Making Sense of Seeming Incomparability

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Introduction

Some comparisons are easy. For instance a kale salad is clearly better than a bag of potato chips with respect to nutritional value, and, a dollar bill is clearly equal to four quarters with respect to monetary value. Other comparisons are hard. Who was better with respect to artistic creativity, Mozart or Michelangelo? What is better with respect to fulfilling one’s moral duties, staying in France to take care of your elderly mother or leaving to join the Free French Forces during World War II? In each of these hard cases, there is an intuitive resistance to claims that one of the items is better than, worse than, or equal to the other. Importantly though, the intuitive resistance does not seem to be due simply to ignorance about the items themselves or their attendant consequences. In this way, the items are seemingly incomparable with each other.

This phenomenon of seeming incomparability raises the pressing questions of what is going on in such cases and what we should rationally do when faced with choices between such items. My dissertation examines both of these questions and argues for several surprising conclusions over three chapters. In the first chapter, I consider the Parity account of seeming incomparability, which claims that while seemingly incomparable items are neither better than, worse than, nor equal to each other, there is nonetheless a distinct and previously unrecognized fourth comparative relation of parity that does hold between them. Here I argue that despite appearances, this endorsement of parity between seemingly incomparable items is not actually unique to the Parity account. In the second chapter, I consider how seeming incomparability affects the rationality of our choices over time. Here I argue that while it seems clearly irrational for an agent to make a series of choices that they know will result in a suboptimal outcome, this is surprisingly not the case when seeming incomparability is involved. In the final chapter, I consider seeming incomparability that takes the form of mildly incomplete preferences. Here I
argue that while such preferences violate the standard set of axioms for rational preferences, there is an alternate set of axioms that we can appeal to instead for rationally governing mildly incomplete preferences.

Chapter 1: The Specification Challenge to Parity

In this chapter, I present a challenge to the Parity account of seeming incomparability championed by Ruth Chang. This account claims that in addition to the familiar comparative relations of betterness, worseness, and equality, there is also a fourth relation called ‘parity’. Moreover, it is this fourth relation that holds between seemingly incomparable items. So while Michelangelo and Mozart are neither better than, worse than, nor equal to each other with respect to artistic creativity, they are on a par with each other. The challenge that I pose to the Parity account is to present a non-mysterious analysis of what exactly this fourth relation of parity is supposed to be. That is, what exactly is it for Michelangelo and Mozart to be on a par with respect to artistic creativity? I argue that any plausible analysis of parity that meets this challenge will be such that the relation of parity can also be endorsed by a rival account of seeming incomparability. In this way, the endorsement of parity between seemingly incomparable items is not what uniquely distinguishes the Parity account.

Chapter 2: Seeming Incomparability and Rational Choice

In this chapter, I consider three intuitively plausible principles for rational choice and how they apply to choice situations involving seemingly incomparable items. That is, I examine how these principles help us to determine what it is rational to do when faced with a choice between, say, apples and oranges. The first principle claims that in situations where the only thing that matters
is some particular value, the rationality of a choice depends solely on how the available options compare with respect to that value. So if we have to choose between an apple and an orange, and the only thing that matters is tastiness, then which item it is rational to choose will depend solely on how the apple and orange compare with respect to tastiness. The second principle claims that the rationality of a sequence of choices depends solely on the rationality of the individual choices within that sequence. So the rationality of the sequence of choices where one first chooses an apple at t1 and then later trades it for an orange at t2 will depend solely on the rationality of those individual choices at t1 and t2. The third principle claims that in cases where risk is not involved, it is less rational to perform a sequence of choices that we know will result in a worse outcome than some other available sequence than it is to perform a sequence that does not have that feature. So if we know that the outcome of Sequence I is worse than the outcome of at least one other available sequence, but the outcome of Sequence II is not worse than the outcome of any other available sequence, then it is less rational to perform Sequence I than Sequence II. I show that these three principles are actually incompatible and argue that the correct response is to reject the third principle. That is, when seeming incomparability is involved, it need not be less rational to perform a sequence of choices that one knows will lead to a suboptimal outcome.

Chapter 3: Axioms for Mildly Incomplete Preferences

In this chapter, I examine how seeming incomparability can crop up in the form of mildly incomplete preferences. An agent has mildly incomplete preferences when there are some items between which they have neither preference nor indifference, but where they do have preferences between each of those items and some common third item. While such preferences fail to satisfy the standard set of axioms for rational preferences, in particular the axioms of Completeness and
Continuity, I argue that this should not be worrying because they can satisfy alternative axioms that are similar in spirit. Indeed, I argue that these alternative axioms along with the expected utility axioms of Transitivity, Better Chances, and Better Prizes, form a new set of core axioms that mildly incomplete preferences both can and should satisfy. In this way, I identify the start of a new standard by which to judge the rationality of mildly incomplete preferences. Moreover, I also identify two ways of further developing this standard by incorporating two additional principles that are both intuitively compelling and which we would seem to want to add to our new set of axioms for governing mildly incomplete preferences. The problem is that these principles are jointly incompatible, so we cannot accept them both. It turns out then that we have to either accept that it is rational to have a preference between two lotteries without having any preference between the outcomes of those lotteries, or, accept that we need not be indifferent between two lotteries that have the exact same probabilities of the exact same outcomes.

In summary, each chapter of my dissertation examines an aspect of seeming incomparability with an eye towards shedding light on this familiar yet mysterious phenomenon. To be sure, there is still much more to learn about seeming incomparability. For instance, there remain many further issues to explore with respect to the nature of seeming incomparability, its implications for decision-making, and its application to real world issues. There are also many parallels between seeming incomparability and other phenomenon, like imprecise credences in epistemology and transformative experiences in rational choice theory, which merit further investigation. However, I hope that with this dissertation we are now a few steps closer to making sense of seeming incomparability.
Chapter 1  
The Specification Challenge to Parity

Abstract

When comparing two items, we sometimes face an intuitive resistance to claims that either item is better than, worse than, or equal to the other. Rather, there are cases where the items just seem to be incomparable. Ruth Chang explains such instances of seeming incomparability by appealing to the existence of a fourth comparative relation called ‘parity’, which is distinct from betterness, worseness, and equality. I here present a challenge to Chang’s account that demands specification of this fourth comparative relation in such a way that it makes clear exactly how her account is distinct from and superior to its main rival. The purpose of this challenge though is not to undermine Chang’s account, but to understand how it might be most plausibly articulated and defended. I argue that this challenge reveals how the distinction between Chang’s account and its main rival is far narrower than it initially appears.
1. Introduction

Some comparisons are easy cases. For instance a kale salad is better than a bag of potato chips with respect to nutritional value; Newark, NJ is worse than Glacier National Park with respect to aesthetic value; and a dollar bill is equal to four quarters with respect to monetary value. Other comparisons though are hard cases. Who was better with respect to artistic creativity, Mozart or Michelangelo?\(^1\) What is better with respect to fulfilling one’s moral duties, staying in France to take care of your elderly mother or leaving to join the Free French Forces in England during World War II?\(^2\) Which is better with respect to the public good, the subsidization of public transportation or pre-kindergarten education? In these hard cases, there is an intuitive resistance to claims that one of the items is better than, worse than, or equal to the other. Importantly though, this intuitive resistance does not seem to be due to ignorance about the items themselves or their attendant consequences. Indeed, it seems that these cases could be maximally specified in ways that would not thereby resolve this intuitive resistance. In this way, the items are seemingly incomparable to each other. What are we to make of such seemingly incomparable items?

Ruth Chang claims that in such cases, none of the three comparative relations of betterness, worseness, or equality holds between the items. However, she also claims that the items are not incomparable because there is a distinct fourth comparative relation of *parity* that does hold between them.\(^3\) Chang’s account is controversial and has been widely debated. In this paper, I present a new challenge to Chang’s account that demands specification of this fourth comparative relation in such a way that it makes clear exactly how her account is distinct from and superior to its main rival account. However, the purpose of this challenge is not so much to

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\(^1\) Example from Chang (2002), pg 659.  
\(^3\) See Chang (2002) for discussion of this view.
undermine Chang’s account as it is to understand how it might be most plausibly articulated and defended. This will be seen as I examine how two recently proposed analyses of parity fare in meeting this challenge. In so doing, I argue that any plausible analysis of parity will be one that the rival account can likewise endorse and so is not unique to Chang’s account. It turns out then that the distinction between Chang’s account and her rival’s is not in whether they endorse the relation of parity between seemingly incomparable items, but in whether they take this relation to be necessary for exhausting the conceptual space of comparability. Whether Chang’s account is superior to her rival then will depend on whether this distinction makes a difference.

2. The Phenomenon of Seeming Incomparability

In order to understand the phenomenon of seeming incomparability, it is helpful to start with a paradigmatic example provided by Ruth Chang:

Suppose you must determine which of a cup of coffee and a cup of tea tastes better to you. The coffee has a full-bodied, sharp, pungent taste, and the tea has a warm, soothing, fragrant taste. It is surely possible that you rationally judge that the cup of Sumatra Gold tastes neither better nor worse than the cup of Pearl Jasmine and that although a slightly more fragrant Jasmine would taste better than the original, the more fragrant Jasmine would not taste better than the cup of coffee. In this case, it is plausible to suppose that you know everything that is relevant to comparing the drinks and that in this case you have first-person authority over which tastes better to you.4

Here, there is an attempt to compare two particular items, the cup of Sumatra Gold coffee and the cup of Pearl Jasmine tea, with respect to some covering value, tastiness, that is applicable to both items. The covering value specifies the particular dimension along which the items are being compared and is crucial, though often implicit, in any comparative claim. That is, there is no way to compare things simpliciter without reference to some dimension of evaluation.5

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4 Chang (2002), pg 669.
5 See Chang (2002) for discussion of this requirement.
Moreover, it is important to note that in this comparison, both of the items fall within the domain of the specified covering value (i.e., tastiness is applicable to both the coffee and tea). In this way, there is no formal failure of comparison here as there would be if the covering value was not applicable to one or more of the items under comparison (e.g., if Sumatra Gold and the number three were compared with respect to tastiness).⁶

Now despite the fact that the covering value of tastiness clearly applies to both the Sumatra Gold and the Pearl Jasmine, there can nonetheless be an intuitive resistance to claims that either item is better than, worse than, or equal to the other with respect to tastiness. That is, you can be reluctant to assent to all of the following claims:

[1] Pearl Jasmine is better than Sumatra Gold with respect to tastiness.  
[2] Pearl Jasmine is worse than Sumatra Gold with respect to tastiness.  
[3] Pearl Jasmine and Sumatra Gold are equal with respect to tastiness.⁷

This is not yet to say that you would hold these claims to all be false. That is a stronger commitment and one that is not, I take it, as intuitively clear. Rather, the phenomenology of such cases seems to be an intuitive reluctance to assent to claims of betterness, worseness, and equality rather than an outright rejection of those claims. It may ultimately be that the best explanation of this intuitive reluctance is that the claims are all false, but this is not the phenomenological starting point of such cases. So I take the first key feature of seeming incomparability to be this intuitive resistance to claims of betterness, worseness, and equality.

The second key feature of seeming incomparability is that this intuitive resistance persists even if one of the items is slightly improved or worsened. That is, replacing one of the items with a slightly improved or worsened version does not break the intuitive resistance towards claims of betterness, worseness, and equality. So in addition to being reluctant to assent to claims [1] – [3]

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⁶ Chang calls such formal failures of comparability instances of noncomparability, holding them to be distinct from both comparability and incomparability. See, Chang (2015) pp 215-6.  
above, one can also be reluctant to assent to any of the following claims where the Pearl Jasmine has been replaced with a slightly better tea, call it Pearl Jasmine$^+$:

1. Pearl Jasmine$^+$ is better than Sumatra Gold with respect to tastiness.
2. Pearl Jasmine$^+$ is worse than Sumatra Gold with respect to tastiness.
3. Pearl Jasmine$^+$ and Sumatra Gold are equal with respect to tastiness.

The third key feature of seeming incomparability is that while the intuitive resistance towards claims of betterness, worseness, and equality persists when one of the items is slightly improved/worsened, it does not persist if one of the items is significantly improved/worsened. That is, replacing one of the items with a significantly improved or worsened version does break the intuitive resistance towards claims of betterness, worseness, and equality. So if Pearl Jasmine were replaced with a much better tea, call it Pearl Jasmine$^{+++\cdots}$, we would no longer be reluctant to assent to all of the following claims:

1. Pearl Jasmine$^{+++\cdots}$ is better than Sumatra Gold with respect to tastiness.
2. Pearl Jasmine$^{+++\cdots}$ is worse than Sumatra Gold with respect to tastiness.
3. Pearl Jasmine$^{+++\cdots}$ and Sumatra Gold are equal with respect to tastiness.

Rather, we would hold [1**] to be true and [2**] and [3**] to both be false. Now that the phenomenon of seeming incomparability has been discussed, we can turn to possible explanations of this phenomenon.

3. Accounts of Seeming Incomparability

There are four accounts of seeming incomparability that have been defended in the literature: **EPISTEMICISM, INDETERMINACY, INCOMPARABILITY, and PARITY.** Each gives a different explanation of the phenomenon of seeming incomparability. In discussing these accounts, I will

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8 This feature is not illustrated in Chang’s example, but is discussed separately as what she calls nominal-notable comparisons. See Chang (1997), pp 14-5.
assume the above example of seeming incomparability between Pearl Jasmine and Sumatra Gold.

The **Epistemicism** account holds that in cases of seeming incomparability, exactly one of the claims of betterness, worseness, and equality is true, we just do not know which.\(^9\) So in our example, one of claims [1] – [3] is true and the others false, but we do not know which is which. Our intuitive reluctance to assent to claims [1] – [3] then is due to our ignorance as to which claim is true and which are false.

The **Indeterminacy** account holds that in cases of seeming incomparability, all of the claims of betterness, worseness, and equality are indeterminate in truth value.\(^10\) One way this might happen is if the covering value being appealed to is vague and admits of multiple legitimate sharpenings. Cases of seeming incomparability then might obtain when claims of betterness, worseness, and equality are true on some sharpenings, but false on others. So in our example, [1] might be true on some sharpenings of betterness with respect to tastiness, but false on other sharpenings and likewise for [2] and [3]. Given that none of the sharpenings are privileged, the claims themselves are indeterminate in truth value. Our intuitive reluctance to assent to claims [1] – [3] then is due to the fact that none of them is determinately true (or false).

The **Incomparability** account holds that in cases of seeming incomparability, all of the claims of betterness, worseness, and equality are false.\(^11\) So in our example, all of [1] – [3] are false and this is what explains our intuitive reluctance to assent to those claims. Moreover, this account endorses what Chang calls the Trichotomy Thesis.\(^12\) This thesis holds that if two items are comparable with respect to a given covering value, they must stand in one of three relations

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\(^9\) See Regan (1997) for defense of such a view.  
\(^10\) See Broome (1997) for defense of such a view.  
\(^11\) See Raz (1986) for defense of such a view.  
\(^12\) See Chang (1997) for discussion of the Trichotomy Thesis and (2002) for her rejection of the view. Also note that while the Trichotomy Thesis is not essential to **Epistemicism** and **Indeterminacy**, it is in fact endorsed by both.
with respect to that covering value: betterness, worseness, or equality. The idea here is that the relations of betterness, worseness, and equality jointly exhaust the conceptual space of comparability. Items that do not stand in one of these relations with respect to some covering value are incomparable with respect to that same value. **INCOMPARABILITY** then holds seemingly incomparable items to be actually incomparable.

The **PARITY** account, like **INCOMPARABILITY**, also holds that in cases of seeming incomparability, all of the claims of betterness, worseness, and equality are false. However, this account differs from **INCOMPARABILITY** in its rejection of the Trichotomy Thesis, holding that betterness, worseness, and equality do not in fact jointly exhaust the conceptual space of comparability. Rather, that conceptual space is only jointly exhausted once a fourth relation, called ‘parity’, is added to the original trichotomy.\(^{13}\) Moreover, it holds that this fourth relation of parity obtains between seemingly incomparable items. So while **PARITY** holds that [1] – [3] are all false in our example, it also holds that [4] below is true:

[4] Pearl Jasmine and Sumatra Gold are on a par with respect to tastiness.

Our intuitive reluctance to assent to claims [1] – [3] then is due to the fact that all of them are false. However, the items are not incomparable because there is some positive comparative relation that holds between the items.

### 4. The Specification Challenge to **PARITY**

Each of the above accounts of seeming incomparability has its advantages and disadvantages, but a full discussion of their comparative merits is beyond the scope of this paper. Rather, I will present a particular challenge to the **PARITY** account. This Specification Challenge is at its core a

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\(^{13}\) See Chang (1997) and (2002) for defense of such a view. Also note that I hereafter use “**PARITY**” to refer to the account of seeming incomparability and “parity” to refer to the fourth relation that is endorsed by **PARITY**.
challenge for proponents of PARITY to explain exactly what they take this fourth relation of parity to consist in. That is, it requires an analysis of parity to help us understand what it is for two items to be on a par. Such an analysis is needed for two reasons. The first is to help alleviate the mystery surrounding parity. The second is to fully understand the distinctions between PARITY and INCOMPARABILITY as well as the possible advantages that might arise from these distinctions. Each of these motivations informs the particular demands of the challenge and so is worth further elaboration.

4.1 Alleviating the Mystery of Parity

It is clear that PARITY holds the relation of parity to be necessary for exhausting the conceptual space of comparability in conjunction with betterness, worseness, and equality. Apart from this though, it is not yet clear what the relation of parity is actually supposed to be. While betterness, worseness, and equality are familiar comparative relations, the supposed comparative relation of parity seems mysterious. An analysis of parity then is needed to help alleviate this mysteriousness and allow us to understand what it means for two items to be on a par. Without such an analysis, the notion of parity would appear to be more of a placeholder for a relation than an actual one.

4.2 Understanding the Distinction between INCOMPARABILITY and PARITY

An analysis of parity is also needed to fully understand the distinctions between INCOMPARABILITY and PARITY. To see why this is the case, we can first understand PARITY as the account of seeming incomparability that endorses the following three claims:
PARITY Claim 1 – Seemingly incomparable items are neither better than, worse than, nor equal to each other.

PARITY Claim 2 – Seemingly incomparable items are on a par with each other.

PARITY Claim 3 – The relation of parity is needed to exhaust the conceptual space of comparability in conjunction with betterness, worseness, and equality.

The first claim denies that any of the original trichotomy of comparative relations obtains between seemingly incomparable items. The second claim holds that the relation of parity obtains between seemingly incomparable items. The third claim denies the Trichotomy Thesis by positing parity as a fourth possible way in which items might be comparable that is distinct from betterness, worseness, and equality.

We can now take this understanding of PARITY and contrast it with INCOMPARABILITY to determine exactly how the two accounts are distinct. That is, we can ask whether INCOMPARABILITY endorses or rejects each of these key claims in turn. Here, we see that INCOMPARABILITY agrees with PARITY in endorsing PARITY Claim 1, but disagrees in rejecting PARITY Claim 3. As discussed above, INCOMPARABILITY denies that any of the original trichotomy of relations holds between seemingly incomparable items and so endorses PARITY Claim 1. However, INCOMPARABILITY endorses the Trichotomy Thesis, and so rejects PARITY Claim 3. So we know that there is at least one distinction between INCOMPARABILITY and PARITY, namely in their endorsement of PARITY Claim 3.

What though about PARITY Claim 2? Does INCOMPARABILITY endorse or reject the claim that seemingly incomparable items are on a par with each other? Here, one might think that INCOMPARABILITY must reject the claim that parity holds between seemingly incomparable items, arguing as follows:
P1 – *Incomparability* holds that seemingly incomparable items are neither better than, worse than, nor equal to each other.

P2 – *Incomparability* holds that betterness, worseness, and equality jointly exhaust the conceptual space of comparability.

C1 – *Incomparability* holds that no basic comparative relation obtains between seemingly incomparable items.

P3 – Parity is a basic comparative relation.

C2 – *Incomparability* denies that parity obtains between seemingly incomparable items.

P1 is just the claim that *Incomparability* endorses Parity Claim 1, which we’ve already established. P2 is just the claim that *Incomparability* endorses the Trichotomy Thesis, which we’ve also already established. From these two premises, it is inferred that *Incomparability* must deny that any basic comparative relation holds between seemingly incomparable items. The notion of a basic comparative relation being used here comes from Chang, who defines such relations as those that are part of a set of relations which jointly exhausts the conceptual space of comparability:

> A set of value relations is basic if it exhausts the conceptual space of comparability between two items with respect to [some covering value]. A value relation is ‘basic’ if it is a member of a basic set. So, for example, ‘x is better than y’ belongs to a basic set, while ‘x is better than y but only slightly worse than z’ does not. 14

To say then that no basic comparative relation obtains between seemingly incomparable items is to say that no relation that is a member of a set of relations (minimally) exhausting the conceptual space of comparability obtains between seemingly incomparable items. This claim then follows from *Incomparability*’s rejection of the original trichotomy of relations obtaining between seemingly incomparable items (P1) and *Incomparability*’s endorsement of the Trichotomy Thesis (P2). This is because each possible basic set must exhaust the same conceptual space of comparability. So if none of the possible relations of one basic set obtains in

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14 Chang (2015), pg 208. While Chang does not make this explicit, I interpret her as holding basic sets to be minimally exhaustive. That is, they exhaust the conceptual space of comparability, but are such that no proper subset is also exhaustive.
a given case, it must be the case that none of the possible relations in any basic set obtains in that case.

P3 is the claim that parity is a basic comparative relation. That is, parity is a member of a (minimal) set of relations that jointly exhausts the conceptual space of comparability. In particular, it is a member of the basic set that includes betterness, worseness, and equality. From C1 and P3 though, it follows that Incomparability must deny that seemingly incomparable items are on a par. So, INCOMPARABILITY must reject PARITY Claim 2.

While this argument is valid, the third premise is potentially problematic. Indeed, it seems strange to assert that parity is a basic comparative relation since the one clear area of disagreement between INCOMPARABILITY and PARITY is in their view of the Trichotomy Thesis. INCOMPARABILITY endorses the Trichotomy Thesis and so would reject P3, which is incompatible with the Trichotomy Thesis. Nonetheless, it may be that the Trichotomy Thesis is false and that P3 is true. One might then argue separately for the truth of P3 and in so doing also argue against the truth of INCOMPARABILITY in addition to supporting the inference to C2.

But perhaps P3 is a conceptual claim that is true by definition. That is, maybe part of what the relation of parity itself is includes that it be a basic comparative relation. If this is right, then an independent argument supporting P3 is not even necessary and the above argument is sound. But does the relation of parity include in its definition that it is a basic comparative relation? As discussed above, one of the motivations of the Specification Challenge is that some analysis of parity be given to alleviate the mystery surrounding the relation. We can then use this non-mysterious analysis, whatever it turns out to be, as at least part of the definition of parity. The question then is whether parity should be conceived of minimally and defined using this yet to be determined non-mysterious analysis alone, or conceived of non-minimally and defined
using both this non-mysterious analysis and the claim that it also be a basic comparative relation. More precisely, the minimal conception of parity can be defined as follows:

Parity $\equiv_{df}$ TBD Non-Mysterious Analysis

The non-minimal conception of parity on the other hand can be defined as follows:

Parity $\equiv_{df}$ TBD Non-Mysterious Analysis & is a Basic Comparative Relation

Which of these conceptions of parity should we use? Here, I argue that we should use the minimal conception of parity and its corresponding definition. I take there to be three reasons in favor of the minimal conception of parity over the non-minimal conception. The first is that none of the other comparative relations seem to include in their definition that they are basic comparative relations. That is, it does not seem that betterness, worseness, and equality are basic comparative relations by definition. But if none of the original trichotomy of relations is basic by definition, then why should the fourth relation of parity be basic by definition?

The second reason to use the minimal conception of parity appeals to a claim made by Chang in her argument against the Trichotomy Thesis. There, she claims that the question of which relations are basic is a substantive matter, not a conceptual one:

Our ordinary concept of comparability does not have Trichotomy built into it; it is an open, substantive question which relations exhaust the conceptual space of comparability between two items. Indeed, we might define our ordinary notion of comparability neutrally as follows: two items are comparable with respect to some covering value just in case there is some basic value relation that holds between them with respect to that covering value. They are incomparable with respect to a covering value just in case no such relation holds. Which relations are basic is a matter for substantive debate.\(^\text{15}\)

I agree with Chang that the notion of comparability does not itself include or entail the Trichotomy Thesis. But if the notion of comparability is neutral with respect to which relations are needed to exhaust the conceptual space of comparability, so too should the relations

\(^{15}\text{Chang (2016b), pg 398.}\)
themselves. That is, if there is to be substantive debate about which relations are basic, it won’t do to include in the definitions of different possible relations that they be basic. So we should not include in the definition of parity that it is a basic comparative relation. We should instead use the minimal conception of parity.

The third reason to use the minimal conception of parity is that it can be used to help adjudicate disputes between those who disagree about the Trichotomy Thesis. To see why, we can look at a toy example that Chang provides involving a dispute between a dichotomist and trichotomist. The dichotomist holds that betterness and worseness jointly exhaust the conceptual space of comparability and so denies that equality is a basic comparative relation. The trichotomist on the other hand holds that equality is also needed to jointly exhaust the conceptual space of comparability and so endorses equality as a basic comparative relation. Here, Chang claims that the trichotomist can try to persuade the dichotomist that equality is a basic comparative relation by appealing to neutral notions that both parties share:

Note, the trichotomist might say, that the way an apple and its duplicate compare in tastiness, call it R, is importantly different from the way an apple and an orange compare in tastiness, call it T. Indeed, the logical properties of R differ from the logical properties of T. Given that R holds between the apple and its duplicate, it is possible to substitute the duplicate for the apple in all comparisons of tastiness in which the apple figures without altering the comparative facts. Not so for the orange... By leveraging shared notions, such as neutral notions of comparability and incomparability and the idea of substitutability, the trichotomist can show that the dichotomist overlooks a third basic way in which items can be compared.17

The trichotomist’s strategy here is to first get the dichotomist to agree that the relation between two identical apples, R, is distinct from the relation between an apple and an orange, T. The dichotomist might agree to this despite denying that equality is a basic relation because she recognizes something about relations R and T that is independent of whether equality is a basic

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16 Chang (2016b), pg 399.
17 Chang (2016b), pg 399.
relation. This is that items that bear relation R to each other are substitutable while items that bear relation T are not. Once the dichotomist recognizes this distinction between the relations, the trichotomist can then go on to try to persuade the dichotomist that it would be a mistake to lump them both in the same category of incomparability. In this way, the trichotomist uses common ground to try to convince the dichotomist to accept equality as a basic comparative relation.

This strategy only works though if the relations that the trichotomist appeals to are independent of the claim that equality is a basic comparative relation. For instance, if the claim that R is a basic comparative relation was included in the notion of R being appealed to by the trichotomist, the dichotomist would simply reject that R obtained between identical apples and no progress in the debate would be made.

This same point though holds in the dispute between the trichotomist and the tetrachotomist. So a parallel strategy that a tetrachotomist might pursue for convincing the trichomist that parity is a fourth basic relation would also have to appeal to a relation that does not itself presuppose parity being a basic comparative relation. The tetrachotomist then would have to appeal to the minimal conception of parity rather than the non-minimal conception.

I conclude then that parity should be conceived of minimally and not include as part of its definition that it is a basic comparative relation. If this is right, then INCOMPARABILITY need not reject Parity Claim II above. Rather, whether INCOMPARABILITY allows for seemingly incomparable items to be on a par with each other will depend on how the minimal analysis of parity is cashed out. In this way, the full extent of the distinction between PARITY and INCOMPARABILITY, as well as the possible advantages each holds, will not become clear until this analysis is provided.
4.3 The Three Demands of the Specification Challenge

We can now use the motivations discussed above to break the Specification Challenge down into the following three interrelated demands.

4.3.1 The Characterization Demand

The first demand is to provide a non-mysterious analysis of parity that characterizes exactly what this relation is supposed to be. Moreover, this analysis should be minimal, making no presuppositions about parity being a basic comparative relation. Call this the Characterization Demand.

4.3.2 The Distinction Demand

The second demand is to give a full accounting of the distinctions between PARITY and INCOMPARABILITY given the analysis of parity used to satisfy the Characterization Demand. As discussed above, we already know that PARITY is distinct from INCOMPARABILITY insofar as the former rejects the Trichotomy Thesis while the latter endorses it. The question is whether this is the only distinction between the two accounts or whether they also differ in their endorsement of parity obtaining between seemingly incomparable items. Once we have an analysis of parity, we can then assess whether this is another point of distinction between the two. Call this the Distinction Demand.

4.3.3 The Advantage Demand

The third demand is to show how PARITY has an advantage over INCOMPARABILITY given the full accounting of distinctions between PARITY and INCOMPARABILITY provided in satisfying the
Distinction Demand. That is, how does the distinction between \textit{Parity} and \textit{Incomparability}, whether it is only with respect to the Trichotomy Thesis or also with respect to endorsing parity between seemingly incomparable items, make \textit{Parity} a superior account? Without such a demonstration, the separation between \textit{Parity} and \textit{Incomparability} amounts to a distinction without a difference. Call this the Advantage Demand.

5. Two Recent Analyses of Parity

Chrisoula Andreou and Ruth Chang have both recently proposed their own analyses of parity in order to help shed light on this otherwise mysterious relation.\footnote{I interpret the account presented in Andreou (2015) as amenable to \textit{Parity}. However, Chang interprets it as assuming the Trichotomy Thesis and so incompatible with \textit{Parity} [from correspondence]. In any case, I take the analysis of parity that I attribute to Andreou here to be entirely neutral with respect to the Trichotomy Thesis.} In this section, I examine each of their analyses to understand how parity might be cashed out as a non-mysterious relation.\footnote{Other analyses of parity have been put forward by Gert (2004), Rabinowicz (2008), and Carlson (2010). However, I here follow Chang (2016b) in setting aside Gert’s and Rabinowicz’s analyses since they both make the controversial assumption that value is to be analyzed in terms of fitting attitudes. Carlson does not make this assumption, but does seem to crucially assume that items on a par must be comparable. If this is so, then he is presenting a non-minimal conception of parity. However, his formal analysis might be separable from this assumption and so be understood as offering a minimal conception of parity as well. Thanks to an anonymous referee for this point.} In the next section, I discuss how these analyses fare in meeting the demands of the Specification Challenge and what it ultimately means for both the distinction between \textit{Parity} and \textit{Incomparability} and what possible advantages the former might have over the latter.

5.1 Andreou’s Analysis

Andreou’s analysis of parity revolves around a distinction between categorical judgments and relational judgments that she borrows from David Papineau.\footnote{Papineau (2015) uses this distinction for his work on color perception, but I here focus only on Andreou’s use.} Categorical judgments are judgments that some item belongs in a particular appraisal category with respect to some
covering value. The idea is that for each covering value, there are different appraisal categories in which we can place different bearers of that value. For instance, with gustatory value (tastiness), we might place bearers of this value in the categories: Tastes Great, Tastes Good, Tastes Ok, etc…

For Andreou’s purposes, it does not matter how many categories there are or exactly what we call them. The important point is that there are such categories, which we can judge particular individual bearers as belonging in. So one might taste the Pearl Jasmine tea and judge that it belongs in the category of Tastes Good. This categorical judgment does not compare the Pearl Jasmine to any other item, it just says of that tea that it tastes good.

Relational judgments are judgments that some particular item is better than, worse than, or equal to some other item with respect to some covering value. These judgments then are the ordinary sorts of comparative judgments that we resist in cases of seeming incomparability. So claims [1] – [3], discussed earlier and reproduced below, are all relational judgments:

[1] Pearl Jasmine is better than Sumatra Gold with respect to tastiness.
[2] Pearl Jasmine is worse than Sumatra Gold with respect to tastiness.
[3] Pearl Jasmine and Sumatra Gold are equal with respect to tastiness.

Andreou’s key claim is that these two types of judgments can be made directly, without reference to each other. That is, we can make a relational judgment between two items without first making categorical judgments of each of them. In tasting two different teas, we might be able to judge that one is better than the other without first judging which category each tea belongs in. We can also make categorical judgments of individual items without first making a relational judgment between those items. In tasting two different coffees, we might be able to judge which category each coffee belongs in without first judging which is better. If this is right, then there might also be cases where one sort of judgment obtains but the other does not. This,

Andreou suggests a different set of possible relevant categories with respect to gustatory value, but my use of these alternative categories makes no difference for our purposes.
Andreou claims, is what is going on when items are on a par. More specifically, she claims that two items are on a par when no relational judgment obtains between those items, but categorical judgments do obtain with regards to each item individually and in particular each item is judged to be in the same category.\textsuperscript{22} So Pearl Jasmine and Sumatra Gold are on a par with respect to tastiness if claims [1] – [3] are false, but claim [5] below is true:

\[
[5] \text{Pearl Jasmine and Sumatra Gold are in the same category with respect to tastiness.}\textsuperscript{23}
\]

It is important to stress here that Andreou does not take sameness of category to be sufficient for parity since items that belong in the same category can also be better than, worse than, or equal to each other.\textsuperscript{24} Rather parity only obtains when it is both the case that the items are in the same category and also that neither item is better than, worse than, or equal to the other with respect to the relevant covering value. So Andreou’s analysis of parity in this case can be understood as [4-Andreou] below, which is simply the conjunction of the falsity of [1] – [3] and the truth of [5]:

\[
[4-Andreou] \text{Pearl Jasmine and Sumatra Gold are neither better than, worse than, nor equal to each other, but are in the same category with respect to tastiness.}
\]

\textbf{5.2 Chang’s Analysis}

Chang’s analysis of parity appeals first to the idea of qualitative dimensions of value and second to the idea of neighborhoods of overall value. Her notion of qualitative dimensions here is simply the commonsensical one that items can differ not only quantitatively, but also qualitatively with respect to some covering value. For instance, a heartbreak is not only quantitatively different

\textsuperscript{22} Andreou actually calls this the narrow interpretation of parity in contrast to the broad interpretation that holds parity as obtaining just in case two items simply share in the same category. However, it is the narrow interpretation that is relevant to PARITY, so we need not discuss the broad interpretation here.

\textsuperscript{23} I am abstracting away from the particular category that each item belongs in here. So, claim [5] is true just in case the categorical judgments “Pearl Jasmine belongs in category X” and “Sumatra Gold belongs in category X” are both true where ‘X’ refers to one of the possible categories with respect to tastiness.

\textsuperscript{24} See Andreou (2015), pg 14.
from stubbing one’s toe with respect to painfulness, but also qualitatively different. Both of these experiences are painful, but they differ in both degree and kind.

Her notion of neighborhoods of overall value is very similar to Andreou’s notion of categories of value. The idea here is that there can be different neighborhoods of overall value where all the items within the same neighborhood are around the same in terms of overall quantitative value. Chang alternatively describes items being in the same neighborhood as equivalent to being in the same ‘rank’, ‘league’, ‘division’, ‘category’, or ‘level’. The only difference between Chang’s neighborhoods and Andreou’s categories seems to be that Chang does not take her neighborhoods to be hierarchical in the way that Andreou takes her categories to be.

Chang then uses these two notions to present sufficient, but not necessary, conditions for parity:

I suggest that parity holds when two items are (i) qualitatively very different with respect to the covering value, and yet (ii) in the same neighborhood of value overall with respect to the covering value. These two features – being qualitatively very different and in the same neighborhood of value with respect to the covering value – together provide sufficient conditions for parity.

Put another way, two items are on a par when they are quantitatively around the same with respect to the covering value, but are qualitatively quite different with respect to that same value. Given these conditions for parity, Pearl Jasmine and Sumatra Gold are on a par with respect to tastiness if claims [1] – [3] are false, but claims [6] and [7] below are true:

[6] Pearl Jasmine and Sumatra Gold are qualitatively very different with respect to tastiness.

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25 See Chang (2016b), pg 405.
26 See Chang (2016b), pg 405 footnote. I admit though that I’m not quite sure I see the importance of this distinction. In conversation, Chang claims that using hierarchical categories will result in an endorsement of the Trichotomy Thesis, though I don’t see why this would follow. In any case, I take the distinction between Andreou’s hierarchical categories and Chang’s non-hierarchical neighborhoods to be unimportant for our purposes.
27 Chang (2016b), pg 404.
[7] Pearl Jasmine and Sumatra Gold are in the same neighborhood overall with respect to tastiness. It is important to stress here that Chang does not take either qualitative difference or sameness of neighborhood to be individually sufficient for parity. This is because items can be qualitatively quite different and still be better than, worse than, or maybe even equal to each other (e.g., Pearl Jasmine \[+++\] is qualitatively quite different from, but still better than, Sumatra Gold). Likewise, items can be in the same neighborhood overall and still be better than, worse than, or equal to each other. Rather parity obtains on this view when it is the case that the items are qualitatively quite different, in the same neighborhood, and also neither better than, worse than, nor equal to each other, all with respect to the relevant covering value. So Chang’s analysis of parity in this case can be understood as [4-Chang] below, which is simply the conjunction of the falsity of [1] – [3] and the truth of [6] and [7]:

[4-Chang] Pearl Jasmine and Sumatra Gold are neither better than, worse than, nor equal to each other, but are qualitatively very different from each other and in the same neighborhood overall, all with respect to tastiness.

6. Meeting the Specification Challenge

In this section, I examine how Andreou’s and Chang’s analyses of parity might meet the three demands of the Specification Challenge. In so doing, I show that INCOMPARABILITY can, and likely does, endorse the claim that seemingly incomparable items are on a par under both analyses. Further, I argue that this will be the case for any other plausible analysis of parity. If this is correct, then PARITY and INCOMPARABILITY both endorse the relation of parity holding between seemingly incomparable items (i.e., PARITY Claim 2), and differ only in whether they take that relation to be necessary for exhausting the conceptual space of comparability (i.e., PARITY Claim 3). That is, the difference between PARITY and INCOMPARABILITY is not in
whether they take seemingly incomparable items to be on a par, but in whether they take parity to be a basic comparative relation. Given that this is the only distinction between Parity and Incomparability, any advantage that one account might have over the other must stem from this difference. As we will see, some such advantages may be in the offing, though I refrain from a full evaluation of their merits.

6.1 Meeting the Characterization Demand

The Characterization Demand is to provide a non-mysterious analysis of parity that characterizes exactly what this relation is supposed to be without presupposing that it is a basic comparative relation. Here Andreou and Chang meet this demand by providing analyses of parity that are clear, appeal to non-mysterious notions, and include no mention of being a basic comparative relation. Andreou’s analysis takes parity to be the relation that obtains when items are neither better than, worse than, nor equal to each other, but nonetheless are in the same category with respect to the relevant covering value. The notion of items being neither better than, worse than, nor equal to each other is familiar, so the only new notion being appealed to here is that of being in the same category. However, neither that notion nor its conjunction with the failure of the original trichotomy of comparative relations is mysterious. So Andreou’s analysis is non-mysterious.

Chang’s analysis takes parity to be the relation that obtains when items are neither better than, worse than, nor equal to each other, but are nonetheless qualitatively different from each other and in the same neighborhood overall with respect to the relevant covering value. The only new notions being appealed to here are that of qualitative difference and sameness of neighborhood. Here too though, neither of these notions nor their conjunction with the failure of
the original trichotomy of comparative relations is mysterious. So Chang’s analysis is also non-mysterious.

**6.2 Meeting the Distinction Demand**

The Distinction Demand is to give a full accounting of the distinction between PARITY and INCOMPARABILITY given the analysis of parity used to satisfy the Characterization Demand. Recall that one distinction between PARITY and INCOMPARABILITY has already been established. That is, it is already clear that the two accounts differ in their endorsement of the Trichotomy Thesis (i.e., PARITY Claim 3). What is not clear is whether the two accounts also differ in their endorsement of the claim that seemingly incomparable items are on par (i.e., PARITY Claim 2). While PARITY endorses this claim, it is not clear whether INCOMPARABILITY does as well. If INCOMPARABILITY does, then there is only one distinction between PARITY and INCOMPARABILITY. If INCOMPARABILITY doesn’t, then there are two distinctions. The question then is whether INCOMPARABILITY endorses the relation of parity obtaining between seemingly incomparable items given Andreou’s and Chang’s analyses of parity.

Here it turns out that when parity is understood using Andreou’s and Chang’s analyses, INCOMPARABILITY certainly can and likely does endorse the claim that seemingly incomparable items are on par. To see this, note that both of their analyses understand parity as consisting of the failure of the original trichotomy of relations along with the addition of certain other relations. Now INCOMPARABILITY of course endorses the failure of the original trichotomy of relations between seemingly incomparable items. But INCOMPARABILITY can also endorse the additional relations that Andreou and Chang appeal to in their analyses. The only reason INCOMPARABILITY would not be able to endorse these additional relations would be if doing so
conflicted with INCOMPARABILITY’s commitment to the Trichotomy Thesis. However, one can endorse these additional relations without abandoning the Trichotomy Thesis. To see this, we’ll examine each of the additional relations in turn.

Take first Andreou’s analysis where the additional relation appealed to is sameness of category. As mentioned above though, items can be in the same category without being on a par. That is, items that are better than, worse than, or equal to each other with respect to the relevant covering value can still be in the same category. For instance, we might say that Bill Gates and Warren Buffett are in the same category with respect to net worth (i.e., they are both in the category of ridiculously wealthy), even though it is also the case that Gates is ‘better than’ Buffett with respect to net worth (i.e., Gates has a higher net worth than Buffett). But in endorsing the claim that Gates and Buffett are in the same category, one need not be rejecting the Trichotomy Thesis. Indeed, the Trichotomy Thesis is clearly true with respect to the covering value of net worth. That is, for any two items that are comparable with respect to net worth, one must be either better than, worse than, or equal to the other in net worth. So sameness of category can be endorsed without rejecting the Trichotomy Thesis.

But if sameness of category can be endorsed without violating the Trichotomy Thesis, then there is no obstacle to INCOMPARABILITY endorsing sameness of category in cases of seeming incomparability. There is then no reason that INCOMPARABILITY cannot endorse parity between seemingly incomparable items under Andreou’s analysis. Chang actually seems to recognize this feature of Andreou’s analysis:

[Andreou’s] understanding of parity is one that trichotomists could accept. She says that the main reason to accept parity is that it allows us to say something ‘positive’ about how two items that are not related by the standard trichotomy. The positive thing we can say is that they belong to the same category of value. But this is something trichotomists can also say. Indeed, some trichotomist incomparabilists understand incomparability in the way Andreou understands
parity. Mozart is neither more or less creative than Michelangelo, and nor are they equally creative: they are incomparable. But this judgment does not preclude the further claim that they are in the same category, league, or neighborhood of creativity – they are both creative geniuses.²⁸

Now take Chang’s analysis where the additional relations appealed to are large qualitative differences and sameness of neighborhood. Since sameness of neighborhood is so similar to sameness of category, I will here focus on the relation of large qualitative difference. As mentioned above, items can be qualitatively quite different without being on a par. That is, items that are better than, worse than, or equal to each with respect to the relevant covering can still be qualitatively quite different.

Take John Stuart Mill’s view on higher and lower pleasures. Here Mill claims that the higher intellectual pleasures are not only qualitatively quite different from the lower animal pleasures, but are also such that the higher pleasures are lexically better than the lower pleasures.²⁹ That is, any amount of higher pleasures, however small, is always better than any amount of lower pleasures, however large. On this picture, Mill endorses large qualitative differences in the utility of higher and lower pleasures while also maintaining the Trichotomy Thesis with respect to utility.

Now whether or not one agrees with Mill here, it seems clear that his endorsement of large qualitative differences between higher and lower pleasures is not in tension with his endorsement of the Trichotomy Thesis. So the existence of large qualitative differences between items can be endorsed without rejecting the Trichotomy Thesis. But if the existence of large qualitative differences can be endorsed without violating the Trichotomy Thesis, then there is no obstacle to INCOMPARABILITY endorsing large qualitative differences in cases of seeming incomparability. Combined with INCOMPARABILITY’s ability to endorse sameness of

²⁸ Chang (2016b), pg 406.
²⁹ See Mill (2005), pp 8-11.
neighborhood given its similarity to sameness of category, there is then no reason that INCOMPARABILITY cannot endorse parity between seemingly incomparable items under Chang’s analysis.

The upshot here is that INCOMPARABILITY can endorse the claim that seemingly incomparable items are on a par under both Andreou’s and Chang’s analyses. Moreover, insofar as it is plausible that the additional relations appealed to in Andreou’s and Chang’s analyses hold in cases of seeming incomparability, INCOMPARABILITY likely does endorse the claim that seemingly incomparable items are on a par. Further, INCOMPARABILITY will likely be able to endorse this claim under any other plausible analysis that similarly takes parity to consist in additional relations beyond the failure of betterness, worseness, and equality. This is because any other plausible additional relations will likely be compatible with the Trichotomy Thesis in the same way that the additional relations appealed to by Andreou and Chang are compatible.

If this is right, then the only distinction between PARITY and INCOMPARABILITY is in their commitment to the Trichotomy Thesis. While both PARITY and INCOMPARABILITY endorse parity between seemingly incomparable items, PARITY takes this relation to be a basic comparative relation and so rejects the Trichotomy Thesis, while INCOMPARABILITY does not and so maintains the Trichotomy Thesis. To return to the Pearl Jasmine/Sumatra Gold example, PARITY and INCOMPARABILITY both hold [4], reproduced below, to be true:

[4] Pearl Jasmine and Sumatra Gold are on a par with respect to tastiness.

INCOMPARABILITY just claims that while [4] is true, parity is not a basic comparative relation needed to exhaust the conceptual space of comparability with respect to tastiness. In this way, INCOMPARABILITY endorses [4] in the same way that PARITY would endorse some relation that it took obtained between seemingly incomparable items but that was non-basic. Take for instance
the relation [4-Salty] below which is simply the conjunction of the falsity of [1] – [3] and the truth of the claim that Pearl Jasmine and Sumatra Gold are equally salty:

[4-Salty] Pearl Jasmine and Sumatra Gold are neither better than, worse than, nor equal to each other, but are equally salty with respect to tastiness.

While both PARITY and INCOMPARABILITY would endorse the truth of [4-Salty], neither would take the failure of betterness, worseness, and equality conjoined with equality of saltiness to be a basic comparative relation needed to exhaust the conceptual space of comparability with respect to tastiness. INCOMPARABILITY then endorses [4] in the same way that PARITY endorses [4-Salty].

6.3 Meeting the Advantage Demand

The Advantage Demand is to show how PARITY might have an advantage over INCOMPARABILITY given the full accounting of distinctions between PARITY and INCOMPARABILITY provided in satisfying the Distinction Demand. Given that PARITY and INCOMPARABILITY differ only in whether they take parity to be a basic comparative relation, any advantage that PARITY has over INCOMPARABILITY will have to be due to this distinction. But what difference could this distinction make? Here, we can derive two potential answers from Chang’s previous works.

The first potential advantage of endorsing parity as a basic comparative relation is that such an endorsement is compatible with both rational choice between seemingly incomparable options and a view about the underpinnings of rational choice that Chang calls comparativism.

Comparativism: Comparative facts about the evaluative merits of the options with respect to what matters in a well-formed choice situation [are] that in virtue of which a choice is rational in that situation.\(^3^0\)

According to comparativism, a necessary condition for rational choice is that there be some comparative fact between the available options. If there are no comparative facts between the

\(^{30}\) Chang (2016a), pg 214.
options available in a choice situation, then there is nothing in virtue of which a rational choice can be made. Now Chang explicitly takes comparative facts to be what she calls ‘positive’ in nature:

By “comparative fact,” I mean a positive comparative fact, that is, a fact that describes how something is rather than how it is not… I mean a fact that gives a positive relation between two items in some respect… Thus, being better than, worse than, and equally good are all comparative relations and give rise to corresponding comparative facts. By contrast, being not worse than, not better than, not equal to, and neither better nor worse are not comparative relations and do not give rise to comparative facts. \(^{31}\)

Chang here is claiming that INCOMPARABILITY is unable to allow for comparative facts between seemingly incomparable options, since it does not allow for any positive comparative relation to obtain between them. INCOMPARABILITY only allows for negative comparative relations between seemingly incomparable options. While INCOMPARABILITY endorses parity between seemingly incomparable options, it does not take parity to be a basic comparative relation and so cannot endorse that relation as a positive comparative fact between the items. PARITY on the other hand allows for a positive comparative relation to obtain between seemingly incomparable options, since it endorses parity as a fourth basic comparative relation, which in turn allows for a positive comparative fact.

Chang’s idea then is that if comparativism is true, then INCOMPARABILITY will not be able to allow for rational choice between seemingly incomparable options. This is because if the seemingly incomparable options are as INCOMPARABILITY claims, then there will be no comparative facts between the items that could ground rational choice between them as demanded by comparativism. If on the other hand, a fourth basic comparative relation of parity obtains between seemingly incomparable items, then there is some positive comparative fact that can ground rational choice between them.

\(^{31}\) Chang (2016a), pg 217.
Put another way, the following constitute an inconsistent triad: **INCOMPARABILITY**, comparativism, and rational choice between seemingly incomparable options. Unlike **INCOMPARABILITY**, **PARITY** is compatible with both comparativism and rational choice between seemingly incomparable options. So, **INCOMPARABILITY** is forced to give up either comparativism or rational choice between seemingly incomparable options, while **PARITY** is not. In this way, **PARITY** has an advantage over **INCOMPARABILITY** insofar as one is inclined to accept both comparativism and rational choice between seemingly incomparable options. I leave open here whether comparativism and rational choice between seemingly incomparable options should in fact be accepted, as that is a contentious debate for another time. The main point here is that the subtle distinction between how **PARITY** endorses parity as a fourth basic comparative relation and how **INCOMPARABILITY** endorses it as a non-basic relation can be a distinction with a difference. Moreover, this difference *may* give **PARITY** an advantage, so long as a good case can be made for both comparativism and rational choice between seemingly incomparable options.

The second potential advantage of endorsing parity as a basic comparative relation relates to Chang’s existential argument for parity, which can be rationally reconstructed as follows:  

Q1 – Pearl Jasmine is neither better than, worse than, nor equal to Sumatra Gold with respect to tastiness.  
Q2 – Pearl Jasmine is comparable to Sumatra Gold with respect to tastiness.  
Q3 – Pearl Jasmine is connected to Pearl Jasmine via a series of small unidimensional differences.  
Q4 – A small unidimensional difference cannot result in incomparability where before there was comparability.  
D1 – Therefore, Pearl Jasmine is comparable to Sumatra Gold.  
Q5 – If two items are comparable, then there must be some basic comparative relation that obtains between them.  
D2 – Therefore, some basic comparative relation obtains between Pearl Jasmine and Sumatra Gold that is not betterness, worseness, or equality (i.e., parity).

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32 See Chang (2016a) for discussion.  
33 Reconstructed from Chang (2002).
Chang’s argument here proceeds by first stipulating that seemingly incomparable items are neither better than, worse than, nor equal to each other with respect to the relevant covering value (Q1). This is of course a claim that INCOMPARABILITY also makes. The second premise invokes the phenomenon of seeming incomparability being broken by significantly improving one of the items while leaving the other fixed (Q2). This is the third key feature of seeming incomparability discussed earlier. The third premise claims that the significantly improved item that breaks the seeming incomparability is connected to the original item by a continuum of small unidimensional differences (Q3). INCOMPARABILITY is not committed to this claim, but it seems entirely plausible. The fourth premise is more controversial. This is Chang’s Small Unidimensional Difference Principle, which claims that if two items are comparable, then making a small unidimensional change to one of them is not enough to generate incomparability between the new pair of items (Q4). If all these premises are true though, then it follows that Pearl Jasmine is comparable to Sumatra Gold since the Small Unidimensional Difference Principle can be serially applied to each item on the continuum between Pearl Jasmine and Pearl Jasmine as compared to Sumatra Gold. Further, given the reasonable further assumption that items are comparable just in case some comparative relation obtains between them (Q5), it follows that some basic comparative relation that is not betterness, worseness, or equality holds between Pearl Jasmine and Sumatra Gold.

Now this is meant to be an independent argument for the existence of parity as some fourth basic comparative relation but without further specification of that relation. So it is compatible with either Andreou’s or Chang’s analysis of parity. However, it is of course not compatible with INCOMPARABILITY which denies the existence of any fourth basic comparative relation and whose endorsement of parity can only be as a non-basic relation. So
INCOMPARABILITY is forced to deny at least one of the premises in this argument. As mentioned above, some of these premises cannot be challenged from the perspective of INCOMPARABILITY, while others are independently plausible. The premise most vulnerable to rejection is likely the Small Unidimensional Difference Principle and indeed has been challenged in the literature.\textsuperscript{34} Whether the Small Unidimensional Difference Principle should be accepted though is a contentious debate for another time. The main point here is that the subtle distinction between how PARITY endorses parity as a fourth basic comparative relation and how INCOMPARABILITY endorses it as a non-basic relation can be a distinction with a difference. Moreover, this difference may give PARITY an advantage, so long as a good case can be made for all of the premises in Chang’s existential argument, especially her Small Unidimensional Difference Principle.

7. Conclusion

I have here presented a new challenge to the PARITY account of seeming incomparability that is motivated by the mysteriousness of parity and need to understand how this account differs from the INCOMPARABILITY account. These motivations inform the three interrelated demands of the challenge. The Characterization Demand requires an analysis of parity telling us what exactly this relation is supposed to be without presupposing that it is a basic comparative relation. The Distinction Demand requires a full account of what distinguishes PARITY and INCOMPARABILITY given the analysis used to satisfy the Characterization Demand. The Advantage Demand requires a demonstration of how PARITY is a better account of seeming incomparability given the differences identified in satisfying the Distinction Demand.

\textsuperscript{34} Chang (2002) herself discusses qualifications to the Small Unidimensional Difference Principle. See also Elson (2014) who challenges Chang’s use of this principle.
I have also explored how two recent analyses of parity might be used to meet this challenge. In so doing, I argued that despite initial appearances, INCOMPARABILITY can and likely does endorse the claim that seemingly incomparable items are on par. It turns out then that the only distinction between INCOMPARABILITY and PARITY is in whether they endorse parity as a basic comparative relation. While this distinction is subtle, it might provide PARITY with an advantage over INCOMPARABILITY. Whether it does will depend on if we have decisive independent reason to accept comparativism and rational choice between seemingly incomparable options, or, all the premises in Chang’s existential argument. This last question though is left for another time.
References


Chapter 2
Seeming Incomparability and Rational Choice

Abstract
We sometimes have to choose between options that are seemingly incomparable insofar as they seem to be neither better than, worse than, nor equal to each other. This often happens when the available options are qualitatively quite different. For instance, consider a promising student who is choosing between a career in law and a career in medicine. For her, the distinctive features of each career might make it such that by her own lights, neither career seems to be better than, worse than, nor equal to the other. Such seemingly incomparable options present a problem for rational choice since it is unclear how the agent might rationally choose between them. What we need are some principles to help govern rational choice when faced with seemingly incomparable options. I here present three such principles. While each principle is individually compelling, I show that they are jointly incompatible. I then argue that the correct response to this inconsistent triad is to reject the principle that rationally censures performing a sequence of choices one knows will result in a suboptimal outcome. The upshot is that when seeming incomparability is involved, one can be money pumped without being less rational for it.
1. Introduction

It is not difficult to think of cases where we feel an intuitive resistance towards claims of betterness, worseness, or equality between two options. Indeed, many choices in life involve such options. For instance, our choice of professional careers, romantic partners, vacation destinations, and even casual lunch plans often involve options that seem to be neither better than, worse than, nor equal to each other. In this way, we often have to choose between options that are seemingly incomparable to each other. Given the ubiquity of such cases, it would be helpful to have some principles to help govern rational choice between seemingly incomparable options.

In this paper, I present three such principles. The first principle focuses on the comparative rationality of sequences of choices (i.e., sets of multiple choices over time) rather than the rationality of a single choice. This principle, which I call the Comparative Money Pump Principle, roughly holds that it is less rational to knowingly perform a sequence of choices that will result in a suboptimal outcome than it is to perform a sequence of choices that will not result in a suboptimal outcome. The second and third principles focus on the supervenience base of the rationality of sequences of choices and individual choices, respectively. While each of these principles is individually compelling, I show that they are actually jointly incompatible. I then argue that the correct response to this inconsistent triad is to maintain the two supervenience principles, but reject the Comparative Money Pump Principle. So when seeming incomparability is involved, one can be money pumped without being less rational for it.

This paper proceeds by first examining the phenomenon of seeming incomparability and what I take to be its three key features. After that, I present the three principles and demonstrate
their joint incompatibility. Finally, I consider the consequences of rejecting each of the principles before concluding that the one to reject is the Comparative Money Pump Principle.

2. The Phenomenon of Seeming Incomparability

In order to understand the phenomenon of seeming incomparability, it is helpful to start with a paradigmatic example provided by Ruth Chang (2002, p. 669):

Suppose you must determine which of a cup of coffee and a cup of tea tastes better to you. The coffee has a full-bodied, sharp, pungent taste, and the tea has a warm, soothing, fragrant taste. It is surely possible that you rationally judge that the cup of Sumatra Gold tastes neither better nor worse than the cup of Pearl Jasmine and that although a slightly more fragrant Jasmine would taste better than the original, the more fragrant Jasmine would not taste better than the cup of coffee. In this case, it is plausible to suppose that you know everything that is relevant to comparing the drinks and that in this case you have first-person authority over which tastes better to you.

Here, there is an attempt to compare two particular items, the cup of Sumatra Gold coffee and the cup of Pearl Jasmine tea, with respect to a particular value, tastiness, that is applicable to both items. The appeal to a particular value, also called a *covering value*, is crucial because it specifies the particular dimension along which the items are being compared and without which comparisons are impossible. That is, there is no way to compare things simpliciter without reference to some dimension of evaluation. Moreover, it is important to note that both of the items here fall within the domain of the specified covering value (i.e., tastiness is applicable to both the coffee and tea). In this way, there is no formal failure of comparison as there would be if the covering value was not applicable to some of the items under comparison (e.g., if Sumatra Gold and the number three were compared with respect to tastiness).

Now despite the covering value of tastiness clearly applying to both the Sumatra Gold and Pearl Jasmine, there can nonetheless be intuitive resistance to claims that either is better

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35 See Chang (2002) for her discussion of this requirement.
than, worse than, or equal to the other with respect to tastiness. That is, you can be reluctant to
assent to all of the following claims:36

[1] Pearl Jasmine is better than Sumatra Gold with respect to tastiness.
[2] Pearl Jasmine is worse than Sumatra Gold with respect to tastiness.
[3] Pearl Jasmine and Sumatra Gold are equal with respect to tastiness.37

This is not yet to say that you would hold these claims to all be false. That is a stronger
commitment and one that is not, I take it, as intuitively clear. Rather, the phenomenology of such
cases seems to be an intuitive reluctance to assent to claims of betterness, worseness, and
equality rather than an outright rejection of those claims. So I take the first key feature of
seeming incomparability to be this intuitive resistance to claims of betterness, worseness, and
equality.

The second key feature of seeming incomparability is that this intuitive resistance persists
even if one of the items is slightly improved or worsened. So in addition to being reluctant to
assent to claims [1] – [3] above, one can also be reluctant to assent to any of the following claims
where the Pearl Jasmine has been replaced with a slightly better tea, call it Pearl Jasmine⁺:

[1*] Pearl Jasmine⁺ is better than Sumatra Gold with respect to tastiness.
[2*] Pearl Jasmine⁺ is worse than Sumatra Gold with respect to tastiness.
[3*] Pearl Jasmine⁺ and Sumatra Gold are equal with respect to tastiness.

The third key feature of seeming incomparability is that while the intuitive resistance
towards claims of betterness, worseness, and equality persists when one of the items is slightly
improved/worsened, it does not persist if one of the items is significantly improved/worsened.38

36 The same goes for claims where the place of the items is switched (e.g., Sumatra Gold is better than Pearl Jasmine
with respect to tastiness).
37 The implausibility of claim [3] is often also argued for using the Small Improvement Argument. See Chang
38 This feature is not illustrated in Chang’s example above, but it is discussed separately as what she calls nominal-
So if Pearl Jasmine were replaced with a much better tea, call it Pearl Jasmine+++…, we would no longer be reluctant to assent to all of the following claims:

[1**] Pearl Jasmine+++… is better than Sumatra Gold with respect to tastiness.
[2**] Pearl Jasmine+++… is worse than Sumatra Gold with respect to tastiness.
[3**] Pearl Jasmine+++… and Sumatra Gold are equal with respect to tastiness.

Rather, we would hold [1**] to be true and [2**] and [3**] to both be false. Before moving on, it is worth mentioning the ubiquity of plausible instances of seeming incomparability. Below are just a few other examples of seeming incomparability:

(i) Comparing joining the army to joining the church with respect to goodness as a career.39
(ii) Comparing leaving for England to join the Free French Forces during World War II to staying in France to take care of your elderly mother with respect to moral duty.40
(iii) Comparing Michelangelo and Mozart with respect to artistic creativity.41
(iv) Comparing a cheaper Chinese restaurant to a closer Indian restaurant with respect to your dinner preferences.42
(v) Comparing a policy proposal that prioritizes public transportation infrastructure to one that prioritizes pre-kindergarten education with respect to the public good.
(vi) Comparing the GRW interpretation to the Many-Worlds interpretation with respect to goodness as an interpretation of quantum mechanical phenomena.
(vii) Comparing the Russian and Pashto languages with respect to their relative similarity to English.

What is particularly interesting here is the wide variation displayed by these cases. Some of these comparisons would be practically important for significant life changing choices (e.g., i, ii, and v), while others would only be relevant for fairly unimportant decisions (e.g., iv) or have no immediate practical import whatsoever (e.g., iii, vii). Some of these comparisons invoke covering values that seem to be objective (e.g., ii, v, vi), while others seem to be subjective (e.g., i, iv). Some invoke covering values that seem to be normative (e.g., ii, v, vi) while others seem to be clearly non-normative (e.g. vii). This variability underscores just how widespread the

41 Example from Chang (2002), pp. 659.
42 Example from Hare (2010), pp. 238.
phenomenon of seeming incomparability is and how it can crop up in areas well beyond value and decision theory. What these cases all have in common though is that they all exhibit the three key features of seeming incomparability discussed above.43

Now that the phenomenon of seeming incomparability and its ubiquity has been discussed, we can turn to the question of what to do in choice situations involving seemingly incomparable options.

3. Three Principles

Given the widespread phenomenon of seeming incomparability in general and choice situations involving seemingly incomparable options in particular, it would be nice to have some principles to help govern rational choice between such options. I will not here defend a complete set of such rational principles, but rather will present three independently compelling principles that we would intuitively want to include in any such complete set.

3.1 Comparative Money Pump Principle

The first principle compares the rationality of performing different sequences of choices (i.e., sets of multiple choices over time). Before presenting the principle itself though, it’s helpful to first look at two choice situations involving seemingly incomparable options that help motivate it:

I should note here a similarity between seeming incomparability and imprecise credences in epistemology. Miriam Schoenfield (2014) has pointed out that both can be understood to be cases of orderings that cannot be represented using a single function. With seeming incomparability, there is no single function that assigns to each item a real number representing its value. With imprecise credences, there is no single function that assigns to each proposition a real number representing one’s degree of belief in that proposition. Given this similarity, Schoenfield holds that investigations into how decision theory might be extended to accommodate each case will run parallel to each other. I agree with her here and think that much of what I say later on can be applied to the project of extending decision theory to cover imprecise credences. I mention some applications in footnote 22, but a full discussion of the parallels between these projects is beyond the scope of this paper.

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In this first choice situation, an agent is faced with a single choice at t1 between four options: A, B-, A-, and B. Here the A options are seemingly incomparable with the B options (e.g., you might imagine them to be Chang’s cups of coffee and tea). So, each individual option is better than or worse than one of the other options, but seemingly incomparable with the other two. For example, A is better than A-, but seemingly incomparable with both B and B-.  

Now, whatever one thinks about the rationality of choosing between seemingly incomparable options, it seems clearly less rational to choose A- or B- rather than A or B. After all, in choosing A- or B-, one is choosing an option that they know to be worse than some other available option and in this way would be suffering a sure loss. While A and B are not better than or equal to every other option, they are at least not worse than any other option and so are not suboptimal, unlike A- and B-. More generally, it seems less rational to choose options that are suboptimal than it is to choose options that are not suboptimal, even when seeming
incomparability is involved.\textsuperscript{44} This is of course not surprising. Take now a more complicated choice situation involving a sequence of two choices:

\textit{Figure 2. Motivating Choice Situation 2}

![Diagram of Motivating Choice Situation 2]

Here, an agent is faced with a choice between seemingly incomparable options A and B at t1. They then face another choice at t2 between keeping their previously chosen option (A or B), or, trading it for an option that is slightly worse than the option they did not choose at t1 (B\textsuperscript{-} or A\textsuperscript{-}). Importantly, the choice at t2 is also between seemingly incomparable options given the second key feature of seeming incomparability previously discussed. (You might imagine the agent facing an initial choice between Chang’s coffee and tea, then immediately thereafter being offered to trade their chosen option for a slightly worse version of their unchosen option.)

There are then four different sequences that the agent can perform in this choice situation: A/A, A/B\textsuperscript{-}, B/A\textsuperscript{-}, and B/B. Two of these sequences involve staying with the option chosen at t1 (A/A, B/B), while the other two involve switching from that option to one that is slightly worse than the option they rejected at t1 (A/B\textsuperscript{-}, B/A\textsuperscript{-}).\textsuperscript{45}

\textsuperscript{44} Note that the intuition appealed to here is entirely comparative in nature. That is, it is only about the relative rationality of the available options. In this way, no stance is taken on whether choosing A or B in Figure 1 is fully rational, only that choosing A or B is more rational than choosing A\textsuperscript{-} or B\textsuperscript{-}.

\textsuperscript{45} It is important to note here that it is part of the stipulations of this choice situation that the A and B items remain seemingly incomparable throughout, regardless of what the agent chooses at t1 or t2. Here it may be helpful to
Now whatever one thinks about the rationality of choosing between seemingly incomparable options, it seems clearly less rational to perform one of the Switch sequences rather than one of the Stay sequences. After all, in performing one of the Switch sequences, one would end up with an outcome that they know is definitely worse than another they could have had if only they had performed one of the Stay sequences. In performing one of the Stay sequences on the other hand, one would not end up with an outcome that they know to be definitely worse than another they could have had by performing a different sequence. So agents who switch are making a series of choices they know will result in a suboptimal outcome, while agents who stay are not. In this way, agents who switch are in effect money pumping themselves (i.e., making a series of choices that results in a sure loss), while agents who stay are not.\(^{46}\) It seems then that just as it is less rational to make a single choice whose outcome is suboptimal as compared to a single choice whose outcome is not suboptimal, so too when it comes to sequences of choices.\(^{47}\) This seeming difference in the rational statuses of sequences of choices is plausibly explained by the following general principle:

*Comparative Money Pump Principle*

In choice situations where risk is not involved, it is less rational to perform a sequence of choices whose outcome one knows in advance will be worse compared to that of some other available sequence than it is to perform a sequence of choices whose outcome one does not know will be worse compared to that of any other available sequence.

This principle captures the intuition that it is less rational to perform a Switch sequence than a Stay sequence in Figure 2. However, it is important to note that this principle only applies to

\(^{46}\) Chang (1997), p. 11 and Broome (2000, 2001) also discuss the intuitive irrationality of being money pumped in this way, though they have different accounts of how problematic it is.\(^{47}\) Another way to approach this is to ask yourself what advice you would give to someone about to face this choice situation. I take it that most of us would advise them to choose either A or B at t1 and stick with that choice at t2.
choice situations where the outcomes are not subject to risk. That is, it only applies when the agent knows which outcomes will result from her choices with certainty and where those outcomes are not lotteries.\textsuperscript{48} It is also important to note that this principle is entirely comparative in nature, making no claim whatsoever about the rationality or irrationality of performing any particular sequence of choices as such. It only claims that certain sequences of choices are \textit{less} rational to perform than others. It does not follow from this that any sequence is fully rational or irrational to perform.

\textbf{3.2 Supervenience Principle I}

The second principle claims that when evaluating the rationality of some sequence of choices, we need only look at the rationality of the individual choices within that sequence. So if we want to evaluate the rationality of a sequence of choices \( C \) consisting of two individual choices \([c_1, c_2]\), we need only look at the rationality of those individual choices. That is, the rationality of \( c_1 \) and \( c_2 \) together determine the rationality of sequence \( C \).

The general idea here is that a sequence of choices is nothing more than a series of particular, individually assessable choices, and that it is the rationality of these constituent choices that determine the rationality of the sequence of choices. So, if two sequences of choices differ in rational status, it must be because of some difference in the rationality of their constituent choices. More specifically, this idea can be understood as the following supervenience principle:

\textsuperscript{48} This is to rule out the applicability of this principle in cases where it seems more rational for an agent to accept a slightly suboptimal outcome for sure than accept a lottery that could result in the optimal outcome or one that is much worse (e.g., a choice between $1000 for sure, $999 for sure, or a lottery between $1000 or $0 with even chances).
Supervenience Principle I

If there is a difference in the rationality of performing two sequences of choices favoring one over the other, there must be a difference in the rationality of some individual choices made within those sequences, favoring the relevant sequence.

This supervenience principle seems quite plausible. To deny it would be to endorse the possibility of two sequences of choices differing in rationality despite being composed of constituent choices that were identical in rational status. But that would be strange. After all, what else would the rationality of a sequence of choices depend on besides the rationality of their constituent choices? As we will see in section 5 though, one might nonetheless hold that the rationality of a sequence of choices can depend on things besides the rationality of their constituent choices. However, I will argue there that this possibility is seriously problematic because it would result in either a picture of rationality that is fundamentally fractured, or, a rational assessment of sequences of choices that is devoid of normative force, or, a commitment to the rejection of Supervenience Principle II.

3.3 Supervenience Principle II

The third principle makes a claim about the rationality of choosing particular options in individual choice situations where only value considerations matter. These are choice situations where there are no non-value considerations (e.g., categorical imperatives, deontic side constraints, etc...) at play. That is, the only thing that matters is how the options fare with respect to certain covering values that are stipulated to be relevant to the choice at hand. In such choice situations, this principle claims that the comparative rationality of choosing a particular option depends only on the comparative relations that obtain between the currently available

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49 It is worth noting that this principle in no way rules out the possibility of past choices affecting the rationality of future choices. Indeed, this principle says nothing at all about how the rationality of individual choices within sequences is to be determined.

50 I take it though that many, if not most or even all, choice situations are actually like this.
options with respect to the relevant covering values. So in a choice situation involving options [o1, o2] and where covering value V is the only relevant consideration, the comparative rationality of choosing either option depends only on how o1 and o2 compare with respect to V.

The general idea here is that if there is a difference in the rationality of choosing between two available options in a choice situation, there has to be a reason that justifies that difference. In cases where only value considerations are relevant, this reason must appeal to some difference between the available options with respect to the relevant covering values that favors one option over the other. Options that are perfectly symmetrical with respect to the relevant covering values will not allow for any reason that could justify a rational difference in choosing one rather than the other. In this way, there must be some comparative asymmetry between the available options with respect to the relevant covering values that can then allow one option to be rationally favored over the others. More specifically, this idea can be understood as the following supervenience principle:

*Supervenience Principle II*

If there is a difference in the rationality of choosing between two options in a choice situation where only value considerations matter, there must be some comparative asymmetry between the available options with respect to the relevant covering values that favors the more rational option.

So if there is a difference between the rationality of choosing o1 and o2, there must be some comparative asymmetry between the two with respect to V, favoring the more rational option. This supervenience principle also seems quite plausible. After all, in choice situations where only V matters, how could there be a rational difference between choosing o1 and o2 unless

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51 More technically, two items are comparatively asymmetrical with respect to some covering value just in case the truth value of *some* comparative claim between those items under that covering value (e.g., claims of betterness, worseness, equality, or perhaps parity) changes when switching the positions of those items in that claim. In contrast, two items are comparatively symmetrical with respect to some covering value just in case the truth value of *any* comparative claim between those items under that covering value does not change when switching the positions of those items in any of those claims.
there was some sort of comparative asymmetry between them with respect to $V$? As we will see in section 5 though, one might object that this supervenience principle is overly restrictive insofar as it only considers currently available options and does not also include previously available options. However, I will argue that this restriction is entirely appropriate as broadening the scope to include previously available options amounts to committing the sunk cost fallacy.

4. Incompatibility of the Three Principles

I take each of the three principles discussed in the previous section to be intuitively compelling. However, I will show that they are jointly incompatible. To do this, I will use two choice situations involving seemingly incomparable options and show that the three principles entail a contradiction. These choice situations are simply variants of Figure 2 above. They are structurally identical insofar as they both consist of a first choice between two options at $t_1$, followed by a second choice at $t_2$ between keeping their previously chosen option, or, switching it for one that is slightly worse than the option they did not previously choose. As with Figure 2, each of the choices at $t_1$ and $t_2$ are between seemingly incomparable options. That is, I am assuming that each of $A^+$, $A$, and $A^-$ are seemingly incomparable with each of $B^+$, $B$, and $B^-$ with respect to some covering value that is stipulated to be the only one that is relevant in this choice situation.\footnote{Strictly speaking, I only need to assume the seeming incomparability of $A$ and $B$ to get the contradiction. It follows from that assumption that the choices at nodes $n_1$, $n_3$, $n_4$, and $n_5$ will all be between seemingly incomparable options and the sequences needed to derive the contradiction go through only these nodes.} The choice nodes ($n_1$–$n_6$) and the possible choice sequences (Stay I-IV and Switch I-IV) are labeled for ease of reference.
Incompatibility Argument

P1 – In choice situations where risk is not involved, it is less rational to perform a sequence of choices whose outcome one knows in advance will be worse compared to that of some other available sequence than it is to perform a sequence of choices whose outcome one does not know will be worse compared to that of any other available sequence. [Comparative Money Pump Principle]

C1 – It is less rational to perform Switch II than Stay II. [from P1 and Situation 1]
C2 – It is less rational to perform Switch III than Stay III. [from P1 and Situation 2]

P2 – If there is a difference in the rationality of performing two sequences of choices favoring one over the other, there must be a difference in the rationality of some individual choices made within those sequences, favoring the relevant sequence. [Supervenience Principle I]

C3 – There must be some difference in the rationality of choosing between A and B at node n3, favoring B. [from C1 and P2]
C4 – There must be some difference in the rationality of choosing between A and B at node n5, favoring A. [from C2 and P2]

P3 – If there is a difference in the rationality of choosing between two options in a choice situation where only value considerations matter, there must be some comparative asymmetry between the available options with respect to the relevant covering values that favors the more rational option [Supervenience Principle II]

C5 – There must be some comparative asymmetry between A and B, favoring B. [from C3 and P3]
C6 – There must be some comparative asymmetry between A and B, favoring A. [from C4 and P3]

\[\therefore\] Contradiction

This argument begins by first invoking the Comparative Money Pump Principle (P1). This principle is then applied to the two example choice situations to conclude that it is less rational to
perform certain sequences of choices rather than others in both cases (C1 and C2). Combining these conclusions with Supervenience Principle I (P2), it follows that there must be some difference in the rationality of choosing between particular choices within those sequences that explains their differing rational statuses. In particular, the rational difference must be between the choices made at t2 since the choices made at t1 are identical in the differing sequences. Therefore, there must be a difference in the rationality between choosing A and choosing B at node n3 favoring B, and between choosing A and choosing B at node n5, favoring A (C3 and C4). This is not yet a contradiction though. Rather, the contradiction arises only once we also invoke Supervenience Principle II (P3) which requires that any difference in the rationality between choosing two options must be due to some comparative asymmetry between the available items favoring the more rational option. It follows that there must be both a comparative asymmetry between A and B that favors B (C5) and a comparative asymmetry between A and B that favors A (C6). But this is impossible, so the three principles are incompatible.

One point to note here is how this argument relates to the different accounts of seeming incomparability. These accounts purport to explain what is going on in cases of seeming incomparability. For instance one account holds that seemingly incomparable items are in fact either better than, worse than, or equal to each other, but appear to be incomparable because of our ignorance as to which of these relations holds between them. Another account claims that it is determinately false that seemingly incomparable items are better than, worse than, or equal to each other, but that there is a fourth comparative relation of parity that holds between the items.

While interesting in its own right, a full discussion of these accounts is best saved for another

53 See Regan (1997) for defense of such a view.
54 See Chang (1997) for defense of such a view.
The important point for our purposes is that the argument presented here is independent of any particular account of seeming incomparability since the premises invoked neither presuppose nor imply any particular account of seeming incomparability. In this way, the incompatibility of these three principles cannot be avoided by simply appealing to one account rather than another. These three principles are incompatible regardless of which account of seeming incomparability is ultimately correct.

Finally, it is worth reiterating that the three principles here only make claims about the comparative rationality of choosing between options or sets of options and what must be the case when they differ in rational status. The principles however make no claims about the particular rational statuses of those options and whether any are ever fully rational to choose. These principles then are incompatible regardless of whether choices between seemingly incomparable options can be fully rational to choose or instead always fall short of full rationality.

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55 See Chang (1997) for an introduction to the various accounts of seeming incomparability.
56 While my discussion here is focused on the incompatibility of these three principles given choice situations involving seeming incomparability, a similar incompatibility arises given choice situations involving imprecise credences as well. Elga (2010) comes close to identifying this incompatibility, but his focus there is importantly different. Elga is concerned with whether having imprecise credences is compatible with perfect rationality, arguing that they are incompatible because there is no adequate account of how such imprecise credences can constrain rational choice. My concern is not whether seeming incomparability is compatible with perfect rationality, but rather how we should rationally respond to choice situations involving seemingly incomparable options. The parallel question then would be how we should rationally respond to choice situations given imprecise credences, regardless of whether it is perfectly rational to have such credences. That said, many of the considerations Elga invokes against the compatibility of imprecise credences with perfect rationality mirror the three principles I discuss. Moreover, I think that various responses to Elga in the literature can be interpreted as rejecting one of these principles. For instance, I think we can interpret Rinard (2015) as rejecting Supervenience Principle I and Williams (2014) as rejecting Supervenience Principle II.
57 My principles then are significantly weaker than those presented by Peterson (2007) in his argument against the possibility of rational choice between items that are on a par. There he explicitly assumes that it is rationally permissible to choose either of two items that are on a par. I take the generality of my weaker principles to be an asset as they better identify the source of the tension that arises when trying to determine what it is rational to do given seemingly incomparable options.
5. Which Principle to Reject?

In this section, I consider rejecting each of the principles before concluding that the one we should reject is the Comparative Money pump principle.

5.1 Rejecting Supervenience Principle I?

Supervenience Principle I claims that the rationality of a sequence of choices supervenes on the rationality of the choices made within that sequence. So the rational statuses of two sequences of choices cannot differ if the rational statuses of their constituent choices are identical. While this principle seems quite plausible, one might try to deny it by claiming that the rationality of a sequence of choices is a rational assessment distinct from and independent of the rational assessment of its constituent choices. How might this work? Take again Motivating Choice Situation 2, reproduced here with choice nodes and sequence names added:

*Figure 4. Motivating Choice Situation 2 Redux*

The idea here is that one might claim that the rational assessment of the agent’s choices at t1 and t2 individually are distinct from and independent of the rational assessment of the agent’s choices at t1 and t2 collectively. That is, there are three different rational assessments of the agent’s choices in Figure 4. There is one rational assessment of the agent’s choice at t1, another...
rational assessment of the agent’s choice at t2, and a third rational assessment of the agent’s choices at both t1 and t2. Importantly this third rational assessment does not supervene on the first two rational assessments. In this way, the rational assessments of all the possible choices the agent might make in Figure 4 could be as follows:

Figure 5: Three Rational Assessments of Motivating Choice Situation 2

<table>
<thead>
<tr>
<th>Rational Assessment of the Agent’s Choice(s) at:</th>
<th>t1</th>
<th>t2</th>
<th>t1 &amp; t2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[A/A]</td>
<td>Rational</td>
<td>Rational</td>
<td>Rational</td>
</tr>
<tr>
<td>[A/B-]</td>
<td>Rational</td>
<td>Rational</td>
<td>Irrational</td>
</tr>
<tr>
<td>[B/A-]</td>
<td>Rational</td>
<td>Rational</td>
<td>Irrational</td>
</tr>
<tr>
<td>[B/B]</td>
<td>Rational</td>
<td>Rational</td>
<td>Rational</td>
</tr>
</tbody>
</table>

I am assuming here, purely for the sake of simplicity, that when faced with a choice between seemingly incomparable options, it is rational to choose either option.\(^58\) Given this, all of the agent’s possible choices assessed at t1 and t2 individually are rational. However, the agent’s possible sequences of choices assessed at t1 and t2 jointly are not all rational. Rather, the performance of sequences [A/B-] and [B/A-] are irrational since each results in an outcome that is worse than that of some other sequence. These sequences are judged to be irrational even though each of the constituent choices within the sequences is individually rational. Now if this model is correct, then Supervenience Principle I is false.

There is some intuitive plausibility to this model, but it is actually incomplete as stated. There is a further question that needs to be answered, namely what the agent should rationally do in cases of conflict between these rational assessments. For instance, suppose the agent chose A at t1 and is now at node n2, facing a choice between A and B-. What rationally should she do now? On this model, there are two conflicting rational assessments of her current options. The assessment of her options at t2 alone holds that it is rational to choose either A or B-. The

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\(^58\) This assumption could be replaced with the claim that such choices are all irrational or indeterminately rational without affecting the ensuing discussion.
assessment of her options at \( t_1 \) & \( t_2 \) jointly holds that in choosing A, she would be completing a rational sequence of choices, whereas in choosing B, she would be completing an irrational sequence of choices. Given these conflicting rational assessments, what should she rationally do all things considered? Is there even an all things considered rational assessment here?

There are two ways to go and both, I argue, are problematic. Either there is an all things considered rational assessment or there isn’t. Take first the possibility that there is no all things considered rational assessment. If this is right, then in cases of conflict between different rational assessments, the agent will be simply left with the conflicting assessments and no further rational guidance as to which takes precedence or how they might be balanced. Rationality then offers no univocal assessment, consisting instead of a fractured series of different assessments vulnerable to irresolvable conflict. Indeed, on this view when we ask what it is rational to do, we are actually asking an ambiguous question. We could be asking about the rationality of options assessed only at the time they are available or assessed as the completion of any sequence of choices of arbitrary length. After all, every choice we make results in the completion of a great many different sequences of choices, ranging from relatively short up to the sequence composed of every choice we have ever made. Rationality then consists of an awful lot of different rational assessments with no all things considered assessment of what the agent should rationally do. This splintered view of rationality is neither attractive nor acceptable.

Take now the possibility that there is an all things considered rational assessment. If this is right, then there are two possibilities of how it would rule in cases of conflict between different rational assessments. Either this all things considered rational assessment at a given time always agrees with the rational assessment of the individual choice at that time, or it does not. So in the case above where the agent is at node \( n_2 \) and asking what it is all things considered rational for
her to do, either that assessment at t2 agrees with the individual rational assessment of the agent’s choices at t2, or it does not. If it always agrees with the rational assessment of the individual choice at a given time, then any conflicting rational assessment of some sequence of choices ending at that time is simply irrelevant for determining the all things considered rational assessment. In this way, the rational assessment of any sequence of choices carries no real rational weight since such assessments are effectively ignored with respect to all things considered rationality. The rational assessment of a sequence of choices then turns out to be rationally toothless.

However, the all things considered rational assessment at a given time might not always agree with the rational assessment of the individual choice at that time. Rather, the all things considered rational assessment at a given time might instead side with the rational assessment of some sequence of choices ending at that time. For instance, it may be that when faced with the conflicting rational assessments at node n2 above, the all things considered rational assessment sides with the rational assessment of the sequence of choices made at t1 and t2. This possibility is intuitively appealing since it would allow for an all things considered rational judgment that forbids the agent from switching and being money pumped. The problem with this possibility though is that such an all things considered rational assessment would violate Supervenience Principle II. This is because it would allow for something besides the comparative values of the available options at the time to influence the rationality of choosing between them. After all, this is precisely what allows the rational assessment of some sequence of choices ending at a given time to differ from the rational assessment of the individual choice at that time. So this possibility is also committed to the rejection of Supervenience Principle II. As I will argue in the
next section, there are serious problems with rejecting Supervenience Principle II, which this possibility then inherits.

The upshot here is that if one wants to reject Supervenience Principle I, they will have to either endorse a fundamentally splintered view of rationality, admit that the rational assessments of sequences of choices have no all things considered rational bite, or also be committed to rejecting Supervenience Principle II.

5.2 Rejecting Supervenience Principle II?

Supervenience Principle II claims that in choice situations where only value considerations matter, the rationality of choosing a particular option supervenes on the comparative relations that obtain between the available options with respect to the relevant covering values. In particular, if there is some difference in the rationality of choosing among the available options, there must be some comparative asymmetry between the available options with respect to the relevant covering values that justifies this rational difference. While this principle seems quite plausible, one might try to reject it by claiming that there are other factors that can justify a difference in rational status among the available options. What might these other factors be? The most natural answer here is to appeal to the comparison of the currently available options to those options that were previously available to the agent. Indeed, what seems to be driving the intuition that it is less rational to perform the Switch sequences than the Stay sequences is that in performing the Switch sequences the agent ends up with an option that is worse than one they previously rejected. We might then conclude that the switching agent’s rational mistake is at t2 because even though their chosen option is not worse than any of those currently available, it is worse than one that was previously available to the agent. That is, Supervenience Principle II
might be incorrect because the rationality of choosing a particular option depends not only on the comparative relations that obtain between the currently available options, but also on the comparative relations that obtain between both currently and previously available options as well.

The problem with this response though is that it is not clear why the comparative relations between currently available options and previously available options are at all relevant for determining the rationality of choosing between only currently available options. In deciding which option it is rational for us to choose now, why should we consider how they compare to options that are no longer available? Given that previously available options are not currently available, they seem to be simply irrelevant to what we should rationally do now. Indeed, taking previously available options into consideration in this manner seems to be committing the sunk cost fallacy.

The sunk cost fallacy is a mistake where one irrationally takes into account previously sunk costs in determining what to do now. Take the following example provided by David Ramsey Steele (1996, p. 609):

[I]f Hillary has paid to commence the building of a canal, which is now half completed, this is sometimes believed to provide a reason for Hillary to complete the canal, even if, in an alternative scenario, the half-completed canal existed as a natural geographical feature, and, knowing what Hillary now knows, she would not think it worthwhile to “complete” the canal. The economist says that these two scenarios are alike in all relevant respects: past expenditures do not justify future expenditures.59

The idea here is that the fact that Hillary has incurred a certain cost in the past is not relevant to whether or not she should complete the canal. Rather, the rationality of her decision to complete

59 Steele actually takes there to be two forms of the sunk cost fallacy, with this example being an instance of what he calls the “Concorde form”. We need not be concerned with the other form for our purposes here.
the canal remains the same regardless of how high or low her previous costs were. All that matters for her decision now are the expected future costs/benefits of each possible action.

Turning back to the sequential choice situations considered above, notice that by taking the options that the agent did not previously choose at t1 into account for the rationality of their choice at t2, one is effectively holding sunk costs to be rationally relevant. This is because at the t2 choice nodes, the option that the agent did not choose at t1 is effectively a sunk cost. To be sure, the sunk cost here takes the form of an opportunity cost wherein what is lost is an opportunity to receive an item rather than an item one already had. That is, in choosing A (or B), they are forgoing the opportunity to receive B (or A). But since this opportunity cost is already sunk at t2, it should have no bearing on the rationality of their current possible choices.

Take a more concrete case, again using Chang’s example of the seemingly incomparable coffee and tea. Here, the question can be formulated as whether in a choice at t2 between Sumatra Gold and Pearl Jasmine, an agent has more reason to choose Sumatra Gold in the scenario where she had previously chosen Sumatra Gold over Pearl Jasmine at t1 as compared to the scenario where she was simply given the Sumatra Gold at t1. Now the agent in the first scenario has certainly incurred a cost at t1 (i.e., the opportunity of getting Pearl Jasmine) that the agent in the second scenario has not. But is this sunk cost relevant to the agent’s choice between Sumatra Gold and Pearl Jasmine at t2? If so, why is it relevant here but not in the case of Hillary’s canal? The upshot is that if one wants to reject Supervenience Principle II by appealing to the comparative relations between currently available options and previously available options, one would have to either show how this is not an instance of the sunk cost fallacy, or, hold that the sunk cost fallacy is not a fallacy.
5.3 Rejecting the Comparative Money Pump Principle?

The Comparative Money Pump Principle claims that when comparing possible sequences of choices where risk is not a factor, if one sequence is known to result in a suboptimal outcome and another sequence is not, then the former is less rational to perform than the latter. While this principle seems compelling, I will argue that it is false. In the realm of game theory, there are well known examples where two players can predictably and rationally end up with a collectively suboptimal outcome. The most famous is the prisoner’s dilemma:

Figure 6. Prisoner’s Dilemma

![Figure 6](image)

Figure 6 represents the choices (Cooperate or Defect) available to two players (1 and 2) and outcomes for each player in utility numbers (1 – 4, higher is better). For each outcome (box), the left number is Player 1’s utility and the top number is Player 2’s utility. Which outcome eventuates depends on the players’ choices (i.e., when both Cooperate, both receive 3, when Player 1 Cooperates and Player 2 Defects, Player 1 receives 1 and Player 2 receives 4, etc…).

When first presented with the prisoner’s dilemma, many have the intuition that it is irrational for the players to Defect since they can achieve an outcome that is better for both of them if they instead Cooperate. More technically, the [Defect/Defect] outcome is Pareto inferior.
to the [Cooperate/Cooperate] outcome. However, the only rational strategy for the agents in such a one-shot prisoner’s dilemma is for both to Defect. This is because [Defect/Defect] is the only Nash equilibrium strategy pair in the game (indicated by the asterisk). That is, it is the only set of strategies such that neither player can improve her outcome by unilaterally deviating. Moreover, Defect dominates Cooperate for both players in this case. That is, it is always in a player’s interest to Defect regardless of what the other player does. To see this, take the perspective of each player in turn. In deciding what to do, Player 1 can reason that Player 2 will either Defect or Cooperate, but in each case it is better for Player 1 to Defect rather than Cooperate. If Player 2 Defects, then it is better for Player 1 to also Defect since a utility of 2 is better than a utility of 1. If Player 2 Cooperates, it is still better for Player 1 to Defect since a utility of 4 is better than a utility of 3. (This is represented by the one-way lines in Figure 6.) So it is better for Player 1 to Defect than Cooperate. Player 2 can reason similarly to conclude that it is better for her to Defect than Cooperate as well. Because of this, and contrary to our initial intuition, it is not irrational for the agents to both Defect even though the outcome of [Defect/Defect] is Pareto inferior to that of [Cooperate/Cooperate]. Now consider the following game:
Figure 7. Equilibria Everywhere

This game is set up such that every strategy pair is a Nash equilibrium, but that the outcome of one pair [Up/Left] is Pareto superior to the outcome of all other pairs. Here too it may seem irrational for the agents to choose, say, [Down/Right], but as with the prisoner’s dilemma, it is still an equilibrium point whereby neither player can improve her outcome by unilaterally deviating. Moreover, it is clear that what a player chooses will be entirely irrelevant to the utility she receives. Rather, each player’s choice determines only the utility that the other player will receive. In deciding what to do, Player 1 can reason that Player 2 will either choose Left or Right, but in each case it is neither better nor worse for her to choose Up rather than Down. If Player 2 chooses Left, then Player 1 will receive a utility of 3 regardless of whether she chooses Up or Down. If Player 2 chooses Right, then Player 1 will receive a utility of 1 regardless of whether she chooses Up or Down. (This is represented by the two-way lines in Figure 7.) So it is neither better nor worse for Player 1 to choose Up than Down. Player 2 can reason similarly to conclude that it is neither better nor worse to choose Left than Right. In this way, both players are indifferent between their available choices. Because of this, and contrary to our initial
intuition, it is not irrational for the agents to choose, say, [Down/Right] even though that outcome is Pareto inferior to that of [Up/Left].

The intuition common to both Prisoner’s Dilemma and Equilibria Everywhere is that it seems irrational for the players to end up with a Pareto inferior outcome. However, once we recognize that these outcomes result from Nash equilibria strategy pairs and that neither player has incentive to play their part of a Pareto optimal strategy pair, we then reject that intuition. This lesson can be applied to Motivating Choice Situation 2 illustrated in Figure 2.\(^6\) Consider the following game involving seemingly incomparable outcomes:

*Figure 8. Equilibria 2*

This game is structurally identical to Equilibria Everywhere insofar as all of the possible strategy pairs are in equilibrium. Here, I am assuming that both players have the same preference ordering, taking the A’s to be seemingly incomparable to the B’s. Unlike Equilibria Everywhere, these players are not indifferent between their choices as they do not take the outcomes to be equal. Nonetheless, they do not prefer one outcome over the other. Given this, neither player can

\(^6\) See page 7.
knowingly improve her outcome by unilaterally deviating from any strategy pair. Rather, each player knows that whatever the other player does, she will not prefer the outcome of one of her choices over another. That is, Player 1 can reason that Player 2 will choose either Stay or Switch, but in each case it is neither better nor worse to choose A rather than B. If Player 2 chooses Stay, then Player 1’s choice will determine whether she gets A or B. But A and B are seemingly incomparable to her, so she does not prefer one over the other. If Player 2 chooses Switch, then Player 1’s choice will determine whether she gets B- or A-. But B- and A- are seemingly incomparable to her, so she does not prefer one over the other. So it is neither better nor worse for Player 1 to choose A rather than B. Player 2 can reason similarly to conclude that it is neither better nor worse to choose Stay rather than Switch. In this way, both players have no incentive to choose one of their available choices rather than the other. Because of this, it is not irrational for the agents to choose [A/Switch] even though that outcome is Pareto inferior to that of [B/Stay]. Likewise, it is not irrational for the agents to choose [B/Switch] even though that outcome is Pareto inferior to that of [A/Stay]. So we should reject the idea that the players are less rational if they choose [A/Switch] or [B/Switch] rather than [A/Stay] or [B/Stay].

If this is right, then we should also reject the idea that the single agent in Motivating Choice Situation 2 would be less rational in performing one of the Switch sequences rather than Stay sequences. This is because Motivating Choice Situation 2 is simply a sequential version of Equilibria 2 where the players are two time-slices of the same agent rather than different individuals (i.e., Player 1 is the agent at t1 and Player 2 is the same agent but at t2). This modification though does not change which choices it is rational for the players to make. Take first the change from two players making one choice each to one player making two choices. In decision theory, what is important is not the number or identity of the agents involved in a choice.
situation or game. Rather, it is the outcomes and preferences of the agents involved that matter. In Equilibria 2, even though there are two players, the outcomes and preferences of those players are exactly the same. Their preferences are identical by stipulation and the payouts to each player are the same for each possible outcome. So even though each player is only concerned with satisfying their own preferences, they are in effect acting as if they are just as concerned with satisfying the preferences of the other player as well. In this way, they could never be at cross purposes as they could in other games (e.g., Prisoner’s Dilemma). Rather, the players in Equilibria 2 are already acting as if they were a single person interested in one preference ordering, but faced with two different decisions. We could make this even clearer by modifying the game to the following:

*Figure 9. Equilibria 3*

![Equilibria 3 Diagram]

The only difference between this game and Equilibria 2 is that here the payout goes not to the two players, but to some third party. Moreover, each of the players have the same preference rankings as they did before, but now they have them with respect to what they want for this third
party. For instance, imagine that Players 1 and 2 are parents who are choosing on behalf of their child and have exactly the same preferences for her. So even though there are two different agents making decisions here, the fact that the outcomes and preferences are the same for both agents makes the rationality of their choices the same as it would be if it were only a single agent making both choices.

Take now the change from the simultaneous nature of Equilibria 2 and 3 to the sequential nature of Motivating Choice Situation 2. The important difference here is that there is an epistemic asymmetry between the agent at t1 and t2 in Motivating Choice Situation 2 that does not exist between the players in Equilibria 2 and 3. That is, the agent at t2 knows the decision she made at t1, whereas Player 2 does not know the decision made by Player 1. However, introducing this asymmetry into Equilibria 2 and 3 does not change the rationality of the player’s choices. For instance, suppose that Player 2 gets to find out what Player 1 has chosen before making her choice. In this case, Player 2 will face a choice between A and B-, or, between B and A-, and she will know exactly which of these choices she faces. But this additional knowledge does not change the rationality of her possible strategies because it is still the case that both form equilibrium pairs with what Player 1 has chosen and that neither Stay nor Switch will result in an outcome that Player 2 prefers over the other. If she knows that Player 1 has chosen A, she knows that she cannot unilaterally improve her outcome by choosing Stay rather than Switch and vice versa. Likewise if she knows that Player 1 has chosen B. Since the rational statuses of the player’s strategies do not differ in the sequential and simultaneous versions of Equilibria 2 and 3, this difference between Motivating Choice Situation 2 and Equilibria 2 and 3 is not relevant for determining the rationality of the agent’s sequences of choices.

61 The same holds if Equilibria Everywhere is turned into a sequential game. More generally, changing a simultaneous game into a sequential game will not change the Nash equilibria in cases where each strategy pair in the simultaneous game is in equilibrium.
The upshot is that because we do not hold the players in Equilibria 2 and 3 to be less rational for playing strategy pairs that result in Pareto suboptimal outcomes, we should not hold the single agent in Motivating Choice Situation 2 to be less rational for performing a choice sequence that they know will result in a suboptimal outcome. The agent in Motivating Choice Situation 2 is simply in a sequential version of Equilibria 2 and 3 where the players are different time-slices of the same agent. So if the two players are not less rational for playing a suboptimal strategy pair, then neither is the single agent in Motivation Choice Situation 2 when she performs a suboptimal sequence. The Comparative Money Pump principle then is false.

The appeal here to game theoretic principles also nicely explains why the Comparative Money Pump Principle seemed so compelling in the first place and why it is ultimately mistaken. The intuition that it is less rational to switch than stay in Motivating Choice Situation 2 is compelling for the same reason that it seemed clearly less rational to defect than cooperate in the prisoner’s dilemma. In both cases, we see that one set of choices will result in a worse outcome by the lights of everyone involved than some other set of choices and so infer that the former must be less rational to perform than the latter.

However, this is a mistake since we are ignoring the rational status of each individual choice within those sets. In the prisoner’s dilemma, we ignore the fact that both choices in the set [Defect, Defect] are rationally required and neither choice in the set [Cooperate, Cooperate] is rationally permitted. In Motivating Choice Situation 2, we ignore the fact that the rational statuses of each choice in all possible sets are the same since each possible individual choice never improves or worsens the agent’s outcome. Ignoring the rational status of individual choices in a set has been realized to be a mistake when the choices are made by different agents, as in the
prisoner’s dilemma, but it should now also be realized to be a mistake when the choices are made by a single agent over time.

6. Conclusion

I have here discussed the phenomenon of seeming incomparability, identified its three key features, and established its ubiquity. I have also presented three intuitively compelling principles for governing rational choice between seemingly incomparable options: the Comparative Money Pump Principle, Supervenience Principle I, and Supervenience Principle II. While each is individually plausible, I have demonstrated that they are jointly incompatible and argued that the correct response is to reject the Comparative Money Pump Principle. Despite its initial appeal, we should ultimately reject the idea that it need be less rational for an agent to perform a sequence of choices that she knows in advance will result in a suboptimal outcome.

The problem with the Comparative Money Pump Principle is that it takes an idea that is applicable in synchronic choice situations and illegitimately extends it to diachronic choice situations. While it is less rational to knowingly choose an option that will result in a suboptimal outcome in an individual choice situation, it need not be less rational to make a series of choices that will result in the same suboptimal outcome.\(^{62}\) In particular, it is not less rational when none of the individual choices in the series involves choosing an option with a suboptimal outcome, which as we have seen is possible when seeming incomparability is involved. So even though the ultimate result of a sequence may be suboptimal, the performance of that sequence is not less rational if none of the individual choices that constitute the sequence are less rational. Just as individual choices made by different agents can result in a suboptimal outcome without rational defect, individual choices made by the same agent over time can also result in a suboptimal

\(^{62}\) Here it may be helpful to compare the situations depicted in Figure 1 and Figure 2 on pages 6 and 7.
outcome without rational defect. Such suboptimal outcomes may be unfortunate and even avoidable, but neither of these things entails that the sequences leading to such outcomes are less rational to perform.
References


Chapter 3
Axioms for Mildly Incomplete Preferences

Abstract
Our preferences are sometimes mildly incomplete. That is, when considering two items, we can neither prefer one to the other, nor be indifferent between them. For example, when comparing a bold cup of coffee to a soothing cup of tea, we might not prefer one to other, nor find them to be equally good. While we have no preference between these items, we can still prefer or disprefer both to some third item, like a cup of swamp water. Such mildly incomplete preferences are standardly taken to be irrational because they fail to satisfy all of the expected utility axioms. However, it is not clear that such preferences are in fact irrational to have. Moreover, mildly incomplete preferences seem to be ubiquitous. We then have reason to look for some alternative set of axioms to either use as a new standard of rational preferences, or, as a second best standard of rationality that our mildly incomplete preferences should satisfy given our inability to satisfy the expected utility axioms. I here identify a core set of five axioms that mildly incomplete preferences can and should satisfy as well as two additional candidate principles that might be added to this core set.
1. Introduction

There seem to be many cases where in comparing two things, we neither prefer one to the other nor are perfectly indifferent between them. Rather, there does not seem to be any comparative relation whatsoever between the items with respect to our preferences. This often happens when the things being compared are qualitatively quite different from one another (e.g., comparing the warm embrace of a fire on a cold night to the sweet relief of fresh water after an exhausting hike). In this way, our preferences are mildly incomplete. They are incomplete because there are some items between which we have neither preference nor indifference. However, they are only mildly incomplete because we do have preferences between each of these items and some third item.

On the standard view, such mildly incomplete preferences are irrational because they fail to satisfy a series of formal conditions called the expected utility axioms. However, it is far from obvious that such preferences must be irrational. Moreover, mildly incomplete preferences seem to be widespread, if not ubiquitous. We then have reason to look for an alternative set of axioms that such preferences can and should satisfy. The idea is that either mildly incomplete preferences are rational or they are not. If they are rational, then we should reject the expected utility axioms as the correct criterion for rational preferences and find some alternative axioms to serve that role. If they are not rational, then we should still find some alternative axioms that such preferences can satisfy in order to be at least as rational as possible. Such axioms would serve as a set of second best axioms that mildly incomplete preferences should satisfy given their inability to satisfy the expected utility axioms. After all, it is plausible that the rationality of preferences comes in degrees, with some irrational preferences being more rational than others.
These axioms then can be used to measure how far from ideal rationality a set of irrational preferences falls.

In this paper, I examine the nature of mildly incomplete preferences, explore ten potential axioms, and argue that five of them compose a core set of axioms that mildly incomplete preferences can and should satisfy. Further, I argue that this core set should be expanded to include one of two additional principles. Each of these principles is individually plausible, but jointly incompatible, so we will have to determine which is more compelling. However, the selection of this additional principle is set aside for another day. Until then, the core set of five axioms can be used as a standard evaluating the rationality of mildly incomplete preferences.

2. Mildly Incomplete Preferences

To understand mildly incomplete preferences, it is helpful to start with an example provided by Caspar Hare (2010):

It is dinner-time. Should we go to the Indian restaurant or the Chinese restaurant? We have visited both many times. We know their pluses and minuses. The Indian restaurant is less far to walk. It serves up a sublime mango lassi. The Chinese restaurant is cheaper. Its raucous atmosphere is more child-friendly. All in all it is a wash for me. I have no all-things considered preference between:

B: Our going to the Indian restaurant.
And C: Our going to the Chinese restaurant.

And learning that it is dollar-off day at either restaurant will not give me an all-things-considered preference. When I compare C to:

B+: Our going to the Indian restaurant and saving $1.

And B to:

C+: Our going to the Chinese restaurant and saving $1.
It remains a wash for me. I have no all-things-considered preference between B+ and C, C+ and B, though I do prefer B+ to B, C+ to C. This isn’t just me. I take it that we all have patterns of preference like this, all the time.63

Hare’s example here is a paradigm case of mildly incomplete preferences that illustrates two of its three key features. The first is that mildly incomplete preferences involve the absence of preference relations between two items. Here, Hare has no preference between going to the Indian restaurant and the Chinese restaurant. This is not to say that he takes the two options to be equally good with respect to his preferences. Rather, he neither prefers one to the other nor is indifferent between them. Taking the symbols ‘≻’, ‘≺’, and ‘∼’ to represent the preference relations of preferring, dispreferring, and indifference between, respectively, and the symbols ‘≾’, ‘≼’, and ‘≁’ to represent the absence of those same preference relations, respectively, we can represent Hare’s lack of preferences as follows:

[1] Indian Restaurant ≾ Chinese Restaurant
[2] Indian Restaurant ≼ Chinese Restaurant
[3] Indian Restaurant ≁ Chinese Restaurant

The second feature is that this absence of preference relations persists even if one of the items is slightly improved or worsened. In the example, Hare also has no preference between going to the now improved Indian option and the original Chinese option. Nor is he indifferent between the two. Hare’s lack of preferences here can be represented as follows:

[1*] Indian Restaurant+ ≾ Chinese Restaurant
[2*] Indian Restaurant+ ≼ Chinese Restaurant
[3*] Indian Restaurant+ ≁ Chinese Restaurant

The third feature is that this absence of preference relations does not persist if one of the items is greatly improved or worsened. That is, Hare would have a preference between going to a significantly improved Indian option (say by being free) over the original Chinese option.

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63 I have changed the labels used in Hare’s example here to be consistent with those I use in my Figure 1 below.
feature is not explicitly mentioned in Hare’s example above, but is both plausible and in line with other examples in the wider literature on incomparability. Taking this on board, Hare’s preference here can be represented as follows:

\[
\begin{align*}
[1^{**}] & \text{ Indian Restaurant+++… } \succ \text{ Chinese Restaurant} \\
[2^{**}] & \text{ Indian Restaurant+++… } \preceq \text{ Chinese Restaurant} \\
[3^{**}] & \text{ Indian Restaurant+++… } \preceq \text{ Chinese Restaurant}
\end{align*}
\]

More generally, these three features of mildly incomplete preferences can be schematically represented as follows\(^{65} \):

Figure 1. Schematic Illustration of Mildly Incomplete Preferences

This figure represents an agent’s mildly incomplete preferences. Here the agent neither prefers, disprefers, nor is indifferent between the B items and the C items. However, the agent does have preferences among the B items (i.e., \( B^+ > B > B^- \)) and among the C items (i.e., \( C^+ > C > C^- \)). Moreover, the agent prefers A to all other items and disprefers D to all other items.

\(^{64}\) See Chang (1997) for a discussion of this third feature, which is brought out in her examples of nominal/notable comparisons.

\(^{65}\) See Morton (1991) who uses a similar schematic representation for the possibility of incomparable values.
I take it that many of us, like Hare, have preferences that are mildly incomplete. After all, it seems somewhat implausible to think that for any two items we do in fact either prefer one to the other or are precisely indifferent between them. In addition to Hare’s restaurant example, it seems entirely plausible to neither prefer, nor be indifferent, between various possible professional careers, vacation destinations, works of art, philanthropic causes, etc… Moreover, such mildly incomplete preferences do not seem to be a mark of irrationality on our part. After all, in comparing, say, the deaths of your devoted dog to that of your cherished cat, must it be irrational to neither prefer one to the other nor be indifferent between the two?

On the standard view, a set of preferences is rational just in case they satisfy all of the expected utility axioms. As we will see in the next section though, mildly incomplete preferences violate two of these axioms. So either mildly incomplete preferences are irrational to have, or the satisfaction of all of the expected utility axioms is not a necessary condition for rational preferences. Either way, it will be instructive to find an alternative set of axioms that mildly incomplete preferences can and should satisfy. If the expected utility axioms are not the correct criterion for rational preferences, then this alternative set might fit the bill. On the other hand, if the expected utility axioms are indeed the correct criterion for rational preferences, it will still be useful to have an alternative set of axioms that helps us see how mildly incomplete preferences might meet a more tolerant rational standard. Finding such an alternative set of axioms will be the focus of the next section.

3. Potential Axioms for Mildly Incomplete Preferences

In this section, I examine ten potential axioms for governing mildly incomplete preferences. These include the expected utility axioms as it is instructive to see exactly which ones mildly
incomplete preferences violate and which they can satisfy.\textsuperscript{66} I have organized these potential axioms into three categories to be discussed in turn: Basic Axioms, Axioms Governing Lotteries with Ordered Outcomes, and Axioms Governing Lotteries with Unordered Outcomes.

\textit{Basic Axioms}

The basic axioms here include those principles that if satisfied by a preference ordering result in a weak order (total preorder). In particular, they include the expected utility axioms of Completeness and Transitivity, as well as a weaker version of Completeness. Start first with regular Completeness. The Completeness axiom holds that for any two items, one either prefers one to the other, or, is indifferent between them. More formally:

\textbf{Completeness:} For all items $x$ and $y$, either $x \succ y$, $x \prec y$, or $x \sim y$.

The basic idea here is that we must make up our minds with respect to every possible pair of items such that some preference relation obtains between them. This is a strong requirement and one that mildly incomplete preferences must clearly violate since they by definition involve an absence of preference relations between at least some items.

However, while mildly incomplete preferences involve an absence of preference relations between some items, those items are not entirely disconnected insofar as they are both preferred or dispreferred to some common third item (see again Figure 1). This contrasts with what we might call radically incomplete preferences. Radically incomplete preferences also involve an absence of preference relations between some items, but those items share no preference relation with any third item. In this way the items are completely disconnected from each other. Figure 2 below schematically represents such radically incomplete preferences:

\textsuperscript{66} I here use the formulation of the Von Neumann-Morgenstern axioms as presented by Michael Resnik (1987).
This difference between mildly and radically incomplete preferences can be more precisely formulated using the following axiom:

Completeness*: For all items \( x \) and \( y \), either \( x \succ y \), or \( x \prec y \), or \( x \sim y \), or, there is some \( z' \) such that \( z' \succ x \) and \( z' \succ y \), or, there is some \( z'' \) such that \( z'' \prec x \) and \( z'' \prec y \).

So while mildly incomplete preferences violate Completeness, they do not violate Completeness*. Radically incomplete preferences on the other hand violate both Completeness and Completeness*.

It is plausible though that preferences that are radically incomplete are less rational than those that are only mildly incomplete. That is, even if an agent neither prefers nor is indifferent between two items, it does seem like there should be some possible item that the agent prefers or disprefers to both. After all, it seems that any item that combines the positive (or negative) aspects of both original items should then be preferred (or dispreferred) to those original items. For instance, even if you neither prefer nor are indifferent between the death of your dog and the death of your cat, you should surely prefer the third outcome where they each continue to live to both outcomes involving the death of one.
If this is right, then we have reason to adopt Completeness* as an axiom for rational preference. This is so regardless of whether we ultimately accept Completeness as well. If we reject Completeness as an appropriate constraint on rational preference, we should nonetheless accept Completeness* since it is independently plausible. If on the other hand we do accept Completeness, it is nevertheless plausible that those preferences that satisfy Completeness* are at least more rational than those that do not. In this way, we want preferences to satisfy Completeness*, even if they cannot satisfy Completeness.

Turning now to Transitivity, this axiom holds that for any three items, if certain preference relations obtain between the first two, and certain preference relations obtain between the second two, then certain preference relations must also obtain between the first and the third. For instance, if the first item is preferred to the second, and the second is preferred to the third, then the first must also be preferred to the third. More formally:

Transitivity: For any items x, y, and z:
   If x \succeq y, and, y \succeq z, then x \succeq z.
   If x \succeq y, and, y \sim z, then x \succeq z.
   If x \sim y, and, y \succeq z, then x \succeq z.
   If x \sim y, and, y \sim z, then x \sim z.\textsuperscript{67}

The basic idea here is that preference relations ought to be similar to the numerical relations of greater than and equal to. For example, if number x is greater than number y, then x must also be greater than any number that y is also greater than. This axiom is intuitively very compelling. After all, in expressing our preferences between two items, we often say that our preference for one item is greater than or more than our preference for the other, or that we prefer them equally.

\textsuperscript{67} There are more efficient ways of formulating this axiom, but I have chosen this formulation because it does not require the introduction of additional disjunctive preference relations and because it makes it easier to see how Transitivity is being invoked in the argument presented later demonstrating the incompatibility of mildly incomplete preferences with Continuity. Also I have left out consideration of dispreference because it is equivalent to preference in the reverse direction.
Mildly incomplete preferences need not violate this axiom. This is in part because mildly incomplete preferences involve the absence of preference relations rather than the presence of them needed to invoke the requirements of this axiom. It is also because any time there are two items between which the agent has neither preference nor indifference, and those items are each preferentially related to some common third item, it must be that both initial items must share the same preference relation to that third item. That is, both initial items must be either preferred or dispreferred to the third item (see again Figure 1). In this way, there are never three items in a set of mildly incomplete preferences whose preference relations violate Transitivity. So mildly incomplete preferences have no difficulty in satisfying Transitivity.

*Axioms Governing Lotteries with Ordered Outcomes*

The next axioms to be discussed govern preferences involving lotteries. These are items with multiple possible outcomes, each of which has some fixed probability of occurring, but where only one will be actualized. For example, the toss of a fair coin where heads results in a $1 gain and tails results in $0 is a lottery. Such lotteries can be represented more simply using the notation \([p, x, y]\) where ‘x’ and ‘y’ are the possible outcomes of the lottery and ‘p’ is the probability of x actualizing (and so 1-p is the probability of y actualizing). So the lottery just mentioned can also be represented as [0.5, $1, $0].

However, these particular axioms apply only to preferences between lotteries with ordered outcomes. These are pairs of lotteries wherein the agent has preferences between all the possible outcomes of the two lotteries. For example, if we assume the preferences indicated in Figure 1, then the pair of lotteries \([p, A, B]\) and \([p, A, D]\) are such that the agent has preferences between every possible outcome of the two lotteries since the agent prefers A to B and B to D. However, the pair of lotteries \([p, A, B]\) and \([p, D, C]\) are not such that the agent has preferences
between every possible outcome of the two lotteries since the agent neither prefers nor is indifferent between B and C.

The axioms that govern lotteries with ordered outcomes include the expected utility axioms of Better Chances and Continuity, as well as a weaker version of Continuity. Start first with Better Chances. The Better Chances axiom holds that for any two lotteries involving the same two outcomes, one of which is preferred to the other, the lottery with the higher probability of the preferred outcome is to be preferred over the lottery with the lower probability of the preferred outcome. More formally:

**Better Chances:** For any items \( x, y \), if \( x \succ y \), then \([p, x, y] \succ [p^*, x, y] \) just in case \( p > p^* \).

The basic idea here is that if you prefer one item over another, then your preferences over lotteries involving those two items should be such that you prefer the lottery that gives you the higher chance of getting your preferred item. This axiom is intuitively compelling. After all, surely you should prefer a higher probability of getting your preferred outcome to a lower probability. If you prefer $100 to $0, then surely you should prefer the lottery \([0.9, $100, $0]\) to the lottery \([0.1, $100, $0]\) and failing to do so certainly seems irrational.

Mildly incomplete preferences need not violate this axiom. This is because this axiom only applies in cases where the agent has preferences between all the outcomes of the lottery, saying nothing about cases where the agent lacks preference or indifference between the outcomes. So even if an agent has mildly incomplete preferences, they can surely hold that when they have a preference between the possible outcomes of a lottery, they prefer the lottery with the higher chance of their preferred outcome. So mildly incomplete preferences have no difficulty in satisfying Better Chances.
Turning now to Continuity, this axiom holds that for any three items where the first is preferred to the second and the second preferred to the third, we must be indifferent between the second item and some lottery where the first and third items are the possible outcomes. More formally:

**Continuity:** For any items \( x, y, z \), if \( x \succ y \succ z \), then there is some \( p \) such that \( y \sim [p, x, z] \).

The basic idea here is that there should be no sharp breaks between the items in one’s preference ordering. That is, there should be no items that are infinitely preferred to another item such that the preferred item is also preferred to every lottery where the dispreferred item occurs as a possible outcome. For example, even if you prefer \$100 to \$0, you should not prefer \$100 to every possible lottery that has \$0 as a potential outcome. This is because some such lotteries should be able to offset the risk of getting the dispreferred item with, for example, a high probability of an outcome that is much better than the original preferred item. Indeed, it’s plausible that even though the agent prefers \$100 to \$0, they should nonetheless prefer a lottery like \([0.99, \$1000, \$0]\) to \$100 for sure since that lottery gives the agent a very high chance (99%) of getting an even better item (\$1000).

Continuity rules out such sharp breaks in one’s preference ordering by requiring that for any three items of decreasing preference, there must be some particular lottery that balances out the possibility of getting the most preferred item and the least preferred item such that the agent is indifferent between that lottery and getting the middle item for sure. Given that there must always be such a point of indifference, there can never be a sharp break in one’s preference ordering where one infinitely prefers some item to another.

As it turns out, mildly incomplete preferences violate Continuity. To see this, assume again the mildly incomplete preferences illustrated in Figure 1 along with the axioms of Better
Chances and Transitivity, which I have earlier argued that mildly incomplete preferences can and should satisfy. Combining these assumptions with Continuity, we derive the following reductio:

P1 – (B $\nlozenge$ C) & (B $\nabla$ C) & (B $\blacklozenge$ C) (from Figure 1)
P2 – A $\succ$ B $\succ$ D (from Figure 1)
P3 – A $\succ$ C $\succ$ D (from Figure 1)
P4 – For any options x, y, z, if x > y > z, then there is some p such that y $\sim$ [p, x, z] (Continuity)

C1 – B $\sim$ [p’, A, D] (from P2, P4)
C2 – C $\sim$ [p”, A, D] (from P3, P4)
C3 – Either p’ > p”, p’ = p”, or p’ < p” (from C1, C2)
C4 – If p’ > p”, then [p’, A, D] > [p”, A, D], and so B $\succ$ C (from C1, C2, Better Chances and Transitivity)
C5 – If p’ = p”, then [p’, A, D] $\sim$ [p”, A, D], and so B $\sim$ C (from C1, C2, Better Chances and Transitivity)
C6 – If p’ < p”, then [p’, A, D] < [p”, A, D], and so B $\prec$ C (from C1, C2, Better Chances and Transitivity)
C7 – Therefore, either (B $\succ$ C) or (B $\sim$ C) or (B $\prec$ C). (from C3, C4, C5, C6)
\[
\therefore\text{Contradiction with P1}
\]

The first three premises are drawn directly from the mildly incomplete preferences illustrated in Figure 1. Recall that while the agent neither prefers, disprefers, nor is indifferent between items B and C, they still prefer A to both items, which are in turn both preferred to D. Continuity is then assumed (for reductio) as the fourth premise and used to derive the conclusion that there must be some lottery with A and D as the possible outcomes such that the agent is indifferent between B and that lottery. Call this lottery [p’, A, D]. Likewise, the agent must also be indifferent between C and some lottery with outcomes A and D, call it [p”, A, D]. Now p’ and p” are the probabilities of getting A in those two lotteries and so are real numbers. As real numbers though, it must be that either one is larger than the other or they are precisely equal. However, if one probability is larger than the other, then by Better Chances it must be that its respective lottery is preferred to the other. For example, if p’ > p”, then it must be that [p’, A, D] is preferred to [p”, A, D] since the former lottery has a higher chance of resulting in the preferred outcome. But if one of the lotteries is preferred to the other, then by Transitivity this
preference relation extends to any items that the agent is indifferent to with respect to those lotteries. So because the agent is indifferent between B and \([p', A, D]\), prefers \([p', A, D]\) to \([p'', A, D]\), and is indifferent between \([p'', A, D]\) and C, it must be that the agent then prefers B to C. However, this is contrary to our initial assumption that the agent neither prefers, dis prefers, nor is indifferent between B and C. By parallel reasoning, we can also derive a contradiction when the probabilities of the two lotteries are equal to each other. Since there is no way to avoid contradiction, mildly incomplete preferences must violate Continuity.

While mildly incomplete preferences cannot accommodate Continuity, they can accommodate a weaker principle that still captures the motivating force of the original axiom. Recall that the motivation for Continuity was to rule out the possibility of items being infinitely preferred to one another in one’s preference ordering. Continuity achieves this by requiring that the agent be precisely indifferent between a middle ranked item and some particular lottery with higher and lower ranked items as possible outcomes. However, this is not the only way to rule out breaks in one’s preference ordering. Rather, the following weaker principle also rules out the possibility of items being infinitely preferred to others:

Continuity*: For any options \(x, y, z\), if \(x \succ y \succ z\), then there is some \(p^*\) such that \(y \succ [p^*, x, z]\) and there is some \(p^{**}\) such that \(y \prec [p^{**}, x, z]\)

The basic idea here is that instead of requiring a precise point of indifference between the middle ranked item and lotteries with higher and lower ranked items as outcomes, all that is required is that there be some such lotteries that are preferred to that middle ranked item and some such lotteries that are dis preferred. So take again the agent who has no preference between the Indian and Chinese restaurants without being indifferent between them, but who does prefer both restaurants to a significantly worse Indian restaurant \([\text{Indian}---\ldots]\), and, dis prefers both restaurants to a significantly better Indian restaurant \([\text{Indian}+++\ldots]\). Continuity* does not require that the
agent be precisely indifferent between the original Chinese restaurant and some particular lottery 
\([p, \text{Indian}^{+++\ldots}, \text{Indian}^{---\ldots}]\) with better and worse items as the possible outcomes. Rather, all that is required is that there be some lottery such that they prefer it to the original Chinese restaurant (e.g., \([0.99, \text{Indian}^{+++\ldots}, \text{Indian}^{---\ldots}]\)), and that there be some lottery such that they disprefer it to the original Chinese restaurant (e.g., \([0.01, \text{Indian}^{+++\ldots}, \text{Indian}^{---\ldots}]\)). This requirement is weaker than Continuity, but still serves to rule out the preference order breaks that motivated Continuity. This is because Continuity* still requires that the agent allow for tradeoffs between higher and lower ranked items. It just does not require that those tradeoffs be precise, resulting in perfect indifference with some middle ranked item. But by removing the requirement of precise indifference, we also remove the condition that prevented mildly incomplete preferences from satisfying Continuity. Mildly incomplete preferences cannot satisfy Continuity because it demands a precise point of indifference. But since Continuity* does not make such a demand, mildly incomplete preferences have no difficulty in satisfying this weaker principle. Moreover, since Continuity* still rules out preference order breaks, we have good reason to accept Continuity* as an axiom that mildly incomplete preferences can and should satisfy.

**Axioms Governing Lotteries with Unordered Outcomes**

The last axioms to be discussed govern preferences between lotteries where the outcomes are not all ordered. These are pairs of lotteries which have as their possible outcomes items that the agent has no preference between, but are also such that the agent is not indifferent between them. For example, assuming again the preferences indicated in Figure 1, the pair of lotteries \([p, A, B]\) and \([p, A, C]\) are such that there are some outcomes of those lotteries that the agent has no preference between, but are also such that the agent is not indifferent between them (i.e., B and C).
The axioms that govern lotteries with unordered outcomes include the expected utility axiom of Better Prizes and three other intuitively plausible principles that are not expected utility axioms. Start first with one of these principles that I call Can’t Lose/Might Win. This principle holds that if one lottery is such that its outcomes are never dispreferred to any of the outcomes of another particular lottery, but that some of the outcomes of the former lottery are preferred to some of the outcomes of the latter, then the former lottery is to be preferred over the latter. More formally:

Can’t Lose/Might Win: If none of the possible outcomes of lottery x are dispreferred to any of the outcomes of lottery y, and some outcomes of x are preferred to some outcomes of y, then x is preferred to y.

The basic idea here is that if a lottery might result in a preferred outcome and carries no risk of resulting in a dispreferred outcome, then surely one should prefer that lottery over the alternative. After all, it gives them a chance of ‘winning’ (i.e., getting a preferred outcome) without any risk of ‘losing’ (i.e., getting a dispreferred outcome). The alternative on the other hand only gives them a chance to lose. To see this principle in action, assume again the preferences illustrated in Figure 1 and consider the lotteries [p, A, B] and [p, C, C]. Note that [p, C, C] is still considered a lottery even though it guarantees that C will result and indeed is equivalent to C itself. Here we can apply Can’t Lose/Might Win as none of the outcomes of [p, A, B] is dispreffered to any of the outcomes of [p, C, C] since neither A nor B are dispreffered to C, but some of its outcomes are preferred since A is preferred to C. This principle then holds that in this case the agent should prefer [p, A, B] to [p, C, C] and so to C as well.
This principle and its application to the lotteries above are both intuitively quite plausible. However, mildly incomplete preferences must violate this principle if they are to satisfy Continuity* and Transitivity. This is demonstrated in the following reductio:

P1 – A > B+ > B (from Figure 1)
P2 – B+ ∨ C (from Figure 1)
P3 – For any options x, y, z, if x > y > z, then there is some p* such that y > [p*, x, z] and there is some p** such that y < [p**, x, z] (Continuity*)
C1 – B+ > [p*, A, B] (from P1, P3)
P4 – If none of the possible outcomes of lottery x are dispreferred to any of the outcomes of lottery y, and some outcomes of x are preferred to some outcomes of y, then x is preferred to y. (Can’t Lose/Might Win)
C2 – [p*, A, B] > C (from P4, Figure 1)
P5 – For any items x, y, and z, if x > y, and, y > z, then x > z. (Transitivity)
C3 – B+ > C (from C1, C2, P5)

Contradiction with P2

The first two premises are drawn directly from the mildly incomplete preferences illustrated in Figure 1. Continuity* is then assumed, which when applied to P1 results in the conclusion that there is some lottery, call it [p*, A, B] such that B+ is preferred to it. Can’t Lose/Might Win is then invoked, which when applied to the preferences in Figure 1 results, as we saw previously, in the conclusion that [p*, A, B] is to be preferred to C. Finally, Transitivity is used to conclude that B+ is preferred to C, which contradicts the mildly incomplete preferences that were initially assumed. So it turns out that an agent with mildly incomplete preferences cannot simultaneously satisfy Continuity*, Transitivity, and Can’t Lose/Might Win.

However, I don’t think this incompatibility should be worrying because Can’t Lose/Might Win should actually be rejected as an axiom for governing mildly incomplete preferences. As compelling as it seemed, its plausibility stems from cases where an agent’s preferences are actually complete. In such cases, this principle is actually just a dominance

\[68\] For example, Adam Bales, Daniel Cohen, and Toby Handfield defend a principle they call Strong Competitiveness, which is similar to Can’t Lose/Might Win, except that it is articulated in terms of rational choice rather than rational preferences. The following argument then serves as an objection to Strong Competitiveness as well as Can’t Lose/Might Win.
principle that requires agents to prefer non-dominated lotteries to dominated ones. As a dominance principle, it is clearly correct. However, when an agent’s preferences are mildly incomplete, application of this principle overreaches to lotteries where neither is dominated. This overreach effectively conflates cases where an agent lacks any preference relation between two items with cases where the agent is perfectly indifferent between them. In this way, Can’t Lose/Might Win fails to take the incompleteness of an agent’s preferences seriously. So while mildly incomplete preferences cannot accommodate Can’t Lose/Might Win, this should not be concerning if we are to take seriously the incompleteness of such preferences.

The problem with this axiom can also be demonstrated more intuitively by comparing the lottery \([0.0001, A, C]\) to [B]. Can’t Lose/Might Win holds that the former lottery must be preferred to the latter even though there is only a very small chance of it resulting in superior item A. Indeed, Can’t Lose/Might Win would have to hold that the former lottery is preferable to the latter for any non-zero probability of A, however infinitesimally small. But allowing for such a small chance of a superior outcome to create a preference between the two lotteries flies in the face of the second feature of mildly incomplete preferences whereby the absence of preference relations between two items persists even if one of them is slightly improved or worsened. That is, the addition of a small chance of getting A is exactly the sort of small improvement that mildly incomplete preferences are supposed to be resistant to. So adding that small chance to C, as \([0.0001, A, C]\) does, should not break the absence of preference relations between it and B.

Turn now to a principle that I call No Preferences without Preferences.\(^ {69}\) This principle holds that if two lotteries are such that no preference relations ever obtain between outcomes of

\(^{69}\) This principle is similar in spirit to Miriam Schoenfield’s (2014) LINK principle and Rabinowicz’s Complementary Dominance.
those lotteries, then no preference relations can obtain between the lotteries themselves. More formally:

No Preferences without Preferences: If there are no states of the world where any preference relation obtains between the outcomes of lottery x and lottery y, then no preference relation obtains between x and y.

The basic idea is that in order for an agent to prefer or be indifferent between two lotteries, they must prefer or be indifferent between at least some of the outcomes of those lotteries. After all, lotteries are just combinations of different possible outcomes and it is the outcomes themselves that ultimately matter. So if some preference relation obtains between two lotteries, it must be because some preference relation obtains between the outcomes of those lotteries. But which outcomes should we compare? Well, since the outcome of any given lottery is determined by the state of the world that obtains, it seems reasonable to divide the possible states of the world into mutually exclusive and jointly exhaustive partitions and then compare the outcomes of the different lotteries under each partition. If it turns out that there is no possible state of the world where some preference relation obtains between the outcomes of two lotteries, then it must be that no preference relation obtains between the lotteries themselves. Put another way, if for every possible state of the world, the agent neither prefers one of the outcomes of the lotteries to the other nor is indifferent between them, then the agent must also neither prefer one of the lotteries as a whole to the other nor be indifferent between them. Indeed, where would a preference between the lotteries come from if not from preferences between their possible outcomes? An agent who did have a preference relation between two lotteries despite having no preference relations between their outcomes would seem to be somehow conjuring preferences from their absence and in this way be failing to take seriously their own lack of preferences over the outcomes of those lotteries.
This principle itself seems quite plausible and it also seems to yield plausible verdicts. For example, consider two lotteries whose outcomes are determined by which of two possible states of the world (S1 or S2) obtains. If S1 obtains, the first lottery (L1) results in B, while the second lottery (L2) results in C. If S2 obtains, L1 results in C, while L2 results in B. However, the probability of S1 or S2 obtaining is unknown. Figure 3 summarizes these conditions and outcomes:

Figure 3: Two States, Two Lotteries, Unknown Probabilities

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>B</td>
</tr>
<tr>
<td>L2</td>
<td>C</td>
</tr>
</tbody>
</table>

In this case, No Preferences without Preferences holds that the agent should neither prefer nor be indifferent between L1 and L2. This is because for each possible state of the world (S1 or S2), the agent neither prefers nor is indifferent between the outcomes of the two lotteries. If S1 obtains, then the agent neither prefers nor is indifferent between the outcomes of L1 and L2 because she neither prefers nor is indifferent between B and C. Likewise if S2 obtains. So there is no state of the world where any preference relation obtains between the outcomes of L1 and L2. It follows then, according to No Preferences without Preferences, that there must also be no preference relation between L1 and L2 themselves.

While this particular verdict does seem plausible, there are other cases that demonstrate potential problems with No Preferences without Preferences. For example, suppose we modify the previous case by making it known that S1 and S2 are equiprobable. Here we can take the two states of affairs to be that of a fair coin landing heads or tails. Figure 4 summarizes this updated situation:
Here, No Preferences without Preference will again conclude that the agent should neither prefer nor be indifferent between L1 and L2. This is because knowing the probability of the states does not change the fact that for each state of the world, it is such that the agent neither prefers nor is indifferent between the outcomes of the two lotteries. However, there is a case to be made that the agent should in fact be indifferent between these two lotteries. Such a position is supported by a plausible principle I call Indifference between Probabilistically Identical Lotteries. This principle holds that if two lotteries have the same probability of the same outcomes, then one must be indifferent between them. More formally:

\[
\text{Indifference between Probabilistically Identical Lotteries: For any items } x \text{ and } y, \text{ and any probability } p, [p, x, y] \sim [1-p, y, x].
\]

The basic idea here is that all that matters in evaluating a lottery are its possible outcomes and the probability of those outcomes obtaining. The particulars of the states of affairs that bring about those outcomes are irrelevant. Two lotteries are probabilistically identical if they have the same probability of the same outcomes. Given that the value of a lottery depends only on its possible outcomes and the probability of those outcomes obtaining, it follows that probabilistically identical lotteries must be exactly equal in value. But if they are exactly equal in value, then the agent must be indifferent between them. So agents must be indifferent between probabilistically identical lotteries.

\[70\, \text{This principle is pretty much identical to Rabinowicz’s Permutation. It is also a weaker version of the reduction of compound lotteries.}\]
This principle is intuitively quite plausible. After all, it does seem that once we know that the probabilities and possible outcomes of two lotteries are identical, we don’t need any further information in order to be indifferent between them. For example, if we are told that two lotteries each offer even chances of winning $1 and winning nothing, we don’t need further information about what particular states of affairs determine the winning and losing conditions in order to be indifferent between the lotteries. If we are then told that one of the lotteries depends on the flip of a fair coin and the other depends on the roll of a fair die, this should not change our indifference between them.

However, this principle directly conflicts with No Preferences without Preferences in the case illustrated in Figure 4. No Preferences without Preferences requires that we neither prefer nor be indifferent between L1 and L2, while Indifference between Probabilistically Identical Lotteries requires that we be indifferent between them. So mildly incomplete preferences will obviously not be able to satisfy both these principles. They can though satisfy each individually as neither the features of mildly incomplete preferences nor the principles we’ve identified for governing mildly incomplete preferences thus far conflict with either. We will then have to determine which of these principles to accept for governing mildly incomplete preferences and which to reject. Unfortunately, resolving this question will have to wait for another time as it would take us too far afield from the scope of this paper.\(^{71}\) I here then just flag these two principles as a decision point for augmenting the set of principles governing mildly incomplete preferences.

The last principle to be discussed is the expected utility axiom of Better Prizes. This principle holds that if one item is preferred to another, then any lottery that has the preferred item

\(^{71}\) See Hare (2010) and Schoenfield (2014) for competing views on this matter.
as an outcome will also be preferred to a lottery that has instead the dispreferred item but is otherwise identical. More formally:

Better Prizes: For any items x, y, and z, and any probability p, x > y, iff [p, x, z] > [p, y, z] and [p, z, x] > [p, z, y].

The basic idea here is that the possible outcomes of a lottery are separable insofar as they contribute to the overall value of the lottery in an additive manner. Because of this, we can consider the contribution of any particular possible outcome of a lottery independently of the other possible outcomes. It follows then that if one possible outcome of a lottery is replaced with a preferred outcome, keeping all other aspects of the lottery fixed, then the resulting lottery must also be preferred to the original lottery. This is so regardless of what the other unchanged aspects of the lottery are. In improving one part of the lottery, we improve the lottery as a whole.

This principle is intuitively compelling and also results in verdicts that seem to be clearly correct. To see it in action, we can take the situation illustrated in Figure 4 and simply add a third lottery (L2+) that results in C+ if the coin lands heads, and B if the coin lands tails. Figure 5 summarizes this updated situation:

Figure 5. One Coin, Two States, Three Lotteries

<table>
<thead>
<tr>
<th>Coin 1</th>
<th>Heads (p = 0.5)</th>
<th>Tails (p = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>L2</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>L2+</td>
<td>C+</td>
<td>B</td>
</tr>
</tbody>
</table>

Here, Better Prizes holds that L2+ must be preferred to L2. This is because C+ is preferred to C, and L2+ is identical to L2 except that one has C+ and the other has C when heads comes up. So the lottery with the preferred outcome, L2+, must be preferred to the lottery with the dispreferred outcome, L2. This verdict is clearly correct. There is a question though about whether this
principle also entails that L2+ must be preferred to L1. This certainly follows if Indifference between Probabilistically Identical Lotteries is true since one would then have to be indifferent between L2 and L1 and so L2+ would have to be preferred to L1 via Transitivity. However, it doesn’t seem that Better Prizes by itself entails that L2+ must be preferred to L1. That is, it seems coherent to maintain that Better Prizes does not entail that L2+ must be preferred to L1 because while one has C+ as an outcome and the other has C, they are not otherwise identical lotteries since the states in which these outcomes obtain are different. In this way, Better Prizes seems to be compatible with No Preferences without Preferences if Indifference between Probabilistically Identical Lotteries is rejected.

If this is right then Better Prizes is compatible with either Indifference between Probabilistically Identical Lotteries or No Preferences without Preferences. Moreover, neither the features of mildly incomplete preferences nor the principles we’ve identified earlier for governing such preferences conflict with the acceptance of Better Prizes. So mildly incomplete preferences both can and should accommodate Better Prizes regardless of whether we ultimately accept Indifference between Probabilistically Identical Lotteries or No Preferences without Preferences.

4. Conclusion
The goal of this paper was to identify a set of axioms suitable for governing mildly incomplete preferences. To this end, ten potential principles, including the expected utility axioms, were examined. I argued that of these principles, mildly incomplete preferences cannot satisfy either Completeness or Continuity, but that this should not be worrying because they can satisfy alternative principles that are similar in spirit. I also argued that an initially plausible principle,
Can’t Lose/Might Win, should actually be rejected because it fails to take an agent’s lack of preferences seriously. In the end, I identified five principles that mildly incomplete preferences both can and should satisfy: Completeness*, Transitivity, Better Chances, Continuity*, and Better Prizes. Further, this core set of axioms should be augmented with either No Preferences without Preferences or Indifference between Probabilistically Identical Lotteries, but not both. Which of these should ultimately be added to the set of axioms governing mildly incomplete preferences though is a question to be settled another time. In any case, these axioms give us at least the start of a standard by which to judge the rationality of mildly incomplete preferences.
References


Rabinowicz, W (Unpublished) “Incommensurability Meets Risk”
