

Abstract of “Wave propagation, instabilities, and pattern transformation in dielectric elastomer composites: Application to tunable soft phononic crystals”

by Michael Anthony Jandron, Ph.D., Brown University, May 2019.

Phononic crystals are periodic, composite solids that exhibit phononic band gaps – frequency ranges in which elastic waves are prohibited. When made from soft elastomers, phononic band gaps may be reversibly manipulated through large elastic deformation of the periodic structure. By using dielectric elastomers, which undergo large, reversible deformations when subjected to an applied electric field, the frequency ranges of band gaps may be adjusted, and new band gaps may be created. This thesis addresses several problems related to the tunability of phononic crystals comprised of dielectric elastomer composites.

First, we present our finite-element-based numerical simulation capability for designing electrically-tunable, soft phononic crystals. Our finite-element tools address both nonlinear quasi-electrostatic processes and the linearized dynamics of electroelastic wave propagation through a pre-deformed state and may be applied to general composite unit-cell geometries subjected to arbitrary far-field electromechanical preloading. We apply our simulation capability to electrically-tunable, soft phononic crystals consisting of periodic lattices of aligned circular-cross-section fibers embedded in a matrix and demonstrate the shifting of band gaps with electrical preloading parallel to the fibers and the opening, closing, and shifting of band gaps with electrical preloading perpendicular to the fibers.

Second, we use our numerical simulation capability to detect electromechanical instabilities in dielectric elastomer composites. We consider all known electromechanical instability types: pull-in, microscopic, macroscopic, and interfacial instabilities, while surveying a comprehensive parameter space. The result is an overarching view of the instability landscape in periodic dielectric elastomer composites.

Third, using our numerical simulation capability, we show that microscopic electromechanical instabilities may be harnessed to invoke reversible, electric-field-triggered pattern transformations in dielectric elastomer composites. We then assess pattern transformation as a mechanism for enhanced tunability of phononic band gaps. Additionally, we consider composites with conductive elastomer inclusions and conductive voids. Finally, we show that electric-field-triggered pattern transformation and hence band-gap tunability may be achieved in phononic crystals under complete, far-field mechanical constraint.

**Wave propagation, instabilities, and pattern transformation in
dielectric elastomer composites: Application to tunable soft
phononic crystals**

by

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DEDICATED TO MY DAD, WAYNE A. JANDRON
(APRIL 6, 1956 – JANUARY 13, 2016)

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Chapter 1

Introduction

1.1 Overview

Soft, periodic materials may be designed to exhibit unusual properties – such as auxetic behavior – and novel functionalities – such as acoustic band gaps and pattern transformation – that are not observed in conventional materials. (See the review of Bertoldi et al. [1] and the references therein for further discussion and examples.) Moreover, due to their compliant nature, periodic materials with tunable properties may be achieved. As a novel route to tunability, dielectric elastomers – materials in which large deformation is induced through the application of an electric field – may be used in soft periodic materials, opening the door for electrically-actuated tunability. The overarching motivation of this thesis is to numerically explore and understand the functionality and tunability of periodic composite materials made from dielectric elastomers.

1.1.1 Objectives

This thesis is framed around the following objectives:

1. Develop a finite-element-based numerical simulation capability for infinite, periodic dielectric elastomer composites that addresses both nonlinear quasi-electrostatic processes and linearized, incremental processes;

2. Apply the simulation capability to electrically-tunable soft phononic crystals, in which phononic band gaps may be tuned through the application of an electric field;
3. Apply the simulation capability to identify and detect all electromechanical instabilities that can arise in dielectric elastomer composites; and
4. Apply the simulation capability to electric-field-triggered pattern transformation in dielectric elastomer composites and explore the use of pattern transformations to manipulate electroelastic waves.

1.1.2 Significance

The opportunities afforded by the design of materials to control acoustic waves are exciting and numerous. We envision a future where structural components may be replaced with tunable phononic crystals for superior wave propagation and vibration control. Potential applications are schematically illustrated in Fig. 1.1 and include vibration mounts, wave guides, and acoustic cloaks. The unique feature of *tunable* phononic crystals is that they may be reversibly adapted during use – meaning optimal isolation can be adjusted to suit the immediate requirements – opening up a new world of possibilities. To reach such a future, the development of modeling technology to numerically study these materials is needed.

1.2 Background

1.2.1 Dielectric elastomers

Dielectric elastomers, or soft dielectrics, are rubbery materials that undergo large elastic deformations in response to an applied electric field [cf., e.g., 2–6]. For example, an unconstrained dielectric elastomer layer undergoes large, reversible mechanical deformation when subjected to an electric field applied through a set of compliant surface-mounted electrodes as illustrated in Fig. 1.2. These materials were first reported by Pelrine and coworkers [2, 7–9] and have since

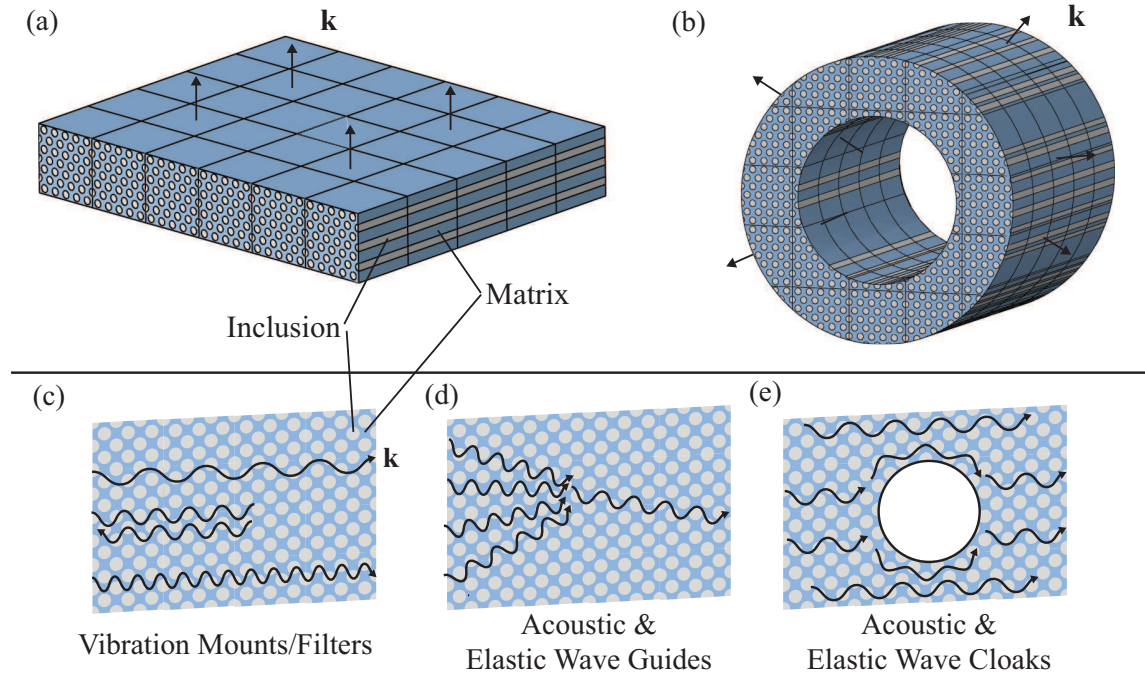


Figure 1.1: Illustrations of potential applications of phononic crystals. On top, schematic phononic crystals comprised of dielectric elastomers in (a) slab and (b) cylindrical configurations, showing representative elastic wave propagation directions by the wave vector $\mathbf{k}$. On bottom, schematic use of periodic composites as (c) vibration mounts/filters, (d) acoustic and elastic wave guides, as well as (e) acoustic cloaks are shown.

then garnered interest as electromechanical transducers for a wide variety of applications [10], such as robotics, biomedical engineering and energy harvesting. Dielectric elastomers differ from piezoelectric materials in that dielectric elastomers are capable of large, reversible deformations and are non-polar, amorphous materials, while piezoelectrics are typically only capable of small, elastic deformations and are polar crystalline materials. Soft dielectrics are also comparatively lighter, more compliant and less expensive, increasing their appeal. At a microscopic level, dielectric elastomers are electrically-insulating, polymeric materials made up of long-chain molecules, possessing charge imbalances that align, or polarize, in the presence of an electric field. Since these materials can achieve both large mechanical deformations and electrically polarize, they are capable of converting energy between mechanical and electrical forms. In particular, actuators that convert an applied electric field into mechanical motion have been used in robotics, including as artificial

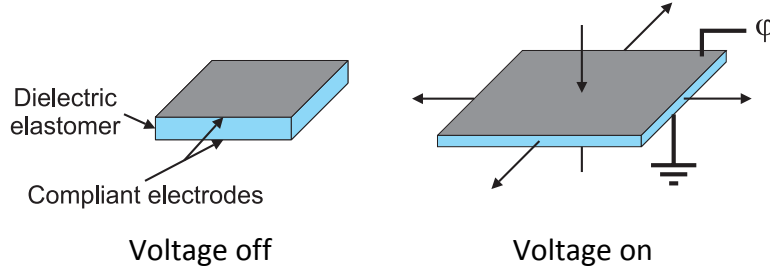


Figure 1.2: Schematic of the basic operation of a dielectric elastomer actuator subjected to a voltage difference φ .

muscles for biomimetic robots and prosthetics [11–14]. In contrast, soft dielectric devices that convert mechanical energy into electrical energy are referred to as energy harvesting devices [15, 16]. In these applications, mechanical energy from motion such as human walking or ocean waves is converted to electrical energy and stored. Work towards developing electromechanically-coupled constitutive theories for dielectric elastomers goes back many decades [cf., e.g., 17–25], and in recent years the mechanics community has come to a relative agreement regarding the formulation of a theory in a thermodynamic framework [26–31].

1.2.2 Phononic crystals

A solid phononic crystal is a periodic composite material that harnesses constructive interference through Bragg scattering to give rise to frequency ranges in which elastic waves cannot propagate – phononic band gaps. Interest in these materials goes back several decades, and phononic crystals made from stiff, linear elastic materials have been extensively researched [32–36]. These materials have shown promise in a wide variety of applications [cf., e.g., 37], including wave guides [38], elastic wave diodes [39], and superlenses [40]. Despite these advances, one of the limitations of such phononic crystals is a lack of tunability – i.e., once the material is fabricated, it is difficult to substantially shift band gaps during use. Tunability of stiff phononic crystals has been explored using thermal property mismatches [41], piezoelectrics [42] and magneto-elasticity [43], but tunability is limited in the small-strain regime. In recent work, Bertoldi and Boyce [44] showed that it is possible

to use soft elastomeric materials to enable large shape changes that permit the opening, closing, and shifting of band gaps, and hence, significant tunability. This work has led to a resurgence of interest in tunable phononic crystals where the focus is no longer on obtaining the widest, fixed band gaps, but rather on obtaining highly tunable ones. Additionally, recent work has shown that by triggering instabilities, it is possible to generate even larger changes in band gap structure, enabling phononic crystal switches [45–49].

1.3 Thesis contributions

1.3.1 Wave propagation in dielectric elastomer composites

For phononic crystals made from soft elastomers, phononic band gaps may be reversibly manipulated through large elastic deformation of the periodic structure. By using dielectric elastomers, the frequency ranges of band gaps may be adjusted, or new band gaps may be created, through electrical stimuli. In Chapter 2, we present our finite-element-based numerical simulation capability for designing electrically-tunable, soft phononic crystals. Our finite-element tools address both nonlinear quasi-electrostatic processes and the linearized dynamics of electroelastic wave propagation through a pre-deformed state and may be applied to general composite unit-cell geometries subjected to arbitrary far-field electromechanical preloading. We apply our simulation capability to electrically-tunable, soft phononic crystals consisting of periodic lattices of aligned circular-cross-section fibers embedded in a matrix and demonstrate the shifting of band gaps with electrical preloading parallel to the fibers and the opening, closing, and shifting of band gaps with electrical preloading perpendicular to the fibers. The roles of large-stretch chain-locking behavior, material parameter contrasts, and fiber volume fraction are also investigated. A schematic overview of the numerical simulation capability for dielectric elastomer composite phononic crystals is illustrated in Fig. 1.3.

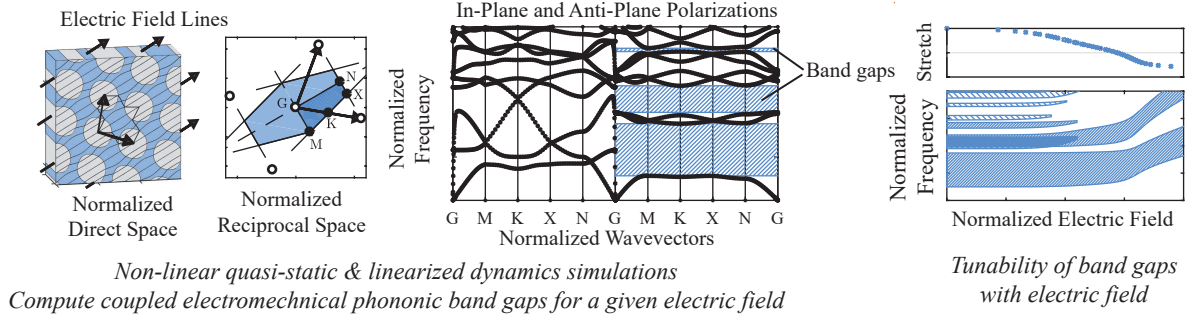


Figure 1.3: Schematic overview of the numerical simulation capability for dielectric elastomer composite phononic crystals developed in Chapter 2.

1.3.2 Electromechanical instabilities in dielectric elastomer composites

Several types of electromechanical instabilities emerge in periodic dielectric elastomer composites. The electromechanical pull-in instability is common in monolithic dielectric elastomers, but additional instabilities – microscopic, macroscopic, and even interfacial instabilities such as electrocreasing – arise in laminates and general dielectric elastomer composites. While closed-form expressions exist for the prediction of pull-in in monolithic dielectric elastomers, for general dielectric elastomer composites undergoing inhomogeneous deformation, numerical simulation is required for the detection of all types of instabilities. In Chapter 3, we apply our numerical simulation capability to the detection of electromechanical instabilities in dielectric elastomer composites consisting of periodic lattices of aligned circular-cross-section fibers embedded in a

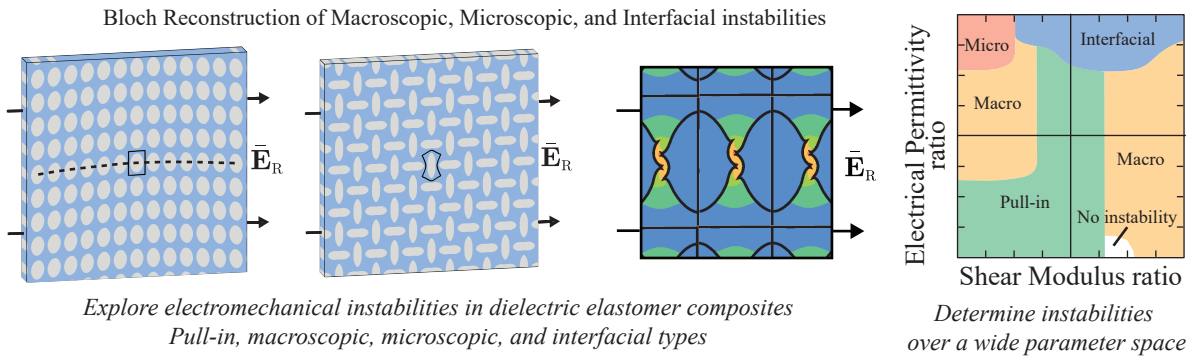


Figure 1.4: Schematic overview of the numerical study of electromechanical instabilities in dielectric elastomer composites presented in Chapter 3.

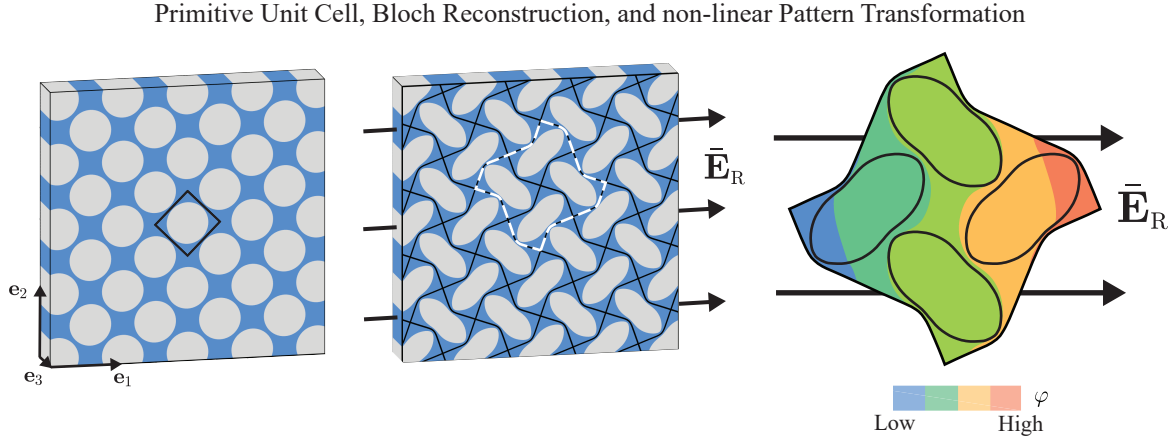


Figure 1.5: Schematic overview of the numerical study of electric-field-triggered pattern transformation in dielectric elastomer composites presented in Chapter 4.

matrix. We report on the existence of all known electromechanical instability types: (i) pull-in, (ii) microscopic, (iii) macroscopic, and (iv) interfacial instabilities while surveying a comprehensive parameter space. Namely, the roles of shear-modulus contrast, electric-permittivity contrast, fiber volume fraction, large-stretch chain-locking behavior, mechanical prestress, prestretch, and confinement are considered. The result is an overarching view of the instability landscape in periodic dielectric elastomer composites as well as a general strategy for exploring the stability of these materials. A schematic overview of the numerical study of electromechanical instabilities in dielectric elastomer composites is illustrated in Fig. 1.4. We conclude the chapter by showing that electric-field-triggered pattern transformations are possible, opening the door for additional study of electrically-tunable periodic materials.

1.3.3 Electric-field-triggered pattern transformations

Recent work has shown that by triggering instabilities, it is possible to generate even larger changes in band gap structure, enabling phononic crystal switches [45–49]. Notably, Shim et al. [49] showed that reversible elastic instabilities on the order of the periodicity – microscopic instabilities

– can be triggered to alter the band gap frequencies. In Chapter 4, we demonstrate that we can harness microscopic instabilities and apply an electric field to invoke pattern transformations in soft dielectric elastomer composites. These shape changes, in turn, induce phononic band gap changes. Interestingly, we also show that pattern transformation in macroscopically-constrained composites can be harnessed to achieve tunability – a functionality that cannot be achieved by mechanical means alone. A schematic overview of the numerical study of electrical-field-triggered pattern transformation in dielectric elastomer composites is illustrated in Fig. 1.5.

Chapter 2

A numerical simulation capability for electroelastic wave propagation in dielectric elastomer composites: Application to tunable soft phononic crystals

Note: A version of this chapter is published in the International Journal of Solids and Structures. Data and figures have been used with all co-authors' consent.

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2.1 Introduction

Using dielectric elastomers as a route to achieving electrically-tunable, soft phononic crystals was first considered by Gei et al. [51] who derived the analytical dispersion relation for flexural waves in pre-stressed dielectric elastomer plates. Shmuel and deBotton [52] – later corrected by Galich

and Rudykh [53] – developed analytical solutions for bulk electroelastic wave propagation in infinite, periodic, two-phase laminates undergoing large deformations in response to an electric field, and subsequently, Shmuel and coworkers [54, 55] expanded upon this work to obtain solutions – by extending the plane-wave-expansion technique of Kushwaha et al. [34, 56] – for dielectric elastomer composites made up of a periodic array of cylindrical fibers in a matrix but only considered the case of an electrostatic preloading along the direction of the fibers. This simplification was made so that deformations induced by the electrostatic preloading were *homogeneous* and hence analytically tractable. In order to address combinations of periodic composite structure and quasi-electrostatic preloading that give rise to inhomogeneous deformations, numerically-based tools are needed. However, finite-element computational procedures are relatively new to dielectric elastomers undergoing nonlinear quasi-electrostatic processes [31, 57–64] and have not been reported for problems of linearized electroelastic wave propagation through arbitrary base states of deformation and electric field. The work of this chapter is aimed at filling this need.

The first objective of this chapter is to report on our numerical simulation capability for infinite, periodic dielectric elastomer composites that addresses both types of problems: (1) nonlinear quasi-electrostatic processes and (2) linearized electroelastic wave propagation. First, to model the quasi-electrostatic response of dielectric elastomer composites of arbitrary structure, we use the finite-element implementation developed by Henann et al. [31] and generalize the approach of Boyce and coworkers [44, 65, 66] for applying arbitrary far-field loadings to electromechanical settings. Second, in order to consider arbitrary electroelastic plane waves propagating through deformed, infinite, periodic composite materials, we base our numerical implementation on the incremental governing equations of Dorfmann and Ogden [67] and utilize electromechanical Bloch-Floquet boundary conditions – a generalization of the approach of Bertoldi and coworkers [44, 47, 49] for purely mechanical problems. The second objective of this chapter is to apply our numerical simulation capability to electrically-tunable soft phononic crystals. Specifically, we consider infinite dielectric elastomer composites consisting of periodic lattices – both square and hexagonal – of aligned prismatic fibers in a matrix, subjected to applied electric fields both parallel to and

perpendicular to the axis of the fibers. In addition, we show that significant tunability may be achieved through the application of an electric field. In particular, inhomogeneous deformations induced by the applied electric field can cause phononic band gaps to shift, open, or close. Further, we show that the electromechanical pull-in instability [68] may be harnessed to achieve enhanced band gap tunability. We also explore the effects of large-stretch chain-locking behavior, material property contrasts, and fiber volume fraction on phononic band gap tunability.

The remainder of this chapter is organized as follows. In §2.2, we overview the governing equations used to describe both nonlinear quasi-electrostatic processes (§2.2.1) and linearized incremental processes (§2.2.2), such as electroelastic wave propagation. Then, in §2.3, we present our finite-element procedures for infinite, periodic dielectric elastomer solids. We discuss infinite, periodic composite materials subjected to arbitrary, macroscopic quasi-electrostatic loading (§2.3.1) as well as linearized electroelastic wave propagation through the deformed composites (§2.3.2). In §2.4, we demonstrate the numerical simulation capability by considering composites consisting of periodic lattices of aligned prismatic fibers in a matrix and show that phononic band gaps in these materials may be shifted, opened, and closed through the application of an electric field. We illustrate electrical tunability (§2.4.3) as well as examine the roles of large-stretch chain-locking behavior (§2.4.4), material property contrasts, and fiber volume fraction (§2.4.5). We close with a summary of the results §2.5.

2.2 Governing equations

2.2.1 Nonlinear quasi-electrostatic processes

We begin by summarizing the governing equations for coupled electromechanical deformation of dielectric elastomers under quasi-electrostatic conditions [24–31]. Consider a material point $\mathbf{x}_R$ in a fixed reference body $\mathcal{B}_R$ undergoing a motion to a point $\mathbf{x} = \chi(\mathbf{x}_R, t)$ in the deformed body $\mathcal{B}$ and subjected to an electric potential $\varphi(\mathbf{x}_R, t)$. The deformation gradient and referential electric field

are then given by¹

$$\mathbf{F} = \nabla \chi \quad \text{and} \quad \mathbf{E}_R = -\nabla \phi, \quad (2.2.1)$$

respectively, with $J = \det \mathbf{F} > 0$. The displacement is denoted as $\mathbf{u}(\mathbf{x}_R, t) = \chi(\mathbf{x}_R, t) - \mathbf{x}_R$, and the right and left Cauchy-Green deformation tensors are $\mathbf{C} = \mathbf{F}^\top \mathbf{F}$ and $\mathbf{B} = \mathbf{F} \mathbf{F}^\top$, respectively. In light of (2.2.1)₂, the referential electric field obeys $\text{Curl } \mathbf{E}_R = \mathbf{0}$, which is the referential form of Faraday's law under electrostatic conditions. The spatial electric field $\mathbf{E} = -\text{grad } \phi$ is related to its referential counterpart by $\mathbf{E} = \mathbf{F}^{-\top} \mathbf{E}_R$ and satisfies the spatial form of Faraday's law, $\text{curl } \mathbf{E} = \mathbf{0}$.

For a quasi-electrostatic process, in the absence of body forces and free body charges, equilibrium and Gauss's law in $\mathcal{B}_R$ are given by

$$\text{Div } \mathbf{T}_R = \mathbf{0} \quad \text{and} \quad \text{Div } \mathbf{D}_R = 0, \quad (2.2.2)$$

respectively, where $\mathbf{T}_R$ denotes the first Piola-Kirchhoff stress and $\mathbf{D}_R$ is the referential electric displacement. The standard Cauchy stress is then related to the Piola stress by $\mathbf{T} = J^{-1} \mathbf{T}_R \mathbf{F}^\top$ and satisfies the spatial form of equilibrium, $\text{div } \mathbf{T} = \mathbf{0}$ in $\mathcal{B}$, and the spatial electric displacement is given by $\mathbf{D} = J^{-1} \mathbf{F} \mathbf{D}_R$ and satisfies the spatial form of Gauss's law, $\text{div } \mathbf{D} = 0$ in $\mathcal{B}$.

We consider a non-dissipative material so that the stress and electric displacement are determined constitutively through a free energy per unit volume of the reference space $\phi_R(\mathbf{F}, \mathbf{E}_R)$ by

$$\mathbf{T}_R = \frac{\partial \phi_R(\mathbf{F}, \mathbf{E}_R)}{\partial \mathbf{F}} \quad \text{and} \quad \mathbf{D}_R = -\frac{\partial \phi_R(\mathbf{F}, \mathbf{E}_R)}{\partial \mathbf{E}_R}. \quad (2.2.3)$$

The free energy density ϕ_R represents an electrical ‘‘Gibbs’’ free energy density, in which the natural independent variables are the deformation gradient $\mathbf{F}$ and the referential electric field $\mathbf{E}_R$. This is in contrast to the Helmholtz free energy density $\psi_R(\mathbf{F}, \mathbf{D}_R)$, in which the Piola stress and referential electric field are given by $\mathbf{T}_R = \partial \psi_R(\mathbf{F}, \mathbf{D}_R) / \partial \mathbf{F}$ and $\mathbf{E}_R = \partial \psi_R(\mathbf{F}, \mathbf{D}_R) / \partial \mathbf{D}_R$, respectively, for a non-dissipative material. The two free energy potentials are related through the Legendre transform

¹Our notational conventions follow those of Gurtin et al. [69]. The symbols ∇ , Div and Curl denote the gradient, divergence, and curl with respect to the material point $\mathbf{x}_R$ in the reference body $\mathcal{B}_R$; grad , div , and curl denote these operators with respect to the point $\mathbf{x} = \chi(\mathbf{x}_R, t)$ in the deformed body $\mathcal{B}$.

$\phi_{\mathbf{r}} = \psi_{\mathbf{r}} - \mathbf{E}_{\mathbf{r}} \cdot \mathbf{D}_{\mathbf{r}}$. We choose to work with the electric field as the independent variable because this approach is more amenable to our purpose.

In the present work, we consider infinite, periodic, composite materials, which do not possess external boundaries subjected to tractions and free charges, and hence, it is not necessary to discuss the jump conditions that arise at external boundaries. However, the composite materials under consideration do possess internal, traction-free and charge-free interfaces between dissimilar materials. Let $\mathcal{S}_{\mathbf{r}}$ denote such an interface between two materials a and b in the reference body $\mathcal{B}_{\mathbf{r}}$, and write $\mathbf{n}_{\mathbf{r}}$ for the unit normal field on $\mathcal{S}_{\mathbf{r}}$ directed from material a to material b. For a perfectly-bonded interface, we assume that the motion χ is continuous across the interface $\mathcal{S}_{\mathbf{r}}$, i.e., $[[\chi]] = \chi^{(a)} - \chi^{(b)} = \mathbf{0}$, where $[[\bullet]]$ is the jump operator on the interface, defined as the difference in the quantity between materials a and b. Unlike the motion, the deformation gradient $\mathbf{F}$ may undergo a jump discontinuity across $\mathcal{S}_{\mathbf{r}}$ with the caveat that $[[\mathbf{F}]]\mathbf{s}_{\mathbf{r}} = \mathbf{0}$ for any vector field $\mathbf{s}_{\mathbf{r}}$ tangent to $\mathcal{S}_{\mathbf{r}}$. The referential interface $\mathcal{S}_{\mathbf{r}}$ then deforms to $\mathcal{S}$ in the deformed body $\mathcal{B}$ with unit normal field $\mathbf{n} = \mathbf{F}^{-\top} \mathbf{n}_{\mathbf{r}} / |\mathbf{F}^{-\top} \mathbf{n}_{\mathbf{r}}|$. Analogous to the motion, we assume that the electric potential field is continuous across the interface $\mathcal{S}_{\mathbf{r}}$, i.e., $[[\phi]] = 0$. Note that this ensures that $[[\mathbf{E}_{\mathbf{r}}]] \times \mathbf{n}_{\mathbf{r}} = \mathbf{0}$ across $\mathcal{S}_{\mathbf{r}}$, or equivalently $[[\mathbf{E}]] \times \mathbf{n} = \mathbf{0}$ across $\mathcal{S}$. Finally, equilibrium and Gauss's law applied to the interface $\mathcal{S}_{\mathbf{r}}$ require

$$[[\mathbf{T}_{\mathbf{r}}]]\mathbf{n}_{\mathbf{r}} = \mathbf{0} \quad \text{and} \quad [[\mathbf{D}_{\mathbf{r}}]] \cdot \mathbf{n}_{\mathbf{r}} = 0. \quad (2.2.4)$$

The jump conditions (2.2.4) may be expressed spatially in terms of the Cauchy stress and spatial electric displacement as $[[\mathbf{T}]]\mathbf{n} = \mathbf{0}$ and $[[\mathbf{D}]] \cdot \mathbf{n} = 0$, respectively, across $\mathcal{S}$.

2.2.2 Linearized incremental processes

Next, we consider a time-dependent, incremental process superimposed upon a time-independent base state of deformation and electric potential [67]. We allow for spatially-inhomogeneous base states, characterized by the fields $\chi(\mathbf{x}_{\mathbf{r}})$ and $\phi(\mathbf{x}_{\mathbf{r}})$, which are consistent with the governing equations of §2.2.1. Denoting small increments with a superposed dot, incremental displacements

and electric potentials, described referentially, are given by $\dot{\mathbf{u}}(\mathbf{x}_R, t)$ and $\dot{\phi}(\mathbf{x}_R, t)$. Like the fields $\chi(\mathbf{x}_R)$ and $\varphi(\mathbf{x}_R)$, the incremental fields $\dot{\mathbf{u}}(\mathbf{x}_R, t)$ and $\dot{\phi}(\mathbf{x}_R, t)$ are continuous across internal, traction-free and charge-free interfaces, i.e., $[[\dot{\mathbf{u}}(\mathbf{x}_R, t)]] = \mathbf{0}$ and $[[\dot{\phi}(\mathbf{x}_R, t)]] = 0$ across $\mathcal{S}_R$. The associated incremental deformation gradient and referential electric field are

$$\dot{\mathbf{F}} = \nabla \dot{\mathbf{u}} \quad \text{and} \quad \dot{\mathbf{E}}_R = -\nabla \dot{\phi}. \quad (2.2.5)$$

Increments in the first Piola stress and referential electric displacement are obtained by linearizing the constitutive equations (2.2.3):

$$\begin{aligned} \dot{\mathbf{T}}_R &= \mathbb{C}_R \dot{\mathbf{F}} + \boldsymbol{\Gamma}_R \dot{\mathbf{E}}_R, \\ \dot{\mathbf{D}}_R &= \boldsymbol{\Lambda}_R \dot{\mathbf{F}} + \mathbf{A}_R \dot{\mathbf{E}}_R, \end{aligned} \quad (2.2.6)$$

where

$$\begin{aligned} \mathbb{C}_R(\mathbf{x}_R) &= \frac{\partial^2 \phi_R}{\partial \mathbf{F} \partial \mathbf{F}}, \quad \boldsymbol{\Gamma}_R(\mathbf{x}_R) = \frac{\partial^2 \phi_R}{\partial \mathbf{F} \partial \mathbf{E}_R}, \\ \boldsymbol{\Lambda}_R(\mathbf{x}_R) &= -\frac{\partial^2 \phi_R}{\partial \mathbf{E}_R \partial \mathbf{F}}, \quad \mathbf{A}_R(\mathbf{x}_R) = -\frac{\partial^2 \phi_R}{\partial \mathbf{E}_R \partial \mathbf{E}_R}, \end{aligned} \quad (2.2.7)$$

are the material moduli, evaluated in the spatially-inhomogeneous base state. We consider incremental processes that are *elastodynamic but electrostatic*, so that the incremental forms of the equations of motion and Gauss's law in $\mathcal{B}_R$ are

$$\text{Div } \dot{\mathbf{T}}_R = \rho_R \frac{D^2 \dot{\mathbf{u}}}{Dt^2} \quad \text{and} \quad \text{Div } \dot{\mathbf{D}}_R = 0, \quad (2.2.8)$$

where ρ_R is the referential mass density and $D(\bullet)/Dt$ is the material time derivative, and the jump conditions at internal interfaces $\mathcal{S}_R$ are

$$[[\dot{\mathbf{T}}_R]] \mathbf{n}_R = \mathbf{0} \quad \text{and} \quad [[\dot{\mathbf{D}}_R]] \cdot \mathbf{n}_R = 0. \quad (2.2.9)$$

We subsequently focus attention on quantities expressed with respect to the deformed body $\mathcal{B}$,

so that the displacement increments and electric potential increments described spatially are given through the time-independent base state by $\dot{\mathbf{u}}(\chi^{-1}(\mathbf{x}), t)$ and $\dot{\phi}(\chi^{-1}(\mathbf{x}), t)$. Using the chain rule, the incremental spatial displacement gradient and electric field are given by

$$\dot{\mathbf{H}} = \text{grad } \dot{\mathbf{u}} = \dot{\mathbf{F}}\mathbf{F}^{-1} \quad \text{and} \quad \dot{\mathbf{E}} = -\text{grad } \dot{\phi} = \mathbf{F}^{-\top} \dot{\mathbf{E}}_{\text{R}}, \quad (2.2.10)$$

respectively. Stress and electric displacement increments, pushed forward to the deformed body [67, 70], are given through

$$\begin{aligned} \dot{\mathbf{T}} &= J^{-1} \dot{\mathbf{T}}_{\text{R}} \mathbf{F}^{\top} = \mathbb{C} \dot{\mathbf{H}} + \Gamma \dot{\mathbf{E}}, \\ \dot{\mathbf{D}} &= J^{-1} \mathbf{F} \dot{\mathbf{D}}_{\text{R}} = \Lambda \dot{\mathbf{H}} + \mathbf{A} \dot{\mathbf{E}}, \end{aligned} \quad (2.2.11)$$

where

$$\begin{aligned} \mathbb{C}_{ijkl}(\mathbf{x}) &= J^{-1} F_{jn} F_{lq} \frac{\partial^2 \phi_{\text{R}}}{\partial F_{in} \partial F_{kq}}, \quad \Gamma_{ijl}(\mathbf{x}) = J^{-1} F_{jn} F_{lq} \frac{\partial^2 \phi_{\text{R}}}{\partial F_{in} \partial E_{\text{R},q}}, \\ \Lambda_{jkl}(\mathbf{x}) &= -J^{-1} F_{jn} F_{lq} \frac{\partial^2 \phi_{\text{R}}}{\partial E_{\text{R},n} \partial F_{kq}}, \quad A_{jl}(\mathbf{x}) = -J^{-1} F_{jn} F_{lq} \frac{\partial^2 \phi_{\text{R}}}{\partial E_{\text{R},n} \partial E_{\text{R},q}}, \end{aligned} \quad (2.2.12)$$

are the spatial moduli, evaluated at the base state. The incremental forms of the equations of motion and Gauss's law expressed spatially in $\mathcal{B}$ are then

$$\begin{aligned} \text{div } \dot{\mathbf{T}} &= \text{div} (\mathbb{C} \dot{\mathbf{H}} + \Gamma \dot{\mathbf{E}}) = \rho \frac{\partial^2 \dot{\mathbf{u}}}{\partial t^2}, \\ \text{div } \dot{\mathbf{D}} &= \text{div} (\Lambda \dot{\mathbf{H}} + \mathbf{A} \dot{\mathbf{E}}) = 0, \end{aligned} \quad (2.2.13)$$

where $\rho = J^{-1} \rho_{\text{R}}$ is the spatial mass density and $\partial(\bullet)/\partial t$ is the spatial time derivative. Note that since the base state is time-independent, $D^2 \dot{\mathbf{u}}/Dt^2 = \partial^2 \dot{\mathbf{u}}/\partial t^2$. The spatial representation of the interface jump conditions across $\mathcal{S}$ are

$$[[\dot{\mathbf{T}}]] \mathbf{n} = \mathbf{0} \quad \text{and} \quad [[\dot{\mathbf{D}}]] \cdot \mathbf{n} = 0. \quad (2.2.14)$$

Finally, we further focus on an incremental process consisting of wave motion with angular frequency ω :

$$\dot{\mathbf{u}}(\mathbf{x}, t) = \tilde{\mathbf{u}}(\mathbf{x}) \exp(-i\omega t) \quad \text{and} \quad \dot{\phi}(\mathbf{x}, t) = \tilde{\phi}(\mathbf{x}) \exp(-i\omega t), \quad (2.2.15)$$

where $\tilde{\mathbf{u}}(\mathbf{x})$ and $\tilde{\phi}(\mathbf{x})$ represent the spatially-dependent magnitudes of the incremental displacement and electric potential, respectively. Then, using (2.2.10), (2.2.13), and (2.2.14), the frequency-domain incremental governing equations in $\mathcal{B}$ are

$$\begin{aligned} \operatorname{div} [\mathbb{C}(\operatorname{grad} \tilde{\mathbf{u}}) + \mathbf{\Gamma}(\operatorname{grad} \tilde{\phi})] &= -\rho \omega^2 \tilde{\mathbf{u}}, \\ \operatorname{div} [\mathbf{\Lambda}(\operatorname{grad} \tilde{\mathbf{u}}) + \mathbf{A}(\operatorname{grad} \tilde{\phi})] &= 0, \end{aligned} \quad (2.2.16)$$

and the frequency-domain jump conditions across $\mathcal{S}$ are

$$\begin{aligned} [[\mathbb{C}(\operatorname{grad} \tilde{\mathbf{u}}) + \mathbf{\Gamma}(\operatorname{grad} \tilde{\phi})]] \mathbf{n} &= \mathbf{0}, \\ [[\mathbf{\Lambda}(\operatorname{grad} \tilde{\mathbf{u}}) + \mathbf{A}(\operatorname{grad} \tilde{\phi})]] \cdot \mathbf{n} &= 0, \end{aligned} \quad (2.2.17)$$

which represent a linear system of differential equations and interfacial jump conditions for the fields $\tilde{\mathbf{u}}(\mathbf{x})$ and $\tilde{\phi}(\mathbf{x})$. We note that the base state enters the frequency-domain incremental governing equations (2.2.16) and (2.2.17) in two ways: (1) the geometry of the deformed body $\mathcal{B}$ and (2) the deformation and electric field dependence of the linearized spatial moduli (2.2.12).

2.2.3 Specialization of the free energy – the ideal dielectric elastomer

We consider the free energy corresponding to the ideal dielectric elastomer [28]:

$$\phi_{\mathbf{R}}(\mathbf{F}, \mathbf{E}_{\mathbf{R}}) = \phi_{\mathbf{R}}^{\text{mech}}(\mathbf{F}) - \frac{1}{2} \varepsilon \mathbf{J} \mathbf{E}_{\mathbf{R}} \cdot \mathbf{C}^{-1} \mathbf{E}_{\mathbf{R}}, \quad (2.2.18)$$

where $\phi_{\mathbf{R}}^{\text{mech}}(\mathbf{F})$ is the purely mechanical contribution to the free energy, which we leave unspecified at this point, and ε is the constant dielectric permittivity. While the permittivity can vary with the state of deformation [6, 71], the effect of such dependence is often small [72] and is neglected in

the case of the ideal dielectric elastomer. Henceforth focusing on spatial quantities, the Cauchy stress and spatial electric displacement are given by

$$\begin{aligned}\mathbf{T} &= J^{-1} \frac{\partial \phi_{\text{R}}(\mathbf{F}, \mathbf{E}_{\text{R}})}{\partial \mathbf{F}} \mathbf{F}^{\top} = \mathbf{T}^{\text{mech}} + \varepsilon \left[\mathbf{E} \otimes \mathbf{E} - \frac{1}{2} (\mathbf{E} \cdot \mathbf{E}) \mathbf{1} \right], \\ \mathbf{D} &= -J^{-1} \mathbf{F} \frac{\partial \phi_{\text{R}}(\mathbf{F}, \mathbf{E}_{\text{R}})}{\partial \mathbf{E}_{\text{R}}} = \varepsilon \mathbf{E},\end{aligned}\tag{2.2.19}$$

where $\mathbf{T}^{\text{mech}} = J^{-1}(\partial \phi_{\text{R}}^{\text{mech}}(\mathbf{F})/\partial \mathbf{F}) \mathbf{F}^{\top}$ is the mechanical contribution to the Cauchy stress, and $\mathbf{1}$ is the identity tensor. Furthermore, straightforward calculations using (2.2.12) and (2.2.18) yield the spatial moduli for the ideal dielectric elastomer:

$$\begin{aligned}\mathbb{C}_{ijkl} &= \mathbb{C}_{ijkl}^{\text{mech}} + \varepsilon \left(-\delta_{il} E_j E_k - E_i \delta_{jk} E_l \right. \\ &\quad \left. - E_i E_k \delta_{jl} + \delta_{ij} E_k E_l + E_i E_j \delta_{kl} \right. \\ &\quad \left. + \frac{1}{2} E_m E_m \delta_{il} \delta_{jk} - \frac{1}{2} E_m E_m \delta_{ij} \delta_{kl} \right), \\ \Gamma_{ijl} &= \varepsilon (\delta_{il} E_j + E_i \delta_{jl} - \delta_{ij} E_l), \\ \Lambda_{jkl} &= -\varepsilon (\delta_{jk} E_l + E_k \delta_{jl} - E_j \delta_{kl}), \\ A_{jl} &= \varepsilon \delta_{jl},\end{aligned}\tag{2.2.20}$$

where $\mathbb{C}_{ijkl}^{\text{mech}} = J^{-1} F_{jn} F_{lq} (\partial^2 \phi_{\text{R}}^{\text{mech}} / \partial F_{in} \partial F_{kq})$ are the spatial mechanical moduli.

Remark: For the case of an infinite, periodic composite material made of ideal dielectric elastomers with free energy and spatial moduli given through (2.2.18) and (2.2.20), respectively, if the permittivity ε is *spatially-constant* within each phase, straightforward but lengthy calculations involving (2.2.16) and (2.2.20) lead to the following governing equation for the incremental displacement field magnitude within each phase:

$$\text{div} [\mathbb{C}^{\text{mech}}(\text{grad } \tilde{\mathbf{u}})] = -\rho \omega^2 \tilde{\mathbf{u}}.\tag{2.2.21}$$

Furthermore, if the permittivity does not undergo discontinuities across internal interfaces $\mathcal{S}$ so that it is constant over the entire composite material, analogous calculations involving (2.2.17) and (2.2.20) lead to the following jump condition for the incremental displacement field magnitude across $\mathcal{S}$:

$$[[\mathbb{C}^{\text{mech}}(\text{grad } \tilde{\mathbf{u}})]]\mathbf{n} = \mathbf{0}. \quad (2.2.22)$$

In this case, the incremental mechanical problem decouples from the electrical potential increments (which are in general non-zero). Therefore, beyond influencing the base state of deformation – and hence the deformed shape and $\mathbb{C}^{\text{mech}}(\mathbf{x})$ – electromechanical coupling does not affect linearized wave propagation – an observation consistent with but broader than the result of [73] for plane waves propagating through homogeneous base states of deformation and electric field. This result holds for any inhomogeneous base state of deformation and electric field, for any mechanical free energy function $\phi_{\text{R}}^{\text{mech}}$, and for discontinuous mechanical free energy functions and mechanical properties across internal interfaces $\mathcal{S}$. However, if the permittivity does undergo discontinuities across internal interfaces $\mathcal{S}$ – as will be the case in the examples considered in §2.4 – the incremental problem does not decouple in this way, and one must solve the coupled system of equations (2.2.16) and (2.2.17) with (2.2.20) when considering linearized incremental problems.

2.3 Finite-element modeling of infinite periodic solids

In this section, we discuss our finite-element-based numerical approach for analyzing infinite, periodic dielectric elastomer solids. We consider composites consisting of periodic lattices of aligned prismatic fibers in a matrix, in which the fibers and the matrix are made from two different ideal dielectric elastomers.² As an illustration, consider the composites and associated undeformed primitive unit cells shown in Fig. 2.1(a) and (b), which consist of aligned circular-cross-section fibers in a square array and a regular hexagonal array, respectively. As shown in Fig. 2.1, we introduce a fixed Cartesian coordinate system with orthonormal basis vectors $\{\mathbf{e}_i | i = 1, 2, 3\}$ and

²We also consider simpler composites consisting of two-phase planar laminates in Appendix A.1 in order to verify the numerical simulation capability described in §2.3 against an analytical dispersion relation.

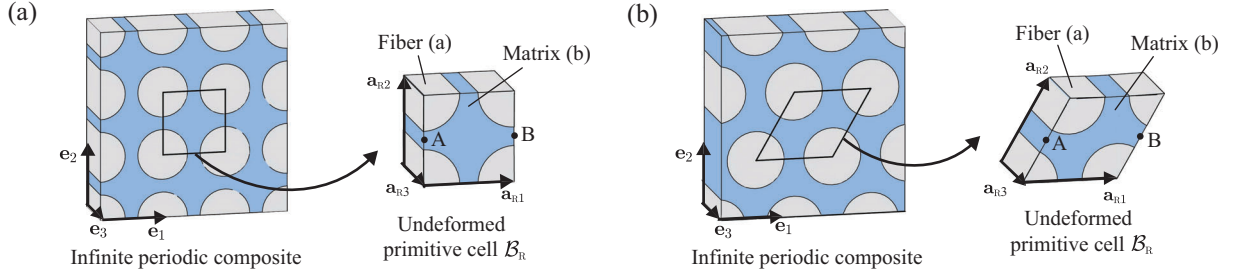


Figure 2.1: Schematic of infinite, periodic dielectric elastomer solids consisting of aligned circular-cross-section fibers in a matrix arranged in (a) a square array and (b) a regular hexagonal array. For each case, the undeformed primitive unit cell $\mathcal{B}_R$ spanned by referential lattice vectors $\mathbf{a}_{R1}$, $\mathbf{a}_{R2}$, and $\mathbf{a}_{R3}$ is shown.

with $\mathbf{e}_3$ parallel to the axis of the fibers. As further shown in Fig. 2.1, the undeformed primitive unit cells $\mathcal{B}_R$ are spanned by the referential lattice vectors $\mathbf{a}_{R1}$, $\mathbf{a}_{R2}$, and $\mathbf{a}_{R3}$. In the present work, we limit attention to composites that possess reflection symmetry with respect to the $\mathbf{a}_{R1}$ - $\mathbf{a}_{R2}$ -plane, so that $\mathbf{a}_{R1} \cdot \mathbf{a}_{R3} = 0$ and $\mathbf{a}_{R2} \cdot \mathbf{a}_{R3} = 0$; however, we place no such restriction on the pair of referential lattice vectors $\mathbf{a}_{R1}$ and $\mathbf{a}_{R2}$. For later use in computing the dispersion relations of dielectric elastomer composites, the referential reciprocal lattice vectors are

$$\begin{aligned} \mathbf{b}_{R1} &= 2\pi \frac{\mathbf{a}_{R2} \times \mathbf{a}_{R3}}{\mathbf{a}_{R1} \cdot (\mathbf{a}_{R2} \times \mathbf{a}_{R3})}, \\ \mathbf{b}_{R2} &= 2\pi \frac{\mathbf{a}_{R3} \times \mathbf{a}_{R1}}{\mathbf{a}_{R2} \cdot (\mathbf{a}_{R3} \times \mathbf{a}_{R1})}, \\ \mathbf{b}_{R3} &= 2\pi \frac{\mathbf{a}_{R1} \times \mathbf{a}_{R2}}{\mathbf{a}_{R3} \cdot (\mathbf{a}_{R1} \times \mathbf{a}_{R2})}, \end{aligned} \quad (2.3.1)$$

so that the relation $\mathbf{a}_{Ri} \cdot \mathbf{b}_{Rj} = 2\pi\delta_{ij}$ holds.

2.3.1 Nonlinear quasi-electrostatic processes

In the absence of body forces and free body charges, the governing partial differential equations expressed in the deformed body $\mathcal{B}$ (deformed unit cell) for an infinite, periodic dielectric elastomer composite undergoing a nonlinear, quasi-electrostatic process consist of the balance of momentum,

$\text{div} \mathbf{T} = \mathbf{0}$, and Gauss's law, $\text{div} \mathbf{D} = 0$, with $\mathbf{T}$ and $\mathbf{D}$ given through the constitutive equations (2.2.19). These coupled equations represent the strong form of the spatial boundary-value problem for the displacement field $\mathbf{u}$ and the electric potential field φ . With $\mathbf{w}_1$ and w_2 denoting two weighting fields, the weak form of the boundary-value problem is expressed as

$$\int_{\mathcal{B}} \mathbf{T} : \frac{\partial \mathbf{w}_1}{\partial \mathbf{x}} dv = \mathbf{0} \quad \text{and} \quad \int_{\mathcal{B}} \mathbf{D} \cdot \frac{\partial w_2}{\partial \mathbf{x}} dv = 0, \quad (2.3.2)$$

which we utilize as the basis of our two-field ($\mathbf{u}$ - φ) finite-element formulation [31, 64]. The deformed body is spatially discretized using finite elements, $\mathcal{B} = \cup \mathcal{B}^e$, and the finite-element approximations of the displacement and electric potential fields inside the element $\mathcal{B}^e$ are given by $\mathbf{u} = \sum_A \mathbf{u}^A N^A$ and $\varphi = \sum_A \varphi^A N^A$ with the index $A = 1, 2, \dots$ denoting the nodes of the element, $\mathbf{u}^A$ and φ^A the nodal displacements and electric potentials, respectively, and N^A the element shape functions. By employing a standard Galerkin approach, in which the weighting fields, $\mathbf{w}_1$ and w_2 , are interpolated by the same shape functions, standard arguments involving (2.3.2) lead to the following element-level residuals for the displacement and electric potential:

$$\begin{aligned} (\mathbf{R}_{\mathbf{u}})^A &= - \int_{\mathcal{B}^e} \mathbf{T} \frac{\partial N^A}{\partial \mathbf{x}} dv, \\ (\mathbf{R}_{\varphi})^A &= \int_{\mathcal{B}^e} \mathbf{D} \cdot \frac{\partial N^A}{\partial \mathbf{x}} dv. \end{aligned} \quad (2.3.3)$$

These element-level residuals are assembled into a set of global residuals, which when set equal to zero, represent a nonlinear system of equations for the nodal degrees of freedom, which when accompanied by appropriate tangents, may be solved iteratively using a Newton-Raphson procedure. For a detailed discussion of the implementation and verification of our quasi-electrostatic finite-element procedures in Abaqus [74] using a user-element (UEL) subroutine, see Henann et al. [31].

To apply our finite-element-based simulation capability to model the general electromechanically-coupled response of infinite, periodic dielectric elastomer composites, we

generalize the procedure of Boyce and coworkers [44, 65, 66] for applying arbitrary far-field deformations to the unit cell of an arbitrary periodic material. Following Danielsson et al. [65], periodicity is maintained and the far-field loading is applied through the use of twelve generalized degrees of freedom, which represent the Cartesian components of the macroscopic deformation gradient and referential electric field, denoted with an overbar, i.e.,

$$[\bar{\mathbf{F}}] = \begin{bmatrix} \bar{F}_{11} & \bar{F}_{12} & \bar{F}_{13} \\ \bar{F}_{21} & \bar{F}_{22} & \bar{F}_{23} \\ \bar{F}_{31} & \bar{F}_{32} & \bar{F}_{33} \end{bmatrix} \quad \text{and} \quad [\bar{\mathbf{E}}_{\mathbf{R}}] = \begin{bmatrix} \bar{E}_{\mathbf{R}1} \\ \bar{E}_{\mathbf{R}2} \\ \bar{E}_{\mathbf{R}3} \end{bmatrix}. \quad (2.3.4)$$

These twelve generalized degrees of freedom then relate the displacements and electric potentials of periodic pairs of nodes through a system of constraint equations. Specifically, for a periodic pair of nodes on the boundary of the unit-cell mesh, denoted as A and B (see Fig. 2.1), with referential coordinates $\mathbf{x}_{\mathbf{R}}^{\mathbf{A}}$ and $\mathbf{x}_{\mathbf{R}}^{\mathbf{B}}$, the nodal displacements, $\mathbf{u}^{\mathbf{A}}$ and $\mathbf{u}^{\mathbf{B}}$, and nodal electric potentials, $\varphi^{\mathbf{A}}$ and $\varphi^{\mathbf{B}}$, are related through

$$\begin{aligned} \mathbf{u}^{\mathbf{B}} - \mathbf{u}^{\mathbf{A}} &= (\bar{\mathbf{F}} - \mathbf{1})(\mathbf{x}_{\mathbf{R}}^{\mathbf{B}} - \mathbf{x}_{\mathbf{R}}^{\mathbf{A}}), \\ \varphi^{\mathbf{B}} - \varphi^{\mathbf{A}} &= \bar{\mathbf{E}}_{\mathbf{R}} \cdot (\mathbf{x}_{\mathbf{R}}^{\mathbf{B}} - \mathbf{x}_{\mathbf{R}}^{\mathbf{A}}), \end{aligned} \quad (2.3.5)$$

where the same nine components of $\bar{\mathbf{F}}$ and three components of $\bar{\mathbf{E}}_{\mathbf{R}}$ are used for all periodic pairs in the finite-element calculation. As a result of the macroscopic deformation, the referential lattice vectors $\mathbf{a}_{\mathbf{R}i}$ transform to their spatial counterparts through $\mathbf{a}_i = \bar{\mathbf{F}}\mathbf{a}_{\mathbf{R}i}$, and the referential reciprocal lattice vectors transform as $\mathbf{b}_i = \bar{\mathbf{F}}^{-\top}\mathbf{b}_{\mathbf{R}i}$, so that $\mathbf{a}_i \cdot \mathbf{b}_i = 2\pi\delta_{ij}$. Further generalizing the work of Danielsson et al. [65], using the principle of virtual work, the generalized forces conjugate to the generalized degrees of freedom (2.3.4) – when divided by the volume of the undeformed primitive unit cell $\mathcal{B}_{\mathbf{R}}$ – represent the nine components of the macroscopic first Piola stress and the three

components of the macroscopic referential electric displacement, also denoted with an overbar, i.e.,

$$[\bar{\mathbf{T}}_{\mathbf{R}}] = \begin{bmatrix} \bar{T}_{\mathbf{R}11} & \bar{T}_{\mathbf{R}12} & \bar{T}_{\mathbf{R}13} \\ \bar{T}_{\mathbf{R}21} & \bar{T}_{\mathbf{R}22} & \bar{T}_{\mathbf{R}23} \\ \bar{T}_{\mathbf{R}31} & \bar{T}_{\mathbf{R}32} & \bar{T}_{\mathbf{R}33} \end{bmatrix} \quad \text{and} \quad [\bar{\mathbf{D}}_{\mathbf{R}}] = \begin{bmatrix} \bar{D}_{\mathbf{R}1} \\ \bar{D}_{\mathbf{R}2} \\ \bar{D}_{\mathbf{R}3} \end{bmatrix}. \quad (2.3.6)$$

Using this unit-cell-based modeling approach, we are able to calculate the nonlinear macroscopic response of arbitrary, infinite, periodic composite materials under any macroscopic state of loading prescribed through a combination of deformation gradient, first Piola stress, referential electric field, and referential electric displacement components, enabling the consideration of more general electromechanical loading states than have been previously considered in the literature [cf., e.g., 60, 75].

2.3.2 Linearized wave propagation

Next, we consider linearized wave propagation through an arbitrary time-independent base state of deformation and electric potential. For a linearized incremental process, the frequency-domain incremental governing equations in the deformed unit cell $\mathcal{B}$ are (2.2.16) with the spatially-inhomogeneous moduli for a given base state given through (2.2.20). The incremental governing equations may then be expressed in the weak form using weighting fields $\mathbf{w}_1$ and w_2 as

$$\begin{aligned} \int_{\mathcal{B}} \left[\mathbb{C}_{ijkl}(\mathbf{x}) \frac{\partial \tilde{u}_k}{\partial x_l} + \Gamma_{ijl}(\mathbf{x}) \frac{\partial \tilde{\varphi}}{\partial x_l} \right] \frac{\partial w_{1i}}{\partial x_j} dv &= \int_{\mathcal{B}} \rho \omega^2 \tilde{u}_i w_{1i} dv, \\ \int_{\mathcal{B}} \left[\Lambda_{jkl}(\mathbf{x}) \frac{\partial \tilde{u}_k}{\partial x_l} + A_{jil}(\mathbf{x}) \frac{\partial \tilde{\varphi}}{\partial x_l} \right] \frac{\partial w_2}{\partial x_j} dv &= 0. \end{aligned} \quad (2.3.7)$$

Analogous to the discussion of §2.3.1, the finite-element approximations of the incremental displacement and electric potential magnitudes inside the element $\mathcal{B}^e$ are given by $\tilde{\mathbf{u}} = \sum_A \tilde{\mathbf{u}}^A N^A$ and $\tilde{\varphi} = \sum_A \tilde{\varphi}^A N^A$, where $\tilde{\mathbf{u}}^A$ and $\tilde{\varphi}^A$ are the nodal degrees of freedom. Interpolating the weighting

fields, $\mathbf{w}_1$ and w_2 , using N^A as before, (2.3.7) leads to the following element-level residuals:

$$\begin{aligned} R_{u_i}^A &= - \int_{\mathcal{B}^e} \left[\mathbb{C}_{ijkl}(\mathbf{x}) \frac{\partial \tilde{u}_k}{\partial x_l} + \Gamma_{ijl}(\mathbf{x}) \frac{\partial \tilde{\varphi}}{\partial x_l} \right] \frac{\partial N^A}{\partial x_j} dv + \int_{\mathcal{B}^e} \rho \omega^2 \tilde{u}_i N^A dv, \\ R_{\varphi}^A &= \int_{\mathcal{B}^e} \left[\Lambda_{jkl}(\mathbf{x}) \frac{\partial \tilde{u}_k}{\partial x_l} + A_{jl}(\mathbf{x}) \frac{\partial \tilde{\varphi}}{\partial x_l} \right] \frac{\partial N^A}{\partial x_j} dv, \end{aligned} \quad (2.3.8)$$

which when assembled into a global set of linear equations and set equal to zero represent an eigenvalue problem with eigenfrequencies ω and mode shapes given through the degrees of freedom, $\tilde{\mathbf{u}}^A$ and $\tilde{\varphi}^A$.

Finally, in order to consider arbitrary electroelastic plane waves propagating through the deformed infinite, periodic composite material, it is necessary to specify arbitrary wave vectors $\mathbf{k}$ that do not share the periodicity of the deformed composite. To do so, we utilize Bloch-Floquet boundary conditions, and generalizing the work of Bertoldi and coworkers [44, 47, 49], the nodal displacement and electric potential magnitudes for periodic pairs of nodes A and B with deformed coordinates $\mathbf{x}^A$ and $\mathbf{x}^B$ are related through the following complex-valued relations:

$$\begin{aligned} \tilde{\mathbf{u}}^B &= \tilde{\mathbf{u}}^A \exp \left[i \mathbf{k} \cdot (\mathbf{x}^B - \mathbf{x}^A) \right], \\ \tilde{\varphi}^B &= \tilde{\varphi}^A \exp \left[i \mathbf{k} \cdot (\mathbf{x}^B - \mathbf{x}^A) \right]. \end{aligned} \quad (2.3.9)$$

We emphasize that $\mathbf{k}$ denotes wave vectors in the *deformed* reciprocal space. Alternatively, one may consider wave vectors $\mathbf{k}_R$ in the *undeformed* reciprocal space, which are related to their deformed counterparts through $\mathbf{k} = \bar{\mathbf{F}}^{-\top} \mathbf{k}_R$. Since $\mathbf{k}_R \cdot (\mathbf{x}_R^B - \mathbf{x}_R^A) = \mathbf{k} \cdot (\mathbf{x}^B - \mathbf{x}^A)$, the Bloch-Floquet boundary conditions (2.3.9) expressed in terms of deformed or undeformed quantities are equivalent. To account for the complex nature of these boundary conditions within the confines of the commercial finite-element code Abaqus [74], calculations are carried out using two separate, identical meshes of the deformed unit cell. The degrees of freedom of the two meshes represent the real and imaginary parts of the displacement and electric potential magnitude – i.e., $\{\tilde{\mathbf{u}}^{A,\text{real}}, \tilde{\varphi}^{A,\text{real}}\}$ and $\{\tilde{\mathbf{u}}^{A,\text{imag}}, \tilde{\varphi}^{A,\text{imag}}\}$, respectively. The two meshes are then coupled through the following form of

the Bloch-Floquet boundary conditions (2.3.9) for periodic pairs of nodes A and B:

$$\begin{cases} \tilde{\mathbf{u}}^{\text{B,real}} = \tilde{\mathbf{u}}^{\text{A,real}} \cos [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] - \tilde{\mathbf{u}}^{\text{A,imag}} \sin [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] , \\ \tilde{\mathbf{u}}^{\text{B,imag}} = \tilde{\mathbf{u}}^{\text{A,real}} \sin [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] + \tilde{\mathbf{u}}^{\text{A,imag}} \cos [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] , \\ \tilde{\phi}^{\text{B,real}} = \tilde{\phi}^{\text{A,real}} \cos [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] - \tilde{\phi}^{\text{A,imag}} \sin [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] , \\ \tilde{\phi}^{\text{B,imag}} = \tilde{\phi}^{\text{A,real}} \sin [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] + \tilde{\phi}^{\text{A,imag}} \cos [\mathbf{k} \cdot (\mathbf{x}^{\text{B}} - \mathbf{x}^{\text{A}})] . \end{cases} \quad (2.3.10)$$

Operationally, the Bloch-Floquet boundary conditions (2.3.10) are prescribed in Abaqus [74] through a user-defined multi-point constraint (MPC). Then, for a given $\mathbf{k}$, the corresponding eigenfrequencies ω may be calculated using an eigenvalue extraction technique, and the dispersion relation $\omega(\mathbf{k})$ may be assembled. In calculating the dispersion relation of a given composite under a given macroscopic loading, we consider Bloch wave vectors $\mathbf{k}$ along the boundary of the *deformed* first irreducible Brillouin zone [76] – a choice that we discuss in the context of individual examples in §2.4.

2.4 Numerical simulations

In this section, we apply our numerical simulation capability to composites consisting of periodic lattices of aligned circular-cross-section fibers in a matrix and arranged in square and regular hexagonal arrays, shown in Fig. 2.1(a) and (b) and described at the beginning of §2.3. The fibers and matrix are perfectly-bonded at their interfaces and are made of materials a and b, respectively. The volume fractions of the two phases are denoted by $\nu^{(a)}$ and $\nu^{(b)}$, and as a baseline, we consider the case of $\nu^{(a)} = \nu^{(b)} = 0.5$. We take the constitutive response of the phases α ($\alpha = a, b$) to be described by the ideal dielectric elastomer model (2.2.18). To this point, we have left the mechanical free energy density function for each phase $\phi_{\text{r}}^{\text{mech}(\alpha)}$ unspecified, since our numerical simulation capability is sufficiently general so that any form may be adopted. Following recent theoretical work on electrically-tunable soft phononic crystals [53–55], we specify a nearly-incompressible

form of the Gent [77] free energy density function for both phases. Defining the distortional left Cauchy-Green tensor, $\mathbf{B}_{\text{dis}} = J^{-2/3}\mathbf{B}$, and the distortional first stretch invariant, $\bar{I}_1 = \text{tr}(\mathbf{B}_{\text{dis}})$, we take the referential free energy density function for phase α to be

$$\phi_{\text{r}}^{\text{mech}(\alpha)} = -\frac{1}{2}G^{(\alpha)}I_{\text{m}}^{(\alpha)} \ln \left(1 - \frac{\bar{I}_1^{(\alpha)} - 3}{I_{\text{m}}^{(\alpha)}} \right) + \frac{1}{2}K^{(\alpha)}(J^{(\alpha)} - 1)^2, \quad (2.4.1)$$

where $G^{(\alpha)}$ and $K^{(\alpha)}$ are the ground-state shear and bulk moduli, respectively, of each phase α . The parameter $I_{\text{m}}^{(\alpha)}$ represents the upper limit of $(\bar{I}_1^{(\alpha)} - 3)$ for each phase α , and as $(\bar{I}_1^{(\alpha)} - 3)$ approaches $I_{\text{m}}^{(\alpha)}$, the magnitude of the mechanical stress rapidly increases and diverges. As shown by Boyce [78], predictions of the phenomenological Gent model are similar to those of the entropic-network-based Arruda-Boyce hyperelasticity model with the parameter $I_{\text{m}}^{(\alpha)}$ accounting for the increase in mechanical stress associated with the locking of polymer chains at large-stretches. From a practical perspective, the presence of large-stretch chain-locking behavior introduces substantial nonlinearity to the large-deformation mechanical response, and consequently, the spatial mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ for the Gent model depend strongly upon the base state of deformation. As $I_{\text{m}}^{(\alpha)}$ increases, chain-locking behavior is delayed to larger stretches, and in the limit $I_{\text{m}}^{(\alpha)} \rightarrow \infty$, the Gent model reduces to the simpler Neo-Hookean model with comparatively less deformation-dependent spatial mechanical moduli.

We take the bulk modulus to be the same in both phases, i.e., $K = K^{(\text{a})} = K^{(\text{b})}$, and in order to obtain nearly-incompressible conditions, we set K to be three orders of magnitude greater than the larger of the two ground-state shear moduli, $G^{(\text{a})}$ or $G^{(\text{b})}$. The assumption of nearly or ideally incompressible behavior is commonly invoked when modeling dielectric elastomers and is motivated by the mechanical behavior of real materials. For example, while the ground-state shear modulus of common silicone-based and polyurethane-based dielectric elastomers is typically less than 1 MPa [10], the bulk modulus is on the order of 1 GPa. While we consider nearly-incompressible situations in the present work, our numerical simulation capability is sufficiently general to be applied to more compressible materials.

The baseline set of material property contrasts used in our numerical simulations is $G^{(a)}/G^{(b)} = 20$, $\rho^{(a)}/\rho^{(b)} = 10$, $\epsilon^{(a)}/\epsilon^{(b)} = 2$, and $I_m^{(a)} = I_m^{(b)} = 10$. Our choices of $G^{(a)}/G^{(b)} = 20$ and $\epsilon^{(a)}/\epsilon^{(b)} = 2$ are based on realistic material properties for dielectric polymers. For example, Getz et al. [55] envisioned a composite with fibers made from Polyurethane PT6100S in a matrix of Silicone CF19-2186 in which $G^{(a)}/G^{(b)} = 17$ and $\epsilon^{(a)}/\epsilon^{(b)} = 2.5$ [10]. The choice of density mismatch $\rho^{(a)}/\rho^{(b)} = 10$ is made for illustrative purposes so that phononic band gaps for the undeformed composite are reasonably wide – a choice that will be revisited in §2.4.5. Finally, for simplicity, we take the locking parameters of the phases, $I_m^{(a)} = I_m^{(b)} = 10$, to be equal and sufficiently large³ so that the electromechanical pull-in instability is not suppressed with the intent of showing in our subsequent simulations that the pull-in instability may be harnessed to achieve enhanced band gap tunability.

Our numerical simulations involve first computing the quasi-electrostatic pre-deformation followed by a linearized dynamic procedure to calculate the dispersion relation. We discuss these procedures in detail in the context of both square and regular hexagonal composite lattices and the material property contrasts discussed above in §2.4.1 and §2.4.2, respectively. Then, in §2.4.3, we demonstrate how an applied electric field may be used to shift, open, and close phononic band gaps and explore the effect of electric field direction on electrical tunability. In §2.4.4, we investigate the role of large-stretch chain-locking behavior on electrical tunability. In particular, we consider the limit in which $I_m^{(a)}, I_m^{(b)} \rightarrow \infty$, and the Gent hyperelastic constitutive model reduces to the simpler Neo-Hookean model. Finally, in §2.4.5, we explore the effects of the material parameter contrasts and fiber volume fraction on band gap tunability.

2.4.1 Nonlinear quasi-electrostatic preloading

We consider nonlinear quasi-electrostatic preloading cases that correspond to a macroscopic referential electric field $\bar{\mathbf{E}}_r$ applied along several different directions relative to the composite structure

³For homogeneous, one-dimensional electrical actuation of an incompressible Gent material, the pull-in instability is suppressed for $I_m^{(\alpha)} \lesssim 7.308$ [31].

for both square and regular hexagonal composite lattices. Denote the direction of the macroscopic electric field through the unit vector $\bar{\mathbf{m}} = \bar{\mathbf{E}}_R / |\bar{\mathbf{E}}_R|$. Then, for both the square and hexagonal composite lattices shown in Fig. 2.1(a) and (b), we consider the following macroscopic electric field directions: (1) parallel to the fibers, i.e., along the $\mathbf{a}_{R3}$ -direction, (2) perpendicular to the fibers and along the closest-packed, in-plane direction, i.e., along the $\mathbf{a}_{R1}$ -direction, and (3) perpendicular to the fibers and along the least-packed, in-plane direction, i.e., along the $(\mathbf{a}_{R1} + \mathbf{a}_{R2})$ -direction. These cases are summarized in Table 2.1 along with the respective representations of $\bar{\mathbf{m}}$ in terms of the Cartesian basis vectors.⁴ Since the composite geometries possess reflection symmetry with respect to the $\mathbf{a}_{R1}$ - $\mathbf{a}_{R2}$ -plane and the preloading directions $\bar{\mathbf{m}}$ described above are either normal to or lie within the $\mathbf{a}_{R1}$ - $\mathbf{a}_{R2}$ -plane, the resulting quasi-electrostatic preloading will take place under generalized plane-strain conditions, in which $\bar{F}_{13} = \bar{F}_{31} = \bar{F}_{23} = \bar{F}_{32} = 0$ but $\bar{F}_{33}$ may differ from one. Further, since all electrical preloading directions $\bar{\mathbf{m}}$ considered here act normal to planes of symmetry, no macroscopic rotation is expected to occur during preloading, and hence, we set $\bar{F}_{12} = \bar{F}_{21}$. Finally, we assume far-field, mechanically-traction-free boundary conditions so that all components of $\bar{\mathbf{T}}_R$ equal zero.

Operationally, rather than prescribing the macroscopic electric field $\bar{\mathbf{E}}_R$ in our simulations, we specify the macroscopic referential electric displacement $\bar{\mathbf{D}}_R$. This is analogous to electrical loading through “charge-control” rather than “voltage-control.” For the preloading cases described above, in which $\bar{\mathbf{m}}$ is normal to a plane of symmetry, these two methods of electrical actuation are, for the most part, identical. The only situation in which they are different is in the presence of an electromechanical pull-in instability. In this case, a voltage-control approach does not admit numerical solutions past the onset of the pull-in instability, while for a corresponding charge-control approach, the pull-in instability does not manifest. This is desirable for our purposes since we are interested in exploring the possibility of harnessing the pull-in instability to achieve

⁴While it is straightforward to envision an electric field applied parallel to the fibers by compliant electrodes on the front and back faces of the composites shown in Fig. 2.1, applying macroscopic in-plane electric fields while maintaining the periodicity of the composite microstructure may pose practical challenges, such as the placement of electrodes or finite-size effects. Such issues might necessitate different in-plane actuation strategies than considered in the present work, such as compliant electrodes embedded in the composite microstructure. However, our numerical simulation capability is sufficiently general to be straightforwardly applied to other types of in-plane electrical loading.

Lattice	Direction $\bar{\mathbf{m}} = \bar{\mathbf{E}}_r / \bar{\mathbf{E}}_r $	Description
Square	$\mathbf{e}_3$	parallel to the fibers (i.e., along the $\mathbf{a}_{r3}$ -direction)
	$\mathbf{e}_1$	perpendicular to the fibers along the closest-packed direction (i.e., along the $\mathbf{a}_{r1}$ -direction)
	$\frac{1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2)$	perpendicular to the fibers along the least-packed direction (i.e., along the $(\mathbf{a}_{r1} + \mathbf{a}_{r2})$ -direction)
Hexagonal	$\mathbf{e}_3$	parallel to the fibers (i.e., along the $\mathbf{a}_{r3}$ -direction)
	$\mathbf{e}_1$	perpendicular to the fibers along the closest-packed direction (i.e., along the $\mathbf{a}_{r1}$ -direction)
	$\frac{\sqrt{3}}{2}\mathbf{e}_1 + \frac{1}{2}\mathbf{e}_2$	perpendicular to the fibers along the least-packed direction (i.e., along the $(\mathbf{a}_{r1} + \mathbf{a}_{r2})$ -direction)

Table 2.1: Directions of the applied macroscopic electric field for quasi-electrostatic preloading for both square and hexagonal composite lattices. The representations of the unit vector $\bar{\mathbf{m}} = \bar{\mathbf{E}}_r / |\bar{\mathbf{E}}_r|$ in terms of Cartesian basis vectors are in reference to the orthonormal basis $\{\mathbf{e}_i | i = 1, 2, 3\}$ defined in Fig. 2.1.

enhanced band-gap tunability, and a charge-control approach enables our simulations to traverse states that are unstable in voltage-control. A similar approach to computationally handling the pull-in instability was utilized by Park and Nguyen [79].

Regarding the finite-element mesh, the unit cells shown in Fig. 2.1 are meshed using a single layer of three-dimensional, eight-node hexahedral elements with one element through the thickness (i.e., in the $\mathbf{e}_3$ -direction). The same mesh is used for both quasi-electrostatic preloading and the linearized wave propagation analysis described in §2.4.2. Defining the magnitude of the in-plane referential lattice vectors as $a_r = |\mathbf{a}_{r1}| = |\mathbf{a}_{r2}|$, we first consider a relatively dense average mesh resolution of $0.02a_r$, which is used for the verification tests described in §2.4.2. Subsequently, we take $0.07a_r$ to be a good compromise between computational expense and accuracy, and all results presented in §2.4.3, §2.4.4, and §2.4.5 utilize a mesh resolution of $0.07a_r$.

Simulated deformed composite structures at illustrative levels of electrical preloading for each of the three electrical preloading directions $\bar{\mathbf{m}}$ described in Table 2.1 are shown for square and

hexagonal arrays in the first columns of Figs. 2.2(b)-(d) and 2.3(b)-(d), respectively, with the undeformed composite structures shown in Figs. 2.2(a) and 2.3(a). The composite structures are shown in normalized direct space, in which the in-plane coordinates, x_1 and x_2 , are normalized by the referential lattice constant a_r . The out-of-plane thicknesses of the undeformed composites shown in Figs. 2.2(a) and 2.3(a) are arbitrary since we have limited attention to generalized plane-strain conditions, but the thicknesses of the deformed composites relative to their undeformed counterparts are true-to-scale. Defining the volume-averaged ground-state shear modulus and permittivity as $\bar{G} = \nu^{(a)}G^{(a)} + \nu^{(b)}G^{(b)}$ and $\bar{\epsilon} = \nu^{(a)}\epsilon^{(a)} + \nu^{(b)}\epsilon^{(b)}$, the normalized macroscopic referential electric field magnitude, $|\bar{\mathbf{E}}_r|/\sqrt{\bar{\epsilon}/\bar{G}}$, and the macroscopic stretch along the electrical preloading direction, $\bar{\lambda} = \sqrt{\bar{\mathbf{m}} \cdot \bar{\mathbf{F}}^T \bar{\mathbf{F}} \bar{\mathbf{m}}}$, are noted in Figs. 2.2 and 2.3 for each case. Furthermore, the deformed primitive unit cell is outlined, and the deformed, in-plane lattice vectors $\mathbf{a}_1 = \bar{\mathbf{F}}\mathbf{a}_{r1}$ and $\mathbf{a}_2 = \bar{\mathbf{F}}\mathbf{a}_{r2}$ are shown. Finally, the electric field lines are also plotted on each deformed composite structure. Importantly, we note that for electrical preloading parallel to the fibers – Figs. 2.2(b) and 2.3(b) – the state of deformation and electric field is *homogeneous* throughout the deformed composite microstructure. This feature makes the analysis of wave propagation through the deformed composite tractable using other techniques, such as the plane-wave-expansion method [34, 56], and hence previous works [54, 55] have focused on this electrical preloading direction. In contrast, for in-plane electrical preloading – Figs. 2.2(c) and (d) and 2.3(c) and (d) – both the deformation and the electric field are *inhomogeneous* throughout the deformed composite microstructure, necessitating our computational tools.

2.4.2 Linearized wave propagation

Next, we calculate dispersion relations $\omega(\mathbf{k})$ for given electrically-preloaded composites obtained through the quasi-electrostatic computational procedure described in §2.4.1. To do so, it is necessary to choose an appropriate set of Bloch wave vectors $\mathbf{k}$. First, we restrict attention to wave propagation

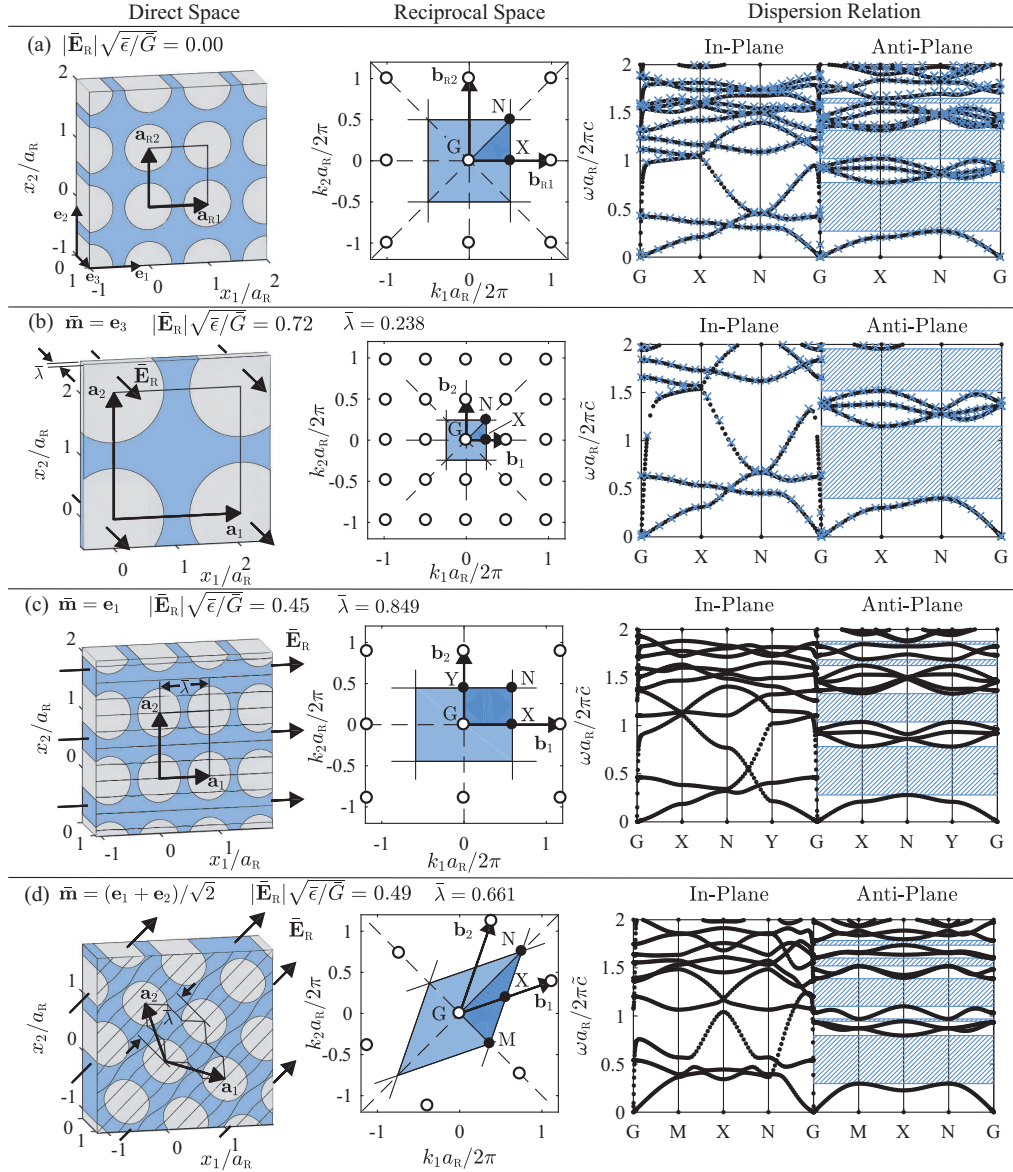


Figure 2.2: Representative simulation results for the square-lattice composite made from Gent materials with property contrasts $G^{(a)}/G^{(b)} = 20$, $\epsilon^{(a)}/\epsilon^{(b)} = 2$, $\rho^{(a)}/\rho^{(b)} = 10$, and $I_m^{(a)} = I_m^{(b)} = 10$. Left column: Direct-space composite structure with the primitive cell, lattice vectors, and electric field lines shown. Middle column: Reciprocal space with the first Brillouin zone, the irreducible Brillouin zone (IBZ), and the reciprocal lattice vectors shown. Symmetry planes of the reciprocal space are illustrated by dashed lines, and the paths along the boundaries of the respective IBZs are denoted with letters. Right column: Dispersion relations $\omega(\mathbf{k})$ with in-plane polarizations on the left and anti-plane polarizations on the right and phononic band gaps shown. (a) Undeformed composite. (b)-(d) Deformed composites corresponding to electrical preloading parallel to the fibers, perpendicular to the fibers along the closest-packed direction, and perpendicular to the fibers along the least-packed direction, respectively. Mesh density: $0.02a_R$.

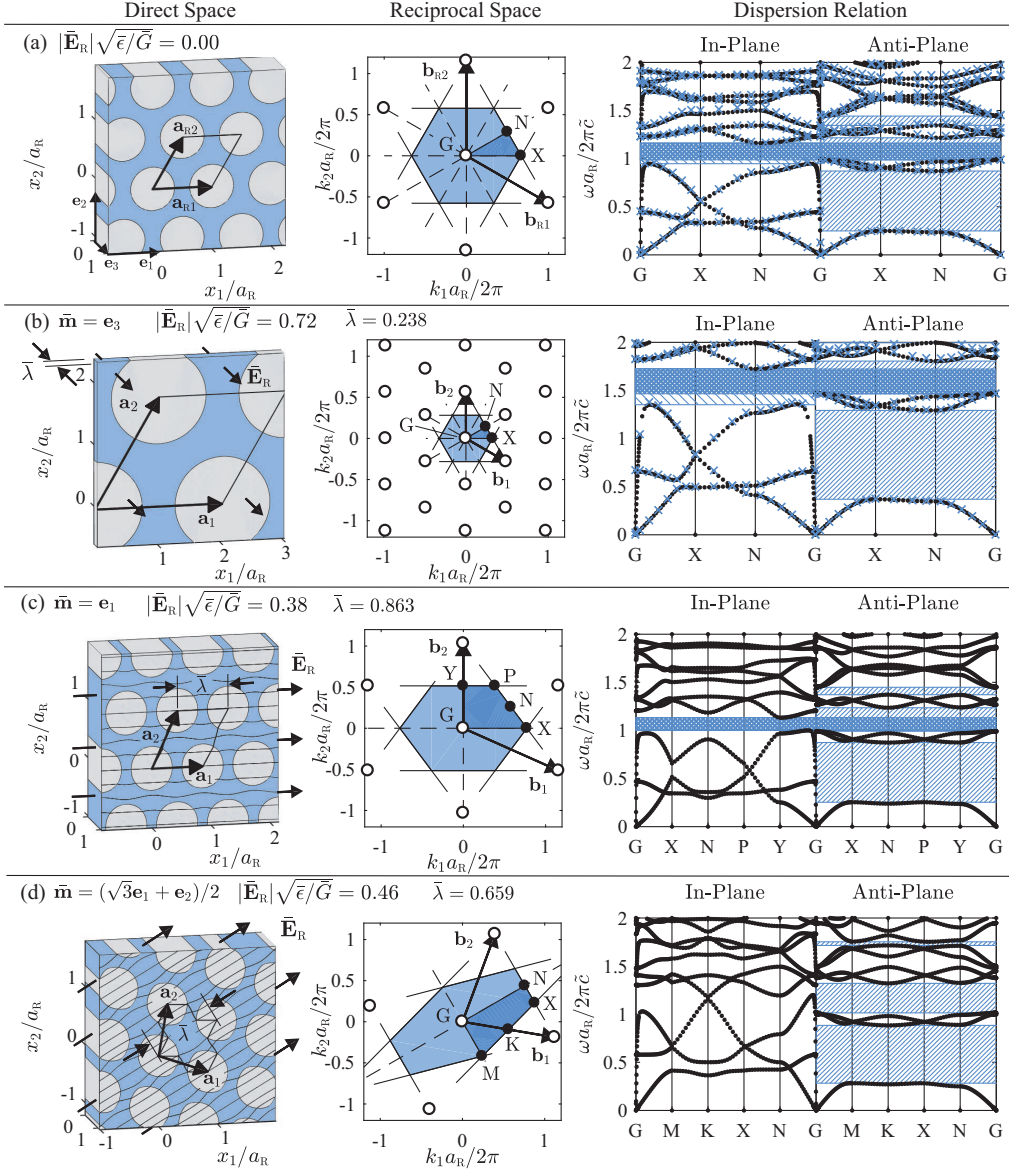


Figure 2.3: Representative simulation results for the hexagonal-lattice composite made from Gent materials with property contrasts $G^{(a)}/G^{(b)} = 20$, $\epsilon^{(a)}/\epsilon^{(b)} = 2$, $\rho^{(a)}/\rho^{(b)} = 10$, and $I_m^{(a)} = I_m^{(b)} = 10$. Results are organized as described in the caption of Fig. 2.2 for the square-lattice composite. Mesh density: $0.02a_R$.

perpendicular to the fibers, so that $k_3 = 0$ throughout.⁵ Next, since the deformed composite structures are periodic, it is only necessary to consider a subset of Bloch wave vectors that span the first Brillouin zone. The reciprocal spaces for the undeformed and deformed composite structures

⁵This is not a limitation of our numerical simulation capability, which may be applied to general three-dimensional phononic crystals.

described in §2.4.1 are shown for square and hexagonal arrays in the second columns of Figs. 2.2 and 2.3, respectively. The reciprocal lattice points are displayed as unfilled circles; solid lines represent the first Bragg planes, which define the boundaries of the first Brillouin zones, shown as lightly-shaded regions; and for the cases with electrical preloading, the deformed reciprocal lattice vectors, $\mathbf{b}_1 = \bar{\mathbf{F}}^{-\top} \mathbf{b}_{R1}$ and $\mathbf{b}_2 = \bar{\mathbf{F}}^{-\top} \mathbf{b}_{R2}$, are shown. Furthermore, since the undeformed and deformed primitive unit cells are invariant under certain mirror-reflection transformations, these symmetries also exist in the corresponding reciprocal spaces, and hence, it is only necessary to consider part of the first Brillouin zone – the irreducible Brillouin zone (IBZ). Symmetry planes for each reciprocal space are shown as dashed lines in the second columns of Figs. 2.2 and 2.3, and the resulting IBZs are shown as darkly-shaded regions. The corners of the IBZs are each labeled by a letter. Importantly, since the symmetries of each deformed primitive unit cell are due not only to the symmetry of the corresponding undeformed primitive unit cell but also the direction of electrical preloading, the IBZs for each of the three preloading directions in Figs. 2.2 and 2.3 are different. For example, for the undeformed square lattice (Fig. 2.2(a)), the IBZ is the triangle GXNG. Then, for electrical preloading parallel to the fibers (Fig. 2.2(b)), the IBZ shrinks but all symmetries of the undeformed lattice are maintained, so that the IBZ remains the triangle GXNG. However, for electrical preloading perpendicular to the fibers, certain symmetries are lost for each case, and the IBZ expands to GXNYG for preloading along the closest-packed direction (Fig. 2.2(c)) and GMXNG for preloading along the least-packed direction (Fig. 2.2(d)). The situation is analogous for the undeformed and deformed IBZs for the hexagonal array (Fig. 2.3). Finally, following standard practice [cf., e.g., 34, 56], for each case, we consider a set of Bloch wave vectors $\mathbf{k}$ associated with the discretized boundary of the respective deformed IBZs. The discretization process involves dividing the path along the undeformed IBZ boundary into equal increments of $|\Delta \mathbf{k}_R| a_R / 2\pi = 0.025$ and considering the deformed wave vectors $\mathbf{k}$ corresponding to each point. An exception is made for small wave vector magnitudes of $|\mathbf{k}_R| a_R / 2\pi \leq 0.05$ – i.e., the region of the IBZ near the point G – where finer increments of $|\Delta \mathbf{k}_R| a_R / 2\pi = 0.0025$ are utilized to capture the dispersion relation as the magnitude of the wave vector approaches zero.

Once the set of Bloch wave vectors for a given case has been selected, the eigenfrequencies may be numerically computed using the procedure described in §2.3.2, and the dispersion relation $\omega(\mathbf{k})$ may be assembled. The calculated dispersion relations for each case described above are plotted for square and hexagonal arrays in the third columns of Figs. 2.2 and 2.3, respectively. On the vertical axes, the eigenfrequencies are normalized as $\omega a_r / 2\pi \tilde{c}$, where $\tilde{c} = (\mathbf{v}^{(a)} / c^{(a)} + \mathbf{v}^{(b)} / c^{(b)})^{-1}$ is the reciprocal volume-averaged ground-state shear wave speed and $c^{(\alpha)} = \sqrt{G^{(\alpha)} / \rho^{(\alpha)}}$ is the ground-state shear wave speed of phase α , and the horizontal axes denote paths along the boundaries of the respective IBZs. Each point on the dispersion relation plots represents a real-valued eigenfrequency for a discrete Bloch wave vector. For the generalized plane-strain conditions considered here, the corresponding mode shapes are either in-plane – i.e., $\tilde{u}_3 = 0$, while $\tilde{u}_1(x_1, x_2)$ and $\tilde{u}_2(x_1, x_2)$ are non-zero – or anti-plane – i.e., $\tilde{u}_1 = \tilde{u}_2 = 0$, while $\tilde{u}_3(x_1, x_2)$ is non-zero. The left side of each dispersion plot corresponds to in-plane electroelastic wave polarizations, while the right side corresponds to anti-plane polarizations.

It is important to note that – while mode shapes for anti-plane electroelastic wave polarizations are necessarily incompressible – mode shapes for in-plane wave polarizations in a solid-solid phononic crystal are affected by the compressibility of the material. For the nearly-incompressible composites under consideration, this effect is minimal for all but the smallest Bloch wave vector magnitudes – i.e., for $|\mathbf{k}_r| a_r / 2\pi \gtrsim 0.05$. Indeed, we have verified that the calculated eigenfrequencies are independent of the specific value of $K^{(\alpha)} / G^{(\alpha)}$ so long as it is sufficiently large and that the corresponding mode shapes for in-plane wave polarizations are nearly incompressible. For $|\mathbf{k}_r| a_r / 2\pi \lesssim 0.05$, the value of $K^{(\alpha)} / G^{(\alpha)}$ does play a role in certain in-plane mode shapes. As illustration, referring to the third columns of Figs. 2.2 and 2.3, one may observe two branches emanating from point G for in-plane wave polarizations. The higher branch represents longitudinal-like plane waves in the homogenized composite. The slope of this branch is comparably high and depends upon the bulk moduli $K^{(\alpha)}$ of both phases, and the corresponding mode shapes involve volumetric deformation in the composite microstructure. Similarly, other high-slope branches near point G at higher frequencies are also affected by compressibility. In the limit of ideally incompressible

phases in a solid-solid phononic crystal, these branches become vertical – such as in the work of Getz et al. [55].

Phononic band gaps are defined as frequency ranges in which no eigenfrequencies exist over the set of Bloch wave vectors. Band gaps may be identified for both in-plane and anti-plane wave polarizations. We note that our definition of in-plane band gaps includes both the nearly-incompressible and longitudinal-like mode shapes discussed in the preceding paragraph. Further, we refer to band gaps that span *both* in-plane and anti-plane polarizations as total, or polarization-independent, band gaps. Band gaps are denoted by cross-hatchings for in-plane, anti-plane, and polarization-independent band gaps on the dispersion plots in the third columns of Figs. 2.2 and 2.3. Comparing Figs. 2.2 and 2.3, it becomes clear that the band gap structures for the square- and hexagonal-lattice composites are quite different, even though the volume fraction and material property contrasts are the same in both cases. For example, the undeformed hexagonal-lattice composite possesses a polarization-independent band gap (Fig. 2.3(a)), while the square-lattice composite does not. Moreover, the band gap structure may be significantly altered through electrical preloading. In §2.4.3, we calculate the dependence of the band gap structure on electrical preloading in order to illustrate the electrical tunability of the soft phononic crystals considered in Figs. 2.2 and 2.3. The observed electrical tunability arises due to two mechanisms: (1) the deformation of the composite microstructure during electrical preloading and (2) the dependence of the linearized moduli – specifically, the spatial mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ – on deformation. We do not attempt to isolate these two mechanisms and separately understand their respective roles; however, we do illustrate the crucial role of deformation-dependent mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ in enhancing tunability in §2.4.4 by considering phononic crystals made from Neo-Hookean materials, which possess a substantially less nonlinear mechanical response than Gent materials.

Comment on verification: We have subjected our numerical simulation capability to several verification tests. First, in Appendix A.1, we consider infinite, incompressible, ideal-dielectric-elastomer laminates that are electrically-preloaded perpendicular to the plane of lamination –

a situation that has been previously considered in the literature [52, 53, 55]. The simulated dispersion relation for electroelastic wave propagation perpendicular to the layers matches the analytical dispersion relation (A.1.15) and its lack of dependence on electrical preloading when the layers are made from Neo-Hookean materials (see Fig. A.1(b)). Second, we utilize the plane-wave-expansion (PWE) method – recently extended to electroelastic wave propagation by Shmuel and coworkers [54, 55] – to calculate dispersion relations for composites subjected to electrical preloading parallel to the fibers. As previously noted, since the state of deformation and electric field is homogeneous throughout the deformed composite microstructure for this electrical preloading direction, the PWE method may be straightforwardly employed. In this technique, quantities such as the density, the permittivity, and the linearized shear modulus about a given preloaded state are expanded in two-dimensional Fourier series. Then, after some manipulation, the incremental governing equations may be recast as an algebraic eigenvalue problem for a given Bloch wave vector $\mathbf{k}$ that may be solved for the eigenfrequencies ω . For a more detailed discussion of the PWE method applied to electroelastic wave propagation in two-dimensional composites, see the works of Shmuel and coworkers [54, 55]. Dispersion relations calculated using the PWE method for unloaded and electrically-preloaded composites are shown as $\times$ symbols in Figs. 2.2(a) and (b) and 2.3(a) and (b) for square and hexagonal arrays, respectively. Our PWE results utilize 3721 plane waves – corresponding to $-30 \leq n_1, n_2 \leq 30$, where n_i is the number of terms in the two-dimensional Fourier series expansions. We note that as the number of plane waves is increased, the frequencies typically converge from above,⁶ while for finite-element results, the frequencies typically converge from below as the mesh is refined. The finite-element results using a mesh resolution of $0.02a_R$ match the PWE results using 3721 plane waves well for all cases shown in Figs. 2.2 and 2.3, providing an additional verification of our numerical simulation capability.

Comment on instabilities: Following Bertoldi and Gei [70], we consider three types of instabilities that a dielectric elastomer composite may undergo during quasi-electrostatic preloading: (1)

⁶Notably, since the discontinuous density, permittivity, and linearized shear modulus fields are approximated using Fourier series expansions, these fields will exhibit oscillations near interfaces, i.e., the Gibbs phenomenon, and as such, there is no guarantee that the plane wave expansion method will converge to the exact result.

loss of positive definiteness of the homogenized continuum (pull-in instability), (2) existence of diffuse modes of bifurcation (microscopic instability), and (3) loss of ellipticity of the homogenized continuum (macroscopic instability). First, the electromechanical pull-in instability corresponds to a peak in the $|\bar{\mathbf{E}}_{\text{r}}|$ versus $|\bar{\mathbf{D}}_{\text{r}}|$ response – or equivalently, the $|\bar{\mathbf{E}}_{\text{r}}|$ versus $\bar{\lambda}$ response – during preloading [28, 68, 70]. Based on our choice of the Gent locking parameter $I_{\text{m}}^{(\alpha)}$, we expect to encounter this type of instability and will explore harnessing the pull-in instability to achieve enhanced tunability. Next, while a detailed exploration of microscopic and macroscopic instabilities in dielectric elastomer composites is reserved for Chapter 3, an effort has been made to detect these instability types and ensure that electrical preloading beyond the onset of microscopic or macroscopic instability is not considered. To this end, the theoretical approach of Triantafyllidis and coworkers [80, 81] – later extended to computational settings by Bertoldi and coworkers [44, 49, 70] – may be used to detect bifurcations in infinite, periodic, composite materials using the linearized wave propagation analysis described in this section on the primitive unit cell. Specifically, microscopic instabilities may be detected by identifying zero-valued eigenfrequencies for short-wavelength Bloch wave vectors of the form $\mathbf{k} = \mathbf{b}_1/m_1 + \mathbf{b}_2/m_2$, where m_1 and m_2 are positive integers. For all cases considered herein, no evidence of microscopic instability was observed. Macroscopic instabilities may be identified through the loss of ellipticity of the homogenized continuum [cf., 44, 80] – an approach which has been generalized to electromechanical settings theoretically for laminates by Bertoldi and Gei [70] and computationally for more complex composites by Goshkoderia and Rudykh [75]. Alternatively, exploiting the connection between macroscopic instabilities and long-wavelength, diffuse modes of bifurcation [70, 81], it is also possible to detect macroscopic instabilities in the same way that microscopic instabilities are detected, using a small, but finite-magnitude Bloch wave vector. Accordingly, to detect macroscopic instabilities, we consider a normalized Bloch wave vector with magnitude $|\mathbf{k}_{\text{r}}|a_{\text{r}}/2\pi = 0.001$ and aligned with the macroscopic preloading direction, so that $\mathbf{k}_{\text{r}}a_{\text{r}}/2\pi = 0.001\bar{\mathbf{m}}$, and check that the corresponding eigenfrequencies ω are positive at each point along the electrical preloading path. Macroscopic instability does not occur for electrical preloading parallel to the fibers; however, for most of the subsequently considered cases of electrical

Lattice	Direction $\bar{\mathbf{m}} = \bar{\mathbf{E}}_{\text{R}}/ \bar{\mathbf{E}}_{\text{R}} $	Material	$ \bar{\mathbf{E}}_{\text{R}}^{\text{macro}} \sqrt{\bar{\epsilon}/\bar{G}}$	$\bar{\lambda}^{\text{macro}}$
Square	$\mathbf{e}_1$	Gent	0.472	0.820
		Neo-Hookean	0.491	0.818
	$\frac{1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2)$	Gent	0.518	0.630
		Neo-Hookean	0.457	0.621
Hexagonal	$\mathbf{e}_1$	Gent	0.487	0.782
		Neo-Hookean	0.463	0.779
	$\frac{\sqrt{3}}{2}\mathbf{e}_1 + \frac{1}{2}\mathbf{e}_2$	Gent	—	—
		Neo-Hookean	0.439	0.440

Table 2.2: Macroscopic electric field and stretch along the preloading direction at the onset of macroscopic instability for both the square- and hexagonal-lattice composite materials, for both in-plane preloading directions, and for composites made from both Gent and Neo-Hookean materials. Material property contrasts: $G^{(a)}/G^{(b)} = 20$, $\epsilon^{(a)}/\epsilon^{(b)} = 2$, $\rho^{(a)}/\rho^{(b)} = 10$, and $I_{\text{m}}^{(a)} = I_{\text{m}}^{(b)} = 10$ for Gent materials. Mesh density: $0.07a_{\text{R}}$.

preloading perpendicular to the fibers, macroscopic instability was detected, providing an upper limit for electrical preloading. Table 2.2 summarizes the macroscopic electric field, $|\bar{\mathbf{E}}_{\text{R}}^{\text{macro}}|\sqrt{\bar{\epsilon}/\bar{G}}$, and stretch along the preloading direction, $\bar{\lambda}^{\text{macro}}$, at the onset of macroscopic instability for both the square and hexagonal lattice, for both in-plane preloading directions, and for composites made from both Gent and Neo-Hookean materials. Small increments in “charge-control” were used to accurately determine the onset of macroscopic instability, and the uncertainty associated with the reported values of $|\bar{\mathbf{E}}_{\text{R}}^{\text{macro}}|\sqrt{\bar{\epsilon}/\bar{G}}$ and $\bar{\lambda}^{\text{macro}}$ is ± 0.002 for all cases as a consequence of the incremental detection procedure. The results reported in Table 2.2 were found to be insensitive to mesh resolution, i.e., simulations utilizing meshes with resolutions of $0.02a_{\text{R}}$ and $0.07a_{\text{R}}$ produced nearly identical results. Notably, the hexagonal-lattice composite made of Gent materials and preloaded along the least-packed direction does not exhibit macroscopic instability.

2.4.3 Electrical tunability of soft phononic crystals

Using the procedures described in the preceding sections, we are now equipped to calculate the electrical tunability of phononic crystals – i.e., phononic band gaps as a function of electrical

preloading. To do so, for a given composite and preloading direction $\bar{\mathbf{m}}$, we calculate the dispersion relation at discrete values of the macroscopic stretch $\bar{\lambda}$ in regularly-spaced intervals of 0.02 – achieved using a user-amplitude subroutine (UAMP) in Abaqus since electrical preloading simulations are conducted under “charge-control.” In this section, we continue to consider square- and hexagonal-lattice composites with volume fractions $\nu^{(a)} = \nu^{(b)} = 0.5$, made from Gent materials with property contrasts $G^{(a)}/G^{(b)} = 20$, $\varepsilon^{(a)}/\varepsilon^{(b)} = 2$, $\rho^{(a)}/\rho^{(b)} = 10$, and $I_m^{(a)} = I_m^{(b)} = 10$, and preloading parallel to the fibers and perpendicular to the fibers along the least-packed direction. For brevity, we do not report results for in-plane preloading along the closest-packed direction, since the results are qualitatively similar to preloading along the least-packed direction except for being more limited by the onset of macroscopic instability (Table 2.2).

Electrical preloading parallel to the fibers: Figs. 2.4(a) and (b) show the dependence of the macroscopic stretch $\bar{\lambda}$ and phononic band gaps on the applied electric field, $|\bar{\mathbf{E}}_R|\sqrt{\bar{\varepsilon}/\bar{G}}$, for the case of electrical preloading parallel to the fibers for square and hexagonal lattices, respectively. Representative deformed composite structures are shown, and the markers, $\circ$, $\square$, $\triangle$, and $\diamond$, indicate the corresponding points on the $\bar{\lambda}$ versus $|\bar{\mathbf{E}}_R|\sqrt{\bar{\varepsilon}/\bar{G}}$ plot. Since the state of deformation and electric field is homogeneous throughout the deformed composite microstructure, the analytical solutions for quasi-electrostatic preloading may be calculated [31] and are shown as solid lines in Figs. 2.4(a) and (b) as additional verification. For both the square and hexagonal lattices, subjected to an increasing macroscopic electric field, the states $\circ \rightarrow \square \rightarrow \diamond$ are followed, where a jump from state $\square$ to $\diamond$ occurs due to the existence of the electromechanical pull-in instability subsequently stabilized by stiffening associated with large-stretch chain-locking. This process may be reversed – albeit with hysteresis [28] – as long as the material exhibits sufficient resistance to dielectric breakdown, and upon electrical unloading, the states $\diamond \rightarrow \triangle \rightarrow \circ$ are traversed with a similar jump from state $\triangle$ to $\circ$. As is evident in Figs. 2.4(a) and (b), phononic band gaps only shift with electrical preloading, i.e., bands gaps do not open or close, which is true in general for electrical preloading parallel to the fibers when $I_m^{(a)} = I_m^{(b)}$. Prior to the pull-in instability, the electrical

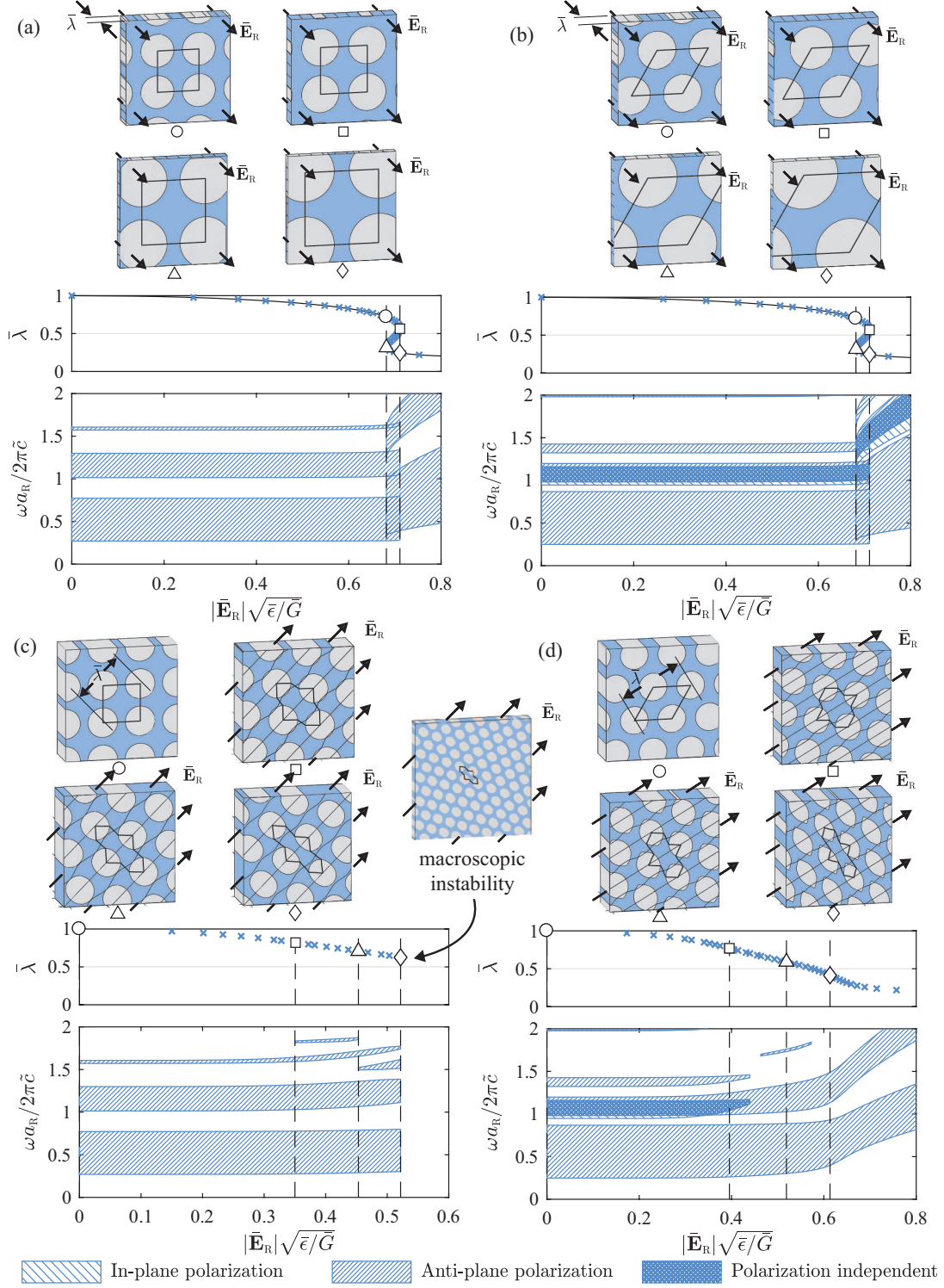


Figure 2.4: Electrical tunability of dielectric elastomer composites made from Gent materials, arranged in (a,c) a square lattice and (b,d) a hexagonal lattice and preloaded (a,b) parallel to the fibers and (c,d) perpendicular to the fibers along the least-packed direction. Mesh density: $0.07a_R$.

tunability of band gaps is minimal; however, due to the pull-in instability, the phononic crystal exhibits a “switch-like” property, in which large, discrete jumps in phononic band gaps may be achieved when preloaded in “voltage-control,” leading to enhanced tunability.

Electrical preloading perpendicular to the fibers along the least-packed direction:

Figs. 2.4(c) and (d) show the electrical tunability of the same phononic crystals for the case of electrical preloading perpendicular to the fibers along the least-packed direction for square and hexagonal lattices, respectively. In contrast to electrical preloading parallel to the fibers, the base state of deformation and electric field is inhomogeneous, as evidenced by the non-uniform electric field lines plotted on the deformed composite structures. Moreover, upon electrical preloading perpendicular to the fibers, phononic band gaps are not only shifted but are also opened and closed. This qualitative change in the nature of band gap tunability is due to the inhomogeneous base state of deformation and electric field as well as changes in the inclusion shape – both of which are absent for electrical preloading parallel to the fibers. For the square lattice, we observe an anti-plane band gap that opens at $\square$ followed by the closing and opening of anti-plane band gaps at $\triangle$. The onset of a macroscopic instability – occurring just after $\diamond$ at $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/\bar{G}} = 0.518$ and $\bar{\lambda} = 0.630$ (Table 2.2) – limits the range of electrical tunability of this specific phononic crystal. The corresponding mode shape of macroscopic instability is shown in the inset of Fig. 2.4(c).⁷ The importance of the lattice structure on electrical tunability is clear when comparing Figs. 2.4(c) and (d). In particular, for the hexagonal lattice preloaded along the least-packed direction, no macroscopic instability occurs, extending the range of electrical tunability. Furthermore, in Fig. 2.4(d), we observe a polarization-independent band gap and an anti-plane band gap that close shortly after $\square$, one anti-plane band gap that opens and closes around $\triangle$, and a significant shift in band gaps occurring around $\diamond$.⁸

For completeness, in Fig. 2.5, we show the dispersion relations for each deformed state denoted by

⁷For display purposes, in order to exaggerate the mode shape of macroscopic instability, the mode shape shown in the inset of Fig. 2.4(c) corresponds to a Bloch wave vector magnitude of $|\mathbf{k}_R|a_R/2\pi = 0.05$, rather than the Bloch wave vector magnitude of $|\mathbf{k}_R|a_R/2\pi = 0.001$ used to detect the onset of macroscopic instability.

⁸While the discrete, switch-like property is absent in this specific case, it is possible to choose material property contrasts so that the electromechanical pull-in instability – and hence the switch-like property – is observed. We have found that a small value of $G^{(a)}/G^{(b)}$ is necessary; however, since $G^{(a)}/G^{(b)}$ is associated with the quantity of band gaps, a tradeoff arises. This point is discussed further in §2.4.5.

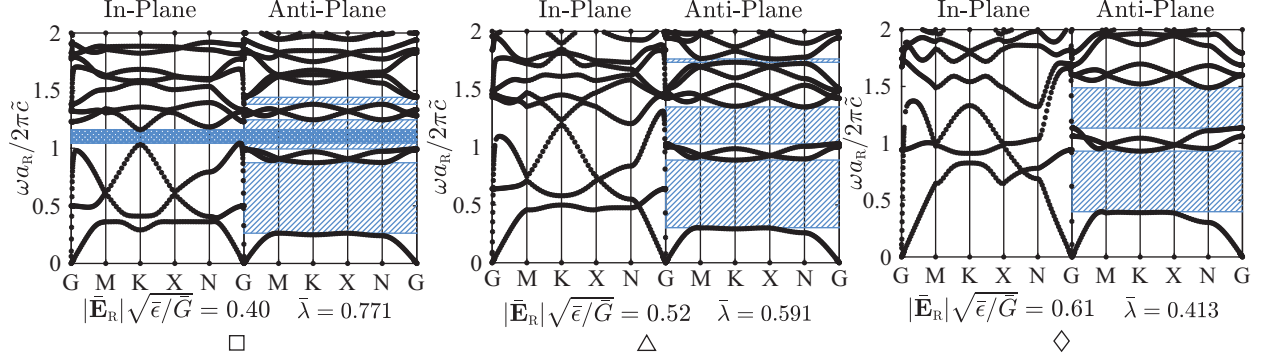


Figure 2.5: Dispersion relations for the hexagonal-lattice composite made from Gent materials and electrically preloaded perpendicular to the fibers in the least-packed direction. Markers correspond to the deformed composite structures shown in Fig. 2.4(d), and the Bloch wave vector path GMKXNG is in reference to the boundary of the IBZ shown in 2.3(d). Mesh density: $0.07a_R$.

the markers in Fig. 2.4(d). Features such as the polarization-independent band gap that is present at $\square$ and subsequently closes by $\triangle$ as well as the anti-plane band gap that appears at $\triangle$ may be seen in the dispersion relations.

2.4.4 Electrical tunability of soft phononic crystals exhibiting a lack of large-stretch chain-locking behavior

To illustrate the role of large-stretch chain-locking behavior – and more broadly, the role of the deformation-dependence of the mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ – on electrical tunability of phononic band gaps, we consider a composite material made of materials that do not exhibit this property. Specifically, we consider the limit in which $I_m^{(a)}, I_m^{(b)} \rightarrow \infty$ and the Gent hyperelastic constitutive model reduces to the Neo-Hookean model. As previously noted, Neo-Hookean materials are less mechanically nonlinear than Gent materials, and the spatial mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ are comparatively less deformation-dependent. Otherwise, we continue to consider the same composite materials previously described – i.e., square and hexagonal lattices with $\nu^{(a)} = \nu^{(b)} = 0.5$, $G^{(a)}/G^{(b)} = 20$, $\epsilon^{(a)}/\epsilon^{(b)} = 2$, and $\rho^{(a)}/\rho^{(b)} = 10$.

Electrical preloading parallel to the fibers: Figs. 2.6(a) and (b) show the calculated dependence of the macroscopic stretch $\bar{\lambda}$ and phononic band gaps on the applied electric field, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, as symbols, along with the analytical solutions as solid lines for the case of electrical preloading parallel to the fibers. Both the square- and hexagonal-lattice composites experience the electromechanical pull-in instability. However, in the absence of large-stretch chain-locking, there is no mechanism for stabilization, and therefore, the composites cannot sustain further electrical loading in “voltage-control.” Moreover, for Neo-Hookean materials subjected homogeneous base states of deformation and electric field, the dispersion relation does not depend on the magnitude of preloading, and therefore, band gaps may not be tuned – an observation consistent with previous works [44, 73] and the discussion of Appendix A.1. Therefore, the large-stretch chain-locking behavior of the Gent model and the associated deformation-dependence of the mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ are essential to achieve the electrical tunability demonstrated in Fig. 2.4(a) and (b). This observation is consistent with the work of Getz et al. [55] who also illustrated the important role of the Gent locking parameter I_m on the electrical tunability of phononic band gaps in similar types of composites for electrical preloading parallel to the fibers.

Electrical preloading perpendicular to the fibers along the least-packed direction:

Figs. 2.6(c) and (d) show the electrical tunability of the same phononic crystals for the case of electrical preloading perpendicular to the fibers along the least-packed direction. While the pull-in instability sets the maximum attainable value of $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$ for the case of preloading parallel to the fibers, macroscopic instability limits $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$ for in-plane preloading – e.g., $|\bar{\mathbf{E}}_R^{\text{macro}}| \sqrt{\bar{\epsilon}/\bar{G}} = 0.457$ and $|\bar{\mathbf{E}}_R^{\text{macro}}| \sqrt{\bar{\epsilon}/\bar{G}} = 0.439$ (Table 2.2) for the square- and hexagonal-lattice composites, respectively. The mode shapes of macroscopic instability are illustrated in the insets of Figs. 2.6(c) and (d). As expected, the phononic band gaps at $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 0$ are identical for corresponding composites made from Gent and Neo-Hookean materials – i.e., comparing Fig. 2.4(c) and Fig. 2.6(c) and comparing Fig. 2.4(d) and Fig. 2.6(d) at $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 0$. Then, as the macroscopic electric field is increased, in contrast to the case of electrical preloading parallel to

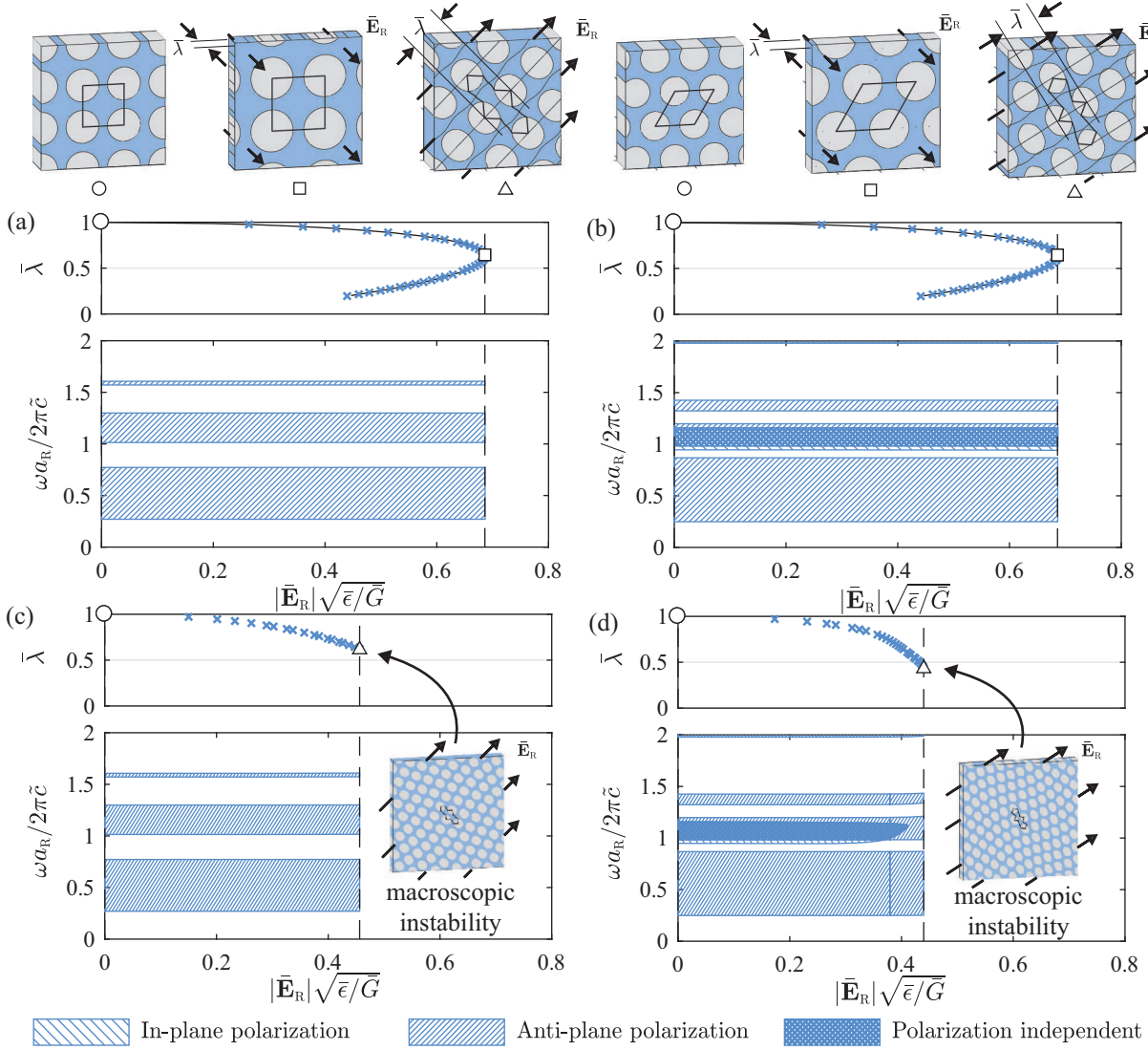


Figure 2.6: Electrical tunability of dielectric elastomer composites made from materials exhibiting a lack of large-stretch chain-locking behavior – Neo-Hookean materials – arranged in (a,c) a square lattice and (b,d) a hexagonal lattice and preloaded (a,b) parallel to the fibers and (c,d) perpendicular to the fibers along the least-packed direction. Mesh density: $0.07a_R$.

the fibers, phononic band gaps shift and close due to the inhomogeneous state of deformation and electric field in the composite microstructure. However, significantly less tunability is observed for the composites made from Neo-Hookean materials compared to their Gent-based counterparts, highlighting that large-stretch chain-locking behavior and the deformation-dependence of the mechanical moduli $\mathbb{C}_{ijkl}^{\text{mech}}$ play a crucial role in enhancing the electrical tunability of soft phononic

crystals.

2.4.5 Parametric study of material parameter contrasts and fiber volume fraction

Finally, we investigate the effects of material parameter contrasts and fiber volume fraction on band gap tunability for the case of a dielectric elastomer composite made from Gent materials arranged in a hexagonal lattice and preloaded perpendicular to the fibers along the least-packed direction. Specifically, we independently vary the contrasts $G^{(a)}/G^{(b)}$, $\epsilon^{(a)}/\epsilon^{(b)}$, $\rho^{(a)}/\rho^{(b)}$, and $I_m^{(a)}/I_m^{(b)}$ to be half and double their baseline values. We also consider several lower value of the density contrast as well as fiber volume fractions of $\nu^{(a)} = 0.25$ and $\nu^{(a)} = 0.75$. The results are presented in Figs. 2.7-2.11 and may be compared to the electrical tunability of the baseline case, shown in Fig. 2.4(d). In Figs. 2.7-2.11, the deformed composite structures are shown at a macroscopic stretch along the preloading direction of $\bar{\lambda} = 0.50$ for all cases and with electric field lines superimposed. The cross-hatching patterns used in Figs. 2.7-2.11 for in-plane, anti-plane, and polarization-independent band gaps are the same as those identified in Fig. 2.4.

Effect of $G^{(a)}/G^{(b)}$: The ground-state shear modulus contrast $G^{(a)}/G^{(b)}$ plays a key role in both quasi-electrostatic preloading and linearized wave propagation. In Figs. 2.7(a) and (b), two effects appear. First, increasing $G^{(a)}/G^{(b)}$ tends to increase the widths of band gaps for the undeformed composite, while decreasing $G^{(a)}/G^{(b)}$ decreases the quantity and widths of band gaps – eliminating the polarization-independent band gap altogether in this example. Second, regarding tunability, for higher $G^{(a)}/G^{(b)}$, band gaps tend to gradually close as the composite is electrically preloaded, while reducing $G^{(a)}/G^{(b)}$ concentrates band gap tunability in a narrower, high-magnitude range of $|\bar{\mathbf{E}}_r|/\sqrt{\bar{\epsilon}/\bar{G}}$. This behavior is similar to the switch-like property associated with the pull-in instability discussed in §2.4.3, and indeed, if $G^{(a)}/G^{(b)}$ is further reduced, the pull-in instability will arise. Therefore, a tradeoff emerges: increasing the quantity and width of band gaps by increasing $G^{(a)}/G^{(b)}$ comes at the cost of sacrificing switch-like electrical tunability.

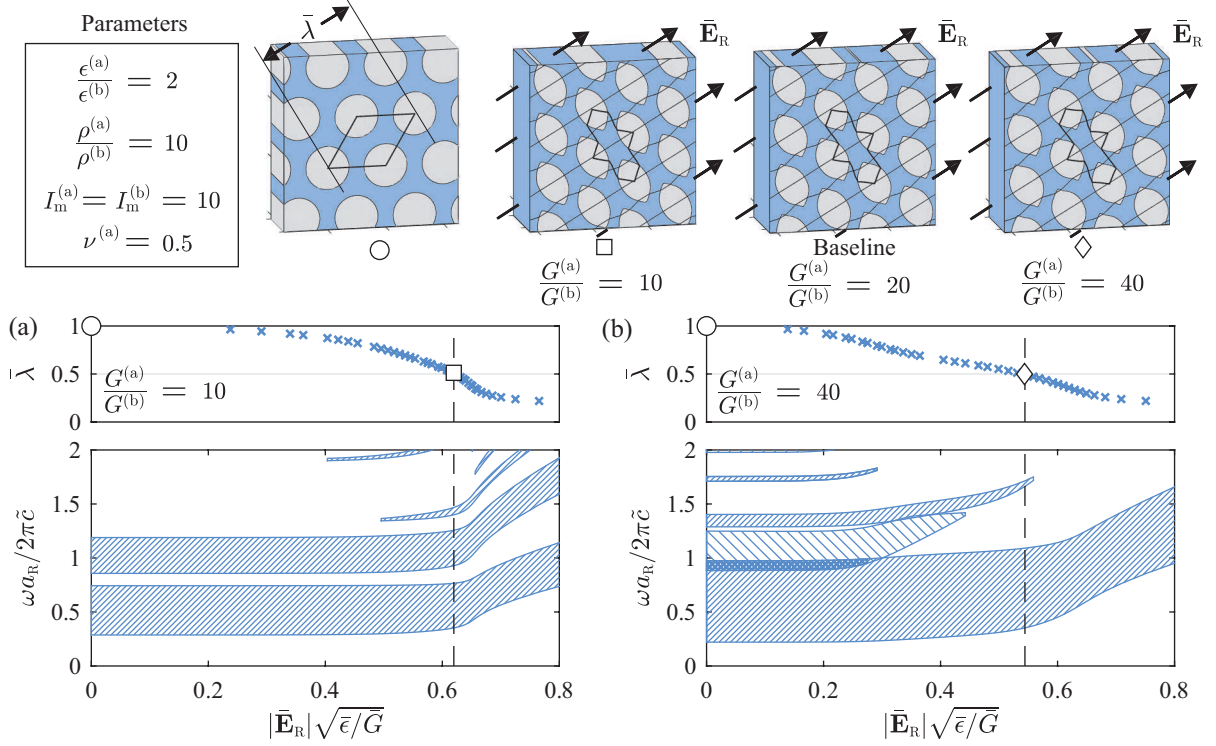


Figure 2.7: The effect of ground-state shear modulus contrast – (a) $G^{(a)}/G^{(b)} = 10$ and (b) $G^{(a)}/G^{(b)} = 40$ – on electrical tunability of dielectric elastomer composites made from Gent materials arranged in a hexagonal lattice and preloaded perpendicular to the fibers along the least-packed direction. Electric tunability of the baseline case – $G^{(a)}/G^{(b)} = 20$ – is shown in Fig. 2.4(d). The deformed composite structures are shown at a macroscopic stretch along the preloading direction of $\bar{\lambda} = 0.50$. Cross-hatchings for in-plane, anti-plane, and polarization-independent band gaps are identified in Fig. 2.4. Mesh density: $0.07a_R$.

Effect of $\epsilon^{(a)}/\epsilon^{(b)}$: Figs. 2.8(a) and (b) illustrate the clear effect of the permittivity contrast $\epsilon^{(a)}/\epsilon^{(b)}$ on electrical preloading – evidenced in both the $\bar{\lambda}$ versus $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/G}$ response and the electric field lines plotted on the deformed composite structures. The permittivity contrast has no effect on phononic band gaps at $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/G} = 0$ but does affect electrical tunability as the electric field is increased. However, the effect of $\epsilon^{(a)}/\epsilon^{(b)}$ on band gap tunability is much more modest than that of $G^{(a)}/G^{(b)}$ for the contrast values considered in the present work.

Effect of $\rho^{(a)}/\rho^{(b)}$: In Fig. 2.9, we consider density contrasts of half and double the baseline value of $\rho^{(a)}/\rho^{(b)} = 10$ as well as lower values of $\rho^{(a)}/\rho^{(b)} = 1$ and $\rho^{(a)}/\rho^{(b)} = 2$, which are more

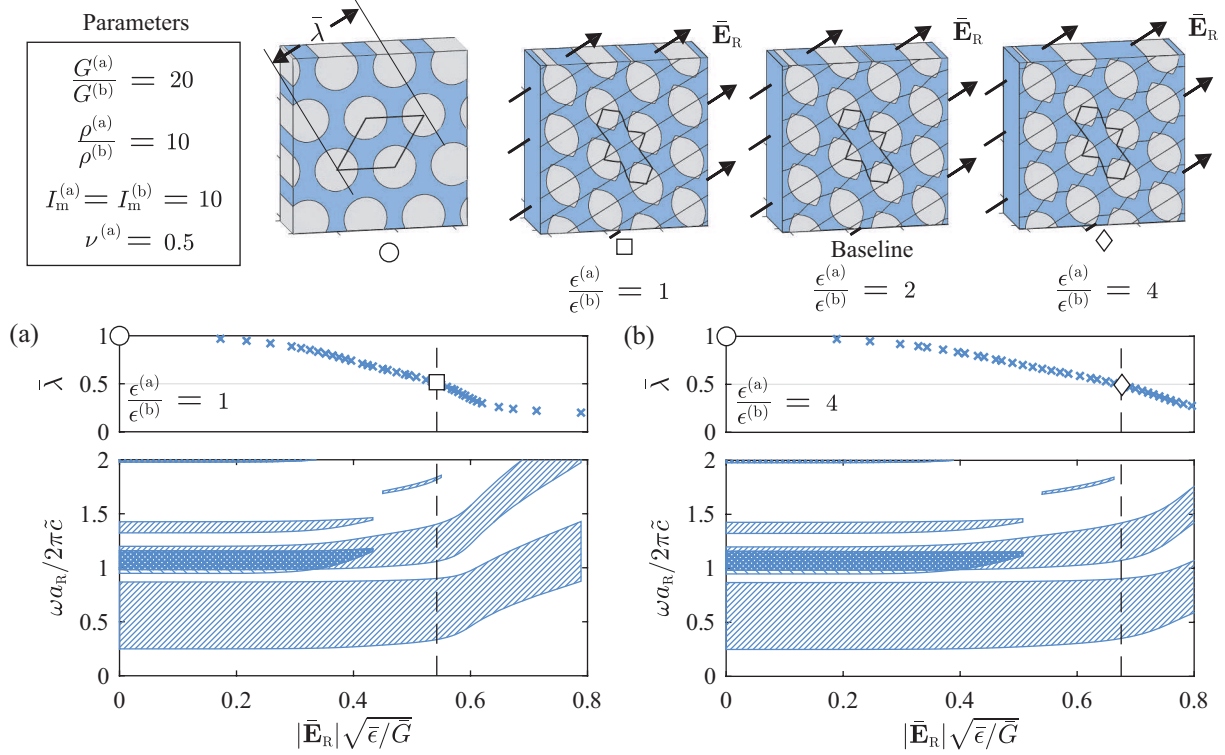


Figure 2.8: The effect of permittivity contrast – (a) $\epsilon^{(a)}/\epsilon^{(b)} = 1$ and (b) $\epsilon^{(a)}/\epsilon^{(b)} = 4$ – on electrical tunability of dielectric elastomer composites made from Gent materials arranged in a hexagonal lattice and preloaded perpendicular to the fibers along the least-packed direction. Electric tunability of the baseline case – $\epsilon^{(a)}/\epsilon^{(b)} = 2$ – is shown in Fig. 2.4(d). Results are organized as described in the caption of Fig. 2.7. Mesh density: $0.07a_R$.

readily achievable in practice. The density contrast has no effect on quasi-electrostatic preloading – only on wave propagation. Therefore, the $\bar{\lambda}$ versus $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}}$ responses in Figs. 2.9(a)-(d) and Fig. 2.4(d) are identical. In Fig. 2.9, as expected, it is observed that increasing $\rho^{(a)}/\rho^{(b)}$ increases the quantity of band gaps. However, even modest density contrasts of $\rho^{(a)}/\rho^{(b)} = 1$ and $\rho^{(a)}/\rho^{(b)} = 2$ show the existence of band gaps and tunability. Therefore, while it can be beneficial to maximize this contrast parameter, it is not essential to achieving electrically-tunable phononic band gaps.

Effect of $I_m^{(a)}/I_m^{(b)}$: The contrast in Gent locking parameter $I_m^{(a)}/I_m^{(b)}$ has no effect on phononic band gaps at $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}} = 0$ – instead influencing electrical tunability at higher electric fields, as recently illustrated by Getz et al. [55] for the case of electrical preloading parallel to the fibers.

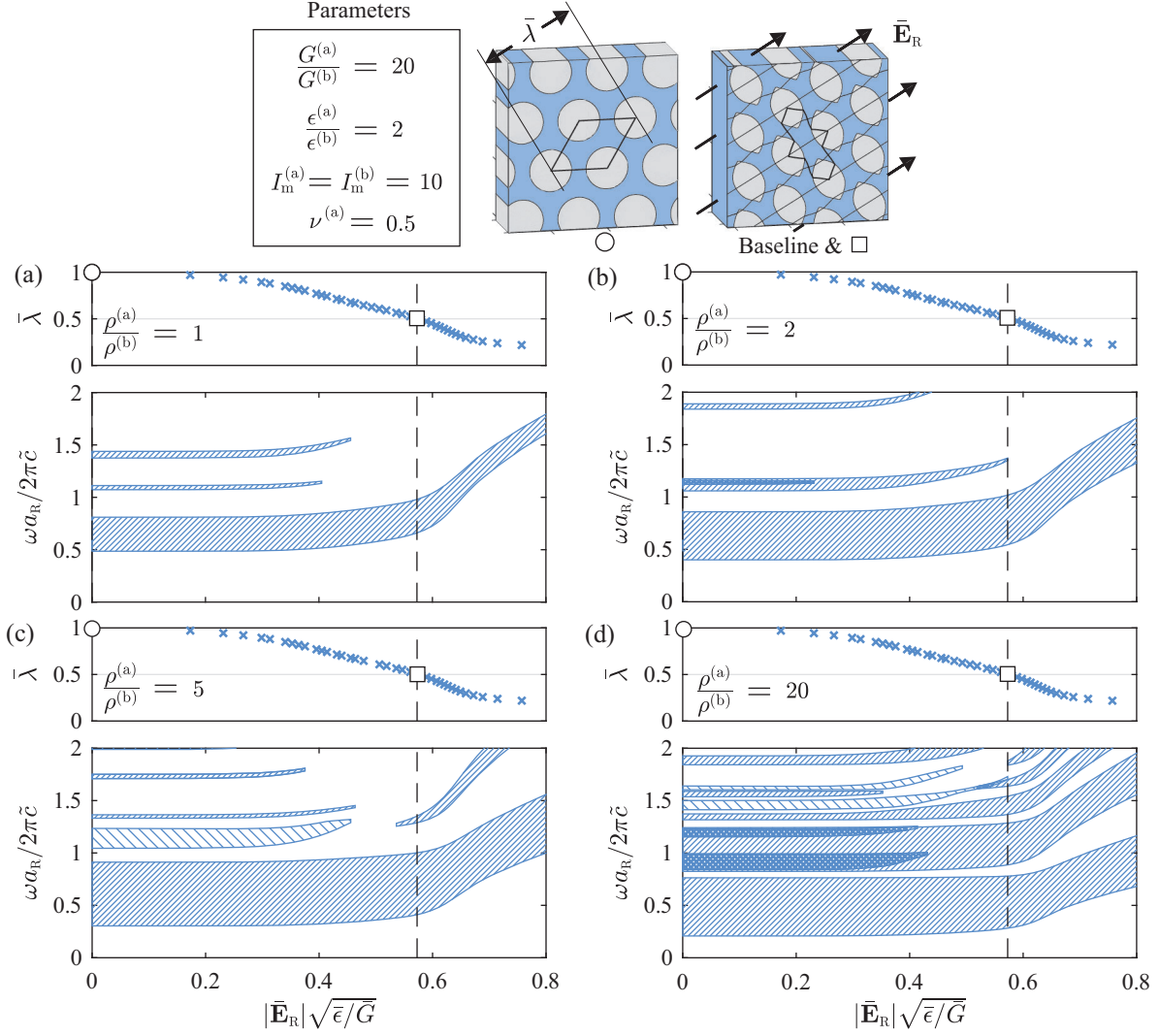


Figure 2.9: The effect of density contrast – (a) $\rho^{(a)}/\rho^{(b)} = 1$, (b) $\rho^{(a)}/\rho^{(b)} = 2$, (c) $\rho^{(a)}/\rho^{(b)} = 5$, and (d) $\rho^{(a)}/\rho^{(b)} = 20$ – on electrical tunability of dielectric elastomer composites made from Gent materials arranged in a hexagonal lattice and preloaded perpendicular to the fibers along the least-packed direction. Electric tunability of the baseline case – $\rho^{(a)}/\rho^{(b)} = 10$ – is shown in Fig. 2.4(d). Results are organized as described in the caption of Fig. 2.7; however, since density contrast has no effect on nonlinear quasi-electrostatic preloading, all deformed composite structures for a given $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$ are identical and only one is pictured. Mesh density: $0.07a_R$.

For $I_m^{(a)} > I_m^{(b)}$, the fibers stiffen less rapidly than the matrix material. This is shown to give rise to nearly switch-like tunability in Fig. 2.10(b) but does not strongly influence the number or width of the band gaps.

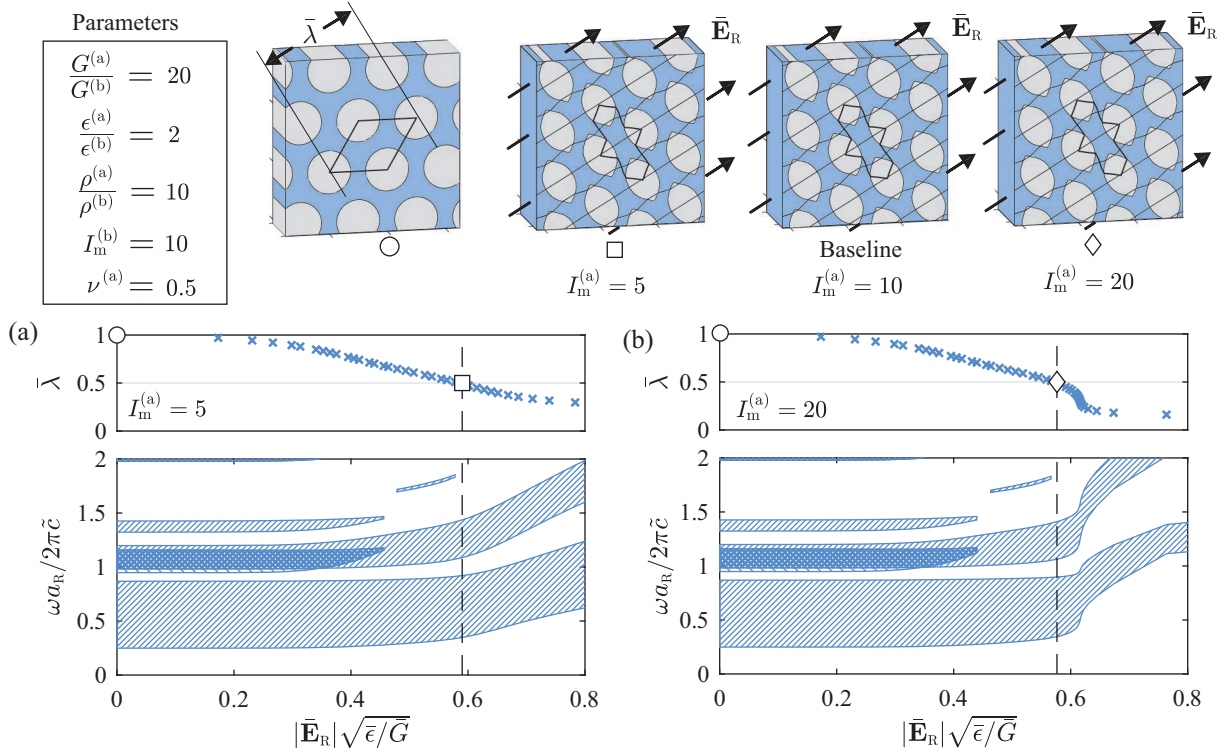


Figure 2.10: The effect of Gent locking parameter for the fibers – (a) $I_m^{(a)} = 5$ and (b) $I_m^{(a)} = 20$ – on electrical tunability of dielectric elastomer composites made from Gent materials arranged in a hexagonal lattice and preloaded perpendicular to the fibers along the least-packed direction. Electric tunability of the baseline case – $I_m^{(a)} = 10$ – is shown in Fig. 2.4(d). Results are organized as described in the caption of Fig. 2.7. Mesh density: $0.07a_R$.

Effect of $\nu^{(a)}$: Fiber volume fraction has an important impact on preloading as well as on the number and tunability of band gaps. For this specific case, smaller fibers ($\nu^{(a)} = 0.25$) lead to a decrease in the width of the polarization-independent band gap (Fig. 2.11(a)). However, a macroscopic instability arises – the mode shape of which is shown in the inset of Fig. 2.11(a) – limiting electrical tunability. For larger fibers ($\nu^{(a)} = 0.75$), only anti-plane band gaps occur, but there is no macroscopic instability for this case, enabling nearly switch-like tunability (Fig. 2.11(b)).

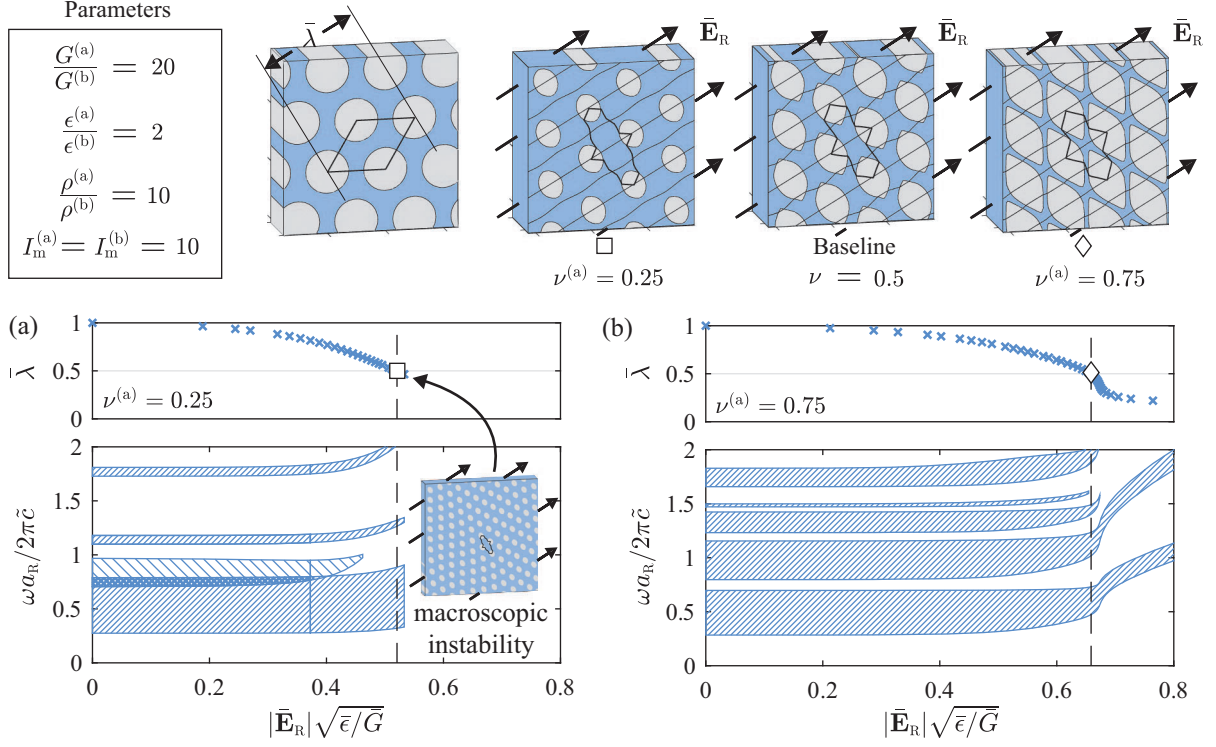


Figure 2.11: The effect of fiber volume fraction – (a) $\nu^{(a)} = 0.25$ and (b) $\nu^{(a)} = 0.75$ – on electrical tunability of dielectric elastomer composites made from Gent materials arranged in a hexagonal lattice and preloaded perpendicular to the fibers along the least-packed direction. Electric tunability of the baseline case – $\nu^{(a)} = 0.5$ – is shown in Fig. 2.4(d). Results are organized as described in the caption of Fig. 2.7. Mesh density: $0.07a_R$.

2.5 Concluding remarks

In this chapter, we have presented our numerical simulation capability for infinite, periodic dielectric elastomer composites, capable of addressing both nonlinear quasi-electrostatic processes and linearized electroelastic wave propagation. Additionally, we have applied this capability to electrically-tunable soft phononic crystals, specifically considering infinite dielectric elastomer composites consisting of periodic lattices – both square and hexagonal – of aligned prismatic fibers in a matrix, subjected to electric fields both aligned with and perpendicular to the axis of the fibers. By computing the dispersion relations at various levels of electrical preloading, we characterized the electrical tunability – the dependence of phononic band gaps on the applied macroscopic electric field – of representative composite materials. Several key observations are summarized below:

- The electromechanical pull-in instability may be exploited to achieve enhanced electrical tunability in the form of a phononic crystal switch.
- Electrical preloading perpendicular to the fibers gives rise to opening and closing in addition to shifting of phononic band gaps unlike the case of electrical preloading parallel to the fibers with $I_m^{(a)} = I_m^{(b)}$. This is due to the inhomogeneous base state of deformation and electric field in the composite microstructure.
- Large-stretch chain-locking behavior and the enhanced deformation-dependence of the spatial mechanical moduli C_{ijkl}^{mech} associated with this behavior is responsible for the delayed onset of macroscopic instability and for enhancing the electrical tunability of phononic band gaps, which can enable switch-like tunability.
- Regarding material parameter contrasts, $G^{(a)}/G^{(b)}$ and $\rho^{(a)}/\rho^{(b)}$ have the most significant effects on the number of band gaps, while $G^{(a)}/G^{(b)}$ and $I_m^{(a)}/I_m^{(b)}$ most strongly influence tunability. The role of $\varepsilon^{(a)}/\varepsilon^{(b)}$ was observed to be more modest for the range of contrast values considered in this study.

This work may be used as a platform for further study of dielectric elastomer composites. Navigating the landscape of macroscopic instabilities is an important problem [cf., 60, 70, 75], and our work opens the door for further simulation-based study of macroscopic instabilities for general composite geometries and loading conditions. Additionally, while microscopic instabilities were not observed in the cases considered in this chapter, recent work has shown they may be used to enhance tunability in purely mechanical phononic crystals [cf., 49], and therefore, it is of interest to use our numerical simulation capability to seek situations in which electrically-induced microscopic instabilities arise. These aims are the subject of the next chapter.

Chapter 3

Electromechanical instabilities in periodic dielectric elastomer composites

Note: A version of this chapter is being prepared for submission to the International Journal of Solids and Structures.

M. A. Jandron and D. L. Henann. Electromechanical instabilities in periodic dielectric elastomer composites. In preparation.

3.1 Introduction

As soft dielectrics deform under the action of an applied electric field, it is not uncommon to encounter one of several modes of instability – a topic which has received significant attention in the recent literature [cf., 68, 82]. The review of Zhao and Wang [68] pointed out three generic modes of instabilities in *homogeneous* soft dielectrics: (1) the aforementioned pull-in instability [4, 28], (2) electrocreasing [83–85], and (3) electrocavitation [86], with the mode of instability dependent upon the geometric configuration and boundary conditions. Electromechanical instabilities in homogeneous dielectric elastomers have been harnessed to achieve enhanced deformations [87], as control valves in microfluidic devices [88], for tunable surfaces [85], and in anti-bio-fouling applications [89]. As observed in Chapter 2, the instability landscape for dielectric elastomer composites is

richer and more complex than that of homogeneous dielectric elastomers. Similar to periodic composites subjected to purely mechanical loads [cf., e.g., 80, 81, 90, 91], additional instability modes emerge. In addition to the pull-in instability – considered in Chapter 2 as a potential route to enhanced tunability of soft phononic crystals – microscopic instabilities, macroscopic instabilities, and even interfacial instabilities – electrocreasing-type instabilities at solid-solid interfaces – can arise in dielectric elastomer composites.

Much of the existing work in the literature regarding instabilities in periodic dielectric elastomer composites focuses on the simple case of laminates [70, 92–94]. Bertoldi and Gei [70] were the first to analytically study the electromechanical instability landscape of laminates, considering laminates subjected to an electric field perpendicular to the plane of lamination and prestressed or prestretched within the plane of lamination. The work of Bertoldi and Gei [70] highlighted the significant effect of prestress and prestretch on the instability landscape and how prestress or prestretch can delay the onset of instability as well as induce instabilities to switch from one type to another. Rudykh and coworkers [92, 93] extended this work to situations, in which the electric field is applied at arbitrary angles with respect to the plane of lamination.

Recent work on instabilities in periodic dielectric elastomer composites has begun to consider composites consisting of aligned fibers in a matrix. Siboni and Ponte Castaneda [95] analytically explored instabilities in square-lattice composites of elliptical cross-section fibers. This analysis was performed using an approximate homogenization technique and focused on electromechanical pull-in and macroscopic instabilities. To go further and analyze instabilities in general settings, numerical tools are necessary. Li and Landis [60] were the first to numerically explore electromechanical instabilities in periodic dielectric elastomer composites, demonstrating the need for numerical tools and, importantly, suggesting the existence of interfacial instabilities in certain cases. More recently, Goshkoderia and Rudykh [75] reported on a numerical simulation capability within COMSOL Multiphysics and considered macroscopic instabilities in square lattices of circular and elliptical cross-section fibers in a matrix under plane-strain conditions with and without mechanical prestretch. Finally, Polukhov et al. [96] presented numerical simulations of

macroscopic and microscopic instabilities in square-lattice composites comprised of stiff circular and elliptical cross-section fibers in a matrix – also under plane-strain conditions. While these works provided a step forward in the understanding of electromechanical instabilities in dielectric elastomer composites, a more complete picture accounting for all types of electromechanical instabilities and their dependence upon composite structure and material properties is currently lacking. This chapter aims to fill this need.

The objective of this chapter is to utilize the numerical simulation capability presented in Chapter 2 to extend the scientific knowledge of instability landscapes from the previously-studied dielectric elastomer laminates [70] to more complex dielectric elastomer composites consisting of periodic lattices of aligned prismatic fibers in a matrix – more specifically, square- and hexagonal-lattices of circular inclusions.¹ We consider four types of electromechanical instability: (1) pull-in instability (loss of positive definiteness of the homogenized continuum), (2) existence of diffuse modes of bifurcation (microscopic instability), (3) loss of ellipticity of the homogenized continuum (macroscopic instability), and (4) interfacial instability – i.e., electrocreasing-type instability occurring at solid-solid interfaces. We study the effects of the two most important material property contrasts – i.e., ground-state shear modulus and electric permittivity contrast – on the onset and type of instability, and additionally, we consider the effects of fiber volume fraction, chain-locking behavior, prestress, prestretch, and confinement. Of note, we highlight interfacial instabilities that can occur in solid-solid dielectric elastomer composites when electric fields become especially large in thin regions. Furthermore, we highlight that applying an electric field to a fully-confined dielectric elastomer composite induces microstructural shape changes and potential instabilities that would not be possible using purely mechanical actuation. This has not been previously reported and motivates applications of electrically-tunable phononic crystals in confined geometries.

The result of this chapter is a comprehensive view of instabilities in periodic dielectric elastomer composites as well as a general strategy for exploring and designing these materials. The remainder of this chapter is organized as follows. In §3.2 we describe our numerical detection procedure for

¹While some recent research considers elliptical inclusion geometries [e.g., 75, 95–97], we limit the present study to circular inclusions for simplicity.

(1) pull-in (loss of positive definiteness), (2) diffuse modes (microscopic), (3) loss of ellipticity (macroscopic), and (4) interfacial electromechanical instabilities in composites consisting of periodic lattices of aligned, circular-cross-section fibers in a matrix. In §3.3, we demonstrate the numerical simulation capability. First, in §3.3.1, we show representative results for a square-lattice composite actuated perpendicular to the fibers along the closest-packed direction and define a phase space for electromechanical instabilities, and in §3.3.3, we present results for baseline cases of actuation in both square- and hexagonal-lattices along both the closest-packed and the least-packed directions. In §3.3.4, we consider several parametric studies. Specifically, we consider the effect of fiber volume fraction, large-stretch chain-locking behavior, mechanical prestress, and prestretch and confinement. Finally, we close with some concluding remarks in §3.4.

3.2 Electromechanical instabilities

As in Chapter 2, we consider composites consisting of periodic lattices of aligned prismatic fibers in a matrix – specifically, circular-cross-section fibers in square and regular hexagonal arrays as shown in Fig. 2.1 – in which the fibers and the matrix are made from two different ideal dielectric elastomers. We continue to denote the referential lattice vectors as $\mathbf{a}_{r1}$, $\mathbf{a}_{r2}$, and $\mathbf{a}_{r3}$ and the referential reciprocal lattice vectors as $\mathbf{b}_{r1}$, $\mathbf{b}_{r2}$, and $\mathbf{b}_{r3}$, which are given by (2.3.1). Nonlinear quasi-electrostatic finite-element simulations are performed using the numerical simulation capability described in Chapter 2, and far-field loading is applied through the use of generalized degrees of freedom [44, 65, 66] as described in §2.3.1. We continue to denote the macroscopic deformation gradient, referential electric field, first Piola stress, and referential electric displacement as $\bar{\mathbf{F}}$, $\bar{\mathbf{E}}_r$, $\bar{\mathbf{T}}_r$, and $\bar{\mathbf{D}}_r$, respectively, and the spatial lattice vectors and reciprocal lattice vectors as $\mathbf{a}_i = \bar{\mathbf{F}}\mathbf{a}_{ri}$ and $\mathbf{b}_i = \bar{\mathbf{F}}^{-\top}\mathbf{b}_{ri}$.

We consider four types of electromechanical instabilities that may occur in a dielectric elastomer composite undergoing a quasi-electrostatic process: (1) pull-in instability (loss of positive definiteness of the homogenized continuum), (2) existence of diffuse modes of bifurcation (microscopic

instability), (3) loss of ellipticity of the homogenized continuum (macroscopic instability), and (4) interfacial instability – i.e., electrocreasing-type instability occurring at solid-solid interfaces. Our process for detecting the onset of each instability type is discussed further below.

Pull-in instability (Loss of positive definiteness of the homogenized continuum): In the classical electromechanical pull-in instability, a *homogeneous*, thin dielectric elastomer layer is sandwiched between two compliant electrodes, and an electric potential difference is applied across the layer, causing its thickness to reduce. Due to thickness reduction, the spatial electric field $\mathbf{E}$ grows faster than its referential counterpart $\mathbf{E}_R$, and the electrostatic contribution to the Cauchy stress consequently grows quite rapidly. If this positive feedback outpaces the material's ability to mechanically resist deformation, the applied referential electric field $\mathbf{E}_R$ reaches a maximum value, unimpeded thinning commences, and the pull-in instability occurs [4, 98].² Similarly, for a periodic composite material subjected to a macroscopic referential electric field $\bar{\mathbf{E}}_R$, the electromechanical pull-in instability is identified when a peak in the $|\bar{\mathbf{E}}_R|$ versus $|\bar{\mathbf{D}}_R|$ response – or equivalently, the $|\bar{\mathbf{E}}_R|$ versus $\bar{\lambda} = \sqrt{\bar{\mathbf{m}} \cdot \bar{\mathbf{F}}^T \bar{\mathbf{F}} \bar{\mathbf{m}}}$ response, where $\bar{\mathbf{m}} = \bar{\mathbf{E}}_R / |\bar{\mathbf{E}}_R|$ – occurs during quasi-electrostatic loading.

Diffuse modes of bifurcation (microscopic instability): As discussed in Chapter 2, diffuse modes of bifurcation (microscopic instability) in infinite, periodic, composite materials may be detected using the linearized wave propagation analysis on the primitive unit cell, described in Section 2.4.2 [44, 49, 70, 80, 81]. Specifically, microscopic instabilities may be detected by identifying zero-valued eigenfrequencies for short-wavelength Bloch wave vectors of the form $\mathbf{k} = \mathbf{b}_1/m_1 + \mathbf{b}_2/m_2$, where $\mathbf{b}_1$ and $\mathbf{b}_2$ are the spatial reciprocal lattice vectors, and m_1 and m_2 are

²For perspective, the pull-in instability is mathematically associated with the loss of positive definiteness of the electro-elastic constitutive tangent operator [28, 68, 70, 99]. As discussed by Bertoldi and Gei [70], for a homogeneous base state of deformation and electric field, $\mathbf{F}$ and $\mathbf{E}_R$, subjected to a homogeneous an incremental process, $\dot{\mathbf{F}}$ and $\dot{\mathbf{E}}_R$, the critical condition occurs when $\bar{\mathbf{T}}_R : \dot{\mathbf{F}} + \bar{\mathbf{D}}_R \cdot \dot{\mathbf{E}}_R = 0$ for some non-trivial combination of $\dot{\mathbf{F}}$ and $\dot{\mathbf{E}}_R$ – i.e., in which one or both of $\dot{\mathbf{F}}$ and $\dot{\mathbf{E}}_R$ are non-zero. Using (2.2.6), the critical condition may be expressed as

$$\dot{\mathbf{F}} : \mathbb{C}_R \dot{\mathbf{F}} + \dot{\mathbf{E}}_R \cdot \mathbf{A}_R \dot{\mathbf{E}}_R = 0, \quad (3.2.1)$$

where based on (2.2.7), we have made use of the property $\Gamma_{R,ijl} = -\Lambda_{R,lij}$. For a composite material, $\bar{\mathbb{C}}_R = \partial \bar{\mathbf{T}}_R(\bar{\mathbf{F}}, \bar{\mathbf{E}}_R) / \partial \bar{\mathbf{F}}$ and $\bar{\mathbf{A}}_R = \partial \bar{\mathbf{D}}_R(\bar{\mathbf{F}}, \bar{\mathbf{E}}_R) / \partial \bar{\mathbf{E}}_R$, and when $\bar{\mathbb{C}}$ and $\bar{\mathbf{A}}_R$ are used in (3.2.1) in place of $\mathbb{C}$ and $\mathbf{A}_R$, (3.2.1) represents the critical condition for loss of positive definiteness of the homogenized continuum.

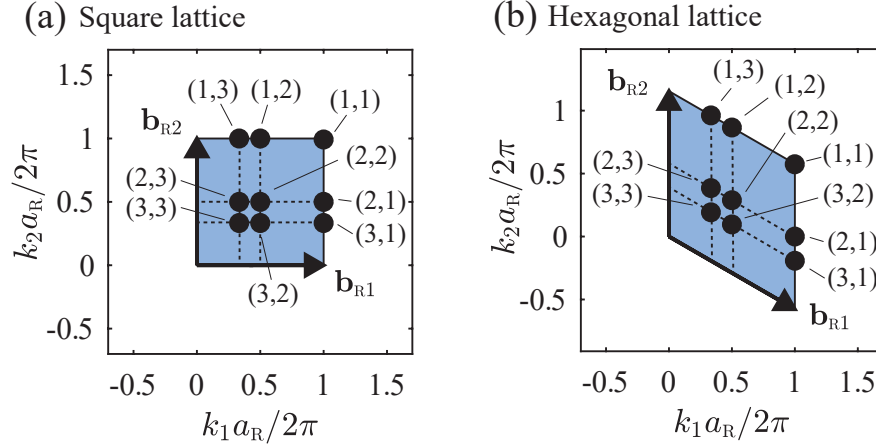


Figure 3.1: Bloch wave vectors considered for the detection of microscopic instability in both the (a) square and (b) regular hexagonal lattice composites. For each case, nine Bloch wave vectors of the form $\mathbf{k} = \mathbf{b}_1/m_1 + \mathbf{b}_2/m_2$ with $m_1 = 1, 2, 3$ and $m_2 = 1, 2, 3$ are considered, which are denoted by filled circles in the undeformed reciprocal space above.

positive integers. In this chapter, to detect microscopic instability, we consider all permutations of $m_1 = 1, 2, 3$ and $m_2 = 1, 2, 3$. The nine Bloch wave vectors under consideration are illustrated as filled circles in the undeformed reciprocal spaces in Fig. 3.1 for (a) square and (b) regular hexagonal lattice composites. We find this set of wave vectors to be sufficient to detect microscopic instability in the present study. Then, for each Bloch wave vector, we calculate the corresponding eigenfrequency ω^- , where the $-$ superscript denotes eigenfrequencies corresponding to microscopic modes, and its dependence on quasi-electrostatic loading. When one of the eigenfrequencies ω^- reaches zero, a microscopic instability is deemed to have occurred.

Loss of ellipticity (macroscopic instability): Macroscopic instability corresponds to the loss of ellipticity of the homogenized continuum – a condition that has been applied to composites under purely mechanical loading [cf., 44, 80] as well as electromechanical loadings [70, 75, 96]. Geymonat et al. [81] showed that for composite materials, the condition for the loss of ellipticity of the homogenized continuum is equivalent to the condition for long-wavelength diffuse modes of bifurcation. Accordingly, to detect macroscopic instabilities, we consider normalized Bloch wave vectors with magnitude $|\mathbf{k}|a_R/2\pi = 0.001$ – corresponding to a long wavelength of $1000a_R$ – and

for all in-plane wave vector directions in reciprocal space in angular increments of $\pi/12$. Then, for each of these wave vectors, we check that the corresponding eigenfrequencies ω^+ , where the $+$ superscript denotes eigenfrequencies corresponding to macroscopic modes, are positive at each point along the quasi-electrostatic loading path. When one of the eigenfrequencies reaches zero, a macroscopic instability is deemed to have occurred. While we have checked for macroscopic instability over the full range of wave vector directions, we consistently find – with only a few exceptions – that when macroscopic instability occurs, the critical wave vector direction coincides with the direction of the macroscopic electric field $\bar{\mathbf{m}}$.

Interfacial instability: Dielectric elastomer composites are also prone to nonlinear, surface-based instabilities at solid-solid interfaces – which we broadly classify as *interfacial* instabilities. Interfacial instabilities are related to other surface-based instabilities in dielectric elastomers. For example, when a homogeneous soft dielectric layer is sandwiched between a rigid electrode on one side and a compliant electrode on the opposite side, lateral expansion is prevented by the rigid electrode, and the pull-in instability is suppressed. In this case, instability occurs through the formation of creases on the unconstrained surface of the soft dielectric film - *electrocreasing* – an instability that has been extensively studied both experimentally [83, 84] and numerically [60, 64, 83, 84, 100–102].³ Another example of interfacial instability is *electrocavitation*, in which an isolated, fluid-filled, conductive droplet in an otherwise homogeneous dielectric elastomer under far-field mechanical constraint is subjected to an electric field. At some critical value of the applied far-field electric field, the deformation around the droplet becomes unstable, and sharp cusps form at the poles of the droplet with respect to the direction of the applied electric field. The electrocavitation instability has been experimentally observed [85] as well as numerically studied [64, 79, 85]. As a final example, Li and Landis [60] numerically observed an *electropinching* type of instability in dielectric elastomers with a periodic array of insulating voids, in which the voids collapse on themselves and form sharp cusps near their equators with respect to the direction of the

³The electrocreasing instability may be thought of as being related to the *ruga*-mechanics of purely mechanical substrates [e.g., 103].

applied electric field.

In the present work, we observe sharp cusps that form on solid-solid interfaces in dielectric elastomer composites – typically in cases involving high permittivity inclusions. To numerically simulate interfacial instabilities, time-dependence must be introduced, either through inertia or material time-dependence, since time-independent simulations will fail to converge at the onset of instability. Following previous works on surface instabilities in dielectric elastomers [100–102], we utilize a dynamic (but electrostatic) finite-element approach, replacing the equations of equilibrium (2.2.2) with the equations of motion in our nonlinear simulations. The equations of motion are integrated using an implicit, backward-Euler time-stepping procedure, and in order to approximate quasi-static conditions, a very slow loading rate is utilized in all situations. Using this implicit, dynamic approach, we are able to more robustly extract the onset of interfacial instability and resolve the post-instability response, while still ensuring that quasi-electrostatic conditions prevail. The identification of the onset of interfacial instability is guided by detecting a small jump in the kinetic energy at some point during quasi-electrostatic loading but manually determined by assessing deformation in the unit cell and checking for when surface cusps begin to form.

Comment on dielectric breakdown: Dielectric breakdown occurs when the electric field exceeds the dielectric strength of the material, mobilizing bound-charge and producing a path for electrical conduction – an irreversible material failure process in solid dielectrics. The consideration of dielectric breakdown is common in the analysis of monolithic dielectric elastomers [cf., e.g., 68, 99, 104] as well as layered dielectric elastomers, [cf., e.g., 70, 93]. However, dielectric strength is an inherent material property, and because of this, we do not include dielectric breakdown when constructing instability landscapes for dielectric elastomer composites in Section 3.3. Instead, since dielectric breakdown does remain an issue in the design of dielectric elastomer composites, we report maximum electric field magnitudes occurring in the matrix and the inclusion at the onset of instability for several cases in this chapter, which may be compared to the dielectric strength of candidate materials.

Comment on verification: We have subjected our numerical simulation capability to several verification tests. First – instabilities aside – as discussed in Chapter 2, our numerical simulation capability has been verified by reproducing analytical dispersion relations for electroelastic wave propagation in dielectric elastomer laminates (Fig. A.1) as well as dispersion relations in both square and hexagonal-lattice composites calculated using the plane-wave expansion technique (Figs. 2.2 and 2.3). Second, we ensured that our numerical simulation capability could reproduce predictions of purely mechanical instabilities in periodic solids reported in the literature – specifically, the microscopic instabilities and post-buckling response reported by Bertoldi and Boyce [44] for square lattices of voids and both the microscopic and macroscopic instabilities reported by Shim et al. [49], including their Bloch mode reconstructions and post-buckling supercell responses for square and hexagonal lattices of voids. Finally, we applied our numerical simulation capability to electromechanical instabilities in dielectric elastomer composites reported in the literature. Specifically, we reproduced the instabilities reported by Li and Landis [60] for dielectric elastomer composites with conducting holes and rigid conducting inclusions and confirmed the interfacial instabilities observed in their work. Additionally, we used our numerical simulation capability to reproduce the analytical pull-in, macroscopic, and microscopic instability predictions of Bertoldi and Gei [70] for infinite, incompressible, dielectric elastomer laminates.

3.3 Numerical simulations

In this section, we consider electromechanical instabilities in composites consisting of periodic lattices of aligned circular-cross-section fibers in a matrix and arranged in square and regular hexagonal arrays, shown in Figs. 2.1(a) and (b) and described in Chapter 2. The fibers and matrix are perfectly-bonded at their interfaces and are made of materials a and b, respectively. The volume fractions of the two phases are denoted by $\nu^{(a)}$ and $\nu^{(b)}$, and as a baseline, we consider the case of $\nu^{(a)} = \nu^{(b)} = 0.5$. We take both phases to be ideal-dielectric, Gent materials, constitutively described by the free-energy function (2.2.18) and (2.4.1), so that the material

parameters for each phase α consist of the ground-state shear modulus $G^{(\alpha)}$, the bulk modulus $K^{(\alpha)}$, the Gent locking parameter $I_m^{(\alpha)}$, and the dielectric permittivity $\epsilon^{(\alpha)}$. We take the bulk modulus to be the same in both phases, and in order to obtain nearly-incompressible conditions, we take $K^{(a)} = K^{(b)} = 10^3 \max(G^{(a)}, G^{(b)})$. For simplicity, we take the locking parameters of the phases, $I_m^{(a)} = I_m^{(b)} = 10$, to be equal and sufficiently large so that the electromechanical pull-in instability is not suppressed. Then, for composites of a given lattice structure and applied macroscopic electric-field direction, we explore the onset of instability across a parameter space spanning the two remaining material parameter contrasts: $G^{(a)}/G^{(b)}$ and $\epsilon^{(a)}/\epsilon^{(b)}$. We note that for some of the cases considered, none of the four instability types discussed in §3.2 occur. In these cases, deformation of the composite material is limited by the large-stretch chain-locking behavior introduced by the Gent model. We detect locking in our numerical simulations when the slope of the $|\bar{\mathbf{E}}_r| \sqrt{\bar{\epsilon}/\bar{G}}$ versus $\bar{\lambda}$ response exceeds 20.⁴

First, details regarding our electromechanical instability phase diagrams are introduced (§3.3.1) along with a high-level summary of the four main base cases considered in this chapter (§3.3.2). We take a closer look at the state at the onset of the pull-in, microscopic, macroscopic, and interfacial instabilities that appear in §3.3.3. We then consider several parametric studies (§3.3.4), in which we explore the effect of fiber volume fraction, large-stretch chain-locking behavior, mechanical prestress, prestretch, and confinement.

3.3.1 Electromechanical instability phase diagram

In this section, we introduce our process for constructing electromechanical instability phase diagrams – focusing first on the square-lattice composite electrically loaded perpendicular to the fibers and along the closest-packed, in-plane direction, i.e., along the $\mathbf{a}_{r1}$ -direction. We assume far-field, mechanically-traction-free boundary conditions so that all components of $\bar{\mathbf{T}}_r$ equal zero. As a result, quasi-electrostatic deformation proceeds under generalized plane-strain conditions, in

⁴The volume-averaged ground-state shear modulus and permittivity are defined as $\bar{G} = \nu^{(a)}G^{(a)} + \nu^{(b)}G^{(b)}$ and $\bar{\epsilon} = \nu^{(a)}\epsilon^{(a)} + \nu^{(b)}\epsilon^{(b)}$, respectively.

which $\bar{F}_{13} = \bar{F}_{31} = \bar{F}_{23} = \bar{F}_{32} = 0$ but $\bar{F}_{33}$ may differ from one, and since electrical loading is applied normal to a plane of symmetry, no macroscopic rotation occurs, so that $\bar{F}_{12} = \bar{F}_{21}$.

Our numerical simulations involve the following steps. First, we utilize a “charge-control” procedure, in which the macroscopic referential electric displacement $\bar{\mathbf{D}}_R$ is prescribed, so that numerical solutions may be obtained past the onset of the pull-in instability. Quasi-electrostatic deformation is then computed in small charge increments targeting macroscopic stretch increments of $\Delta\bar{\lambda} = 0.01$, where $\bar{\lambda} = \sqrt{\bar{\mathbf{m}} \cdot \bar{\mathbf{F}}^T \bar{\mathbf{F}} \bar{\mathbf{m}}}$ is measured along the direction of electrical loading. Second, at each increment, eigenfrequencies for potential microscopic modes of bifurcation ω^- are computed using the Bloch wave vectors illustrated in Fig. 3.1. Similarly, eigenfrequencies for potential macroscopic modes of bifurcation ω^+ are also computed using long wavelength Bloch wave vectors as described in §3.2. The simulation results are then post-processed to detect pull-in, microscopic, and macroscopic instabilities as described in §3.2 or, absent all instabilities, the point of chain-locking. For interfacial instability, in almost all cases, the onset of interfacial instability coincides with the failure of the fully electrostatic numerical simulation – i.e., the lack of convergence of the nonlinear Newton-Raphson solver. Therefore, to robustly address interfacial instability, for each case, a corresponding implicit dynamic simulation is carried out at a very slow electrical loading rate in order to detect the onset of interfacial instability. Both the static and dynamic simulations are used to construct the instability landscape for each analysis discussed in this chapter.

Regarding the finite-element mesh, the unit cells are meshed using a single layer of three-dimensional, eight-node hexahedral elements with one element through the thickness. The same mesh is used for both static and dynamic versions of the simulations. Unless otherwise noted, all results presented in this section utilize a mesh resolution of $0.05a_R$, where $a_R = |\mathbf{a}_{R1}| = |\mathbf{a}_{R2}|$.

In Fig. 3.2, we introduce the electromechanical instability phase diagram for the square-lattice composite electrically loaded perpendicular to the fibers and along the closest-packed, in-plane direction – for which the unit-cell is shown in the right central panel of Fig. 3.2. Stability is explored across a broad range of practical values of the ground-state shear modulus contrast $G^{(a)}/G^{(b)}$ and permittivity contrast $\epsilon^{(a)}/\epsilon^{(b)}$. In particular, we consider the following values for

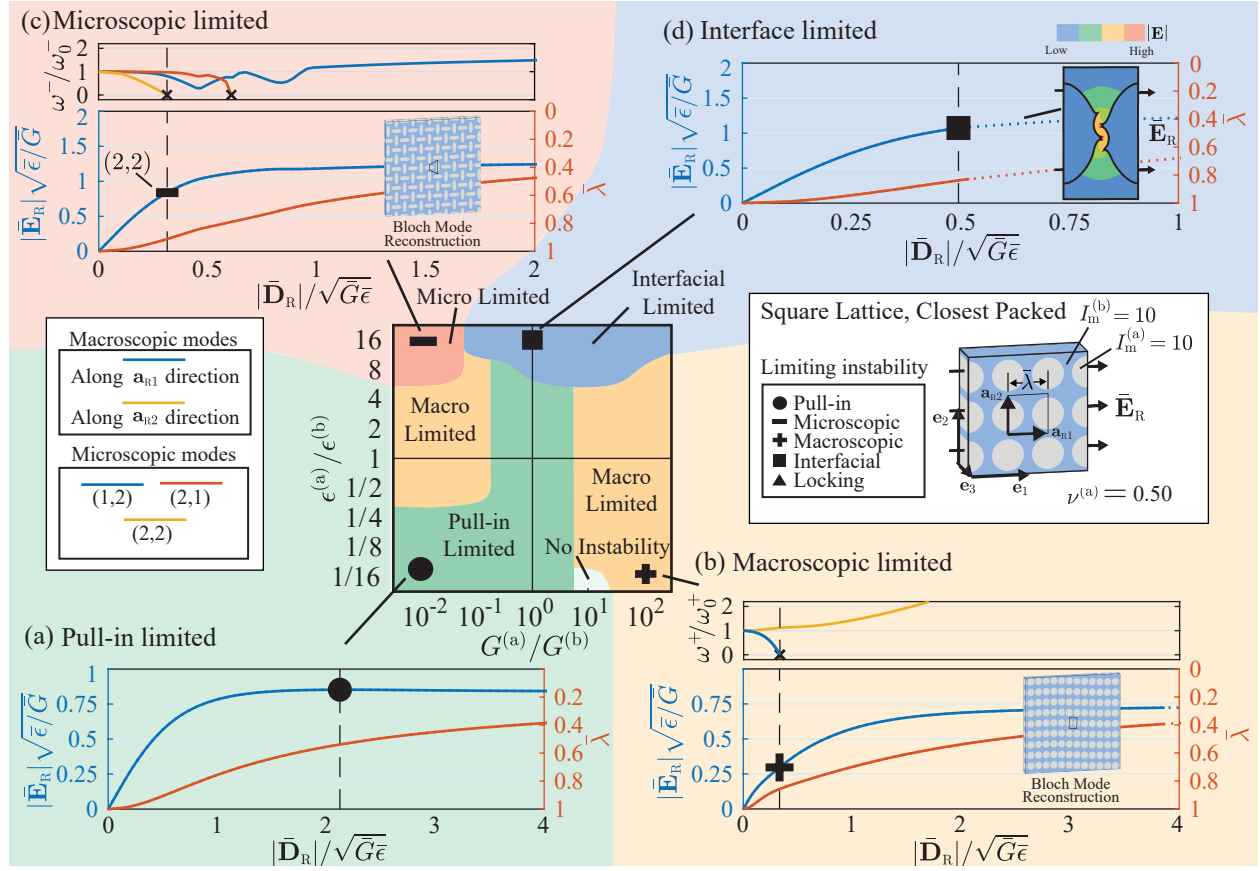


Figure 3.2: Electromechanical instability phase diagram for the square-lattice composite made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$ and electrically loaded perpendicular to the fibers along the closest-packed direction. Shaded regions denote the type of instability: (a) pull-in, (b) macroscopic, (c) microscopic, and (d) interfacial instability. In all four cases, the macroscopic referential electric field magnitude and the macroscopic stretch along the electrical loading direction are plotted as a function of the macroscopic referential electric displacement magnitude. In (b) and (c), eigenfrequencies for potential (b) macroscopic modes and (c) microscopic modes of bifurcation are plotted as a function of the macroscopic referential electric displacement magnitude. In (d), dotted lines correspond to implicit dynamic simulations that proceed beyond the onset of interfacial instability. Mesh density: $0.05a_R$.

the contrasts:

$$G^{(a)}/G^{(b)} = [1/100, 1/31.6, 1/10, 1/3.16, 1, 3.16, 10, 31.6, 100],$$

$$\epsilon^{(a)}/\epsilon^{(b)} = [1/16, 1/8, 1/4, 1/2, 1, 2, 4, 8, 16].$$

Numerical simulations are carried out for each combination of $G^{(a)}/G^{(b)}$ and $\epsilon^{(a)}/\epsilon^{(b)}$, and the

type of instability that first occurs – the limiting instability – is indicated in the phase diagram in the central panel of Fig. 3.2. Representative simulation results for cases corresponding to each of the four instability types are shown in Fig. 3.2 for (a) pull-in, (b) macroscopic, (c) microscopic, and (d) interfacial instability. In all four cases, the normalized macroscopic referential electric field magnitude $|\bar{\mathbf{E}}_{\text{R}}|/\sqrt{\bar{\epsilon}/\bar{G}}$ and the macroscopic stretch along the electrical loading direction $\bar{\lambda}$ are plotted as a function of the normalized macroscopic referential electric displacement magnitude $|\bar{\mathbf{D}}_{\text{R}}|/\sqrt{\bar{G}\bar{\epsilon}}$.

In Fig. 3.2(a), an example of a pull-in instability limited case is illustrated for contrasts of $G^{(a)}/G^{(b)} = 1/100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1/16$. A peak in the $|\bar{\mathbf{E}}_{\text{R}}|/\sqrt{\bar{\epsilon}/\bar{G}}$ versus $|\bar{\mathbf{D}}_{\text{R}}|/\sqrt{\bar{G}\bar{\epsilon}}$ response indicates the onset of pull-in, which is identified by the marker $\bullet$. Figure 3.2(b) shows an example of a macroscopic instability limited case for contrasts of $G^{(a)}/G^{(b)} = 100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1/16$. In this case, the eigenfrequencies for potential macroscopic modes of bifurcation ω^+ along the $\mathbf{a}_{\text{R}1}$ and $\mathbf{a}_{\text{R}2}$ directions – each normalized by their respective values in the undeformed configuration ω_0^+ – are plotted as a function of $|\bar{\mathbf{D}}_{\text{R}}|/\sqrt{\bar{G}\bar{\epsilon}}$. At a critical value of electrical loading, the eigenfrequency corresponding to the $\mathbf{a}_{\text{R}1}$ direction – i.e., the direction aligned with the electrical loading direction $\bar{\mathbf{m}}$ – goes to zero, indicating the onset of macroscopic instability, which is denoted by the marker $+$. The corresponding mode shape of macroscopic instability is shown in the inset of Fig. 3.2(b).⁵ An example of a microscopic instability limited case for contrasts of $G^{(a)}/G^{(b)} = 1/100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 16$ is illustrated in Fig. 3.2(c). The eigenfrequencies for potential microscopic modes of bifurcation ω^- – each normalized by their respective values in the undeformed configuration ω_0^- – are plotted as a function of $|\bar{\mathbf{D}}_{\text{R}}|/\sqrt{\bar{G}\bar{\epsilon}}$. While we consider all nine potential microscopic modes illustrated in Fig. 3.1, for clarity, only the eigenfrequencies for three of these modes are included in Fig. 3.2(c) – $(m_1 = 1, m_2 = 2)$, $(m_1 = 2, m_2 = 1)$, and $(m_1 = 2, m_2 = 2)$. At a critical value of electrical loading for this case, the eigenfrequency corresponding to the microscopic mode for $(m_1 = 2, m_2 = 2)$ goes to zero, indicating the onset of microscopic instability, which is denoted

⁵While the simulations used to detect the onset of macroscopic instability use a Bloch wave vector magnitude of $|\mathbf{k}|a_{\text{R}}/2\pi = 0.001$, for display purposes, the mode shapes of macroscopic instability pictured in the figures throughout this chapter utilize a wave vector magnitude of $|\mathbf{k}|a_{\text{R}}/2\pi = 0.05$ – corresponding to a wavelength of $20a_{\text{R}}$.

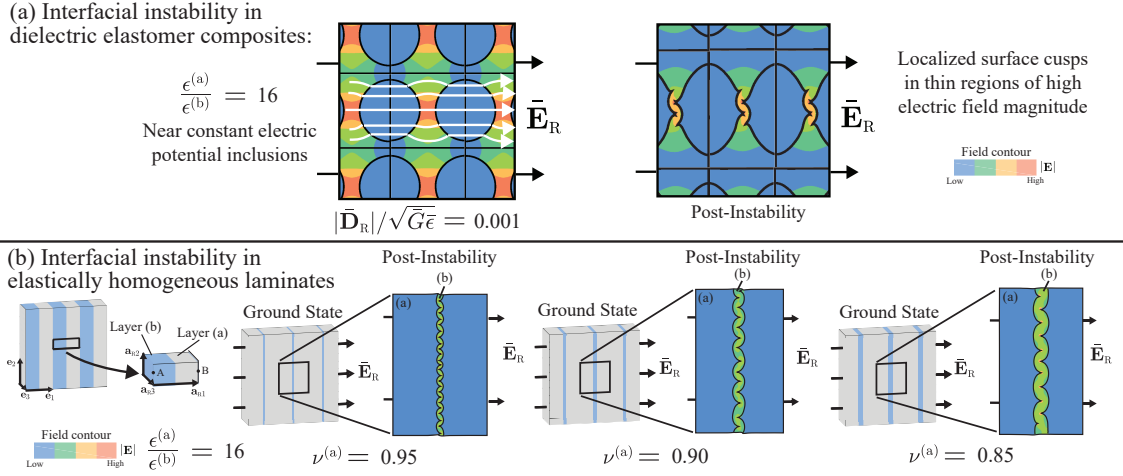


Figure 3.3: Illustration of the morphology of interfacial instability in dielectric elastomer composites. (a) Interfacial instability in the square-lattice composite made from Gent materials and electrically loaded perpendicular to the fibers along the closest-packed direction for the case of high electric permittivity contrast but no ground-state shear modulus contrast. (b) A demonstration of interfacial instability in dielectric elastomer laminates made from neo-Hookean materials and electrically loaded perpendicular to the plane of lamination for a high electric permittivity contrast of $\epsilon^{(a)}/\epsilon^{(b)} = 16$ but no ground-state shear modulus contrast and three different volume fractions of the high-permittivity phase.

by the marker —. The corresponding mode shape of microscopic instability is shown in the inset of Fig. 3.2(c). Finally, Fig. 3.2(d) shows an example of an interfacial instability limited case, corresponding to contrasts of $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 16$. At a critical value of electrical loading, cusp-like features form at the interface between the two phases, indicating the onset of interfacial instability, which is denoted by the marker ■. At the onset of interfacial instability, the fully electrostatic numerical simulation fails to converge, and corresponding implicit dynamic simulations – carried out at a very slow electrical loading rate in order to approximate quasi-electrostatic conditions – allow the response after the onset of interfacial instability to be calculated, which is indicated by the dotted lines in Fig. 3.2(d). A snapshot of the deformed microstructure after the onset of interfacial instability is shown in the inset, illustrating the cusp-like morphology.

Comment on interfacial instabilities: Further discussion of interfacial instabilities is warranted. For large permittivity contrast $\epsilon^{(a)}/\epsilon^{(b)}$, we encounter interfacial instability over a wide range of

ground-state shear modulus contrasts, as shown in the phase diagram of Fig. 3.2. For contrasts of $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 16$, the deformed microstructures during the initial stages of electrical loading ($|\bar{\mathbf{D}}_r|/\sqrt{G\bar{\epsilon}} = 0.001$) and after the onset of interfacial instability are shown in Fig. 3.3. Contours of the spatial electric field magnitude $|\mathbf{E}|$ along with electric field lines are plotted on the deformed microstructures.⁶ Inside the high permittivity inclusions, the electric field is nearly zero and the electric potential is nearly constant, which leads to very high electric field magnitudes in the matrix-material ligaments between the high-permittivity inclusions. Ultimately this high-magnitude electric field acting normal to the interface causes localized cusps to form along the interface. We also explored this phenomenon in dielectric elastomer laminates subjected to a macroscopic electric field perpendicular to the plane of lamination and zero macroscopic stress and observed analogous behavior, as shown in Fig. 3.3(b), which has not been previously reported. In particular, laminates with a high permittivity contrast of $\epsilon^{(a)}/\epsilon^{(b)} = 16$ but no shear modulus contrast $G^{(a)}/G^{(b)} = 1$ are considered for three different volume fractions of the high permittivity phase: $v^{(a)} = 0.95$, $v^{(a)} = 0.90$, and $v^{(a)} = 0.85$. These cases involve very high electric fields that develop in the thin layers of the low permittivity phase – similar to the thin matrix-material ligaments in Fig. 3.3(a). In all three laminate cases, cusp-like interfacial instability is encountered beyond a critical value of the electrical loading, as illustrated in Fig. 3.3(b). One may interpret these interfacial instabilities as an extension of the classical electrocreasing instability for a dielectric elastomer confined between one rigid and one compliant electrode [83, 84]. The electromechanical instabilities encountered in dielectric elastomer laminates are discussed in further detail in Appendix A.2.

3.3.2 Effect of lattice and electrical loading direction

In this section, we expand the results of the preceding section to consider both square- and hexagonal-lattice composites electrically loaded perpendicular to the fibers along both the least- and

⁶Throughout this chapter, contour plots of the spatial electric field magnitude are shown using a relative scale, in which the highest electric field magnitude for a given case is displayed as red, and zero electric field magnitude is displayed as blue.

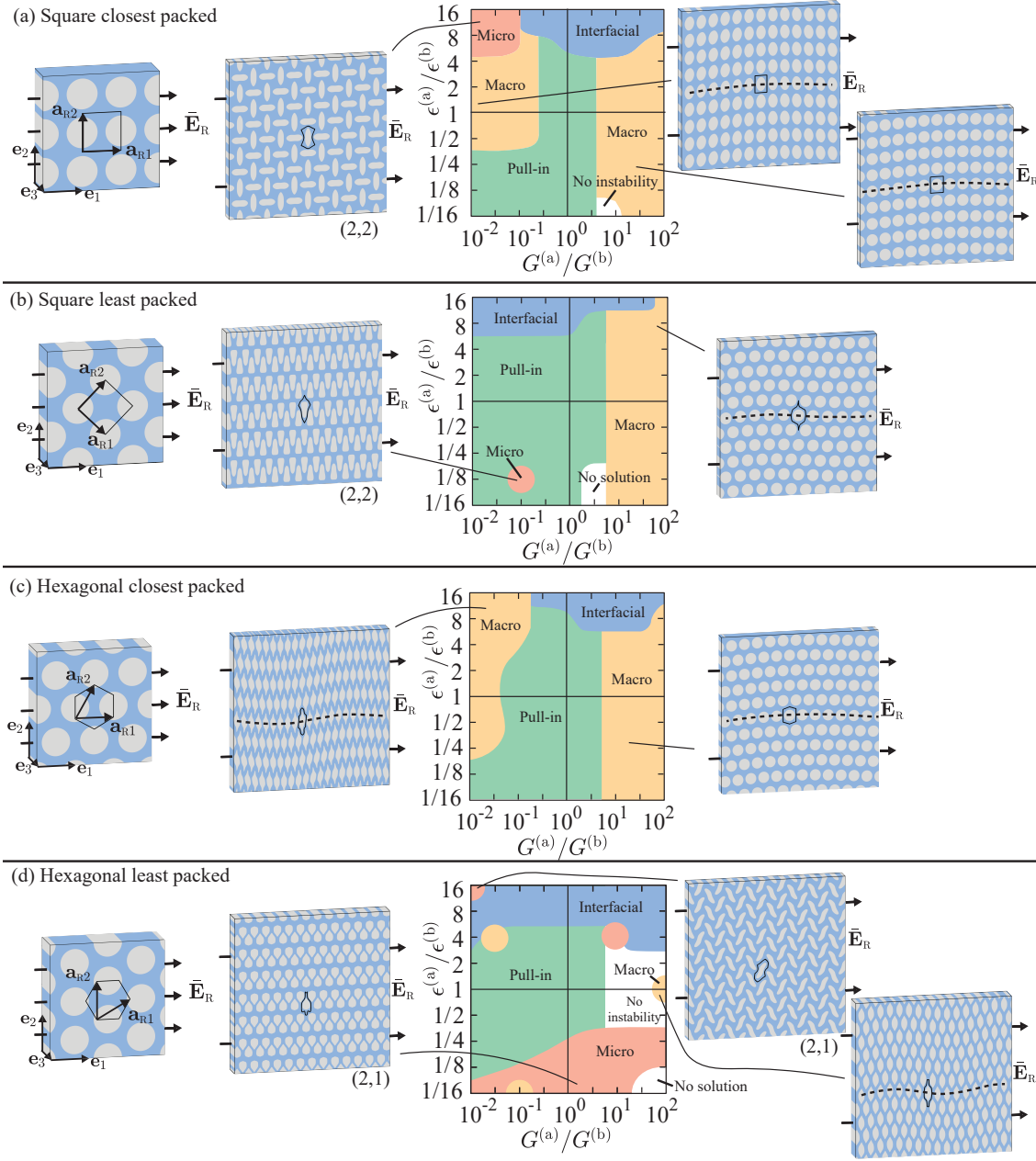


Figure 3.4: Electromechanical instability phase diagrams for the square-lattice composite made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$ and electrically loaded perpendicular to the fibers along (a) the closest-packed direction and (b) the least-packed direction and the hexagonal-lattice composite electrically loaded along (c) the closest-packed direction and (d) the least-packed direction. Bloch mode shape reconstructions are shown for selected cases to highlight representative modes of microscopic and macroscopic electromechanical instability. For examples of microscopic instability, the corresponding values of (m_1, m_2) are shown.

closest-packed directions. In Fig. 3.4, we present the electromechanical instability phase diagrams as a function of $G^{(a)}/G^{(b)}$ and $\epsilon^{(a)}/\epsilon^{(b)}$ for the four cases considered in this chapter – i.e., (a) the square-lattice composite loaded along the closest-packed direction, (b) the square-lattice composite loaded along the least-packed direction, (c) the hexagonal-lattice composite loaded along the closest packed direction, and (d) the hexagonal-lattice composite loaded along the least packed direction. In addition, Bloch mode reconstructions for several illustrative cases in each of the phase spaces are shown. We note that for all cases shown in Fig. 3.4, the wave-vector direction for macroscopic instability coincides with the direction of electrical loading $\bar{\mathbf{m}}$.

Based on the results shown in Fig. 3.4, we make several broad observations:

- Instances of all four types of instability appear in our numerical simulations. Moreover, for $I_m^{(a)} = I_m^{(b)} = 10$ and $\nu^{(a)} = \nu^{(b)} = 0.5$, most cases are limited by instability with only a few instances of composites that may be electrically loaded up until the point of locking. Therefore, it is indeed important to consider all instability types when assessing the performance of a given dielectric elastomer composite, and it is clear that the type of electromechanical instability may be tailored by controlling the contrasts $G^{(a)}/G^{(b)}$ and $\epsilon^{(a)}/\epsilon^{(b)}$.
- In all cases, in the absence of material parameter contrasts – i.e., $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1$ – the simple case of a homogeneous dielectric elastomer is attained, which is limited by the pull-in instability.
- In general, for large ground-state shear modulus contrast – i.e., stiff inclusions with $G^{(a)}/G^{(b)} \gtrsim 10$ – the composite materials display a tendency towards macroscopic instability. The notable exception is the hexagonal-lattice composite electrically loaded along the least-packed direction (Fig. 3.4(d)), which displays a comparable resistance to macroscopic instability.
- For small ground-state shear modulus contrasts – i.e., soft inclusions with $G^{(a)}/G^{(b)} \lesssim 0.01$ – the cases of electrical loading along the closest-packed direction are limited by macroscopic instabilities for both square- and hexagonal-lattice composites. However, in this regime, the

cases of electrical loading along the least-packed direction are limited by pull-in instability for both lattices.

- For cases with high electric permittivity contrast – i.e., $\epsilon^{(a)}/\epsilon^{(b)} \gtrsim 10$ – interfacial instability arises in all four cases considered in Fig. 3.4. In all of these cases, we observe a situation similar to that discussed in the preceding section – i.e., very high electric field magnitudes arising in thin matrix-material ligaments between high-permittivity inclusions.
- Instances of microscopic instability are observed in all cases except the case of the hexagonal-lattice composite electrically loaded along the closest-packed direction. For periodic materials comprised of planar lattices of circular heterogeneities, microscopic instability has most commonly been observed for the case of voids [e.g., 105], so the observation of microscopic instability in a solid-solid composite is notable.
- Microscopic instability only arises in situations in which a permittivity mismatch is present. Moreover, based on additional simulations (not shown), microscopic instability does not appear in any of these lattices for purely mechanical far-field loading – i.e., $\bar{\mathbf{E}}_R = \mathbf{0}$ but $\bar{\mathbf{T}}_R \neq \mathbf{0}$. That is to say, electrical loading in the presence of a permittivity mismatch is required to induce microscopic instability in these nearly-incompressible solid-solid composites.
- A sizable regime of no instabilities – a locking-limited regime – appears for the hexagonal-lattice composite electrically loaded along the least-packed direction for stiff inclusions $G^{(a)}/G^{(b)} \gtrsim 10$ with similar permittivity as the matrix $1/4 \lesssim \epsilon^{(a)}/\epsilon^{(b)} \lesssim 4$. It appears that this combination of lattice and electrical loading direction is particularly resistant to instability, and for this reason, we largely focused on cases within this regime in the parametric studies of Chapter 2.4.5.

Comment on numerical challenges: For small electric permittivity contrast $\epsilon^{(a)}/\epsilon^{(b)}$, we often encounter numerical difficulties for cases involving very large deformation, which are not preceded by pull-in, microscopic, or macroscopic instability. In particular, at a critical value of the electrical

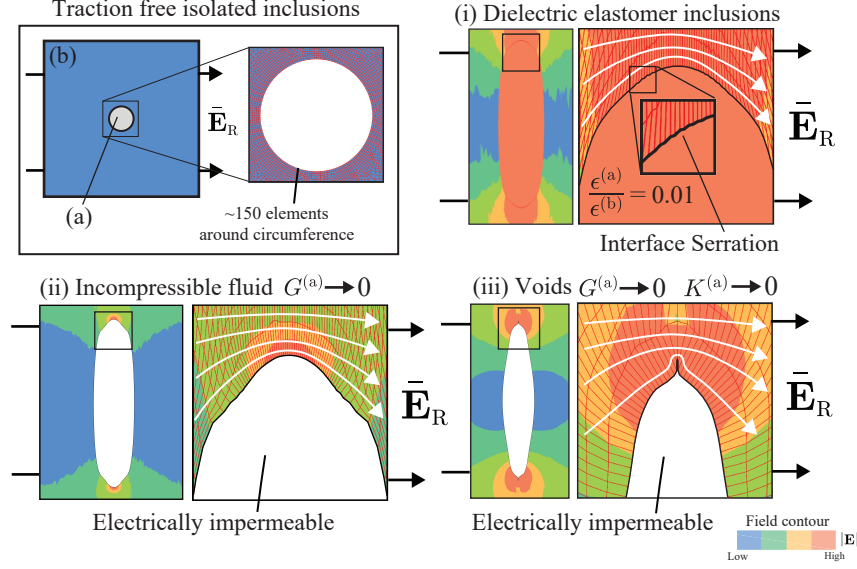


Figure 3.5: Illustration of a numerical issue that can arise in situations involving low permittivity contrast, $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow 0$, and very large deformations. Serrated, mesh-level distortion of the interface at the equator of the inclusion with respect to the direction of electrical loading for (i) a dielectric elastomer inclusion with $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 0.01$ and (ii) an incompressible fluid-filled, electrically-impermeable inclusion. (iii) Electro-pinching for an electrically-impermeable void.

loading, we observe serrated mesh-level distortion of the interface in the regions of the inclusion equator with respect to the direction of electrical loading. This issue is not ameliorated by mesh refinement, since interface serration remains on the scale of the mesh size. Upon further exploration, we observe that even isolated, low-permittivity inclusions display this issue, which is illustrated in Fig. 3.5(i). We rationalize this effect as an electro-pinching event, in which the inclusion interface wants to pinch near its equator, as shown for the case of electrically impermeable voids in Fig. 3.5(iii),⁷ but is frustrated by the incompressible inclusion. Even in the case of an incompressible fluid inclusion,⁸ shown in Fig. 3.5(ii), interface pinching is frustrated by the incompressibility constraint, and mesh-scale serration occurs, as in the solid-solid composite case. Since this effect appears to be a numerical issue and not a physical instability, in the phase diagrams of Figs. 3.2, we denote the cases that exhibit this behavior by “no solution.”

⁷This result for an electrically impermeable void was also reported by Li and Landis [60].

⁸Modeled using Abaqus’ built-in fluid cavity approach.

3.3.3 State at the onset of electromechanical instability

In this section, we expand the results of the preceding section and report on additional quantities of interest at the onset of instability. Specifically, we present contour plots of (i) the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, (ii) the normalized macroscopic electric field, $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/\bar{G}}$, (iii) the normalized macroscopic electric displacement, $|\bar{\mathbf{D}}_R|/\sqrt{\bar{G}\bar{\epsilon}}$, (iv) the maximum electric field magnitude inside the inclusion, $\max|\mathbf{E}^{(a)}|/\sqrt{\bar{\epsilon}/\bar{G}}$, and (v) the maximum electric field magnitude inside the matrix, $\max|\mathbf{E}^{(b)}|/\sqrt{\bar{\epsilon}/\bar{G}}$, as a function of the ground-state shear modulus contrast, $G^{(a)}/G^{(b)}$, and the electric permittivity contrast, $\epsilon^{(a)}/\epsilon^{(b)}$ – all at the onset of instability. The results are shown in Figs. 3.6 and 3.7 for the square- and hexagonal-lattice composites, respectively, and results for electrical loading perpendicular to the fibers along the closest-packed and least-packed directions are shown in parts (a) and (b) of each figure. White regions of the contour plots represent values outside the range of the contour-plot scale and correspond to cases exhibiting near-locking behavior – in which the behavior approaches that of a near-rigid dielectric and electrical loading will be limited by dielectric breakdown instead of instability. The markers (●, –, +, ■), introduced in §3.3.1 and referenced in figure legends throughout this chapter, are included in part (i) of each figure and indicate the limiting instability type for each case: pull-in (●), microscopic instability (–), macroscopic instability (+), and interfacial instability (■). In addition, the marker (▲) denotes cases in which large-stretch locking is observed. Markers are omitted for cases exhibiting the numerical issue highlighted in the preceding section; however, for these cases, we do include the values of $\bar{\lambda}$, $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/\bar{G}}$, $|\bar{\mathbf{D}}_R|/\sqrt{\bar{G}\bar{\epsilon}}$, $\max|\mathbf{E}^{(a)}|/\sqrt{\bar{\epsilon}/\bar{G}}$, and $\max|\mathbf{E}^{(b)}|/\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset interface serration in the numerical simulations in the contour plots.

The images along the top row of each figure show the deformed composite structures and contours of the spatial electric field magnitude $|\mathbf{E}|$ at the onset of instability for extreme cases of the parameter set – specifically, (A) $G^{(a)}/G^{(b)} = 1/100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1/16$, (B) $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1/16$, (C) $G^{(a)}/G^{(b)} = 100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1/16$, (D) $G^{(a)}/G^{(b)} = 1/100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 16$, (E) $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 16$, and (F) $G^{(a)}/G^{(b)} = 100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 16$.

Based on the results shown in Figs. 3.6 and 3.7, we make several broad observations:

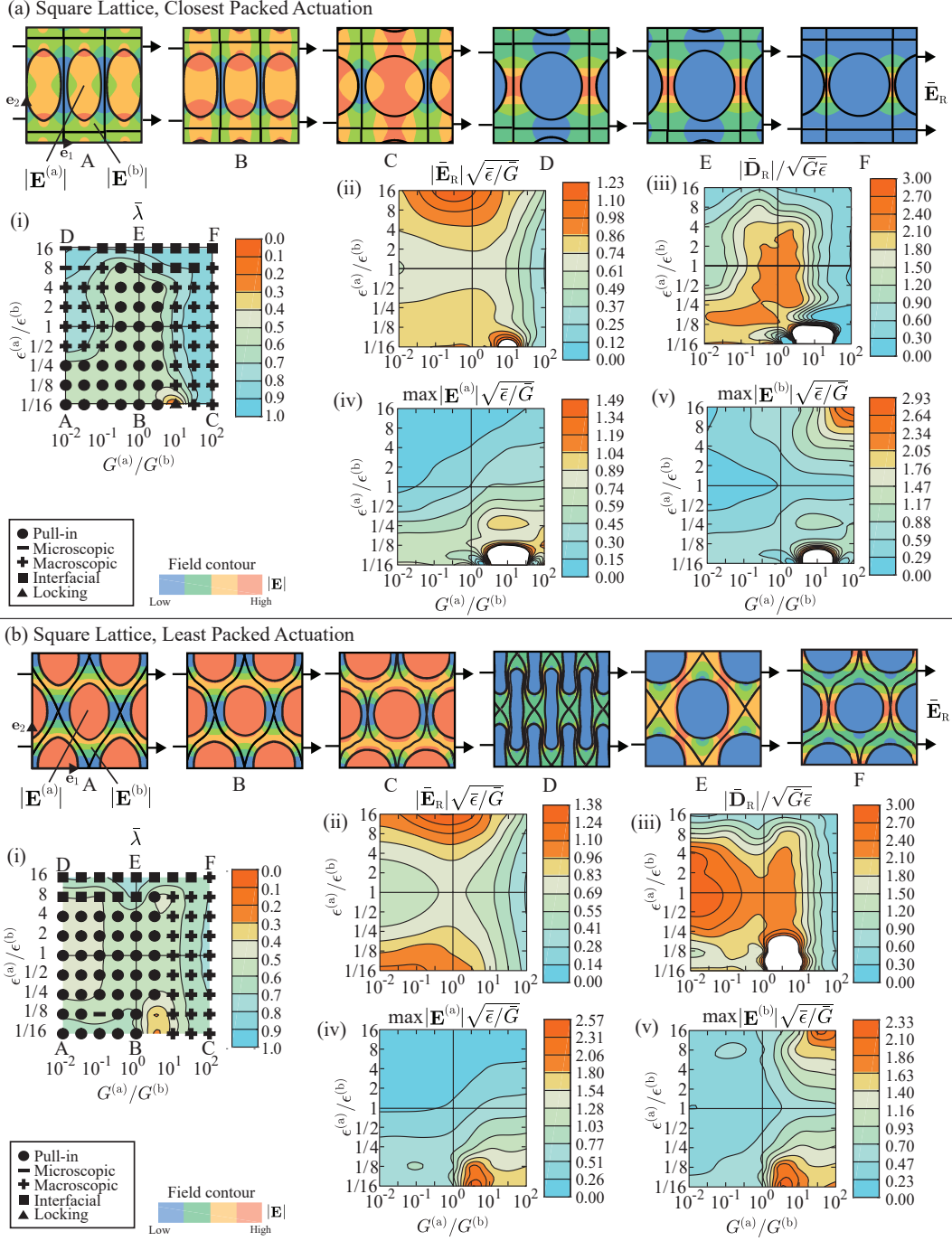


Figure 3.6: State at the onset of electromechanical instability for the square-lattice composite electrically loaded along (a) the closest-packed and (b) the least-packed directions. Contour plots are shown for (i) macroscopic stretch, (ii) normalized macroscopic electric field, (iii) normalized macroscopic electric displacement, and the maximum normalized electric field magnitude in (iv) the inclusion and (v) the matrix. Top row: Deformed composite structures and contours of the spatial electric field magnitude at the onset of instability for selected cases. Mesh density: $0.05a_R$.

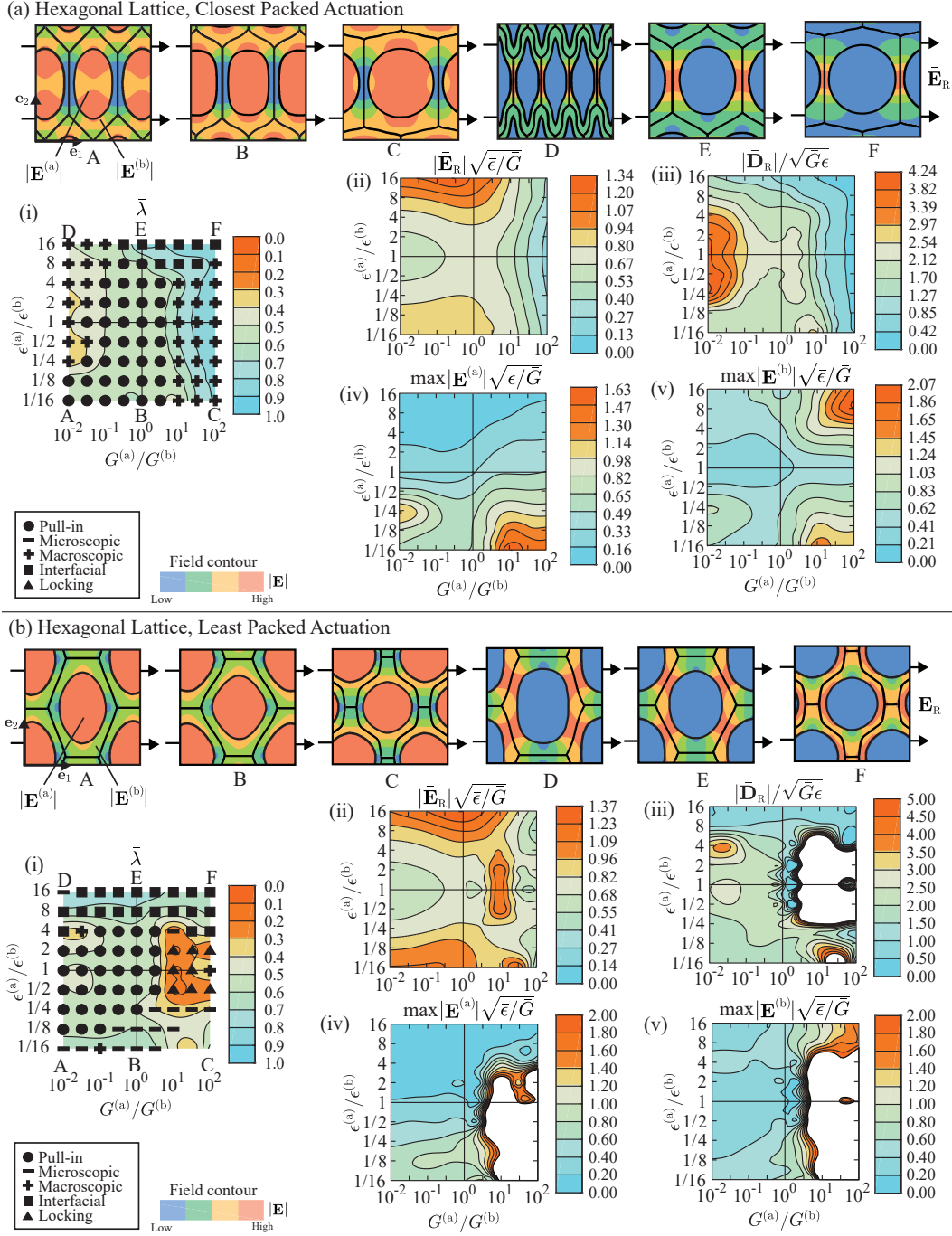


Figure 3.7: State at the onset of electromechanical instability for the hexagonal-lattice composite electrically loaded along (a) the closest-packed and (b) the least-packed directions. Contour plots are shown for (i) macroscopic stretch, (ii) normalized macroscopic electric field, (iii) normalized macroscopic electric displacement, and the maximum normalized electric field magnitude in (iv) the inclusion and (v) the matrix. Top row: Deformed composite structures and contours of the spatial electric field magnitude at the onset of instability for selected cases. Mesh density: $0.05a_R$.

- For soft inclusions, the permittivity contrast $\epsilon^{(a)}/\epsilon^{(b)}$ has a significant effect on the deformed shape. For example, to see this, compare the deformed shapes for cases (A) and (D) in Figs. 3.6 and 3.7.
- In contrast, for stiff inclusions, the permittivity contrast has a less strong effect on the deformed shape. For example, to see this, compare the deformed shapes for cases (C) and (F) in Figs. 3.6 and 3.7.
- For cases involving high permittivity inclusions – i.e., in the limit $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$ – the electric potential inside the inclusions is nearly-constant. Consequently, the electric field inside the inclusions is nearly-zero, and the maximum electric field magnitude in the matrix is very high. These high electric field magnitudes can lead to interfacial instabilities (■) at the matrix-inclusion interface.
- Conversely, for cases involving low permittivity inclusions – i.e., in the limit $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow 0$ – the electric field magnitude in the inclusions is higher than that in the matrix. However, because the matrix phase is continuous, a state of nearly-zero electric field and nearly-constant electric potential is never attained in the matrix. Since localized regions of very high electric field magnitude do not arise in these cases, interfacial instability does not occur.

3.3.4 Parametric studies

With a baseline understanding of the instability landscape in dielectric elastomer composites established, in this section, we explore the roles of several additional effects – namely, fiber volume fraction, large-stretch chain-locking behavior, mechanical prestress, and prestretch and confinement.

Volume fraction

The fiber volume fraction has an important effect on the instability landscape. Here, we consider fiber volume fractions of $v^{(a)} = 1/4$ and $v^{(a)} = 2/3$, and the instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability are shown in Figs 3.8(a) and (b), respectively.

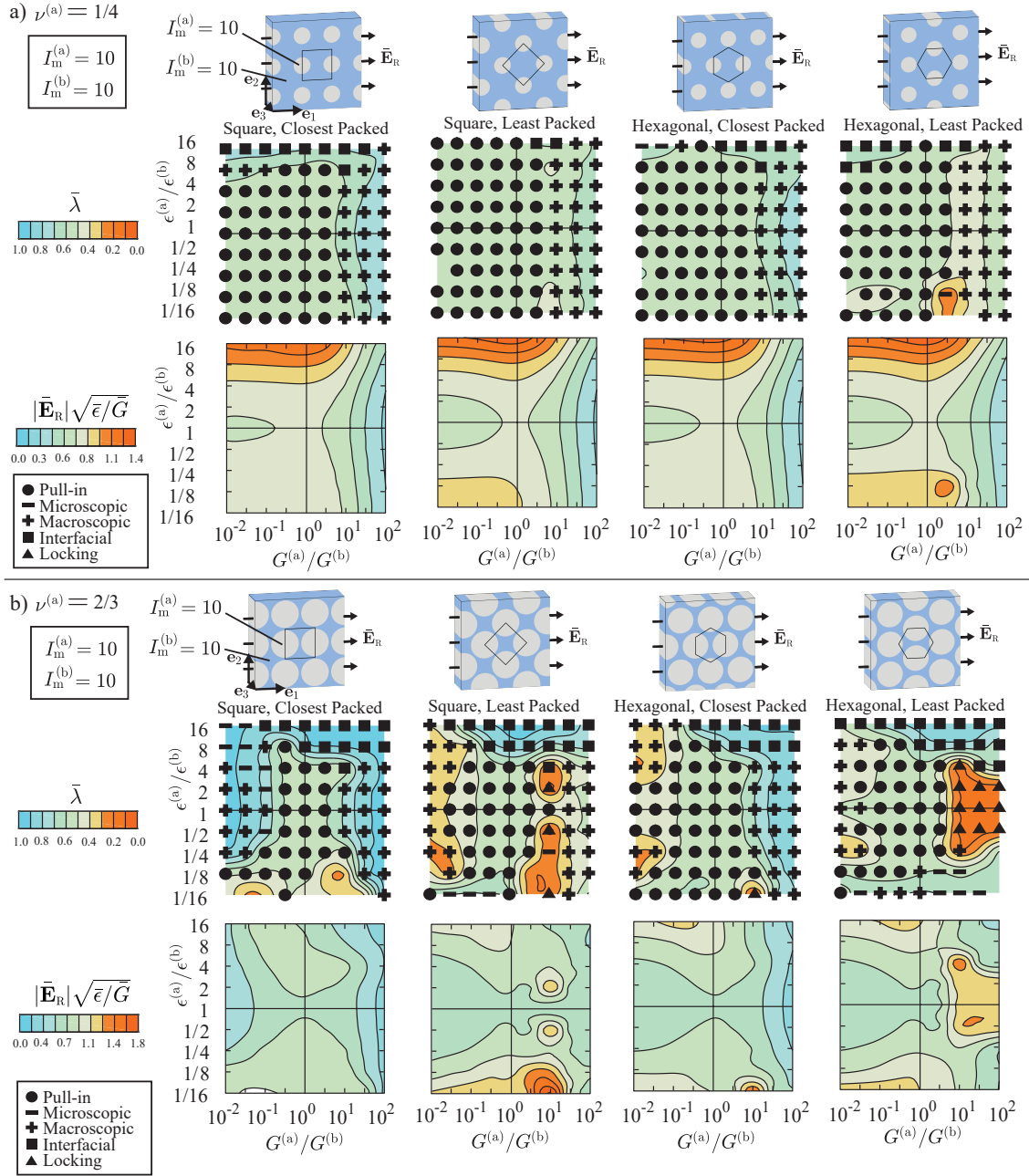


Figure 3.8: The effect of fiber volume fraction – (a) $\nu^{(a)} = 1/4$ and (b) $\nu^{(a)} = 2/3$ – on the electromechanical instability phase diagrams and contour plots of the macroscopic stretch, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: (a) $0.05a_R$ and (b) $0.03a_R$.

For each volume fraction, all four combinations of lattice geometry and electric loading direction are pictured. We note that the finite-element mesh density for the $v^{(a)} = 2/3$ case was increased to $0.03a_R$ to adequately resolve the thin ligaments of matrix material. We observe the following.

- Decreasing the fiber volume fraction leads to the pull-in instability occurring first over a wide range of the material-property-contrast phase space. Furthermore, for cases of pull-in, the macroscopic stretch at the onset of instability, $\bar{\lambda}$, remains relatively close to the value for a homogeneous dielectric elastomer – namely, $\bar{\lambda} = 0.58$.
- While pull-in is the dominant mode of instability for low fiber volume fraction, macroscopic instability still occurs when the inclusions are sufficiently stiff. Also, interfacial instability can still occur when the permittivity of the inclusions is sufficiently high.
- For high fiber volume fraction, the instability phase diagram is more complex, and all types of instability are encountered. Furthermore, as previously noted for the case of $v^{(a)} = 0.5$, for the hexagonal-lattice composite electrically loaded along the least-packed direction, instability may be suppressed for some combinations of material property contrasts.
- For high fiber volume fraction, the quantity of interfacial-instability-limited cases increases, which is due to the very high electric field magnitudes that occur in the thin ligaments of matrix material between inclusions.

To further explore the effect fiber volume fraction has on interfacial instability, we consider the elastically homogeneous case, $G^{(a)}/G^{(b)} = 1$, with a high permittivity contrast $\epsilon^{(a)}/\epsilon^{(b)} = 16$. The macroscopic stretch at the onset of instability is plotted as a function of fiber volume fraction over the range $v^{(a)} = 0.05$ to $v^{(a)} = 0.7$ in Fig. 3.9 with the instability type indicated by marker type. In all combinations of lattice geometry and electrical loading direction, pull-in dominates for small fiber volume fractions while interfacial instability dominates for large volume fractions. For the cases of pull-in, as $v^{(a)}$ decreases, $\bar{\lambda}$ approaches the value corresponding to a homogeneous dielectric elastomer of $\bar{\lambda} = 0.58$. The deformed composite microstructures after the onset of interfacial

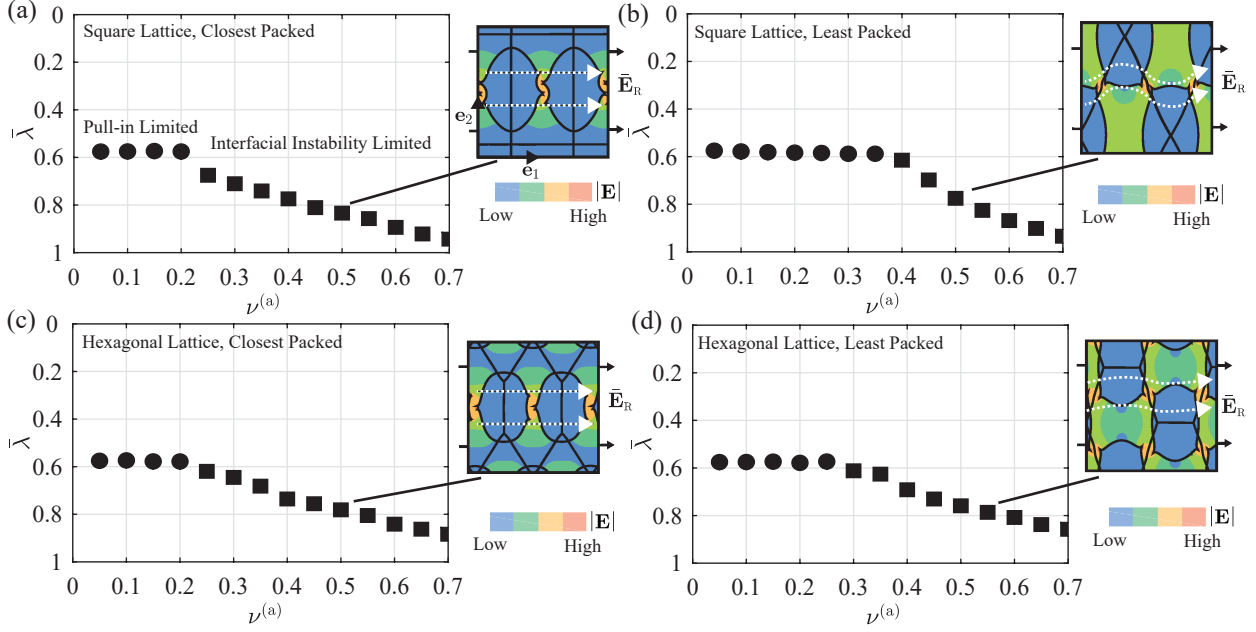


Figure 3.9: The macroscopic stretch at the onset of instability, $\bar{\lambda}$, as a function of fiber volume fraction, $\nu^{(a)}$, for the elastically homogeneous case, $G^{(a)}/G^{(b)} = 1$, with a high permittivity contrast, $\epsilon^{(a)}/\epsilon^{(b)} = 16$, for the square-lattice composite electrically loaded along the (a) closest-packed direction and (b) least-packed direction and similarly for the hexagonal-lattice composite (c,d). Snapshots of the deformed composite microstructures after the onset of interfacial instability are shown for $\nu^{(a)} = 0.50$ in each case. Mesh density: $0.05a_R$.

instability are shown for $\nu^{(a)} = 0.50$ for each case with white dotted lines roughly indicating the direction of the electric field – illustrating that interfacial instabilities do not necessarily occur along the macroscopic electrical loading direction, instead occurring where the inclusions are closest to each other.

Chain-locking behavior

Motivated by the work of Getz et al. [55], who illustrated the effect of large-stretch chain-locking behavior on electroelastic wave propagation in dielectric elastomer composites, we explore the effect of the Gent locking parameter on the instability landscape. To this point, we have focused on the $I_m^{(a)} = I_m^{(b)} = 10$ case. In this section, we consider the case of neo-Hookean inclusions, $I_m^{(a)} \rightarrow \infty$, in a rapidly-stiffening Gent matrix with $I_m^{(b)} = 5$ and vice versa – i.e., $I_m^{(a)} = 5$ with $I_m^{(b)} \rightarrow \infty$ – both shown in Fig. 3.10 with the results organized as in Fig. 3.8. Immediately, we notice a significant

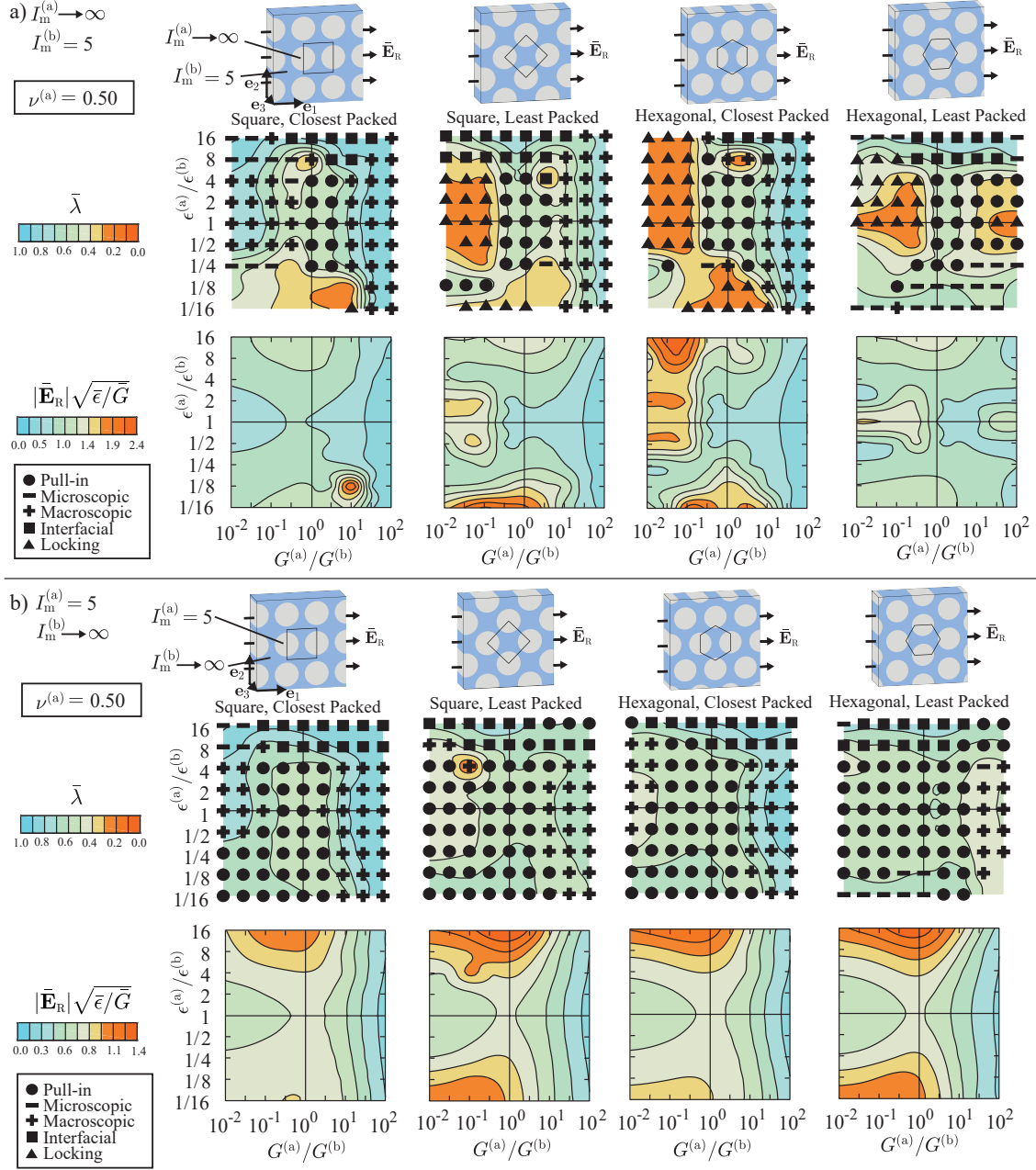


Figure 3.10: The effect of Gent locking parameter – (a) $I_m^{(a)} \rightarrow \infty$ and $I_m^{(b)} = 5$ and (b) $I_m^{(a)} = 5$ and $I_m^{(b)} \rightarrow \infty$ – on the electromechanical instability phase diagrams and contour plots of the macroscopic stretch, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for composites made from Gent materials. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

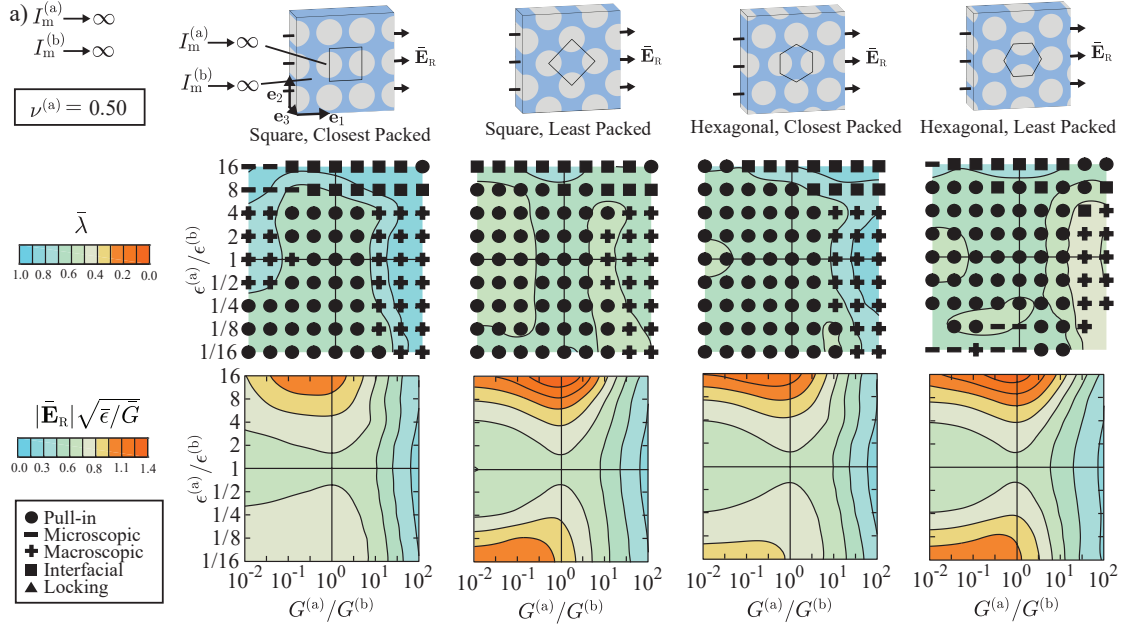


Figure 3.11: Electromechanical instability phase diagrams and contour plots of the macroscopic stretch, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/G}$, at the onset of instability for composites made from Neo-Hookean materials – $I_m^{(a)} \rightarrow \infty$ and $I_m^{(b)} \rightarrow \infty$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown. Mesh density: $0.05a_R$.

qualitative difference between Figs. 3.10(a) and (b). Focusing first on Fig. 3.10(a), for the case of a rapidly-stiffening Gent matrix with neo-Hookean inclusions, we observe a complex instability landscape with significant regions of locking limited cases – due to the low value of $I_m^{(b)} = 5$. For the case of rapidly-stiffening Gent inclusions in a neo-Hookean matrix in Fig. 3.10(b), when comparing to the $I_m^{(a)} = I_m^{(b)} = 10$ case of Figs. 3.6 and 3.7, we observe that the presence of a neo-Hookean matrix eliminates locking-limited cases in spite of the presence of rapidly-stiffening inclusions with $I_m^{(a)} = 5$. For completeness, the case of dielectric elastomer composites comprised solely of neo-Hookean materials is illustrated in Fig. 3.11. The instability landscape is dominated by pull-in with macroscopic instability occurring for sufficiently stiff inclusions and interfacial instability occurring for sufficiently high permittivity inclusions.

Prestress

Prestress or prestretch are commonly applied to monolithic dielectric elastomer actuators as a means to lower the required applied voltage and to suppress pull-in [cf., e.g., 68, 106]. These effects also play an important role in composites [c.f., e.g., 75], and, for dielectric elastomer laminates, Bertoldi and Gei [70] showed that prestress and prestretch alter the critical instability landscape. Here, we consider the effects of prestress and prestretch on the electromechanical instability landscape in fiber-based composites.

Focusing first on prestress, we explore the effect of an applied uniaxial tensile prestress in the out-of-plane, $\mathbf{e}_3$ -direction in Fig. 3.12. For this case, we first calculate the purely mechanical response during prestressing over the full range of ground-state shear-modulus contrasts, $G^{(a)}/G^{(b)}$, considered previously. The out-of-plane prestress, $\bar{T}_{r,33}^0/\bar{G}$, required to attain a given stretch along the prestressing direction, $\bar{\lambda}^0$, over the full range of $G^{(a)}/G^{(b)}$ is shown in Fig. 3.12(a) for both square- and hexagonal-lattice composites. Because $\bar{T}_{r,33}^0$ is normalized by $\bar{G}$, this response is independent of $G^{(a)}/G^{(b)}$, and since prestressing is applied along the fibers, and pre-deformation is homogeneous, the response is independent of the lattice type for a fixed fiber volume fraction. Then, we apply a prestress corresponding to a value of $\bar{\lambda}^0 = 1.10$, as shown by the dashed lines in Fig. 3.12(a), and apply an electric field along the $\mathbf{e}_1$ -direction, while holding $\bar{T}_{r,33}^0$ fixed. Since $\bar{T}_{r,33}^0$ remains fixed, the stretch along the $\mathbf{e}_3$ -direction *does* change during electrical loading. Figure 3.12(b) shows the electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_r|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability. All four combinations of lattice geometry and electrical loading direction are pictured, and the macroscopic stretch along the electrical loading direction $\bar{\lambda}$ is still defined relative to the undeformed configuration, not the prestressed configuration. Comparing these results to those of Figs. 3.6 and 3.7, we see that the presence of prestress along the $\mathbf{e}_3$ -direction leads to a modest increase in the amount of electrical loading that may be applied prior to the onset of instability and changes some of the pull-in instability-limited cases to macroscopic-limited or locking-limited cases. For completeness, we have also considered the cases of uniaxial tensile prestress in the $\mathbf{e}_2$ -direction as well as uniaxial compressive prestress in the $\mathbf{e}_1$ -direction, and results

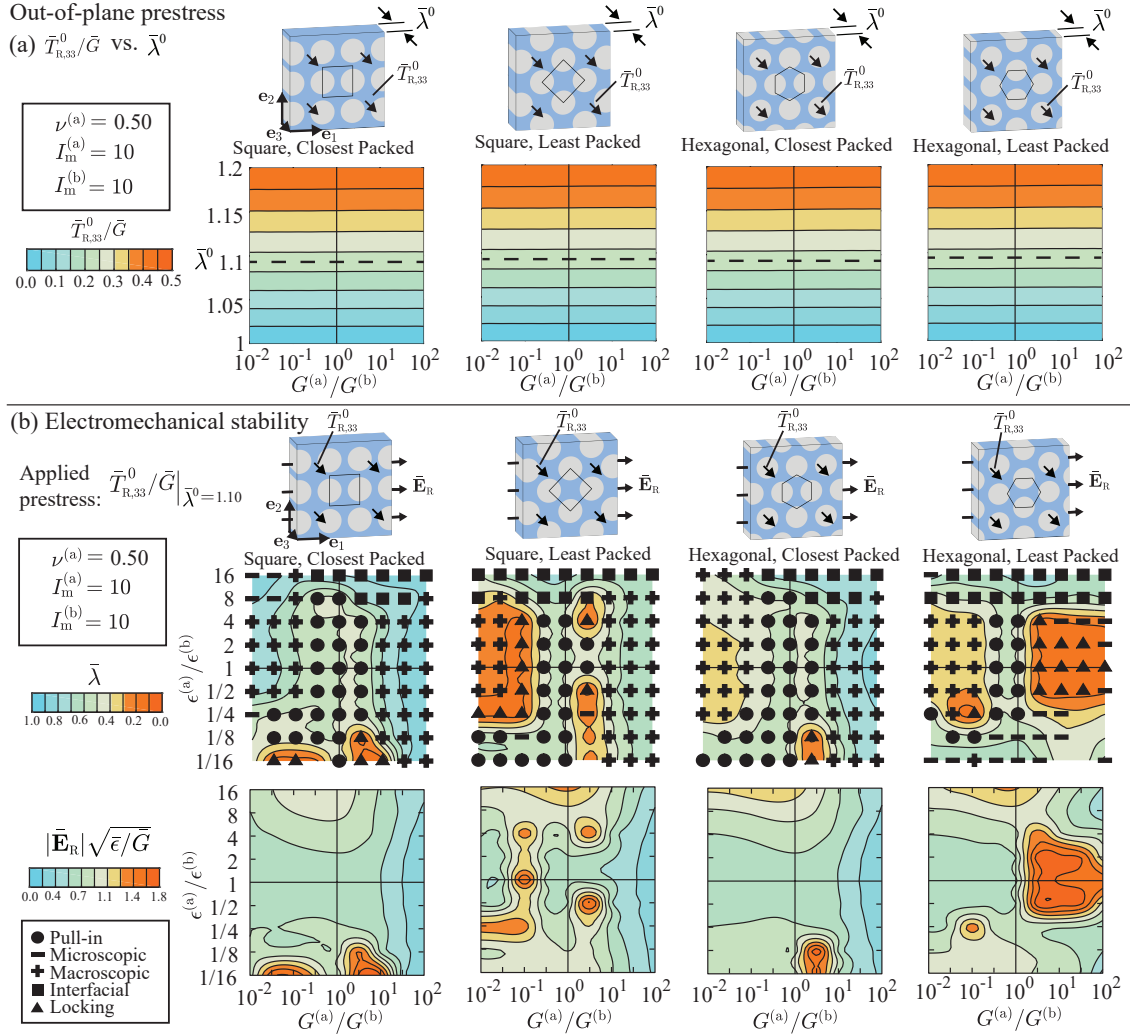


Figure 3.12: The effect of mechanical prestress applied along the $\mathbf{e}_3$ -direction with electrical loading along the $\mathbf{e}_1$ -direction on electromechanical instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. (a) Mechanical response during prestressing in the absence of a macroscopic electric field. Specifically, the macroscopic normal stress along the $\mathbf{e}_3$ -direction, $\bar{T}_{R,33}^0/\bar{G}$, required to obtain a given stretch along the $\mathbf{e}_3$ -direction, $\bar{\lambda}^0$, and for a given shear modulus contrast, $G^{(a)}/G^{(b)}$. (b) Electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for a prestress corresponding to $\bar{\lambda}^0 = 1.10$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

for these cases are presented in Appendix B.

In practice, prestressing is often applied in a biaxial manner, rather than the uniaxial prestressing considered in Fig. 3.12. Therefore, we also consider the case of equibiaxial prestresses applied

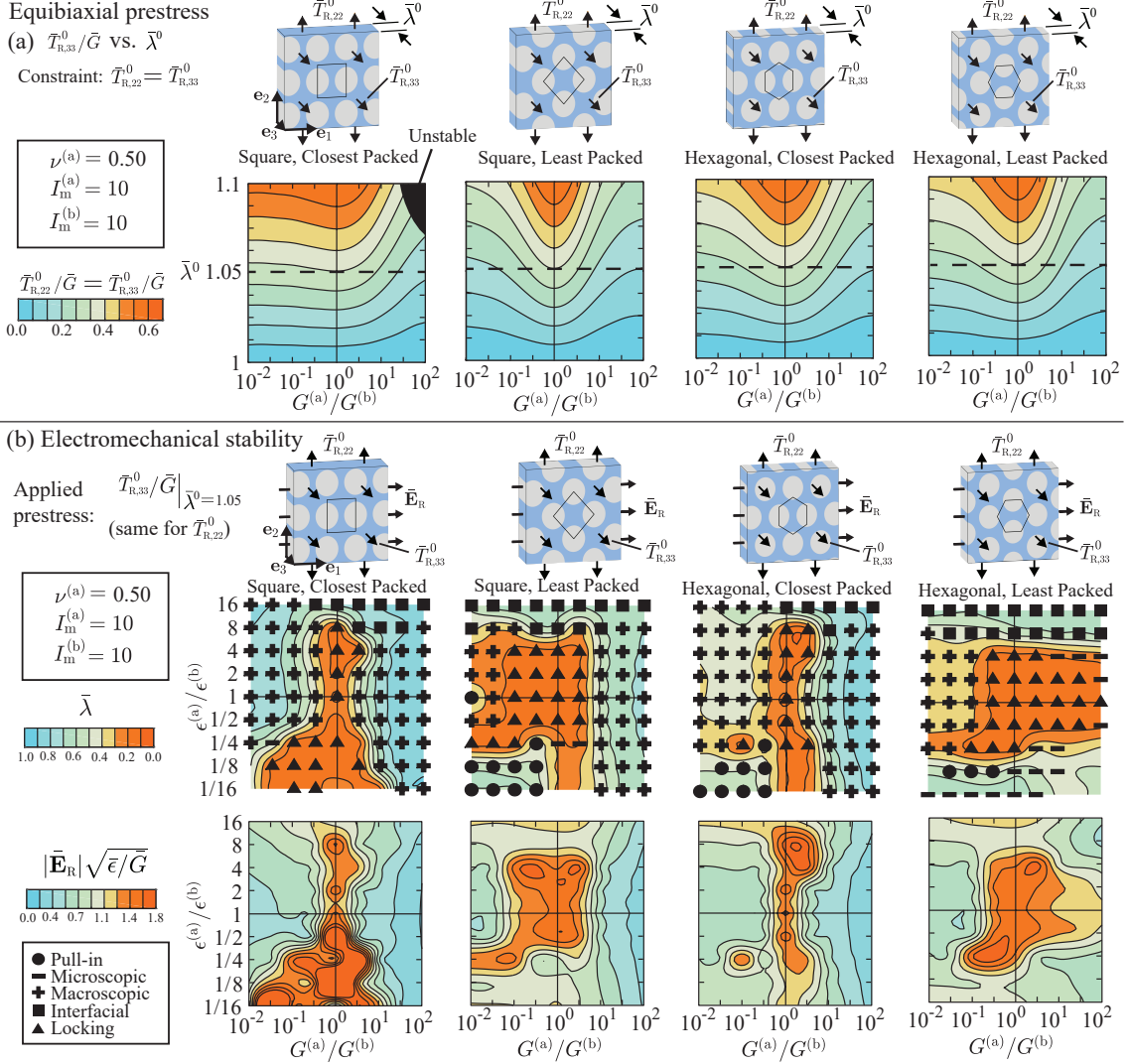


Figure 3.13: The effect of equibiaxial mechanical prestress applied along the $\mathbf{e}_2$ - and $\mathbf{e}_3$ -directions on electromechanical instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. (a) Mechanical response during equibiaxial prestressing in the absence of a macroscopic electric field. Specifically, the macroscopic equibiaxial normal stress along the $\mathbf{e}_2$ - and $\mathbf{e}_3$ -directions, $\bar{T}_{R,22}^0/\bar{G} = \bar{T}_{R,33}^0/\bar{G}$, required to obtain a given stretch along the $\mathbf{e}_3$ -direction, $\bar{\lambda}^0$, and for a given shear modulus contrast, $G^{(a)}/G^{(b)}$. (b) Electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for an equibiaxial prestress corresponding to $\bar{\lambda}^0 = 1.10$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

along the $\mathbf{e}_2$ - and $\mathbf{e}_3$ -directions simultaneously, as shown in Fig. 3.13. The purely mechanical response during prestressing is shown in Fig. 3.13(a). The macroscopic equibiaxial normal stress along the $\mathbf{e}_2$ - and $\mathbf{e}_3$ -directions, $\bar{T}_{R,22}^0/\bar{G} = \bar{T}_{R,33}^0/\bar{G}$, required to attain a given stretch along the $\mathbf{e}_3$ -direction, $\bar{\lambda}^0$, over the full range of $G^{(a)}/G^{(b)}$ is shown for both square- and hexagonal-lattice composites and for the $\mathbf{e}_1$ -direction aligned with the closest-packed and least-packed directions. Since the composite material is anisotropic, the macroscopic stretch along the $\mathbf{e}_2$ - and $\mathbf{e}_3$ -directions are different, but Fig. 3.13(a) only plots the macroscopic stretch along the $\mathbf{e}_3$ -direction. Unlike in Fig. 3.12, the response depends on both $G^{(a)}/G^{(b)}$ and the lattice configuration. Moreover, purely mechanical instability can arise during prestretching, denoted by the black region in Fig. 3.13(a). Then, we consider an equibiaxial prestress corresponding to a value of $\bar{\lambda}^0 = 1.05$, as shown by the dashed lines in Fig. 3.13(a), and apply an electric field along the $\mathbf{e}_1$ -direction, while holding $\bar{T}_{R,22}^0/\bar{G} = \bar{T}_{R,33}^0/\bar{G}$ fixed. Since the macroscopic prestress components remain fixed, the stretches along the $\mathbf{e}_1$ - and $\mathbf{e}_3$ -directions do change during electrical loading and are different from one another. Figure 3.12(b) shows the electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability. All four combinations of lattice geometry and electrical loading direction are pictured. An equibiaxial mode of prestress results in dramatically increased regions of instability suppression. In fact, even the pull-in instability of the homogeneous case, $G^{(a)}/G^{(b)} = 1$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1$, is suppressed. As a result, prestress can be very useful in suppressing the pull-in instability. However, macroscopic and interfacial instabilities remain prevalent in the presence of equibiaxial prestress.

Prestretch and confinement

Next, we consider the effects of kinematic constraints – namely, prestretch and confinement. We first explore the effect of a constraint along the $\mathbf{e}_3$ -direction, which imposes plane-strain conditions in the $\mathbf{e}_1$ - $\mathbf{e}_2$ -plane and enforces the macroscopic out-of-plane stretch to be $\bar{\lambda}^0 = 1.00$. Figure 3.14(a) shows the electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability, when a macroscopic electric field is applied along the $\mathbf{e}_1$ -direction. All

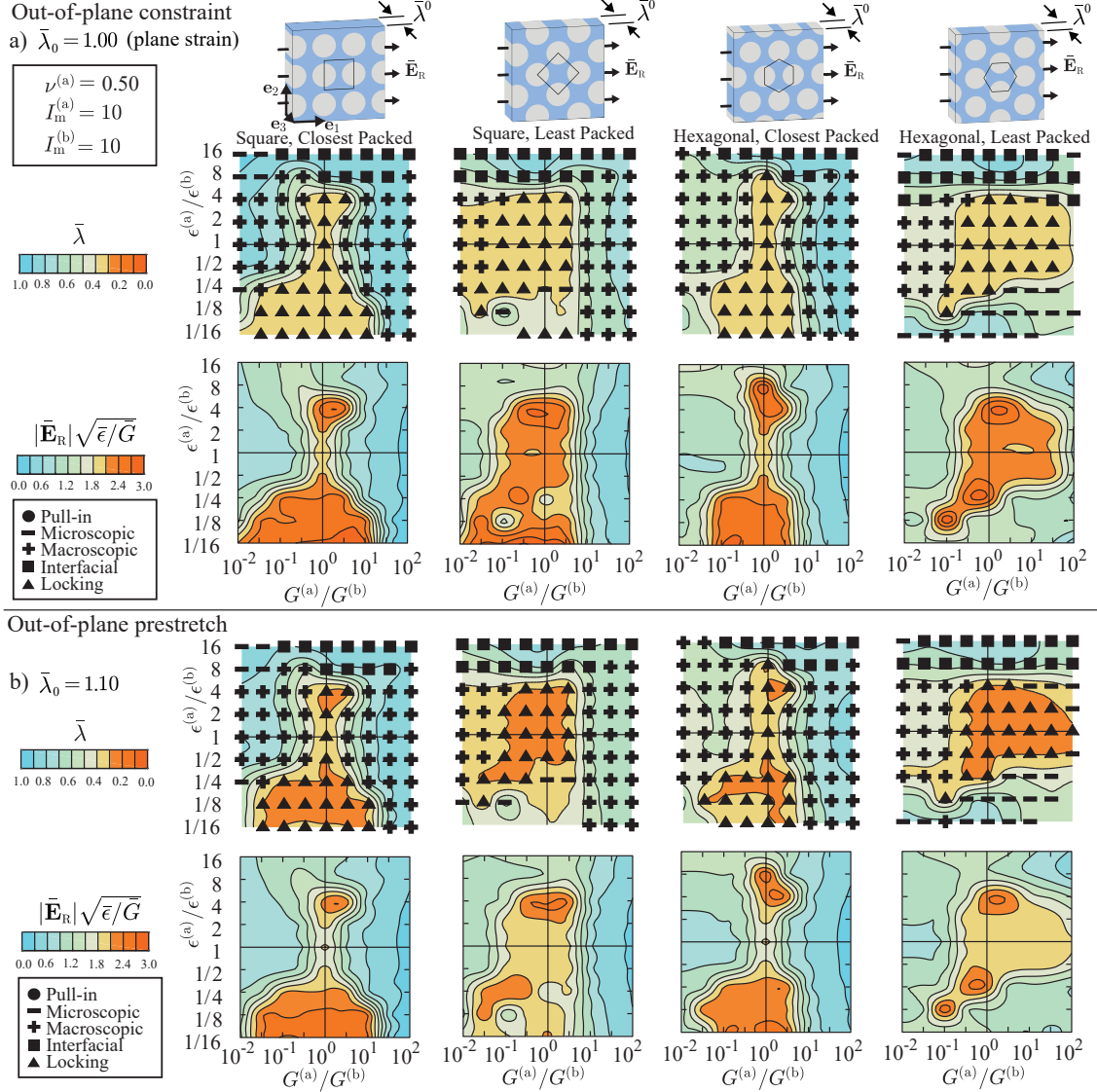


Figure 3.14: The effect of mechanical prestretch applied along the $\mathbf{e}_3$ -direction on electromechanical instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. Electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for (a) out-of-plane plane-strain confinement – i.e., $\bar{\lambda}^0 = 1.00$ – and (b) a fixed, out-of-plane prestretch of $\bar{\lambda}^0 = 1.10$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

four combinations of lattice geometry and electrical loading direction are pictured. Under plane-strain conditions, pull-in is completely suppressed, and comparably large amounts of macroscopic deformation and electric field magnitudes may be attained. Locking-limited behavior is observed

for modest material property contrasts, while macroscopic instability still arises for large and small shear-modulus contrasts, and interfacial instability still arises for high permittivity contrasts. Figure 3.14(b) displays analogous results for the case of a fixed, macroscopic, out-of-plane stretch of $\bar{\lambda}^0 = 1.10$. The electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability are quite similar to the plane-strain case. For completeness, we have also considered the cases of prestretch in the in-plane $\mathbf{e}_2$ -direction, and the results are included in Appendix B.

Next, we consider the situation in which all macroscopic deformation is constrained – i.e., a fully-confined dielectric elastomer composite with $\bar{\mathbf{F}} = \mathbf{1}$.⁹ Importantly, in spite of the macroscopic confinement, a macroscopic electric field can induce deformation within the composite microstructure, and instability can arise – albeit not of the pull-in type. For the fully-confined case, Fig. 3.15(a) shows the electromechanical instability phase diagrams and contour plots of $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability with the macroscopic electric field applied along the $\mathbf{e}_1$ -direction. All four combinations of lattice geometry and electrical loading direction are pictured. We note that since the electric field is spatially-constant when $\epsilon^{(a)}/\epsilon^{(b)} = 1$, no instability occurs in these cases, which are denoted by the hatched regions in Fig. 3.15(a). However, for permittivity contrasts different from unity, despite confinement, microscopic, macroscopic, and interfacial instabilities arise. In particular, for all four lattice/loading configurations, cases with $\epsilon^{(a)}/\epsilon^{(b)} < 1$ are prone to macroscopic instability, while cases with $\epsilon^{(a)}/\epsilon^{(b)} > 1$ are prone to interfacial instability. Microscopic instability can arise for cases with large permittivity contrast and small shear modulus contrast.

Importantly, the combination of permittivity mismatch and electrical loading gives rise to interesting microstructural deformation within the unit cell even when all macroscopic deformation is constrained. To illustrate, the mode shapes for several cases of microscopic and macroscopic instability for the square-lattice composite electrically loaded along the closest-packed direction are shown in Fig. 3.15(b) with the deformed unit cell just prior to the onset of instability shown as an inset for each case. Notably, we observe that the critical direction for macroscopic instability

⁹For completeness, the case in which macroscopic deformation is confined along the direction of the macroscopic electric field but unconstrained in the lateral directions is considered in Appendix B.

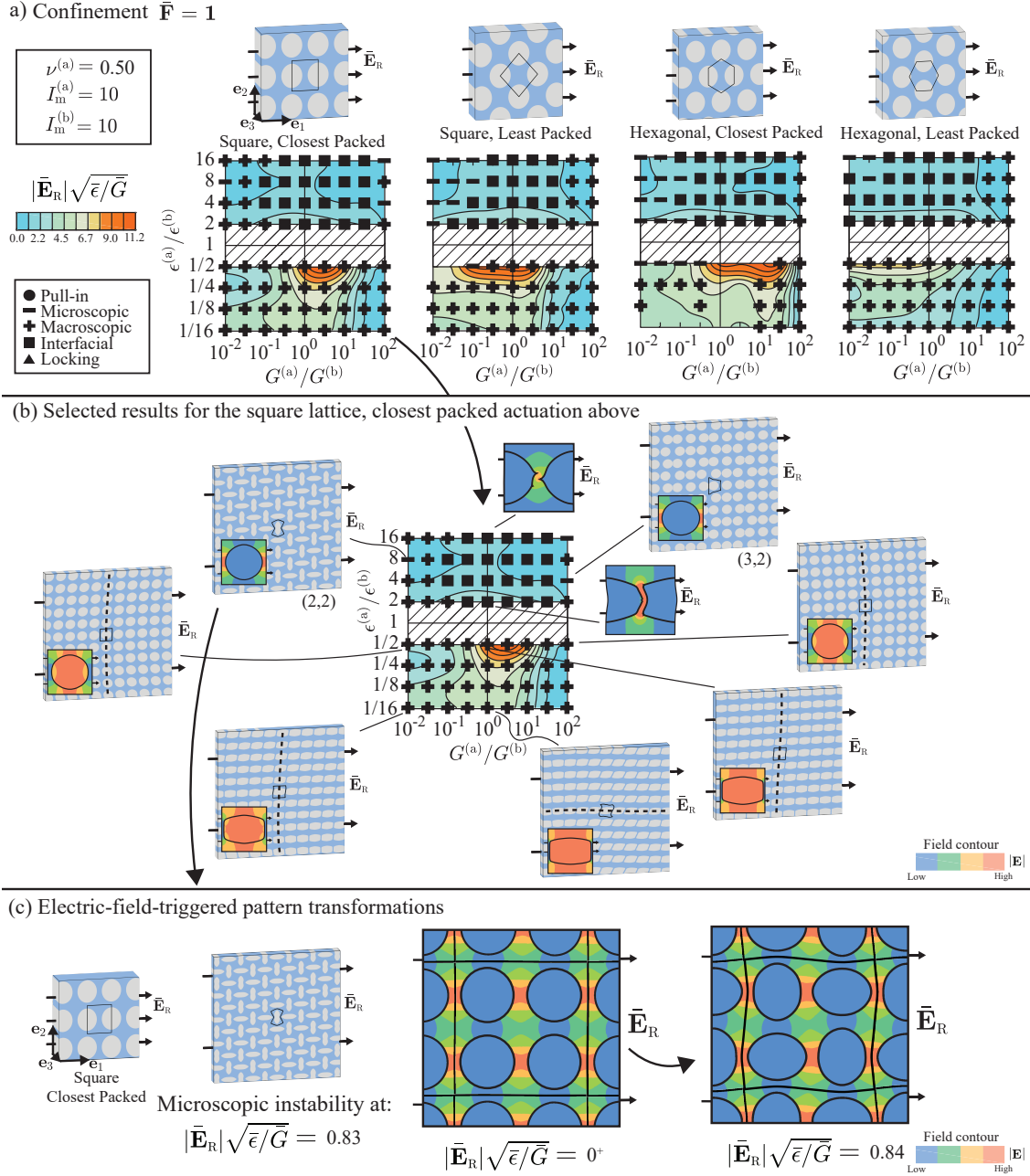


Figure 3.15: (a) Electromechanical instability phase diagrams and contour plots of the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for a fully-confined composite material made from Gent materials. (b) Mode shape reconstructions and deformed composite microstructures for selected cases for the square-lattice composite electrically loaded along the closest-packed direction to highlight representative modes of microscopic, macroscopic, and interfacial instability. (c) A representative case of pattern transformation corresponding to a microscopic instability. Mesh density: $0.05a_R$.

can be either along the closest-packed or the least-packed direction, depending on the case and denoted by the dashed lines in Fig. 3.15(b). Several examples of deformed microstructures after the onset of interfacial instability are also pictured in Fig. 3.15(b). Finally, we note that microscopic instability modes may be harnessed to achieve electrically-triggered pattern transformations, as illustrated in Fig. 3.15(c) for the square-lattice composite electrically-loaded along the closest-packed direction and contrasts of $G^{(a)}/G^{(b)} = 1/100$ and $\varepsilon^{(a)}/\varepsilon^{(b)} = 8$. Figure 3.15(c) shows the result of a nonlinear post-buckling analysis beyond the onset of microscopic instability, illustrating that pattern transformation, in which the unit cell after the onset of microscopic instability enlarges a two-by-two version of the primitive unit cell, may theoretically be achieved. This functionality will be explored in further detail in Chapter 4.

3.4 Concluding remarks

In this chapter, we applied our numerical simulation capability for infinite, periodic dielectric elastomer composites to exhaustively explore electromechanical instabilities in composites consisting of aligned, cylindrical fibers arranged in square and hexagonal lattices and subjected to electrical loading along both the closest- and the least-packed directions. Specifically, we observed cases of pull-in, microscopic, macroscopic, and interfacial instability. Additionally, we considered the effects of fiber volume fraction and large-stretch chain-locking behavior as well as prestress, prestretch, and confinement. Several of our key observations are summarized below:

- For cases with small material property contrasts, **pull-in instability** is typically the limiting instability type.
- In general, the likelihood of **macroscopic instabilities** increases with the distance from the $G^{(a)}/G^{(b)} = 1$ axis. A notable exception is that macroscopic instabilities are almost completely suppressed for $G^{(a)}/G^{(b)} < 1$ when square- and hexagonal-lattice composites are electrically loaded along the least-packed direction.

- Notably, **microscopic instabilities** occur as $G^{(a)}/G^{(b)} \rightarrow 0$ and as $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$ for the square-lattice composite electrically loaded along the closest-packed direction, and also as $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow 0$ for the hexagonal-lattice composite electrically loaded along the least-packed direction.
- In general, the likelihood of **interfacial instabilities** increases with permittivity contrast, $\epsilon^{(a)}/\epsilon^{(b)}$, and fiber volume fraction, $v^{(a)}$. These instabilities are triggered when very strong electric fields are concentrated in thin ligaments between regions with lower electric field magnitudes. We showed that interfacial instability also occurs in laminates, and we interpret this type of instability as an extension of the electrocreasing instability [83, 84].
- The likelihood of pull-in instability increases as the **fiber volume fraction**, $v^{(a)}$, decreases, and the macroscopic stretch along the direction of the macroscopic electric field tends towards the homogeneous limit of $\bar{\lambda} = 0.58$. Conversely, as $v^{(a)}$ increases, the likelihood of pull-in occurring reduces in favor of macroscopic, microscopic and interfacial instabilities as well as large-stretch chain-locking.
- The interplay between the **large-stretch chain-locking behavior** of the two phases, quantified by the Gent parameters $I_m^{(a)}$ and $I_m^{(b)}$, suggests that the role of inclusion chain-locking behavior, $I_m^{(a)}$, is secondary to the role of matrix chain-locking behavior, $I_m^{(b)}$. Further, the likelihood of pull-in occurring also increases as $I_m^{(a)}$ and $I_m^{(b)}$ increase, and in the limit of a composite made from neo-Hookean materials, pull-in dominates the instability landscape.
- As in monolithic dielectric elastomers and dielectric elastomer laminates, **prestress** and **prestretch** can provide a route to suppressing the pull-in instability and increasing the amount of deformation and the magnitude of the macroscopic electric field that may be applied to the composite material.
- The application of an electric field to a **fully-confined** dielectric elastomer composite ($\bar{\mathbf{F}} = \mathbf{1}$) induces deformation and electromechanical instabilities within the composite microstructure,

which are not attainable through purely mechanical loading.

This work may be used as a platform for future study of periodic dielectric elastomer composites. We envision this work being useful to design dielectric elastomer composites with higher permissible deformations while navigating their inherently complex electromechanical instability landscape, and at the same time, requiring smaller electric fields. While the most straightforward application of this work is a composite made from two different dielectric elastomer materials, one may also consider composites with stiff, high-permittivity glass or ceramic inclusions, low-permittivity plastic inclusions, or even fluid-filled inclusions. These possibilities are discussed further in Appendix C.

An interesting extension of this work is to seek pattern transformation opportunities by harnessing the microscopic instabilities that were identified in this chapter. This is motivated by recent work, which has shown that pattern transformation may be used to enhance tunability in purely mechanical phononic crystals [cf., 44, 49, 50]. Additionally, our observation of deformation and instability in fully-confined composite materials subjected to an electric field suggests that tunability may still be achieved despite fixed far-field boundaries. This functionality is interesting from a design perspective for applications, in which dielectric elastomers are sandwiched between fixed boundaries. We consider this potential functionality further in the next chapter.

Chapter 4

Phononic band gap tunability through pattern transformation in dielectric elastomer composites

4.1 Introduction

Carefully designed periodic materials have been shown to exhibit pattern transformations under mechanical loading [105, 107], and when these materials are comprised of soft elastic materials, pattern transformations are reversible and repeatable. These materials constitute a class of elastic mechanical metamaterials that exhibit properties and functionalities that are not present in conventional materials [e.g., 1, 108] and have found use in designing systems with tunable properties such as negative Poisson's ratio [107, 109], effective negative swelling [110], phononic [44, 111, 112] and photonic [113] switches, and switchable chirality [114].

Typically, pattern transformations in mechanical metamaterials are invoked using macroscopic mechanical forces or displacements. This presents a challenge in certain environments – in particular, in situations that are mechanically confined. Additionally, almost all examples of pattern transformation involve macroscopically compressible materials – in particular, ones containing

periodic voids. This motivates the search for reversible pattern transformations in macroscopically (nearly) incompressible periodic solids, as well non-mechanical means to invoke them.

Interestingly, the work of Chapter 3 demonstrated that certain composite lattices and electrical actuation arrangements enable pattern transformation in nearly-incompressible, solid-solid dielectric elastomer composites.¹ The work of Chapter 3 further showed that pattern transformation is also possible in macroscopically-confined composites and can therefore be harnessed to achieve tunability – a functionality that cannot be achieved by mechanical means alone. In this chapter, we explore this functionality further. We show how dielectric elastomer composites may be designed to achieve electric-field-triggered pattern transformation and, in turn, how pattern transformations affect electroelastic wave propagation. This research opens the door for the realization of electromechanical metamaterials.

The remainder of this chapter is organized as follows. In §4.2 we describe our numerical approach to detect microscopic instabilities and subsequently simulate pattern transformations in composites consisting of periodic lattices of aligned, circular-cross-section fibers in a matrix. In §4.3, we establish pattern transformation base cases and consider the effect of fiber volume fraction. We additionally consider several cases of composites with conductive inclusions and voids. In §4.4, we reintroduce electroelastic wave propagation and demonstrate how pattern transformation can be used to manipulate electroelastic waves in these composites. We subsequently assess the effect of density mismatch, volume fraction, as well the role of conductive inclusions and voids on tunability. Finally, we close with some concluding remarks in §4.5.

4.2 Numerical strategy

Following Chapters 2 and 3, we consider composites consisting of periodic lattices of aligned prismatic fibers in a matrix, in which the fibers and the matrix are made from two different ideal dielectric elastomers. As an illustration, consider the composites and associated undeformed

¹The recent work of Polukhov et al. [96] has also illustrated electric-field-triggered pattern transformation in composites with nearly-rigid elliptical-cross-section inclusions.

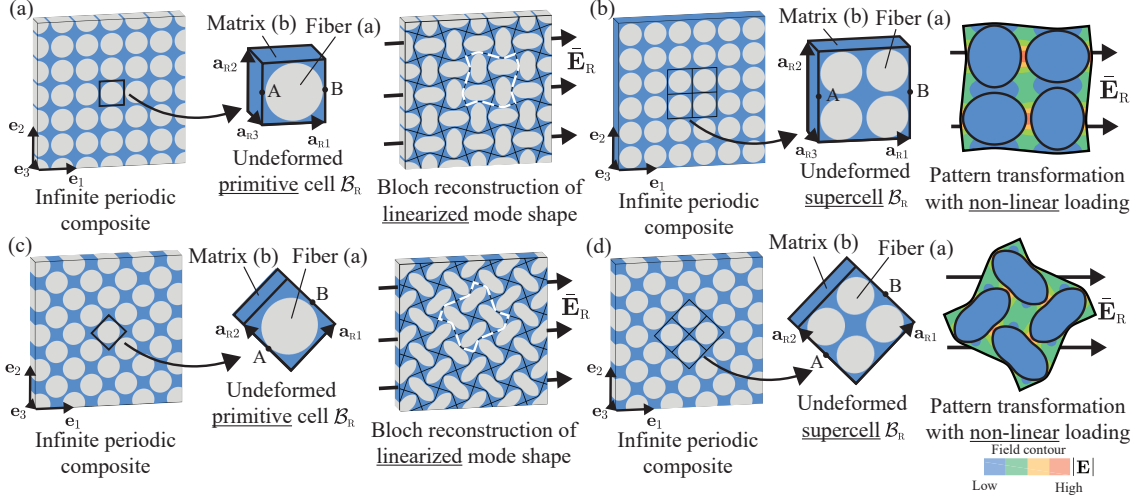


Figure 4.1: Illustration of the numerical strategy for detecting microscopic instabilities and subsequent simulation of pattern transformation in dielectric elastomer composites consisting of aligned circular-cross-section fibers in a matrix arranged in a square array. Schematics of infinite, periodic dielectric elastomer solids and undeformed unit cells for electrical loading along the (a) closest-packed and (c) least-packed directions. In both cases, a representative mode shape of microscopic instability is shown with the enlarged supercell outlined by dashed white lines. (b,d) Undeformed supercells, seeded by the microscopic instability mode shape, for electrical loading along the (b) closest-packed and (d) least-packed directions. In both cases, the deformed supercell after the onset of instability – calculated using nonlinear quasi-electrostatic simulations – is shown.

primitive unit cells shown in Fig. 4.1(a) and (c), which consist of aligned circular-cross-section fibers in a square array. As shown in Fig. 4.1, we introduce a fixed Cartesian coordinate system with orthonormal basis vectors $\{\mathbf{e}_i | i = 1, 2, 3\}$ and with $\mathbf{e}_3$ parallel to the axis of the fibers. As further shown in Fig. 4.1, the undeformed primitive unit cells $\mathcal{B}_R$ are spanned by the referential lattice vectors $\mathbf{a}_{R1}$, $\mathbf{a}_{R2}$, and $\mathbf{a}_{R3}$. We continue to limit attention to composites that possess reflection symmetry with respect to the $\mathbf{a}_{R1}$ - $\mathbf{a}_{R2}$ -plane, so that $\mathbf{a}_{R1} \cdot \mathbf{a}_{R3} = 0$ and $\mathbf{a}_{R2} \cdot \mathbf{a}_{R3} = 0$.

We consider macroscopic electrical loading along the $\mathbf{e}_1$ -direction, which corresponds to the closest-packed and least-packed directions for Figs. 4.1(a) and (c), respectively. Upon loading, we identify and detect critical electromechanical instabilities in accordance with the methodology presented in Chapter 3 – i.e., pull-in, microscopic, macroscopic, and interfacial instabilities are detected. We specifically seek microscopic modes enabling pattern transformation. Bloch reconstructions of representative microscopic modes are shown in Figs. 4.1(a) and (c). When a

microscopic instability is detected at a critical electric field magnitude, a *supercell* of the primitive cell is generated based on the Bloch wave vector associated with microscopic instability – i.e., the supercell consists of a m_1 by m_2 version of the primitive cell. The supercell geometry is seeded with a small perturbation based on the microscopic mode shape. Upon quasi-electrostatic electrical loading of the supercell, pattern transformation is subsequently observed at the critical electrical field magnitude determined during the analysis on the primitive cell, as illustrated in Figs. 4.1(b) and (d) for macroscopically fully-confined cases in which $\bar{\mathbf{F}} = \mathbf{1}$.

In order to ensure that the supercell is stable after the onset of pattern transformation, we perform an additional electromechanical instability analysis on the supercell. Specifically, using the supercell, we thoroughly check for pull-in, macroscopic, interfacial, and further microscopic instabilities that occur after the onset of pattern transformation.

4.3 Pattern Transformations

In this section, we illustrate pattern transformation in the square-lattice composite for cases of electrical loading along both the closest-packed and least-packed directions and both macroscopically traction-free and fully-confined cases. In the present study, we focus on square-lattice composites because these cases exhibit the potential for robust pattern transformation, while we have omitted hexagonal-lattice composites because these cases exhibit secondary instability almost immediately after the occurrence microscopic instability – thus making it difficult to ensure robust pattern transformations. For simplicity, and motivated by physical materials,² we focus on Neo-Hookean elastomers, which correspond to Gent elastomers in the limit $I_m \rightarrow \infty$. Based on the results in §3.3.4, the instability landscape for Neo-Hookean composites is dominated by electromechanical pull-in; however, as shown in Fig. 3.11, microscopic instability occurs in the macroscopically traction-free case for sufficiently compliant and high-permittivity inclusions.

²For instance, the Gent locking parameter for the dielectric elastomer Silicone CF19-2186 has been estimated to be $I_m = 46.3$ [55].

4.3.1 Base cases

Following the work presented in Chapter 3, we recognize that the likelihood of enabling pattern transformation increases as $G^{(a)}/G^{(b)}$ decreases and $\varepsilon^{(a)}/\varepsilon^{(b)}$ increases. To this end, as a base case, we consider dielectric elastomer inclusions in a dielectric elastomer matrix – both modeled as ideal-dielectric neo-Hookean materials – with a low ground-state shear-modulus contrast of $G^{(a)}/G^{(b)} = 1/100$, a high permittivity contrast of $\varepsilon^{(a)}/\varepsilon^{(b)} = 10$, and a fiber volume fraction of $v^{(a)} = 0.65$. We consider both closest-packed and least-packed electrical loading as well as both macroscopically traction-free ($\bar{\mathbf{T}}_r = \mathbf{0}$) and fully-mechanically-confined ($\bar{\mathbf{F}} = \mathbf{1}$) cases.³ Notably, among these four combinations, the macroscopically traction-free square-lattice composite electrically loaded along the least packed direction does not exhibit pattern transformation. The remaining three combinations are shown in Fig. 4.2 – (a) a macroscopically traction-free composite electrically loaded along the closest-packed direction, (b) a fully-confined composite electrically loaded along the closest-packed direction, and (c) a fully-confined composite electrically loaded along the least-packed direction. In all three cases, the deformed supercell microstructures with contours of the electric potential field are shown at the onset of pattern transformation as well as just prior to the onset of secondary instability, which limits further deformation. We observe pattern transformation corresponding to microscopic modes shapes with $(m_1 = 2, m_2 = 2)$ in all three cases – similar to the widely-studied case of a square array of circular voids in an elastomeric matrix [105] but here for a nearly-incompressible solid-solid composite. Regarding secondary instability, for Fig. 4.2(a), a secondary microscopic instability occurs for a macroscopic electric field magnitude of $|\bar{\mathbf{E}}_r| \sqrt{\bar{\varepsilon}/\bar{G}} = 0.71$; for Fig. 4.2(b), a secondary macroscopic instability occurs for a macroscopic electric field magnitude of $|\bar{\mathbf{E}}_r| \sqrt{\bar{\varepsilon}/\bar{G}} = 0.60$; and for Fig. 4.2(c), a secondary interfacial instability occurs for an electric field magnitude of $|\bar{\mathbf{E}}_r| \sqrt{\bar{\varepsilon}/\bar{G}} = 1.20$. Figure 4.2(d) shows the macroscopic electric field magnitude, $|\bar{\mathbf{E}}_r| \sqrt{\bar{\varepsilon}/\bar{G}}$, and the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, versus the macroscopic electric displacement magnitude, $|\bar{\mathbf{D}}_r|/\sqrt{\bar{G}\bar{\varepsilon}}$, for each case. The solid and dashed lines represent the

³Mechanical confinement was shown to enable pattern transformation in Fig. 3.15.

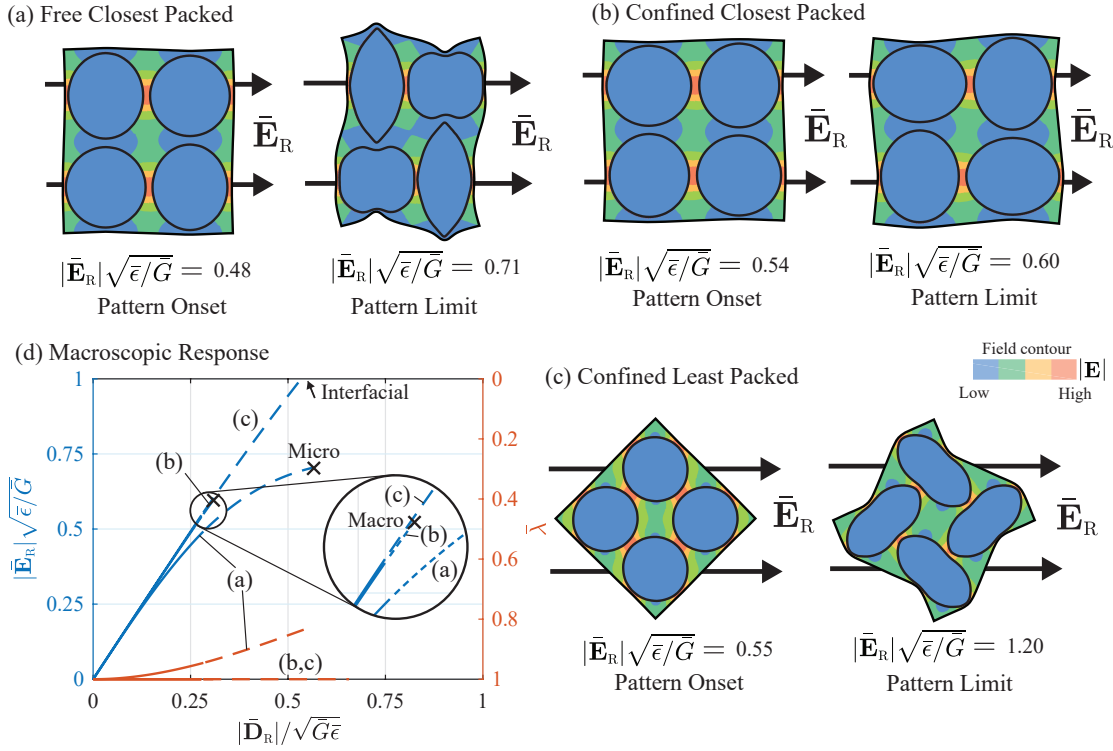


Figure 4.2: Representative examples of electrical-field-triggered pattern transformation in solid-solid dielectric elastomer composites made from neo-Hookean materials for (a) a macroscopically traction-free composite electrically loaded along the closest-packed direction, (b) a fully-confined composite electrically loaded along the closest-packed direction, and (c) a fully-confined composite electrically loaded along the least-packed direction. In all three cases, $G^{(a)}/G^{(b)} = 1/100$, $\epsilon^{(a)}/\epsilon^{(b)} = 10$, and $\nu^{(a)} = 0.65$. (d) Macroscopic electric field magnitude and macroscopic stretch along the electrical loading direction versus macroscopic electric displacement magnitude for each case. Solid lines represent the response before the onset of pattern transformation, and dashed lines represents the response after the onset of pattern transformation. Mesh density: $0.02a_R$.

response prior to and after pattern transformation, respectively,⁴ obtained from nonlinear, quasi-electrostatic numerical simulations using the enlarged supercell. The fully-confined composite electrically loaded along the closest-packed direction (Fig. 4.2(b)) exhibits secondary instability soon after pattern transformation – so that there is little potential for tunability beyond the onset of pattern transformation. In contrast, the cases of Fig. 4.2(a) and (c) do display a wide range of the macroscopic electric field magnitude over which the enlarged supercell is stable – indicating the potential for robust pattern transformation.

⁴For cases (b) and (c), the composite is fully-confined, and the macroscopic stretch, $\bar{\lambda}$, remains equal to one.

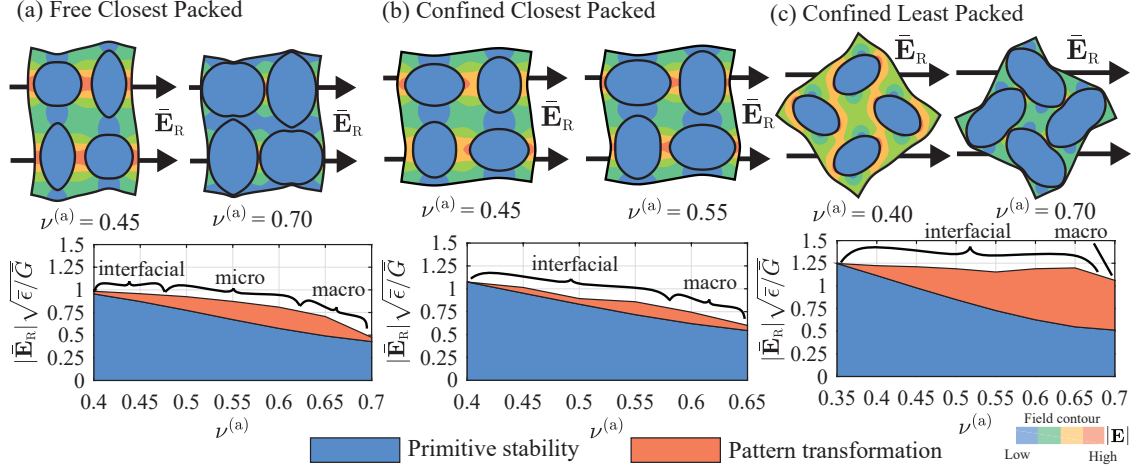


Figure 4.3: Effect of fiber volume fraction on the onset of pattern transformation and subsequent secondary instability in solid-solid dielectric elastomer composites made from neo-Hookean materials for (a) a macroscopically traction-free composite electrically loaded along the closest-packed direction, (b) a fully-confined composite electrically loaded along the closest-packed direction, and (c) a fully-confined composite electrically loaded along the least-packed direction. The blue regions indicate stability of the primitive unit cell, while the red regions indicate stability of the enlarged supercell after the onset of pattern transformation. Additionally, the limiting secondary instability type is identified for each case. Illustrative snapshots of the deformed supercell after the onset of pattern transformation are shown for selected fiber volume fractions. Material parameter contrasts: $G^{(a)}/G^{(b)} = 1/100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 10$. Mesh density: $0.02a_R$.

4.3.2 Effect of fiber volume fraction

Next, we consider the effect of fiber volume fraction on the onset and subsequent robustness of pattern transformation. Figure 4.3 shows the macroscopic electric field magnitude at the onset of pattern transformation and at the onset of secondary instability as a function of fiber volume fraction for each of the three cases from Fig. 4.2. In each figure, blue regions indicate combinations of fiber volume fraction and electric field magnitude in which the primitive unit cell is stable; red regions indicate stability of the enlarged supercell after the onset of pattern transformation; and the limiting secondary instability type is identified for each case. We observe that the highest $\nu^{(a)}$ cases are limited by macroscopic instability, while the smallest $\nu^{(a)}$ cases are limited by interfacial instability. Interestingly, Fig. 4.3(a) shows that the unconstrained lattice displays a range of $\nu^{(a)}$ in which a secondary microscopic instability occurs. We have not explored this observation further, but

these cases could potentially enable sequential pattern transformations. For each case, illustrative snapshots of the deformed supercell after the onset of pattern transformation are shown for selected fiber volume fractions, showing that the $(m_1 = 2, m_2 = 2)$ mode of pattern transformation continues to arise in all cases. Larger red regions in Fig. 4.3 correspond to wider ranges of macroscopic electric field magnitude over which the enlarged supercell is stable – and hence greater robustness of pattern transformation. The results of Fig. 4.3 indicate that the greatest range of stable pattern transformation is observed for the case of a fully-confined square-lattice composite electrically loaded along the least-packed direction.

4.3.3 Composites with conductive inclusions and voids

In this section, we consider two different composite types – namely, composites with electrically-conductive, nearly-incompressible, elastic inclusions as well as composites with electrically-conductive voids.

Electrically-conductive inclusions: Electrically-conductive inclusions may be achieved in composites in which the inclusions are made from a conductive elastomer. This situation is numerically simulated by constraining the electric potential degree-of-freedom to be constant within the inclusions. As before, we take the mechanical response of the inclusion material to be governed by the nearly-incompressible neo-Hookean constitutive model. This case may be interpreted as the limit in which $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$, resulting in a constant-electric-potential inclusion. We continue to assume $G^{(a)}/G^{(b)} = 1/100$, but since $\epsilon^{(a)}$ is no longer a relevant quantity, the normalized macroscopic electric field magnitude is redefined as $|\bar{\mathbf{E}}_r| \sqrt{\epsilon^{(b)}/\bar{G}}$. Figures 4.4(a-c) show simulation results for the cases of square-lattice composites with electrically-conductive inclusions organized as in Fig. 4.3 – i.e., (a) a macroscopically traction-free composite electrically loaded along the closest-packed direction, (b) a fully-confined composite electrically loaded along the closest-packed direction, and (c) a fully-confined composite electrically loaded along the least-packed direction. Illustrative snapshots of the deformed composite microstructure after the onset of pattern transformation are shown

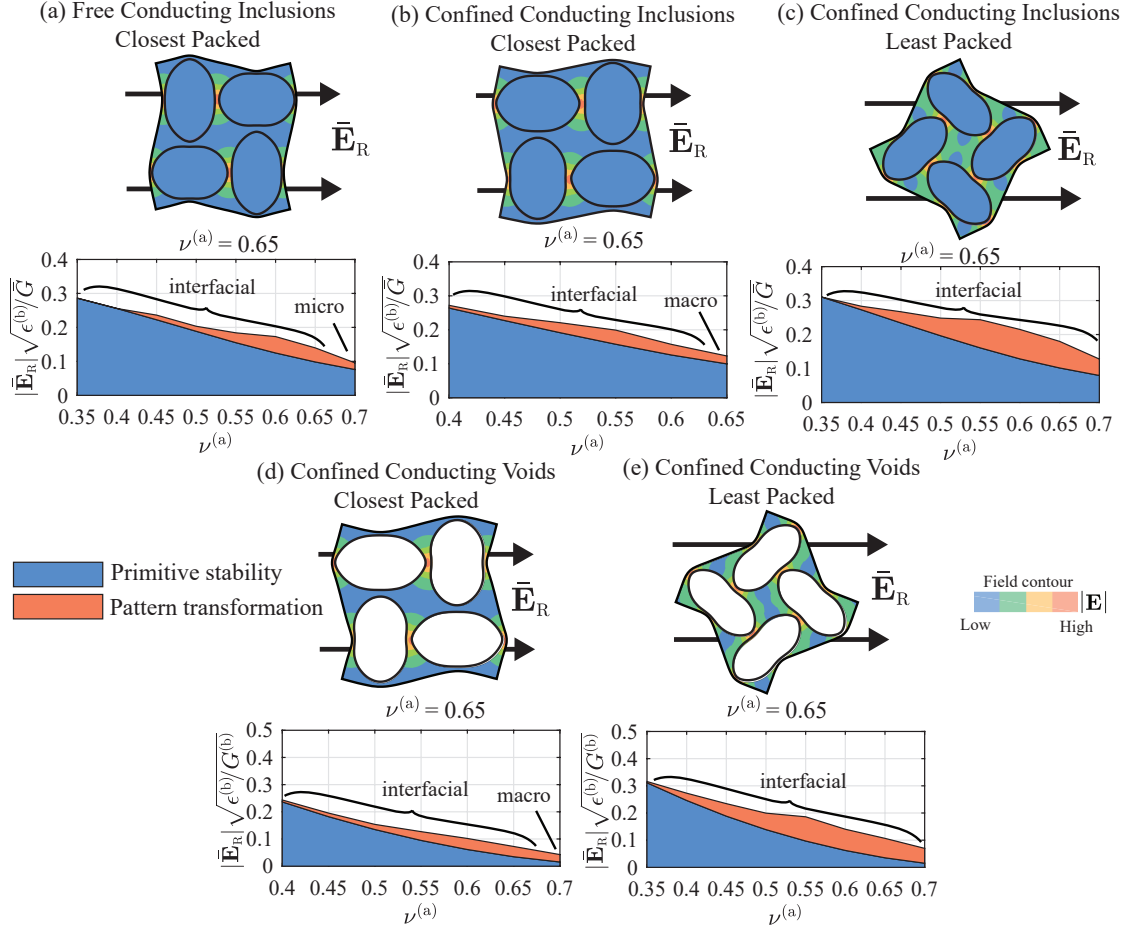


Figure 4.4: Onset of pattern transformation and subsequent secondary instability in dielectric elastomer composites with electrically-conductive inclusions for (a) a macroscopically traction-free composite electrically loaded along the closest-packed direction, (b) a fully-confined composite electrically loaded along the closest-packed direction, and (c) a fully-confined composite electrically loaded along the least-packed direction. Analogous results for dielectric elastomer composites with electrically-conductive voids for (d) a fully-confined composite electrically loaded along the closest-packed direction and (e) a fully-confined composite electrically loaded along the least-packed direction. The blue regions indicate stability of the primitive unit cell, while the red regions indicate stability of the enlarged supercell after the onset of pattern transformation. Additionally, the limiting secondary instability type is identified for each case. Illustrative snapshots of the deformed supercell after the onset of pattern transformation are shown for a fiber/void volume fraction of $\nu^{(a)} = 0.65$. Mesh density: $0.02a_R$.

for each case for a fiber volume fraction of $\nu^{(a)} = 0.65$, illustrating the $(m_1 = 2, m_2 = 2)$ microscopic mode shape, and the macroscopic electric field magnitudes at the onset of pattern transformation and at the onset of secondary instability are plotted as a function of the fiber volume fraction $\nu^{(a)}$.

Comparing these results to Fig. 4.3, we observe that the region of stable pattern transformation is slightly reduced. Also, the limiting secondary instability types are more dominated by interfacial instability. Since the case of conductive inclusions is approached in the limit $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$ for general dielectric elastomer composites, these results illustrate that the permittivity contrast influences the onset of pattern transformation and the subsequent limiting behavior.

Electrically-conductive voids: Electrically-conductive voids were previously considered by Li and Landis [60] and can arise in cases in which the surfaces of voids are coated with a thin layer of a compliant conductive material. This case is numerically simulated by constraining the electric potential degree-of-freedom to be constant on the void boundary and may be interpreted as the limit in which $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$, $G^{(a)}/G^{(b)} \rightarrow 0$, and $K^{(a)}/K^{(b)} \rightarrow 0$. Since $G^{(a)}$ and $\epsilon^{(a)}$ are no longer relevant quantities, the normalized macroscopic electric field magnitude is redefined as $|\bar{\mathbf{E}}_R| \sqrt{\epsilon^{(b)}/G^{(b)}}$. In our numerical simulations, we observe that macroscopically traction-free cases with voids do not undergo pattern transformation – instead, the voids simply shrink. However, pattern transformation does arise in the fully-confined case, since the constraint prevents the voids from shrinking. Figures 4.4(d) and (e) show the cases of fully-confined composites with electrically-conductive voids arranged in a square lattice and electrically loaded along (d) the closest-packed and (e) least-packed directions. Snapshots of the deformed composite microstructure after the onset of pattern transformation are shown for each case, illustrating the $(m_1 = 2, m_2 = 2)$ microscopic mode shape, and the macroscopic electric field magnitudes at the onset of pattern transformation and at the onset secondary instability are plotted as a function of the void volume fraction $v^{(a)}$. Comparing these results to Fig. 4.3, we observe similar regions of stable pattern transformation and secondary instability type.

4.4 Enhanced Tunability through Pattern Transformations

In §4.3, we established cases, in which pattern transformation in dielectric elastomer composites is possible. In this section, we reintroduce electroelastic wave propagation and explore how pattern

transformation may be used to manipulate electroelastic waves in these composites.

4.4.1 Base cases

Here, we focus on two base cases: (a) a macroscopically traction-free, square-lattice composite with a fiber volume fraction of $v^{(a)} = 0.65$ and electrically loaded along the closest-packed direction and (b) a fully-confined, square-lattice composite with a fiber volume fraction of $v^{(a)} = 0.70$ and electrically loaded along the least-packed direction for material property contrasts of $G^{(a)}/G^{(b)} = 1/100$ and $\epsilon^{(a)}/\epsilon^{(b)} = 10$. For the base cases, we take the mass densities of the fiber and matrix to be equal, so that $\rho^{(a)}/\rho^{(b)} = 1$. Both cases exhibit microscopic instability with $(m_1 = 2, m_2 = 2)$ and hence pattern transformation and are chosen because they display a wide range of macroscopic electric field magnitudes between the onset of pattern transformation and secondary instability (see Fig. 4.3). The top left panels of Figs. 4.5(a) and (b) show the undeformed composites with the $\mathbf{e}_1$ -direction aligned with the electrical loading direction $\bar{\mathbf{m}}$ and with the enlarged supercell outlined. The top right panels of Figs. 4.5(a) and (b) show snapshots of the deformed supercells for each case during quasi-electrostatic preloading (i) just after and (ii) well after the onset of pattern transformation and (iii) just before the onset of secondary instability.

We then calculate the tunability of phononic band gaps in these two cases using the approach described in Chapter 2 – with the exception that we utilize the supercell with $(m_1 = 2, m_2 = 2)$ rather than the primitive unit cell. The referential basis vectors $\mathbf{a}_{ri}$ are updated to account for the supercell periodicity, as shown in the top left panels of Figs. 4.5(a) and (b), but we continue to denote the magnitude of the primitive-cell basis vectors as a_r . The undeformed microstructure geometry is seeded with a small perturbation based on the microscopic mode corresponding to pattern transformation. The perturbation is sufficiently small so that it does not affect any of the calculated results prior to the onset of pattern transformation. Following the discussion of Chapter 2, the referential reciprocal lattice vectors for the supercell are then defined by

$$\mathbf{b}_{r1} = 2\pi \frac{\mathbf{a}_{r2} \times \mathbf{a}_{r3}}{\mathbf{a}_{r1} \cdot (\mathbf{a}_{r2} \times \mathbf{a}_{r3})}, \quad \mathbf{b}_{r2} = 2\pi \frac{\mathbf{a}_{r3} \times \mathbf{a}_{r1}}{\mathbf{a}_{r2} \cdot (\mathbf{a}_{r3} \times \mathbf{a}_{r1})}, \quad \text{and} \quad \mathbf{b}_{r3} = 2\pi \frac{\mathbf{a}_{r1} \times \mathbf{a}_{r2}}{\mathbf{a}_{r3} \cdot (\mathbf{a}_{r1} \times \mathbf{a}_{r2})}. \quad (4.4.1)$$

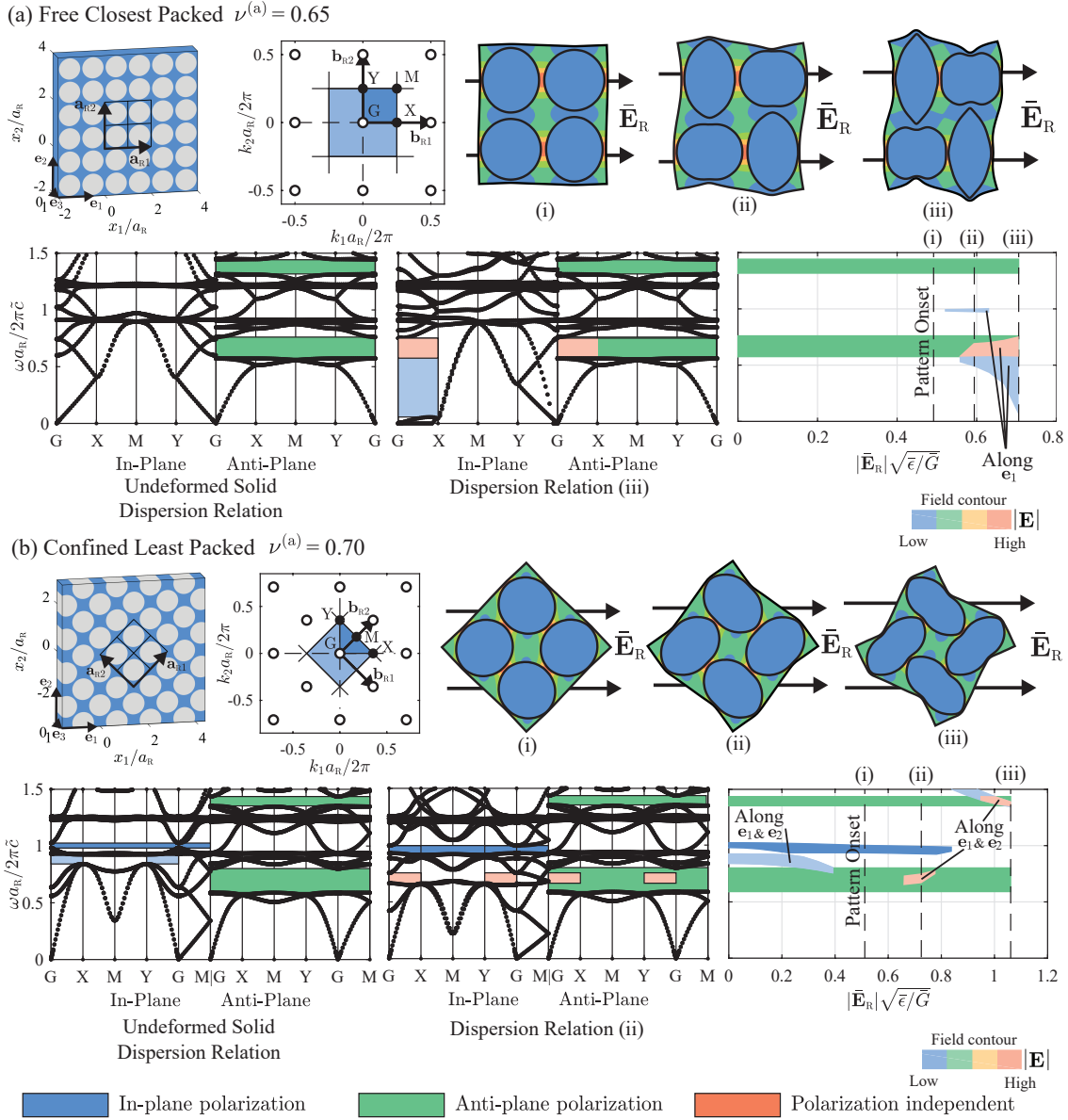


Figure 4.5: Pattern transformation and electrical tunability for two cases of the square-lattice composite made from neo-Hookean materials with property contrasts $G^{(a)}/G^{(b)} = 1/100$, $\epsilon^{(a)}/\epsilon^{(b)} = 10$, and $\rho^{(a)}/\rho^{(b)} = 1$: (a) a macroscopically traction-free composite with a fiber volume fraction of $\nu^{(a)} = 0.65$ and electrically loaded along the closest-packed direction and (b) a fully-confined composite with a fiber volume fraction of $\nu^{(a)} = 0.70$ and electrically loaded along the least-packed direction. Top left: The undeformed direct-space structure with the supercell outlined. Top middle: Undeformed reciprocal space. Top right: Snapshots of the deformed supercell (i) just after and (ii) well after the onset of pattern transformation and (iii) just before the onset of secondary instability. Bottom left: Dispersion relations $\omega(\mathbf{k})$ for the undeformed and deformed composites. Bottom right: Electrical tunability of phononic band gaps. Mesh density: $0.02a_R$.

The undeformed reciprocal spaces for each case are shown in the top middle panels of Figs. 4.5(a) and (b). The reciprocal lattice points are displayed as unfilled circles; solid lines represent the first Bragg planes, which define the boundaries of the first Brillouin zones, shown as lightly-shaded regions. Furthermore, due to the symmetries of the deformed supercell after pattern transformation, we may continue to only consider part of the first Brillouin zone – the irreducible Brillouin zone (IBZ). Symmetry planes for each reciprocal space are shown as dashed lines, and the resulting IBZs are shown as darkly-shaded regions. The corners of the IBZs are each labeled by a letter. Note that while there are more symmetry planes for the undeformed and unloaded composite, the symmetry planes denoted in Fig. 4.5 correspond to the symmetry planes of the deformed and electrically-loaded supercell after pattern transformation. As such, the IBZs of the two cases shown in Figs. 4.5 are different. Finally, we consider a set of Bloch wave vectors $\mathbf{k}$ associated with the discretized boundary of the respective deformed IBZs. The discretization process involves dividing the path along the undeformed IBZ boundary into equal increments of $|\Delta\mathbf{k}_R|a_R/2\pi = 0.025$ and considering the deformed wave vectors $\mathbf{k} = \bar{\mathbf{F}}^{-\top}\mathbf{k}_R$ corresponding to each point. An exception is made for small wave vector magnitudes of $|\mathbf{k}_R|a_R/2\pi \leq 0.05$ – i.e., the region of the IBZ near the point G – where finer increments of $|\Delta\mathbf{k}_R|a_R/2\pi = 0.0025$ are utilized.

Using this set of Bloch wave vectors, the corresponding eigenfrequencies are numerically computed as in Chapter 2, and the dispersion relation $\omega(\mathbf{k})$ is assembled. Calculated dispersion relations for the two base cases are plotted in the bottom left panels of Figs. 4.5 (a) and (b) for both the undeformed composite and a representative example of the deformed composite after the onset of pattern transformation. On the vertical axes, the eigenfrequencies are normalized as $\omega a_R/2\pi\tilde{c}$, where $\tilde{c} = (\nu^{(a)}/c^{(a)} + \nu^{(b)}/c^{(b)})^{-1}$ is the reciprocal volume-averaged ground-state shear wave speed and $c^{(\alpha)} = \sqrt{G^{(\alpha)}/\rho^{(\alpha)}}$ is the ground-state shear wave speed of phase α , and the horizontal axes denote paths along the boundaries of the respective IBZs. Each point on the dispersion relation plots represents an eigenfrequency for a discrete Bloch wave vector. The left side of each dispersion plot corresponds to in-plane electroelastic wave polarizations, while the right side corresponds to

anti-plane polarizations.⁵ In the dispersion relations, symmetry is observed in the undeformed solid along the G-X and G-Y branches (i.e., along the $\mathbf{e}_1$ -direction and $\mathbf{e}_2$ -direction, respectively) but differences can emerge as electrically-induced deformation progresses.

Phononic band gaps are defined as frequency ranges in which no eigenfrequencies exist over the set of Bloch wave vectors and are identified for both in-plane and anti-plane wave polarizations. Band gaps are represented by shaded regions – blue for in-plane polarization band gaps, green for anti-plane polarization band gaps, and red for polarization-independent band gaps. We also consider band gaps for electroelastic waves propagating along the direction of and perpendicular to the applied electric field $\bar{\mathbf{m}}$ -direction – i.e., along the $\mathbf{e}_1$ -direction and the $\mathbf{e}_2$ -direction, respectively. These partial band gaps are represented using lighter shades of the same colors used to represent the band gaps across the full set of Bloch wave vectors and are explicitly labeled. We then calculate the dispersion relation and identify phononic band gaps at discrete values of electrical preloading. The bottom right panels of Figs. 4.5 (a) and (b) show the dependence of phononic band gaps on the applied electric field, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, for each base case. The onset of pattern formation is denoted by (i); a point after the onset of a partial band gap is denoted by (ii); and the state just prior to secondary instability is denoted by (iii). Prior to pattern transformation, very little tunability is observed in the macroscopically traction-free case (Fig. 4.5(a)) – consistent with the observations of Fig. 2.6 for dielectric elastomer composites made from neo-Hookean materials; however, several partial band gaps along the $\mathbf{e}_1$ -direction open after the onset of pattern transformation due to the substantial changes in the geometry of the composite microstructure. On the other hand, more gradual tunability is exhibited by the case of a fully-confined composite electrically loaded along the least-packed direction (Fig. 4.5(b)). Partial band gaps along both the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions

⁵**Comment on compressibility:** In Chapter 2, we considered a nearly-incompressible composite with $K^{(a)} = K^{(b)} = 10^3 \max(G^{(a)}, G^{(b)})$ and found that longitudinal-like plane waves in the homogenized composite, which involve volumetric deformation in the composite microstructure, arise for $|\mathbf{k}_R|a_R/2\pi \lesssim 0.05$. In this chapter, we take $K^{(a)} = K^{(b)} = 10^6 \max(G^{(a)}, G^{(b)})$, so that longitudinal-like plane waves are very fast. We do not attempt to resolve these longitudinal-like plane waves in the dispersion diagrams of Fig. 4.5, and in-plane longitudinal-like mode shapes are *not* considered in the definition of phononic band gaps used in this chapter. Furthermore, we note that very fast longitudinal-like waves do not arise in the composites with the conductive voids, considered later in this chapter, since the presence of voids makes the composite macroscopically compressible.

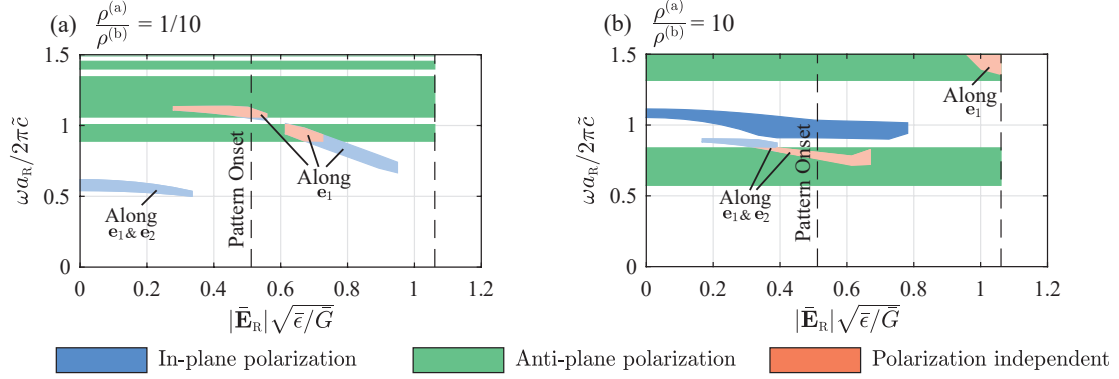


Figure 4.6: The effect of mass density contrast – (a) $\rho^{(a)}/\rho^{(b)} = 1/10$ and (b) $\rho^{(a)}/\rho^{(b)} = 10$ – on electrical tunability of fully-confined dielectric elastomer composites electrically loaded along the least-packed direction and for $G^{(a)}/G^{(b)} = 1/100$, $\epsilon^{(a)}/\epsilon^{(b)} = 10$, and $\nu^{(a)} = 0.70$. These figures may be compared to the $\rho^{(a)}/\rho^{(b)} = 1$ case of Fig. 4.5(b). Mesh density: $0.02a_R$.

emerge and persist through pattern transformation. There are also two instances of polarization-independent partial band gaps that open after the onset of pattern transformation.

4.4.2 Parametric studies

In this section, we consider the effects of density ratio and fiber volume fraction on the tunability of phononic band gaps.

Effect of mass density contrast: Figure 4.6 shows the effect of mass density contrast, focusing on the case of the fully-confined composite electrically loaded along the least-packed direction for brevity. The dependence of phononic band gaps on the applied electric field, $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}}$, for mass density contrasts of $\rho^{(a)}/\rho^{(b)} = 1/10$ and $\rho^{(a)}/\rho^{(b)} = 10$ are shown in Figs. 4.6(a) and (b), respectively, and may be compared to the $\rho^{(a)}/\rho^{(b)} = 1$ base case of Fig. 4.5(b). The results of Fig. 4.6 show that either increasing or decreasing the density mismatch leads to favorable effects – namely, wider and more persistent polarization-independent band gaps. Note that before pattern transformation, partial band gaps appear for both the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions, but after pattern transformation, they only appear for the $\mathbf{e}_1$ -direction. The mechanical confinement leads to the geometry of pattern deformation being nearly symmetric; however, because the electrical loading

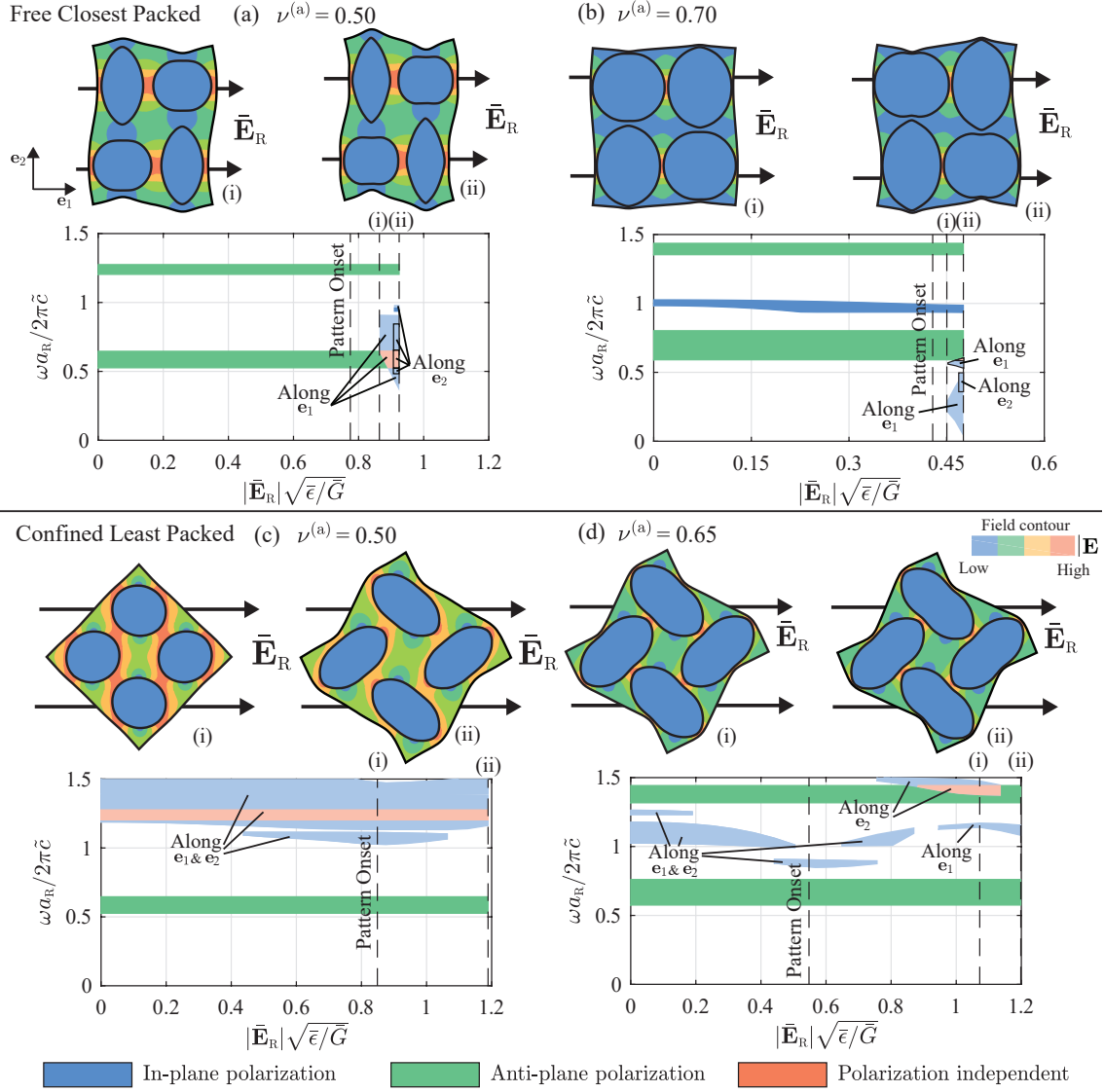


Figure 4.7: The effect of fiber volume fraction on electrical tunability of dielectric elastomer composites with $G^{(a)}/G^{(b)} = 1/100$, $\epsilon^{(a)}/\epsilon^{(b)} = 10$, and $\rho^{(a)}/\rho^{(b)} = 1$. For the macroscopically traction-free composite electrically loaded along the closest-packed direction, (a) $\nu^{(a)} = 0.50$ and (b) $\nu^{(a)} = 0.70$. These figures may be compared to the $\nu^{(a)} = 0.65$ case of Fig. 4.5(a). For the case of the fully-confined composite electrically loaded along the least-packed direction, (c) $\nu^{(a)} = 0.50$ and (d) $\nu^{(a)} = 0.65$. These figures may be compared to the $\nu^{(a)} = 0.70$ case of Fig. 4.5(b). Mesh density: $0.02a_R$.

is along the $\mathbf{e}_1$ -direction, the dispersion relations are not the same along the G-X and G-Y branches.

Effect of fiber volume fraction: Figure 4.7 illustrates the effect of the fiber volume fraction for both the cases of a macroscopically traction-free composite electrically loaded along the closest-packed direction and a fully-confined composite electrically loaded along the least-packed direction. For the traction-free, closest-packed case, Figs. 4.7(a) and (b) show the dependence of phononic band gaps on the applied electric field, $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}}$, for fiber volume fractions of (a) $\nu^{(a)} = 0.50$ and (b) $\nu^{(b)} = 0.70$. The deformed composite microstructures with contours of the electric field magnitude are shown at selected levels of electrical preloading. We continue to observe the opening of in-plane and polarization-independent partial band gaps along the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions, but the opening of polarization-independent band gaps in the $\nu^{(a)} = 0.70$ case is nearly eliminated. However, the $\nu^{(a)} = 0.70$ case shows an in-plane band gap near $\omega a_R/2\pi\tilde{c} = 1$ that does not exist in the other volume fraction cases. The result is that each volume fraction produces similar band gap tunability, and there is no optimal volume fraction. For the fully-confined, least-packed case, Figs. 4.7(c) and (d) show the dependence of phononic band gaps on the applied electric field for fiber volume fractions of (c) $\nu^{(a)} = 0.50$ and (d) $\nu^{(b)} = 0.65$ and suggest much broader electrical tunability. Lowering the volume fraction from the base case of $\nu^{(a)} = 0.70$, shown in Fig. 4.5(b), leads to persistent, polarization-independent partial band gaps in the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions. Symmetry between the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions is nearly preserved through pattern transformation. Figure 4.7(d) shows a variety of instances of opening and closing of in-plane partial band gaps and offers similar electrical tunability to the $\nu^{(b)} = 0.65$ case of Fig. 4.5(b). The larger pattern transformation region – i.e., separation between the pattern transformation onset and secondary instability – aids in this broader electrical tunability.

4.4.3 Composites with conductive inclusions and voids

Finally, we consider the electrical tunability of phononic band gaps in the two previously-considered composite types – composites with electrically-conductive, nearly-incompressible, elastic inclusions as well as composites with electrically-conductive voids.

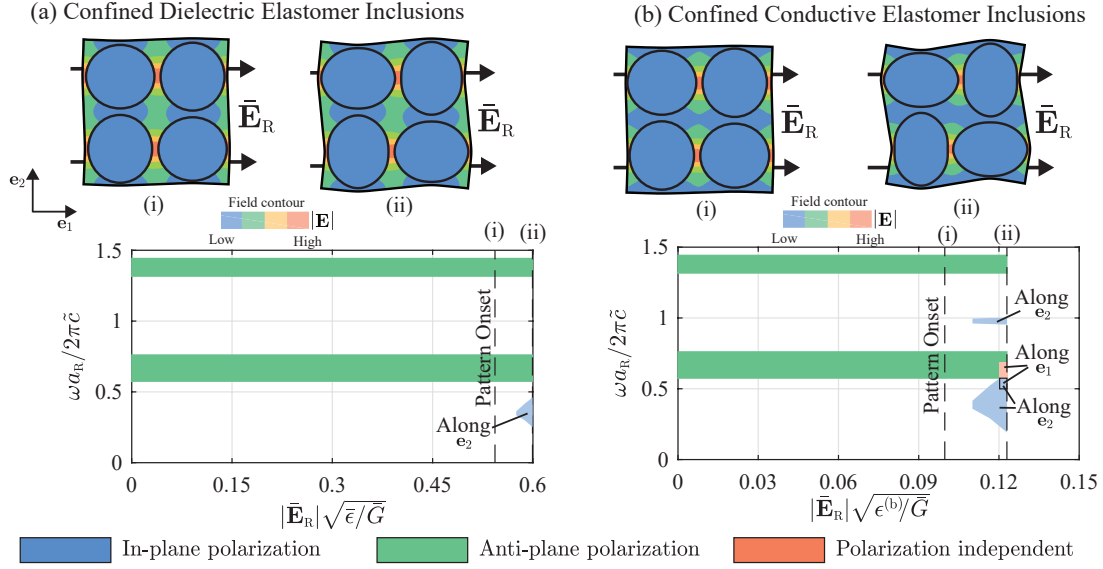


Figure 4.8: Electrical tunability of the fully-confined square-lattice dielectric elastomer composites with (a) dielectric elastomer inclusions with $\epsilon^{(a)}/\epsilon^{(b)} = 10$; and (b) conductive elastomer inclusions – i.e., the limit in which $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$. For both cases, $G^{(a)}/G^{(b)} = 1/100$, $\rho^{(a)}/\rho^{(b)} = 1$, and $\nu^{(a)} = 0.65$. Mesh density: $0.02a_R$.

Electrically-conductive inclusions: Here, we consider composites with electrically-conductive inclusions and explore the tunability of phononic band gaps for the case of the fully-confined square-lattice composite electrically loaded along the closest-packed direction. Figure 4.8 shows the dependence of phononic band gaps on the applied electric field for (a) a case in which the inclusions are made from dielectric elastomers with a high permittivity of $\epsilon^{(a)}/\epsilon^{(b)} = 10$ and (b) a case of electrically-conductive inclusions – i.e., the limit in which $\epsilon^{(a)}/\epsilon^{(b)} \rightarrow \infty$. For both cases, $G^{(a)}/G^{(b)} = 1/100$, $\rho^{(a)}/\rho^{(b)} = 1$, and $\nu^{(a)} = 0.65$, and the deformed composite microstructures with contours of the electric field are shown at selected levels of electrical preloading. The tunability displayed by both cases is quite similar. For the case of electrically-conductive inclusions, we find that band gaps open along both the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions, but only the later appear for the dielectric elastomer composite case (Fig. 4.8(a)). By replacing the dielectric elastomer inclusion with a conductive elastomer inclusion, we find that band gap tunability is slightly enhanced.

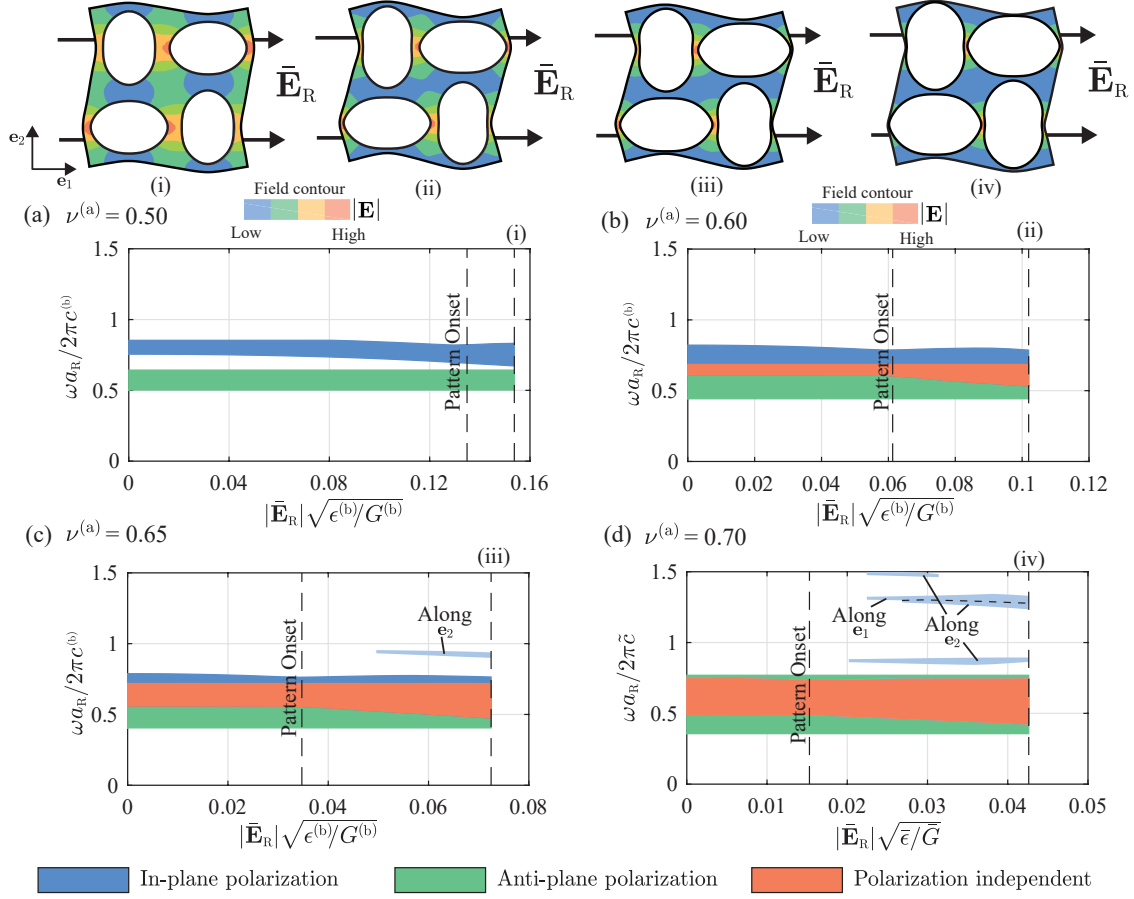


Figure 4.9: Electrical tunability of fully-confined dielectric elastomer composites with electrically-conductive voids for void volume fractions of (a) $\nu^{(a)} = 0.50$, (b) $\nu^{(a)} = 0.60$, (c) $\nu^{(a)} = 0.65$, and (d) $\nu^{(a)} = 0.70$. Mesh density: $0.02a_R$.

Electrically-conductive voids Finally, we consider fully-confined dielectric elastomers with periodic arrays of electrically-conductive voids. For this case, the tunability of phononic band gaps with the applied electric field is presented in Fig. 4.9. We assess band-gap tunability across a range of volume fractions – namely, (a) $\nu^{(a)} = 0.50$, (b) $\nu^{(a)} = 0.60$, (c) $\nu^{(a)} = 0.65$, and (d) $\nu^{(a)} = 0.70$ – and immediately observe wide polarization-independent band gaps. These band gaps decrease in width (i.e., range of frequencies affected) as the void volume fraction decreases. Also observed is the separation between the electric field at the pattern transformation onset and the pattern transformation limit – which also decreases as the void volume fraction decreases. We note that the total band gaps widen as the electric field magnitude increases, which is favorable from the perspective

of electric-field tunability. Additionally, the $v^{(a)} = 0.70$ case reveals the opening of several in-plane partial band gaps along the $\mathbf{e}_1$ - and $\mathbf{e}_2$ -directions.

4.5 Concluding remarks

In this chapter, we explored electric-field-triggered pattern transformations in dielectric elastomer composites. To do so, we applied the numerical simulation capability of Chapter 2 to consider both primitive cell and supercell stability and established the onset of pattern transformation and the subsequent onset of secondary instability for lattices of (i) dielectric elastomer inclusions, (ii) conductive elastomer inclusions, and (iii) conductive voids – for both macroscopically traction-free composites and fully-confined composites. Regarding secondary instability, interfacial instabilities limited pattern transformations for all cases at low volume fractions, while macroscopic instabilities often limited pattern transformations at large volume fractions. Interestingly, microscopic instabilities limited pattern transformation for the traction-free composite electrically loaded along the closest-packed direction for volume fractions between $0.5 \leq v^{(a)} \leq 0.6$, which may be further explored to invoke sequential pattern transformation.

We then considered electroelastic wave propagation in the presence of pattern transformation and studied the dependence of phononic band gaps on the applied electric field. First, we established base cases, in which we considered closest-packed electrical loading in a traction-free square-lattice composite and least-packed electrical loading in a fully-confined square-lattice composite. Importantly, we showed the opening of partial band gaps after the onset of pattern transformation in both cases. We also considered the effects of density contrast and fiber volume fraction as well as band gap tunability in composites with conductive inclusions and conductive voids. As anticipated by previous work on wave propagation through elastomers with voids [cf, e.g., 49], cases with voids revealed dramatic total band gaps and also showed electric-field-induced band-gap tunability. These observations motivate future work to design composites with either conductive inclusions or compliant-electrode-lined voids that may be used as electrodes to applied near-field electrical

loading.

In sum, this chapter showed that electric-field triggered pattern transformation and subsequent enhanced band gap tunability in dielectric elastomer composites is possible. Additionally, this work provides the groundwork and motivation to further explore band gap tunability in materials that are mechanically confined – which preclude mechanical tuning strategies – and can potentially arise in mechanically reinforced dielectric elastomer composites.

Chapter 5

Conclusions and outlook

5.1 Summary

This thesis developed a simulation capability for numerically exploring and designing dielectric elastomer composites. Electroelastic wave propagation in dielectric elastomer composites was studied in Chapter 2, followed by a detailed survey of the electromechanical instability landscape in Chapter 3. We concluded by showing how electrically-triggered pattern transformations can invoke changes in the wave propagation characteristics in these materials in Chapter 4. Specifically, the main conclusions and contributions of this thesis are summarized in the following:

1. A finite-element-based numerical simulation capability for infinite, periodic dielectric elastomer composites that addresses both nonlinear quasi-electrostatic processes and linearized, incremental processes was developed.
2. The simulation capability was applied to electrically-tunable soft phononic crystals, in which phononic band gaps may be tuned through the application of an electric field. Conclusions:
 - (a) The electromechanical pull-in instability can be used enable switch-like band-gap tunability;
 - (b) An applied electric field can induce inhomogeneous deformations, and in turn, cause

- phononic band gaps to shift, open, or close;
- (c) The shear modulus contrast, electrical permittivity contrast, and large stretch chain locking behavior of the constituent materials influences the nonlinear quasi-electrostatic deformation of the composites;
 - (d) The shear modulus, density, and electrical permittivity influence the phononic band gaps;
 - (e) The presence of macroscopic electromechanical instabilities limits tunability.
3. The simulation capability was used to identify and detect all electromechanical instabilities that can arise in dielectric elastomer composites. Conclusions:
- (a) We found that four instability types completely describe the electromechanical instability landscape: pull-in, microscopic, macroscopic, and interfacial instabilities;
 - (b) Interfacial instabilities are related to electro-creasing in homogeneous dielectric elastomers, and we showed they also occur in laminates in addition to composites comprised of aligned fibers in a matrix;
 - (c) Even fully confined dielectric elastomer composites show tunability – enabling opportunities not possible with mechanical actuation alone; and
 - (d) The presence of microscopic modes are observed, and these may be harnessed to invoke pattern transformation.
4. The simulation capability was used to demonstrate electric-field-triggered pattern transformation in dielectric elastomer composites and explore the use of pattern transformations to manipulate electroelastic waves. Conclusions:
- (a) Electric-field-triggered pattern transformation may be harnessed to manipulate phononic band gaps in dielectric elastomer composites;
 - (b) Total, or polarization independent, band gaps can emerge due to pattern transformation;

- (c) Secondary instabilities, occurring after the onset of pattern transformation, set a limit on the amount of tunability that may be achieved for a given composite;
- (d) Pattern transformation in hexagonal lattices is difficult to harness because of nearly simultaneous, and hence competing, microscopic instabilities;
- (e) Electric-field-triggered pattern transformation, and hence tunability, can be achieved in fully-confined composites; and
- (f) The observation of tunability in fully-confined composites provides motivation for electrically-tunable phononic band gaps in structurally reinforced dielectric elastomer composites.

5.2 Future research directions

While significant progress on the development on a numerical simulation capability for dielectric elastomer composites has been made in this thesis, there are numerous directions for future work. Several of these directions are highlighted below:

Structural reinforcement: While the use of soft solids for vibration isolation is standard, we envision the broader use of these materials, in which one is able to support significant structural loads while maintaining some form of band gap tunability. On one extreme is the seminal work on linear elastic phononic crystals [32–36], which demonstrates band gaps for metallic-metallic phononic crystal composites, and the other extreme is the research on purely elastomeric phononic crystals as described in this thesis. The work of Chapters 3 and 4 demonstrated that tunability may be achieved in mechanically confined composites, and we further speculate that phononic band gaps might be manipulated in reinforced composites. An illustration of these composites is envisioned for several lattices in Fig. 5.1. In the figure, structural load is supported by the reinforcement shown in red.

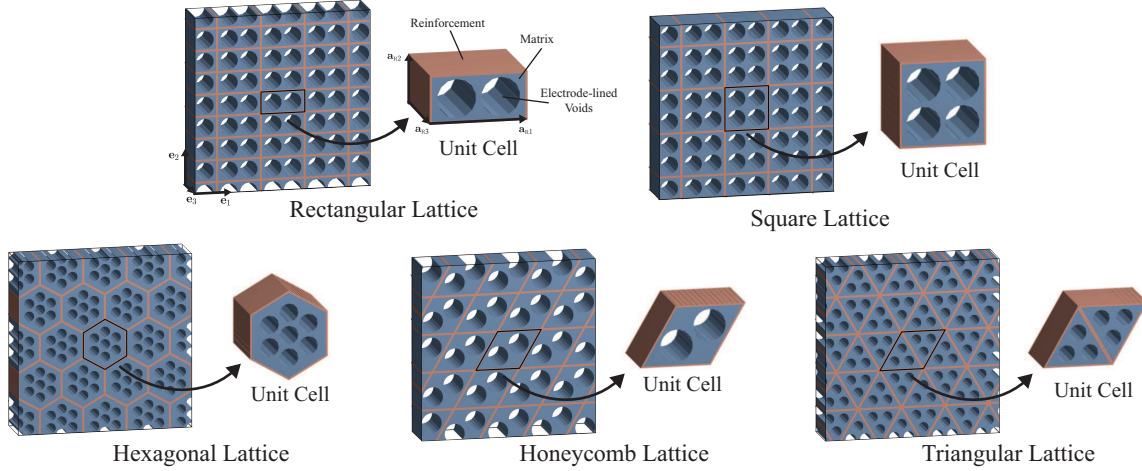


Figure 5.1: Schematic illustration of potential structurally-reinforced tunable phononic crystals. Here, the structural load is supported by a stiff reinforcement material, shown in red. We envision applications, in which phononic band gaps can be tuned by the application of an electric field.

Deformation dependent permittivity: The assumption of an “ideal dielectric” may be relaxed to incorporate stretch-dependence. Experimental evidence of this dependence has been reported [e.g., 115, 116] and analytically studied [e.g., 6, 71, 117, 118]. However, often this dependence is small [72] and neglected. Either way, it would be straightforward to extend the ideal dielectric constitutive model utilized in this thesis to a deformation dependent model to assess its effect and perhaps provide more accurate numerical simulations.

Viscoelastic effects: Typically elastomers possess some viscosity and exhibit time- and frequency-dependent material properties. The treatment developed in this thesis neglected these effects. In problems involving dynamic loading, one may invoke a form of non-linear viscoelasticity, such as the one considered by Wang et al. [64]. The role of damping also affects the linearized wave propagation properties, which we have neglected in the present work. For illustrative purposes, Fig. 5.2 qualitatively illustrates the effect of structural damping on a purely mechanical phononic crystal. In this example, the structural damping coefficient η assumes that the damping forces are proportional to the forces caused by stressing of the structure and are opposed to the velocity. See Abaqus [74] for further details.

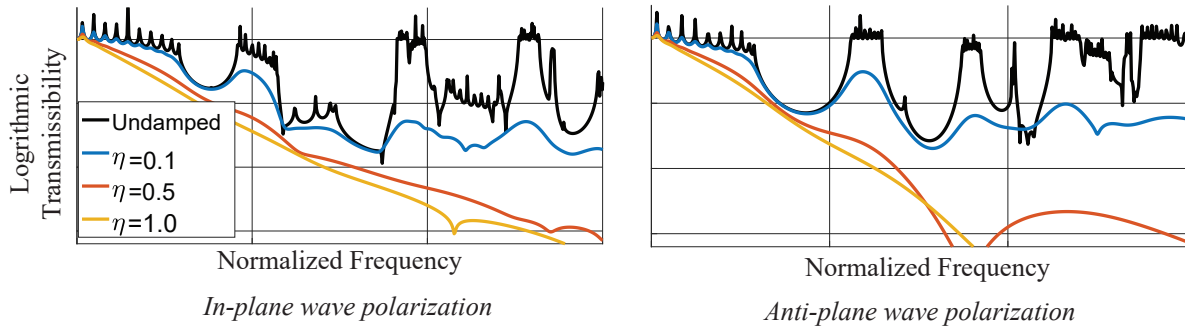


Figure 5.2: Qualitative illustration of the effect of structural damping on a mechanical phononic crystal.

Electrets: An electret, analogous to a magnet, is a dielectric material that has a quasi-permanent electric charge or dipole polarization. They can be constructed by curing a dielectric material in the presence of a strong electric field. Upon cure and removal of the external electric field, the dielectric elastomer will remain polarized – creating its own residual electric field. This effect has been historically demonstrated with carnauba wax [e.g., 119], and these materials are commonly employed in electret microphones. It would be possible to account for electrets in this present work, which could provide interesting avenues for design and tunability.

Magnetostrictive elastomers: Magnetostrictive elastomers provide an analogous alternative to dielectric elastomers, in which a magnetic field is used to induce deformation opposed to an electric field. Working with high voltages is mitigated, and this feature is often of substantial appeal. Additionally, much of the theoretical framework for modeling dielectric elastomers has been extended to magnetostrictive elastomers [cf., e.g., 120, 121]. As a result, it would be possible to extend the numerical procedures to simulate magnetostrictive elastomers and compare against the existing work.

Electromechanical metamaterials: The numerical framework developed in this thesis may be applied to other metamaterial concepts, such as elastic cloaks [e.g., 122–124] and gradient-index devices [e.g., 125–127]. We envision designing tunable cloaks and wave steering devices with

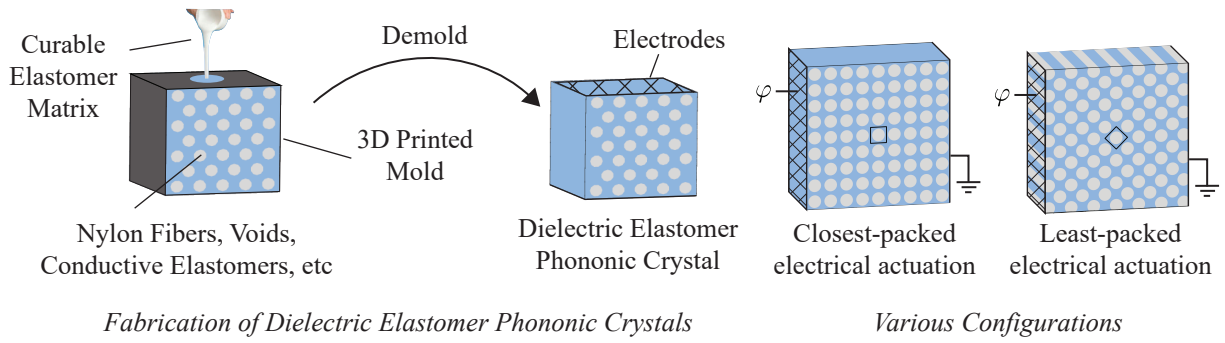


Figure 5.3: Potential fabrication strategy for dielectric elastomer composites. On the left, a 3D-printed mold and various materials may be used to fabricate dielectric elastomer composites and, on the right, various lattices and electrode arrangements may be considered.

dielectric elastomers.

5.3 Fabrication opportunities

The work of this thesis showed that periodic dielectric elastomer composites may be designed to realize tunable soft phononic crystals, while navigating the complex instability landscape in these materials. However, to move beyond the current state-of-the-art and achieve electrically-tunable soft phononic crystals in practice, novel strategies for fabricating these materials are required. As shown in Fig. 5.3, we envision utilizing 3D printed molds that may be used to cast and cure a silicone-based dielectric elastomer (e.g., BJB Enterprises TC-5005) around inclusions such as fibers. Additionally, electrodes can be attached to surfaces or even embedded within the composite microstructure, using conductive metals (e.g., Copper, Aluminum), conductive polymers (e.g., Stockwell SNEF-60), or conductive greases (e.g., MG Carbon Conductive Grease). As demonstrated throughout this thesis, these composites may be arranged with various lattices, with the square-lattice closest-packed and least-packed arrangements illustrated in Fig. 5.4.

As a further guide and motivation, Appendix C outlines several cases of composites comprised from specific materials, showing that each of the observed electromechanical instabilities as well as a case enabling pattern transformation may occur in realistic scenarios. Additionally, we specialize and

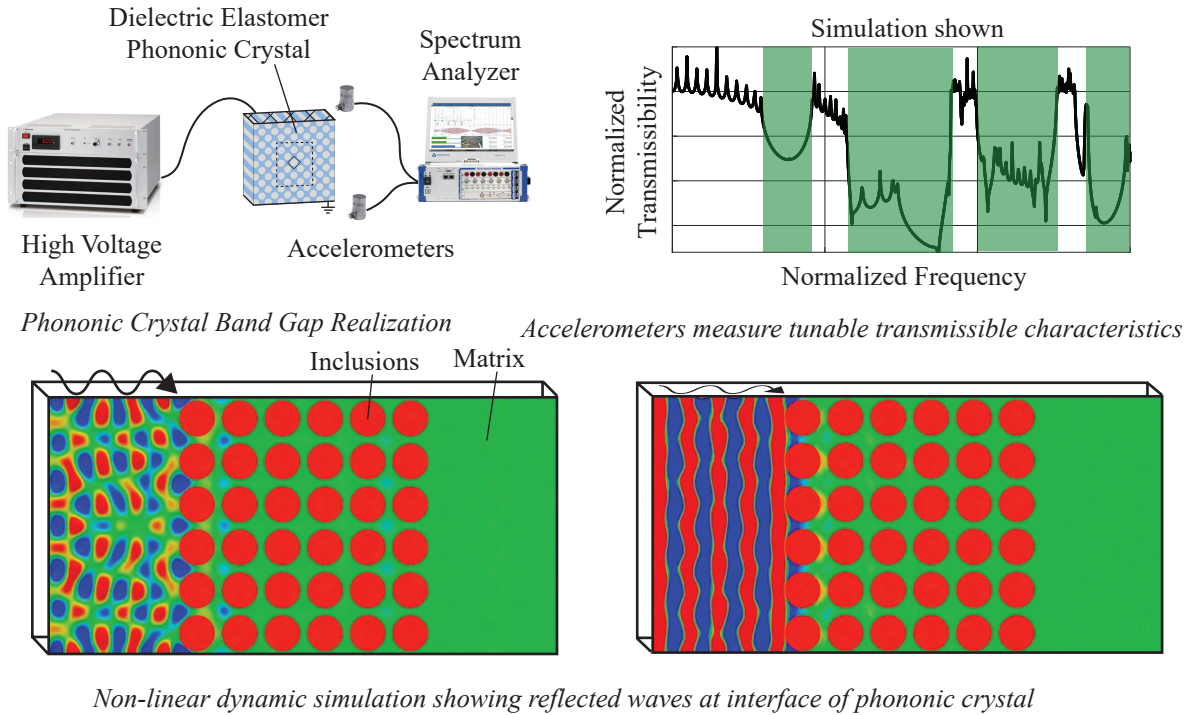


Figure 5.4: Potential experimental realization of soft tunable phononic crystals. Top left shows notional vibration testing, while the top right shows representative frequency domain results of the transmissibility as a function of frequency with band gaps denoted in green. The bottom shows an example non-linear dynamic simulation of an incoming wave at the boundary of a phononic crystal. Here the contours represent the vibration amplitude for an incoming in-plane polarized wave (left) and an anti-plane polarized wave (right).

compute actual required voltages for each case and conclude these are within current technological limits. We found that by introducing soft, low permittivity (polyethylene) fibers, we are able to suppress the pull-in instability and cause the material to instead be limited by chain-locking. We also show that dielectric elastomer composites with glass-fiber inclusions exhibit macroscopic instabilities and that composites with cylindrical water-filled cavities exhibit pattern transformation.

5.4 Experimental opportunities

An experimental validation of predictions of phononic band gaps in dielectric elastomer composites is undoubtedly required. However, several challenges must be first overcome. The extremely high

electric potentials required to deform the dielectric elastomer composite necessitate very small electrode spacing – typically on the order of millimeters – which presents a challenge of embedding them into the dielectric elastomer composite without affecting the response. Alternatively, one could consider using the inclusions as electrodes themselves. This could be easily explored numerically and our preliminary investigation of this showed little qualitative difference between far-field and near-field electrical actuation. The required voltage must be balanced with the expected phononic band gap frequencies. Typically, these frequencies will be high, on the order of kilohertz. Once a design candidate is selected for fabrication, we envision the application of an electric field with superimposed structural vibration as schematically illustrated in Fig. 5.4. Accelerometers may be used to measure the vibration transmissibility across a finite structure and may be compared to numerical simulations, analogous to the approach used for purely mechanical phononic crystals [e.g., 111, 128]. Figure 5.4(bottom) shows a non-linear dynamic simulation of an incoming wave at the boundary of a phononic crystal – demonstrating the effectiveness of a phononic band gap with a finite-size 6×6 phononic crystal.

5.5 Final remarks

This thesis provides a fundamental numerical simulation capability for periodic dielectric elastomer composites. The periodic nature of these materials defines a phononic crystal, in which electroelastic waves may be manipulated. We showed how the application of an electric field may be used to invoke deformation, instabilities, and pattern transformation in dielectric elastomer composites. We showed how large phononic band gap tunability may be realized by harnessing instabilities, and we concluded by guiding future research and providing suggestions for experimental validations. Additionally, the ideas developed in this thesis may be readily adapted to other smart materials such as elastic cloaks and gradient-index devices – opening the door for electromechanical metamaterials.

Appendix A

Dielectric elastomer laminates

A.1 Wave propagation in dielectric elastomer laminates

In order to provide a simple verification of our numerical simulation capability, we consider an infinite, incompressible, multilayered body undergoing finite deformations under the action of an applied electric field, as first considered by Shmuel and deBotton [52] and later corrected by Galich and Rudykh [53]. The solid consists of alternating, perfectly-bonded, ideally-incompressible layers of materials a and b with initial thicknesses $H^{(a)}$ and $H^{(b)}$ in the undeformed configuration and infinite in the lateral directions. As shown in Fig. A.1(a), we introduce a fixed Cartesian coordinate system with orthonormal basis vectors $\{\mathbf{e}_i | i = 1, 2, 3\}$ and with $\mathbf{e}_3$ perpendicular to the plane of lamination in the reference configuration. The volume fractions of the two phases are denoted by $v^{(a)}$ and $v^{(b)}$ – i.e., $v^{(a)} = H^{(a)}/H$ and $v^{(b)} = H^{(b)}/H$ with $H = H^{(a)} + H^{(b)}$. We take the constitutive response of the phases α ($\alpha = a, b$) to be described by the ideal dielectric elastomer model (2.2.18) with the mechanical free energy given by the Neo-Hookean model,

$$\phi_r^{\text{mech}(\alpha)} = \frac{1}{2} G^{(\alpha)} (\text{tr} \mathbf{B}^{(\alpha)} - 3), \quad (\text{A.1.1})$$

where $G^{(\alpha)}$ is the ground-state shear modulus of phase α , and the corresponding spatial mechanical moduli for a Neo-Hookean material are given by $\mathbb{C}_{ijkl}^{\text{mech}(\alpha)} = G^{(\alpha)} \delta_{ik} B_{jl}^{(\alpha)}$.

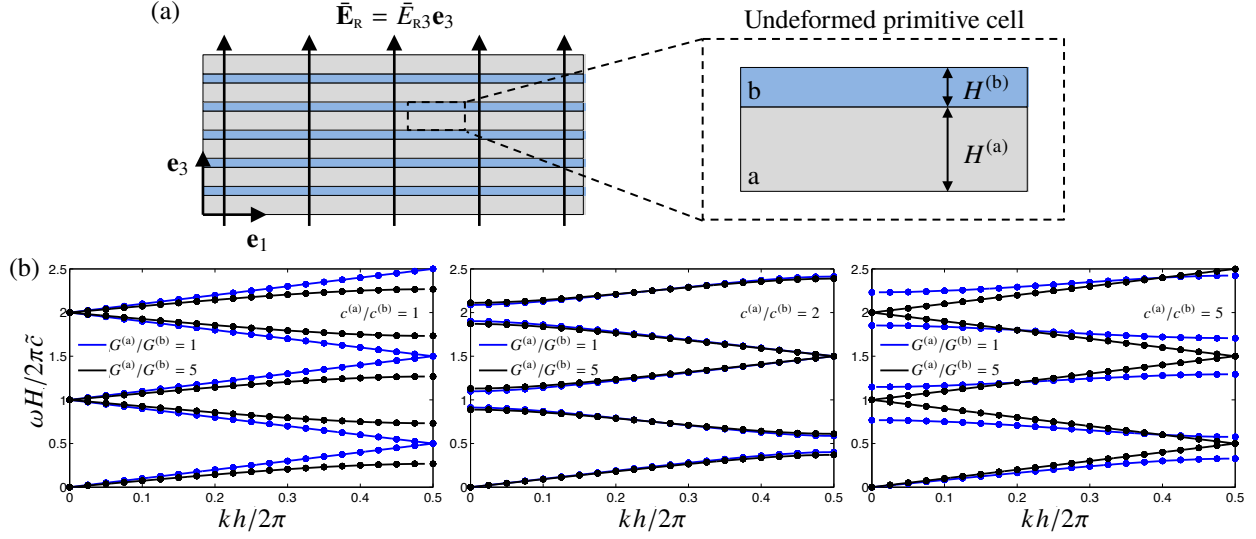


Figure A.1: (a) Schematic of an infinite, two-phase laminate, subjected to quasi-electrostatic preloading consisting of an electric field in the $\mathbf{e}_3$ -direction and the corresponding undeformed primitive unit cell. (b) Dispersion relations for electroelastic waves propagating through the composite perpendicular to the plane of lamination for several combinations of ground-state shear wave speed contrast, $c^{(a)}/c^{(b)}$, and ground-state shear modulus contrast, $G^{(a)}/G^{(b)}$, and for $\nu^{(a)} = \nu^{(b)} = 0.5$. Solid lines represent the analytical dispersion relation (A.1.15), and symbols represent the results of corresponding finite-element simulations.

A.1.1 Nonlinear quasi-electrostatic preloading

We consider a nonlinear quasi-electrostatic preloading compatible with compliant electrodes parallel to the plane of lamination, i.e., $\bar{\mathbf{E}}_R = \bar{E}_{R3}\mathbf{e}_3$, and assume far-field, mechanically-traction-free boundary conditions in all three directions, so that the problem is axially-symmetric about the $\mathbf{e}_3$ -direction. As in the main body of the thesis, we use an overbar to denote macroscopic quantities averaged over the primitive cell shown in Fig. A.1(a). The macroscopically-applied electric field is related to the electric field in each phase through

$$\bar{E}_{R3} = \nu^{(a)}E_{R3}^{(a)} + \nu^{(b)}E_{R3}^{(b)}. \quad (\text{A.1.2})$$

Invoking incompressibility, the electrostatic loading induces a homogeneous deformation in each phase α with principal stretches given by

$$\lambda_1^{(\alpha)} = \lambda_2^{(\alpha)} = \lambda^{(\alpha)}, \quad \lambda_3^{(\alpha)} = \left(\lambda^{(\alpha)}\right)^{-2}. \quad (\text{A.1.3})$$

Perfect bonding between layers requires that both phases share the same lateral stretch and consequently that $\lambda^{(\alpha)} = \lambda$. Next, it follows from the relation $\mathbf{E} = \mathbf{F}^{-\top} \mathbf{E}_R$ and the constitutive equation (2.2.19)₂ that the components of the spatial electric displacement vector are

$$D_1^{(\alpha)} = D_2^{(\alpha)} = 0, \quad D_3^{(\alpha)} = \varepsilon^{(\alpha)} \lambda^2 E_{R3}^{(\alpha)}, \quad (\text{A.1.4})$$

where $\varepsilon^{(\alpha)}$ is the permittivity of phase α . Continuity of the normal component of the electric displacement vector across the interface implies that $D_3^{(a)} = D_3^{(b)}$, yielding

$$\varepsilon^{(a)} E_{R3}^{(a)} = \varepsilon^{(b)} E_{R3}^{(b)}. \quad (\text{A.1.5})$$

Using (2.2.19)₁ and (A.1.1), the non-zero components of the Cauchy stress in each phase are given by

$$\begin{aligned} T_{11}^{(\alpha)} &= T_{22}^{(\alpha)} = G^{(\alpha)} \lambda^2 - \frac{1}{2} \varepsilon^{(\alpha)} \lambda^4 \left(E_{R3}^{(\alpha)}\right)^2 - p^{(\alpha)}, \\ T_{33}^{(\alpha)} &= \frac{G^{(\alpha)}}{\lambda^4} + \frac{1}{2} \varepsilon^{(\alpha)} \lambda^4 (E_{R3}^{(\alpha)})^2 - p^{(\alpha)}, \end{aligned} \quad (\text{A.1.6})$$

where $p^{(\alpha)}$ is a constitutively-indeterminate pressure in phase α , introduced to account for incompressibility. Continuity of traction at the interface requires that $T_{33}^{(a)} = T_{33}^{(b)}$, and since far-field traction-free boundary conditions are considered,

$$\begin{aligned} \bar{T}_{11} &= \bar{T}_{22} = \nu^{(a)} T_{11}^{(a)} + \nu^{(b)} T_{11}^{(b)} = 0, \\ \bar{T}_{33} &= T_{33}^{(a)} = T_{33}^{(b)} = 0. \end{aligned} \quad (\text{A.1.7})$$

Upon combining (A.1.5), (A.1.2), (A.1.6), and (A.1.7) and rearranging, we obtain the quasi-electrostatic actuation response of the laminate:

$$\bar{E}_{R3} \sqrt{\frac{\tilde{\epsilon}}{\bar{G}}} = \frac{1}{\lambda} \sqrt{\left(1 - \frac{1}{\lambda^6}\right)}, \quad (\text{A.1.8})$$

with $\bar{G} = \mathbf{v}^{(a)} G^{(a)} + \mathbf{v}^{(b)} G^{(b)}$ and $\tilde{\epsilon} = (\mathbf{v}^{(a)} / \epsilon^{(a)} + \mathbf{v}^{(b)} / \epsilon^{(b)})^{-1}$.

A.1.2 Linearized wave propagation

Next, we consider the propagation of small-amplitude waves through the time-independent base state (A.1.8). As in Shmuel and deBotton [52], we limit attention to electroelastic waves propagating perpendicular to the plane of lamination, i.e., in the $\mathbf{e}_3$ -direction. Further, led by Shmuel and deBotton [52] and [53], for waves propagating through incompressible constituents in the $\mathbf{e}_3$ -direction, we anticipate that displacement increments only occur in the plane of lamination, so that $\dot{u}_3 = 0$, and that pressure increments are also zero. Since the problem is axisymmetric about the $\mathbf{e}_3$ -direction, for simplicity, we only consider waves in the $\mathbf{e}_1$ - $\mathbf{e}_3$ plane. Then, in each layer α , we expect solutions to the frequency-domain incremental governing equations (2.2.16) in the form

$$\begin{aligned} \tilde{u}_1^{(\alpha)} &= U^{(\alpha)} \exp\left(i q^{(\alpha)} x_3\right), \\ \tilde{u}_2^{(\alpha)} &= \tilde{u}_3^{(\alpha)} = 0, \\ \tilde{\phi}^{(\alpha)} &= \Phi^{(\alpha)} \exp\left(i q^{(\alpha)} x_3\right), \end{aligned} \quad (\text{A.1.9})$$

where $U^{(\alpha)}$ and $\Phi^{(\alpha)}$ collectively represents the magnitude of the electroelastic wave in layer α , and $q^{(\alpha)}$ is the wave number in layer α . Substituting (A.1.9) into (2.2.16) leads to two nontrivial governing equations – the balance of momentum in the $\mathbf{e}_1$ -direction and Gauss's law:

$$\begin{aligned} \mathbb{C}_{1313}^{(\alpha)} q^{(\alpha)2} U^{(\alpha)} + \Gamma_{133}^{(\alpha)} q^{(\alpha)2} \Phi^{(\alpha)} &= \rho^{(\alpha)} \omega^2 U^{(\alpha)} \\ \Lambda_{313}^{(\alpha)} q^{(\alpha)2} U^{(\alpha)} + A_{33}^{(\alpha)} q^{(\alpha)2} \Phi^{(\alpha)} &= 0, \end{aligned} \quad (\text{A.1.10})$$

where $\rho^{(\alpha)}$ is the mass density of phase α , and we have made use of the fact that the spatial moduli (2.2.12) are homogeneous in each layer α . For an ideal dielectric elastomer with base-state electric field directed along the $\mathbf{e}_3$ -direction, (2.2.20) gives that $\mathbb{C}_{1313} = \mathbb{C}_{1313}^{\text{mech}(\alpha)}$, $\Gamma_{133}^{(\alpha)} = 0$, $\Lambda_{313}^{(\alpha)} = 0$, and $A_{33}^{(\alpha)} = \varepsilon$, so that (A.1.10) simplifies to

$$\mathbb{C}_{1313}^{\text{mech}(\alpha)} q^{(\alpha)2} U^{(\alpha)} = \rho^{(\alpha)} \omega^2 U^{(\alpha)} \quad \text{and} \quad \Phi^{(\alpha)} = 0. \quad (\text{A.1.11})$$

Therefore, for this problem, the electric potential increments are zero, and it follows that the electric displacement increments are also zero, so that the jump condition (2.2.14)₂ is trivially satisfied. Hence, the base-state electric field and the permittivities of the two phases will not enter the final dispersion relation. In other words, for waves propagating perpendicular to the plane of lamination, electroelastic wave propagation through an electrostatically-deformed laminate is identical to purely elastic wave propagation through a mechanically-deformed laminate for equivalent base states of deformation.

For Neo-Hookean layers, using the base state of deformation (A.1.3), we have that $\mathbb{C}_{1313}^{\text{mech}(\alpha)} = G^{(\alpha)}/\lambda^4$, and it follows from (A.1.11), that there are two permissible values of q in each layer α ,

$$q_1^{(\alpha)} = \omega \lambda^2 \sqrt{\frac{\rho^{(\alpha)}}{G^{(\alpha)}}}, \quad q_2^{(\alpha)} = -\omega \lambda^2 \sqrt{\frac{\rho^{(\alpha)}}{G^{(\alpha)}}}. \quad (\text{A.1.12})$$

Defining the ground-state shear wave speed of phase α as $c^{(\alpha)} = \sqrt{G^{(\alpha)}/\rho^{(\alpha)}}$, it then follows that the displacement and shear stress increments in each layer α are

$$\begin{aligned} \tilde{u}_1^{(\alpha)} &= U_1^{(\alpha)} \exp\left(\iota \frac{\omega \lambda^2 x_3}{c^{(\alpha)}}\right) + U_2^{(\alpha)} \exp\left(-\iota \frac{\omega \lambda^2 x_3}{c^{(\alpha)}}\right), \\ \tilde{T}_{13}^{(\alpha)} &= \mathbb{C}_{1313}^{\text{mech}(\alpha)} \frac{\partial \tilde{u}_1^{(\alpha)}}{\partial x_3} = \iota \frac{\omega}{\lambda^2} \frac{G^{(\alpha)}}{c^{(\alpha)}} \left[U_1^{(\alpha)} \exp\left(\iota \frac{\omega \lambda^2 x_3}{c^{(\alpha)}}\right) - U_2^{(\alpha)} \exp\left(-\iota \frac{\omega \lambda^2 x_3}{c^{(\alpha)}}\right) \right], \end{aligned} \quad (\text{A.1.13})$$

where $U_1^{(\alpha)}$ and $U_2^{(\alpha)}$ are constants representing the wave amplitude in layer α for wavenumbers $q_1^{(\alpha)}$ and $q_2^{(\alpha)}$, respectively.

Finally, it remains to apply the appropriate continuity and Bloch-Floquet boundary conditions. First, continuity of displacement and traction at the interface requires that $\tilde{u}_1^{(b)}(h^{(a)}) = \tilde{u}_1^{(a)}(h^{(a)})$ and $\tilde{T}_{13}^{(b)}(h^{(a)}) = \tilde{T}_{13}^{(a)}(h^{(a)})$ where $h^{(\alpha)} = H^{(\alpha)}/\lambda^2$ is the deformed thickness of layer α . Second, Bloch-Floquet boundary conditions require that $\tilde{u}_1^{(b)}(h) = \tilde{u}_1^{(a)}(0) \exp(\imath kh)$ and $\tilde{T}_{13}^{(b)}(h) = \tilde{T}_{13}^{(a)}(0) \exp(\imath kh)$ where $h = h^{(a)} + h^{(b)}$ and k is the spatial Bloch wave number. Using (A.1.13), these four conditions may be expressed through the following system of equations:

$$\begin{bmatrix} \exp\left(\imath \frac{\omega H^{(a)}}{c^{(b)}}\right) & \exp\left(-\imath \frac{\omega H^{(a)}}{c^{(b)}}\right) & -\exp\left(\imath \frac{\omega H^{(a)}}{c^{(a)}}\right) & -\exp\left(-\imath \frac{\omega H^{(a)}}{c^{(a)}}\right) \\ \frac{G^{(b)}}{c^{(b)}} \exp\left(\imath \frac{\omega H^{(a)}}{c^{(b)}}\right) & -\frac{G^{(b)}}{c^{(b)}} \exp\left(-\imath \frac{\omega H^{(a)}}{c^{(b)}}\right) & -\frac{G^{(a)}}{c^{(a)}} \exp\left(\imath \frac{\omega H^{(a)}}{c^{(a)}}\right) & \frac{G^{(a)}}{c^{(a)}} \exp\left(-\imath \frac{\omega H^{(a)}}{c^{(a)}}\right) \\ \exp\left(\imath \frac{\omega H}{c^{(b)}}\right) & \exp\left(-\imath \frac{\omega H}{c^{(b)}}\right) & -\exp(\imath kh) & -\exp(\imath kh) \\ \frac{G^{(b)}}{c^{(b)}} \exp\left(\imath \frac{\omega H}{c^{(b)}}\right) & -\frac{G^{(b)}}{c^{(b)}} \exp\left(-\imath \frac{\omega H}{c^{(b)}}\right) & -\frac{G^{(a)}}{c^{(a)}} \exp(\imath kh) & \frac{G^{(a)}}{c^{(a)}} \exp(\imath kh) \end{bmatrix} \begin{bmatrix} U_1^{(b)} \\ U_2^{(b)} \\ U_1^{(a)} \\ U_2^{(a)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{A.1.14})$$

To obtain a nontrivial solution, we set the determinant of the matrix appearing in (A.1.14) to zero, yielding the following dispersion relation:

$$\cos(kh) = \cos\left(\frac{\omega H^{(a)}}{c^{(a)}}\right) \cos\left(\frac{\omega H^{(b)}}{c^{(b)}}\right) - \frac{1}{2} \left(\frac{G^{(a)} c^{(b)}}{G^{(b)} c^{(a)}} + \frac{G^{(b)} c^{(a)}}{G^{(a)} c^{(b)}} \right) \sin\left(\frac{\omega H^{(a)}}{c^{(a)}}\right) \sin\left(\frac{\omega H^{(b)}}{c^{(b)}}\right), \quad (\text{A.1.15})$$

which is identical to the dispersion relation derived by Galich and Rudykh [53].¹ As anticipated, the dispersion relation does not depend upon the base-state electric field or the permittivities of the two phases. This property holds regardless of the mechanical free energy density function $\phi_R^{\text{mech}(\alpha)}$ employed for the phases α . For Neo-Hookean phases, the dispersion relation (A.1.15) is independent of deformation, and hence band gaps cannot be tuned through the application of an electric field perpendicular to the plane of lamination (or equivalent purely mechanical

¹We note that the dispersion relation (A.1.15) contradicts the result of Shmuel and deBotton [52] and their claim that phononic band gaps may be shifted downward through the application of an electric field perpendicular to the plane of lamination. Recently, Shmuel and deBotton [129] corrected several errors in their analysis but in doing so, introduced new mistakes leading to the erroneous conclusion that phononic band gaps may be shifted upward with increasing electric field. Our dispersion relation (A.1.15) is corroborated by its agreement with the result of Galich and Rudykh [53].

deformation). As observed by Galich and Rudykh [53], this is because the change in layer thickness and the dependence of $\mathbb{C}_{1313}^{\text{mech}}$ on λ for a Neo-Hookean material exactly cancel one another out. Galich and Rudykh [53] further showed that when the layers are described by a free energy function possessing large-strain stiffening, such as the Gent model, this mathematical simplification does not occur, and the dispersion relation does depend upon λ , yielding phononic band gaps that shift upward with increasing λ .

Since the acoustic response – and hence phononic band gaps – is not tunable for Neo-Hookean layers, this problem is not particularly interesting from an application perspective; however, it does provide a useful analytical solution for the verification of our numerical simulation capability. In Fig. A.1(b), the dispersion relation (A.1.15) is plotted using solid lines for several combinations of ground-state shear wave speed contrast, $c^{(a)}/c^{(b)}$, and ground-state shear modulus contrast, $G^{(a)}/G^{(b)}$, for $\nu^{(a)} = \nu^{(b)} = 0.5$, and for spatial Bloch wave numbers over the range $0 \leq kh/2\pi \leq 0.5$, i.e., the deformed irreducible Brillouin zone for this one-dimensional problem. In normalizing the frequency ω in Fig. A.1(b), as in the body of the thesis, we utilize the reciprocal volume-averaged ground-state shear wave speed $\tilde{c} = (\nu^{(a)}/c^{(a)} + \nu^{(b)}/c^{(b)})^{-1}$. The dispersion relation is also calculated for corresponding combinations of parameters using the finite-element-based numerical simulation capability described in §2.3 and plotted using symbols in Fig. A.1(b). The numerical simulations were performed using a unit cell consisting of a single column of three-dimensional, eight-node hexahedral elements along the $\mathbf{e}_3$ -direction with an undeformed mesh size of $H/200$. The numerical results agree well with the analytical dispersion relation (A.1.15), providing a simple verification of our numerical simulation capability. We also numerically calculated the dispersion relation for several magnitudes of the applied electric field in the $\mathbf{e}_3$ -direction, and identical results were obtained, confirming that band gaps cannot be tuned through the application of an electric field perpendicular to the plane of lamination for Neo-Hookean phases.

A.2 Instabilities in dielectric elastomer laminates

In this section, we demonstrate that interfacial instabilities occur in infinite, incompressible, multi-layered dielectric elastomer laminates undergoing finite deformations under the action of an electric field applied perpendicular to the plane of lamination. We consider a solid consisting of alternating, perfectly-bonded, nearly-incompressible layers of materials a and b that are infinite in the lateral directions. As shown in Fig. A.2(a), we introduce a fixed Cartesian coordinate system with orthonormal basis vectors $\{\mathbf{e}_i | i = 1, 2, 3\}$ and with $\mathbf{e}_3$ perpendicular to the plane of lamination in the reference configuration. The volume fractions of the two phases are denoted by $\nu^{(a)}$ and $\nu^{(b)}$. We take the constitutive response of the phases α ($\alpha = a, b$) to be described by the ideal dielectric elastomer model with mechanical free energy given by the Gent model as described in §2.2, and we take the Gent locking parameter in both phases to be the same and given by $I_m^{(a)} = I_m^{(b)} = 10$.

Figure A.2 shows electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, as a function of the ground-state shear modulus contrast, $G^{(a)}/G^{(b)}$, and the electric permittivity contrast, $\epsilon^{(a)}/\epsilon^{(b)}$ – all at the onset of instability. Figures A.2(b), (c), and (d) correspond to volume fractions of $\nu^{(a)} = 0.95$, $\nu^{(a)} = 0.90$, and $\nu^{(a)} = 0.85$, respectively. Interfacial instabilities emerge when $\nu^{(a)}$ and $\epsilon^{(a)}/\epsilon^{(b)}$ become large – in other words, in situations involving thin layers of low electrical permittivity material surrounded by much larger layers of high permittivity material. Otherwise, pull-in is the limiting instability. Example deformed composite shapes and contours of the electric field magnitude after the onset of interfacial instability are shown for each volume-fraction case. Numerical simulations beyond the onset of interfacial instability were performed using a dynamic, implicit procedure at a very slow electrical loading rate, and a small geometric perturbation was used to trigger the instability towards the center of the unit cell. We note that cases displaying interfacial instability are the opposite of the cases considered by Bertoldi and Gei [70], who considered thin layers of high electrical permittivity material and did not observe interfacial instabilities.

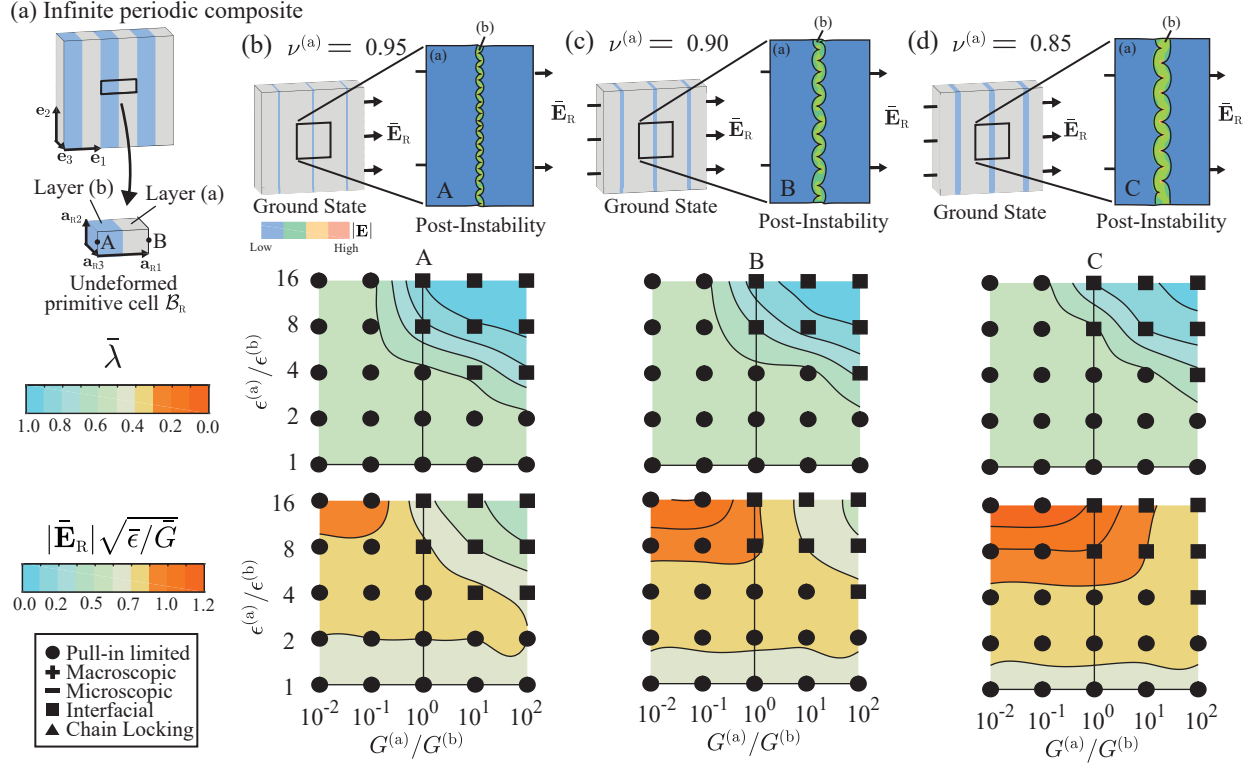


Figure A.2: (a) Undeformed primitive unit cell $\mathcal{B}_R$ spanned by referential lattice vectors $\mathbf{a}_{R1}$, $\mathbf{a}_{R2}$, and $\mathbf{a}_{R3}$ for an infinite, two-phase laminate. (b,c,d) Electromechanical instability phase diagrams and contour plots of the macroscopic stretch, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for infinite, nearly-incompressible, multilayered dielectric elastomer laminates made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$ and for three volume fractions: (b) $\nu^{(a)} = 0.95$, (c) $\nu^{(a)} = 0.90$, and (d) $\nu^{(a)} = 0.85$. For each volume fraction, a snapshot of the deformed composite microstructure and contours of the electric field magnitude are shown for the case of an elastically homogeneous composite, $G^{(a)}/G^{(b)} = 1$, with a high permittivity mismatch, $\epsilon^{(a)}/\epsilon^{(b)} = 16$, after the onset of interfacial instability. Mesh density: $0.02a_R$.

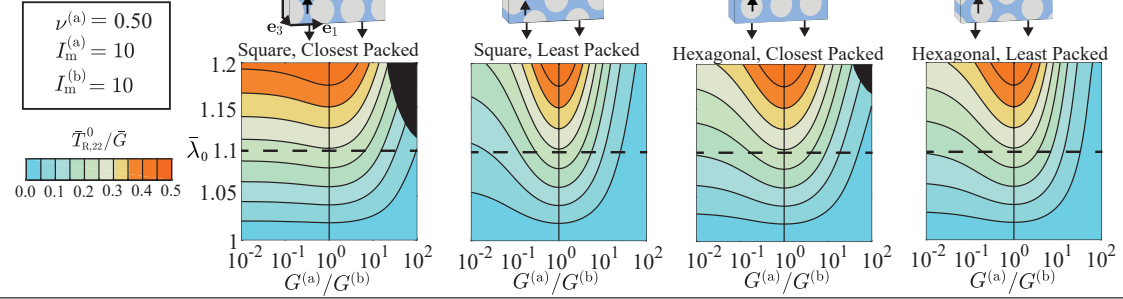
Appendix B

Electromechanical instabilities in dielectric elastomer composites: Additional studies

B.1 Prestress

In this appendix, we include several additional results related to the effect of prestress and prestretch on electromechanical instabilities in dielectric elastomer composites. First, regarding prestress, we explore the effect of an applied uniaxial tensile prestress in the in-plane $\mathbf{e}_2$ -direction in Fig. B.1. The purely mechanical response during prestressing is shown in Fig. B.1(a). The macroscopic normal stress along the $\mathbf{e}_2$ -direction, $\bar{T}_{R,22}^0/\bar{G}$, required to attain a given stretch along the prestressing direction, $\bar{\lambda}^0$, over the full range of $G^{(a)}/G^{(b)}$ is shown in Fig. B.1(a) for both square- and hexagonal-lattice composites and for the $\mathbf{e}_1$ -direction aligned with the closest-packed and least-packed directions. Then, we apply a prestress corresponding to a value of $\bar{\lambda}^0 = 1.10$, as shown by the dashed lines in Fig. B.1(a), and apply a macroscopic electric field along the $\mathbf{e}_1$ -direction, while holding $\bar{T}_{R,22}^0$ fixed. Since $\bar{T}_{R,22}^0$ remains fixed, the stretch along the $\mathbf{e}_2$ -direction does change during electrical loading. Figure B.1(b) shows the electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_R|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability. All four combinations of lattice geometry and electrical loading direction are pictured. Comparing these results to the case of

In-plane prestress

(a) $\bar{T}_{R,22}^0/\bar{G}$ vs. $\bar{\lambda}^0$ 

(b) Electromechanical stability

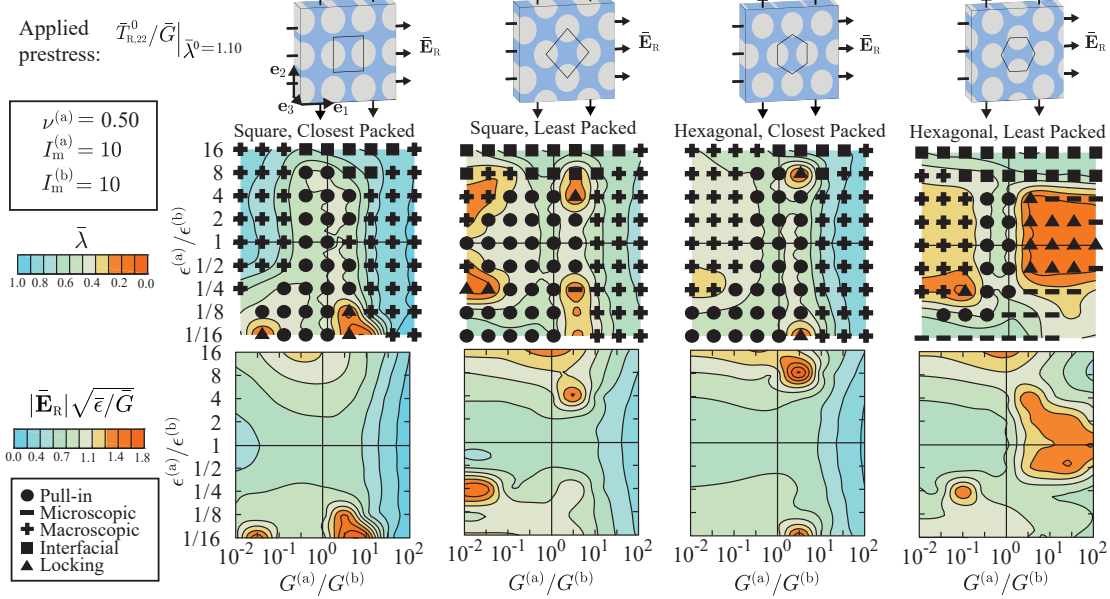


Figure B.1: The effect of mechanical prestress applied along the $\mathbf{e}_2$ -direction with electrical loading along the $\mathbf{e}_1$ -direction on electromechanical instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. (a) Mechanical response during prestressing in the absence of a macroscopic electric field. Specifically, the macroscopic normal stress along the $\mathbf{e}_2$ -direction, $\bar{T}_{R,22}^0/\bar{G}$, required to obtain a given stretch along the $\mathbf{e}_2$ -direction, $\bar{\lambda}^0$, and for a given shear modulus contrast, $G^{(a)}/G^{(b)}$. (b) Electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R|/\sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for a prestress corresponding to $\bar{\lambda}^0 = 1.10$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

prestress in the $\mathbf{e}_3$ -direction of Fig. 3.12, we observe a very similar effect.

Furthermore, we consider the case of an applied uniaxial compressive prestress in the $\mathbf{e}_1$ -direction in Fig. B.2, and the results are arranged as in Fig. B.1. Figure B.2(a) shows the stretch

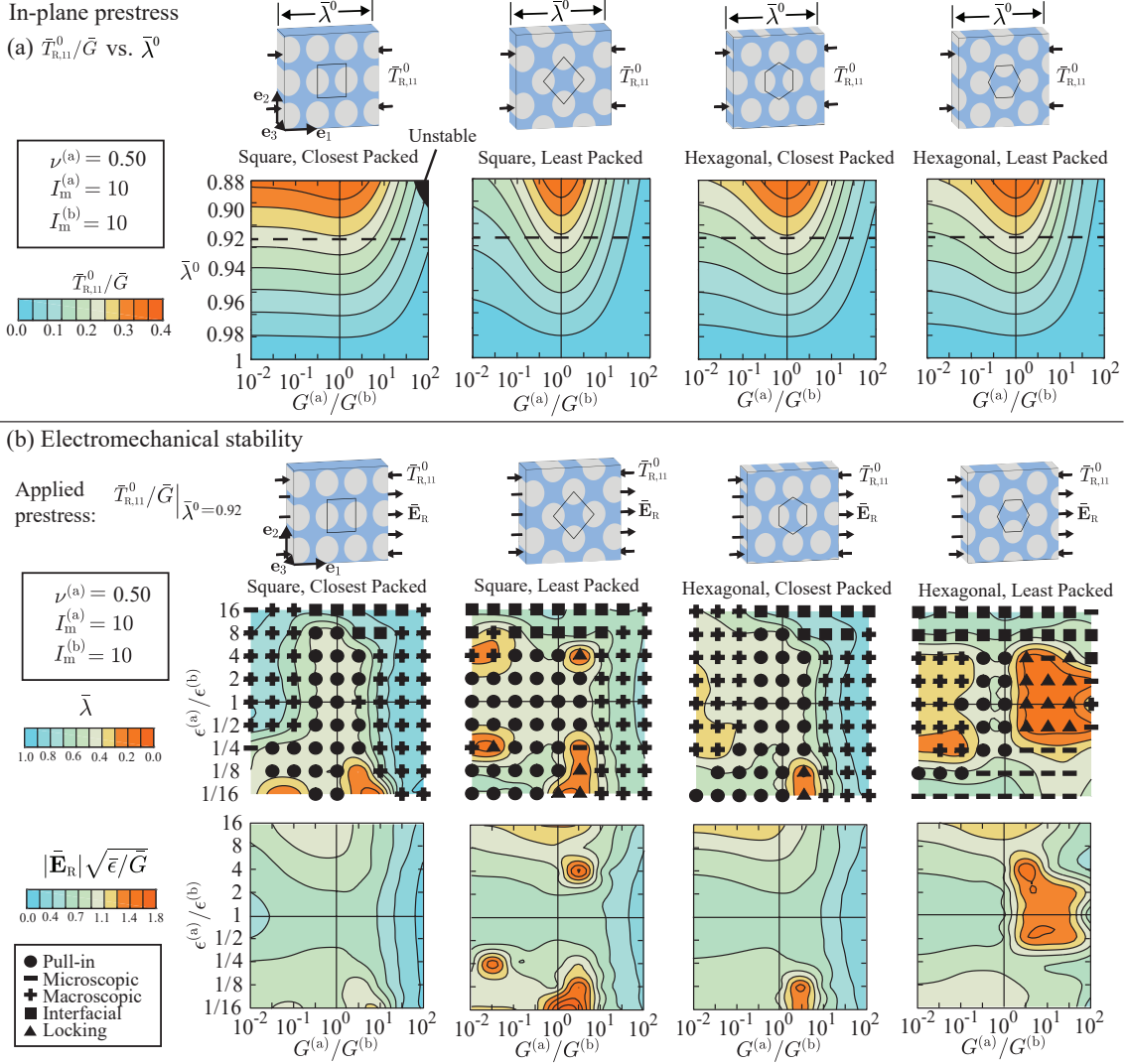


Figure B.2: The effect of compressive mechanical prestress applied along the $\mathbf{e}_1$ -direction with electrical loading also along the $\mathbf{e}_1$ -direction on electromechanical instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. (a) Mechanical response during prestressing in the absence of a macroscopic electric field. Specifically, the magnitude of the macroscopic normal stress along the $\mathbf{e}_1$ -direction, $\bar{T}_{R,11}^0/\bar{G}$, required to obtain a given stretch along the $\mathbf{e}_1$ -direction, $\bar{\lambda}^0$, and for a given shear modulus contrast, $G^{(a)}/G^{(b)}$. (b) Electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for a prestress corresponding to $\bar{\lambda}^0 = 0.92$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

along the prestressing direction, $\bar{\lambda}^0$, as a function of $\bar{T}_{r,11}^0/\bar{G}$ and $G^{(a)}/G^{(b)}$ for all cases. Then, we apply a prestress corresponding to a macroscopic stretch along the $\mathbf{e}_1$ -direction of $\bar{\lambda}^0 = 0.92$ and apply a macroscopic electric field along the $\mathbf{e}_1$ -direction, while holding $\bar{T}_{r,11}^0$ fixed. Figure B.2(b) shows the electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_r|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability. Comparing with Fig. B.2(b), we see that compressive prestress along the $\mathbf{e}_1$ -direction has almost the same effect as tensile prestress applied along the in-plane $\mathbf{e}_2$ direction.

B.2 Prestretch and confinement

Regarding prestretch, we explore the effect of a constraint along the $\mathbf{e}_2$ -direction – specifically cases of a macroscopic stretch of $\bar{\lambda}^0 = 1.00$ and $\bar{\lambda}^0 = 1.10$. Figure B.3 shows the electromechanical instability phase diagrams and contour plots of $\bar{\lambda}$ and $|\bar{\mathbf{E}}_r|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability for (a) $\bar{\lambda}^0 = 1.00$ and (b) $\bar{\lambda}^0 = 1.10$, when a macroscopic electric field is applied along the $\mathbf{e}_1$ -direction. All four combinations of lattice geometry and electrical loading direction are pictured. The results of Fig. B.3 are quite similar to those of Fig. 3.14 for the case of prestretch along the $\mathbf{e}_3$ -direction – namely, the suppression of pull-in and the increase of locking-limited behavior.

Finally, we consider the case in which the composite is confined along the $\mathbf{e}_1$ -direction, free in the $\mathbf{e}_2$ - and $\mathbf{e}_3$ -directions, and subjected to a macroscopic electric field along the $\mathbf{e}_1$ -direction. Figure B.4 shows the electromechanical instability phase diagrams and contour plots of $|\bar{\mathbf{E}}_r|\sqrt{\bar{\epsilon}/\bar{G}}$ at the onset of instability when a macroscopic electric field is applied along the $\mathbf{e}_1$ -direction. All four combinations of lattice geometry and electrical loading direction are pictured. As in the fully-confined case shown in Fig. 3.15, no instability occurs when the electric permittivity contrast is unity, and these cases are denoted by the hatched regions in Figure B.4. Results are quite similar to the fully-confined case – namely, cases with $\epsilon^{(a)}/\epsilon^{(b)} < 1$ are prone to macroscopic instability, while cases with $\epsilon^{(a)}/\epsilon^{(b)} > 1$ are prone to interfacial instability.

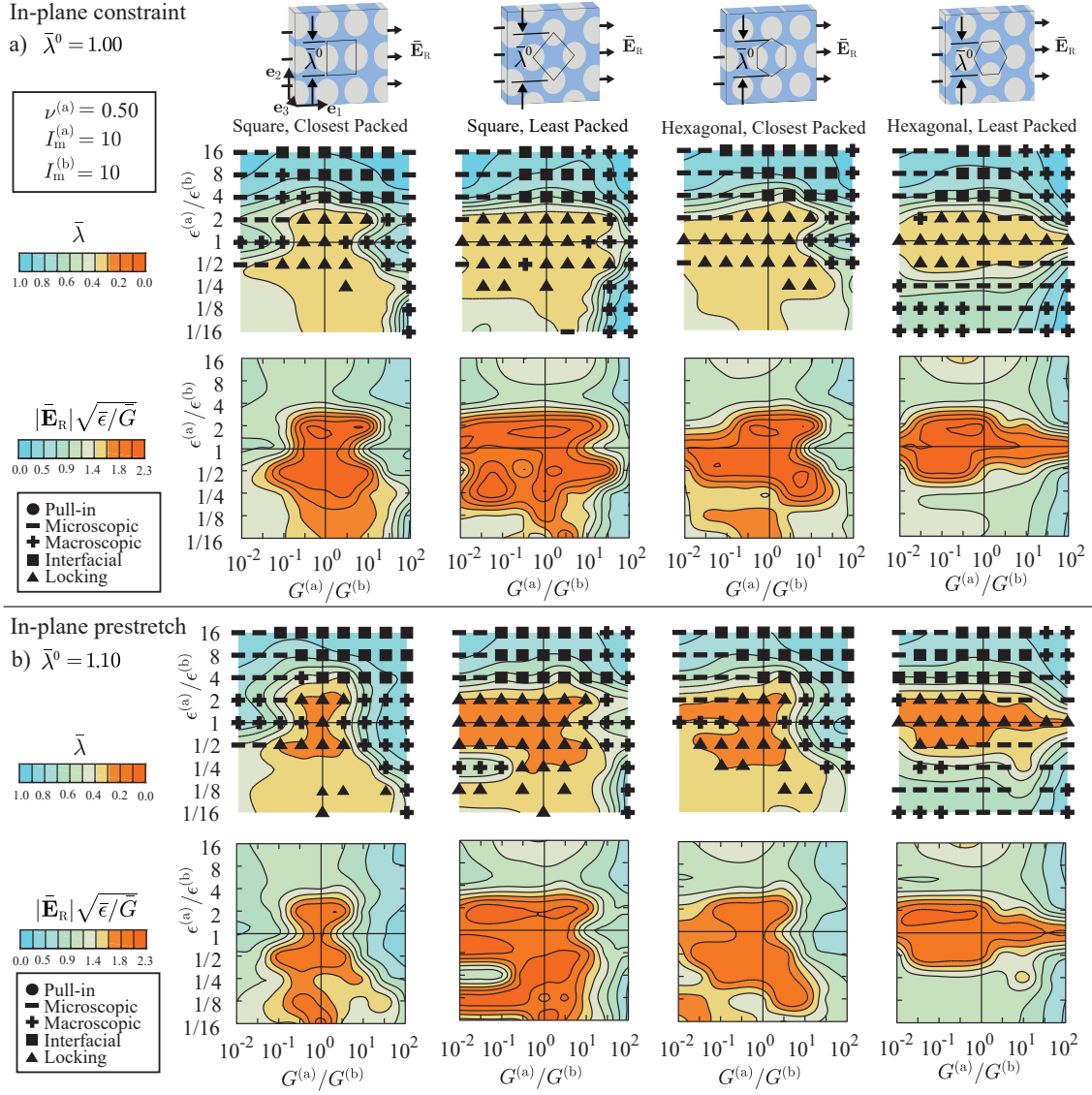


Figure B.3: The effect of mechanical prestretch applied along the $\mathbf{e}_2$ -direction on electromechanical instability for composites made from Gent materials with $I_m^{(a)} = I_m^{(b)} = 10$. Electromechanical instability phase diagrams and contour plots of the macroscopic stretch along the electrical loading direction, $\bar{\lambda}$, and the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for (a) in-plane confinement – i.e., $\bar{\lambda}^0 = 1.00$ – and (b) a fixed, in-plane prestretch of $\bar{\lambda}^0 = 1.10$. Both square- and hexagonal-lattice composites and electrical loading along the closest-packed and least-packed directions are shown for each case. Mesh density: $0.05a_R$.

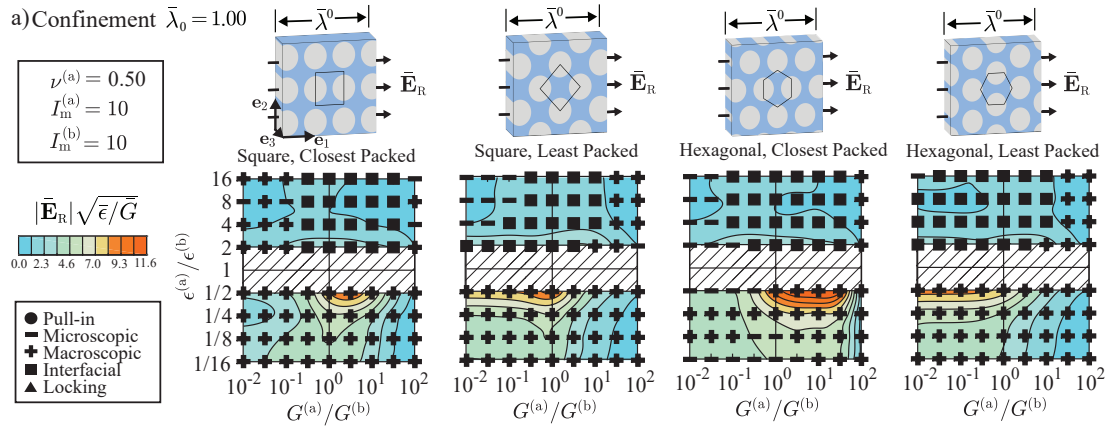


Figure B.4: Electromechanical instability phase diagrams and contour plots of the normalized macroscopic electric field magnitude, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}}$, at the onset of instability for composites made from Gent materials that are confined along the $\mathbf{e}_1$ -direction and subjected to electrical loading along the $\mathbf{e}_1$ -direction. Mesh density: $0.05a_R$.

Appendix C

Design opportunities

In this appendix, we provide design suggestions for dielectric elastomer composites made from specific materials, based on the results of Chapter 3. Chapter 3 focused on composites made from two different dielectric elastomers; however, it is also conceivable to consider composites with inclusions made from stiffer materials such as plastic or glass or – in the opposite limit – fluid-filled inclusions. First, focusing on the matrix material, it is possible to control the stiffness and electric permittivity of a dielectric elastomer by introducing thinning agents, high permittivity particles, or solvents [e.g. cf. 130] into the pre-cured elastomer. Some high permittivity powder-based particles are also stiff, increasing the shear modulus. Examples include inorganic powders such as Titanium Dioxide, Barium Titanate, and Lead Magnesium Niobate Lead Titanate (PMN-PT). An alternative approach is to dissolve organic solvents such as poly(3-hexylthiophene-2,5-diyl) into the pre-cured elastomer. Both approaches allow one to effectively vary the permittivity of the dielectric elastomer – but the accompanying modification of the shear modulus must also be considered [e.g. cf. 13, 131].

Regarding the inclusions, a variety of insulating materials may be considered to obtain the desired material property contrast. For example, high shear modulus contrasts can be achieved using various molecular weight polyethylenes ($G = 100 \text{ kPa}$ to 900 kPa), polytetrafluoroethylenes such as Teflon ($G = 500 \text{ MPa}$), synthetic polymers such as nylon ($G = 2 \text{ GPa}$), glasses such as

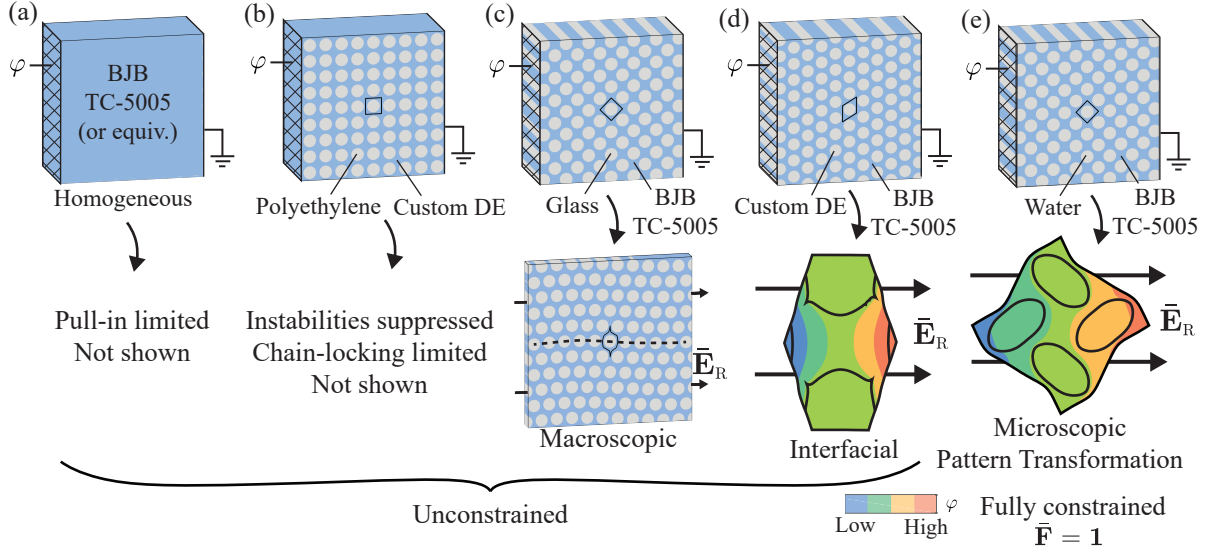


Figure C.1: Potential designs of dielectric elastomer composites. Top: (a) homogeneous dielectric elastomer; (b) polyethylene fibers in a dielectric elastomer matrix, arranged in a square lattice, mechanically unconstrained, and electrically loaded along the closest-packed direction; (c) glass fibers in a dielectric elastomer matrix, arranged in a square lattice, mechanically unconstrained, and electrically loaded along the least-packed direction; (d) a composite made from two dielectric elastomers, arranged in a hexagonal lattice, mechanically unconstrained, and electrically loaded along the least-packed direction; and (e) water-filled cylindrical inclusions in a dielectric elastomer matrix, arranged in a square lattice, fully mechanically constrained, and electrically loaded along the least-packed direction. Bottom: Critical instabilities, post-instability response, and/or resulting pattern transformation.

borosilicate glass ($G = 50$ GPa), or alumina-based ceramics ($G = 100$ GPa to 400 GPa). Lower shear modulus contrasts can be achieved with materials with vanishing shear behavior, such as dielectric fluids or air. Approximate values of the permittivity for the materials listed above are: polyethylenes ($\epsilon/\epsilon_0 = 2$), Teflon ($\epsilon/\epsilon_0 = 2$), Nylon ($\epsilon/\epsilon_0 = 3.5$), borosilicate glass ($\epsilon/\epsilon_0 = 8$), or alumina-based ceramics ($\epsilon/\epsilon_0 = 10$) where $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$ is the permittivity of free space. Moreover, the permittivity of distilled water is high ($\epsilon/\epsilon_0 = 40$). In the examples that follow, we consider several specific combinations – summarized in Table C.1 – and explore the required voltages and critical instabilities for each case.

Figure	Material	G	ϵ/ϵ_0	E_B	Event	$\bar{\lambda}$	$ \bar{\mathbf{E}}_R $	$\max \mathbf{E} $
Fig. C.1(a)	BJB TC-5005	16 kPa	5	35 MV m^{-1}	●	0.58	13.7 MV m^{-1}	23.6 MV m^{-1}
Fig. C.1(b)	Custom DE Polyethylene	30 kPa	36.8	35 MV m^{-1}	▲	0.17	1.71 MV m^{-1}	46.6 MV m^{-1}
		300 kPa	2.3	27 MV m^{-1}				48.4 MV m^{-1}
Fig. C.1(c)	BJB TC-5005 Glass	16 kPa	5	35 MV m^{-1}	+	0.71	30.4 MV m^{-1}	41.4 MV m^{-1}
		26.1 GPa	8	470 MV m^{-1}				20.4 MV m^{-1}
Fig. C.1(d)	BJB TC-5005 Custom DE	16 kPa	5	35 MV m^{-1}	■	0.77	8.91 MV m^{-1}	26.6 MV m^{-1}
		16 kPa	80	35 MV m^{-1}				1.91 MV m^{-1}
Fig. C.1(e)	BJB TC-5005 Water	16 kPa	5	35 MV m^{-1}	—	0.77	6.21 MV m^{-1}	17.7 MV m^{-1}
		≈ 0	40	65 MV m^{-1}				2.23 MV m^{-1}

Table C.1: Potential designs of dielectric elastomer composites. Material properties are approximate and used for illustrative purposes. The material dielectric strength is denoted as E_B , and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$ is the permittivity of free space. For a given case, the limiting event type, macroscopic stretch $\bar{\lambda}$, referential macroscopic electric field magnitude $|\bar{\mathbf{E}}_R|$, and maximum spatial electric field within the composite $\max|\mathbf{E}|$ are reported. Results are presented for $\nu^{(a)} = \nu^{(b)} = 0.5$ and $I_m^{(a)} = I_m^{(b)} = 10$.

Monolithic dielectric elastomer: As a baseline, consider a monolithic BJB TC-5005 dielectric elastomer [10, 132]¹ as shown schematically in Fig. C.1(a). We consider the case of a unconstrained elastomer without prestress or prestretch. This case may be considered analytically and is limited by pull-in instability at a stretch of $\bar{\lambda} = 0.58$ and an electric field magnitude of $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 0.72$. Utilizing the quantitative values of the shear modulus and permittivity given in Table C.1, this implies a macroscopic referential electric field magnitude of $|\bar{\mathbf{E}}_R| = 13.7 \text{ MV m}^{-1}$ and a spatial electric field magnitude of $|\bar{\mathbf{E}}| = 23.6 \text{ MV m}^{-1}$ at pull-in. To support this, a dielectric strength of $\max|\mathbf{E}| = 23.6 \text{ MV m}^{-1}$ is required – which is within the allowable range for BJB TC-5005. Thus, assuming a 1 mm electrode separation, an applied voltage of 13.7 kV is required to reach the pull-in instability. These results are tabulated in Table C.1.

Polyethylene fibers in a dielectric elastomer matrix: Next, we consider low molecular weight polyethylene [133] fibers in a dielectric elastomer matrix arranged in a square lattice and electrically loaded along the closest-packed direction with a fiber volume fraction of $\nu^{(a)} = 0.5$ and without

¹Note that our intent is to consider the pull-in instability of this material, and we assume $I_m = 10$ for this purpose.

prestress or prestretch. This case is shown schematically in Fig. C.1(b). The assumed material property contrasts are $G^{(a)}/G^{(b)} = 10$ and $\epsilon^{(a)}/\epsilon^{(b)} = 1/16$, and for simplicity, we continue to assume $I_m^{(a)} = I_m^{(b)} = 10$. The response of this composite follows from Fig. 3.6(a). We find that all instabilities are suppressed, and this case is limited by large-stretch chain-locking, which we observe at around $\bar{\lambda} = 0.17$ and $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 1.71$. For the quantitative values of the shear moduli and permittivities given in Table C.1, we require a macroscopically applied referential electric field of $|\bar{\mathbf{E}}_R| = 0.53 \text{ MV m}^{-1}$ with peak dielectric breakdown electric fields of $\max|\mathbf{E}^{(a)}| = 1440 \text{ MV m}^{-1}$ and $\max|\mathbf{E}^{(b)}| = 1493 \text{ MV m}^{-1}$ – which are very high. Thus, assuming a 1 mm electrode separation, an applied voltage of 530 V is required to reach the chain-locking behavior with a corresponding macroscopic stretch of $\bar{\lambda} = 0.17$. Interestingly, the instability map of Fig. 3.6(a)(i) shows this case to provide the most amount of deformation while navigating around all the other instabilities. By using a high permittivity matrix material, the applied voltages are significantly reduced, but at the same time, a very large dielectric strength is required to reach the locking limit. This case will likely be limited by dielectric breakdown in practice.

Cylindrical glass fibers in a dielectric elastomer matrix: A composite consisting of glass fibers arranged in a square lattice in a dielectric elastomer matrix without mechanical constraint or prestress and electrically loaded along the least-packed direction is considered next. This case is shown schematically in Fig. C.1(c). We take $\nu^{(a)} = \nu^{(b)} = 0.5$ and assume that the incompressibility assumption remains valid.² For these materials, $G^{(a)}/G^{(b)} \rightarrow \infty$ and $\epsilon^{(a)}/\epsilon^{(b)} \approx 2$. Since $G^{(a)}/G^{(b)}$ is large, when comparing to the instability maps from Chapter 3, we approximate the shear-modulus contrast to be the largest considered value of $G^{(a)}/G^{(b)} \approx 100$. We further assume $I_m^{(a)} = I_m^{(b)} = 10$. The response follows from Fig. 3.6(b), and we find this case limited by a macroscopic instability at $\bar{\lambda} = 0.71$, $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 0.28$. For the quantitative values of the shear moduli and permittivities given in Table C.1, this case requires a macroscopically applied referential electric field of $|\bar{\mathbf{E}}_R| = 30.4 \text{ MV m}^{-1}$ to attain macroscopic instability. Additionally, peak dielectric

²Note that glass is not incompressible, but because glass is essentially rigid for this case, we expect the effect of inclusion compressibility to be negligible.

breakdown electric fields of $\max|\mathbf{E}^{(a)}| = 20.4 \text{ MV m}^{-1}$ and $\max|\mathbf{E}^{(b)}| = 41.4 \text{ MV m}^{-1}$ are required. Thus, assuming a 1 mm electrode separation, an applied voltage of 30.4 kV is required to reach the macroscopic instability with a corresponding macroscopic stretch of $\bar{\lambda} = 0.71$.

A composite of two dielectric elastomers: This case was engineered to reach the interfacial instability, introduced in Chapter 3. The design is shown in Fig. C.1(d) and represents two different dielectric elastomers arranged in a hexagonal lattice actuated along the least packed direction without mechanical constraint or prestress. We also assume $\nu^{(a)} = \nu^{(b)} = 0.5$. According to the prior discussion, we envision blending the matrix material with either high permittivity powders or solvents with thinning agents to achieve $G^{(a)} \approx G^{(b)}$ and $\epsilon^{(a)}/\epsilon^{(b)} \approx 16$. We further assume $I_m^{(a)} = I_m^{(b)} = 10$. The corresponding response follows from Fig. 3.7(b). This case is in fact limited by an interfacial instability at $\bar{\lambda} = 0.77$ and $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 1.37$. For the quantitative values of the shear moduli and permittivities given in Table C.1, we require a macroscopically applied referential electric field of $|\bar{\mathbf{E}}_R| = 8.91 \text{ MV m}^{-1}$ to attain interfacial instability. To support this peak dielectric breakdown electric fields of $\max|\mathbf{E}^{(a)}| = 1.91 \text{ MV m}^{-1}$ and $\max|\mathbf{E}^{(b)}| = 26.6 \text{ MV m}^{-1}$ are needed – which is achievable at least at the onset of the instability. Thus, assuming a 1 mm electrode separation, an applied voltage of 8.91 kV is required to reach the interfacial instability with a corresponding stretch of $\bar{\lambda} = 0.77$. Afterwards, we suspect localized dielectric breakdown will occur. These results are tabulated in Table C.1.

Water inclusions in a dielectric elastomer matrix: We conclude with a case showing pattern transformation, which is motivated by the results in Chapter 4. Here we consider distilled-water-filled cylindrical inclusions [133] in a BJB TC-5005 dielectric elastomer matrix arranged in a square lattice with $\nu^{(a)} = 0.5$ and actuated along the least packed direction, with full mechanical confinement. The design is shown in Fig. C.1(e). Alternatively, one could construct this same case with soft, but high permittivity, dielectric elastomer inclusions. Here we assume $G^{(a)}/G^{(b)} = 10^{-2}$ and $\epsilon^{(a)}/\epsilon^{(b)} = 10$. Additionally, we continue the incompressible assumption and take $I_m^{(a)} = I_m^{(b)} \rightarrow \infty$ (neo-Hookean). The response follows from Fig. 4.3(c) which undergoes microscopic instability

at a macroscopic electric field magnitude of $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 0.85$, enabling a pattern transformation until it is further limited by an interfacial instability at $|\bar{\mathbf{E}}_R| \sqrt{\bar{\epsilon}/\bar{G}} = 1.19$. Specializing for the quantitative material properties given in Table C.1, a macroscopically applied referential electric field of $|\bar{\mathbf{E}}_R| = 5.54 \text{ MV m}^{-1}$ is required to trigger the pattern transformation which is ultimately limited by $|\bar{\mathbf{E}}_R| = 7.75 \text{ MV m}^{-1}$. Additionally, peak dielectric breakdown electric fields –not presented in Chapter 4 – of $\max|\mathbf{E}^{(a)}| = 0.95 \text{ MV m}^{-1}$ and $\max|\mathbf{E}^{(b)}| = 7.94 \text{ MV m}^{-1}$ are required to enable the pattern transformation and avoid dielectric breakdown failure before the interfacial limit. Thus, assuming a 1 mm electrode separation, an applied voltage of 7.75 kV is required to reach the microscopic instability – which requires a lower voltage and permits larger deformation compared to the monolithic case. These results are tabulated in Table C.1.

Summary In this appendix, we illustrated five design cases – one for each of the observed instabilities, pull-in, macroscopic, microscopic, interfacial, as well as a case that suppresses all instabilities. The case enabling pattern transformation was also highlighted. The required voltages and material properties are all within current practical limits. Combined with the richness of the electromechanical instability landscape in these materials, this appendix is intended to guide and further motivate experimental investigation.

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