contact time is defined as

$$\tau = \frac{t_r}{t_c}.$$

Therefore, from Eqns. A.5 and A.9, we can write

$$\tau = \frac{1}{\alpha} \left(\frac{DE^2 U_0 \cos \theta_0}{m^2} \right)^{1/5} \frac{H(4\kappa + 1)}{U_0 \sin \theta_0 + 2\kappa D \omega_0}$$

After simplification, we have

$$\tau = \beta H^* \text{Ca}^{2/5} \frac{(\cos \theta_0)^{1/5}}{\sin \theta_0 + 2\kappa \omega^*} \quad (\text{A.10})$$

where $\beta = 0.6058$ is a numerical constant, $\omega^* = \omega_0 D / U_0$, $H^* = H / D$ and

$$\text{Ca} = \frac{E}{\rho_{ball} U_0^2}$$

where Ca is the Cauchy number and $\rho_{ball} = 6m / (\pi D^3)$.

A.4 Results and discussion

A vast experimental campaign was conducted (Table A.1) to test the conditions set by Eqn. A.10. In all experiments, no initial rotation was considered ($\omega^* = 0$) and only two representative incident angles, θ_0 , were tested. Figure A.2 shows all the experimental results. In these maps, we plot the value of τ for a range of values H^* and Ca. On the plots, the filled symbols indicate the cases nick was observed. A shot was considered to be a nick if the rebound vertical speed was smaller than $0.1 U_0 \sin \theta_0$. Clearly, two conditions can be identified: $\tau < 1$ and $0.6 < H^* < 0.75$.

Even without any fitting parameters, Eqn. A.10 predicts the conditions very well.

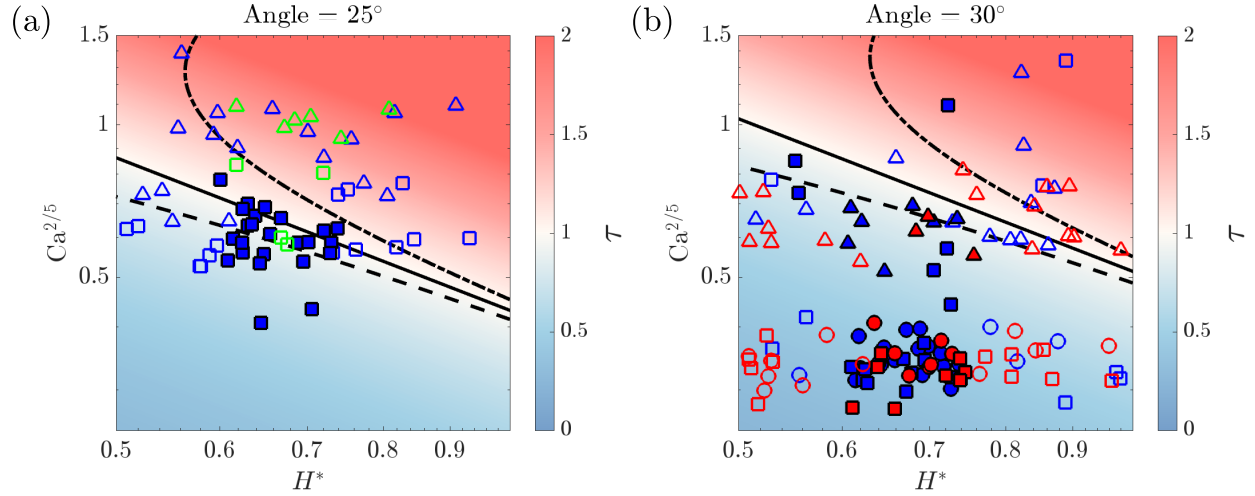


Figure A.2: Map of conditions for nick occurrence: value of the time ratio τ (indicated by the color) as a function of $Ca^{2/5}$ and H^* for a fixed incidence angle θ_0 and no initial rotation $\omega_0 = 0$. (a) $\theta_0 = 25^\circ$; (b) $\theta_0 = 30^\circ$. Symbols according to Table A.1. Filled symbols (resp. empty) show the cases when the nick shot was observed (resp. was not observed). Green symbols on (a) show experiments when the wall was lubricated with a viscous fluid. The dot-dashed and dashed lines consider $\omega_0 = \pm 100 \text{ s}^{-1}$, topspin and backspin, respectively.

Several other conditions were also tested, including ball temperature, ball type, ball speeds, and wall lubrication, all of which are shown in Fig. A.2. A detailed account appears in the SI, including a discussion on the effect of initial ball spin. The conditions for the occurrence of the nick shot hold for all these tests.

Considering a mechanical model (described in the SI), we can explain why the ball does not bounce vertically when the nick shot conditions are satisfied. The rolling process that occurs when the ball is in contact with the vertical wall cannot proceed when a second contact point, with the floor, is imposed. This second contact point induces a torque in the opposite direction of the rolling motion, canceling it, resulting in a mechanical equilibrium condition for which both the linear and rotational momentum of the ball become null. In other words, the ball gets stuck on the corner when the nick conditions are met. Because of the energy stored during the compression phase with the vertical wall, the ball can still rebound horizontally. This is why the ball moves away from the wall after the collision with no vertical momentum.

Understanding the nick mechanics can be used to refine squash technique. Mastering the nick shot requires precision and low τ , achieved with higher speed (lower Ca), steeper angles, or softer balls (lower E). The ideal shot comes from a fully extended arm, hit with force, and when the ball is warm, in accordance to what is observed in professional play.

The theory developed for the mechanism of the nick shot could have practical use beyond squash. Abrupt changes in angular momentum, typically associated with head impacts [164–166], could be reduced. Our model predicts that increasing mass prolongs momentum transfer ($\propto m^{2/5}$), while larger size ($\propto D^{-1/5}$) and greater stiffness ($\propto E^{-2/5}$) shorten it. Thus, a heavier, larger, and more compliant damper that slides on impact would be more effective in reducing large rotational rates.

Statistics of nick shot occurrence To determine how often nick shots are played, one of us (RZ) watched all the matches (men and women, 32 matches) played at the Tournament of Champions of 2023, transmitted by SquashTV. This prestigious tournament, part of the professional tour, is played every January in New York City. We counted the number of times a player won a point with a nick shot. It occurred 8.7 % times. Note that not all of these shots were rolling nicks. Those occurred only 1.2% of all winning shots.

A.5 The mechanical model of the ball contact

A.5.1 Stage 1: Contact with the wall, $t = t_1$

Before making contact with the vertical wall, the ball travels at a velocity $\mathbf{U}_0$, approaching the wall at an angle θ_0 , as depicted in Fig. 1(a) of the main text. When the ball reaches the wall, its horizontal location is $x/D = 0.5$ while its vertical location, $H^* = y/D$, depends on how close to the floor it reaches the vertical wall. The ball makes contact with the wall at a material point on its surface, denoted as A . After making contact, the ball experiences an impulse which changes its center of mass velocity, as well as its angular velocity. From the balance of linear and angular momentum, the change of the ball's center of mass velocity is given by

$$\Delta U_{cm} = D_J \Delta \omega$$

where $\Delta \omega$ is the change of angular velocity before and after the impact and $D_J = 2J/(mD)$, with J being the polar moment of inertia (see Eqn. 5 above).

Assuming that the velocity of a material point of the ball at location A (no slip) is zero, we can conclude that

$$U_0 \sin \theta_0 + \Delta U_{cm} - \omega D/2 = 0$$

and that

$$\Delta\omega = \frac{\frac{1}{2}U_0 \sin \theta_0 - \omega_0 D}{D_J + D}.$$

Therefore, the speed of the center of mass immediately after contact is

$$U_{cm} = \frac{D(U_0 \sin \theta_0 - D\omega_0/2)}{D_J + D}. \quad (\text{A.11})$$

A.5.2 Stage 2: contact with the floor, while already in contact with the wall, $t = t_2$

As described in the main text, when the ball has not yet concluded its rolling motion on the vertical wall but makes contact with the floor, the nick behavior is manifested. In this case the ball is observed to roll on the floor with no vertical velocity.

When the ball makes simultaneous contact with the vertical wall, at point A with the vertical wall and at point B with the floor, it experiences impulses from these two points that will change the ball's velocity. Let the change of the ball's center of mass at $t = t_2$ be

$$\Delta U_{cm}(t_2) = P_B/m + P_A/m$$

where P_B and P_A are the impulses at B and A , respectively.

At point A , considering that at $t = t_2$ the material point is still fixed (non slip), we will have

$$U_{cm}(t_2) = D\omega/2. \quad (\text{A.12})$$

Now, we can also assume that at the material point B the the velocity is null. Therefore, following a similar argument as before, we can conclude that

$$U_{cm}(t_2) = -D\omega/2. \quad (\text{A.13})$$

The only possibility for Eqns. A.12 and A.13 to be both satisfied is for $\omega = 0$. From it, it follows that $U_{cm} = 0$.

A.5.3 Stage 3: restitution

As shown above, if the ball makes simultaneous contact with the vertical wall and floor, while rolling on the vertical wall, its velocity and rotation cancels out. However, due to the impact, the ball still has energy stored in elastic deformation. Since the energy is mostly from deformation in the horizontal direction, the ball will regain kinetic energy in this direction. Due to the contact at point B , the ball will roll as the elastic energy is restored into kinetic energy in the horizontal direction.

A.6 Detailed description of experiments

Experiments when the nick shot is not observed In this section we provide a detailed description of two experiments shown in Fig. 1(c) and 1(d) (case C2 and C3) that do not exhibit the nick behavior.

Our observations indicate that the nick behavior occurs when the balls touches the floor before it has completed the rolling phase with the vertical wall. To illustrate that this is indeed the nick condition, we show two other cases in Fig.1 of the main document, with very similar conditions but reaching the corner at slightly different H^* values. Case C2, shown in Figs. 1(c) and 1(d) by the black markers and the movie below, displays a collision for which $H^* = 0.51$; in this case, the ball makes contact with both vertical wall and floor at approximately the same time ($t_1 \approx t_2$). The compression-decompression cycle occurs with both walls at the same time and rolling does not occur. As explained above, rolling is not possible while having two no-sling contact points. The ball bounces off with at a slightly lower angle that the incoming one, see Fig. 1(d). Case C3, displayed by the blue symbols, also in Figs. 1(c) and 1(d) and movie below, shows an interaction for which $H^* = 0.81$. The collision occurs at a higher location than C1 (nick case). The evolution of the ball position and velocity is very different. The evolution of position and speed, for times soon after $t = t_1$ occurs in a manner similar to Case C1: the ball compresses and rolls on the vertical wall. Since the vertical location is higher, the ball is capable of completing is rolling motion with respect to point A by the time it touches the floor at time t_2 . Hence, the ball is free to continue its vertical motion downwards, starting a compression phase with the floor. During this compression phase with the floor, the ball is not longer in contact with the wall on point A . Therefore, the motion of the center of mass of the ball has a negative velocity even for times longer that t_2 even as the ball is already moving away from the wall in the

horizontal direction, as seen on Fig. 1(d). Subsequently, the ball bounces off from the floor with a certain angle, with both horizontal and vertical momentum.

Testing several collision conditions A vast experimental campaign was conducted, including three balls, two temperatures, two incident angles, for a range of impact speeds and initial contact locations. Furthermore, for one test we added a small amount of oil to the wall to lubricate the contact to induce sliding on the wall upon contact. The nick conditions were identified to occur for $0.6 < H^* < 0.75$ and $\tau < 1$. All experiments conducted in this investigation satisfied these conditions, as shown, as shown in Fig. 2 of the main document.

High speed collisions An interesting group of data is that for $\tau < 1$ but $H^* > 0.75$ (data on lower right hand side of Fig. 2(b)). One would expect the condition of $\tau < 1$ to be sufficient since it quantifies the ratio of rolling to contact time.

It is not possible for a ball to reach the horizontal wall for $H^* > 0.78$, if one considers that the maximum possible stretching on the wall upon impact (see above). However, the calculation from Eqn. 2, from the main text, results in values with $\tau < 1$. We note that these experiments have small values of the Cauchy number ($Ca < 0.16$) which correspond for high impact speeds ($U_0 \approx 45$ m/s). For such a range of speeds, the deformation of the ball is significant. We do not expect of the calculation of the contact and rolling times to be valid in such a case.

Experiments at different temperatures For the experiments at $\theta_0 = 30^\circ$, tests were conducted with the same three balls but at a higher temperature (40°C), to replicate the conditions observed in a real squash game. At higher temperatures the elastic properties of the ball change (see Table 1 of the main document), leading to an increase in the coefficient of restitution [167]. The material becomes softer at higher temperatures (the value of E decreases). The results for these tests are shown in Fig. 2(b) of the main text by the red symbols. These data also satisfy the nick occurrence conditions. Since, in general, E decreases with temperature, the attainable values of Ca are smaller.

Collisions over lubricated walls An additional test was conducted for the experiments at $\theta_0 = 25^\circ$ to further test our rolling hypothesis. For a set of experiments involving the double-dot and doubles balls, at

room temperature, the vertical wall was covered by a thin film of viscous fluid (glycerol) with the intention to lower the surface friction. It is well known that when the wall is lubricated by a viscous fluid, sliding (instead of rolling) occurs upon contact[168]. The results from these tests are shown by the green symbols in Fig. 2(a) of the main text. The nick shot was prevented from occurring when the collision corresponded to a value of $\tau < 1$. Upon closer inspection of the high speed videos, for these collisions the ball slid upon contact; hence, the rolling frustration mechanisms did not occur.

A.7 Effect of initial spin

Experiments were not conducted with initial rotation. However, this effect is often present while playing squash. Our estimation, obtained by watching videos from the Professional Squash Association, is that the ball rotation never exceeds 100 s^{-1} during normal playing.

Tangential recoil A ball with backspin could bounce with an angle different from the incident one. Considering both normal and tangential coefficients of restitution and the conservation of angular momentum during an oblique collision with spin, according to Cross [169]:

$$\frac{U_y^{\text{after}}}{U_y^{\text{before}}} = \frac{1 - 4\kappa e_t}{1 + 4\kappa} + \frac{2\kappa(1 + e_t)}{1 + 4\kappa} \left(\frac{D\omega_0}{U_0 \sin \theta_0} \right) \quad (\text{A.14})$$

where U_y^{after} and U_y^{before} are the vertical speeds after and before the collision, $\kappa = 0.1346$ (see calculation above) and e_t is the tangential coefficient of restitution, defined as $e_t = (U_y^{\text{after}} - \omega^{\text{after}} D/2)/(U_y^{\text{before}} - \omega^{\text{before}} D/2)$. From Eqn. (12), the condition required for a rebound with zero vertical speed is:

$$\frac{\omega_0 D}{U_0} = \frac{(4\kappa e_t - 1) \sin \theta_0}{2\kappa(1 + e_t)}. \quad (\text{A.15})$$

Considering, for instance $\theta_0 = 25^\circ$ (as in Fig. 2(a) of the main text), and $e_t = 0.8$ (typical value), leads to

$$\omega^* = \frac{\omega_0 D}{U_0} = -0.49.$$

For ball speeds in a squash game, this condition corresponds to a rotation rate of around $-500 \text{ s}^{-1} = -30000 \text{ RPM}$ (backspin). This value of backspin is not attainable in regular play. Therefore, a collision with backspin cannot explain the occurrence of the nick shot.

Model predictions with initial spin Our model does account for ball spin. The term $2\kappa\omega^*$ in Eqn. (2) of the main text can be varied to analyze the effect of spin. Our observations (from PSA videos), indicate that ω_0 is never larger than 100 s^{-1} , with either top or backspin. Fig. 2 (main text) shows two lines that include the effect of top (dashed-dotted) and backspin (dashed) for $\omega_0 = 100 \text{ s}^{-1}$. When spin is considered, the rotation time is either longer or shorter (top or backspin, respectively); therefore, the condition for $\tau = 1$ is displaced to higher or smaller values of Ca for top and backspin, respectively. In other words, the nick shot is possible but the range of speeds and ball elasticities are slightly different.

A.8 Collision videos

Videos for typical ball collisions have been placed on a data repository. All videos were recorded at 5000 fps, with a shuttle speed of 1/20000, using a Photron FASTCAM SA5 camera.

Movie S1. Case C1: Nick collision. [HERE](#)

Conditions: $\text{Ca} = 0.25$, $H^* = 0.62$, $\theta_0 = 24.43^\circ$ and $\tau = 0.87$.

Movie S2. Case C2: No nick collision. [HERE](#)

Conditions: $\text{Ca} = 0.24$, $H^* = 0.51$, $\theta_0 = 24.34^\circ$, and $\tau = 0.76$.

Movie S3. Case C3: No nick collision. [HERE](#)

Conditions: $\text{Ca} = 0.25$, $H^* = 0.81$, $\theta_0 = 24.31^\circ$, and $\tau = 1.10$.

B BLENDING FLUID MECHANICS AND ART PART I: MAKING A SPLASH WITH HIGH-SPEED PHOTOGRAPHY

Information This chapter is a modified version of the work published in The Physics Teacher [4].

B.1 Introduction - Overview of the Idea

In part I of III of blending fluid mechanics and art[6, 7], students capture a photo of a fluid droplet impact by controlling a peristaltic pump, digital camera, and flash programmed through an Arduino microprocessor. As a fluid droplet impacts a fluid bath, an aesthetically pleasing series of events unfolds. Namely, a fluid crown is formed around the point of contact, and after this crown submerges back into the bath a tall jet emerges and subsequently collapses. The objective of this lab is for students to capture particular moments of this fluid process in an artistic way. The photograph's appeal is enhanced by modifying lighting, timing, and fluid properties. This lab aims to expose students from all fields to the interdisciplinarity of engineering in an exciting and novel way. Students utilize fundamental principles from fluid mechanics in order to manipulate their final images according to their individual vision, gaining physical intuition into properties such as viscosity, surface tension, and density.

B.1.1 Worthington-Edgerton crowns

One of the most recognizable scientific images is the Worthington-Edgerton splash. Worthington obtained the first high speed image of the splash produced by a liquid droplet colliding against the free surface of a stagnant liquid in 1897 [170]. Later on, in 1936, Edgerton improved the technique leading to very clear and astonishing images[171]. A student-created photograph of the same phenomena is shown in Fig. 1 below.

In addition to its visual appeal, the process is of great importance in understanding the dynamic spreading of droplets through hydrodynamic instabilities and surface tension phenomena, which have important implications for other problems in engineering applications and natural phenomena. Hence, the subject has been vastly studied by many authors. The review by Yarin [172] provides an excellent overview of the subject.



Figure B.1: Worthington-Edgerton crown created by a droplet impacting the free surface of a stagnant liquid.

B.1.2 The process of a drop impact on a free surface

The sequence of images shown in Fig. B.2 below depict the splashing process. When the droplet impacts the free surface, its kinetic energy is transferred to the fluid layer. The free surface is therefore deformed downward. During this phase, liquid at the rim of a forming cavity is pushed upwards, forming a thin cylindrical fluid film. Under some conditions, the rim of this film becomes unstable forming radial jets that can fragment into droplets (this structure is called the Worthington crown)[170]. After the crown descends, a fluid crater is formed. This cavity energetically springs back up as a result of unbalanced hydrostatic pressure. The upward flow forms a vertical jet that can become unstable due to the Rayleigh-Plateau instability, fragmenting into one or many droplets[170].

Each step of the behavior described above is dependent on fluid properties such as viscosity, density, and surface tension as well as the speed and size of the free falling droplet [17, 173, 174]. To characterize the splash in terms of all these quantities, it is convenient to use two dimensionless parameters. The first is the Reynolds number and is defined here as

$$\text{Re} = \frac{\rho U D}{\mu} \quad (\text{B.1})$$

where ρ , U , D , and μ are the droplet density, impact velocity, diameter, and viscosity, respectively. The Reynolds number is the most widely used dimensionless quantity in fluid mechanics and represents the

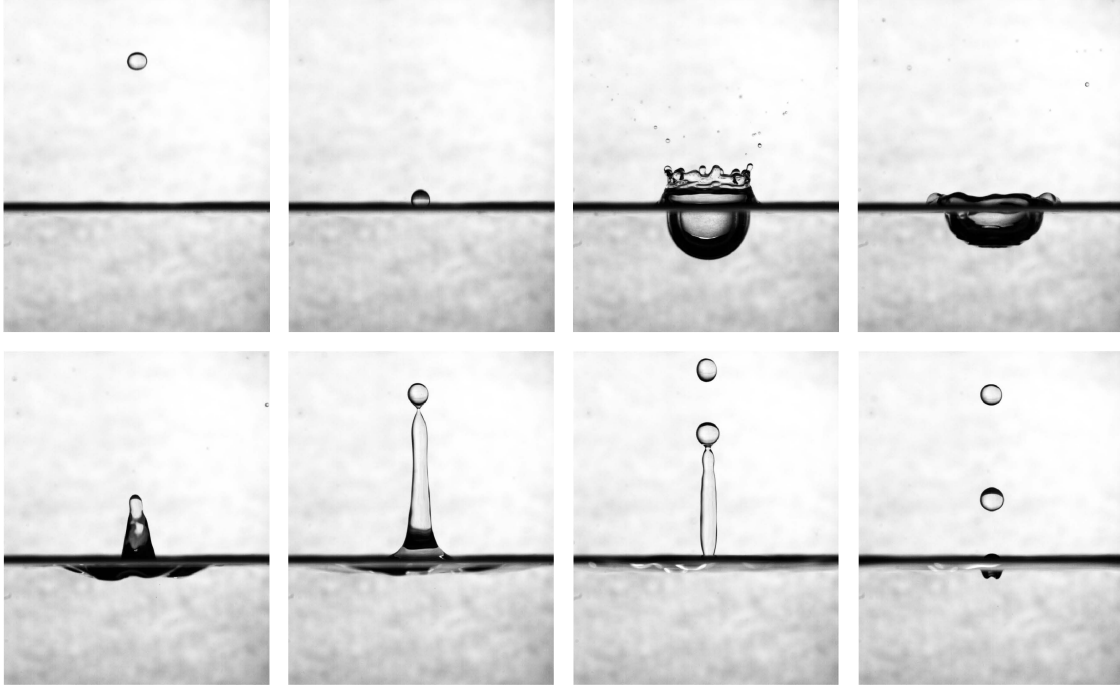


Figure B.2: Image sequence of the collision of a water droplet against the free surface of a stagnant water. A droplet falls from above onto a quiescent liquid bath. At impact, the droplet deforms the free surface downwards causing a crown of droplets to form around the cavity. As the cavity springs back up, a jet emerges from the free surface eventually destabilizing into a line of droplets. Experiment shown here are with a droplet of radius $R = 5$ mm at an impact speed of $U = 3.75$ m/s with corresponding Reynolds and Weber numbers of $Re = 1025$ and $We = 2675$, respectively. The image sequence was obtained with a Photron SA3 model 120K-M3 at 2000 frames per second, though a similar photo series can readily be reproduced with a standard DSLR or smartphone camera.

ratio of viscous to inertial forces. The second dimensionless number relevant for this study is the Weber number. This quantity is defined as

$$We = \frac{\rho U^2 D}{\sigma} \quad (B.2)$$

where σ is the fluid surface tension. This number quantifies the relative importance of inertial and surface tension forces.

B.1.3 Arduino microprocessor

An Arduino is an open hardware microprocessor board that can be readily connected to a computer, and its output ports can be used to control analog and digital devices. In this lab, one Arduino board is used to control a pump and to trigger a digital camera and flash to capture images of a droplet splashing onto a free surface of fluid.

In a dark environment, by leaving a camera's shutter open and exposing the sensor to a small duration of light, a very particular instant in time is captured with high clarity. Therefore, by changing the time delay between the drop release from the pump and the triggering of the flash, the camera will capture different phases of the impact.

B.2 Design and Setup

B.2.1 Circuit

A breadboard circuit, shown in Fig. B.3, is created to control the experiment through an Arduino. The microprocessor is connected to a computer using a USB cable and to a 12 volt power supply. A digital output pin is used to control the peristaltic pump. Note that a number of pumps can be put in parallel, as shown in Fig. B.3, to produce many drops simultaneously or with some time delay. The circuitry required to control the pump is highlighted in Fig. B.3 by the gray boxes labeled Pump 1 and Pump 2. Resistors ($230\ \Omega$) and transistors (5 V) are used to modulate the amount current and voltage sent from the input ports to the pumps. Two digital output ports are used to trigger the camera and the flash. An optocoupler is used to control when the flash and camera fired. As a signal passes through this component of the circuit, it "closes", effectively connecting the leads from the flash or camera.

B.2.2 Operating Procedure

The experimental setup, shown in Fig. B.4 consists of a fluid reservoir, tubing, a peristaltic pump, a nozzle, and a fluid bath. Additionally, a photo camera and a flash are used to capture the image. These are also controlled by the Arduino circuit and code.

To conduct the experiment, a computer connected to an Arduino-breadboard circuit is needed. The Arduino software can be readily downloaded for free [here](#). To ensure the computer is communicating with the Arduino board, select the correct board and port under the "Tools" tab in Arduino. The camera is then focused to the area of interest. In this example, we use a Nikon DSLR. For this or a similar model, we suggest beginning with the following camera settings: shutter speed (2 s), aperture (f11) and ISO-value (ISO160). The experiments should be conducted in a dark environment to prevent excess light from entering the camera sensor.

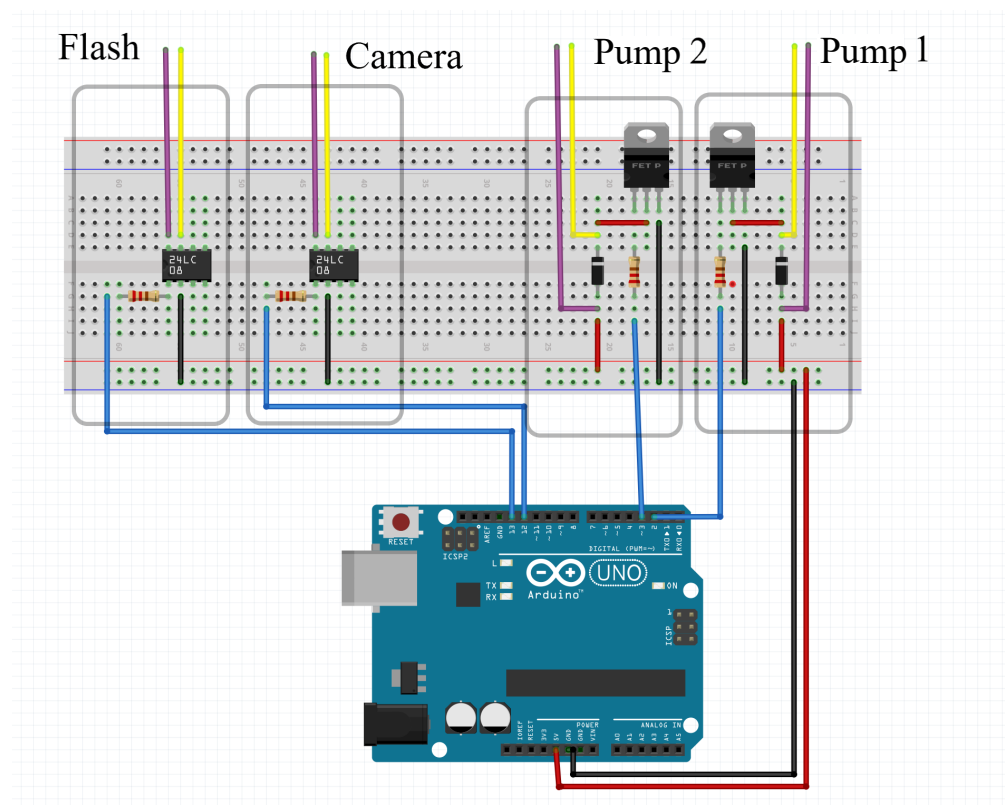


Figure B.3: Arduino and breadboard circuit used to control experimental setup. Output pins send pulses to each element of the setup as dictated by user defined time delays within the Arduino code.

The Arduino code dictates the sequence of signals sent to the experimental components. First, a pulse is sent to the camera which opens the shutter. The shutter remains open for the chosen time selected on the camera. The Arduino then sends a pulse to the pump to produce a droplet which falls from the nozzle. After a time delay (`FlashDelay` in the provided code), the Arduino then sends a pulse that activates the flash. It is at this instant of illumination that a photo is captured. After the flash, the camera shutter closes and the image is stored in the camera memory. By changing the value of `FlashDelay`, the camera will capture different instants of the splash process.

Once familiar with the device and running the experiment, the artistry begins. Students are expected to modify the setup in a number of ways to produce more interesting images. In previous iterations of the course, students have modified the code to release a secondary drop to collide with the jet created by the initial splash. Doing so, as seen in Fig.C.5e, creates an umbrella-like structure. Students are also encouraged to test multiple values of the Reynolds and Weber numbers by using fluids of differing properties. Changing these values can lead to drastically different splashes. Past students have chosen to use viscoelastic fluids.

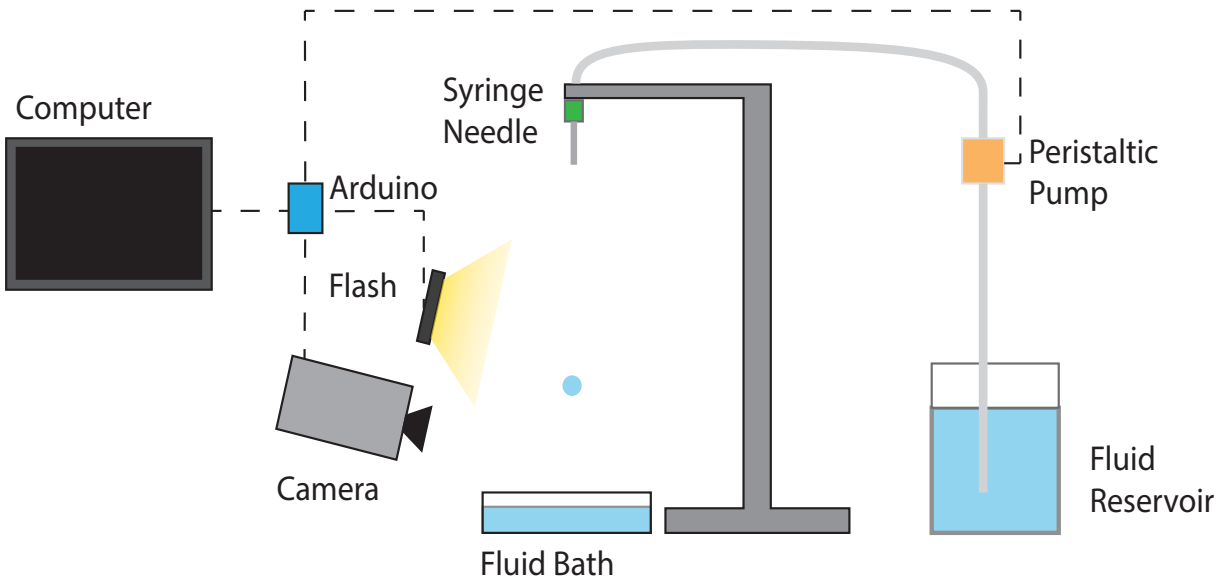


Figure B.4: Experimental setup The setup consists of a peristaltic pump which draws fluid from a fluid reservoir, producing the droplet at the syringe needle. A camera and flash controlled by an Arduino (not shown here) capture the moment of impact of the droplet with the stagnant fluid bath.

Splashes of these fluids often have long, thin filaments that eject from the impact due to the elasticity of the fluid [175], such as in Figure C.5d. Glitter, soap, and food coloring are some of the common choices to improve the aesthetic appeal of the photographs. The viscosity and surface tension of many household fluids and additives can be found online to be used in the calculation of the Reynolds and Weber numbers. In the case in which these properties were not available, the teaching staff would determine viscosity with a rheometer and surface tension with a tensiometer. It is suggested that students use Photoshop or other postprocessing software to improve brightness, contrast and other aesthetic qualities of their images. It is emphasized, however, that photo editing should not obscure or fabricate the physics.

B.3 Results

B.3.1 Examples

Here we discuss example photographs taken by students of the Spring 2023 and 2024 course iterations. Figure 1 (“Defying gravity”) captures a Worthington-Edgerton crown splash resulting from a water on water impact. Students calculated values of $Re = 13,580$ and $We = 572$, indicating we should expect to see

an irregular crown splash with microdroplet ejection[17]. Here we see that a relatively high Reynolds number overcomes surface tension effects, resulting in an instability which ejects droplets away from the crown rather than maintaining symmetric undulations around the crest of the crown. Figure C.5a (“Armless Crawler”) displays the fluid jet resulting from a water impact. Equivalent values of Reynolds and Weber numbers were calculated during this experiment as to in Figure 1, providing for a useful visualization of the timing of the impact dynamics. We see that following the collapse of the Worthington crown, a tall pillar of fluid emerges from the bath as a result of the displaced fluid converging radially after impact. Early on into the growth of the jet, the cylindrical structure is maintained. However, in fractions of a second the column will fragment into droplets due to the Rayleigh-Plateau instability (see bottom row of Fig. 2), which occurs when surface tension acts to minimize the surface area of a given fluid volume. Figure C.5b (“Jelly Friends”) shows milk dropped into ethanol. The original image is then reflected about the diagonal to give the sense of two jellyfish interacting. The students calculated values of $Re = 658$ and $We = 523$. At this low Reynolds number, we do not expect droplet formation, hence the lack of satellite drops (see Figure B.6). We see a distinct structure within the fluid jet arising from the milk not mixing homogeneously within the alcohol, leading to a jellyfish-like effect. This structure can occur when fluids of unequal density are accelerated together, and is known as the Rayleigh-Taylor instability. Figure C.5c (“Two Crowns”) captures two Worthington crowns composed of honey and red (left) or blue (right) dye. The impact occurs at $Re = 14600$ and $We = 6800$. Students utilized shallow fluid films to drop liquid on in order to create larger crowns. They then chose to superpose two images horizontally to create a surreal, illogical effect. Figure C.5d (“Cerulean”) uses 50-50 water glycerol mixture (with a hint of cream for opacity) and captures an impact at $Re = 2585$ and $We = 1263$. Here we see the result of a droplet impacting the rising jet of a previous impact. The collision results in a highly irregular and asymmetric geometry, which gives the image a sense of movement. Students use yellow and blue food dye for contrast. Figure C.5e (“Mycology”) captures a glow stick fluid drop impacting a mixture of water, glycerol and cream at $Re = 2202$ and $We = 1257$. Similar to the previously discussed photo, we again see the result of one drop impacting the jet from an earlier impact. What distinguishes this impact is the symmetry and stability of the fluid film which is ejected from the epicenter of the collision. This is due to the relatively lower Reynolds number and capturing the photo at an earlier time in the impact dynamics.

B.3.2 Analysis

After conducting the experiment, the students must choose three photos based on aesthetic merit to analyze and present in a written report. Each image should be titled and accompanied by a technical and aesthetic assessment, including a brief explanation of the methods used to capture the image. The students describe and justify the creative process they chose to follow, emphasizing how they manipulated fluid properties and the experimental setup to realize an artistic vision.

Students are asked to produce a detailed write-up after conducting the experiment and obtaining photos. As part of the report, students calculate the Reynolds and Weber numbers for three photographs of their choosing. When coupled on a regime map [17] the Reynolds and Weber numbers predict the splash behavior that will be observed in a given experiment. Figure B.6 shows this regime map and identifies where examples from Figure B.1 and Figure C.5 lie. When both numbers are small, the splashing dynamics are dominated by viscous and surface tension forces. Instead of a crown-like geometry the splash will create what is known as a peregrine sheet. This splash is a stable, cylindrical sheet which grows upwards without fragmenting into jets and droplets. When both numbers are high, the flow is dominated by inertial forces, the crown violently emerges from the surface and ejects micro-droplets across the surface of the fluid bath. Using the regime map of Figure B.6, students identify which regime each of their photos is located based on the corresponding Reynolds and Weber numbers. The students report if the observed behavior agrees with the predicted behavior within the regime map.

B.4 Teaching Outcomes

Through this interdisciplinary lab, students from all academic fields are able to contribute and learn in a scientific setting. By combining art and fluid mechanics, students from the sciences and humanities alike are able to contribute their knowledge to produce works of art. Through manipulating physical parameters which can sometimes be foreign to students, empirical insights are gained into how fluid properties such as viscosity and surface tension change the resulting flows. Additionally, students develop experimental techniques such as sample production of fluids with specific material properties and gain early exposure to Arduino code to control an array of digital elements.

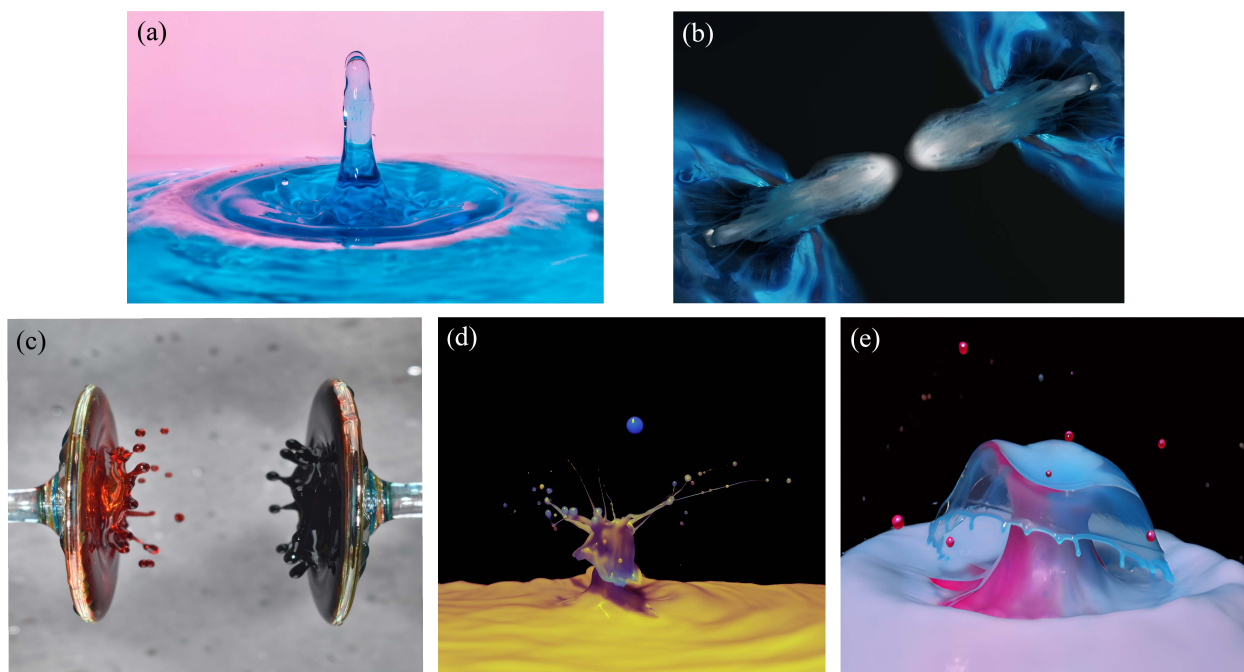


Figure B.5: Artistic representation of droplet splashes. The objective of this lab is for students to produce original and artistic images following the theme of droplet collisions. Shown here are a subset of the final results produced in the 2023 and 2024 iterations of the course. See Appendix III for attribution [16].

B.5 Extensions and Future Directions

Improvements in the robustness and reliability of the experimental setup would help students gather results more efficiently. Often wires may come loose from the breadboard or Arduino board, disrupting laboratory flow and forcing students to do troubleshooting during the lab. Additionally, the peristaltic pump does not always release a droplet at the same time in each run, leading to experimental inconsistencies despite running the same code. Ambitious students in the past have modified the code to make it more interactive, adding a button to the circuit such that when the button is pressed a drop is formed and a picture taken. In doing so the code does not need to compile in each run saving much time over the span of the laboratory exercise. We note the lab could be modified to use a readily available phone camera in place of a digital camera, possibly reducing laboratory costs substantially. In our knowledge, there is no way to manually alter the shutter speed of a phone's camera. To work around this, we recommend taking a slow motion capture video in an illuminated room. Spanning the entire duration of the impact allows the student to select from a range of moments across the impact time. In this case, the camera is operated manually and not with the Arduino, though the droplet is still initiated using the code and Arduino board. The analysis

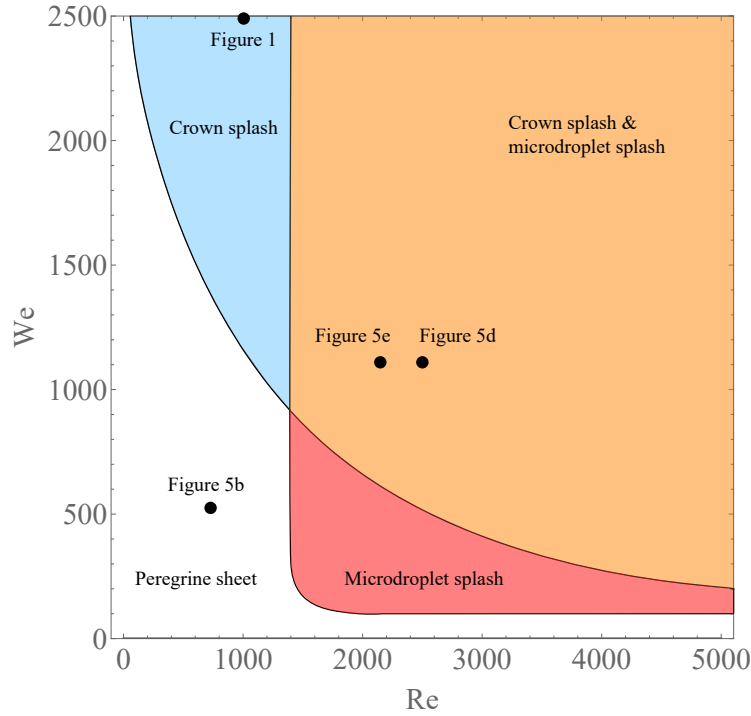


Figure B.6: Splash regime map. This map, proposed by [17], locates the expected splash behavior for given values of the Reynolds and Weber numbers. Examples from students experiments are shown, as discussed in the manuscript.

could be shortened or extended depending on the level of fluid mechanics comprehension of the intended audience. For example, a more advanced study could include the effect of the Weissenberg number, which is a dimensionless number which measures the balance of elastic to inertial forces and is used in the study of viscoelastic fluids.

B.6 Conclusions

We present an undergraduate laboratory exercise which bridges the perceived gap between fluid mechanics and art, allowing students to empirically interact with the inherent beauty of the field. Students interface with an Arduino circuit through code in order to control the experimental process of releasing a fluid drop into a bath and capturing the moment of impact with a digital camera. Students are given freedom over the experiment (e.g. changing viscosity, surface tension, impact speed, etc.) to allow them to produce unique images of splash phenomena. The motive is to produce images of aesthetic merit while simultaneously gaining powerful intuition into fluid mechanics.

B.7 Bill of Materials

The experiment can be reproduced with the following materials. Prices are provided for Amazon products in the USA.

Material	Price	Description
12V DC Peristaltic Pump (Gikfun).	\$11.98	Peristaltic pumps are used to produce the fluid droplet. Additional pumps can be added to produce additional droplets.
Peristaltic Pump tubing (Intllab).	\$13.99	Tubing to connect pump from fluid reservoir to nozzle.
Camera flash (ploture)	\$41.99	Flash to be placed around droplet staging area necessary to perform high speed photography.
Shutter release remote (x2) (Nikon)	\$8.99	The release remotes are connected to the camera and flash separately. The leads from the remote connect to the Arduino board.
Arduino Uno (Arduino)	\$27.99	Microprocessor used to control the experimental process.
Arduino code	Free	Available here . The code is written to work as a test case for producing a single droplet from a height of 30 cm and capturing the crown splash.
Circuit components (Swpeet)	\$25.65	Kit including necessary circuitry components.
Breadboard kit (GeekPi)	\$25.99	Breadboard kit including jumper wires, diodes, and resistors.
Nikon d5500	\$440	Note that any similar camera works. We used this one since it was available. We also note it is possible to modify the lab to use a smartphone instead.
TOTAL	\$596.58	Estimated total cost to run the experiment.

B.8 Attribution Table

Fig.	Title	Students
1	“Defying Gravity”	Angelina Schorr, Grant Adam, Victoria Vo, Ian Becker, Jennifer Cheng. (2024).
5a	“Armless Crawler”	Angelina Schorr, Grant Adam, Victoria Vo, Ian Becker, Jennifer Cheng. (2024).
5b	“Jelly Friends”	Jake Srebnick, Erin Alexander, Mateo Benitez, Makayla Minter, Toshiki Kato. (2024).
5c	“Two Crowns”	Houston Li, Julia Zaborowsky, Mandy He, Raymond Gresalfi. (2023).
5d	“Cerulean”	Lucian Sharpe, Sofia Gilroy, Kiara Vong, Himanssh Pettie. (2024).
5e	“Mycology”	Lucian Sharpe, Sofia Gilroy, Kiara Vong, Himanssh Pettie. (2024).

C BLENDING FLUID MECHANICS AND ART PART II: FLOW VISUALIZATION

Information This chapter is a modified version of the work currently in press in The Physics Teacher [6].

C.1 Introduction - Overview of the Idea

In part II of III of blending fluid mechanics and art[4, 7], students use a table-top setup to explore flow visualization of vortex rings using smoke and a light source. This laboratory introduces students to using dimensionless numbers to characterize fluid flow and gain experience with laser techniques for flow visualization. A particular emphasis is placed on capturing aesthetic imagery and video. The lab described is an example of incorporating the arts into STEM education, sometimes called “STEAM”, allowing students to add creativity into the visualization process with different lighting and colors.

The incorporation of the arts into STEM education has allowed for creative, divergent, thinking associated with the arts to be paired with the technical, convergent thinking associated with STEM [176, 177]. STEAM initiatives can take many forms, with one being the use of visual arts, which serve a two-fold purpose: reinforcing technical understanding of scientific concepts and providing an alternative mode of scientific communication, while also possessing artistic merit [178]. The use of visualization in conveying topics in the sciences and grabbing attention both inside and outside the classroom has previously been demonstrated with physics topics such as quantum mechanics [179] and fluid mechanics [180]. Here, we share a laboratory activity on the flow visualization of vortex rings.

A vortex ring is a torus-shaped flow structure characterized by the rotation of the fluid parcels that form a closed loop [181, 182], shown schematically in Fig. C.1(a). Vortex rings are found throughout nature at a variety of scales such as at the top of a volcano after an eruption [183], to the wake of birds and fishes as they propel forward in air and water, respectively [184]. Vortex rings usually move in a direction that is perpendicular to the plane of the ring, traveling relatively long distances relative to their diameter D , transporting fluid within it. These flow structures are often used as examples to depict the complexity of flows when they transition to turbulence. Vortex dynamics form the makeup of turbulent flow, generally

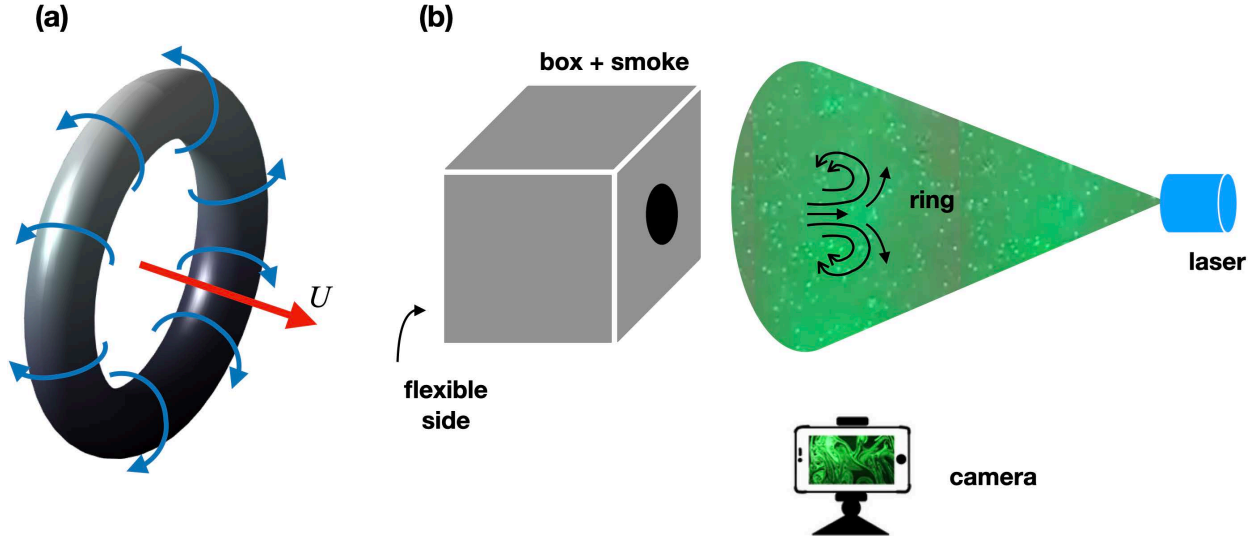


Figure C.1: (a) Sketch of flow in a vortex ring. (b) Experimental setup for generating vortex rings. A box with a cut-out orifice is filled with smoke from a fog/smoke generator. The back side of the box is made of flexible material, plastic wrap. By shining a laser, a plane of interest is illuminated. Tapping the flexible side of the box produces an impulse producing the vortex ring out of the orifice which can be recorded with a phone camera set up close by.

represented as a mixture of vortices, of different sizes, interacting with each other by breaking, merging, and evolving the flow landscape [185, 186].

With the following laboratory activity and fluid mechanics demonstration, we seek to introduce physics and engineering students to flow visualization via a simple tabletop experiment on vortex rings [187]. The objective of the activity is introducing the concepts of flow transition (laminar to turbulent) by considering the use of classical fluid mechanics dimensionless numbers (the Reynolds number). Moreover, given the striking beauty of vortex rings, the lab emphasizes the aesthetic output in the form of images and videos. Follow-up exercises are discussed, such as collisions of vortex rings, interactions with obstacles, and considering non-circular vortices, i.e. produced by using ellipse cross-sections.

C.2 Design and Setup

The experimental setup consists of a 6-sided box with a modular face, a modified laser pointer, smoke from a smoke machine, and a phone camera shown in Fig C.1 (b).

The box consists of 4 square laser-cut acrylic faces of side length $l = 0.152$ m, a modular laser-cut front-face with a prescribed circular cutout, and a back-face that is made from standard plastic wrap from

the grocery store (Optional: one could take use of [makercase.com](https://www.makercase.com) for construction of the box faces). The four fixed sides of the box are assembled with Weld-on Acrylic Cement. When assembling, the box pieces are held in place for a few minutes (typically 2-3 minutes) and then left to fully weld together overnight. The circular cutout, in the front face of the box, allows for the emission of the vortex ring from the box, and the plastic wrap acting as an elastic back face allows for a contraction of volume to occur. The cutout prescribes the diameter D of the vortex ring that is used in subsequent calculations. The front face is slotted into the box to allow for it to be swapped out. This allows students to explore different diameters and can also extend beyond to non-circular orifices. Setups similar in spirit have been documented at larger scales via the construction of “vortex cannons” which take use of garbage bins and larger size orifices [188].

To visualize the emission of the vortex ring, a standard laser pointer for presentations/classroom use is used (less than 1 mW, Class II). A small glass rod segment, from a stirring rod, is taped to the front of the laser. The glass rod converts the beam of the laser pointer into an illuminated 2D plane, rather than a focused beam. By aligning the illuminated plane with the center of the front face of the box, a slice of the vortex ring is visualized. The overhead lights of the classroom are turned off to increase the contrast of the laser light.

Once the vortex ring box is assembled, and the laser and camera are in place, the box is filled with smoke from the smoke machine. The smoke machine, typically used for making an eerie, foggy effect on a lawn for Halloween or in the production of plays, can serve the scientific purpose of adding tracer particles to the flow by generating a cloud inside the box. Now, by flicking the plastic wrap side of the box, a smoke ring is sent forward, with its motion tracked by the phone camera. An example of standard operating procedure with laser illumination is shown both for ambient turbulent flows present in the room by waving one’s arms around (Fig. 2a) and with the presence of a turbulent vortex ring (Fig 2b). Here, two lasers, red and green, were used to illuminate planes perpendicular to one another for additional visualization. While the laser illumination allows for planar flow visualization, one could also opt to use a high-powered flood lamp as an alternative light source to capture the 3D structure of the vortex ring.

Regarding safety, it should be stressed to the participants that working with lasers can be hazardous and laser-safe goggles are *recommended* while conducting the experiment. The recommended laser pointer is a Class II, so it can technically be used without laser safety goggles. Nevertheless, safety goggles should be used when available. See Appendix II for more details on the specific materials used.

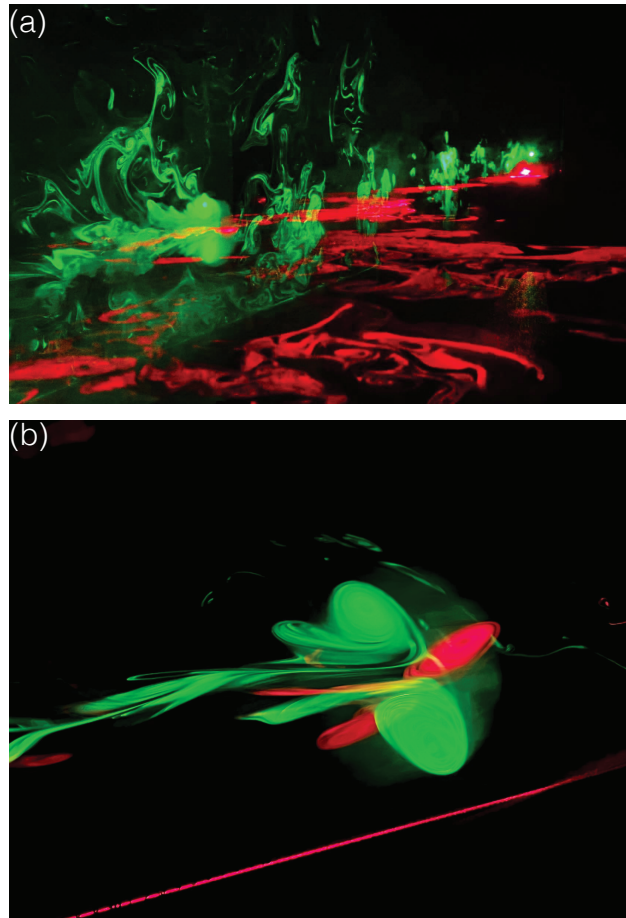


Figure C.2: Plane visualization of turbulence. (a) Flow in the room visualized via the lasers placed a distance away from the vortex ring generator box after waving one's arms around. (b) A vortex ring can be visualized in an equivalent way. All photographs are reproduced with permission from the students. See Appendix III for attribution.

The laboratory activity was used in a cross-listed Engineering and Design course at our university, fostering a collaboration between the arts and sciences. The students were assigned to groups of 4, with generally half of the students per groups coming from a humanities background and the other half coming from a science/engineering backgrounds. The students were given a short training session on using the equipment and then left to work on obtaining images and videos independently.

C.3 Results

The students began their activity by investigating how the illuminated vortex ring's flow structure was affected by the strength of their tapping on the flexible side of the box. By varying the strength of the impulse, the propagation speed of the vortex ring from the orifice varied. Larger impulses resulted in faster propagation speeds accompanied by turbulent, flame-like wakes and small impulses resulted in slower propagation speeds with laminar flow visualized in the toroidal structure of the vortex ring. To accompany their qualitative description of the vortex ring, students were asked to measure both the orifice diameter D , and the propagation speed of the vortex ring U .

The propagation velocity of the vortex ring U was calculated by the students using either ImageJ, Matlab, or Tracker Video Analysis[189] for image processing. Students were instructed to take a video starting at the impulse event (tapping on the flexible side of the container) and ending when the vortex ring traveled some distance (typically 0.5 – 2 m) from the orifice. The horizontal position of the vortex ring is tracked through the video as a function of time ($x(t)$). Using the frame rate (fps) of the camera, the propagation velocity of the vortex U is calculated via $U = \Delta x / \Delta t$, where $\Delta t = 1/\text{fps}$. After some distance, ambient air currents and air conditioning lead to the breakdown of the vortex ring and/or drifting on a nonlinear trajectory. Students either reported the average propagation speed over the vortex ring's trajectory before breakup or the propagation speed at the particular frame shared in their writeup and/or gallery image.

As part of the conducted activity, students were directed to compute a Reynolds number associated with the vortex ring, defined as

$$\text{Re} = \frac{\rho U D}{\mu} \quad (\text{C.1})$$

where D is the diameter of the orifice opening which we assume is the diameter of the vortex ring, U is the calculated propagation velocity, and ρ and μ are the fluid density and dynamic viscosity, respectively.

Tabulated values of the air density and viscosity are $\rho = 1.204 \text{ kg/m}^3$ and $\mu = 1.8 \times 10^{-5} \text{ kg/ms}$. The Reynolds number is the ratio of inertial (ρU^2) to viscous stresses ($\mu U/D$); hence, it can be used to quantify their relative importance. This number naturally emerges when nondimensionalizing the governing equations of motion for Newtonian fluid mechanics —the Navier–Stokes equations [181, 190]. Notably, the value of Re determines the flow type: the flow generally transitions from laminar to turbulent when $\text{Re} \gtrsim \mathcal{O}(10^3)$. The Reynolds number offers a succinct way for students to characterize the flow associated with their experimental activity, highlighting that it is not a single parameter, such as the velocity or the density, setting the overall characteristics of the flow structure observed, but rather the ratio of inertial to viscous stresses characterizing the observed behavior [191]. The Reynolds number, being a relevant dimensionless parameter, allows us to note that if we were to repeat the experiment with an orifice of half the size ($D/2$), we would need to obtain a propagation speed of $2U$ to achieve a qualitatively similar flow structure for the same fluid.

To compute the Reynolds number for our setup, we opted to use the propagation speed U and diameter of the orifice D , respectively, although other choices are viable taking into account alternative length and velocity scales. (for an extended discussion see Saffman [192]). By using standard air properties, most of the vortex rings tracked in the gathered data of the students had Reynolds numbers in the range of $\mathcal{O}(10^2) < \text{Re} < \mathcal{O}(10^4)$. With gentle emission protocol (tapping lightly on the flexible side of the box) and/or small orifice openings, laminar regimes could be reached with azimuthal instabilities appearing as the vortex ring travels away from the orifice. For their lab activity, each student group was instructed to document a laminar and turbulent vortex ring with the computation of the associated Reynolds number. In Fig C.3(a), a laminar vortex ring is shown ($\text{Re} = 3400$) where streamlines can be seen, and the turbulent vortex ring (Fig C.3(b)) looks less structured ($\text{Re} = 8300$). The laminar to turbulent transition was observed around a $\text{Re} \approx 4000$. This formed the first collection of results and analysis.

The second set of results focused on extensions of the initial configuration, with an emphasis on aesthetics. In order to allow for creativity with the experimental setup, multiple additions were suggested to the students. Of those, a few examples include considering the collision of vortex rings with other objects and/or other vortex rings which have been shown to generate objects of striking visual appeal [193]. In addition, deviations from vortex ring generation were suggested. Additions included dragging a model car across the table and visualizing the turbulent wake behind it as well as adding toys to interact with

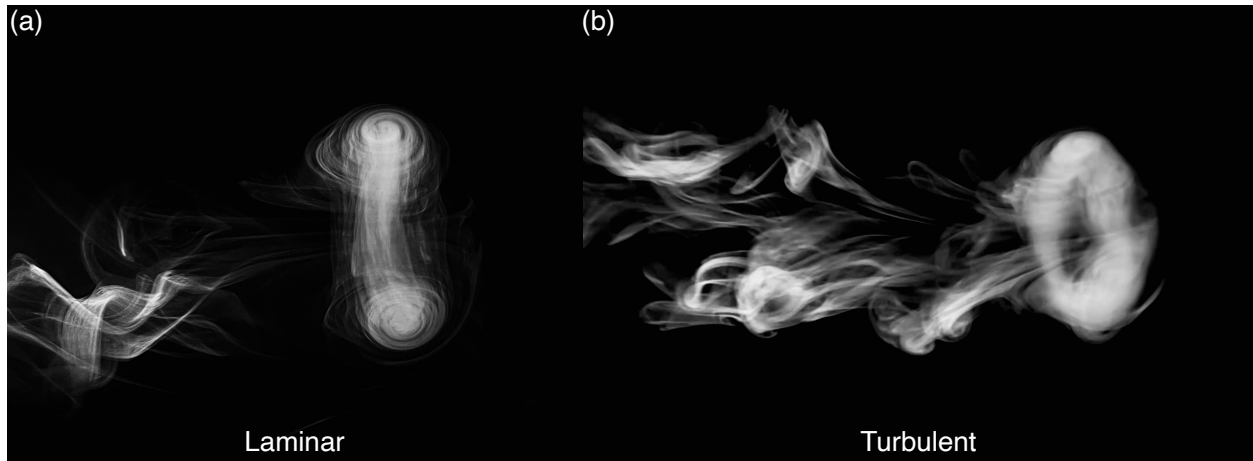


Figure C.3: Generation of laminar and turbulent vortex rings. By changing the intensity of the impulse on the back of the container, both a laminar ($Re = 3400$) (a) and turbulent ($Re = 8300$) (b) vortex rings are generated. While the laminar vortex ring is characterized by visible streamlines in the ring itself, the turbulent vortex ring exhibits an unsteady blob of smoke with a pronounced wake behind the traveling ring. All photographs are reproduced with permission from the students. See Appendix III for attribution.

a laminar vortex ring looming (Fig C.4), with the intention being to mimic Henri Matisse’s 1905 Joy of Life. The use of Adobe Photoshop, ImageJ, or other photo editing applications are suggested to modify the colors, contrast, and brightness of the images. Although there were many submissions of the best flow visualization of a vortex ring, a subset of those are displayed to demonstrate the final output from the 2023 and 2024 iterations of the course (Fig. C.5).

Finally, an additional, optional activity could be conducted with the setup: the propagation of an elliptical vortex ring. When the experimental setup is operated with an elliptical orifice opening, the resulting vortex ring wobbles as it propagates outward. The leading instability of wobbling leads to the oscillation from prolate to oblate elliptical toroids [194]. Students were asked to take videos for different size and different aspect ratio ellipses to determine the period of oscillation, T_{ellipse} , and plot the period of oscillation as a function of the exit velocity U , eccentricity, and absolute size of the semi-major axis to see if any trends emerged. The semi-major axis took on values between 3.2 cm and 4.4 cm, and the semi-minor axis took on values between 1.9 cm and 2.5 cm. This allowed for the activity to not only represent a place to gather striking visualizations of vortex rings with a simple table-top setup, but it also allowed for a platform to gather data to track a quantity of interest (T_{ellipse}) to be tracked over time for different experimental parameters. Students found that T_{ellipse} attained values between 0.2 s and 1.2 s, and that the period of

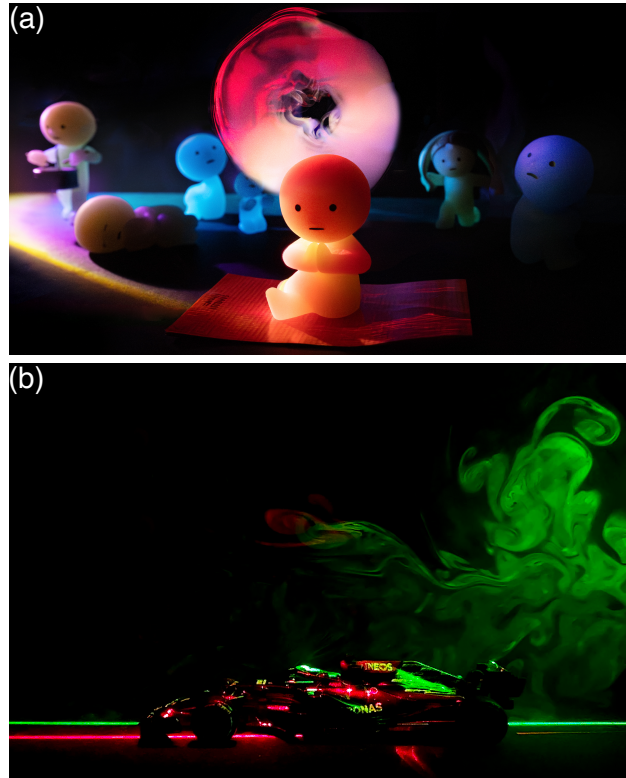


Figure C.4: Visual additions to the vortex ring experiment. A large array of behavior was visualized as the students were given the freedom to adapt the experimental setup. **(a)** A laminar vortex ring is placed within a group of stick figures in the work to mimic Henri Matisse's 1905 *Joy of Life*. **(b)** A toy car is accelerated on the table leading to a large disturbance in the ambient flow fields of the air. All photographs are reproduced with permission from the students. See Appendix III for attribution.

oscillation monotonically decreased with the exit velocity of the vortex. The students did not find a clear monotonic trend for the eccentricity nor absolute size of the semi-major axis.

C.4 Conclusions

Here, the students took different creative approaches both during the experimental setup and after in the post-processing of the photos/videos to create striking works of art. The breakdown of a laminar vortex ring into a heart shaped vortex filament demonstrates the route to instability as the ring traverses from the box (Fig. C.5a). By tapping the elastic side of the box twice, a pair of rings can be created that interact with one another and one another's wakes (Fig. C.5b) with a Reynolds number of $Re = 1900$. Finally, a turbulent (Fig. C.5c), $Re = 6000$, versus laminar vortex ring (Fig. C.5d), $Re = 3800$ are visualized, with image color modification to draw out the different hues in the image.

We have presented a simple table-top experiment that can be used to both introduce flow visualization with a vortex ring and highlight the intrinsic beauty that comes with vortical flows. By focusing on visualizing vortex rings, the students were able to compare and contrast turbulent and laminar fluid flow, creating aesthetic imagery in the process. Through imaging the experiment, students were introduced to simple image analysis techniques, allowing them to track vortex rings over time and calculate both the vortex propagation velocity, U , and the corresponding Reynolds number, Re . By computing the Reynolds number, students were exposed to dimensionless numbers in fluid mechanics, using it to characterize the flow and distinguish between laminar and turbulent behaviors, beyond a qualitative description. The simple experimental setup allows for the introduction of concepts in fluid mechanics to be approached at an earlier time and age than a standard junior-level fluid mechanics course, generally much more focused on the theoretical aspects of fluid flow. The experiment extended beyond the traditional physics curriculum, fostering interdisciplinary collaboration with students in the arts and humanities with an emphasis on producing aesthetically compelling results that are featured as part of an art exhibit on the campus of our university.

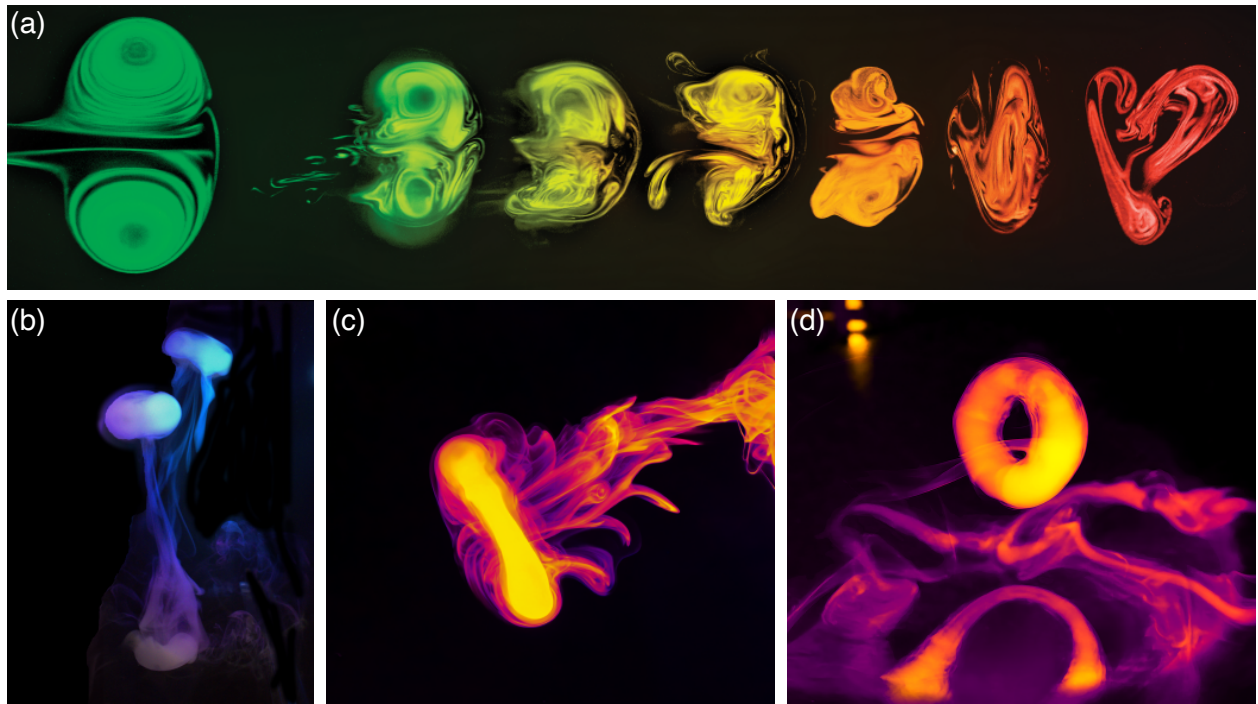


Figure C.5: Artistic representation of vortex rings. At the end of the lab, students presented their favorite images from their exploration of which a small subset of the final results produced in the 2023 and 2024 iterations of the course. All photographs are reproduced with permission from the students. See Appendix III for attribution.

C.5 Bill of Materials

The experiment can be reproduced with the following materials. Prices are provided for Amazon products in the USA. The use of a phone as a camera is assumed.

Material	Price	Description
1/4" Acrylic Sheets (AZM Displays). [x3]	11.99	Acrylic can be laser cut or sawed to make the 5 sides of the vortex ring generator box. Additional sheets of acrylic are useful for making modular front faces.
Plastic Wrap (Amazon basics 200 sq ft)	3.29	Used to form the back face of the box that is flexible and able to generate an impulse.
Laser Pointer (LUEIN)	19.99	Visualization of a 2D slide of the flow field.
Glass Stirring Rods (Burrry Life Science)	8.99	Addition to the front of the laser pointer to illuminate a plane.
Fog Machine (LUEIN)	39.99	Machine to send out fog machine fluid to illuminate fluid flow.
Fog Machine Fluid	19.99	Illumination of fluid flow.
TOTAL	128.22	Estimated total cost to run the experiment.

C.6 Attribution Table

Fig	Title	Students
2(a)	“Untitled”	Amaya Vazquez, Dashaun Simon, Isabel Roth-Dishy, Shivani Mendiratta, Benjamin Newcomb. (2023).
2(b)	“Untitled”	Sahil Bansal, Rachel Rowey, Alara Kalfazade, Peyton Newman. (2023).
3(a)	“Laminar”	Sahil Bansal, Rachel Rowey, Alara Kalfazade, Peyton Newman. (2023).
3(b)	“Turbulent”	Sahil Bansal, Rachel Rowey, Alara Kalfazade, Peyton Newman. (2023).
4(a)	“Joy of Life”	Morgan Holstine, Venkatsai Bellala, Hannah Jeon, Fiona Liu, Gaurav Rathore. (2024).
4(b)	“Dirty Air”	Houston Li, Julia Zaborowsky, Raymond Gresalfi, Mandy He, Yuki Hayashita. (2023).
5(a)	“Laminar Love”	Morgan Holstine, Venkatsai Bellala, Hannah Jeon, Fiona Liu, Gaurav Rathore. (2024).
5(b)	“Ernie and Bernie ”	Victoria Vo, Angelina Schorr, Ian Becker, Jennifer Cheng, Grant Adam. (2024).
5(c)	“Inferno”	Lucian Sharpe, Sofia Gilroy, Kiara Vong, Himanssh Pettie. (2024).
5(d)	“Decay”	Lucian Sharpe, Sofia Gilroy, Kiara Vong, Himanssh Pettie. (2024).

D BLENDING FLUID MECHANICS AND ART PART III: FLUIDS ART PROJECT

Information This chapter is a modified version of the work currently in press in The Physics Teacher [7].

D.1 Introduction - Overview of the Idea

In part III of III of blending fluid mechanics and art[4, 6], students are tasked with creating an art piece that artistically represents a fluid mechanics phenomenon to be showcased at the course art exhibit. Students had the freedom to choose any medium for their artwork while developing a comprehensive plan, considering design integration, layout, and spatial considerations, and adhering to budget constraints. This activity is used to fuel students' concepts of engineering project development and management. The outcomes of this project, which are detailed here, come from participants in a course taught at our University.

Course-based research is effective for student growth and learning in a variety of fields, including nursing, geology, and engineering [195, 196]. The benefits of undergraduate research have been found to encourage students to pursue advanced degrees or continue careers in STEM [197, 198]. To make research more accessible, we introduce research projects into the curriculum, like the course detailed here. This has also been implemented in other classes in the engineering department, such as an advanced fluid dynamics course [196]. The Art-Fluids Engineering course is designed to actively engage students in research while integrating artistic expression with scientific principles. It attracts engineering students as well as those from liberal and the visual arts. This diverse mix fosters an inclusive, welcoming environment where students are challenged beyond their typical academic path.

Similar to previous work, this course asks students to capture the beauty in fluid mechanics by taking photographs [199, 200], however, the course does not limit students to photographs as many projects focused on in-person displays, musical expressions, and permanent installations around campus. This course offers a novel approach to an artistic engineering course work by having students (1) learn the tools to capture these phenomena, may that be photography, design and construction, machining, 3D printing, etc. and (2) to understand the phenomena through the governing parameters, specifically, quantified with dimensionless

numbers. In giving students the incentive to look at these phenomenon through dimensionless numbers it enables them to understand the scales and conditions at which this phenomena will occur, therefore providing art and engineering students an intuition for fluid mechanics that they may not get in a lecture based course.

Here, in part III of III of the series highlighting the connection of art and fluid mechanics, projects were done within a preset group of students from different majors including engineering, international affairs, visual arts, and behavioral sciences, etc. Each preset group contains typically 4-5 students at the junior or senior level, and at least one engineering student. The course typically contains 40% non-engineering students, and 60% engineering students. There are no prerequisites for this course. The topics covered in lectures cover necessary material, however, each group does contain at least one engineering student to ensure each group can reach the academic standards that are set for reports and analysis. The goal was to give the students the freedom to explore a fluid mechanics phenomenon that may interest them, but also challenge them to decide how they would make it come to life as an artistic piece. The example final projects displayed below are the works of students enrolled in this course in the Spring 2023, 2024, and 2025 semesters.

Understanding engineering design, fabrication, and adhering to client constraints is crucial for both engineering students and professional engineers. Many engineering projects require leadership, teamwork, organization, and a comprehensive design plan. Successful project fabrication involves budget allocation, planning, design integration, and spatial considerations. The final project embodies these elements, teaching students to collaborate with individuals outside their discipline, thus introducing them to working with clients or designers. Students not only had to work within the physical constraints of their design plan for the art installation, but also had to manage a budget and a tight schedule are essential skills for effective project management.

D.2 Design and Setup

The project asked students to imagine being commissioned to create an immersive art installation that seamlessly blends the principles of fluid mechanics with artistic expression. The task is to design and justify this unique piece within the constraints of engineering guidelines. Students must consider the following:

1. **Budget Allocation:**

- Develop a detailed budget for this installation, considering the costs of materials, equipment, and any additional resources needed.
- Justify budgetary decisions by explaining how each expenditure contributes to both artistic and engineering elements.

The budget limit for this course was 250 USD per group. The students were encouraged to use the resources at our University such as 3D printers, tools and leftover materials at the machine shop. All groups were able to design their project within this budget, spending varied from large expenses such as Go Pro cameras, or smaller expenses including materials like batteries, wood, tubing, etc.

2. **Planning:**

- Create a comprehensive project plan outlining the key milestones involved in the development of this art piece.
- Discuss the importance of planning to ensure the successful execution of both the artistic and technical aspects.
- Create a contract that includes: a breakdown of leadership and members' roles and strengths, contact information, expected response times, data sharing policy, a decision-making policy, participation expectations, conflict resolution strategies, potential roadblocks, and solutions.

3. **Design Integration:**

- Clearly articulate how the principles of fluid mechanics will be integrated into the artistic design of this installation, specifically discussing fluid characteristics and instabilities that may be causing the art piece to be what it is. Fluid instabilities are typically the driving factor behind the beauty seen in fluid flow. A classical example of this is the Kelvin-Helmholtz instability which can be observed in waves or clouds, where there are two parallel fluids with different densities and velocities. The interface is unstable and susceptible to small perturbations therefore can result in the classical wave shape of the Kelvin-Helmholtz instability [201].

- Discuss how the design choices, such as color, shapes, and materials, reflect and enhance the fluid mechanics concepts incorporated in the piece.

The art piece is evaluated on the students ability to equally convey the important and relevant fluid properties as well as the ability to convince an audience of the artistic merit. To give the students the tools to convey this information, the lectures of this course touched on both aspects including lectures on engineering topics such as surface tension and viscosity, and having guest lectures from active artists. The guest artists were also included in the final project to give advice and guidance for the students to deliver quality artistic pieces.

4. Layout and Spatial Considerations:

- Consider the layout plan for the art installation, discussing mass, length, and time for the physical space it will occupy.
- Discuss how the spatial arrangement enhances the viewer's experience and complements the fluid mechanics principles you aim to highlight.

The students were asked to justify the final project by demonstrating a thoughtful and balanced integration of artistic creativity with engineering precision, showcasing the interdisciplinary nature of this project. Finally, they were asked to provide an artistic assessment of their art piece by considering how their choices contribute to both the aesthetic appeal of the art piece and the underlying fluid mechanics.

The students prototyped, designed, constructed, and analyzed their art installation. Students are asked to meet with the Professor each week once as a check-in with the “vendor”. This allowed time for the students to clarify details of their ideas and work within the constraints of what a buyer might request from them. During this time, students were asked to reflect on the contract they previously designed to make sure each member of their team was staying on track with their set goals. Students were asked to calculate various necessary parameters of their installations, such as pertinent dimensionless numbers, length scales, or viscosities, to predict how, when, or where the observer would witness the fluid phenomena they were trying to display.

D.3 Results

The following examples present two of the successful group projects (see supplement for additional examples). Each project highlights the necessary components for their art installation, including design and artistic elements as well as the fluid phenomena that were explored and the necessary dimensionless parameters or relevant equations used.

D.3.1 Spacewalker: An Artistic Display of Rayleigh-Bénard Convection Cells

The purpose of this project was to visualize thermal convection in an aesthetically pleasing way. To do this, the students used Rayleigh-Bénard convection [202]. This was done by heating a bath of fluid: as the fluid heats from the bottom, the hotter fluid's density will decrease and start to rise to the surface. As it rises, the fluid will begin to cool and the density will increase, therefore making the fluid sink. This rise and fall pattern will create a cell-like structure that can be visualized by heating a bath of vegetable oil seeded with rheoscopic particles. Vegetable oil was used because it is affordable, non-toxic, has a high boiling point, and has a higher density than some other oils such as mineral oil. Mineral oil and cooking oil were both tested but the convection cells produced using vegetable oil were more prominent. Using a fluid with a higher starting density will allow for a larger gradient in density as the fluid is heated, resulting in more apparent convection cells.

Students fabricated the piece, as seen in Fig. D.1, using a griddle for the heating mechanism and a dish that contains both cooking oil and mica powder. The students calculated the Rayleigh number, $Ra = \frac{g\alpha\Delta T b^3}{\nu\kappa}$, the Prandtl number $Pr = \frac{\nu}{\kappa}$, and the net heat transfer, $Q = hA\Delta T$, where g is gravity, α is the coefficient of thermal expansion, ΔT is the temperature difference over the layer, b is the thickness of the layer, ν is the kinetic viscosity, κ is thermal diffusivity, h is the convection heat transfer coefficient, and A is the surface area. These parameters informed the students of the conditions under which they could expect to see convection cells form. When the Rayleigh number is above a critical value for the fluid, then the flow will be governed by convection. At high Prandtl numbers, the flow is dominated by thermal diffusivity, causing cells to form [202]. The net heat transfer quantifies heat transfer from a solid to a moving fluid.

In modifying the temperature from the heating plate and the distribution of heat using an aluminum sheet, the students were able to calculate the amount of heat, or temperature, needed to create beautiful

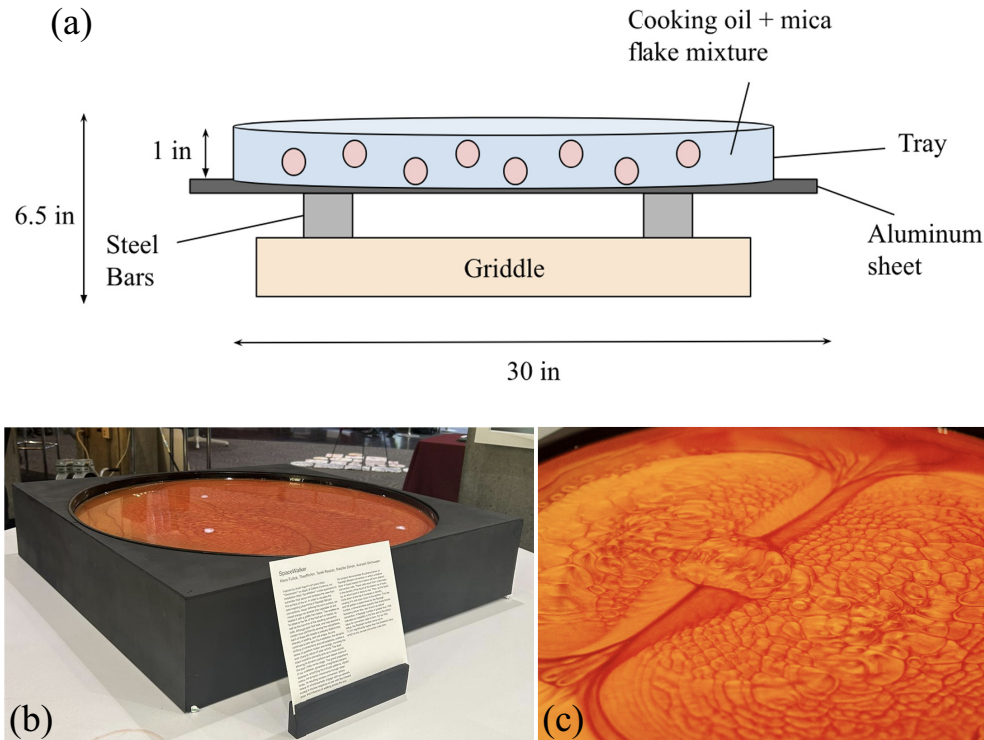


Figure D.1: (a) Rayleigh-Bénard convection cell experimental setup. (b) Displayed installation of Rayleigh-Bénard convection with a diameter of about 75 cm. (c) Zoomed in view of convection cells. Attributions are listed in Appendix II.

convection cells. Temperatures for heating ranged from 32 to 190°C. Fig. D.1 shows the final production of the convection cells, the small repeating patterns, within the vegetable oil bath. For this particular case, to see the convection cells, the temperature difference from the top of the oil and the heating plate was 8.33 Kelvin. The calculated Rayleigh number was 7.12×10^4 . The fluid is orange due to the orange rheoscopic particles in the vegetable oil. The two different lobes in the fluid are due to a non-uniform heating source; the heating was less intense in the center, causing the cells to form on either side. The convection cells were created quickly after the heater was turned on, typically 1-2 minutes. After the convection cells formed, the piece would remain on for roughly 2 plus hours giving viewers time, typically observing for several minutes, to watch the rise and fall evolution of the cells.

Regarding the artistic value of their piece, the students said: “SpaceWalker’s grand proportions generate a heightened sensory experience, amplifying the intricate patterns, vibrant colors, and dynamic movements through sheer scale. The resulting sensory immersion allows viewers to wholeheartedly engage with our

artwork, evoking a visceral response on par with the inherent power and influence of walking across the sun.”

D.3.2 Clouds on the Loose: Creating indoor clouds

This project’s goal was to create large clouds in iconic rooms and spaces on campus. To create indoor clouds, the students poured liquid nitrogen into a container of boiling water. Boiling water was used to increase the humidity in the air, followed by the introduction of liquid nitrogen, which rapidly cooled the air. This sudden cooling caused the humidity to condense into droplets, forming clouds [203]. Assuming standard atmospheric conditions and a relative humidity of approximately 100%, after the addition of warm water, the students estimated the amount of water that could be extracted from the air to form droplets [204]. Using a typical droplet volume fraction value ($\alpha = 10^{-6}$) and a droplet diameter of $d = 10 \mu\text{m}$, they calculated the corresponding cloud volume as $V_{\text{cloud}} = M_{\text{water}}/(\rho_{\text{water}}\alpha)$, where M is the amount of water in air with 100% humidity. The calculation was ultimately compared with the clouds obtained experimentally. The students were able to take photographs of the clouds which were then used to compute rough estimates of the dimensions of each cloud, in turn, calculating the volume. From the volume the students were able to calculate the mass of the water at a corresponding humidity and compare this to a psychrometric chart [205]. The students created a few setups, in various iconic buildings at our university, which involved rapidly pouring liquid nitrogen into a contained area with boiling water. The students experimentally found that if the nitrogen was not poured quickly enough the cloud would not be as large as they wanted. Therefore, using a 3 liter metal bucket with about 2 liters of boiling water, and pouring the liquid nitrogen through a large spouted vessel created the tallest and largest cloud plumes.

The students’ final project included the photographs taken from these experiments to capture the clouds that lasted for approximately 10-15 seconds, these are shown in Figure D.2, which were displayed on the walls of the exhibition.

When asked to comment on the artistic value of their project, the students said: “We knew that we wanted to stage the clouds in iconic, recognizable spaces across the beautiful and picturesque campus. Specifically, we wanted the clouds to interact with the campus and to fill the space of students, creating a sense of awareness and awe for the spaces that students walk through or inhabit daily.”



Figure D.2: Liquid nitrogen clouds in various locations on campus, including our performing arts center, greenhouse, quiet green, and cafe on campus. Attributions are listed in Appendix II.

D.3.3 Love is in the Air: Wingtip Vortices Visualization

The purpose of this project was to visualize trailing vortices formed from the tip of an airplane wing. The geometry of the wing and the angle of attack create a significant pressure difference between the lower and upper surfaces of the airfoil. The faster airflow goes over the top surface, leading to a lower pressure compared to the air below the wing as dictated by Bernoulli's principle [206]. At a positive lift condition, the high-pressure air moves outward from beneath the wing and curls to create wingtip vortices [207]. Although these wingtip vortices are always being produced at sufficient plane speed, to visualize them, the plane must move through opaque droplet or particle suspensions such as clouds.

Before construction, the students needed to understand what factors would influence the creation of the wingtip vortices. Using Prandtl's Lifting Line Theory, students were able to make assumptions about the impact of wing geometry and angle of attack on lift and the resulting vortices [207, 208]. It was found that the most visually impressive wingtip vortices occur when induced drag is high, which can be achieved by

generating a significant pressure difference between the top and bottom surfaces of the wing. They found, according to models, that a clean (laminar, $Re \approx 3000$) [206] and slow (low velocity) airflow was needed. Experimentally, they achieved this by aiming for relatively low Reynolds numbers ($Re = \rho U d / \mu < 3000$). In order to control the airflow, the students found that it would be easier to move the airplane rather than trying to control the airflow around the plane with a wind tunnel. This, therefore, informed the design of the final project to have the airplane on the sliding rail. The velocity of the plane was controlled manually. The velocity that produced the best results was found experimentally.

The final art installation was composed of a foam toy airplane on a rail system, as seen in Figure D.3(a). A fog machine was placed below the rail. The fog was directed into a box with a paper tube coming out of the top to control the direction of the fog. This is the gray box below the rail in Figure D.3(a). This optimized the amount of fog in one area, making the visualization of the wingtip vortices clearer, as seen in Figure D.3(b). Lighting was changed during the showing in order to create more aesthetic images and higher contrast.

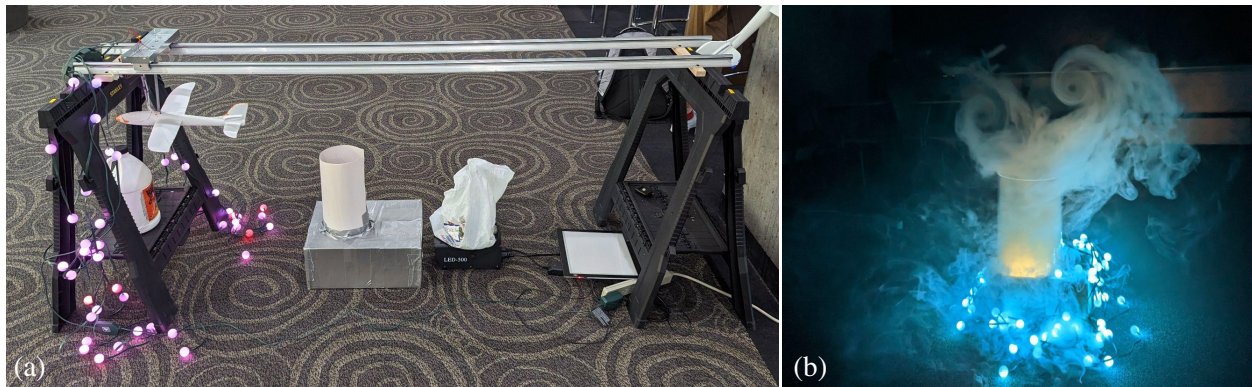


Figure D.3: (a) Final configuration of foam airplane mounted below rail system with fog machine on the floor. (b) Wingtip vortices are visualized with fog and blue lighting. Attributions are listed in Appendix II.

For this project, the students stated: “The composition of the images makes it seem as though the plane is going on an adventure. With the background panel seemingly like fluffy clouds in the sky and the fairy bulbs lighting the plane’s way, the image brings about a feeling of excitement. In the same breath, the images also convey the plane’s speed and force, capturing the dynamic nature of its movement, allowing for a more immersive experience, effectively showcasing the plane’s rapid motion from the use of springs.”

D.3.4 The Flow of Color: Vortex-Shedding–Driven Paper Marbling

The goal of this project was to visualize vortex shedding in fluid flows. The project was inspired by the traditional painting technique called 'paper marbling'. Using bluff bodies moved through various dyes in a fluid bath, the group was able to achieve a multitude of final prints. The team explored how factors such as bluff body size, velocity, and viscosity influenced the resulting patterns captured on Hosho paper. Hosho paper is a professional-grade, thicker paper ideal for absorbing the patterns created in the fluid bath.

To create the vortex shredding prints seen in Fig. D.4, the students dropped marbling ink on the surface of a water bath. Once there was enough ink on the surface, the students would then drag the bluff bodies, such as a PVC pipe, through the ink at a constant rate. This would induce vortex shedding, which would then be captured by immediately placing the paper on top of the ink in the water bath.

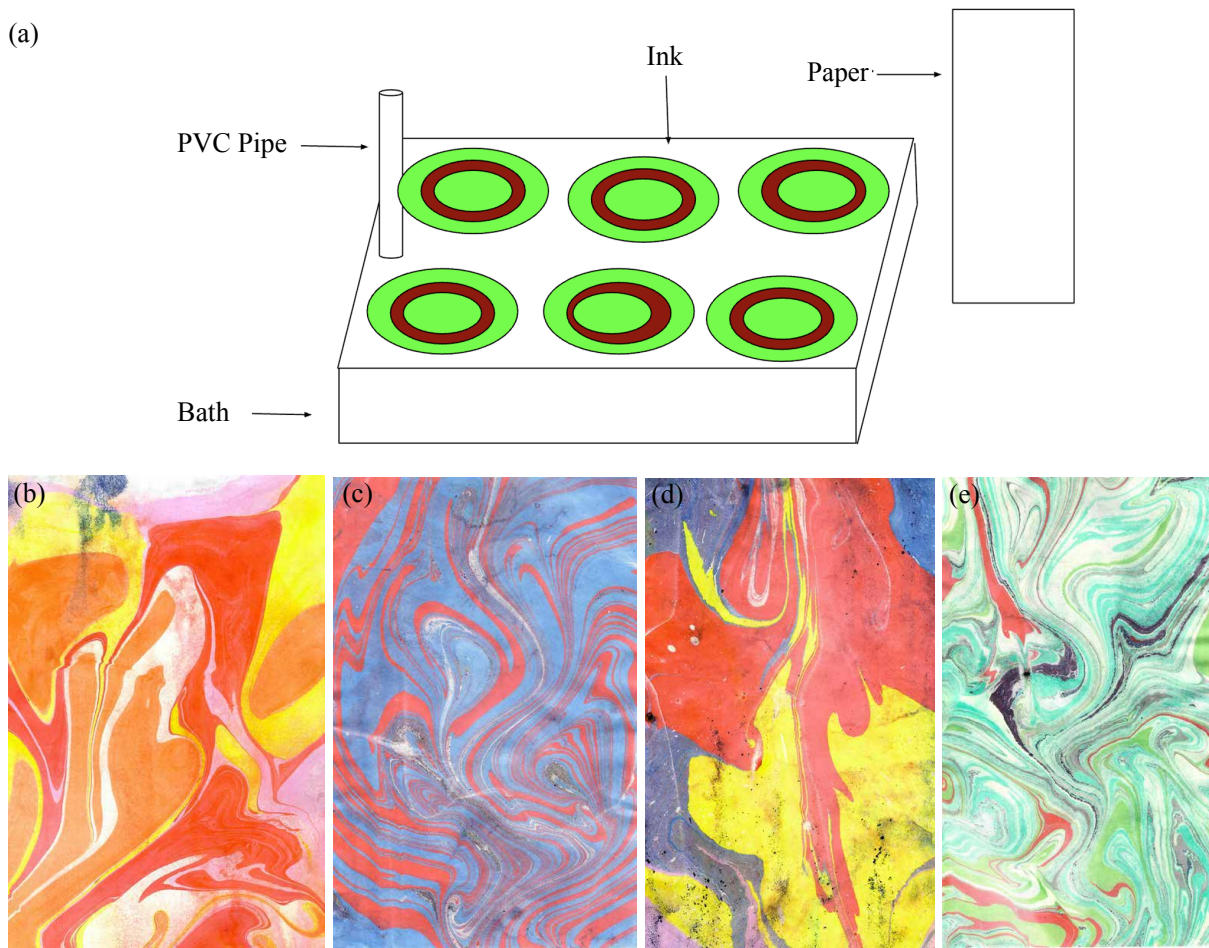


Figure D.4: (a) Experimental schematic for vortex shedding. (b-e) Examples of the final prints, which used subtle, gradual transitions in colors to capture vortices.

Although the setup was overall simple and used as a common artistic technique, the group had to explore how various fluid properties influence their vortex shedding. For instance, the Reynolds number was calculated to determine the speed to move the PVC pipe or other bluff body through the ink water mixture. Here, the Reynolds number, $Re = U\rho d/\mu$, is defined by ρ , μ , d , and U , which are the fluid density, fluid viscosity, bluff body diameter, and characteristic speed, respectively. To determine the rate to manually move the bluff body, the students wrote a simple tracking code on MATLAB that used images captured during tests of the bluff body. From these, the students could find the velocity using a known distance (i.e. the length of the tub) and a time (i.e. the video duration). The ideal range of Reynolds numbers to see the vortex shredding is between 60 and 160; therefore, experiments were conducted within this range [209]. In tests that used a small cylinder at a comparable rate, there was a lack of vortex shedding, showing that the Reynolds number was too low to cause vortices to detach, whereas tests with a large cylinder, caused higher Reynolds numbers which caused the flow to become turbulent making the vortices to irregular and not ideal for visualization or printing. The final product for this project was a display of the vortex shedding prints, which are shown in Fig. D.4(b). Each print used a variety of different color dyes to capture an artistic vision beyond the shedding vortices. The prints were hung on a wall, grouped by color for the art gallery.

This group wrote the following aesthetic statement for their work: "Some pieces feature perfect symmetry, highlighting an elegant balance that encourages contemplation, while others embrace more chaotic, spontaneous bursts of color, capturing the unpredictable energy of the creative process. This thoughtful inclusion enriches the overall composition, resulting in stunning marble papers that reflect a delicate balance between tension and harmony, as well as passion and tranquility. Each piece carries its own narrative within the larger artwork, inviting viewers to delve into the complexities of emotions and uncover the beauty in their interplay."

D.4 Teaching Outcomes

The final projects encouraged students to think creatively and practically as they explored a range of fluid mechanics phenomena while producing visually appealing art installations. Each group's project demonstrated a unique approach, both in the fluid phenomena they chose to visualize and in the presentation of their final art piece. The projects highlighted above represent just a portion of the course's final outputs.

The projects also varied in their presentation style. While many included real-time demonstrations for the audience, this was not a requirement. Some groups, whose work is not shown here, opted to showcase videos or images of their work, while others displayed physical canvases, such as a project where a drone was used to paint multiple canvases. The final projects spanned topics including bubbly flows, soap film interference patterns, musical aspects of rain on different surfaces, visualization of wind, the art of airflow over an airfoil, and even chocolate fountains. Despite the creative freedom, all projects followed a common structure emphasizing not only teamwork, leadership, and organizational skills but also detailed planning, budgeting, design integration, and spatial layout considerations.

D.5 Conclusions

We provided an overview of the final project for the Art Fluids course, which challenges the conventional divide between fluid mechanics and art. Students were tasked with creating art installations centered on fluid mechanics phenomena, emphasizing their inherent beauty to be showcased at an art exhibition. This project introduced the students to the concept of experimental design and project management.

D.6 Tips, Disclaimers, Room for Future Directions

For each project in this course, students were organized into groups of approximately five members. The instructor carefully selected the groups to include at least one engineering major, alongside visual or humanities students. This thoughtful composition ensured that both the engineering and artistic aspects were effectively addressed in the creation of their art pieces.

There was a large portion of this project left for students to be creative and have the freedom to test out their ideas; however, a few stricter guidelines and deadlines may have helped the students create better functioning projects and timely results. Implementing further engineering tools into the course could help this, such as Gantt charts, Conceptual Design Review (CoDR), and Preliminary Design Review (PDR). These tools are often taught in other engineering design courses and encourage personal timelines that work within the semester. These tools will also teach the students how to manage their time in and outside the classroom.

D.7 Attribution Table

No.	Title	Students
1	“SpaceWalker”	Kiera Fullick, Theo McArn, Tarek Razzaz, Maddie Simon, Avinash Srinivasan (2024).
2(a)	“Ghost Rehearsal”	Makayla Minter, Mateo Benitez, Jake Srebnick, Toshi Kato, Erin Alexander (2024).
2(b)	“Sky Garden”	Makayla Minter, Mateo Benitez, Jake Srebnick, Toshi Kato, Erin Alexander (2024).
2(c)	“Blink and You’ll Mist It”	Makayla Minter, Mateo Benitez, Jake Srebnick, Toshi Kato, Erin Alexander (2024).
2(d)	“This Seat’s Taken”	Makayla Minter, Mateo Benitez, Jake Srebnick, Toshi Kato, and Erin Alexander (2024).
3b	“Love is in the Air”	Julia Zaborowsky, Yuki Hayashita, Ray Gresalfi, Mandy He, Houston Li (2023).
4b	“Golden Hour”	Ella Blanco, Lauren Duncan, Jake Cardillo, Adam Lalani (2025).
4c	“Crimson Tides and Sapphire Skies”	Ella Blanco, Lauren Duncan, Jake Cardillo, Adam Lalani (2025).
4c	“Childish Dreaming”	Ella Blanco, Lauren Duncan, Jake Cardillo, Adam Lalani (2025).
4e	“Currents of Verdant”	Ella Blanco, Lauren Duncan, Jake Cardillo, Adam Lalani (2025).