

SOURCE Lesson Plans Detail

Fractals: Measuring fractal dimensionality

Topic	Fractals
Program	Brown Science Prep
Developed by	Alex Loosley
Developer Type	High school students

Overview / Purpose / Essential Questions

This lesson plan is for an interactive and engaging lesson that teaches highschool students about fractals and dimensionality. I tried it in September of 2013 on a grade eleven pre-calc class at Beacon Hill highschool and it was very successful. The lesson raised essential questions such as, what are dimensions; what is a fractional dimension; how can the fractional (fractal) dimensionality of everyday real objects be measured? The first question is answered using humor and brainstorming as part of a chalkboard talk, whereas the latter two questions are discovered by the students through guided interactive experimentation.

Performance / Lesson Objective(s)

The objective is to give high school student a generalized sense of what a dimension is and a quantitative tool for measuring dimensionality.

Lesson Materials

- Blackboard or whiteboard
- Different color chalks or whiteboard markers
- Coastline Paradox Materials:
 - ~18x18" foam board with complex coastlines inscribed (see picture attached)
 - ~100 push pins per island
 - 2 1" rulers per island, 2 2" rulers per island, 2 5" rulers per island, 1 10" ruler per island.
- Computer with Excel connected to a projector

Lesson Motivation

Fractals are an important part of everyday life and most of us do not know it. One interpretation is that fractal dimensions of a coastline or trajectory measure its complexity. In my research, I study the migration of human white blood cells crawling from the blood stream to site of infection or injury. The complexity of these migration trajectories can be measured by measuring their fractal dimensionality. Given that we live in Rhode Island, it is also interesting to measure the complexity of our coastline using the Mandelbrot fractal representation.

Procedure

1. Humorous blackboard introduction (involving FRACTAL anagrams)
 - a. I started with some funny words I came up with and then used the anagram word FLATCARS to get the show rolling.
 - b. I used cardboard bowl to represent a car claiming that the highschool would not allow me to drive my car into the building... I asked the class if this was a 1-D, 2-D, or 3-D car. They answered 3-D. Then I flattened it (as best i could) and took a pool (hands up to vote), is this now a:
 - i. 2-D car
 - ii. 3-D car
 - iii. 2.5-D car
 - c. What does 2.5-D mean? We'll find out later
2. Group brainstorm
 - a. Draw a box: class lists the dimensions (they should figure out that the dim's are height, width, and depth - the parameters necessary to define the box)
 - b. Draw a sphere: class learns that this is only 1-D (radius)
 - c. Draw a stick figure of somebody important (like the principle) - some dimensions included (after I started the students off) - arm length, leg length, girth, percentage of head covered in hair (this exercise was very humorous and engaging)
3. Recognize from the group brainstorm that we're still only working in integer dimensions, though we now have a more general understanding of what an integer dimension is. This was a qualitative exercise, now we'll look at dimensionality quantitatively.
4. Draw line, square, cube... show that when the lengthscale is doubled, the number of lines doubles, the number of squares increases by a factor of 4, and the number of cubes increases by a factor of 8... this mathematically demonstrates basic spatial dimensionality.
5. Show class the Koch Curve (snow flake), lengthscale must be tripled to get a curve that is quadrupled. Therefore, $d \sim 1.26$ (weird)
6. Show class another fractal like Serpinski's triangle if time (I usually leave this out)
7. Introduce the concept measurements of length or area or volume depend on the lengthscale of measurement. If one looks at either of the fractals introduced in 5 or 6 at a big lengthscale, they miss all the details. Leave the class a little confused, get them into groups of 3-4 and hand out the foam board island. **ACTIVITY:** each group should measure the perimeter of their island using a 1" ruler, a 2" ruler, a 5" ruler, and a 10" ruler (they can use two 1" and 2" rulers to speed this process up).
8. Collect their results: weird finding!! Throw the data back at the student -the coastline perimeter changes depending on the ruler used?!?!

9. Using excel spreadsheet (this should be premade), input the class measurements in the spreadsheet and fit the Mandelbrot power law formula to measure the dimensionality of each island. Well done, now everybody in the class has successfully measured their first fractional dimensionality.
10. Which coastline had the highest dimensionality, does this one look the most complex?
11. I finished with a bit about my research, in which I use this same technique to measure the complexity of migration trajectories of human white blood cells as they migrate from the bloodstream to sites of infection and/or injury. There, now everybody in the class can do the "impressive" mathematics that I do :-).

Reference(s)

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Alignment Info

Audience(s)	High school students Undergraduate students
STEM Area(s)	Applied Math
Standard(s)	<i>Physical Sciences (RI GSE) PS1.9–11.1a</i> Students demonstrate an understanding of characteristic properties of matter by ... utilizing appropriate data (related to chemical and physical properties), to distinguish one substance from another or identify an unknown substance.
Activity Type(s)	Hands-on
Grade Level(s)	High School
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