

# Understanding Overtone Interference in Surface wave Measurements: Application to Anisotropic Imaging of the Uppermost Mantle Beneath North America

by

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A Dissertation submitted in partial fulfillment of the  
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“Science, my lad, has been built upon many errors; but they  
are errors which it was good to fall into, for they led to the  
truth.”

Jules Verne, *Journey to the Center of the Earth*

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# Chapter 1 | Introduction

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In 1974, at the Fall AGU (American Geophysical Union) meeting, Keiiti Aki, then a professor at MIT, presented a study showing images of the crust and upper mantle beneath California (Aki *et al.*, 1974). While other studies had used measurements of seismic waves to infer the physical properties of the Earth’s interior, this study was the first to use measurements of the arrival times of seismic waves to present images of the Earth’s interior with the goal of interpreting them in the context of better understanding the geology and tectonics of the region. The value of Aki’s approach was clearly evident to the community; within two years, the same method had been applied by Aki and others to additional geographic regions, including Hawaii and the New Madrid Seismic Zone (Ellsworth, 1978; Mitchell *et al.*, 1977). These studies marked the inception of the field of **seismic tomography**, a term with its root in the greek *tomos* (slice, section) that describes the construction of images of the Earth’s interior.

Fifty years later, studies employing seismic tomography now span a range of length-scales: from models of the entire Earth (Zhang and Tanimoto, 1993; Moulik and Ekström, 2014; Ritsema *et al.*, 2011) to models of a single magma chamber (Jaxybulatov *et al.*, 2014), tomographic images capture and offer insight about a range of geodynamic processes that impact the evolution of the Earth and other planetary bodies. To construct these images, tomographers can make use of nearly every individual component of the full seismic wavefield, a vast release of energy comprised of a multitude of wavepackets that arrive at different times and with energy distributed

across a range of frequencies. Each of these wavepackets provides different information about the Earth’s interior, and to even begin to attempt to catalog this rigorously is out of the scope of this introduction.

The subset of tomographic imaging that this thesis focuses on is derived from measurements of surface waves, broadly described as the longest-period traveling waves radiated from an earthquake. Surface waves propagate along the surface of the Earth, (in contrast to body waves, which travel through the Earth’s interior) and can be separated into two types; Rayleigh waves and Love waves, most easily distinguishable on the basis of their particle motion (retrograde elliptical motion for Rayleigh waves and transverse for Love waves) which decays with depth. There are several frameworks in which we can analyze these waves; for the purposes of seismic imaging, one useful paradigm is to consider them as dispersive wavepackets with frequency-dependent phase and group velocities. Consideration of these frequency-dependent properties is particularly useful for seismologists, who take advantage of the fact that surface waves ‘sense’ different depths of the Earth’s interior at different frequencies. Therefore, measurements of the frequency-dependent propagation speeds of surface waves at different periods are routinely used to solve for images of the depth-dependent properties of the Earth’s interior that best explains the measured observations.

A specific context in which surface-wave tomography is often applied is to data collected by dense, temporary seismic arrays that are designed to investigate, in high resolution, the seismic properties of a specific region. Early datasets used for tomographic imaging were often derived from globally distributed networks of single stations such as the WWSN (world-wide seismic network), which was deployed during the 1960s with the goal of improving nuclear monitoring capabilities, or the GSN (Global Seismographic Network) in the 1980s, which was designed to offer the highest-quality data possible. However, in the early 2000s, endeavors such as PASS-CAL (Program for Array Seismic Studies of the Continental Lithosphere) and the

EarthScope Flexible Array made localized, dense arrays of seismic stations sometimes deployed for  $\approx 2$  years at a time a standard and relatively cost-effective tool to answer fundamental questions about the evolution and formation of the crust and mantle, particularly beneath continents. Surface-wave tomography became a particularly stable and attractive ‘workhorse’ of regional studies using data from these dense arrays due to the proliferation of imaging approaches such as the two-plane wave method (Forsyth and Li, 2005) or the two-station method (Yao *et al.*, 2006), which enabled measured variations in the travel-time (and amplitude, in the case of the two-plane wave method) of a surface wave measured as it propagated through the array, to be ascribed solely to the effects of Earth structure within the array. A family of imaging methods that similarly aimed to exploit observations of the propagation of surface waves across an array that was first applied within the context of the burgeoning field of ambient noise seismology is known as wavefront tracking (Lin *et al.*, 2009), and also began to be applied to surface wave measurements in the early 2010s (Lin and Ritzwoller, 2011), theoretically presenting the possibility to eliminate the impact of subjective damping and smoothing choices chosen by the operator that often impact first-order features of tomographic images. With these and other array-based methods (Pollitz and Snoke, 2010), seismologists no longer needed to account for the effects of structure outside the array on the measured seismic waves, and could therefore achieve much higher-resolution images. The development of freely-available tools such as the ASWMS toolkit (Jin and Gaherty, 2015) allowed these methods to become very accessible, and their rapid application partially marked the transition of surface-wave tomography into a ‘big-data’ discipline, where the sheer number of available measurements meant that performing measurement quality control on the basis of visual examination of seismograms became more difficult in practice.

Many challenges remain in surface-wave tomography. One of these is the difficulty of eliminating noise in the phase and amplitude measurements that are used to construct the images of surface-wave wavespeed. One of these sources of noise

is interference due to scattering and multipathing, both due to interactions between the propagating surface wave and other elastic heterogeneity. This is a source of noise that has been acknowledged by many studies and that many approaches try to account for; the two-plane wave method models multipathing interference as a second plane wave and helmholtz tomography accounts for scattering interference in phase velocities from traveltimes by incorporating a correction based on spatial operators to the amplitude field. In these cases, the interfering waves propagate at the same velocity as the original surface wave that is being measured. Another source of interference in the underlying phase and amplitude measurements is interference from overtones, which correspond to higher-mode (larger  $n$ ) traveling wave solutions to the surface-wave eigenproblem. Overtone interference refers to the condition in which overtones arrive in the same time window in which the fundamental-mode surface wave is being measured, resulting in perturbations to the measured phase and amplitude. Overtone interference has dramatic effects on surface-wave measurements and is one of the main subjects of this thesis.

Overtone interference is a historically significant source of noise due to its role in a long-standing problem in structural seismology — the Rayleigh-Love discrepancy. The Rayleigh-Love discrepancy refers to the inability of a model with the same wavespeed of horizontally and vertically polarized shear waves ( $V_{sh}$  &  $V_{sv}$  respectively) to simultaneously explain measurements of the phase speeds of Rayleigh and Love waves. Requiring differences in  $V_{sh}$  &  $V_{sv}$  implies a requirement for radial anisotropy, with profound implications for the rheology and material properties of the mantle. One of the earliest studies of overtone interference suggested that, due to the similar group speeds of the fundamental mode and first overtone Love waves in a wide period band, overtone interference could lead to a bias to higher Love wave phase velocities (Thatcher and Brune, 1969), potentially explaining the Rayleigh-Love discrepancy. Early studies therefore employed relative caution when interpreting measurements of Love waves, though some were more confident when doing so over large path lengths,

recognizing that the relative impact of overtone contamination diminished over larger path lengths (Knopoff, 1972). Later studies making global-scale maps of Love wave dispersion (Nettles and Dziewoński, 2008; Ritsema *et al.*, 2011) did so under the implicit assumption that interference effects were relatively small, an assumption at least partially justified on the basis that some measurement techniques were designed to eliminate overtone interference (Ekström *et al.*, 1997). Nettles and Dziewoński (2011) conclusively closed the debate regarding the impact of overtone interference on global maps of radial anisotropy by showing that inversions using subsets of data with both long and short path lengths did not result in systematic differences in global phase velocity maps, implying that overtone interference would not likely impact conclusions regarding the existence and extent of radial anisotropy at global scales. However, the same study noted that “a better understanding of the impact of overtone-interference effects on such small-scale regional studies requires further investigation”.

Sure enough, in the wake of surface-wave analyses conducted in the early 2010s focusing on the initial data from the USArray Transportable Array—a network of stations ‘marching’ across America in a densely spaced configuration—and with the goal of obtaining high-resolution regional Love wave phase velocity maps using measurements with small path lengths, there was renewed interest in the phenomenon of overtone interference. The work of Foster *et al.* (2014) catalogued measurement anomalies in regional two-station and mini-array measurements due to overtone interference, described and broadly modeled the character of these anomalies, and showed that measurement methods applied to dense arrays could expect scatter on the order of 10 – 20% and most concerningly, could expect to obtain measurements that were systematically biased. The work of Jin and Gaherty (2015) provided an independent confirmation of these results, using a wavefront-tracking approach. At the time at which this thesis was beginning to be written, little progress has been made on this problem, with certain exceptions; Luo *et al.* (2015) showed that the high-resolution Linear Radon transform could separate the fundamental-mode and overtone Love

waves, and Yang *et al.* (2020) applied this to an array of ocean-bottom seismometers to successfully measure Love wave dispersion averaged across the array. Tao *et al.* (2021) was able to extract Love wave phase velocity maps in the Eastern U.S. through careful visual inspection of seismograms to eliminate interference. Nonetheless, nearly a decade after the initial deployment of USArray, Love wave phase velocity maps from earthquakes had not yet been calculated for the entirety of the U.S., and our understanding of radial anisotropy in the North American upper mantle was relatively limited in comparison to our understanding of variations in isotropic shear wave velocity.

The work presented in this thesis addresses the issue of overtone interference in the context of both Love and Rayleigh waves, and is situated as a natural follow-up to the imaging studies discussed previously that identified overtone interference in Love wave measurements and catalogued its severity. From a geological perspective, the work in this thesis is also positioned to address some of the scientific goals of the USArray project, which enabled an unprecedented, high-resolution examination of the seismic properties of the mantle beneath the continental U.S. The resulting seismic imaging holds the potential to elucidate, in great detail, aspects of the tectonic history of the continental U.S., including the geometry of the subducting Farallon plate beneath western North America, the origin of Basin and Range extension, the relationship between the position of arc volcanism and the location of the Juan de Fuca plate, and the relationship between the topography of the Colorado Plateau and the upper mantle beneath. A binding question throughout these investigations is the extent to which anomalies at depth can be linked to geologic features or activity at the Earth's surface. Images of structure at depth beneath North America are also primed to understand fundamental characteristics of the upper mantle. These include the nature of the lithosphere-asthenosphere boundary, the composition and stratification of cratonic lithosphere, the preservation of lithospheric fabric, the extent to which strain actively takes place in the lithosphere and asthenosphere, and the mechanisms

by which such deformation is accommodated at depth.

The studies of overtone interference in this thesis have improved our understanding of this phenomenon and source of noise in seismic data. Through enabling the identification of higher-quality measurements, they lead to new images of the propagation speeds of surface waves, new images of the depth-dependent anisotropic wavespeed beneath North America, and help suggest different geodynamic hypotheses that can be explored to understand deformation in the crust and upper mantle beneath North America.

Chapter 2 presents the first observations of major-arc overtone interference in Rayleigh wave phase, amplitude, and array-based phase velocities. Using synthetic seismograms, we prove the origin of this interference is due to overtones that take the major-arc path around the Earth from the earthquake to the station, and that this interference mostly impacts measurements made at epicentral distances greater than  $115^\circ$ . The major-arc overtone interference is characterized by dramatic, short-wavelength oscillations in phase, amplitude, and phase velocities. We characterize the signature of this interference in both global and regional-scale phase velocity data sets as well as individual measurements on specific earthquakes, with profound implications for our understanding of the sources and distribution of error in Rayleigh wave phase velocities.

Chapter 3 presents a new framework to catalog and understand overtone interference. Through an empirical analysis of overtone interference in Rayleigh and Love waves with synthetic seismograms and real data in conjunction with theoretical calculations, this study examines the distinct impact of both group velocity separation and the relative excitation of surface wave overtones on errors from interference, and presents a framework to jointly use these to estimate the degree of overtone contamination in Love wave measurements and Rayleigh wave measurements. This work explains nuanced features of the phenomena catalogued in chapter 1, including the precise distribution of errors in array-based Rayleigh wave phase velocity measure-

ments as a function of epicentral distance.

Chapter 4 focuses on Love wave phase velocity measurements, which recent studies have shown to be systematically biased at regional scales due to overtone interference. Building upon the framework developed in the previous chapter, this chapter presents an approach to isolate data relatively uncontaminated by overtone interference, and uses this data to generate the first Love wave phase velocity maps spanning the entirety of the region covered by USArray. We examine the implications of these maps for radial anisotropy, highlighting both consistency with a previous high-resolution study of radial anisotropy in the crust of the western U.S. and changes in radial anisotropy between the crust and upper mantle. We also discuss where globally and at what periods workers can expect to obtain high-quality Love wave measurements, finding that Love wave measurements made in Europe tend to be disproportionately of a high quality.

Chapter 5 of this thesis then uses these new Love wave maps to solve for a three-dimensional model of radial anisotropy and shear velocity in the crust and uppermost mantle. The performance of the inversion is rigorously assessed, and the structural images are interpreted for geologic significance, suggesting a mostly isotropic crust except in the Western and Easternmost U.S. and a generally radially anisotropic upper mantle with strong anisotropic heterogeneity.

The final chapter of this thesis suggests some future directions that studies of overtone interference can explore in order to better understand the implications and impacts of this phenomenon on surface-wave imaging using other observations, and presents new tools to support this process.

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# **Chapter 2** | Evidence of Overtone Interference in Fundamental-Mode Rayleigh Wave Phase and Amplitude Measurements

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## 2.1 Abstract

We present evidence that measurements of minor-arc fundamental-mode (FM) Rayleigh waves experience interference from major-arc overtones, resulting in travel-time and amplitude measurements oscillating along a ray path. The oscillations are present in synthetic seismograms generated in a 3-D Earth model via SPECFEM3D\_GLOBE and in a 1-D Earth model by mode summation. The absence of oscillations in synthetics containing only the FM indicates that the oscillations originate from higher-mode interference. This interference is present across multiple measurement techniques, including multi-channel cross correlation, phase-matched filtering, and cluster analysis. Experiments with 1-D synthetics suggest that contamination from interference is largest at epicentral distances greater than around 120 degrees, where record sections of seismograms show the major-arc overtones intersecting the minor-arc FM Rayleigh wave. The short wavelength of the interference pattern means it is only observable with dense station spacing and high data quality, which may explain why it has not, to our knowledge, been previously recognized. We show the interference is visible in real data. Its overall impact on phase-velocity maps is probably minor due to many measurements from shorter path lengths less prone to interference bias. However, phase-velocity maps constructed only from measurements at epicentral distances prone to interference exhibit significant noise and poor agreement with maps from measurements that include all path lengths; the issue is especially problematic for approaches that use differential travel times between nearby stations. Accounting for interference may diminish noise in measurements and improve the accuracy of images of the upper mantle.

## 2.2 Introduction

Fundamental-mode (FM) Rayleigh wave measurements provide unique and critical information for imaging the seismic structure of the upper mantle. Rayleigh waves are clear on the vertical component of seismograms, well separated from other seismic arrivals, and well dispersed in frequency. Their depth sensitivity to shear velocity depends strongly on period, providing constraints on seismic structure at depths  $< 300$  km. As a result of these properties and the good lateral coverage of surface-wave paths, FM Rayleigh wave measurements form the basis of many global (Shapiro and Ritzwoller, 2002; Zhou *et al.*, 2006; Ritsema *et al.*, 2011) and regional (Weeraratne *et al.*, 2003; Wagner *et al.*, 2010; Shen and Ritzwoller, 2016) models of shear velocity as well as models including azimuthal (Li *et al.*, 2003; Debayle and Ricard, 2013) and radial (Moulik and Ekström, 2014; Beghein *et al.*, 2014) anisotropy. Rayleigh wave phase velocities can also be used in combination with other data types such as Rayleigh wave ellipticity (Gao and Lekić, 2018) and receiver functions (Eilon *et al.*, 2018) to constrain 1-D velocity structure beneath seismic stations. FM Rayleigh wave amplitudes are essential in studies of upper-mantle seismic attenuation (Selby and Woodhouse, 2002; Bao *et al.*, 2016) and increasingly are used in determining velocity models (Forsyth and Li, 2005; Dalton and Ekström, 2006; Hamada and Yoshizawa, 2015; Lin and Ritzwoller, 2011).

In addition to the FM surface waves, earthquakes excite higher-mode surface waves, which in a normal-mode framework correspond to free oscillations with radial order  $n > 0$ . These higher-mode surface waves, or overtones, affect studies of Earth structure in two ways. One, their sensitivity kernels are more complex and sample deeper structure than the kernels for the FM waves, offering potential to image deeper structure such as the transition zone and uppermost lower mantle. Two, the overtones can interfere with the FM when the group arrival windows overlap. This interference

can bias measurements of the FM if it is not accounted for. This has been recognized as a problem for Love waves, particularly in oceanic settings where the group velocity curves of different mode branches overlap (Figure 1b).

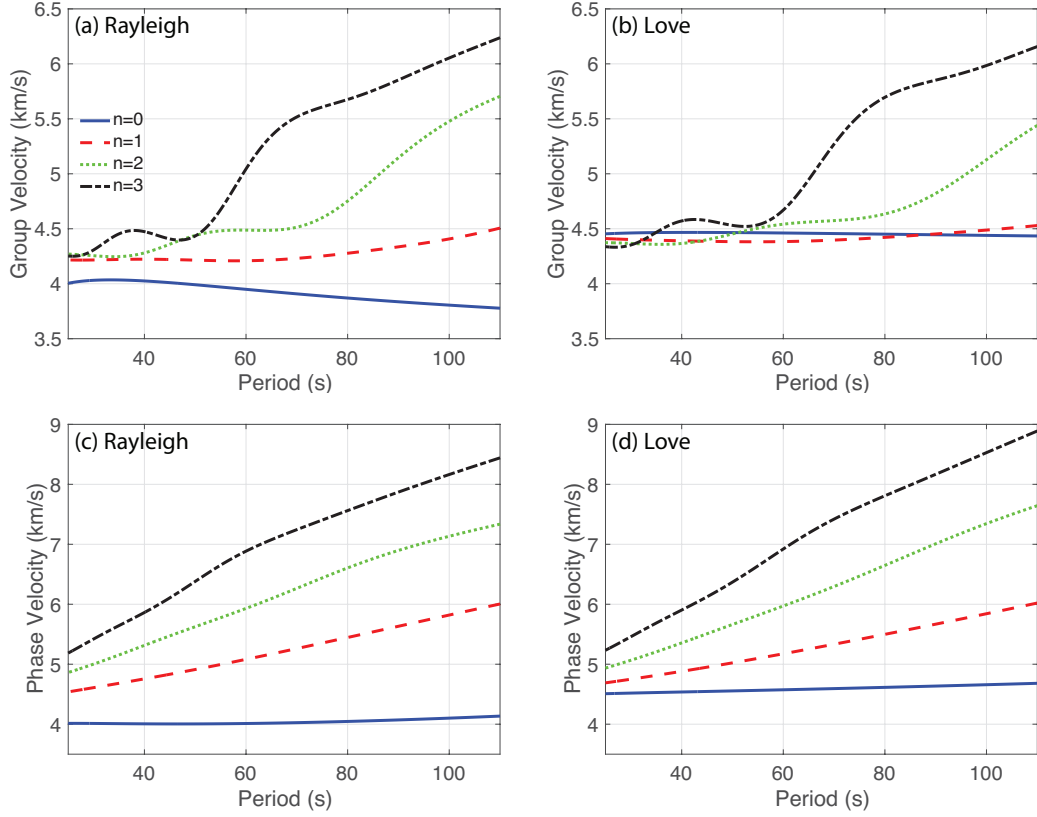


Figure 2.1: Rayleigh and Love wave group and phase velocity for the fundamental mode ( $n=0$ ) and the first three higher modes. Group and phase velocities are calculated with MINEOS for the oceanic model ATL2a (James *et al.*, 2014).

Early studies of higher-mode interference for Love waves debated whether the difficulty reconciling FM Rayleigh wave and Love wave phase velocities was due to overtone interference (Thatcher and Brune, 1969) or radially anisotropic seismic structure (Boore, 1969). Forsyth (1975a) showed that both factors mattered. A statistically significant lower misfit to Love wave phase observations was obtained when the phase velocity of the first higher mode was included as an unknown parameter in inversions (Forsyth, 1975b), but accounting for the overtone interference did not eliminate the inconsistency between Rayleigh wave and Love wave data sets (Forsyth, 1975a). More

recently, Nettles and Dziewoński (2011) demonstrated, using tests with synthetic and real data, that global long-wavelength patterns of upper-mantle radial anisotropy are not an artifact of overtone interference.

Higher-mode interference does not introduce a systematic bias into global maps of FM Love wave phase velocity because such global inversions include measurements from a wide range of path lengths and because the overall perturbation to the total path-integrated travel time is relatively small. The overlapping group velocities of FM and overtone Love waves cause overlapping group arrival times, but the differing phase velocities (Figure 1d) cause the degree of constructive and destructive interference to vary with path length. By modeling a Love wave arrival as the interference between two sinusoidal waves, the wavelength of the interference pattern can be shown to depend on the phase velocities of the two waves (Thatcher and Brune, 1969; Foster *et al.*, 2014; Jin, 2015). Thus, as long as many paths of sufficiently different lengths are used, the bias in the individual measurements can be suppressed (Boore, 1969; Forsyth, 1975a; Nettles and Dziewoński, 2011; Foster *et al.*, 2014). However, the strong dependence of the Love wave interference pattern on path length introduces significant bias into estimates of Love wave phase velocity between pairs or small groups of nearby stations. Foster *et al.* (2014) showed that phase-velocity errors can be up to 10% when the two-station method (interstation distance = 350 – 750 km) is used and up to 20% when a mini-array method (interstation distance < 225 km) is used. Moreover, because the shape of the interference pattern with distance is saw-toothed rather than symmetric, the two-station and mini-array methods can introduce an overall bias toward fast phase velocities (Foster *et al.*, 2014,?).

Unlike for the Love waves, the group velocities of the FM and higher-mode Rayleigh waves are well separated (Figure 1a). A number of methods have been developed to utilize the overtone signal in seismograms in order to improve seismic imaging of the mantle transition zone. These strategies include array-based methods (Nolet,



1975; Matsuzawa and Yoshizawa, 2019), the use of pure-mode synthetic seismograms (Cara and L  v  que, 1987; van Heijst and Woodhouse, 1997), full-waveform inversion (Woodhouse and Dziewonski, 1984; Li and Romanowicz, 1996; Lebedev and Nolet, 2003; Bozda   *et al.*, 2016), linear Radon transform (Luo *et al.*, 2015), and model-space searches (Visser *et al.*, 2007; Yoshizawa and Ekstr  m, 2010; Xu and Beghein, 2019). Nonetheless, approaches designed to isolate the FM Rayleigh wave take steps to avoid overtone interference. These steps include using only shallow earthquakes, which most strongly excite the fundamental modes, and discarding measurements from paths with epicentral distances  $< 20$  degrees and  $> 160$  degrees to avoid interference between the minor-arc FM and overtones along short paths and between the minor-arc and major-arc FM along long paths (Ekstr  m *et al.*, 1997; Levshin *et al.*, 2005).

Here, we show that the well separated Rayleigh wave group-velocity curves cause minor-arc FM Rayleigh wave measurements to be impacted by interference with higher-mode Rayleigh waves on the major arc (sometimes referred to as  $Xn$  phases, where  $n$  is the orbit number of the overtones). Traveling at greater speeds, the higher modes along the major arc arrive in the same time window as the fundamental mode along the minor arc, mostly at distances  $> 120$  degrees for the shallow earthquakes considered in this study. We use synthetic seismograms to characterize the interference. We also show that the interference is present in real data, can affect phase-velocity maps depending on the path lengths used, and results in increased measurement errors for long paths.

## 2.3 Data and Methods

The analysis described in sections 3 and 4 utilizes measurements of Rayleigh wave travel time and amplitude. These measurements are made on synthetic waveforms generated by SPEC-FEM3D-GLOBE and normal-mode summation as well as on ob-

served waveforms at USArray stations. The measurements are made using both inter-station and single-station approaches. Finally, phase-velocity maps are generated from the measurements. Below, the details of the data and methods are discussed.

### 2.3.1 Synthetic Seismograms

Synthetic seismograms used in this study are generated with two different approaches. To include complex wave-propagation phenomena and the effects of realistic Earth structure, we use seismograms generated from Princeton University’s ShakeMovie portal (Tromp *et al.*, 2010) (<http://global.shakemovie.princeton.edu>, last accessed 7-1-2019), created using the spectral-element code SPECFEM3D\_GLOBE and implementing the 3-D Earth model S362ANI (Kustowski *et al.*, 2008), with crustal structure from CRUST2.0 (Bassin *et al.*, 2000).

To perform tests on the influence of different surface-wave overtones in seismograms, we calculate synthetic seismograms using normal-mode summation (Gilbert, 1971) for a known 1-D Earth structure; the mode eigenfunctions and eigenfrequencies are computed with the MINEOS code (Masters *et al.*, 2011). The advantage of mode summation is that we can control which mode branches (e.g., which values of  $n$ ) are included in the seismograms. Given the prevalence of oceanic paths in global data sets, the 1-D Earth structure used in this study is the oceanic model ATL2A (James *et al.*, 2014), which has a 4 km water layer, a 300 m sediment layer, and a Moho depth of 10.3 km.

The examples shown in sections 3 and 4 utilize a few different earthquake sources. The source parameters for these events are from the Global CMT catalog (Ekström *et al.*, 2012).

### 2.3.2 Measurement Approaches

We measure the travel time and amplitude of fundamental-mode Rayleigh waves. We compare measurements made using three different approaches to demonstrate that our results are not dependent on how the measurement is made. The first approach is the Automated Surface-Wave Measurement System (ASWMS) software package (Jin and Gaherty, 2015), which uses multi-channel cross-correlation of waveforms at nearby stations. For each event, single-station travel times are obtained from the inter-station delay times using least-squares inversion, and the single-station amplitudes are determined from the square root of the peak of the auto-correlation function of the windowed Rayleigh wave.

The second approach utilizes a phase-matched filter to produce single-station travel-time and amplitude measurements (Ekström *et al.*, 1997). This method, with which the measurements are made relative to synthetic reference seismograms, has been used to generate the phase-delay data sets described by Ekström *et al.* (1997); Ekström (2011); Foster *et al.* (2014) and Eddy *et al.* (2018). The third approach is the cluster-analysis method of Ma *et al.* (2014), which relies on time-domain multi-channel cross-correlation to extract the relative travel times. With this method, cluster analysis using cross-correlation as a similarity metric is used to generate clustering trees, from which users can manually eliminate measurements below a desired similarity threshold.

While the focus of this study is synthetic seismograms, we also show some Rayleigh wave travel-time and amplitude measurements made on real data, using the data set of Babikoff and Dalton (2019). These measurements, which exist for the period range 25-180 s, were made using ASWMS (Jin and Gaherty, 2015) on waveforms recorded by 1831 USArray stations. They include 549 earthquakes with  $M_w > 6.0$ , depth  $< 50$  km, and a maximum epicentral distance of 160 degrees.

### 2.3.3 Determining Phase Velocity Maps

The phase-velocity maps shown here are generated using either Eikonal tomography (Lin *et al.*, 2009) or Helmholtz tomography (Lin and Ritzwoller, 2011). The governing Helmholtz equation is:

$$\frac{1}{c^2} = |\nabla\tau|^2 - \frac{\nabla^2 A}{\omega^2 A} \quad (2.1)$$

where  $\tau$  and  $A$  are the measured travel-time and amplitude fields,  $\omega$  is angular frequency, and  $c$  represents the phase-velocity variations to be determined. When the second term on the right-hand side is small, Equation 1 is reduced to the Eikonal equation, and  $c$  is determined only from the travel-time field.

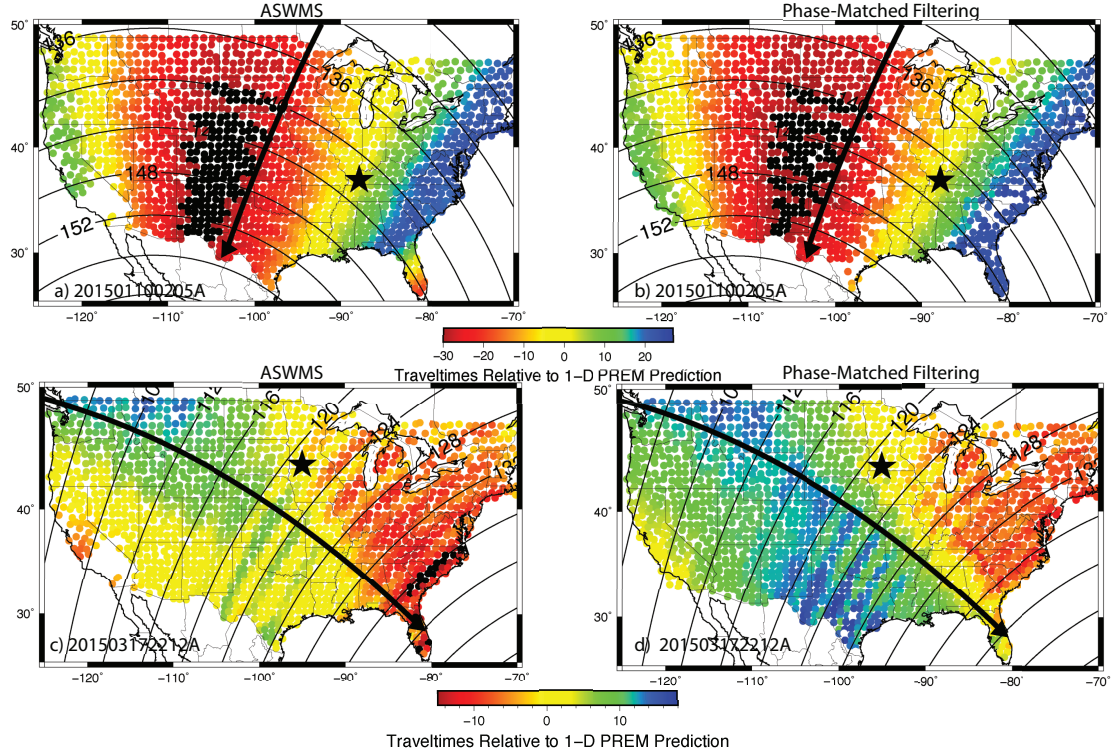
In practice, we average the travel-time and amplitude measurements in  $0.5^\circ \times 0.5^\circ$  cells and interpolate them onto a gridded surface with  $0.25^\circ \times 0.25^\circ$  cells. Our surfaces are smoothed with a Gaussian filter of width 200 km, and the gradient terms are calculated by finite difference in spherical coordinates. This process is performed separately for each event, and event-specific Eikonal phase-velocity maps are determined from the inverse of  $\sqrt{|\nabla\tau|^2}$ . To generate composite Eikonal phase-velocity maps from multiple events, we use the median of the  $|\nabla\tau|^2$  values in each pixel. For Helmholtz phase-velocity maps we use the median of the entire right-hand side of Equation 1.

## 2.4 Results

### 2.4.1 Evidence from Spectral-Element Synthetic Seismograms

Our experiments on synthetic seismograms show that Rayleigh wave measurements from certain events exhibit streaks or banding when plotted in map view. Figures 2-4 show examples of 100 s measurements made on ShakeMovie synthetics for two events. Event 201503172212A is a shallow (41.7 km) event in the Northern Molucca Sea, and

Event 201501100205A is a shallow (12.0 km) event in the Chagos Archipelago region. The two events are located roughly 120 and 140 degrees from the center of the U.S, respectively. In the maps, bands of alternating advanced and delayed traveltimes (Figure 2), high and low amplitude (Figure 3), and slow and fast velocity (Figure 4) are apparent. In Figure 2, the predicted travel times for propagation in a 1-D Earth model have been subtracted. The orientation of the bands is roughly parallel to wavefronts, and the bands have a similar wavelength in all maps.



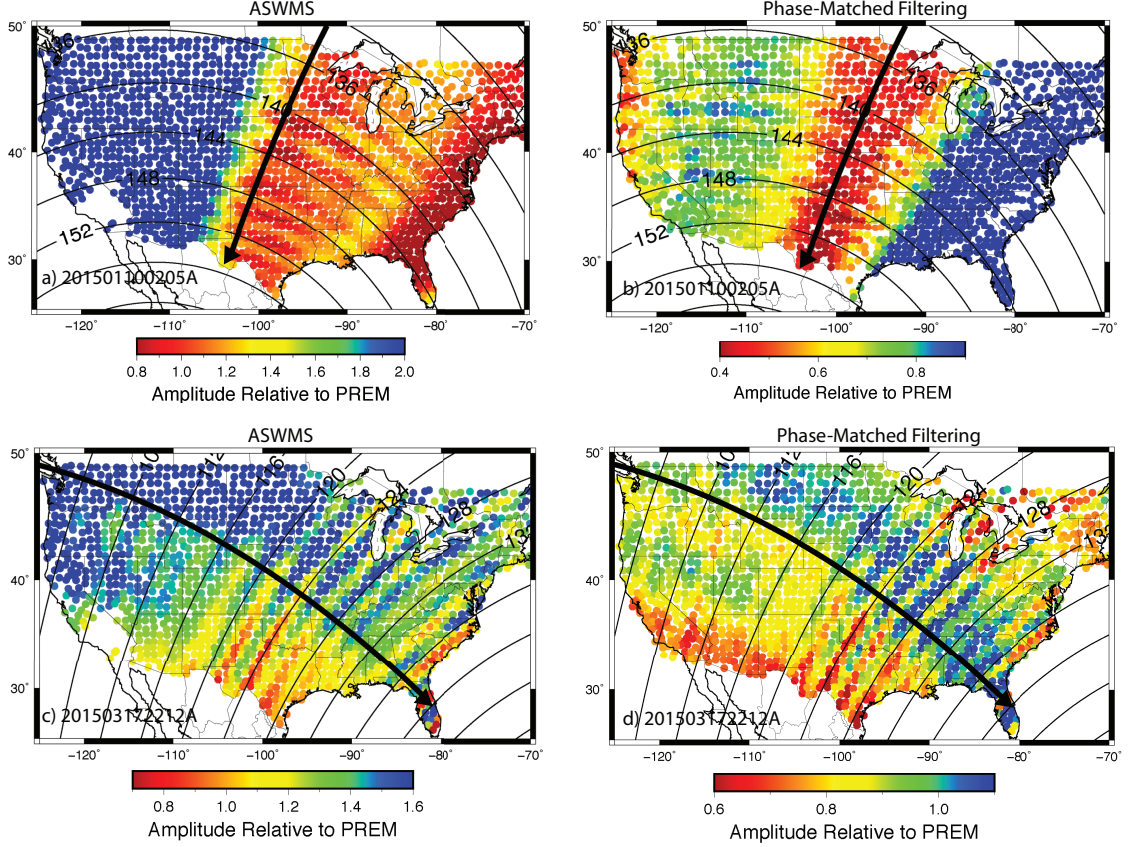


Figure 2.3: As in Figure 2 but for Rayleigh wave amplitude measurements, also at 100 s. The predicted effects of the source radiation pattern have been removed from the ASWMS measurements. Amplitudes are shown relative to the same reference station for both measurement approaches. Different color scales are used to emphasize the amplitude variations. Contours of epicentral distance, spaced 4 degrees apart, are overlain and labeled.

that includes the source radiation pattern, 1-D attenuation, and geometrical spreading with the phase-matched filtering approach. All four examples of the amplitude field (Figure 3) show the oscillatory features. In Figure 5, as an example of a third event and a third measurement approach, we compare Eikonal phase velocities determined from travel times measured with ASWMS and with the cluster-analysis method of (Ma *et al.*, 2014) for a shallow event (12 km) at Papua New Guinea, roughly 115 degrees from the center of the array. The banding is apparent in both sets of measurements from this event, especially at stations located  $> 120$  degrees from the earthquake.

For the events in Figure 4, the streaks obscure the long-wavelength phase-velocity



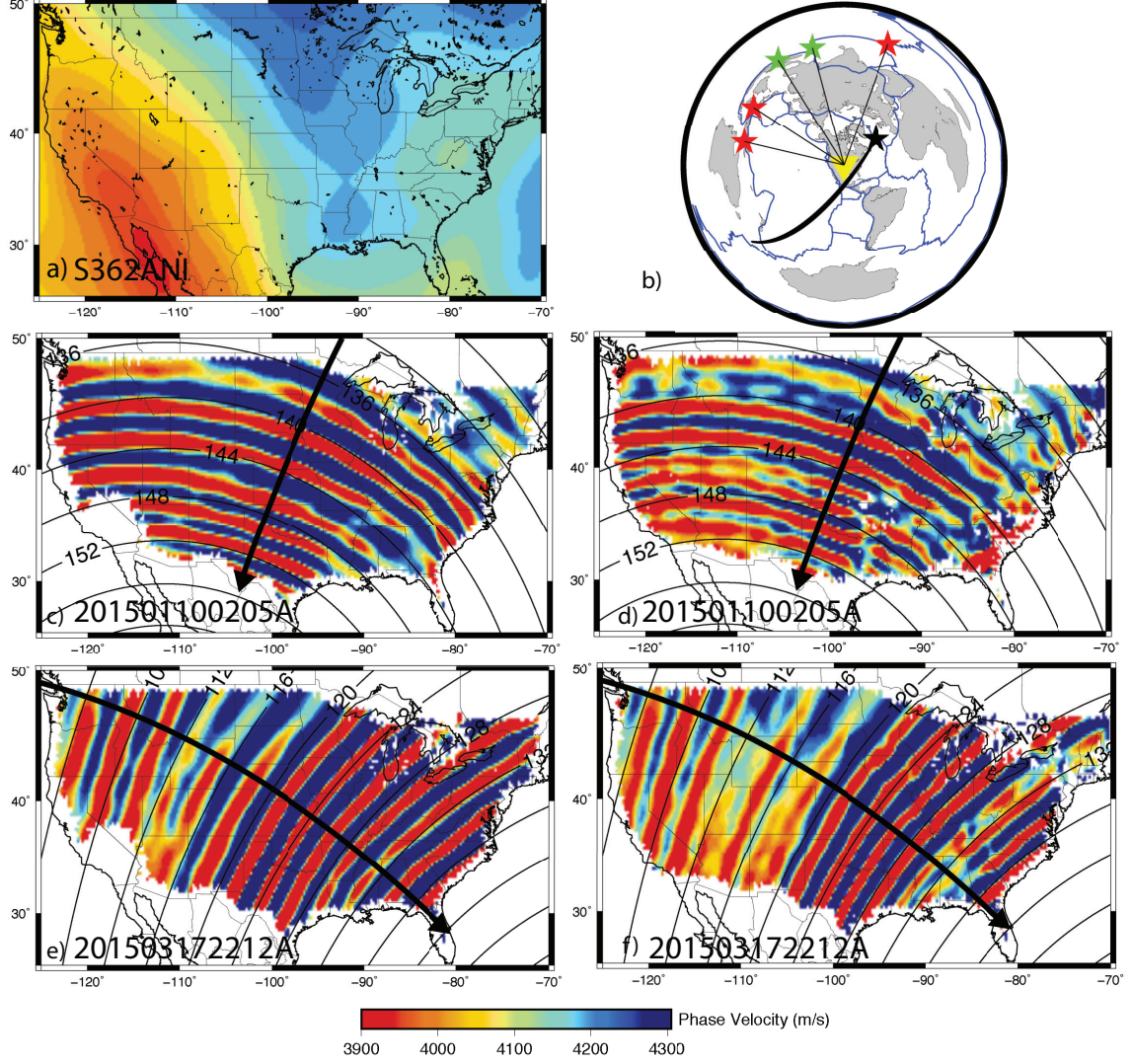


Figure 2.4: Phase-velocity maps for 100-s Rayleigh waves. (a) Predicted phase velocities for the 3-D model S362ANI with CRUST2.0. Since this model is used to calculate the spectral-element synthetics, this map represents the input structure we expect to recover. (b) Event locations used in this study. Red stars correspond to events with corresponding SHAKEMOVIE synthetics analyzed. Green stars correspond to sources recorded on real data, shown later in the paper (Figure 11). The black star corresponds to a source used in Figures 9 and 10, and the black line shows corresponding stations. (c-f) Event-specific Eikonal phase velocities determined for the same events shown in Figure 2, 201501100205A (c,d) and 201503172212A (e,f), using measurements made with ASWMS (c,e) and phase-matched filtering (d,f). The black arrow illustrates a great-circle ray path. Contours of epicentral distance, spaced 4 degrees apart, are overlain and labeled.

features we expect to recover, whereas the example in Figure 5 more successfully recovers the input structure with the exception of pixels in the eastern U.S., which are



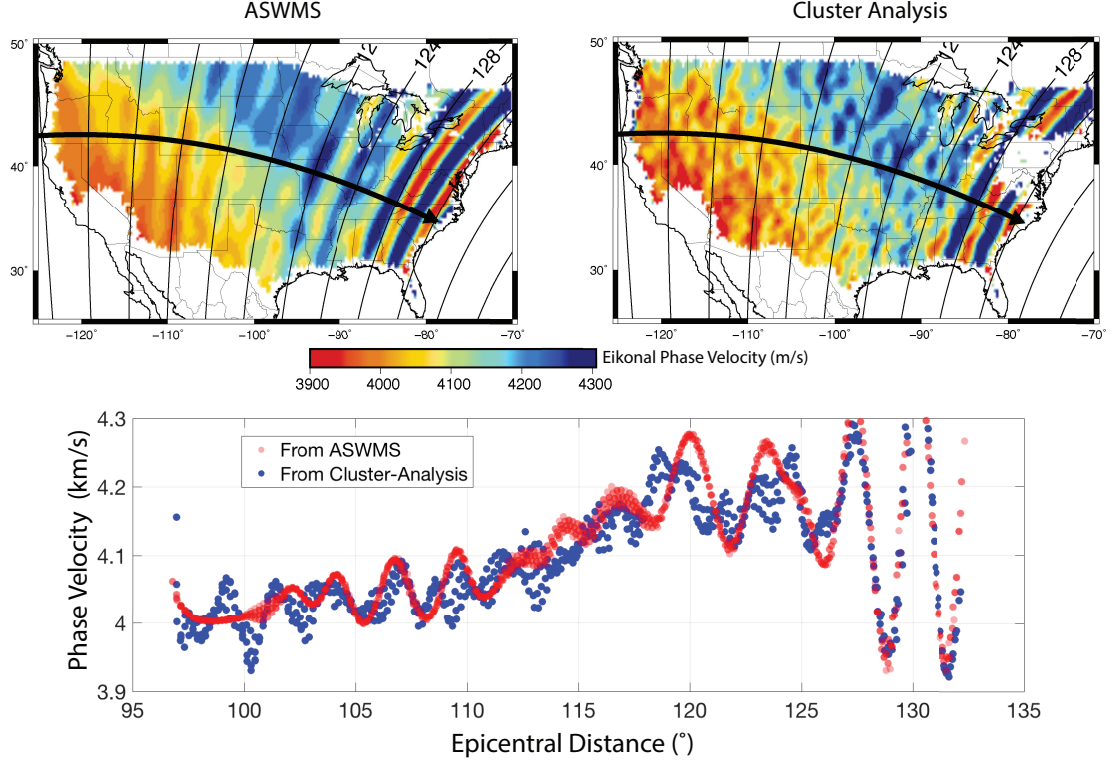


Figure 2.5: Eikonal phase-velocity maps for 100 s Rayleigh waves. Travel times are measured on ShakeMovie synthetics using ASWMS (left) and the cluster-analysis measurement of Ma et al. (2014) (right) for event 201802251744A, which is located at 142.97°E, 6.29°S, and depth=12 km. The bottom panel shows the along-ray transect through both maps. Contours of epicentral distance, spaced 4 degrees apart, are overlain and labeled.

located  $> 120$  degrees from the earthquake. An advantage of using the Eikonal equation is that it utilizes only the travel times; the Eikonal phase-velocity maps therefore represent how the anomalies in the travel-time measurements will impact estimates of velocity. Since the streaks are also present in the amplitude measurements, it is reasonable to wonder whether they could be suppressed if the Helmholtz equation (Equation 1) is instead used. By comparing the Eikonal and Helmholtz phase-velocity maps for all events we have analyzed, we have found that this is true to an extent; the amplitude of the banding is weaker, though still significant, in the Helmholtz phase-velocity maps. Figure 6 shows a comparison of Eikonal and Helmholtz velocities in map view and along a transect for event 201503172212A measured with ASWMS. The inability of the Helmholtz equation (Equation 1) to fully account for the interfer-

ence is likely because the assumption that the interfering waves have the same phase velocity is violated.

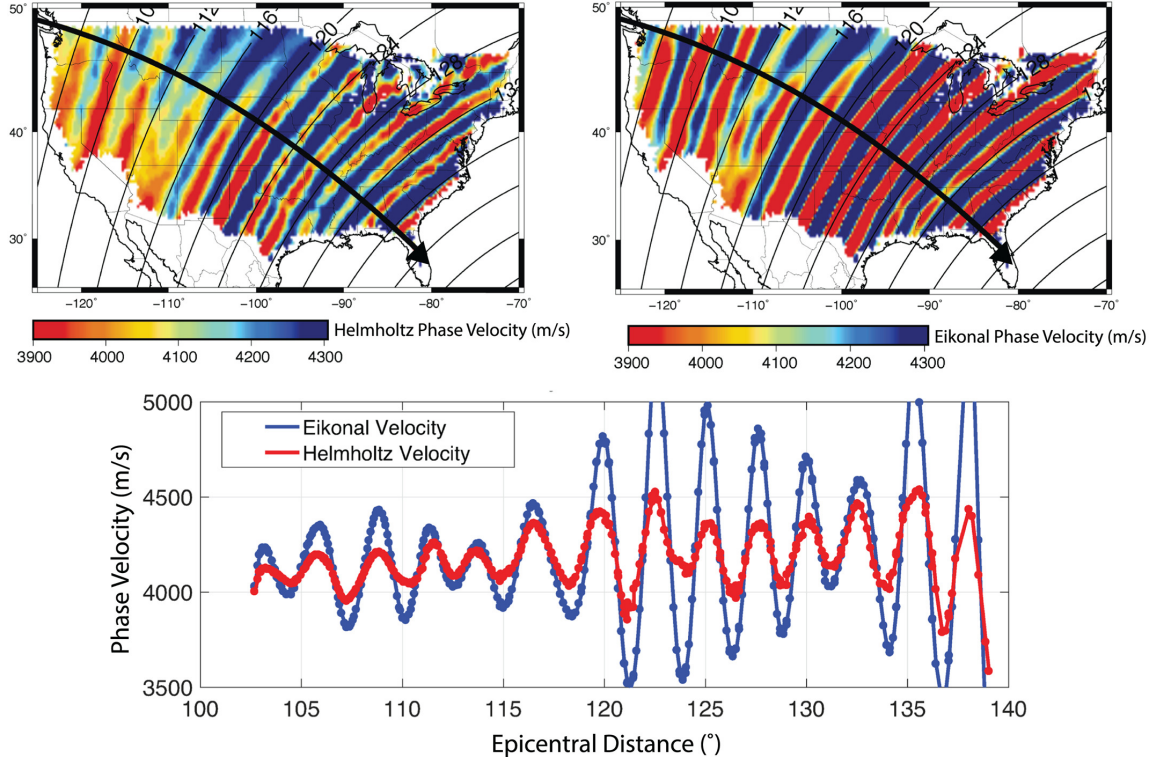


Figure 2.6: Comparison of Helmholtz and Eikonal phase velocities for 100-s Rayleigh waves for event 201503172212A. Travel times and amplitudes were measured using ASWMS. Left: Helmholtz velocities plotted in map view. Right: Eikonal velocities in map view (as in Figure 4c). Bottom: Transect along ray path comparing Helmholtz (red) and Eikonal (blue) phase velocities.

Figures 5 and 7 show along-ray transects of amplitude and Eikonal phase velocity, measured with different methods, as a function of epicentral distance. These figures highlight the agreement between measurements made with different approaches. They also show the approximate wavelength of the bands ( $\approx 3 - 5$  degrees, depending on location) and suggest that the banding becomes stronger at epicentral distances  $> 120$  degrees. The offset between amplitudes measured with the two different approaches (Figures 7b,d) may be because the amplitudes measured with the phase-matched filtering approach have had 1-D attenuation and geometrical spreading removed.

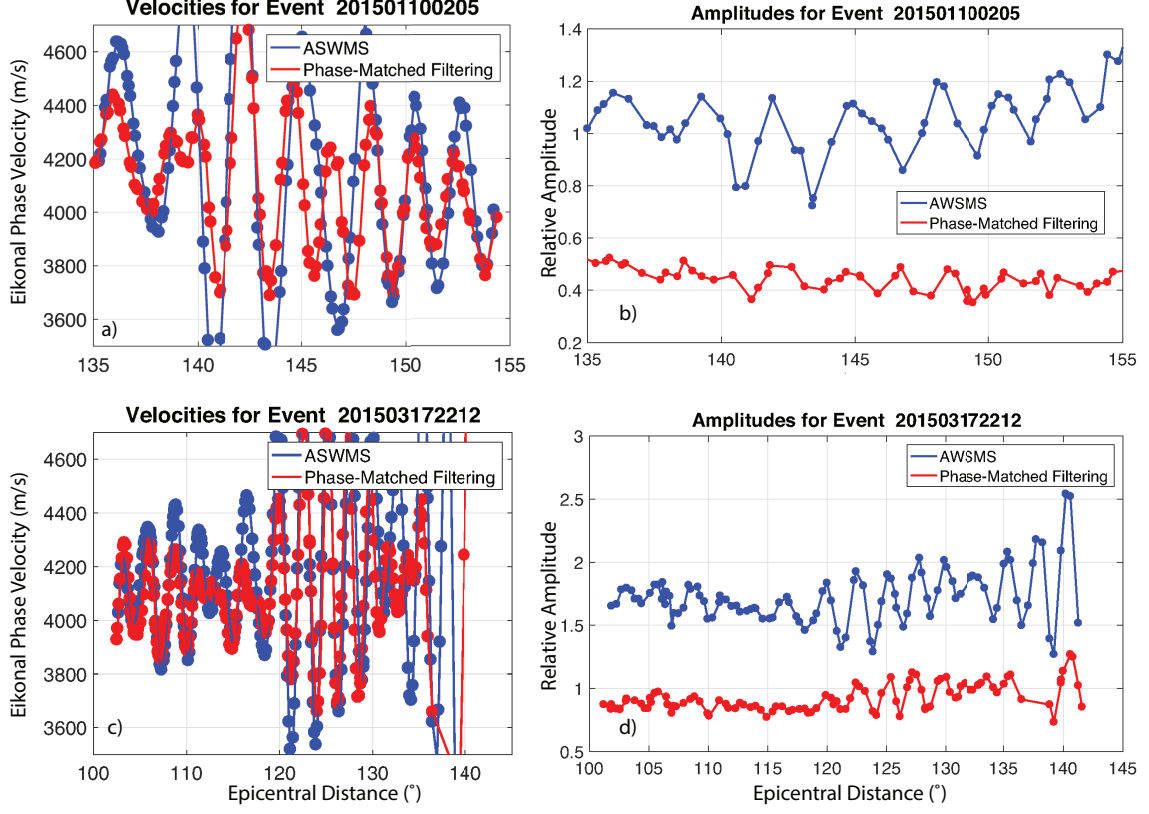


Figure 2.7: Along-ray transects of Eikonal phase velocity (a,c) and Rayleigh wave amplitude (b,d) for the measurements and phase-velocity maps in Figures 3 and 4. While the transects sample an evenly spaced grid for the phase-velocity maps, they sample USArray station locations for the amplitudes, which explains the difference in sampling density between the two sets of transects.

#### 2.4.2 Tests with Normal-Mode Synthetic Seismograms

To explore explanations for the anomalies described in the previous section, we simulate seismograms for the same two events considered in Figures 2-4 using normal-mode summation with the 1-D Earth model ATL2a (James *et al.*, 2014). Two sets of normal-mode seismograms are generated. The first set contains only the fundamental spheroidal modes, and the second set contains the full catalog of spheroidal modes. The summation includes all modes with frequency  $< 50$  mHz and angular order  $< 500$ . We use ASWMS to measure Rayleigh wave travel times and amplitudes for the two events. Figure 8 shows the resulting 100 s Eikonal phase-velocity maps. Maps

corresponding to synthetic seismograms that contain only the fundamental modes successfully recover the homogenous input phase velocity (Figures 8b,d), but maps corresponding to seismograms containing the full mode catalog do not (Figures 8a,c). These maps contain strong bands of alternating slow and fast velocity. These bands are similar to the ones in Figures 4c,e, suggesting a common mechanism. Moreover, the absence of the bands when only the fundamental modes are used suggests that the interference of the higher modes with the fundamental mode causes the banding. The edge effect seen along the East Coast in Figures 8a,b is due to erroneous travel-time measurements, which result from the relative paucity of stations for which interstation cross-correlation can be conducted.

Record sections of vertical-component synthetic seismograms illustrate the likely explanation (Figure 9a). Normal-mode summation is used to calculate these seismograms for the ATL2a model and event 201502131859A, a shallow (depth=25.2 km) strike-slip event on the Reykjanes Ridge. Stations are evenly spaced along a great-circle path spanning epicentral distances 20-175 degrees. Synthetic seismograms containing only the fundamental mode and containing only higher modes are plotted separately, with the latter amplitude exaggerated by a factor of five. At distances greater than roughly 25 degrees the minor-arc higher-mode Rayleigh waves arrive much earlier than the minor-arc fundamental-mode Rayleigh waves as a result of their distinct group velocities (Figure 1a; Figure 9b). However, at distances  $>\approx 120$  degrees, the major-arc higher-mode Rayleigh waves ( $X_2$  phases) arrive within the same window as the minor-arc fundamental mode. We suggest that this interference introduces bias into the Rayleigh wave amplitude and travel-time measurements as the different wavepackets constructively and destructively interfere with each other.

Figure 10 quantifies this interference by measuring the impact on amplitude and travel time for the synthetics in Figure 9. The amplitude and travel time measured from the normal-mode synthetics that contain the full mode catalog are expressed rel-

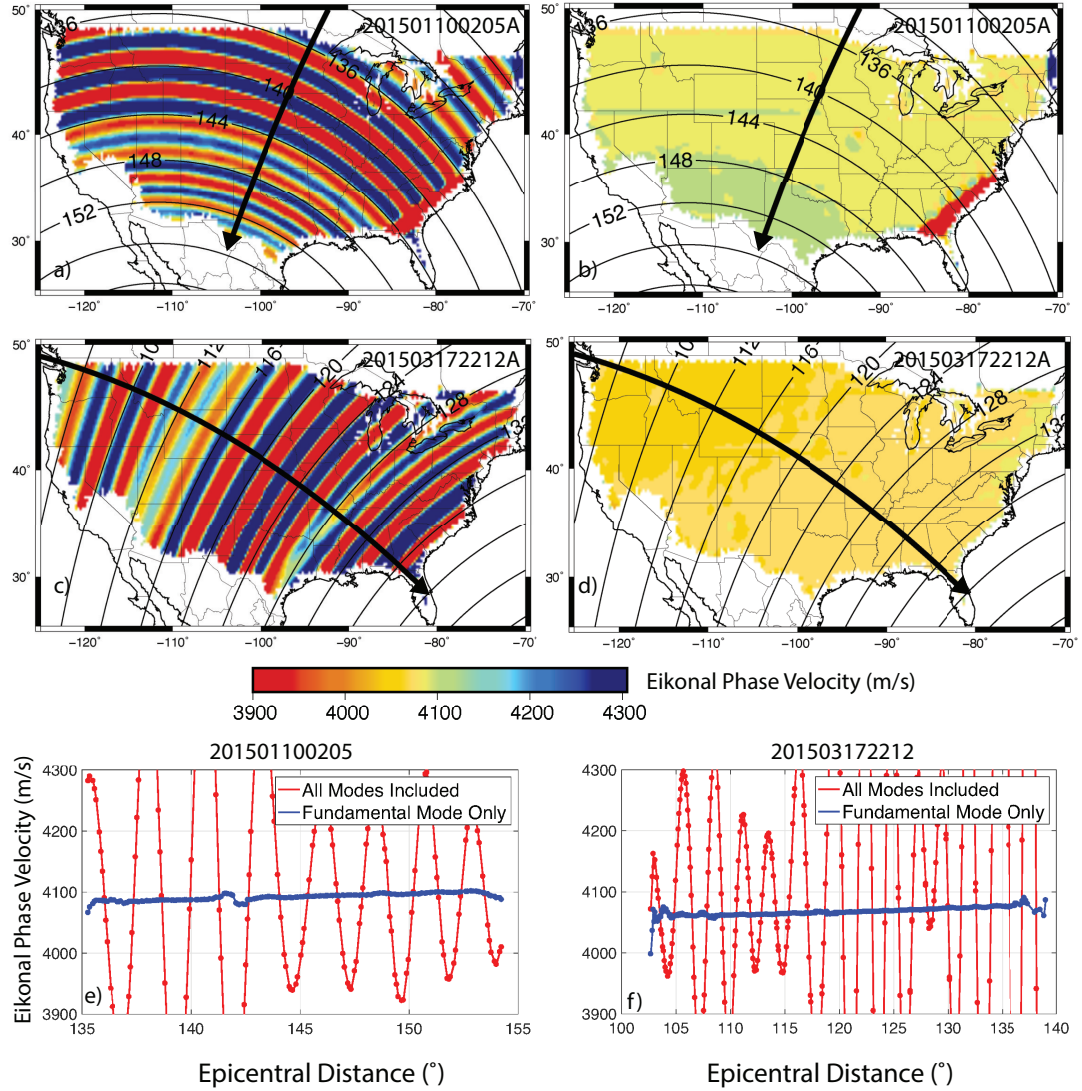


Figure 2.8: Eikonal phase velocity for 100-s Rayleigh waves for events 201501100205A (panels a, b, e) and 201503172212A (panels c, d, f). The travel times are measured from synthetic seismograms generated with normal-mode summation, using the full mode catalog (a,c) and only the fundamental modes (b,d). The black arrow illustrates a great-circle ray path. Contours of epicentral distance, spaced 4 degrees apart, are overlain.

ative to their values measured from the synthetics containing only the fundamental mode. In doing so the effects of the source radiation pattern, attenuation, and geometrical spreading will be cancelled out, allowing the effects of higher-mode interference to be isolated. These comparisons show a slight and long-wavelength perturbation to amplitude and travel time at distances  $< 50$  degrees, presumably due to interfer-



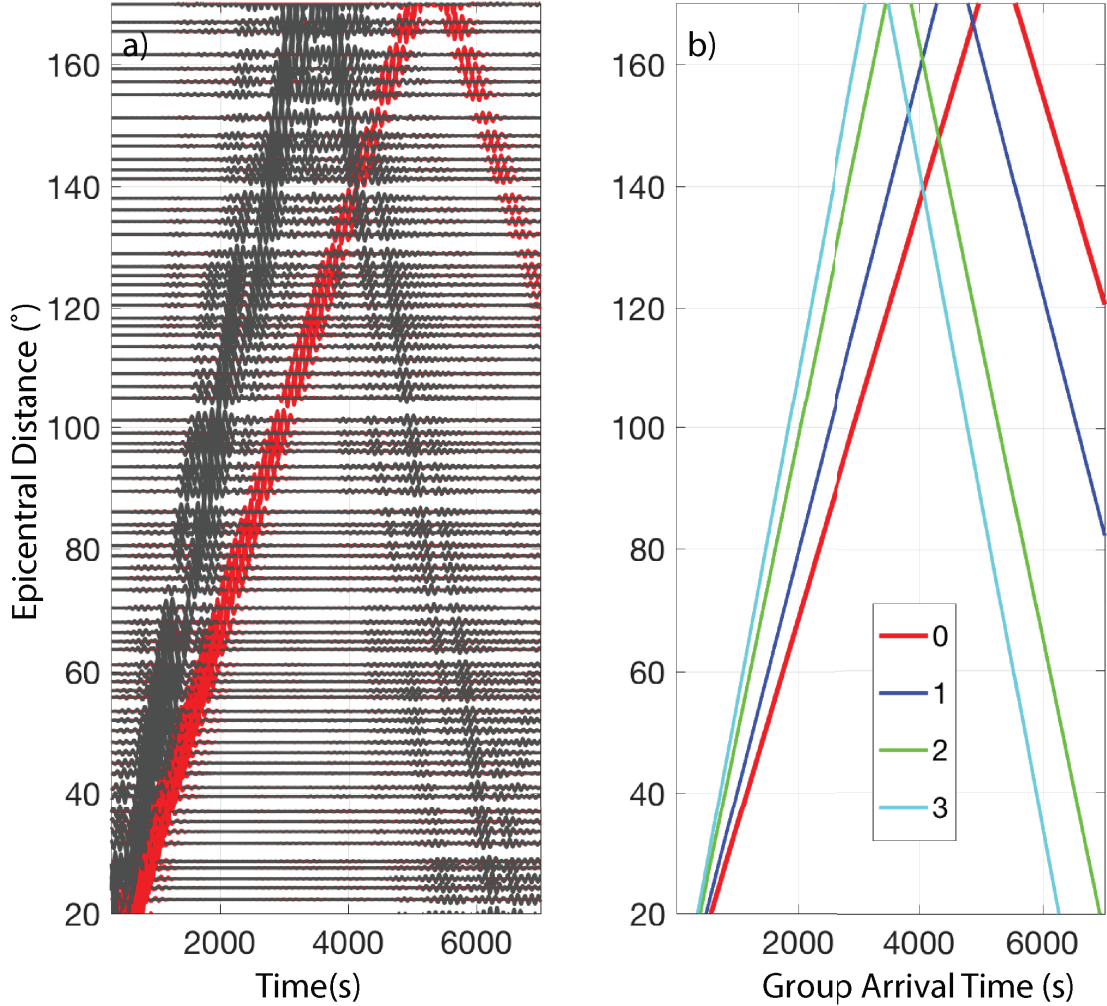


Figure 2.9: (a) Record sections of vertical-component synthetic seismograms calculated using normal-mode summation for event 201502131859A. Event 201502131859A is located at  $32.74^{\circ}\text{W}$ ,  $52.7^{\circ}\text{N}$ , and 25.2-km depth. Two sets of seismograms are shown, bandpass-filtered with a center period at 100 s. The red traces contain only the fundamental mode, and the black traces contain all mode branches except the fundamental mode; for easier visualization the black traces are vertically exaggerated by a factor of 5. (b) Predicted group arrival times for the first four mode branches as a function of epicentral distance using ATL2a-predicted group velocities for period = 100 s.

ence between the minor-arc fundamental and higher modes (Figure 9), and larger and shorter-wavelength perturbations at distances  $> 120$  degrees. These short-wavelength perturbations are largest around 140 degrees, where branches 2 and 3 overlap with

branch 0.

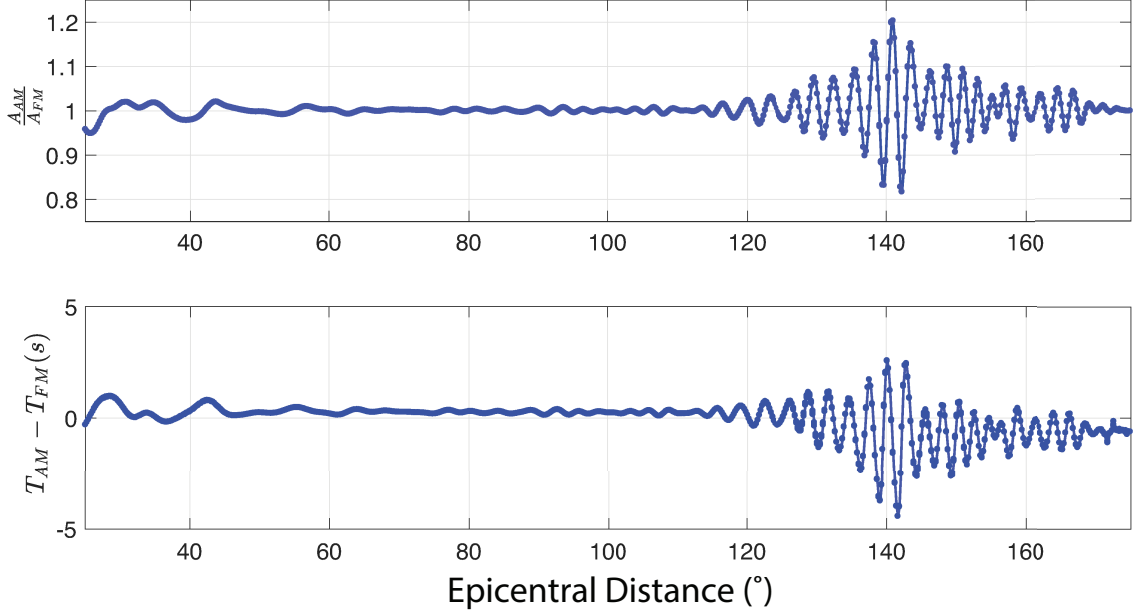


Figure 2.10: Rayleigh wave amplitudes (top) and travel times (bottom) measured at 100 s using ASWMS for the synthetics in Figure 9. The measurements from synthetics containing the full mode catalog (AM) are plotted relative to those from synthetics containing only the fundamental mode (FM).

Foster *et al.* (2014) calculated the interference of higher-mode Love waves with the fundamental-mode Love waves. It differs from the Rayleigh wave higher-mode interference patterns in Figure 10 in several aspects. One, the Love wave interference pattern is present throughout a broad range of epicentral distances, in contrast to the Rayleigh wave pattern, which is most prominent at distances  $> 120$  degrees. Two, the perturbations that characterize the Love wave interference pattern are relatively constant across a broad distance range, whereas the Rayleigh wave perturbations are strongly dependent on distance. Three, the Love wave interference pattern has a longer wavelength ( $\approx 20$  degrees), in contrast to the  $\approx 3$ -degree wavelength of the Rayleigh wave pattern at distances  $> 120$  degrees.

These differences can be understood from the Rayleigh and Love group and phase velocities (Figure 1). For Love waves, the overlapping group velocities cause the minor-arc fundamental and higher modes to interfere across a broad range of dis-

tances, whereas the well separated Rayleigh wave group velocities cause the higher modes to pass into and out of the fundamental-mode window. The wavelength of the interference pattern is dictated by the relative phase velocities of the interfering modes (Thatcher and Brune, 1969; Foster *et al.*, 2014), with a larger difference causing a shorter-wavelength pattern. Although the relative phase velocities between modes are not dramatically different for the Rayleigh waves and Love waves, major-arc higher modes are traveling in the opposite direction from the minor-arc fundamental mode. Thus, the relative phase velocity will appear much larger; for Rayleigh wave branches 2 ( $c=7.1$  km/s) and 0 ( $c=4.1$  km/s), the difference will appear to be 11.2 km/s rather than 3.0 km/s. This can explain the short-wavelength nature of the Rayleigh wave interference pattern.

#### **2.4.3 Interference in Real Data**

Because the higher-mode interference pattern is characterized by short wavelengths and is strongest at epicentral distances  $> 120$  degrees, it will be easiest to observe in real data with long arrays of densely spaced stations. Furthermore, real data will be noisier and have greater wavefield complexity than the rather idealized synthetic seismograms analyzed in Sections 3.1-3.2. Nonetheless, using Rayleigh waves recorded by the EarthScope USArray stations for events with ray paths traveling along the long axis of the Transportable Array deployment, we have found travel-time and amplitude anomalies that are similar to those observed in the synthetic data. Figure 11 shows the Eikonal velocities and amplitudes for two events; these travel-time and amplitude data are from the study of Babikoff and Dalton (2019), in which Rayleigh waves were measured using ASWMS for 549 events in the period range 25-180 s. The real data exhibit oscillations in velocity and amplitude along a ray path. Both the wavelength and the magnitude of the oscillations in the real data are similar to those present in the measurements made on synthetic data, suggesting that the anomalies



are generated by interference of the higher-mode major-arc Rayleigh wave with the minor-arc fundamental mode. In Section 4 we consider how this interference affects Rayleigh wave phase-velocity maps on regional and global scales.

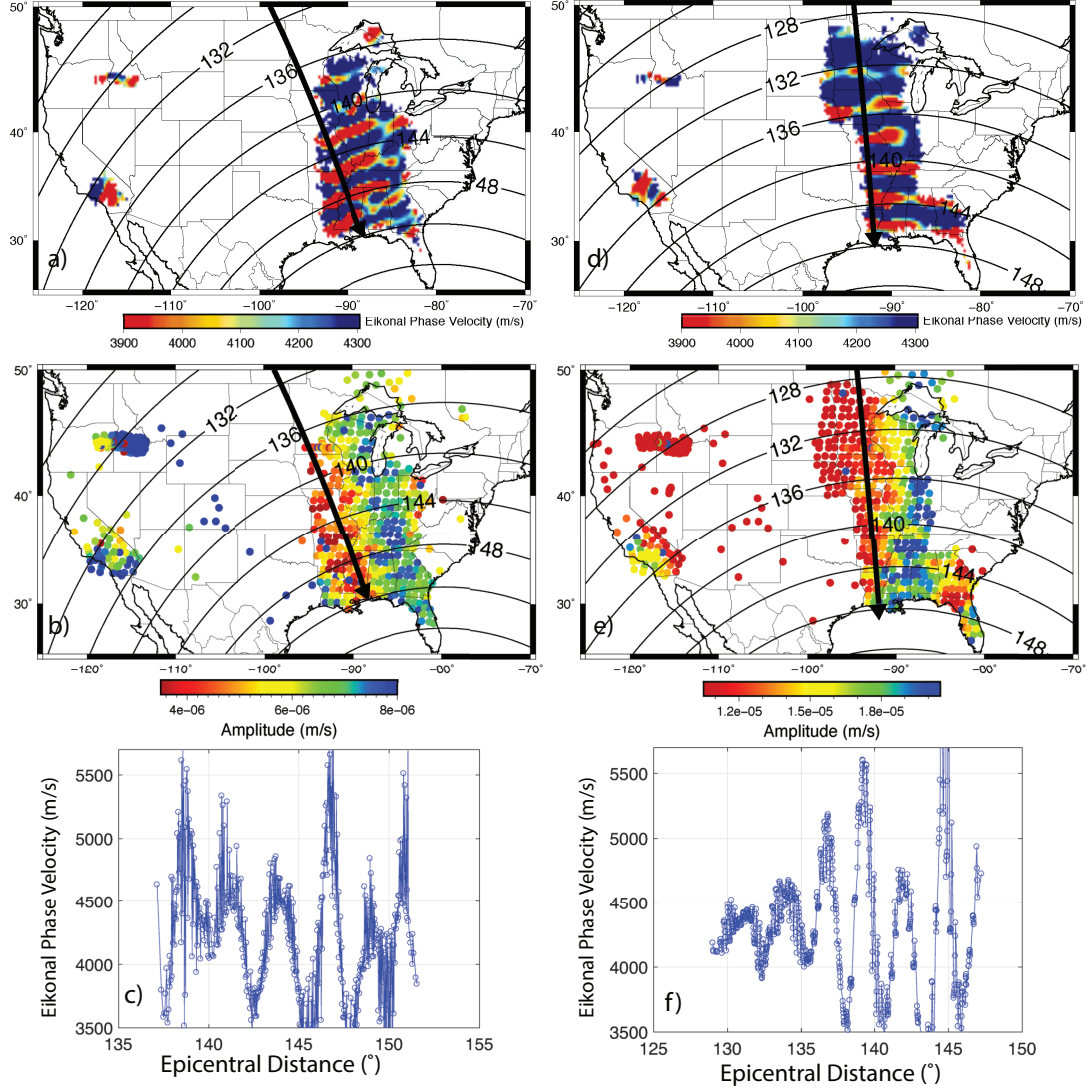


Figure 2.11: Evidence of the interference in real data using the 100-s Rayleigh wave travel-time and amplitude measurements from Babikoff and Dalton (2019) for two events, 201204150557 (a-c) and 201209140451 (d-f). Event 201204150557 is located at  $90.31^{\circ}\text{E}$ ,  $2.49^{\circ}\text{N}$ , and 33.0-km depth. Event 201209140451 is located at  $100.32^{\circ}\text{E}$ ,  $3.58^{\circ}\text{S}$ , and 12.0-km depth. These measurements were made using ASWMS. Eikonal velocities determined from the travel times are shown in map view (a,d) and as transects (c,f), and amplitudes are shown in map view (b,e). The black arrow illustrates a great-circle ray path.

## 2.5 Discussion

### 2.5.1 Implications for USArray Measurements & Phase Velocity Maps

In this section we evaluate the effects of the overtone interference on the 100-s Rayleigh wave travel-time measurements made at USArray stations in the contiguous U.S. by Babikoff and Dalton (2019) and on the Eikonal phase-velocity maps determined from that data set. The effects of overtone interference (i.e., the oscillations with distance) are not visible in the single-station travel-time measurements, mostly because the dominant signal is from 3-D structural heterogeneity. We predict and remove the long-wavelength velocity variations from the travel-time data by integrating along the great-circle path through GDM52, the global phase-velocity maps of Ekström (2011). Since the predictions are path-integrated travel times and the measurements of Babikoff and Dalton (2019) are expressed relative to the travel time at a reference station, we convert the measurements into path-integrated times by fixing the travel time at the reference station to the predicted value and applying this static offset to all other relative travel times. For 90% of the 208,233 paths from Babikoff and Dalton (2019) the path-averaged phase-velocity anomaly is within 0.5% of the GDM52 predictions. Figure 12a shows the median absolute velocity difference in one-degree epicentral distance bins; all bins between 17 degrees and 152 degrees contain at least 200 paths. The calculation shows that paths in the distance range 70-120 degrees are fit best by the GDM predictions, with higher residuals at distances  $< 70$  degrees and  $> 120$  degrees. Comparison of Figures 12a and 10b suggests that the elevated residuals at distances  $> 120$  degrees correspond to interference with the major-arc higher modes; we speculate that the signal for short paths may be due to interference with the minor-arc higher modes (Figure 9).

With Eikonal and Helmholtz tomography (Lin *et al.*, 2009; Lin and Ritzwoller, 2011) individual phase-velocity maps are generated for each event, and then they are

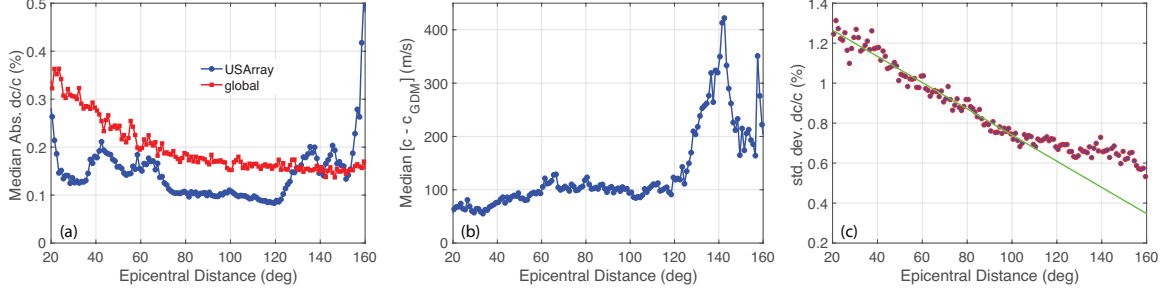


Figure 2.12: Evidence of higher-mode interference in USArray and global data sets of 100-s Rayleigh wave travel times and phase velocities. (a) The median absolute difference of the path-averaged phase-velocity anomalies ( $dc/c$ ), binned by epicentral distance. Path-averaged  $dc/c$  predicted by integrating through the GDM52 phase-velocity map is subtracted from the measured values. Results are shown for 208,233 measurements at USArray stations (Babikoff and Dalton, 2019) and 282,996 measurements at global stations (Ekström, 2011). (b) Median absolute phase-velocity difference (m/s) in the contiguous U.S. between 549 event-specific Eikonal maps of Babikoff and Dalton (2019) and the U.S. portion of the GDM52 global map. (c) Standard deviation of the path-averaged phase-velocity anomalies in the global data set of Ekström (2011), binned by epicentral distance. Green line shows linear regression applied to values in the distance range 20-110 degrees.

combined to yield a single composite phase-velocity map at a given period. Figure 12b shows the median absolute difference between the event-specific Eikonal phase-velocity maps and the portion of the global GDM52 map confined to the contiguous U.S. For this figure distance is calculated from the center of each  $0.25^\circ \times 0.25^\circ$  pixel and the corresponding earthquake location. This calculation makes use of event-specific velocities from 549 earthquakes and a total of 11,932 pixels, although most of the event-specific phase-velocity maps consist of roughly 2100 pixels. The median residual phase velocity is  $\approx 100$  m/s (2%) at distances  $< 120$  degrees; the residual peaks at distance  $\approx 140$  degrees, where it is 422 m/s (10%).

Figure 12b makes clear that the event-specific phase velocities for distances  $> 120$  degrees are characterized by large errors. In Figure 13 we explore how the composite map is affected when velocities that may be biased by the higher-mode interference are excluded. Referring to Equation 1, the composite Eikonal maps are generated from the inverse of the square root of the median of all  $|\nabla\tau|^2$  values in each pixel. In

Figure 13b we only include an event-specific  $|\nabla\tau|^2$  value in the median calculation if the distance between the earthquake and pixel is  $< 115$  degrees, and in Figure 13c only  $|\nabla\tau|^2$  values from earthquakes  $> 115$  degrees are included. For this test we use the Rayleigh wave Eikonal maps of Babikoff and Dalton (2019). Figure 13d shows that the number of distant events ( $> 115$  degrees) is much smaller than the number of nearby events ( $< 115$  degrees). Since the median value may not be as robust if determined from a small number of measurements, we require that the number of distant and nearby events used at each pixel is the same. In practice, in each pixel we randomly select a subset of the nearby events; this results in a speckled appearance of the map in Figure 13b. Because most pixels are dominated by data from events at distances  $< 115$  degrees, the similarity between Figures 13a and 13b shows that the composite map generated from all data is not biased by the inclusion of data from events at distances  $> 115$  degrees. However, Figure 13c illustrates the potential pitfalls of including data biased by interference effects; this composite map differs significantly from the composite map determined from all data. As mentioned earlier, the impact of this interference is somewhat weaker for maps constructed using Helmholtz tomography (Figure 6).

### 2.5.2 Implications for Global Rayleigh Wave Data Sets

To investigate the possible impact of Rayleigh wave higher-mode interference on global studies, in Figure 12a we plot the median residual path-averaged phase-velocity anomaly determined from the global 100-s phase-delay data set of Ekström (2011). As with the analogous USArray calculation, the measured anomalies are expressed relative to predictions obtained from integrating the GDM52 global map along the great-circle path. In general the median residuals are comparable in size to the USArray residuals ( $\approx 0.2\%$ ). However, unlike the USArray residuals, the global data set

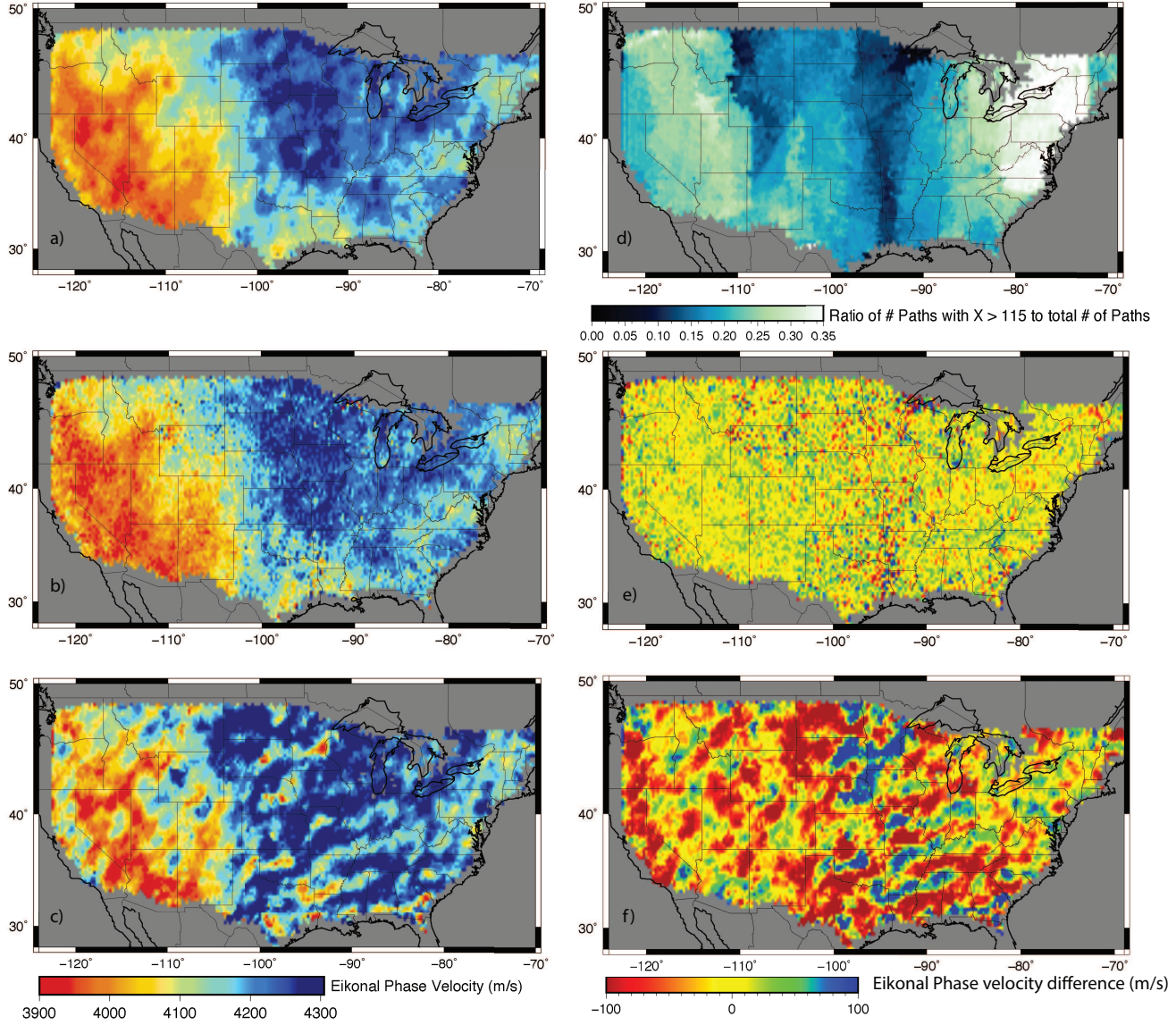


Figure 2.13: Eikonal phase velocities for 100 s Rayleigh waves calculated from the database of Babikoff and Dalton (2019) for measurements grouped into different subsets based on path length. (a) All measurements are used. (b) Only measurements with path lengths  $< 115$  degrees are used. (c) Only measurements with path lengths  $> 115$  degrees are used. (d) Hit count map showing proportion of all measurements with path lengths  $> 115$  degrees. (e) Difference between panels (b) and (a). (f) Difference between panels (c) and (a).

does not show evidence of a larger mismatch between observations and predictions for path length  $> 120$  degrees as might be expected for major-arc higher-mode interference. On the other hand, Figure 12c hints at larger errors in the global data set for

paths longer than 120 degrees. It shows the standard deviation of all measurements in each one-degree distance bin. All bins in the distance range 20-160 degrees contain at least 500 measurements, and this calculation utilizes measurements that have not been corrected for propagation through the GDM52 map. The data reveal a linear decline in standard deviation with distance for path lengths  $< 110$  degrees. A larger spread in measurements for shorter paths is expected, since they can be confined to regions with relatively uniform seismic properties— for example, some short paths will travel only through continents whereas others will travel only through oceans. Long paths necessarily integrate across a range of tectonic settings, which will tend to homogenize the path-averaged phase velocities. The linear decline is interrupted for path lengths  $> 110$  degrees. This may indicate larger uncertainties in these measurements due to higher-mode interference.

Since our results suggest that interference varies as a function of path length, we explore the question of whether measurements made at different locations on Earth are more prone to interference, as the distribution of path lengths sampled at any position on the Earth varies due to the global distribution of earthquake sources. Figure 14a shows the proportion of events with path lengths  $> 115$  degrees. The 971 earthquakes used for this calculation are from the Global CMT catalog (Ekström *et al.*, 2012; Dziewonski *et al.*, 1981) for the years 2013, 2014 and 2015 with  $M_w > 5.5$  and depth  $< 70$  km. For a hypothetical station located along the southern Mid-Atlantic Ridge, roughly 50% of the shallow earthquakes it records will be located  $> 115$  degrees away, and we predict that measurements of the minor-arc Rayleigh waves will therefore be more susceptible to higher-mode interference. Plans for future deployments of ocean-bottom seismometers may want to take this into consideration. Figure 14a also shows, however, that most regions of the Earth are not dominated by long paths, and thus Rayleigh wave measurements in those regions will experience only a nominal impact from interference bias. Figure 14b shows that global Rayleigh



wave data sets do contain these regional variations in path length to earthquakes. We calculate epicentral distance for each event-station pair in the data set of Ekström (2011) and determine the fraction of events located  $> 115$  degrees away from each station. Figure 14b shows that for stations located along the western edge of the Atlantic basin, roughly 50% of the measurements could be prone to interference.

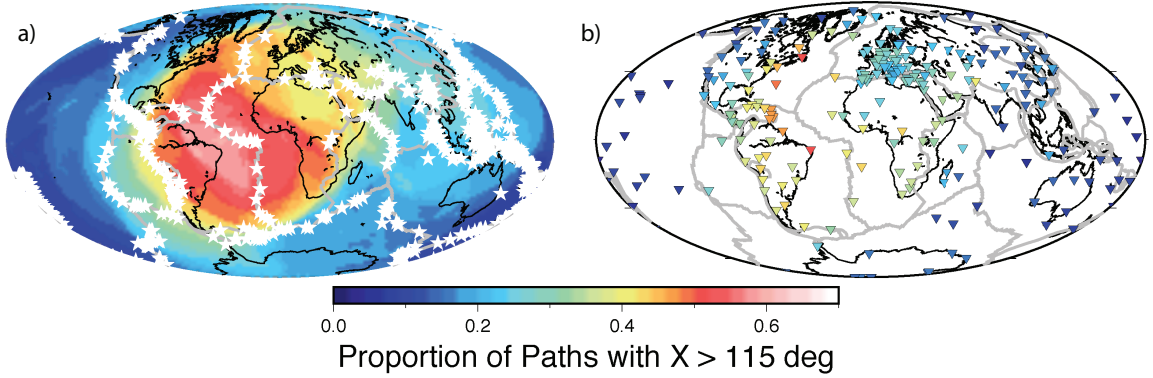


Figure 2.14: (a) Map view showing, in each pixel, the number of events located  $> 115$  degrees away from each pixel, plotted as a fraction of the total number of events considered in the calculation. White stars show earthquakes used in the calculation, which are from the Global CMT catalog with depth  $< 70$  km and  $M_w \geq 5.5$  for the years 2013-2015. Tectonic plate boundaries from Bird (2003) are overlain. (b) As in (a), but calculated using only the source-station event pairs used in the construction of the GDM52 model at 100 s for Rayleigh waves (Ekström, 2011)

Overall we expect that regional and global phase-velocity maps will be minimally affected by the higher-mode interference. First, most data sets are dominated by path lengths  $< 120$  degrees. In addition, the fact that the oscillations vary with distance and have a short wavelength means that systematic bias is unlikely to result given a large number of measurements. Second, for phase-velocity inversions that use path-integrated travel times as data, the  $\pm 5$  second perturbation introduced by higher-mode interference (Figures 2 and 10) will not result in a large bias in inverted phase velocity. Damping and smoothing, which are traditionally applied to these inversions, will help to suppress any streaks or banding. On the other hand, phase-velocity inversions based on travel-time differences between pairs or small groups of nearby stations (Lin *et al.*, 2009; Foster *et al.*, 2014) are more susceptible to bias from

higher-mode interference.

Furthermore, travel-time and amplitude measurements that are strongly biased by the interference may be removed as part of quality control procedures, a notion discussed by Thatcher and Brune (1969) in the context of Love wave interference. An additional consideration is that measurements of fundamental-mode Rayleigh waves on the major arc (R2) may be biased by interference with higher modes traveling on the minor arc after completing one orbit (often termed *X3* phases), and the group velocities in Figure 1a suggest that the interference for R2 will occur over a much wider range of epicentral distances ( $\approx 20 - 90$  degrees) than the minor-arc interference considered here. Finally, we note that our study has used 100 s Rayleigh waves to document the phenomenon of higher-mode interference and describe its general characteristics. Measurements made at other periods also show evidence of the interference. In both Eikonal velocity (Figure S1) and amplitude (Figure S2), it is clear that the wavelength of the interference pattern decreases with decreasing period and that the onset of the strongest interference is consistently around 120 degrees.

Future work will investigate how the earthquake source, including its moment tensor and depth, the velocity structure, and the period affect the characteristics of higher-mode interference. Major-arc overtone interference also holds the potential for studies to image higher-mode wave propagation through a unique lens, as the interference signal preserves information about the phase velocity and amplitude of the higher modes while the large distance range of the major-arc phase ensures the individual mode branches are well separated in time. Previous studies (Forsyth, 1975a; Thatcher and Brune, 1969) have demonstrated the potential to model the relative amplitude and phase velocity of individual modes in a travelling-wave signal using the superposition of plane waves, suggesting that the same principle can be applied to the wavefield identified here. The ability to extract information about overtone phase velocity and amplitude would allow new investigations in the velocity



and attenuation structure of the mantle transition zone and uppermost lower mantle.

## Conclusion

We show that interference of Rayleigh wave major-arc overtones with the minor-arc fundamental mode can be detected in travel-time and amplitude measurements made on synthetic and real waveforms. This interference manifests as oscillations about the underlying value along a ray path. Comparing synthetic seismograms computed by normal-mode summation with a full mode catalog and with only the fundamental modes shows that the higher modes traveling along the major arc overlap with the fundamental mode along the minor arc, thus producing the interference. The interference is most pronounced for relatively long paths (epicentral distance  $> 120$  degrees) and is characterized by a wavelength of  $\approx 3$  degrees at 100 s.

The interference is most easily observed with dense arrays of seismometers, and we have found examples of it in real data recorded by the EarthScope USArray stations. The impact of the interference on estimates of Rayleigh wave phase velocity will be larger for approaches based on relative travel times between stations than for approaches that use path-integrated travel times, such as most global studies. Overall, we expect that the impact of the interference will be small since most data sets are dominated by measurements from path lengths  $< 120$  degrees. We show that, as a consequence of the global distribution of earthquakes, measurements made at stations in and around the Atlantic Ocean are more susceptible to interference bias. Looking forward, modeling various characteristics of the interference pattern, including its distance dependence, wavelength, and amplitude, may allow the phase and group velocities and amplitudes of higher-mode Rayleigh waves to be measured.

## 2.6 Acknowledgments

The authors thank Kazunori Yoshizawa and an anonymous reviewer for helpful and constructive suggestions and reviews that improved the quality of this manuscript. We also thank Don Forsyth and others in the Brown University Seismology and Geophysics Groups for helpful discussions and suggestions. Some figures were made using the generic mapping tools (Wessel and Smith, 1998). We also thank the group at Princeton University maintaining the ShakeMovie Portal for making synthetic seismograms accessible. This work has been funded by NSF grant EAR-1553367 to C.A.D.

Seismograms recorded at USArray stations can be downloaded from the IRIS DMC.

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## 2.8 Supplemental Material

This supporting information contains supplementary figures referred to in the main text that explore the period dependence of the interference pattern from Rayleigh-wave overtone interference. The phase velocities (Fig S1) and corrected amplitudes (Fig S2) shown here are processed and calculated as discussed in the methods section of the main text.

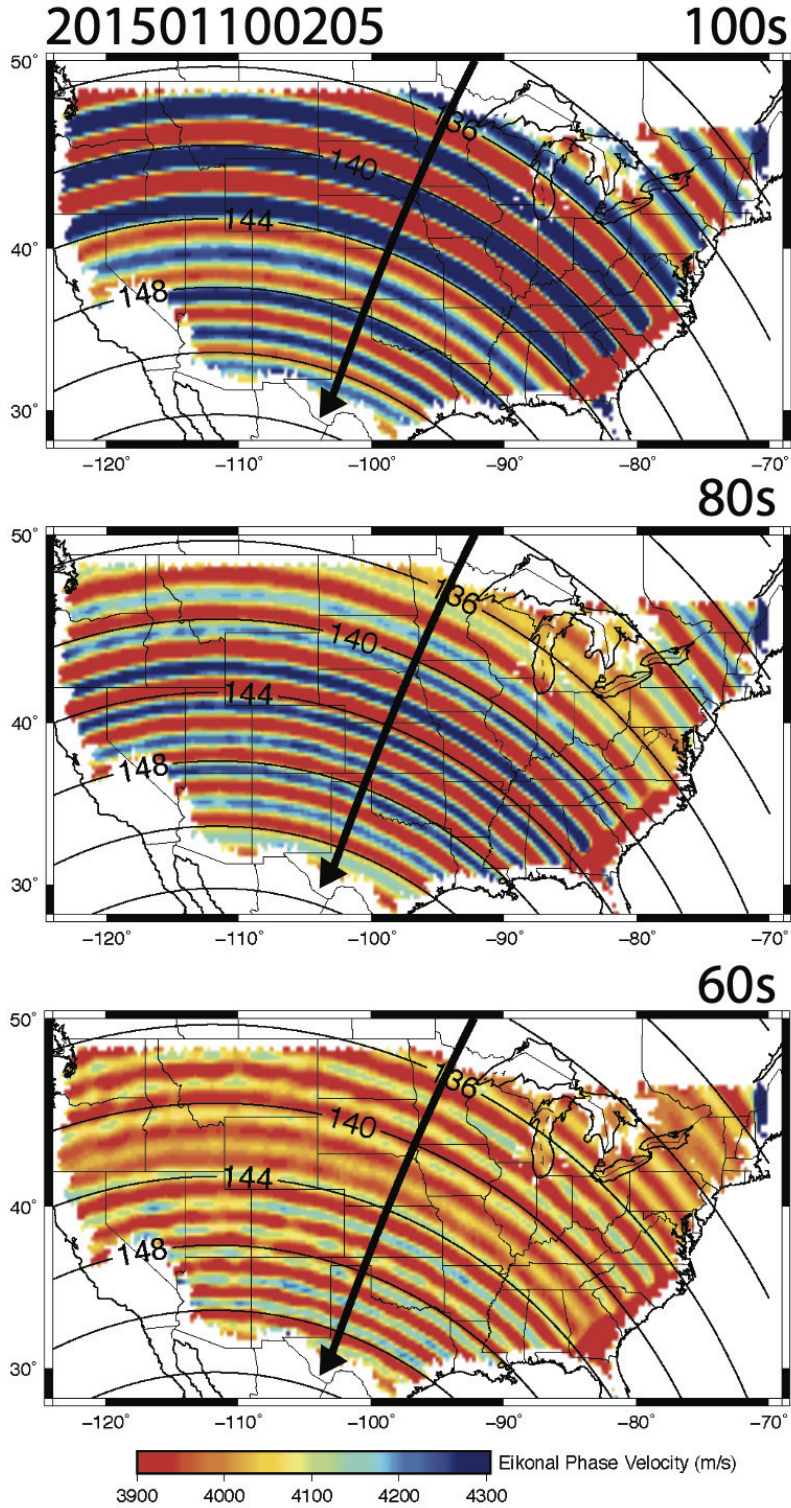


Figure 2.15: **Figure S1:** Map view plots of Rayleigh wave Eikonal phase velocity measured at periods of 100s, 80s, and 60s, for event 201501100205A (as in Figure 8a). The seismograms are generated with normal-mode summation using the full mode catalog. Labeled contours of constant epicentral distance are overlain, spaced 4 degrees apart.

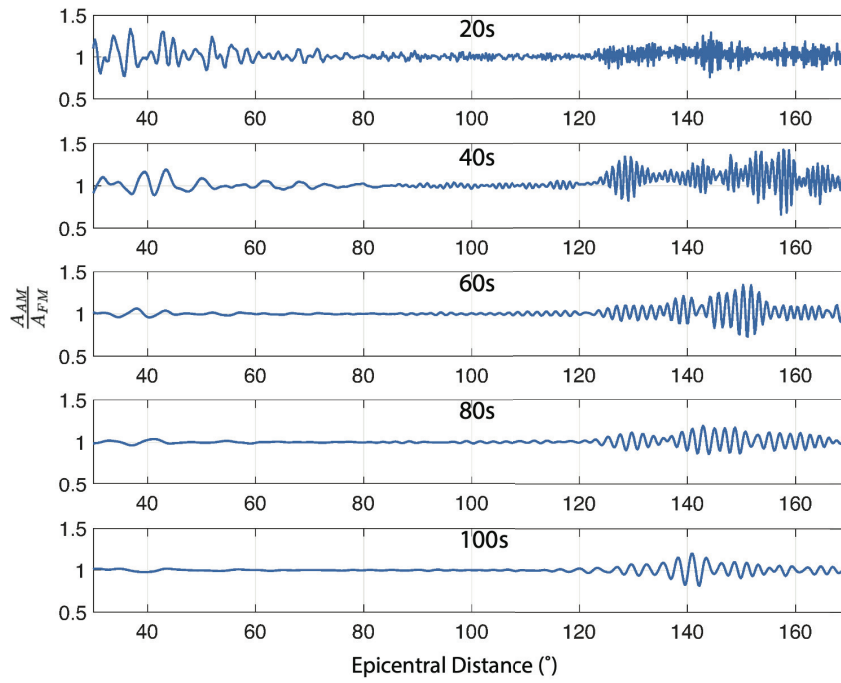


Figure 2.16: **Figure S2:** Plots of corrected amplitude (as in the top panel of Figure 10) for the same event as in Figure 9, measured at different periods as labeled in each panel.

# Chapter 3 | Controls on Surface Wave Overtone Interference

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### 3.1 Abstract

Measurements of fundamental-mode (FM) surface waves along the minor arc are impacted by overtone interference. This interference is primarily due to major-arc overtones for Rayleigh waves and minor-arc overtones for Love waves. In both cases, interference contaminates measurements of phase and amplitude and can introduce bias in seismic images. Here, we use synthetic seismograms computed via normal mode summation to probe how interference can vary as a function of the surface-wave group velocities, source mechanism and depth, which control the relative excitation of the FM and overtones, and period. By comparing seismograms that include all overtones to those that include only the FM, we can quantify the interference, i.e., how the presence of the overtones perturbs the FM phase and amplitude. We compare the strength of this interference to calculations of excitation of the overtones and FM. We show that these calculations explain well the varying strength of interference for different source mechanisms and depths. Notably, these calculations illuminate source depths where Love wave overtone excitation is quite low and therefore interference is unusually weak, and depths where Rayleigh wave FM excitation is low and therefore interference is unusually strong. Our analysis also reinforces the dependence of the interference on the FM and overtone group velocities. For Love waves, this results in weak minor-arc overtone interference at long periods and, for continental paths, short periods. For Rayleigh waves, the differing overtone and FM group velocities and the relative excitation of the overtones and FM explain rapid variations in the strength of Rayleigh wave major-arc overtone interference as a function of epicentral distance. We then show that real data are affected by the relative excitation of the FM and overtones. We find that errors in Rayleigh wave phase velocities determined at the EarthScope USArray stations are larger when the ratios of overtone to FM excitation are larger. We also find a dependence of phase velocity error on excitation ratio for

Love waves, and we identify the presence of major-arc overtone interference in Love wave measurements. Our results highlight opportunities for more nuanced quality control of surface-wave measurements. The relative excitation ratio of the overtones and FM may be a better criterion for event selection than source depth, allowing deeper events that well excite FMs to be included and shallower events with large overtone excitation to be excluded. This would allow the collection of more accurate measurements that will increase the precision of seismic images.

### 3.2 Introduction

Measurements of the phase and amplitude of fundamental-mode (FM) Rayleigh and Love waves are critical to our understanding of the seismic structure of the upper mantle since both types of surface waves have a period-dependent sensitivity to shear velocity (e.g. Nettles and Dziewoński, 2008). Secondary observables calculated from these surface-wave measurements include phase velocities (e.g. Lin and Ritzwoller, 2011; Eddy *et al.*, 2018), H/V ratios (e.g. Tanimoto and Rivera, 2008), and attenuation (e.g. Dalton and Ekström, 2006), all of which inform models of the Earth’s physical, chemical, and anisotropic properties (e.g. Dalton *et al.*, 2017; Altoe *et al.*, 2020; Russell *et al.*, 2019). Improving the quality and quantity of surface-wave observations thus has the potential to augment the accuracy and resolution with which we can image the interior of the Earth.

It is well established that measurements of FM surface-wave phase and amplitude along the minor arc can be biased by higher-mode, or overtone, interference, especially for FM Love waves. This interference is manifest as oscillations in phase and amplitude as a function of epicentral distance, which can then propagate into secondary observables such as phase velocity (e.g. Thatcher and Brune, 1969; Foster *et al.*, 2014; Hariharan *et al.*, 2020). Interference thus poses a challenge to obtaining accurate seismic images, and it is a particular challenge for array-based studies that seek high-resolution models, as demonstrated by Foster *et al.* (2014) and Hariharan *et al.* (2020).

Overtone interference with FM Love waves arises because of the closeness of FM and overtone group velocity, especially at periods  $< 100$  s and for propagation through an oceanic-type velocity model with a well defined low-velocity zone (Fig. 1a,b). Early studies described the oscillatory character of Love wave overtone interference (Thatcher and Brune, 1969; Boore, 1969), showed it was a particular problem in

oceanic settings (Thatcher and Brune, 1969; Yoshida, 1983), and modeled the wave-field as interference between individual modes (Forsyth, 1975). In the 1990s, there was debate about the impact of overtone interference on estimates of radial anisotropy (Polet and Kanamori, 1997; Maupin and Cara, 1992); Nettles and Dziewoński (2011) conclusively showed that overtone interference did not introduce measurable bias into global models of radial anisotropy. Foster *et al.* (2014) demonstrated that the strong distance dependence of overtone interference introduces errors of up to 20% into Love wave phase velocities estimated using differential travel times between nearby stations. Overall, overtone interference remains arguably the largest challenge impeding widespread use of earthquake-derived Love waves in inversions for seismic models; however, few studies have rigorously examined how Love wave overtone interference varies with period or the properties of the earthquake source (e.g. Yoshida, 1983).

For Rayleigh waves, the interference primarily results from overtones on the major arc arriving in the same window as the FM on the minor arc; minor-arc overtones can also cause interference in the special case that their group velocity is close to the FM group velocity. The impact of overtone interference on Rayleigh wave measurements has only recently been recognized (Hariharan *et al.*, 2020), although Tanimoto and Rivera (2008) observed that major-arc overtones complicated their H/V measurements.

Hariharan *et al.* (2020) noted several differences between Rayleigh and Love wave interference. One, the Rayleigh wave interference exists only at epicentral distances where the minor-arc FM and major-arc overtone intersect (generally  $> 120^\circ$ ), and the strength of the interference varies strongly with distance. In contrast, the Love wave interference is both present and relatively constant over a broad distance range. This difference can be understood from considering group velocity: unlike the overlapping Love wave group velocities, the well separated Rayleigh wave group velocities cause the higher modes to pass into and out of the FM window. Two, the Rayleigh



wave interference pattern has a shorter wavelength ( $\approx 3^\circ$  at 100 s) than the Love wave interference pattern ( $\approx 20^\circ$  at 100 s). This can be understood from considering the difference between the FM and overtone phase velocities, which governs the interference wavelength, with a larger difference causing a shorter-wavelength pattern (Thatcher and Brune, 1969; Foster *et al.*, 2014). Since the major-arc overtones are traveling in the opposite direction from the minor-arc FM, the apparent phase-velocity difference is large.

Although the study of Hariharan *et al.* (2020) showed that the characteristics of the Rayleigh wave interference pattern vary strongly with period, it focused mainly on a single period (100 s). It also did not examine which factors control the strength of the Rayleigh wave interference as a function of epicentral distance or by event. The excitation of normal modes can be calculated for any source mechanism and depth (Dziewonski *et al.*, 1981; Dahlen and Tromp, 1998), but previous studies have not comprehensively explored how these effects propagate into surface wave overtone interference, which depends on the excitation of both the FM and overtones. Moreover, earlier studies have not probed the role of the Earth model, which impacts interference in two ways. One is via the source excitation, which depends on the Earth model through the displacement eigenfunctions, and the other is due to the path-integrated group velocity, which dictates the arrival times of the FM and overtones.

Here, we build on the initial study of Hariharan *et al.* (2020) by rigorously and systematically investigating overtone interference effects for both Rayleigh and Love waves. We use tests with synthetic seismograms to quantify and understand how the strength of interference depends on period, source depth, source mechanism, and velocity model. We also show that the overtone interference, including its dependence on source excitation, can be detected in real surface-wave data measured with the EarthScope USArray. Our findings suggest a path to improving data sets of surface-wave measurements by using the relative excitation of overtones as an event-selection

criterion, thereby improving the quality of our images of the Earth’s interior.

### 3.2.1 Theoretical Background

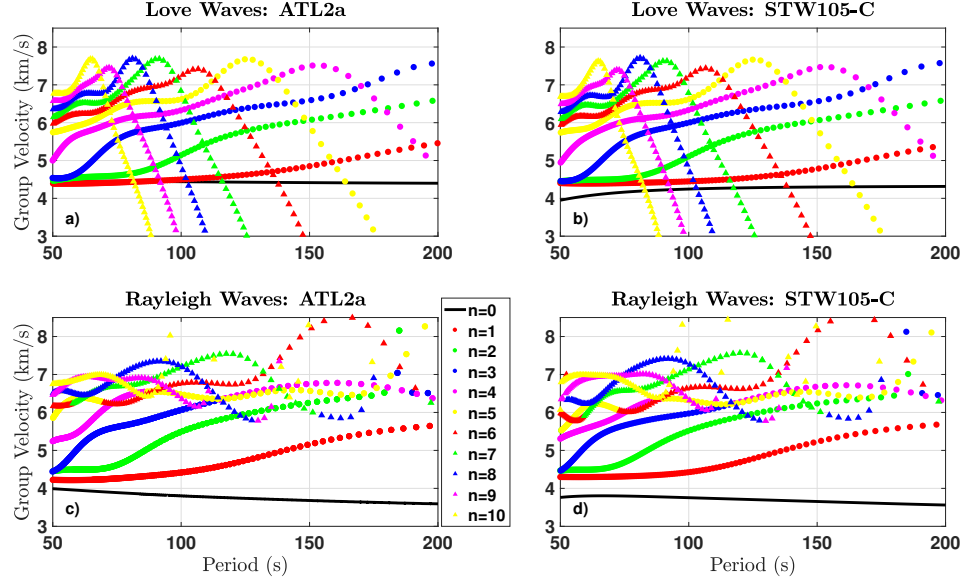


Figure 3.1: Group velocities for the fundamental mode ( $n = 0$ ) and first ten overtones for (a) Love waves in an oceanic model, (b) Love waves in a continental model, (c) Rayleigh waves in an oceanic model, (d) Rayleigh waves in a continental model. Each color/symbol combination corresponds to a different radial order  $n$ .

The fundamental requirement for overtone interference is that the group arrival times of the FM and the overtone overlap. Fig. 1 shows the group velocities of toroidal and spheroidal normal modes for radial orders  $n = 0 - 10$ . The focus of this paper is propagating Love and Rayleigh waves, and in what follows we directly relate the overtone number or branch of the traveling wave to the radial order of the equivalent normal mode. Thus, the first overtone branch corresponds to  $n = 1$  and so forth. We show group velocities for both continental and oceanic settings, using an Earth model we refer to as STW105-C, which is STW105 (Kustowski *et al.*, 2008) modified to have a 42-km-thick crust, for the continental setting and ATL2a (James *et al.*, 2014) for the oceanic setting.

At a given period, group velocities generally increase with overtone number  $n$ , although this relationship is not apparent for Rayleigh waves with  $n > 4$ . Rayleigh wave overtone group velocities that exceed the vertical scale correspond to Stoneley modes, which have sensitivity near the core-mantle boundary and are not well excited by the sources discussed in this study. Similarly, the high- $n$  Love wave group velocities that decrease abruptly at longer periods correspond to the core-reflected  $ScS_{SH}$  phase (Dahlen and Tromp, 1998), and their overlap with the FM does not represent a significant contribution to overtone interference due to the low excitation of these modes.

Fig. 1 shows that the requirement of group arrival time overlap between an overtone and the FM is fulfilled for Love waves at periods less than 120 s, for which the group velocities of the FM and the first overtone are nearly the same. For continental settings, the Love wave group velocities of the first overtone and FM are close at intermediate periods but are well separated at periods less than 60 s (Nettles and Dziewoński, 2011). For the oceanic model, the first, second and perhaps even third Love wave overtones overlap with the FM at the shortest periods. On the other hand, for Rayleigh waves the group arrival times of the minor-arc overtones and minor-arc FM are consistently well separated due to large differences in the group velocities of the overtone and FM branches, except at the shortest periods for oceanic paths.

The large difference in Rayleigh wave group velocities between the FM and overtones causes the major-arc overtones to intersect the FM at intermediate to large epicentral distances, thereby contaminating FM measurements at those distances. The epicentral distance at which this intersection takes place can be estimated using Equation 1, which we arrived at by equating the group arrival times of the minor-arc fundamental-mode and major-arc overtone, where  $U_0$  and  $U_1$  are the group velocities of the FM and overtone of interest at a specific period, in km/s,  $R_e$  is the radius of the Earth in km, and  $X_{intersect}$  is the epicentral distance (in km) at which the overtone

and FM intersect.

$$X_{intersect} = \frac{2\pi R_e U_0}{U_1 + U_0} \quad (3.1)$$

Fig. 2 plots  $X_{intersect}$  for Rayleigh waves, calculated using the oceanic group velocities, as a function of period and overtone number. At shorter periods, the intersection distances tend to be slightly larger as a result of the generally slower overtone group velocities, whereas at longer periods the intersection distances are mostly concentrated between 115-160°.

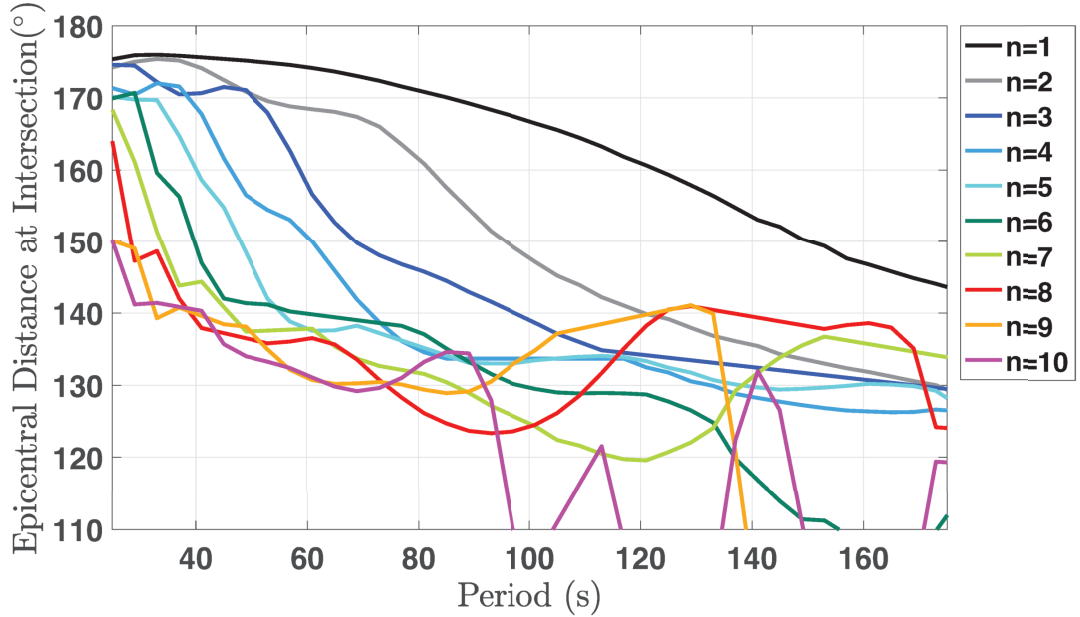


Figure 3.2: Epicentral distance at which the Rayleigh wave minor-arc FM and major-arc overtone intersect, calculated using Equation 1 for the oceanic model ATL2a. Each color corresponds to a different radial order  $n$ . For ease of visualization, Stoneley modes are omitted in this figure.

When the FM and overtone group arrival times overlap, interference can occur. Forsyth (1975) and Thatcher and Brune (1969) presented the following expressions for the effect of two-plane-wave interference on surface-wave phase  $\phi$  (in radians) and amplitude  $A$

$$\phi(\omega, x) = \tan^{-1} \left( \frac{\sin(\omega x/c_0) + \delta a \sin(\omega x/c_1)}{\cos(\omega x/c_0) + \delta a \cos(\omega x/c_1)} \right) \quad (3.2a)$$

$$A(\omega, x, t) = (1 + \delta a^2 + 2\delta a \cos[(\omega/c_0 - \omega/c_1)x])^{\frac{1}{2}} e^{-i(\omega t)} \quad (3.2b)$$

where  $\delta a$  is the amplitude ratio of the overtone to FM,  $c_0$  and  $c_1$  are the FM and overtone phase velocities, respectively,  $\omega$  is angular frequency,  $x$  is epicentral distance, and  $t$  is time. From Equation 2b it is clear that the wavenumber of the interference pattern multiplies  $x$  inside the cosine term and thus the wavelength of the interference pattern can be rewritten as:

$$\lambda = \frac{T}{1/c_0 - 1/c_1}, \quad (3.3)$$

where  $T$  is period. However, the apparent phase velocity of a major-arc overtone is negative, so the expression for the wavelength of the interference pattern in the case of major-arc interference is:

$$\lambda_{major} = \frac{T}{1/c_0 + 1/c_1}. \quad (3.4)$$

The amplitude ratio  $\delta a$  is critical to the strength of interference, and is governed by the relative source excitation of the FM and overtone surface waves. The Rayleigh wave source amplitude  $R$  and Love wave source amplitude  $L$  are given in equation 11.34 in Dahlen and Tromp (1998):

$$\begin{aligned} R = & |\omega[(M_{rr}\dot{U}_s + (M_{\theta\theta} + M_{\phi\phi})r_s^{-1}(U_s - \frac{1}{2}kV_s))e^{i\pi/4} \\ & + (-1)^q(\dot{V}_s - r_s^{-1}V_s + kr_s^{-1}U_s)(M_{r\phi}\sin\Psi + M_{r\theta}\cos\Psi)e^{-i\pi/4} \\ & - kr_s^{-1}V_s[M_{\theta\phi}\sin(2\Psi) + \frac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\cos(2\Psi)]e^{i\pi/4}]| \end{aligned} \quad (3.5)$$

$$L = |\omega((-1)^q(\dot{W}_s - r_s^{-1}W_s)(M_{r\theta}\sin(\Psi) - M_{r\phi}\cos(\Psi))e^{-i\pi/4} - kr_s^{-1}W_s[\frac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\sin(2\Psi) - M_{\theta\phi}\cos(2\Psi)]e^{i\pi/4})| \quad (3.6)$$

where  $\omega$  is the angular frequency,  $k$  is the wavenumber,  $M_n$  represents the components of the moment tensor,  $r_s$  is the source radius,  $\Psi$  is the azimuth measured counterclockwise from south,  $q$  is the wavegroup index, and  $U_s$ ,  $V_s$ ,  $W_s$ ,  $\dot{U}_s$ ,  $\dot{V}_s$ , and  $\dot{W}_s$  are the normalized displacement eigenfunctions and their derivatives, respectively, evaluated at the source depth.

### 3.3 Data and Measurements

#### 3.3.1 Normal-Mode Synthetic Seismograms

We explore the extent of overtone interference for a range of example source mechanisms and depths. We use synthetic seismograms calculated for a known 1-D Earth model via normal-mode summation (Gilbert, 1971). The mode eigenfunctions and eigenfrequencies are computed with the MINEOS code (Masters *et al.*, 2011). The summation includes all modes with frequency  $< 50$  mHz and angular order  $< 500$ . The advantage of mode summation is that it is computationally cheap and we can control which overtone branches (i.e., which values of  $n$ ) are included in the seismograms. For Love waves, the source excitation and group velocity (Fig. 1) of the FM and overtones differ substantially for oceanic and continental models, and we repeat all calculations using ATL2a and STW105-C. For Rayleigh waves, the differences are minor, and we perform simulations using ATL2a only.

We calculate synthetic seismograms for three example focal mechanisms and four source depths. We use focal mechanisms corresponding to a vertical strike-slip fault ( $\delta = 90^\circ$ ,  $\lambda = 0^\circ$ ), a dip-slip fault dipping at  $45^\circ$  ( $\delta = 45^\circ$ ,  $\lambda = 90^\circ$ ), and a dip-slip fault dipping at  $90^\circ$  ( $\delta = 90^\circ$ ,  $\lambda = 90^\circ$ ), where  $\delta$  is the dip angle and  $\lambda$  is the slip

angle. We will refer to the latter as a vertical dip-slip fault. We use source depths of 12, 62, 112, and 162 km. In addition to these three example fault types, we have also considered a low-angle thrust fault and high-angle normal fault. We found that they produced nearly identical results as the  $45^\circ$  dip-slip fault and therefore do not discuss them further. When calculating the synthetic seismograms, stations are placed along a great-circle path at approximately  $1^\circ$  spacing. For consistency, the position of the stations changes for each event and is chosen so that the azimuth of the linear array corresponds to a maximum in the FM radiation pattern for the source of interest. In Section 3 we comment on how our results depend on the take-off angle.

We quantify the impact of overtone interference on the measurements of FM travel time and amplitude following the approach of Hariharan *et al.* (2020) and Foster *et al.* (2014). We generate two sets of synthetic seismograms for every station: one includes the FM and overtones, and the other includes only the FM. FM travel time and amplitude are measured for both sets (Section 2.2). The measurements made on synthetic seismograms that contain overtones and the FM are expressed relative to their values measured from the synthetics containing only the FM; the travel times are differenced and the amplitudes are divided. In doing so the effects of the source radiation pattern, attenuation, and geometrical spreading will be cancelled out, allowing the effects of higher-mode interference to be isolated. We refer to these representations of the measurements as “the travel-time interference pattern” and “the amplitude interference pattern”. Deviations of the travel-time interference pattern from zero and of the amplitude interference pattern from one are evidence of overtone interference effects.

### 3.3.2 Measurement Methods and Phase Velocity Calculations

To make measurements of the travel time and amplitude of the Rayleigh and Love waves in the normal-mode synthetic seismograms, we use Fourier analysis (Forsyth

and Li, 2005). With this approach, a window is defined based on the expected surface-wave arrival. The window is centered on the predicted FM group arrival time (Fig. 1), and its length is equal to twice the period of the wave. A Tukey taper (tapered cosine) is applied to the center half of the window. We have experimented with larger window lengths and find that both the amplitude and travel-time measurements and the resulting interference patterns depend negligibly on the window length apart from a very minor increase in interference as window length increases for Rayleigh waves. After windowing, the seismograms are bandpass filtered with a filter width equal to one-tenth of the center period. After taking the Fourier transform of the windowed and filtered seismogram, we extract the phase from the angle corresponding to the ratio of the real and imaginary components. The phase is unwrapped after sorting with increasing epicentral distance so that it increases with distance instead of being restricted to the range of the arctangent function. The modulus of the real and imaginary components is used to estimate amplitude.

We have found that for almost all waveforms, this Fourier approach and the Automated Surface Wave Measuring System (Jin and Gaherty, 2015), which we have previously used to measure Rayleigh waves on real array-based data, yield nearly identical measurements of surface-wave phase and amplitude. For waveforms characterized by very strong overtone interference, the two approaches can diverge, presumably because the FM is weak and the window is instead dominated by the overtone (Fig. S1).

We quantify how overtone interference impacts phase velocity using Eikonal tomography (Lin *et al.*, 2009), which applies the Eikonal equation (Equation 7) to the measured travel-time field  $\tau$ ,

$$\frac{1}{c^2} = \nabla\tau \cdot \nabla\tau \quad (3.7)$$

where  $c$  is 2-D phase velocity. In practice, the travel-time measurements are first averaged in  $0.5^\circ$  cells, then interpolated onto a smooth surface. We use the algorithm



of Smith and Wessel (1990) implemented in GMT (Wessel and Smith, 1998), which solves for a minimum-curvature surface under tension. We interpolate onto a grid of  $0.25^\circ \times 0.25^\circ$  cells. The gradient term is calculated using a finite-difference method in spherical coordinates. This process is performed separately for each event, and the event-specific Eikonal phase-velocity maps are determined from the reciprocal of  $\sqrt{|\nabla\tau|^2}$ .

## 3.4 Results of Tests with Synthetic Seismograms

In this section, we use measurements on normal-mode synthetic seismograms to characterize patterns in the strength of overtone interference for Rayleigh and Love waves. We examine to what extent these patterns can be explained by the relative excitation of the FM and overtones.

### 3.4.1 Love Wave Interference

For Love waves, the relative excitation of the FM and first overtone is often considered to exert first-order control on interference strength, as a result of the closeness of the group-velocity curves. Fig. 1a shows that at short periods the second and even third overtone may also influence interference for an oceanic velocity model; this is explored later in this section. Using Equation 6, we calculate excitation as a function of source depth for the FM and first overtone for the three example source mechanisms described in Section 2.1. Fig. 3 shows the ratio of the excitation of the first overtone and FM, where larger values correspond to larger relative excitation of the first overtone. Fig. S2 shows the raw excitation as a function of depth and at different periods. Whenever excitation ratio is referred to or shown in this paper, the excitation of the FM is in the denominator.

The excitation ratios for strike-slip and 45-degree dip-slip faults are identical, although the magnitudes of the 45-degree dip-slip excitation are larger. For both of

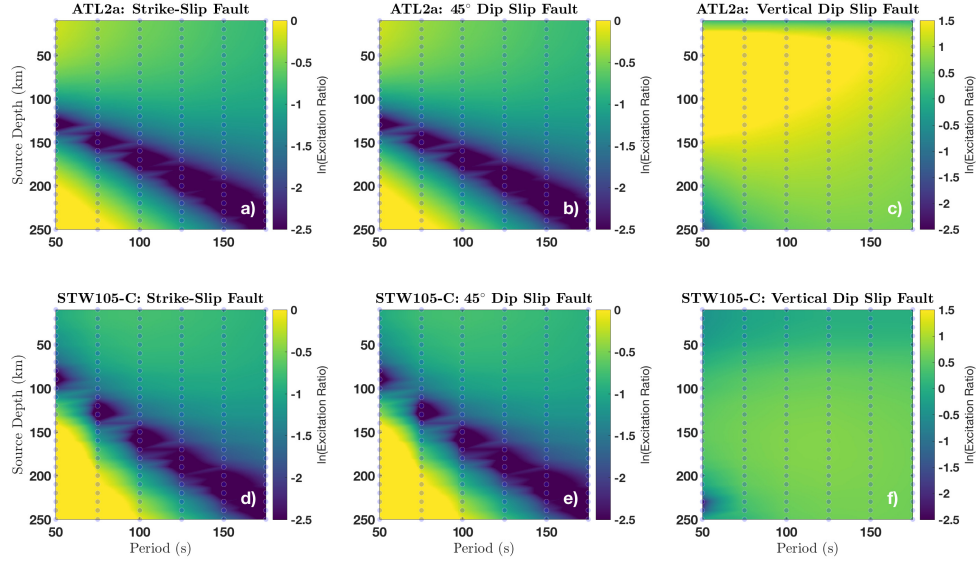


Figure 3.3: Ratio of Love wave excitation: first overtone excitation with respect to the FM excitation. (a-c): Calculations for ATL2a. (d-f): Calculations for STW105-C. Each column is a different example source. The points at which this function is evaluated are plotted as blue dots; these excitation ratios have been interpolated to form the color surface. Note the different range of excitation ratios on the third column as compared to the first and second.

these faults, the excitation of the first overtone exhibits a minimum in excitation at a depth that increases with increasing period, from approximately 125 km at 50 s to around 250 km at 175 s. This minimum also broadens in depth with increasing period. As a result, the excitation ratios for these fault types also contain minima, the depths of which vary systemically with period (Fig. 3). For the vertical dip-slip fault, the excitation of the first overtone is almost always larger than the excitation of the FM, resulting in excitation ratios  $> 1$ .

We have found that variations in the character of overtone interference as a function of source depth, as inferred from normal-mode synthetic seismograms, are well explained by variations in the Love wave excitation ratios. As described in Section 2.1, we computed synthetic seismograms for the three example events at four depths. Fig. 4 shows the amplitude interference patterns that we determined from them. We do not show results for the 45-degree dip-slip event since they are identical to the

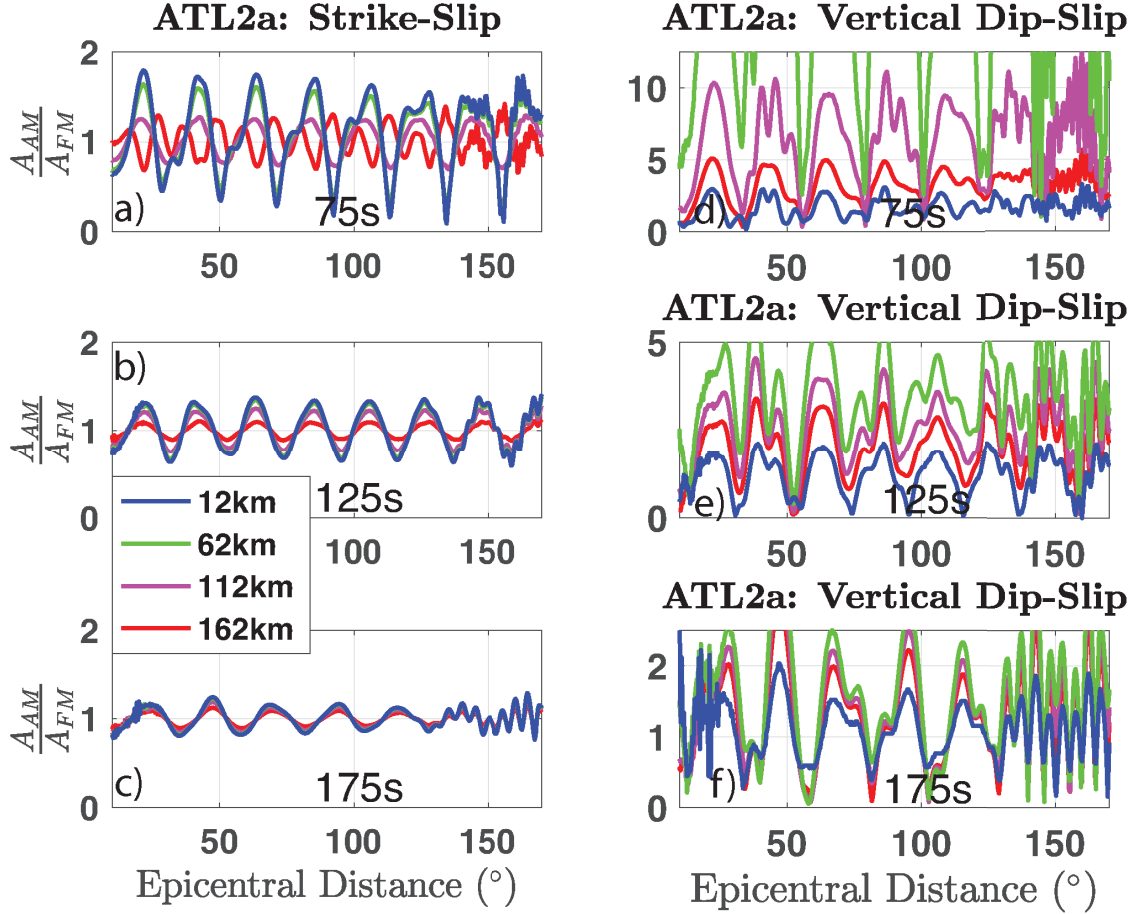


Figure 3.4: Amplitude interference patterns for Love waves, showing the ratio of amplitudes measured from synthetic seismograms containing all modes to those measured from synthetic seismograms containing only the FM. Simulations are for example events and four source depths. Note the different y-axis ranges for panels in the right column.

strike-slip event. Examples of travel-time interference patterns can be found in Fig. S4. At a given period in Fig. 4, all four curves (source depths) have the same group velocity; differences between them are thus due to source excitation. For the strike-slip fault at 75 s, Fig. 3a suggests that overtone interference should be largest for source depths approximately  $< 75$  km and  $> 175$  km, and this is confirmed by amplitude interference patterns: the effect of Love wave overtone interference on amplitude is roughly twice as large for source depths of 12 km and 62 km than for 112 km and 162 km. Fig. 3a also hints that at 125 s the relative excitation of the first overtone is

weak for a strike-slip fault at 162 km, which is corroborated by the small interference effects in Fig. 4b. On the other hand, Fig. 4d-f show clearly that overtone interference is very strong for the vertical dip-slip fault, as expected from Fig. 3c. In particular, measurements at short periods and shallow depths in Fig. 4d show the strongest overtone interference for the vertical dip-slip fault, reflecting correspondingly large excitation ratios at short periods and shallow depths. Fig. 4f shows that at 175 s for vertical dip-slip faults, interference effects are smallest for a source at 12 km, which is consistent with the depth dependence of excitation ratios in Fig. 3c.

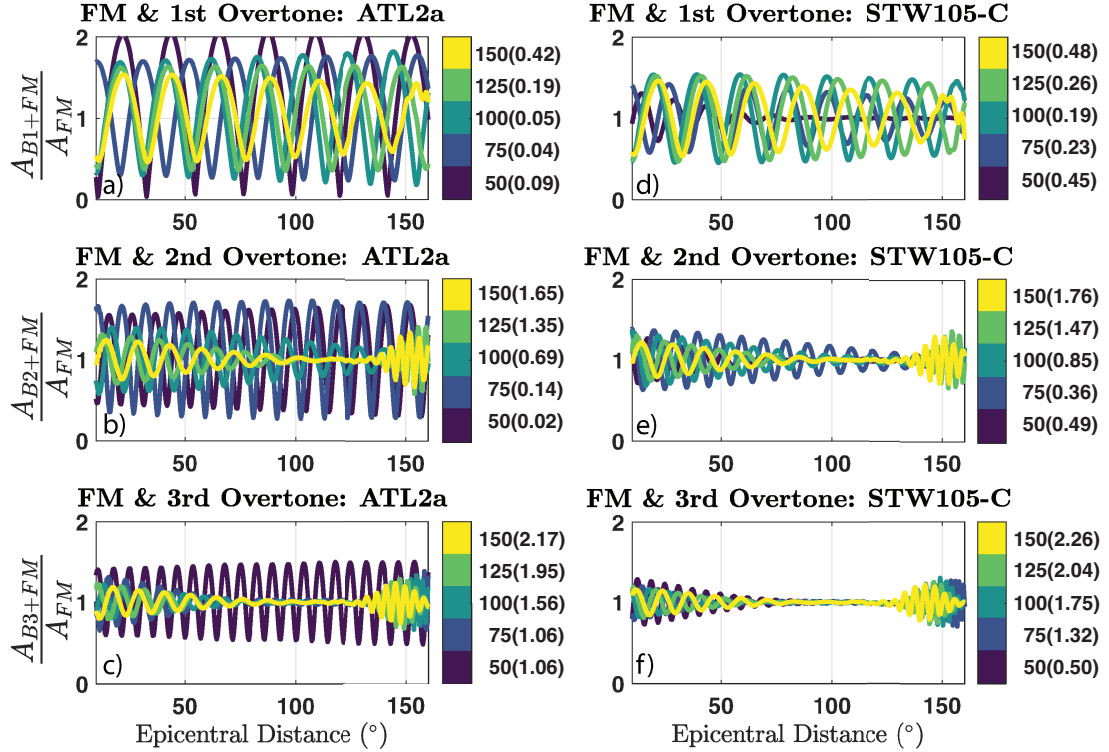


Figure 3.5: Love wave amplitude interference patterns, color-coded by period (s), for the FM and first overtone (a,d), second overtone (b,e), and third overtone (c,f). The absolute difference (in km/s) between the group velocities of the FM and overtone is indicated in parentheses next to the period label. In all cases the source mechanism and depth are chosen such that the ratio of overtone to FM excitation is approximately unity. Thus, differences between the interference patterns in the left-hand and right-hand columns are due entirely to the different group velocities of the oceanic and continental Earth models.

The Love wave interference patterns are affected by the proximity of FM and

overtone group velocities in addition to the relative excitation of the FM and overtones. We explore the role of group-velocity proximity in Fig. 5, where we isolate the interference effects from the first, second, and third overtones, since group-velocity separation generally increases with overtone number (Fig. 1a,b). In each case we choose a source mechanism and depth so that the overtone-to-FM excitation ratio is approximately unity. Fig. 5 makes clear that group-velocity proximity controls both the strength and distance-dependence of interference. If the group velocities are close (within  $\sim 0.25$  km/s), interference strength is constant across the distance range with a magnitude that scales with degree of velocity closeness (Fig. 5a). If the group velocities are better separated, the interference strength depends strongly on distance as the overtone wave packet moves ahead of the FM wave packet. This is illustrated by the interference between the FM and second overtone in Fig. 5b. At the longest periods, interference decays with distance as a consequence of the well separated group velocities. The shortest periods, where group velocities overlap, show stronger interference that varies little with distance. The role of group-velocity separation is especially evident in the comparison of interference patterns computed for the oceanic (Figs. 5a-c) and continental (Figs. 5d-f) velocity models. The short-period group velocities in the continental model are lower and therefore better separated from the overtones than in the oceanic model. As a result, 50-s interference is very weak for STW105-C.

Group-velocity separation has two additional consequences for the Love wave interference patterns in Fig. 4. Firstly, a secondary oscillation in amplitude ratio is superimposed on the dominant signal that is due to interference from the first overtone, which reflects the fact that both the first and second overtones are able to interfere with the FM at short periods (e.g., source depths of 12 km and 162 km in Fig. 4c). Secondly, the short-wavelength signal that modulates the interference pattern at distances  $> 130^\circ$  reflects interference from Love wave overtones traveling

along the major arc.

The velocity model has a second-order effect on the overtone-to-FM excitation ratio (Fig. 3) and therefore a second-order effect on the interference pattern. For example, for the vertical dip-slip fault, the relative excitation of the first overtone is smaller for STW105-C than for ATL2a. As a result, interference is slightly weaker for STW105-C than ATL2a, which can be seen by comparing Figs. 4f and S3l.

In Fig. 6, the effects of velocity model, source depth and mechanism, and period on Love wave interference are summarized using a single quantity, which is the maximum value of the amplitude interference pattern in the epicentral distance range  $30^\circ$ – $90^\circ$ . It illustrates the larger interference for vertical dip-slip faults than strike-slip faults and for the oceanic model than the continental model. The effect of source depth depends on the fault type. For strike-slip faults, the shallowest events produce stronger Love wave overtone interference, whereas for vertical dip-slip faults the deeper events have stronger interference. Although our calculations of synthetic seismograms and excitation ratios are performed for the azimuth corresponding to the maximum in the source radiation pattern, we have found that the Love wave excitation ratios vary little with azimuth, and any variations are typically isolated to azimuths near nodes in the radiation pattern. Thus the results in Figs. 3-6 apply for most take-off angles.

### 3.4.2 Rayleigh Wave Interference

For Rayleigh waves, we must consider many overtone branches, since the interference is not controlled by closeness of group velocities as it is for Love waves, and multiple overtones can intersect the FM at certain distances (Fig. 2). Fig. 7 shows the excitation as a function of depth for the example fault types (Equation 5), calculated for the FM Rayleigh wave and first ten overtones. An important feature of these excitation functions is the minimum in FM excitation for the strike-slip and 45-degree dip-slip faults. The depth of the minimum increases with increasing period. For strike-slip

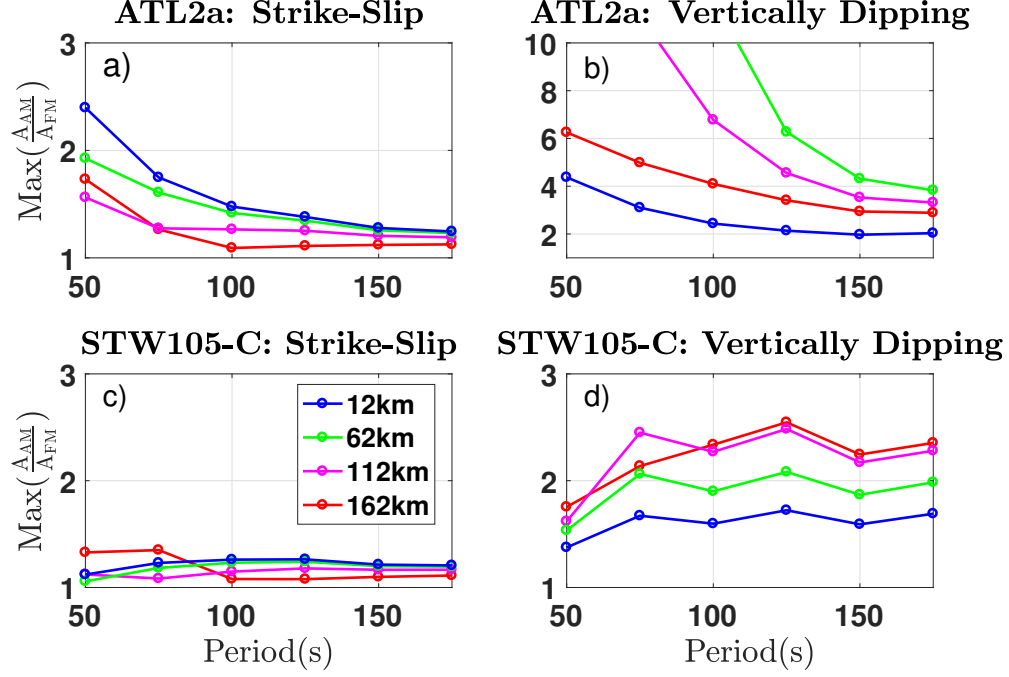


Figure 3.6: Summary of the strength of Love wave amplitude interference as a function of period, velocity model, and source mechanism and depth. Note the different y-axis range on panel b).

faults, the depth of the FM excitation minimum increases from approximately 50 km to 110 km between 50 s and 175 s. For 45-degree dip-slip faults, it increases from approximately 25 km at 50 s to 100 km at 175 s. For vertical dip-slip faults, the FM is more excited than the overtones, except for large depths at short periods. The overtone excitations also contain minima at certain depths, which means that source depth will influence which overtones interfere with the FM at a given period.

Figs. 8 and 9 show that relative excitation of Rayleigh wave overtones and FM exerts strong control on the interference effects in amplitude measurements as determined from normal-mode synthetic seismograms. Fig. 8 focuses on the distance range where major-arc overtone interference dominates, and Fig. 9 focuses on the distance range where minor-arc interference dominates. Unlike the Love wave amplitude interference patterns in Fig. 4, we show only the top half ( $\frac{A_{AM}}{A_{FM}} > 1$ ). It is clear from Fig. 8 that the strength of major-arc overtone interference varies dramatically as a

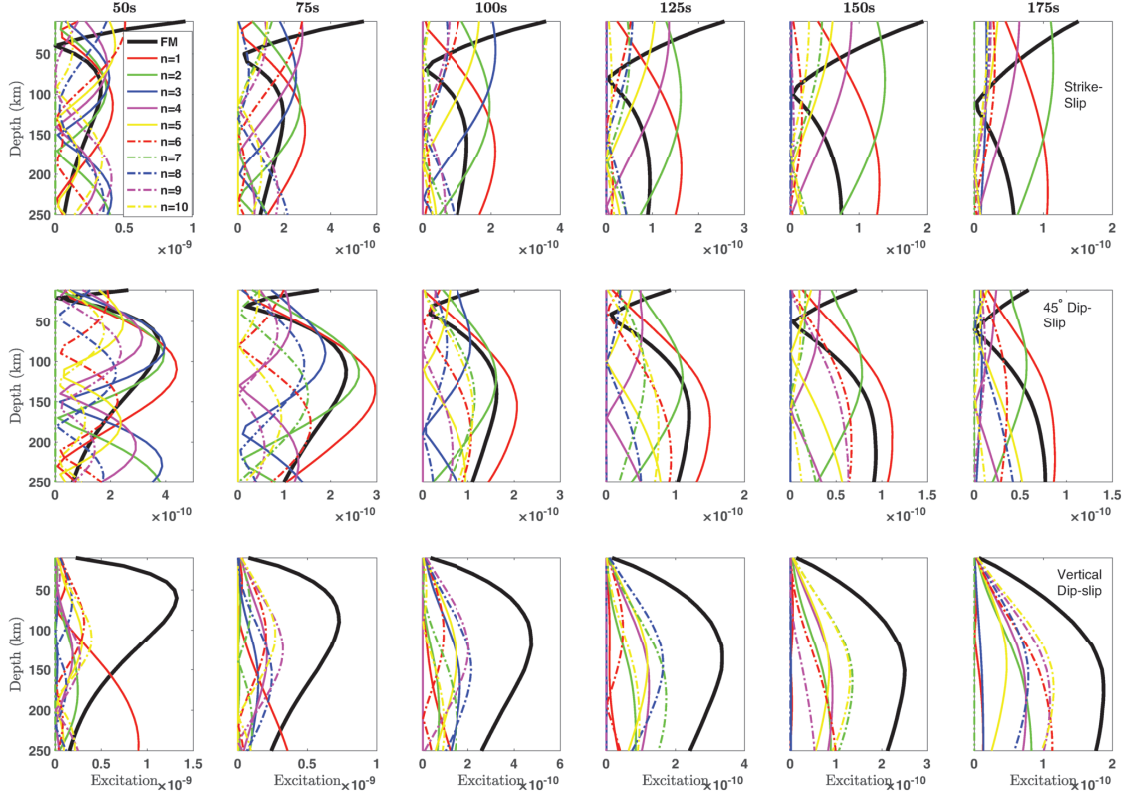


Figure 3.7: Rayleigh wave excitation for the fundamental mode and first ten overtones as a function of depth for the three example source mechanisms. Calculated for the oceanic velocity model.

function of epicentral distance, source depth, and fault type. To evaluate the role of overtone excitation in controlling both the strength and distance dependence of interference, we plot as circles the excitation ratios for the first ten overtones at the distance where they intersect the FM; a large excitation ratio corresponds to large relative excitation of the overtone.

For a given source mechanism and period, Fig. 8 shows that source depth is the dominant control on the strength of major-arc overtone interference. The excitation ratios, shown as circles, also depend strongly on source depth, and the source depths that produce the strongest Rayleigh wave interference also have the largest excitation ratios (e.g., 62 km at 100 s for strike-slip and 45° dip-slip faults, and 112 km at 150 s for a strike-slip fault). Thus, excitation ratio explains the overall strength of interference



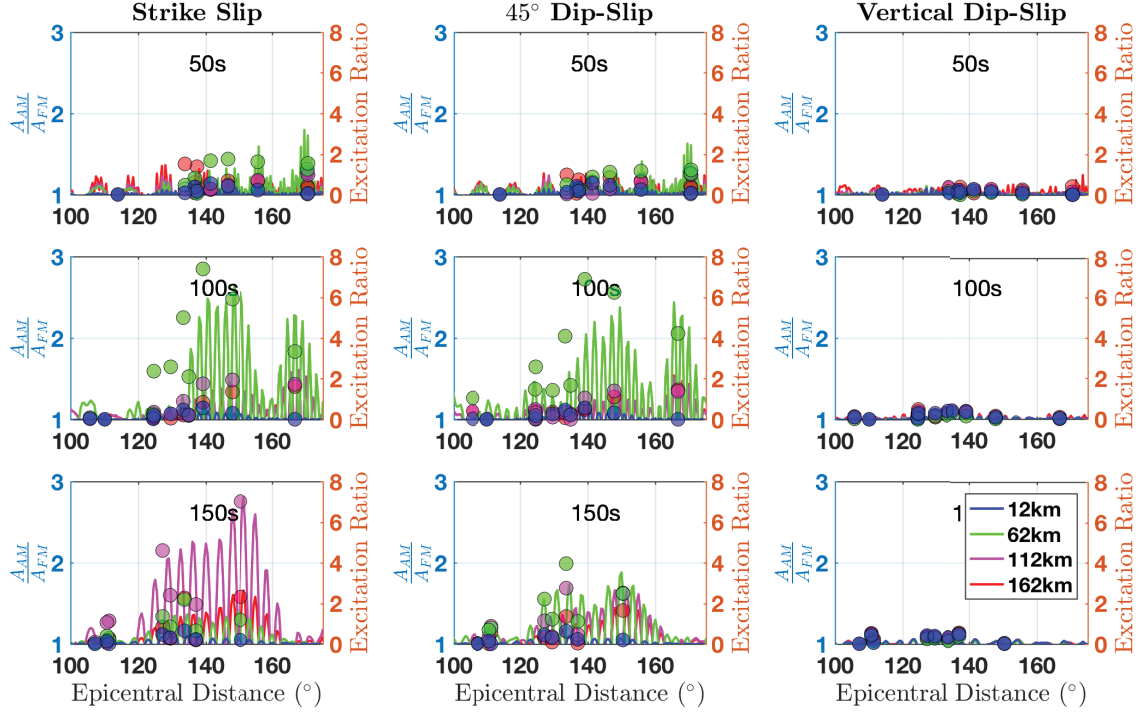


Figure 3.8: Left vertical axis shows Rayleigh wave amplitude interference pattern (top half only) in the distance range prone to major-arc interference. Right vertical axis shows Rayleigh wave excitation ratios (overtone excitation / FM excitation) for the first ten overtones, plotted as circles at the distances at which the overtones intersect the FM and colored by source depth.

in the distance range prone to major-arc interference. Furthermore, the excitation ratios explain the modulation of the interference pattern as a function of distance. For instance, at 100 s for the strike-slip fault, interference is low between 100 and 120° for all source depths, and the excitation ratios for the overtones that intersect the FM in this distance range are very small. On the other hand, the excitation ratios of the overtones that intersect the FM at distances > 120° are much larger, especially for the 62-km source, and peak at approximately 140°, consistent with the peak in the amplitude interference pattern determined from synthetic seismograms.

The source depth that produces the largest interference varies with period and source mechanism, and comparison with Fig. 7 indicates that the relative excitation of the FM is the dominant factor controlling this variation. At 150 s, it is the 112-km

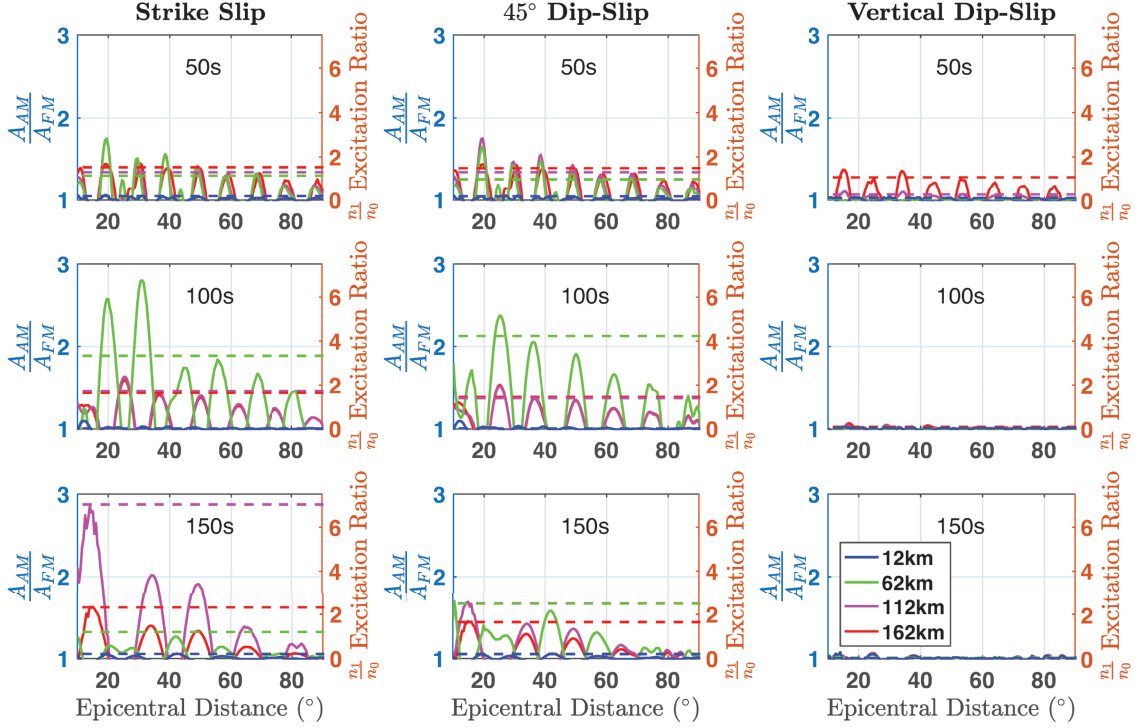


Figure 3.9: As in Fig. 8 but in the distance range prone to minor-arc interference. The Rayleigh wave amplitude interference patterns (top half only) are plotted with solid lines. The excitation ratio of the first overtone relative to the FM is plotted as a horizontal dashed line across the entire distance range.

event for the strike-slip fault and the 62-km event for the 45-degree dip-slip fault. These depths are close to the minima in FM excitation for these two fault types, thereby allowing a larger contribution for overtones. Another factor that influences the shape of the interference pattern is whether any overtones intersect the FM. Many of the interference patterns show little interference in the distance range  $155 - 165^\circ$ , which can be attributed to an absence of overtone intersections in this distance range.

Vertical dip-slip faults generally have very low excitation ratios as the FM is generally the most strongly excited at depths  $< 250$  km. Consequently, the amplitude interference pattern is small. In summary, our analysis shows that the overtone excitation ratios, when considered in context with the distance at which each overtone intersects the FM, can explain the characteristics of the major-arc interference pattern, including its distance dependence and strength.

At shorter epicentral distances, we find that interference of minor-arc Rayleigh wave overtones, especially the first overtone, with the FM can be strong under certain circumstances. Fig. 9 plots the minor-arc interference pattern overlain with the excitation ratio of the first overtone and FM as a horizontal line corresponding to the right-hand vertical axis. As in Fig. 8, the vertical dip-slip faults produce minimal overtone interference with the exception of the deepest source at short periods, where the first overtone has notable excitation (Fig. 7). The event at 12 km well excites the FM and weakly excites the first overtone for all periods and fault types, resulting in low interference for this source depth. Deeper events are characterized by excitation ratios  $> 1$  for the strike-slip and  $45^\circ$ -dip-slip faults, and therefore the Rayleigh wave measurements exhibit substantial minor-arc interference. These excitation ratios control the strength of the interference pattern. For instance, at 150 s for the strike-slip fault, the excitation ratios sorted from largest to smallest are generated by sources at 112 km, 162 km, 62 km, and finally 12 km. The strength of the amplitude interference pattern determined from the synthetic seismograms follows the same order.

As we found for major-arc overtone interference, the presence of a minimum in FM excitation exerts strong control on the excitation ratio of the first overtone with respect to the FM. At 100 s, for the strike-slip and 45-degree dip-slip faults, the FM excitation minimum occurs close to 62 km, resulting in the large minor-arc interference for this source depth. Fig. 9 shows that the separation of FM and first overtone group velocities introduces a decay in minor-arc interference with epicentral distance as the overtone moves out of the time window used in measurements. This decay rate is smaller at shorter periods, due to the relative closeness of the FM and first overtone group velocities.

Although our calculations of synthetic seismograms and excitation ratios are performed for the azimuth corresponding to the maximum in the source radiation pattern, we have found that the Rayleigh wave excitation ratios are largely independent

of azimuth, and thus the results in Figs. 8-9 apply for most take-off angles. The exception is take-off angles near the nodes in the FM radiation patterns, where the excitation of the FM and overtones can change rapidly as a function of azimuth.

Figs. 8 and 9 highlight the sharp contrast in the wavelength of major- and minor-arc overtone interference, with a much larger wavelength for the latter. Using synthetic seismograms for an explosive source and including only the first overtone and the FM to ensure the simplest possible interference pattern, we show in Fig. S5 that Equations 3 and 4 accurately predict the wavelength of the major and minor-arc interference that we measure from the amplitude interference patterns. Wavelength is measured by windowing the interference pattern in the relevant distance range, taking the Fourier transform of the interference pattern, and finding the wavelength that corresponds to the spectral peak.

Figs. S6 and S7 show the complete Rayleigh wave amplitude and travel-time interference patterns, respectively, for several periods and the full range of epicentral distance. When the excitation ratio is very large (e.g., at 100 s for the 62-km source depth and the 45-degree dip-slip fault), the travel-time interference pattern can take on a very different character, with unreasonably large time shifts and the absence of an oscillatory pattern, which we speculate occurs because the windowed seismogram is dominated by the overtone rather than the FM.

### 3.4.3 Interference in a Realistic Event Distribution

Having probed example source mechanisms, we now explore how well the tests in Sections 3.1 and 3.2 capture the diversity of interference in typical surface-wave data sets. We compute normal-mode synthetic seismograms and calculate the interference patterns, for both Rayleigh and Love waves, for 200 events in the Global CMT catalogue (Dziewonski *et al.*, 1981; Ekström *et al.*, 2012). These 200 events are randomly selected from the subset that occurred between 2007 and 2016 and had centroid depths

between 15 and 50 km; 50 km is chosen because it is often used as an upper limit on source depth in surface-wave studies (Bao *et al.*, 2016; Eddy *et al.*, 2018; Babikoff and Dalton, 2019). For each event we space stations every  $1^\circ$  along a great-circle path with an azimuth corresponding to the maximum of the FM radiation pattern. As described in previous sections, we generate synthetic seismograms that contain only the FM and that contain all modes, and calculate the travel-time and amplitude interference patterns. These simulations use the oceanic model, ATL2a.

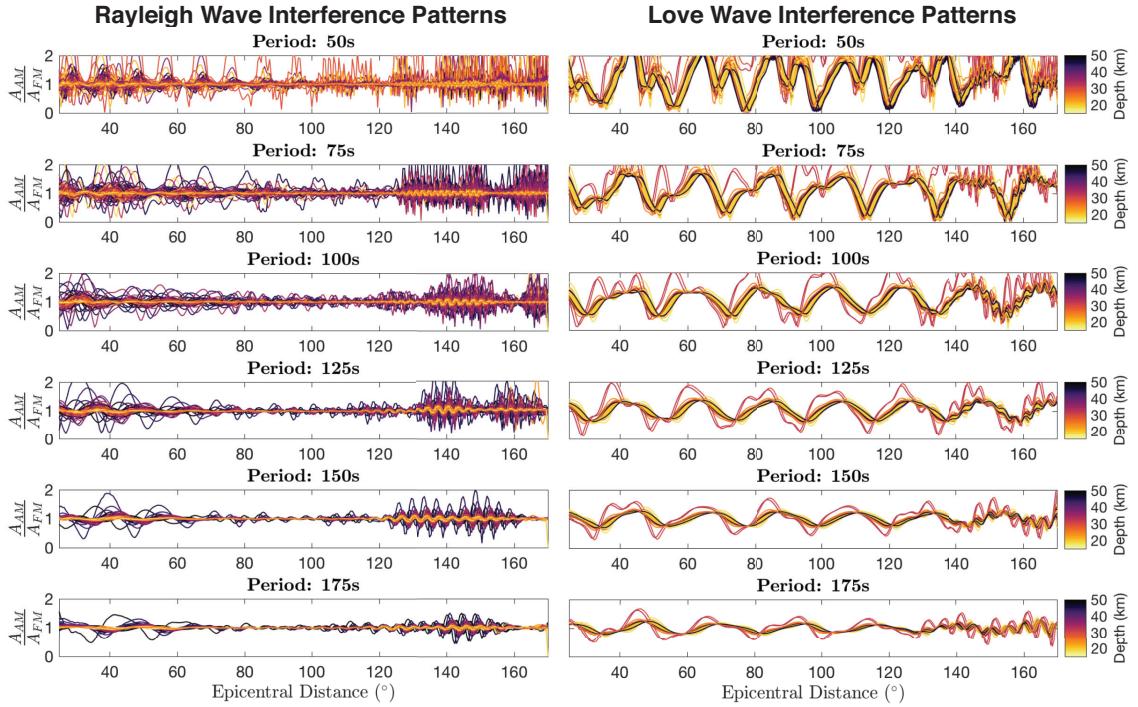


Figure 3.10: Amplitude interference patterns at periods between 50 and 175 s for Rayleigh (left) and Love (right) waves. They are computed for a set of 200 randomly selected events from the Global CMT catalog with source depth between 15 and 50 km. The interference patterns are color-coded by source depth. Simulations use the oceanic earth model ATL2a.

Fig. 10 shows the interference pattern for each event, colored by source depth. This figure highlights some of the features that are visible in our analysis of example events and also reveals some new ones. Fig. S8 shows the corresponding set of travel-time interference patterns. Perhaps the most striking difference between the Love and Rayleigh wave interference patterns is that the Love wave interference patterns

are tightly clustered, indicating little variability in the interference for typical events. However, the Rayleigh wave interference patterns exhibit a much larger variability in morphology and strength. Fig. 10 shows that source depth is the dominant control on Rayleigh wave overtone interference. With the exception of 50 s, the Rayleigh wave measurements show the largest interference for the deepest events. This is due to the depth dependence of FM excitation for strike-slip and 45-degree dip-slip faults (Fig. 7); at most periods, FM excitation decreases as depth increases from 0 km to 50 km. For instance, for a 45-degree dip-slip event at 100 s, the minimum in FM excitation is for source depths at approximately 50 km. On the other hand, at 50 s the FM excitation minimum occurs at 30 km, and therefore the 50-km events show less interference than the 30-km events.

The depth dependence of Rayleigh wave interference is characteristic not only of overtones along the major arc (epicentral distances  $> 120^\circ$ ) but also of overtones along the minor arc ( $< 60^\circ$ ). Fig. 10 makes clear that for many of these realistic source geometries with event depths  $> 40$  km the minor-arc overtone interference can be large, as was also suggested by our test with example fault types (Fig. 9). This minor-arc interference is particularly strong at shorter periods, for which the group velocities of the FM and first overtone are relatively close, and minor-arc interference can be significant even at intermediate epicentral distances ( $0 - 60^\circ$ ). This analysis highlights that our initial conclusions in Sections 3.1 and 3.2, drawn from excitation functions and interference patterns for example events, are applicable to real events found in earthquake catalogs.

### 3.5 Discussion: Interference in Phase Velocity Measurements on Real Data

In this section we explore whether real data sets of FM Love and Rayleigh wave travel-time measurements show evidence of overtone interference and, if so, to what extent it is controlled by the relative excitation of the overtones and FM. The data sets, which are from USArray stations, are described in Section 2.3. For each event, we use Eikonal tomography to generate a phase-velocity map (Equation 7). We estimate the error in each pixel by differencing the Eikonal velocity and the velocity in the GDM52 global phase-velocity map (Ekström, 2011), using the latter as a smoothly varying reference. Thus, each error estimate corresponds to a particular pixel and event, and the errors can be binned to explore dependencies on epicentral distance and source excitation. The Rayleigh wave data set uses between 569 and 591 earthquakes, resulting in  $\approx 1.28$  million error measurements at each period. The Love wave data set uses between 543 and 998 earthquakes, resulting in  $\approx 350,000$  error measurements at each period.

#### 3.5.1 Converting Traveltime Interference into Phase-Velocity Interference

Prior to examining overtone interference in real phase-velocity data, we first explore the consequences of the traveltime interference patterns in Section 3 for phase-velocity interference. Interstation phase velocity can be estimated from the inverse of the travel-time gradient, e.g., Equation 7 for the Eikonal equation. The magnitude of the travel-time gradient will depend on both the amplitude and the wavelength of the travel-time interference pattern, with larger travel-time variations and shorter wavelengths giving rise to stronger interference effects in interstation phase-velocity measurements. The amplitude of the travel-time interference pattern depends on the overtone excitation ratio and, for Love waves, the proximity of the overtone and FM

group velocities; a strongly excited overtone whose group velocity overlaps that of the FM causes a large travel-time bias. The interference wavelength depends on the phase velocities of the FM and overtone (Equations 3 and 4).

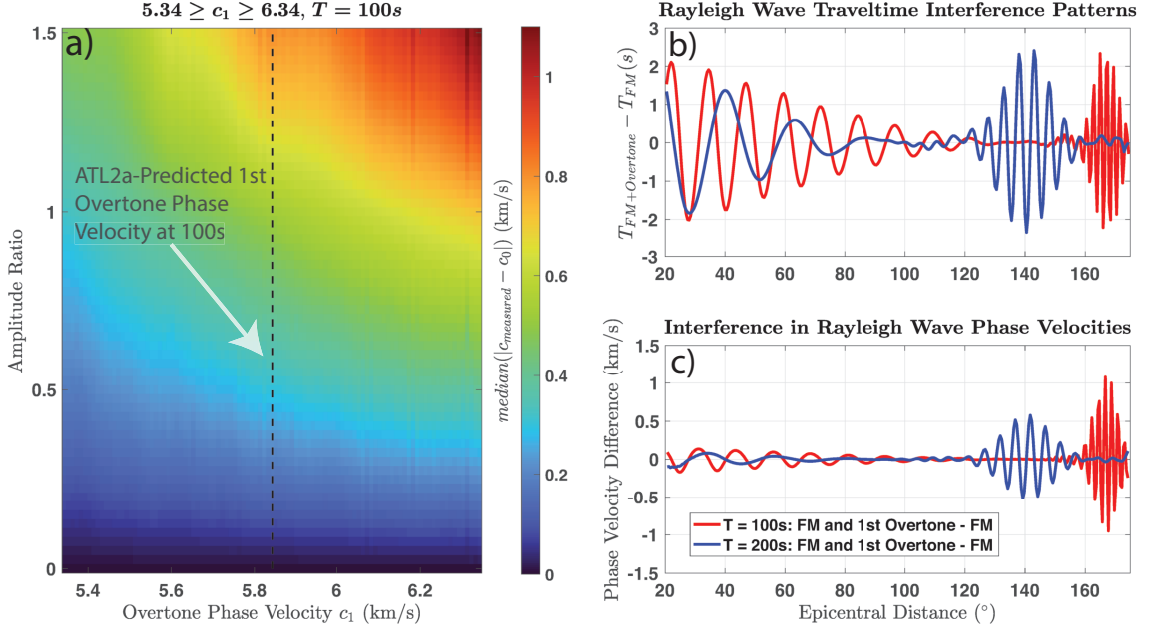


Figure 3.11: (a) Surface of 100 s Love wave FM phase velocity error for two interfering plane waves (Equation 2a) for which the FM phase velocity is fixed while the phase velocity ( $c_1$ ) and relative amplitude ( $\delta a$ ) of the overtone vary within the ranges shown. For the ATL2a model,  $c_0 = 4.66$  km/s and  $c_1 = 5.84$  km/s. (b) Examples of Rayleigh wave traveltime interference patterns at two periods (red = 100 s and blue = 200 s), with source depth tuned such that minor-and major-arc interference at both periods has a similar magnitude ( $\pm 2$  s). (c) Rayleigh wave phase velocities from the traveltimes shown in (b), determined using the Eikonal equation and presented relative to the known FM phase velocity.

In Fig. 11 we demonstrate how these two factors— interference amplitude and wavelength— control how overtone interference is manifest in phase-velocity errors. We use Equation 2a to predict synthetic travel-time interference patterns for a range of overtone phase velocities  $c_1$  and relative amplitudes  $\delta a$ , using a linear array of stations spaced at  $0.5^\circ$  and spanning  $70^\circ$ . Phase velocity as a function of distance is then measured using Equation 7; when overtone interference is negligible, this measurement should be the FM phase velocity. We quantify the phase-velocity error



for each combination of  $c_1$  and  $\delta a$  as the median of the absolute difference between this measurement and the FM phase velocity  $c_0$ . Fig. 11a shows that the phase-velocity error increases with increasing amplitude ratio, due to its effect on the magnitude of the travel-time bias, and with increasing overtone phase velocity, since larger  $c_1$  values cause smaller interference wavelengths (Equation 3).

The range of overtone phase velocities explored in Fig. 11a is much larger than expected from realistic lateral variations in the Earth. However, the large variations in interference wavelength that they represent are indeed relevant, since major-arc interference is characterized by much smaller wavelengths ( $2^\circ$ ) than minor-arc interference ( $12^\circ$ ), as shown in Fig. S5. Figs. 11b,c illustrate the significant role interference wavelength plays in controlling the phase-velocity error. The two Rayleigh wave traveltime interference patterns have a similar magnitude ( $\pm 2s$ ) but visibly different wavelengths. The resulting phase-velocity measurements (Fig. 11c) show that the phase-velocity error is much larger when the interference wavelength is shorter. This can be seen by comparing the 100-s and 200-s results to each other; it can also be seen by comparing the minor-arc interference at short distances to the major-arc interference at large distances. Thus, large errors in Rayleigh wave interstation phase velocity due to major-arc interference are expected, even when the traveltime bias is relatively small (e.g., Fig. S8), as was documented by Hariharan *et al.* (2020)

In summary, the interstation phase-velocity error that is caused by overtone interference is controlled by three factors: the relative excitation of overtones, the proximity of the overtone and FM group velocities (primarily for Love waves), and the wavelength of the interference pattern. In the following sections we examine the error in USArray Love and Rayleigh wave phase-velocity measurements.

### 3.5.2 Love Wave Measurements on Real Data

Fig. 12 plots the lower, middle, and upper quartiles of the errors in USArray Love wave phase velocities, binned as a function of epicentral distance with bin width of  $1^\circ$ . The binned Love wave errors exhibit a regular periodicity with a wavelength of approximately  $20^\circ$ , consistent with previous studies (Foster *et al.*, 2014) and our tests with synthetic seismograms (Figs. 4 and 10). That this oscillatory pattern emerges even after binning is a testament to the fact that Love wave overtone interference is the dominant source of error in this data set. At short periods, error is lower than at other periods. This may be related to the fact that at short periods, FM group velocities for a continental earth model are well separated from overtone group velocities, resulting in a lowered apparent amplitude ratio and relatively diminished interference (Fig. 5).

We then explore whether the excitation ratio exerts control on the Love wave errors. For every event in the data set, we calculate excitation for the FM and first overtone, using the STW105-C eigenfunctions if the event occurs beneath a continent and the ATL2a eigenfunctions otherwise. This associates every error estimate with an excitation ratio, and we bin error as a function of excitation ratio, using a bin width of  $0.2^\circ$ . Fig. 13 summarizes the median absolute value of binned error as a function of excitation ratio. The general trend is that error increases as excitation ratio increases. The correlation between error and excitation ratio suggests that when the source mechanism and depth poorly excite the FM and/or strongly excite the first overtone, measurements of the Love wave travel time are more likely to be biased. For array-based imaging methods that rely on differential travel times over short distances, these travel-time errors propagate into large phase-velocity errors (Foster *et al.*, 2014).

The correlation between error and excitation ratio also indicates that imposing quality control criteria based on excitation ratios may be a path forward to improving

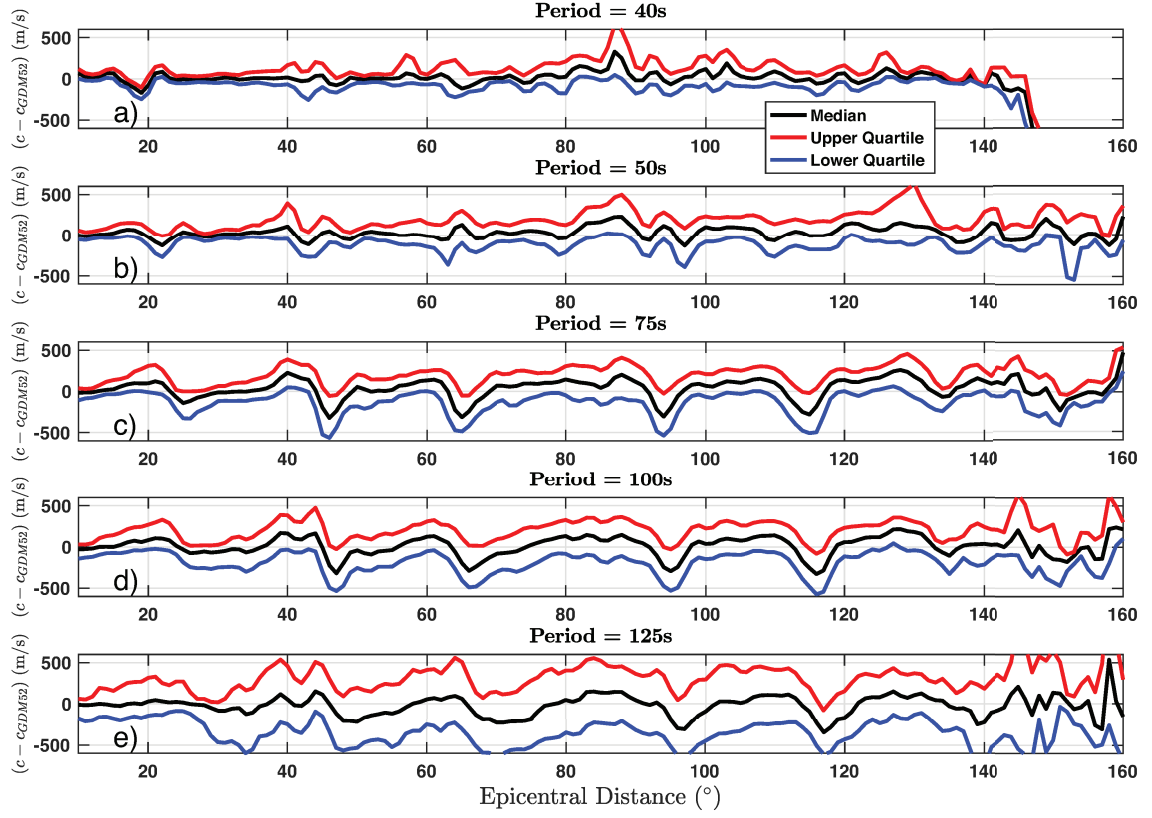


Figure 3.12: Lower, middle, and upper quartile error in USArray Love wave Eikonal phase velocities, binned as a function of epicentral distance with bin width =  $1^\circ$ . Calculation of error is described in Section 4.

the quality of these data sets. Our USArray Love wave data were measured from earthquakes in the depth range 12-50 km. Some of these shallow events have large excitation ratios and correspondingly large phase-velocity errors (Fig. 13), and Fig. 3 hints that some events deeper than 50 km could have small excitation ratios and presumably small phase-velocity errors. Future work will explore whether making measurements on deeper events could increase the number of Love wave measurements with small excitation ratios and therefore small interference biases.

We have also found evidence in real Love wave data that overtones along the major arc interfere with the FM along the minor arc, as was suggested by our tests with synthetic seismograms. In Fig. 12, the wavelength of error decreases and the magnitude of error increases at large epicentral distances ( $\geq 140^\circ$ ). Fig. S9 shows

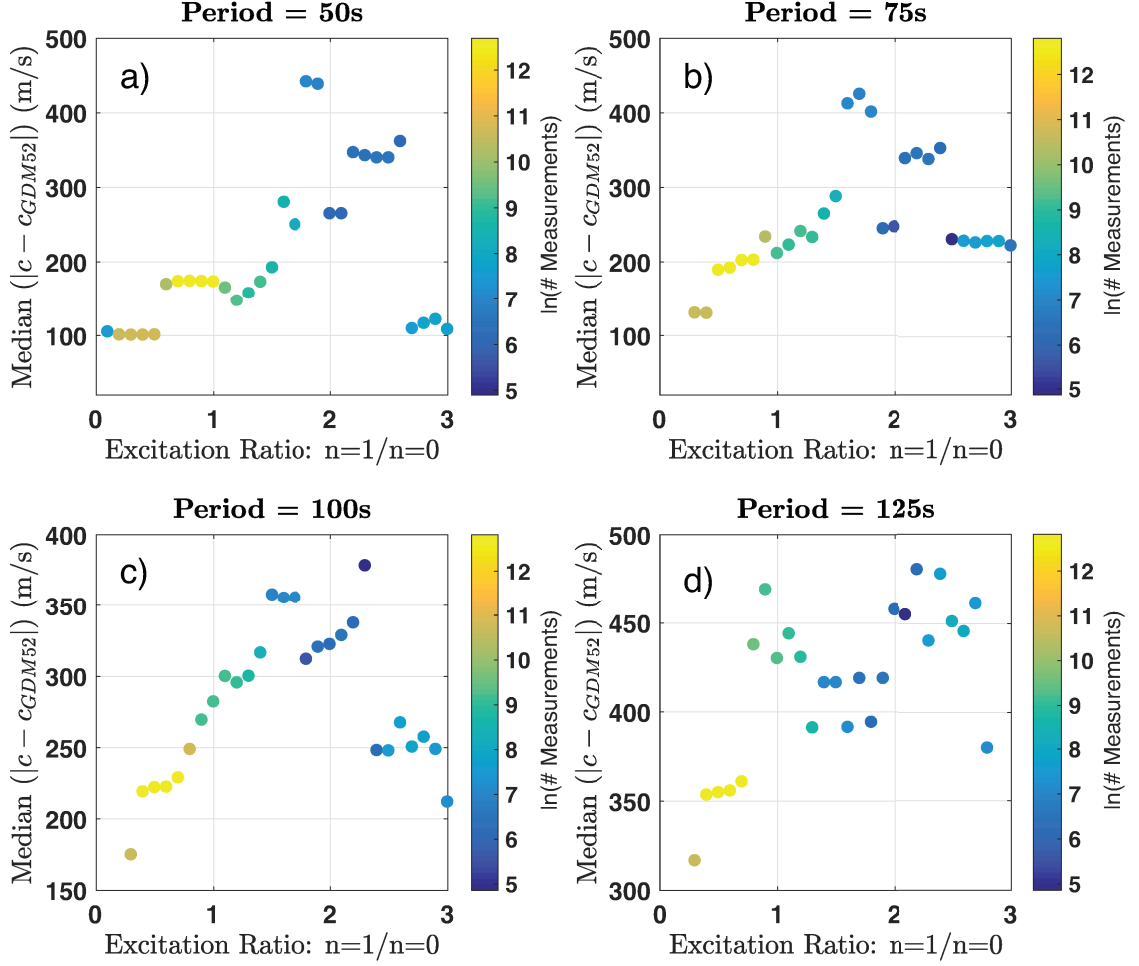


Figure 3.13: Median absolute error in USArray Love wave Eikonal phase velocities, binned as a function of the ratio of the first-overtone excitation to the FM excitation. Color scale shows the number of measurements in each bin.

that this major-arc interference can also be observed in event-specific phase velocity maps constructed from real data measured on USArray stations. In map view and in transect view, oscillations with a wavelength of approximately 3 degrees are clear. This wavelength is consistent with the prediction for major-arc interference (Equation 4) and much smaller than that expected for minor-arc interference ( $\approx 18^\circ$  for Love waves at 75s, using phase velocities from STW105-C).

Overall, our recommendation for data selection for Love waves is to use the excitation ratios of the first overtone and the FM. Retaining only Love wave data from shallow events with low excitation ratios will negatively impact the size of the data

set, which may be ameliorated by including measurements from earthquake sources deeper than 50 km if their excitation ratios are low.

### 3.5.3 Rayleigh Wave Measurements on Real Data

Fig. 14 summarizes the absolute value of the errors in USArray Rayleigh wave phase velocities, binned as a function of epicentral distance with bin width of  $1^\circ$ . The dominant feature in these distributions of measurement error is the marked variation with distance, with a peak in error that is maximized at around  $140^\circ$ , although the width and amplitude of this peak vary slightly between periods, with a larger and sharper peak at longer periods. At 100 s, the median error at  $140^\circ$  is larger by a factor of four than that at  $60^\circ$ . Note that for this calculation, the phase velocity measurements shown at 120 s are differenced with the predictions from GDM52 at 125s, which is the closest period that exists in the GDM52 maps, while the measurements shown at other periods are differenced with predictions from the GDM52 map at exactly the same period. This slight mismatch of periods at 120 s does not appear to impact our conclusions.

To show that the Rayleigh wave phase-velocity errors in Fig. 14 are controlled by the relative excitation of the overtones and FM, we calculate the excitation, using ATL2a, of the FM and first ten overtones for the source mechanism and depth and receiver location corresponding to every phase-velocity measurement in this data set. In Fig. 14 we show the median excitation ratio for each overtone relative to the FM for the entire data set, plotted at the distance where the overtone is predicted to intersect the FM. Overall, the variation of these median excitation ratios with distance mirrors the phase-velocity errors in real data and establishes that at distances  $> 120^\circ$ , major-arc overtone interference is the dominant source of error. For example, at 100 s, the third overtone has the largest excitation ratio and intersects the FM at around  $140^\circ$ , which is where the peak in phase-velocity error occurs. The ramp-up in error

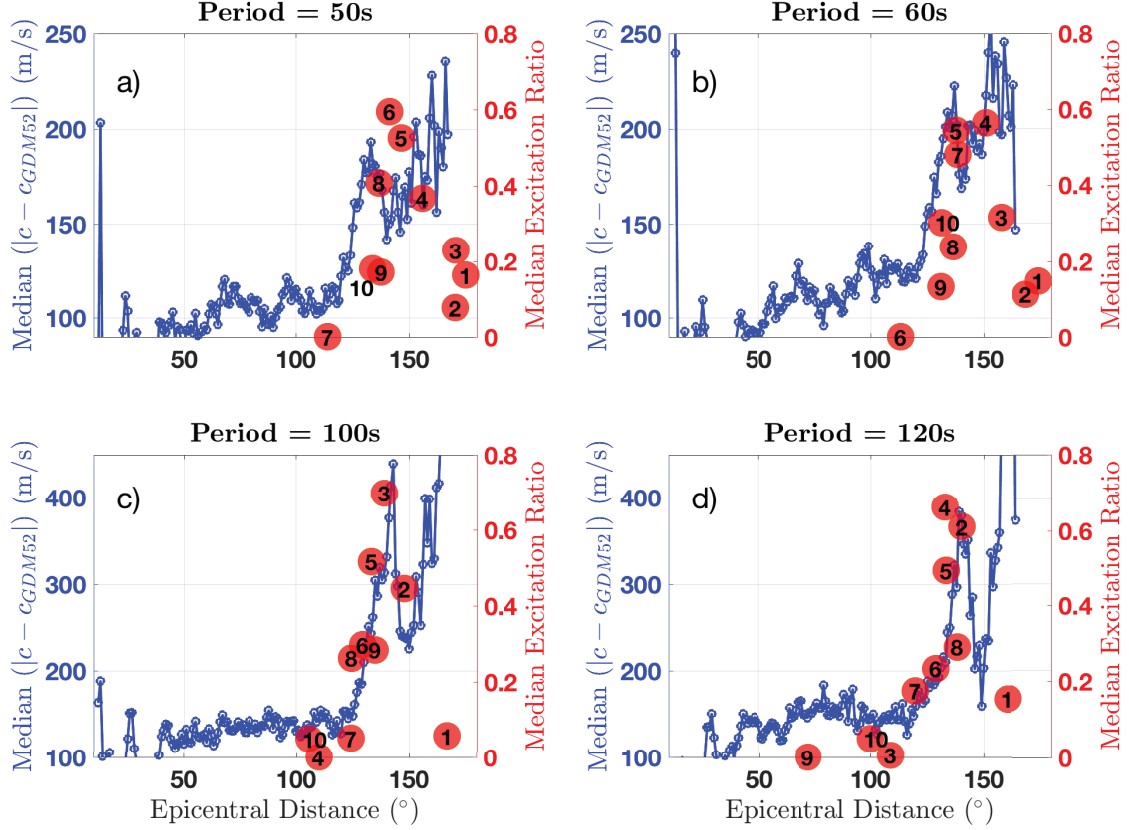


Figure 3.14: Blue curve and left vertical axis: median absolute errors in USArray Rayleigh wave Eikonal phase velocities with respect to the GDM52 maps, binned as a function of distance. Red circles and right vertical axis: median excitation ratios for all events in the data set, labeled by overtone number  $n$ . The ratios show the excitation of overtone  $n$  relative to the FM excitation and are plotted at the distance at which the overtone is predicted to intersect the FM.

from 120° to 140° is reflected in the ascending excitation ratios of the overtones that intersect the FM in this distance range. At 50 s many of the excitation ratios are smaller than at 100 s, and their distribution is not as steeply peaked. This is mirrored in the smaller and less peaked phase-velocity errors.

Fig. 14 shows that the relative contribution of an overtone branch to the Rayleigh wave phase-velocity error depends on period. For example, at 100 s, the peak in error corresponds to the third overtone, but at 120 s it corresponds to the second and fourth overtones, with very small excitation of the third overtone, which corresponds to a Stoneley mode at 120 s. At 60 s, the peak corresponds to the fourth, fifth,

and perhaps seventh overtones. This variation is entirely expected from the period dependence of the excitation functions in Fig. 7. The Eikonal phase-velocity maps, and thus the error, are determined from the same group of earthquakes at all periods, but the relative weight of overtone and FM excitation depends on period.

There are several reasons why we do not expect a perfect correlation between the measurement errors and excitation ratios. Firstly, there will be variation in the distance at which an overtone intersects the FM based on the specific propagation paths and path-averaged group velocities for each measurement. To explore the scope of this variation, we calculated path-integrated FM group velocities using the GDM52 maps (Ekström, 2011) and re-calculated the range of intersection distances for each overtone/FM pair using Equation 1, retaining the same 1-D group velocities for overtones. We find that allowing for heterogeneous FM group velocities can change the intersection distances shown in Fig. 2, which were calculated with a 1-D model, by several hundred kilometers, especially at shorter periods. At 50 s, for 90% of paths the re-calculated intersection distances differ from the 1-D prediction by less than 631 km; at 100 s, the 90% difference in intersection distance is 295 km. Secondly, condensing the excitation ratios for 581 events (on average) with a range of source mechanisms and depths into a single value for the entire data set is an over-simplification. Thirdly, we note also that the agreement between measurement errors and excitation ratios is weaker at distances greater than  $160^\circ$ , where the major-arc FM interferes with the minor-arc FM (e.g. Levshin *et al.*, 2005), which is not accounted for with these excitation ratio calculations. Nonetheless, it is remarkable how well the observed errors can be explained by the combined effects of group velocity, which controls the intersection distance, and source excitation, which controls the strength of overtones and FM.

In Figs. 15 and 16 we bin Rayleigh wave phase-velocity error by excitation ratio rather than distance in the distance ranges most affected by major-arc and minor-arc interference, respectively. In the distance range of major-arc interference, each

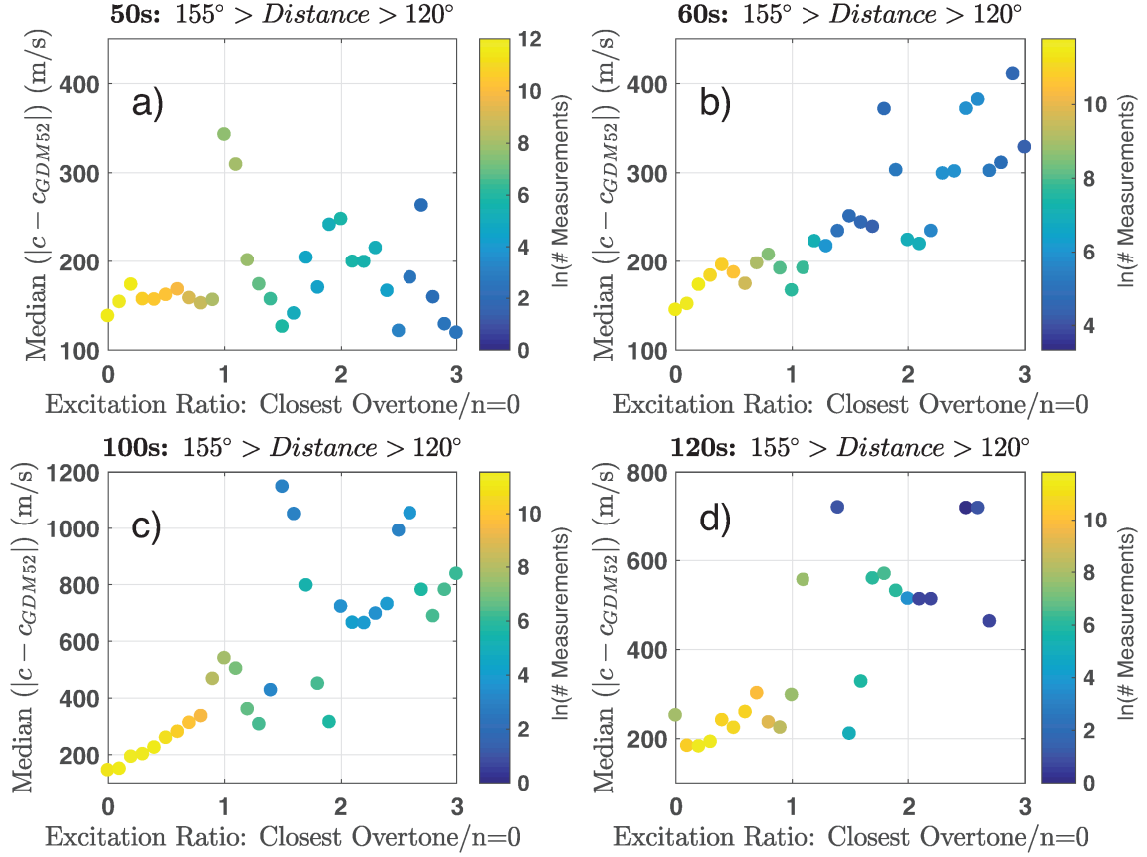


Figure 3.15: Median absolute error in USArray Rayleigh wave Eikonal phase velocities in the distance range prone to major-arc interference, binned as a function of the ratio of closest overtone excitation to FM excitation. Color scale shows the number of measurements in each bin. The “closest overtone” is that that intersects the FM at the distance closest to each measurement; see text.

error measurement is binned based on the excitation of the overtone that intersects the FM at the epicentral distance closest to where the measurement is made. For example, at 100 s, the error measured in a pixel located  $122^\circ$  from the earthquake would be associated with the seventh overtone and therefore binned based on its excitation ratio. A strong correlation between excitation ratio and error is visible at the longer periods (Fig. 15c,d). This is important for two reasons. One, it further establishes that source excitation of the overtones and FM is the primary control on phase-velocity error. Two, it suggests that even in the distance range where major-arc overtone interference is prominent, there is a subset of measurements that are



minimally affected by the interference and could be used. Excitation ratio could be a valuable and more nuanced criterion for quality control rather than a blanket recommendation to exclude all measurements at distances greater than  $120^\circ$ , which may substantially reduce the size of data sets for certain station locations (Hariharan *et al.*, 2020).

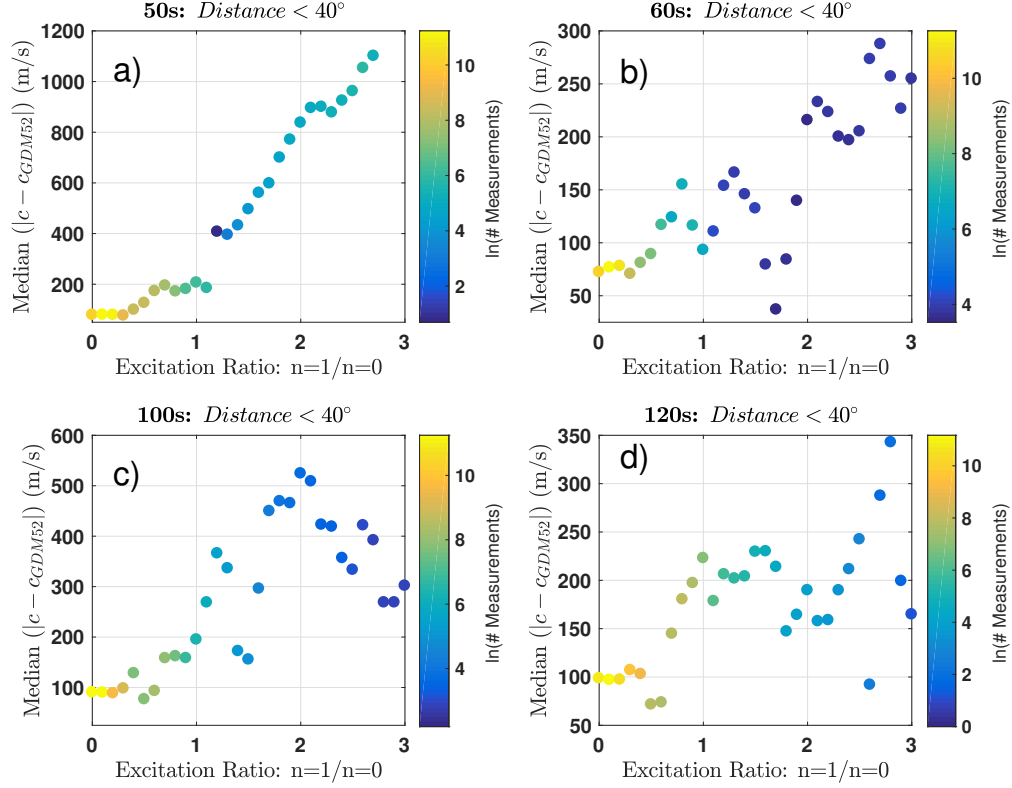


Figure 3.16: Median absolute error in USArray Rayleigh wave Eikonal phase velocities in the distance range prone to minor-arc interference, binned as a function of the ratio of the first-overtone excitation to the FM excitation. Color scale shows the number of measurements in each bin.

Several factors may contribute to the weaker correlation observed at shorter periods in Fig. 15. These include path-dependent variability in the FM intersection distance for an overtone, as mentioned above, as well as the fact that Fig. 15 considers only the closest overtone when in some cases multiple overtones may be responsible for interference effects (Fig. 14). A third consideration is the impact of attenuation.

The sensitivity of a surface wave at some period to depth-dependent shear attenuation depends on overtone number (Dahlen and Tromp, 1998), which can result in different amounts of attenuation for different overtone branches, since shear attenuation varies with depth. Fig. S10d shows the impact of attenuation on the relative FM and overtone amplitudes. This calculation depends primarily on two factors: the attenuation ( $Q^{-1}$ ) and the distances traveled by the FM and overtone. At short periods, the high- $n$  overtones can experience much less attenuation (2-3 times less) than the FM. Thus, our assumption that source excitation ratio alone can control phase-velocity error is incomplete, because for the short-period high- $n$  overtones, the excitation ratio is modulated by a ratio of the attenuation coefficient that can be of the same order of magnitude.

In the distance range of Rayleigh wave minor-arc interference, each error measurement is binned based on the excitation ratio of the first overtone to the FM (Fig. 16). As in Figs. 13 and 15, a strong correlation is visible, particularly at the shortest period of 50 s, where the Rayleigh wave FM and first overtone have similar group velocities. Minor-arc overtone interference for Rayleigh waves is typically considered negligible at epicentral distances  $> 25^\circ$ . However, the analysis in Fig. 16, as well as the interference patterns in Figs. 9 and 10, suggest that minor-arc interference may cause measurable bias up to longer distances. Although the majority of measurements are characterized by small excitation ratios, our analysis suggests that quality control at short to intermediate distances ( $< 40^\circ$ ) based on excitation ratio would yield a cleaner data set.

In the aforementioned analysis, we have used the Eikonal equation to solve for surface-wave phase velocities. While there are surface-wave imaging studies that use the Eikonal equation to construct phase velocity maps (e.g. Ball *et al.*, 2016; Gama *et al.*, 2021; Lehujeur and Chevrot, 2020), many studies instead use the Helmholtz equation (Lin and Ritzwoller, 2011) in order to better incorporate finite-frequency

effects. Hariharan *et al.* (2020) showed that using the Helmholtz equation reduces but does not eliminate bias due to overtone interference in synthetic data. Here, we investigate applying the Helmholtz equation to real Rayleigh wave data. We repeat the analysis in Fig. 14, binning the error in Helmholtz phase velocities by distance at 100 s (Fig. S11). We find that, as with the errors in Eikonal velocities, errors in Helmholtz velocities increase sharply at distances greater than about 120 degrees, coincident with overtones characterized by large excitation ratios that intersect the FM at these distances. However, the binned errors in Helmholtz phase velocities are slightly smaller, reaching a maximum of approximately 270 m/s as opposed to 440 m/s for Eikonal phase velocities. Like Hariharan *et al.* (2020), we conclude that the Helmholtz equation can reduce but not eliminate overtone interference effects in phase velocities.

Overall, our analysis highlights the ability of excitation ratios to explain errors in interstation Rayleigh wave phase velocity measurements. On average, these errors tend to be more pronounced at large epicentral distances due to well excited overtones arriving along the major arc at these distances. Our recommendation for data selection is to consider the excitation ratios of any overtones with intersection distances within roughly four degrees of the epicentral distance of a phase-velocity measurement in order to allow for uncertainty in the predicted intersection distance. We suggest removing measurements associated with large excitation ratios. Depending on the array geometry, it may not be necessary to eliminate all data from a particular event if a subset of the measurements are at distances unlikely to be affected by well excited overtones. Furthermore, it may be worthwhile to include events deeper than 50 km if the FM is well excited and overtone excitation ratios are small.

### 3.6 Conclusion

Using normal-mode synthetic seismograms calculated for 1-D Earth models, we illustrate the nature and diversity of overtone interference in FM Rayleigh wave and Love wave measurements of amplitude and travel time. We show that the strength of overtone interference on travel time and amplitude can span a wide range and varies strongly as a function of period and source mechanism and depth. Although the character of interference in Rayleigh and Love wave measurements is dramatically different, we show that in both cases it is controlled by the group velocities and source excitation of the overtones. We present excitation functions for the FM and first ten overtones for three example source mechanisms. For Rayleigh waves, we show that the distance at which an overtone intersects the FM and its excitation relative to the FM can explain the fine-scale features in travel-time and amplitude interference at both the scale of a single event and the distributions of error for an entire data set. For Love waves, we show that excitation ratio between the first overtone and FM is similarly correlated with the distribution of error in a regional data set, and that the Love wave data suffer from overtone interference along the major arc in addition to the minor arc.

We show that the error introduced by overtone interference into interstation (array-based) phase-velocity measurements depends on the magnitude of the travel-time bias, which is controlled by (1) the relative excitation of an overtone and the closeness of its group velocity to the FM group velocity, and (2) the wavelength of the interference pattern, which is controlled by the phase velocities of the FM and overtone. Thus, the small-wavelength nature of major-arc interference can produce large errors in phase velocity. We note that overtone interference may impact other surface-wave observables and products. For example, regional measurements of attenuation, when made with an interstation approach such as the two-station method

(Zhou *et al.*, 2020) or a wavefront-tracking approach (Bao *et al.*, 2016) may be vulnerable due to interference in amplitudes. Other examples of vulnerable observables are single-station (rather than path-averaged) quantities such as H/V ratios (Tanimoto and Rivera, 2008) and site amplification (Eddy and Ekström, 2014; Schardong *et al.*, 2019).

Therefore, when possible, it is worth taking measures to avoid overtone interference. For measurements of the FM, we suggest quality control measures based on the relative excitation of overtones and the FM and, for Rayleigh waves at distances  $> 120^\circ$ , the predicted intersection distances of overtones with the FM. Data selection by the source excitation ratio could also allow more nuanced choices, for example that Rayleigh wave measurements from certain events could be included if the period and distance range predicts little interference and excluded otherwise. Just as importantly, our calculations suggest that there is no reason to restrict all source depths in a data set to be shallower than 50 km (e.g. Jin *et al.*, 2015; Accardo *et al.*, 2017; Adams *et al.*, 2018; Babikoff and Dalton, 2019). This depth range includes some events with high overtone-to-FM excitation ratios, particularly at shorter periods for Rayleigh waves, and excludes deeper events with low excitation ratios, particularly for Love waves. Thus, such quality-control measures may allow for both the size and quality of typical surface-wave data sets to be increased. Considering the variations in overtone excitation ratio can also benefit studies aiming to extract overtone dispersion of a specific overtone; depending on the overtone targeted, event selection can be optimized to maximize the excitation ratio of the overtone of interest relative to other modes in the window. A careful assessment of how different data selection strategies impact the size and quality of regional FM surface-wave data sets will be the subject of a future publication.

### 3.7 Acknowledgements

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### 3.8 Data Availability

The seismic data from USArray used to make surface-wave measurements in this study can be downloaded from the IRIS DMC. Normal mode synthetic seismograms can be generated with the MINEOS code (Masters *et al.*, 2011), which can be downloaded from CIG at <https://geodynamics.org/cig/software/mineos/>

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### 3.10 Supplemental Material

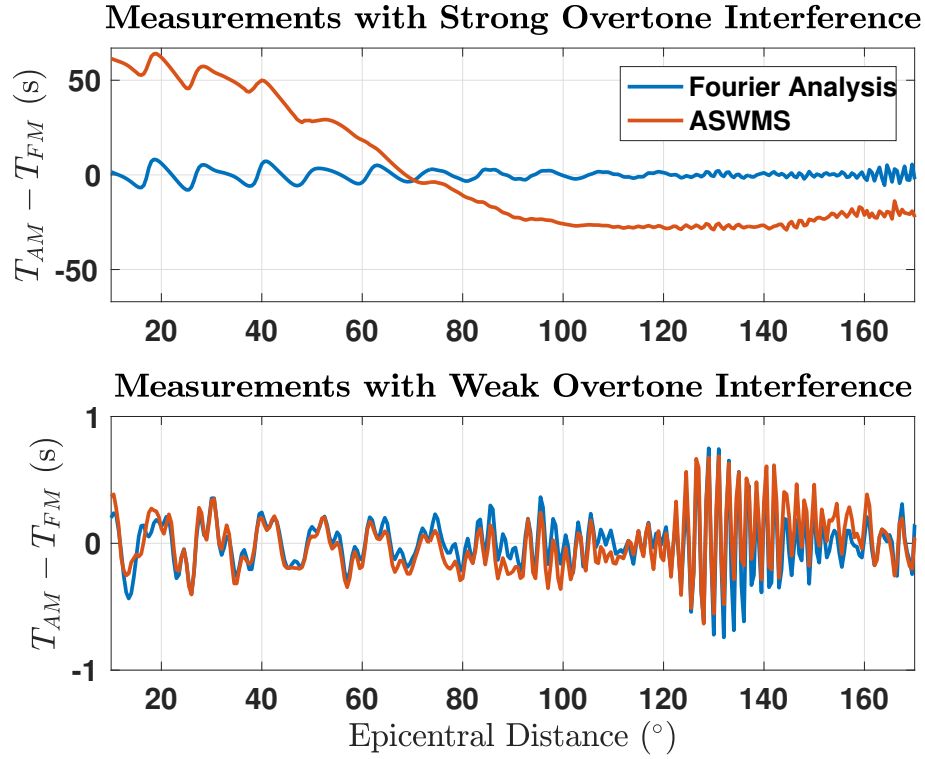


Figure 3.17: **Figure S1:** Travel-time interference pattern, for 100-s Rayleigh waves, for an event with strong overtone interference (top) and weak overtone interference (bottom). The measurements have been repeated for two approaches: using ASWMS (orange) and Fourier analysis (blue). Note the different vertical scales.

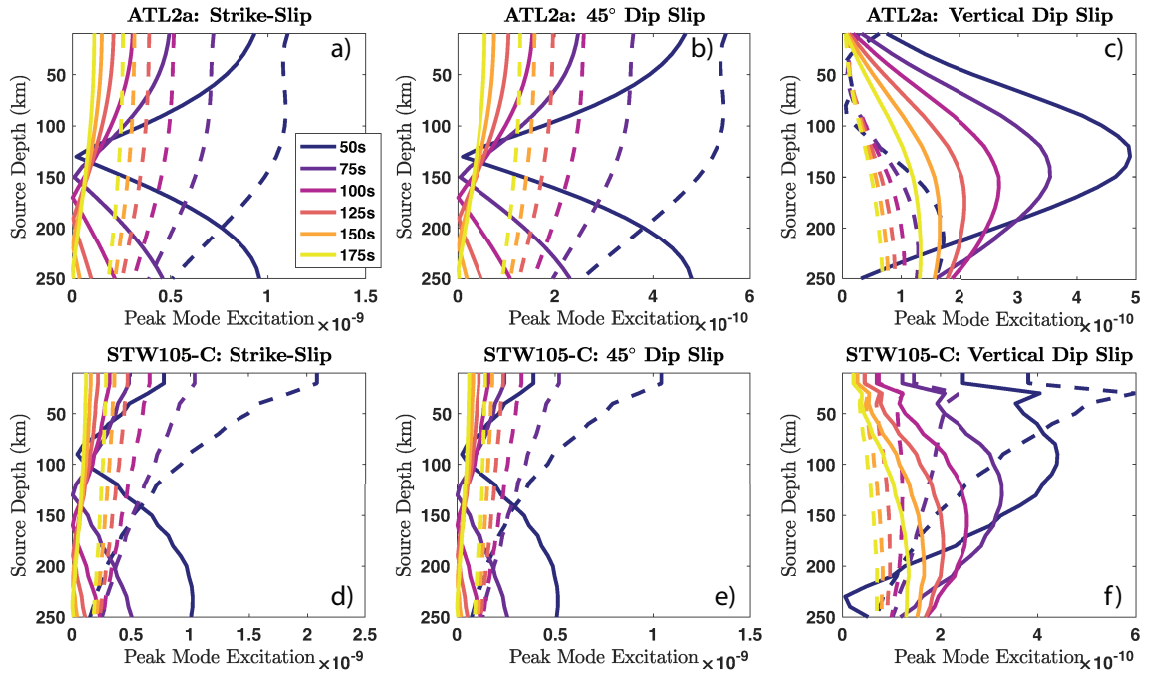


Figure 3.18: **Figure S2:** Love wave excitation as a function of depth for the FM and first overtone. (a-c): Calculations for ATL2a. (d-f): Calculations for STW105-C. Each column is a different example source mechanism. Dashed line: fundamental mode. Solid line: first overtone.

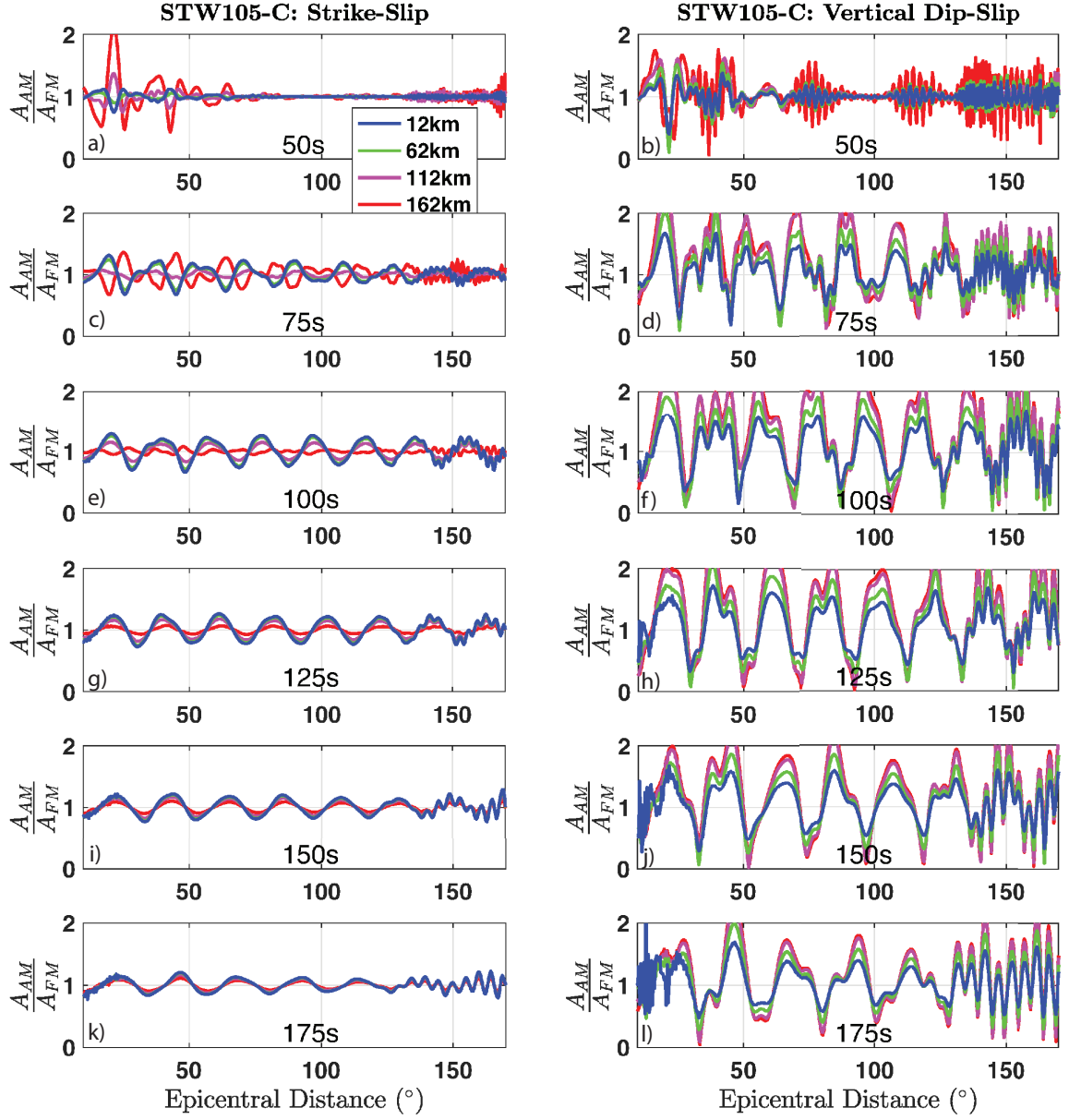


Figure 3.19: **Figure S3:** Plots of the Love wave amplitude interference pattern, color-coded by source depth, for example source geometries (as in Fig. 4 in the main text) and for the continental Earth model STW105-C. Left column: Interference patterns for a strike-slip event. Right column: Interference patterns for a vertical dip-slip event.

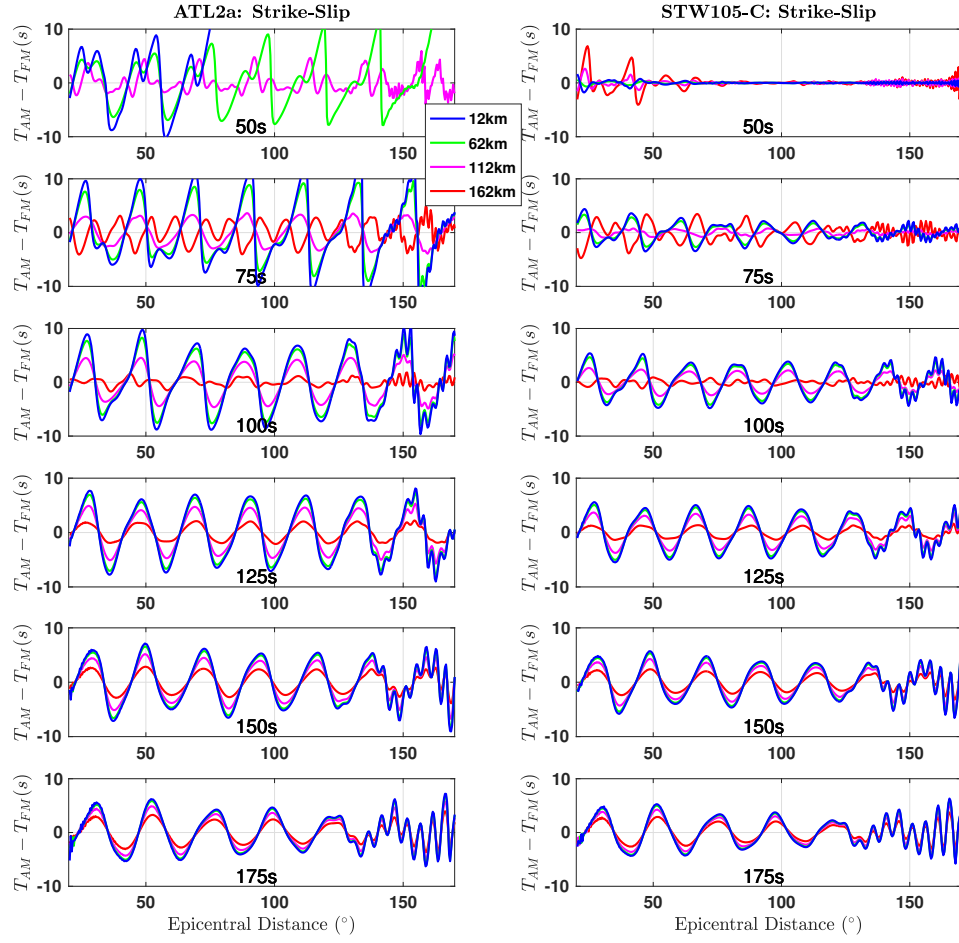


Figure 3.20: **Figure S4:** Love wave travel-time interference patterns at different periods, color-coded by source depth, for a strike-slip source. Left: measurements for ATL2a. Right: measurements for STW105-C.

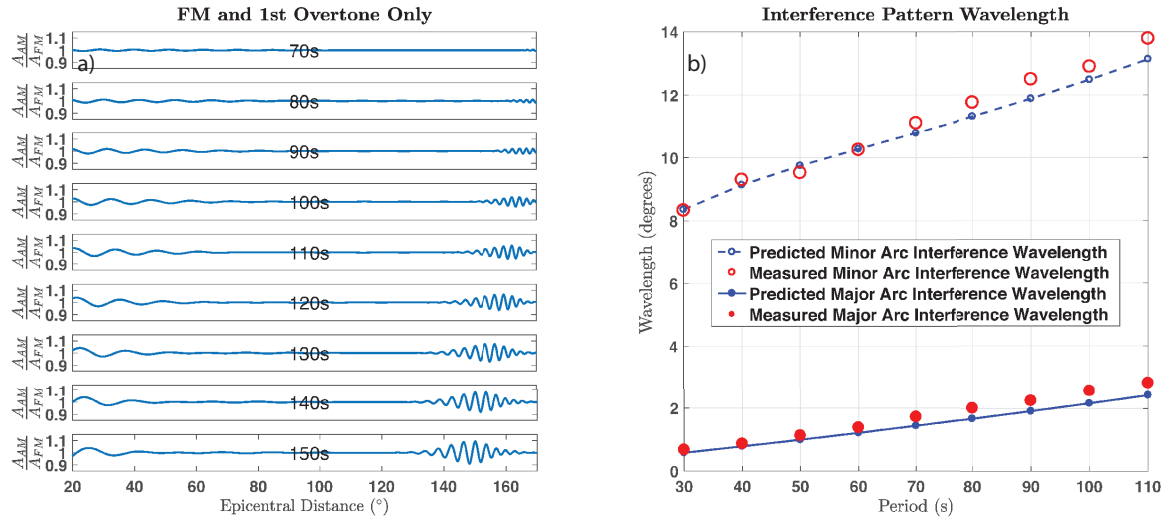
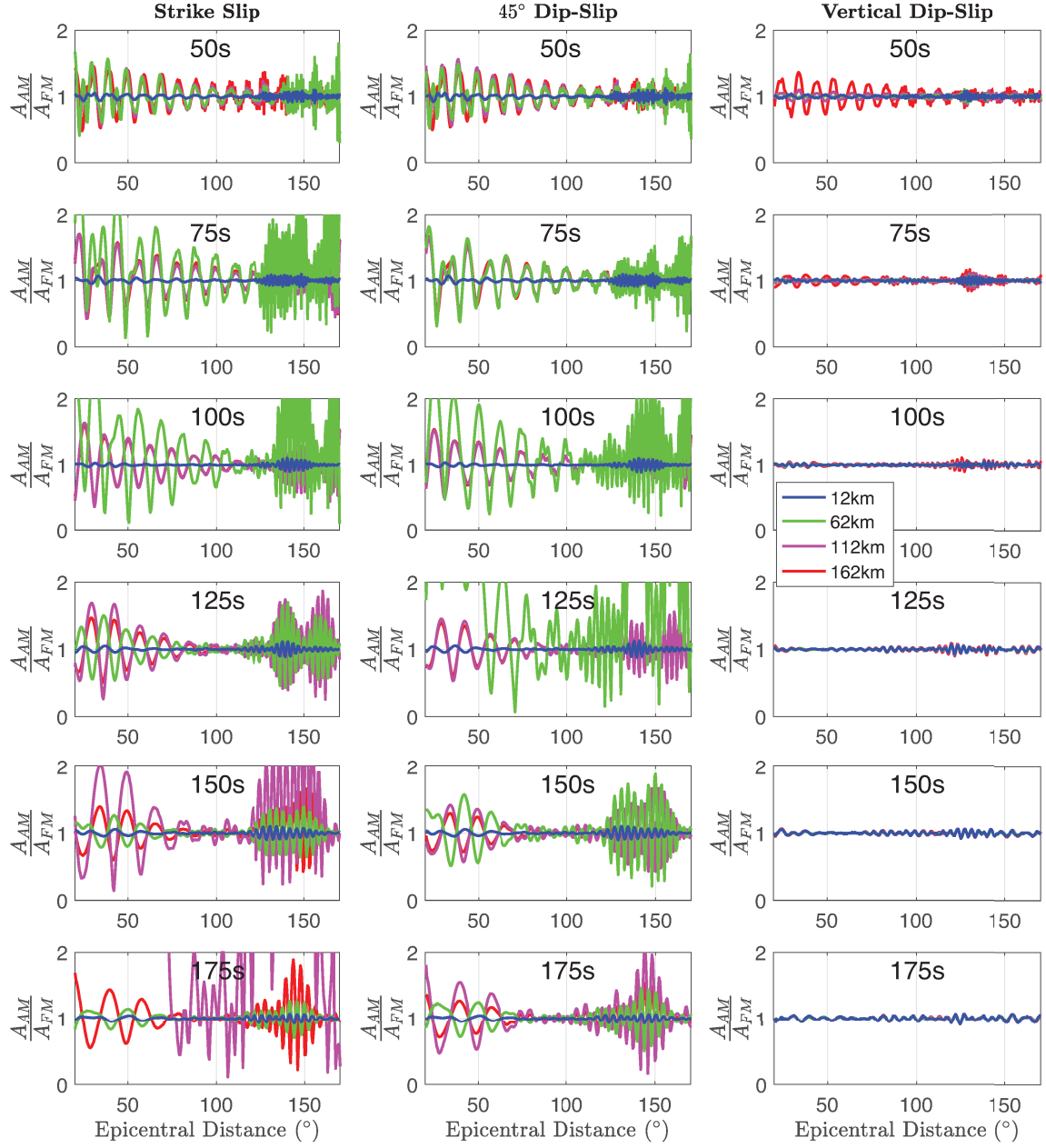


Figure 3.21: **Figure S5:** (a) Rayleigh wave amplitude interference patterns determined from normal-mode synthetic seismograms with an explosive source. The vertical axis shows the ratio of amplitudes measured from synthetic seismograms that contain the first and fundamental modes to those measured from synthetic seismograms containing only the fundamental mode. (b) Comparison of interference wavelength measured from the Rayleigh wave amplitude interference patterns in (a) (see text) to that predicted using Equations 3 and 4.





**Figure S6:** Rayleigh wave amplitude interference patterns across both the distance range prone to minor-arc overtone interference and that prone to major-arc overtone interference, color-coded by depth and shown at all periods analyzed. Each column corresponds to a different example source mechanism.

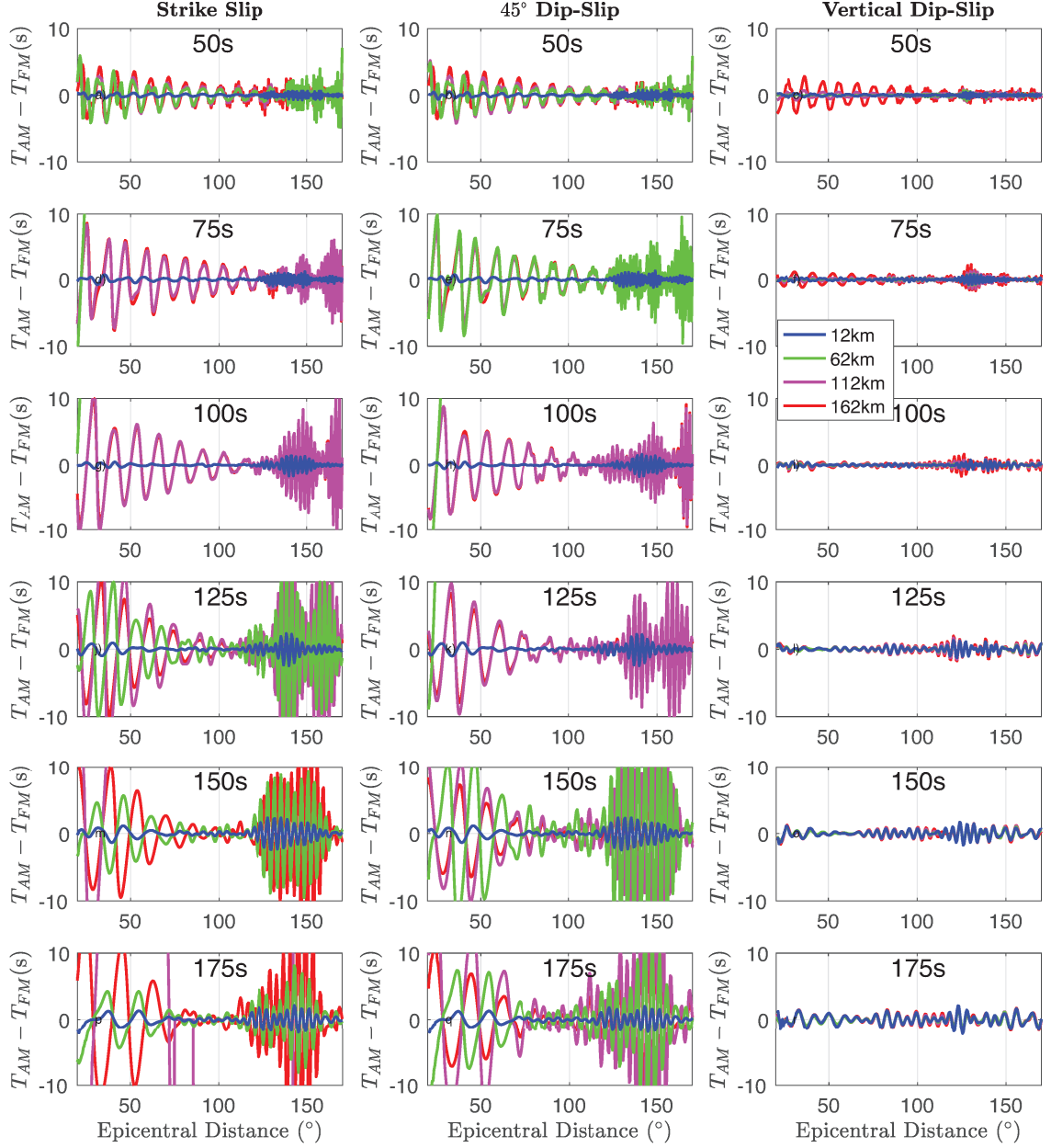


Figure 3.22: **Figure S7:** As in Fig. S6 but showing Rayleigh wave travel-time interference patterns instead of amplitude interference patterns. The measurements that go off-scale are: (1) the 62-km strike-slip event measured at 75 s, (2) the 62-km strike-slip event measured at 100 s, (3) the 62-km 45° dip-slip event measured at 100 s, (4) the 62-km 45° dip-slip event measured at 125 s, (5) the 112-km strike-slip event measured at 150 s, and (6) the 112-km strike-slip event measured at 175 s.

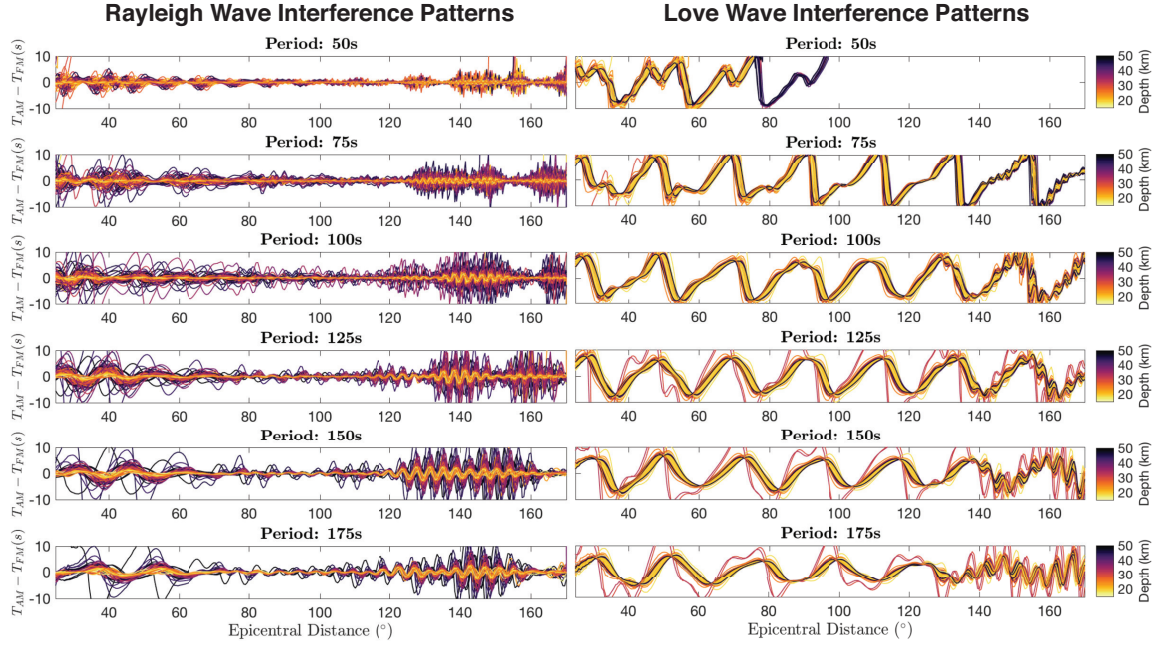


Figure 3.23: **Figure S8:** As in Fig. 10 but showing travel-time interference patterns instead of amplitude interference patterns. Interference patterns are color-coded by depth for Rayleigh (left) and Love (right) waves across a range of periods. The interference patterns are computed for 200 randomly selected events from the Global CMT catalog with source depth between 15 and 50 km.

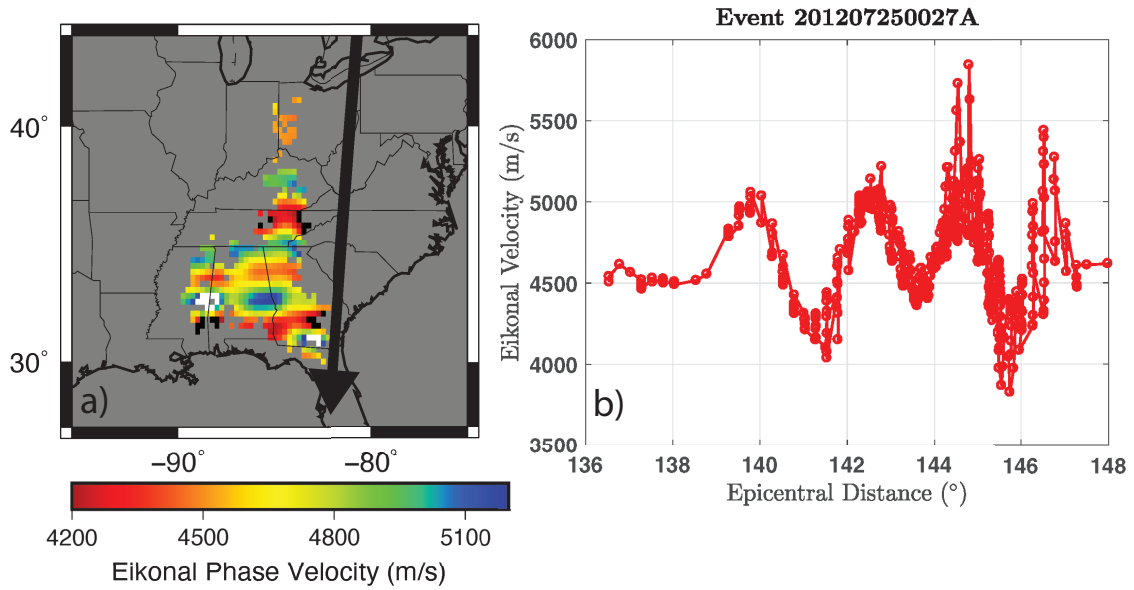


Figure 3.24: **Figure S9:** Example of Love wave major-arc overtone interference at 75 s. (a) Eikonal velocities in map view. (b) Eikonal velocities plotted as a function of epicentral distance. Measurements are made using phase-matched filtering on event ID 201207250027A, which is located at  $95.93^{\circ}\text{E}$ ,  $2.46^{\circ}\text{N}$ , and at a depth of 20 km. The black arrow illustrates a great-circle ray path.

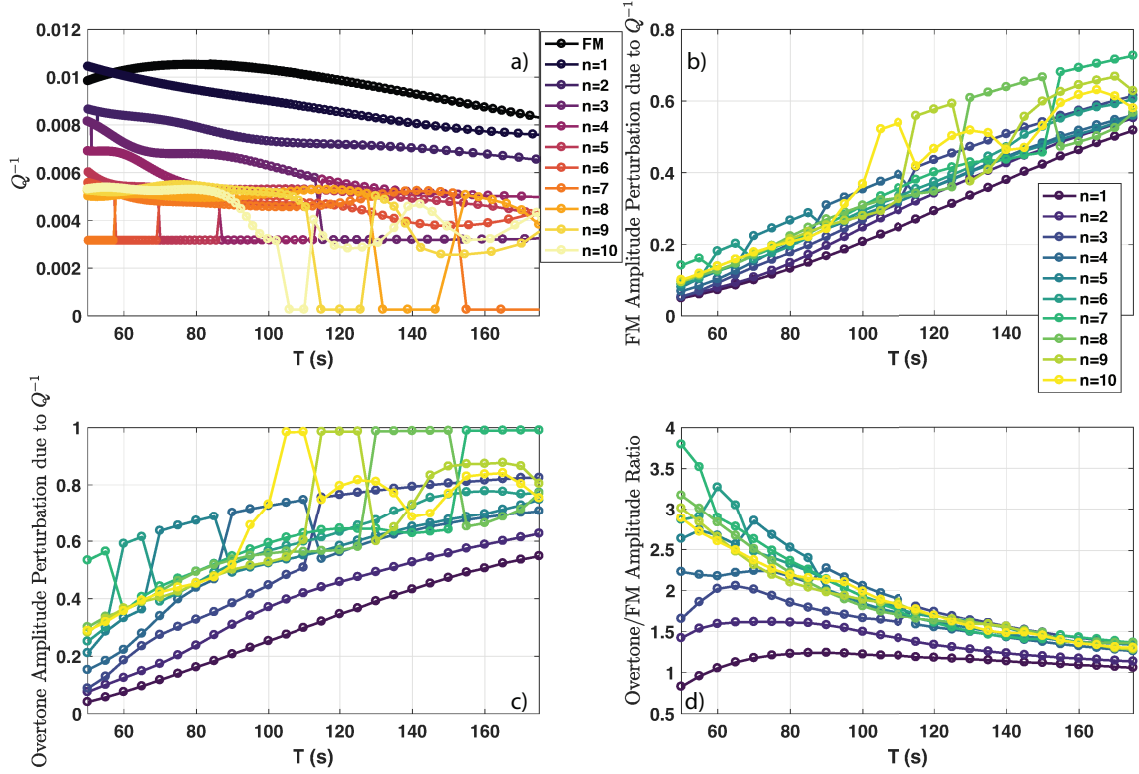


Figure 3.25: **Figure S10:** The impact of attenuation on Rayleigh wave major-arc interference. (a) Rayleigh wave attenuation ( $Q^{-1}$ ) as a function of period for the first 10 overtones and FM. (b) Amplitude perturbation to the FM at the minor-arc distance at which it intersects an overtone, due to attenuation. (c) Amplitude perturbation to the overtone at the major-arc distance at which it intersects the FM, due to attenuation. (d) Ratio of the amplitude calculations in (c) and (b).

### Error in Eikonal and Helmholtz Velocities ( $T = 100\text{s}$ )

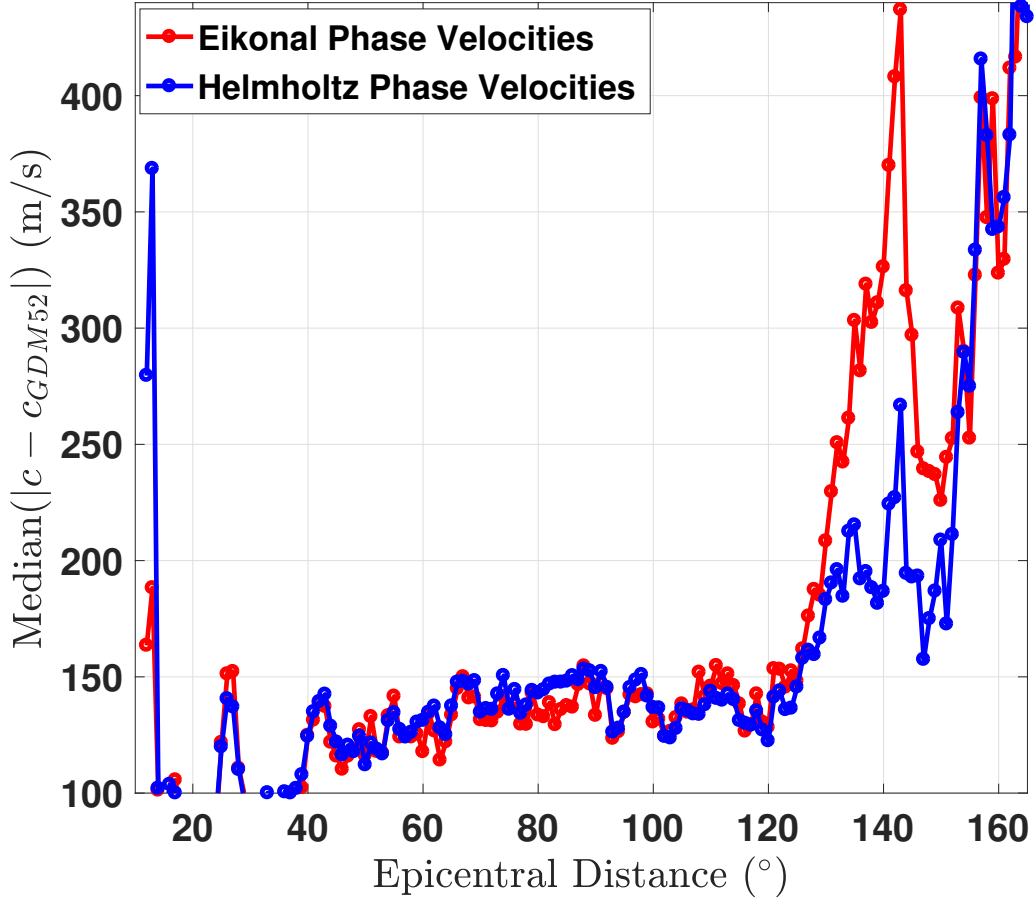


Figure 3.26: **Figure S11:** Median absolute errors in real USArray Rayleigh wave phase velocities at 100 s. As in the left vertical axis in Fig. 14c but showing both Eikonal (red) and Helmholtz (blue) phase velocities. Error in phase velocity measurements, relative to the GDM52 phase velocity maps, is binned as a function of epicentral distance.

## Chapter 4 | Love wave tomography of the United States

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## 4.1 Abstract

Love wave phase velocity maps provide essential constraints on radial anisotropy and deformation in the crust and upper mantle. However, the phenomenon of overtone interference causes scatter and systematic bias in the velocity measurements and impedes efforts to image small-scale anisotropic variations. We develop an approach for identifying Love wave measurements that are biased by overtone interference, demonstrate its efficacy with EarthScope USArray data, and determine the first earthquake-derived Love wave phase velocity maps for the entire conterminous U.S. in the period range 35–75 s. We show that radial anisotropy in parts of the crust and most of the lithospheric mantle is necessary to reconcile these maps with Rayleigh wave phase velocities. Our results convey the impact and geographic variability of overtone interference, offer an easy-to-implement method to ameliorate this impact, and present high-resolution constraints on radial anisotropy beneath North America.

## 4.2 Plain Language Summary

Measurements of Love wave phase speeds at different frequencies provide unique information that can help understand how the Earth’s interior is deforming. These measurements are difficult to make because other seismic waves interfere with the one we are attempting to measure, resulting in low-quality and biased measurements. Here, we calculate the times at which the different waves arrive and use these times to identify and remove measurements impacted by interference. The resulting cleaned data set increases the precision of derived images of the Earth’s interior and reduces bias in these images. Our images suggest shear waves polarized in vertical and horizontal directions are required to have different speeds.



### 4.3 Introduction

As minerals such as olivine undergo finite strain, they develop a crystallographic or lattice-preferred orientation (LPO), which manifests as anisotropy in measurements of seismic wavespeed (Ribe, 1992). Radial seismic anisotropy, which describes a material with transverse isotropy and a vertical axis of symmetry (Babuska and Cara, 1991), causes a difference in the speeds of horizontally ( $V_{SH}$ ) and vertically polarized ( $V_{SV}$ ) shear waves. Love and Rayleigh wave phase velocities are strongly sensitive to  $V_{SH}$  and  $V_{SV}$ , respectively, and provide the best constraints on radial anisotropy in the lithosphere and asthenosphere. Global studies and those focused on the Pacific basin agree reasonably well on the long-wavelength variations in radial anisotropy and have found, for example, that anisotropy can be attributed to both ridge corner flow and shear between the lithosphere and asthenosphere (Eddy *et al.*, 2022), and that a mismatch between observed radial anisotropy and that predicted by geodynamic models of mantle flow implies the presence of small-scale convection in the deeper asthenosphere ( $\approx 250$  km) (Becker *et al.*, 2008).

Much less is known about smaller-scale variations in radial anisotropy, especially in the continental upper mantle, where radial anisotropy is useful for mapping the degree of crustal extension (Moschetti *et al.*, 2010) and may explain a mid-lithospheric discontinuity observed by scattered body waves (Selway *et al.*, 2015; Karato and Park, 2018). A comparison of four recent, state-of-the-art models of radial anisotropy beneath North America (Fig. S1) shows significant disagreement in both the amplitudes and patterns of heterogeneity, even between models developed with a similar full-waveform inversion approach. This disagreement suggests a consensus has not yet been reached on the distribution of radially anisotropic heterogeneity in the upper mantle beneath the U.S., precluding using these models to make robust inferences about geodynamic processes. A path forward is to incorporate information from

regional earthquake-derived Love wave phase velocity maps, which can offer high resolution at the continental scale (e.g. Tao *et al.*, 2021; Li and Li, 2015) but were not used in developing the current generation of models. However, it is challenging to develop these regional-scale Love wave phase velocity maps due to noise on the horizontal components of seismometers (e.g. Rohde *et al.*, 2017; Zürn *et al.*, 2022) and overtone interference (Thatcher and Brune, 1969). In this study, we focus on resolving the latter difficulty.

Overtone interference refers to the presence of higher modes in the time window of a waveform where an operator seeks to measure the phase and amplitude of the fundamental mode (FM). Love waves are particularly vulnerable to overtone interference because the FM Love wave has a similar group velocity to that of the first, and in some cases second and third, overtones at periods less than roughly 120 s (Fig. 1a), particularly for oceanic Earth models. Previous work has demonstrated that overtone interference can introduce bias into Love wave phase velocity measurements by causing phase and amplitude measurements to oscillate about their true value as a function of distance (Foster *et al.*, 2014; Hariharan *et al.*, 2022). Fig. 1b shows an example of this using amplitude measurements made on two sets of synthetic seismograms calculated using normal-mode summation with MINEOS (Masters *et al.*, 2011); one set includes all modes and the other includes only the FM. Amplitude measurements show a long-wavelength oscillation as a function of epicentral distance only when the higher modes are included in the seismograms. The corresponding oscillation in phase (not shown) has a negligible effect on global-scale inversions for radial anisotropy using measurements from a wide range of epicentral distances (Nettles and Dziewoński, 2011), but when phase velocities are measured using differences in phase at nearby stations, as in wavefront-tracking approaches such as Eikonal or Helmholtz tomography (Lin *et al.*, 2009; Lin and Ritzwoller, 2011), or with the two-station method (Foster *et al.*, 2014), the resulting phase velocity measurements oscillate dramatically

about their true value, resulting in error of 10% or more for regional-scale studies (Foster *et al.*, 2014; Hariharan *et al.*, 2022). Furthermore, the phase oscillations are not symmetric but sawtoothed as a function of distance. This asymmetry in phase, when differentiated to produce velocity, causes many Love wave phase velocity measurements to be systematically higher than their true value (Foster *et al.*, 2014). This bias has led phase velocity maps constructed from earthquake-derived Love waves to have systematically higher values than their ambient-noise derived counterparts at the same period (Jin and Gaherty, 2015).

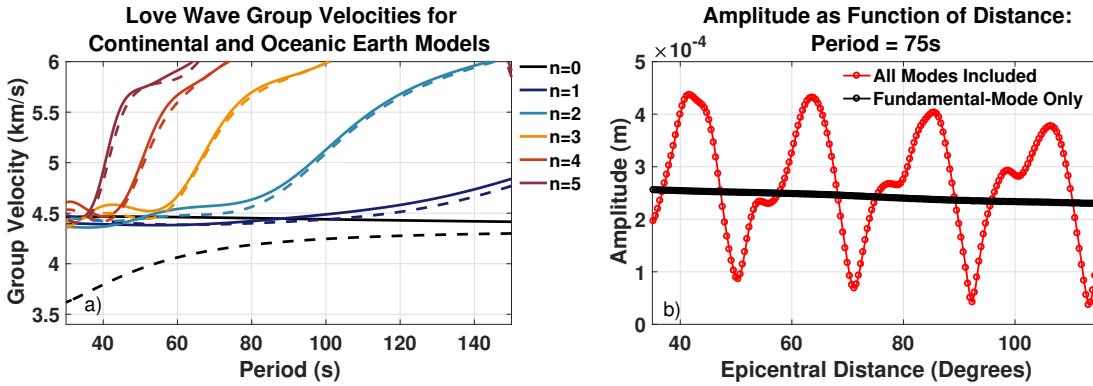


Figure 4.1: Left: Love wave group velocities for the FM and first five higher modes for continental (dashed line) and oceanic (solid line) Earth models. The continental Earth model is STW105-C (Hariharan *et al.*, 2022) and the oceanic Earth model is ATL2a (James *et al.*, 2014). Right: Love wave amplitudes measured using Fourier analysis on seismograms including all modes (red) and including only the FM (black) at a period of 75 s. The source mechanism for the synthetic seismograms is a strike-slip fault at a depth of 12 km.

In this study, we introduce a quality control method based on removing paths with similar FM and overtone group arrival times in order to minimize bias due to overtone interference. We demonstrate its effectiveness via three metrics applied to Love wave phase velocity measurements: (1) the elimination of bias toward anomalously high velocity, (2) the suppression of scatter, and (3) the suppression of strong distance-dependent oscillations. Our approach is couched in intuitive physics and easy to implement. We apply it along with wavefront tracking to determine new earthquake-derived Love wave phase velocity maps for the contiguous U.S. With predictions of

the  $V_{SV}$  model of Shen and Ritzwoller (2016), we test the null hypothesis of isotropy in the North American lithosphere.

## 4.4 Data and Methods

### 4.4.1 Measurements of Love Wave Phase Velocities

We use Eikonal tomography (Lin *et al.*, 2009) to generate maps of Love wave phase velocity for individual earthquakes. We use a data set of Love wave phase delays observed at EarthScope USArray stations (Eddy and Ekström, 2014, 2020), which contains 1100-1600 teleseismic events at each period and was measured with the approach of Ekström *et al.* (1997). The event-specific maps are obtained through several pre-processing steps broadly described by Babikoff and Dalton (2019) and specifically outlined in Text S1. The final map at any period is calculated by taking the median from all the individual event-based phase velocity maps.

### 4.4.2 Description of Our Quality-Control Scheme

We use the difference between the predicted group arrival times of the FM and the first overtone as a proxy for the strength of overtone interference in any phase velocity measurement. We henceforth refer to this value as  $\Delta T$ .

Predicting group arrival times requires globally defined maps of group speed, which have not been published for Love wave overtones. We therefore calculate global group velocity maps for the FM and the first overtone using an *a priori* model of 3-D Earth structure. To generate a global Earth model for this calculation, we overlay the crustal model CRUST1.0 (Laske *et al.*, 2013) on the mantle model S362ANI (Kustowski *et al.*, 2008). Depth-dependent profiles are then sampled on an evenly spaced grid and provided as input to MINEOS (Masters *et al.*, 2011) to calculate group velocities for the Love wave FM and first overtone. For every period, the location-specific

calculations are combined to form group velocity maps, which are used to predict group arrival times for every path in our data set by integrating along the great-circle path from source to receiver. Our predicted FM group velocity maps agree well with the measured GDM52 maps (Ekström, 2011), with an average unsigned difference of less than 0.15 km/s and a correlation coefficient greater than 0.83 at all periods. We calculate  $\Delta T$  as the first overtone arrival time subtracted from the FM arrival time; thus, most  $\Delta T$  values are positive. This yields a  $\Delta T$  value for every phase velocity measurement for every event in the data set.

## 4.5 Results: Application of Quality Control to Love Wave Phase Velocity Measurements

In this section, we demonstrate that selecting high-quality Love wave phase velocity measurements on the basis of their  $\Delta T$  value reduces the negative impacts of overtone interference. As discussed in Section 1, we are concerned with three distinct consequences of overtone interference on Love wave phase velocity measurements: 1) a bias to anomalously high values, 2) increased scatter, and 3) a strong oscillatory distance dependence. To investigate how these consequences are affected by data selection using  $\Delta T$ , we define measurement error relative to the GDM52 global phase velocity maps (Ekström, 2011); we consider GDM52 to be a smoothly varying reference that is largely unbiased by overtone interference since it was built from phase measurements integrated over long propagation paths. Error for event  $i$  in pixel  $j$  is calculated as  $E_{ij} = c_{ij}^{obs} - c_j^{GDM52}$ , where  $c_{ij}^{obs}$  refers to the observed event-specific Eikonal phase velocity maps. In the presence of overtone interference, we expect  $E_{ij}$  to 1) be systematically greater than zero (Foster *et al.*, 2014), 2) exhibit greater scatter than if no overtone interference is present, and 3) show an oscillatory dependence on epicentral distance.

Every error value has a corresponding  $\Delta T$  value. In Fig. 2a we bin error by  $\Delta T$  and plot the median error in each bin. Fig. 2a shows that phase velocity measurements corresponding to lower  $\Delta T$  values are biased high relative to the reference value. The median bias at low  $\Delta T$  is 60-80 m/s (1.5-2.0%), meaning that half of the measurements suffer an even larger bias. However, the median bias decreases noticeably for measurements with larger  $\Delta T$  values and approaches zero bias for most periods as  $\Delta T$  approaches the period of interest. Fig. 2b shows that retaining measurements with large  $\Delta T$  values also reduces the standard deviation of the error in Love wave phase velocities, which is a measure of the amount of scatter in the measurements.

We find that the signed error at low  $\Delta T$  is consistently larger at longer periods (Fig. 2a). The  $\Delta T$  value only measures the separation between the peak of the FM and overtone group envelopes, not the wavepackets themselves. At longer periods, these wavepackets by definition take up more space in the time domain; thus, a larger  $\Delta T$  is required at longer periods to fully separate the two wavepackets. To account for the effect of wavepacket size, we scale  $\Delta T$  by the period of the waveform when choosing a threshold to isolate good measurements. We find that retaining the subset of phase velocity measurements for which  $\Delta T \geq 1.2 \times \text{period}$  (e.g.,  $\Delta T \geq 54$  s for period=45 s and  $\Delta T \geq 72$  s for period=60 s) results in a quality-controlled data set with median signed error less than 15 m/s at all periods, thereby eliminating the majority of the systematic bias while still retaining many measurements. This retains  $\approx 43\%$  of the data set at the shortest periods and  $\approx 14\%$  at the longest periods. We proceed using this threshold for quality control but note that others may choose differently depending on the size, event distribution, and extent to which overtone interference is present in their data set (Section 4.7.1).

Importantly, we find that quality control by group arrival time suppresses, and in some cases eliminates, the oscillations in phase velocity measurements as a function of epicentral distance (Fig. 2c,d). When only measurements with  $\Delta T$  values below

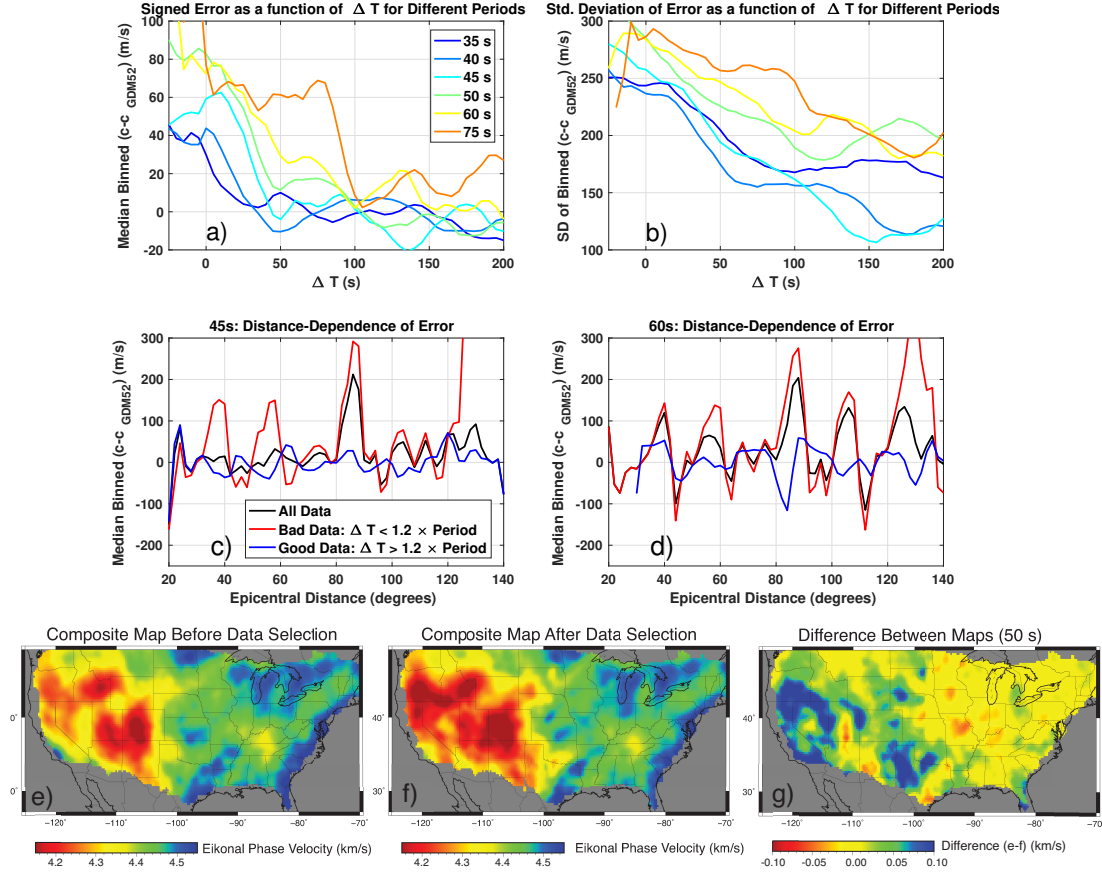


Figure 4.2: a) Median signed errors in USArray Love wave phase velocities with respect to the GDM52 maps, binned as a function of  $\Delta T$ , which is the difference in the group arrival times of the FM and first overtone. b) Standard deviation in the errors in USArray Love wave phase velocities with respect to the GDM52 maps, binned as a function of  $\Delta T$ . c) Distance dependence of phase velocity measurements for period=45 s. Blue line: Only using data that pass our quality control criteria. Red line: Only using data that fail our quality control criteria. Black line: All data. d) As in (c) but for period=60 s. e,f,g): Illustration of the impact of quality control on 50-s Love wave phase velocity map. e) No data selection based on group arrival time difference  $\Delta T$ . f) Data selection based on group arrival time difference  $\Delta T$ . g) Difference, with (f) subtracted from (e). Positive values indicate faster velocity in (e).

the threshold of  $1.2 \times \text{period}$  are used (i.e., selecting for measurements with strong interference), the measurement error has a clear periodic distance dependence with a wavelength of about  $20^\circ$ . This indicates that our metric is successfully able to identify biased measurements. On the other hand, when only measurements with  $\Delta T$  values above the threshold are used, the resulting distance dependence is weak

at all periods and distances, indicating that our metric is successfully able to isolate good measurements. For the full data set with no selection, the binned values show a strong distance dependence at long periods that grows weaker at shorter periods (Fig. S2).

Finally, in Fig. 2e,f,g we show that the quality control approach reduces bias in composite phase velocity maps. We compare maps constructed with (Fig. 2f) and without (Fig. 2e) data selection and find that although the large-scale pattern of heterogeneity is not dramatically affected, the smaller-scale features are altered and the absolute phase velocities can differ by as much as 0.2 km/s in some locations (Fig. 2g). This is consistent with the bias toward anomalously high velocity that overtone interference induces in phase velocity measurements. Fig. 2g also shows that the geographical distribution of data selection is uneven and mostly concentrated in the western U.S., where the impact of overtone interference is more significant (Section 4.7.1).

## 4.6 Results: Composite Phase Velocity Maps

We present the final phase velocity maps made using our quality control approach in Fig. 3. At periods below 45 s, our maps can be compared to the USANT15 phase velocity maps derived from ambient noise (Ekström, 2017). We observe very strong agreement between the two models at periods of 35 and 40 s (Fig. S3), with an average absolute difference of  $\approx 0.03$  km/s.

The distributed depth-sensitivity of Love wave phase velocity measurements renders it impossible to interpret Love wave phase velocity anomalies in terms of shear velocity at specific depths. Nonetheless, the heterogeneity in our Love wave maps exhibits correlations with specific tectonic features (labeled in Fig. 3). At short periods ( $T \leq 45$  s), our measurements are very sensitive to crustal structure. At these periods, the lowest velocity anomalies occur along and south of the Southern Rocky



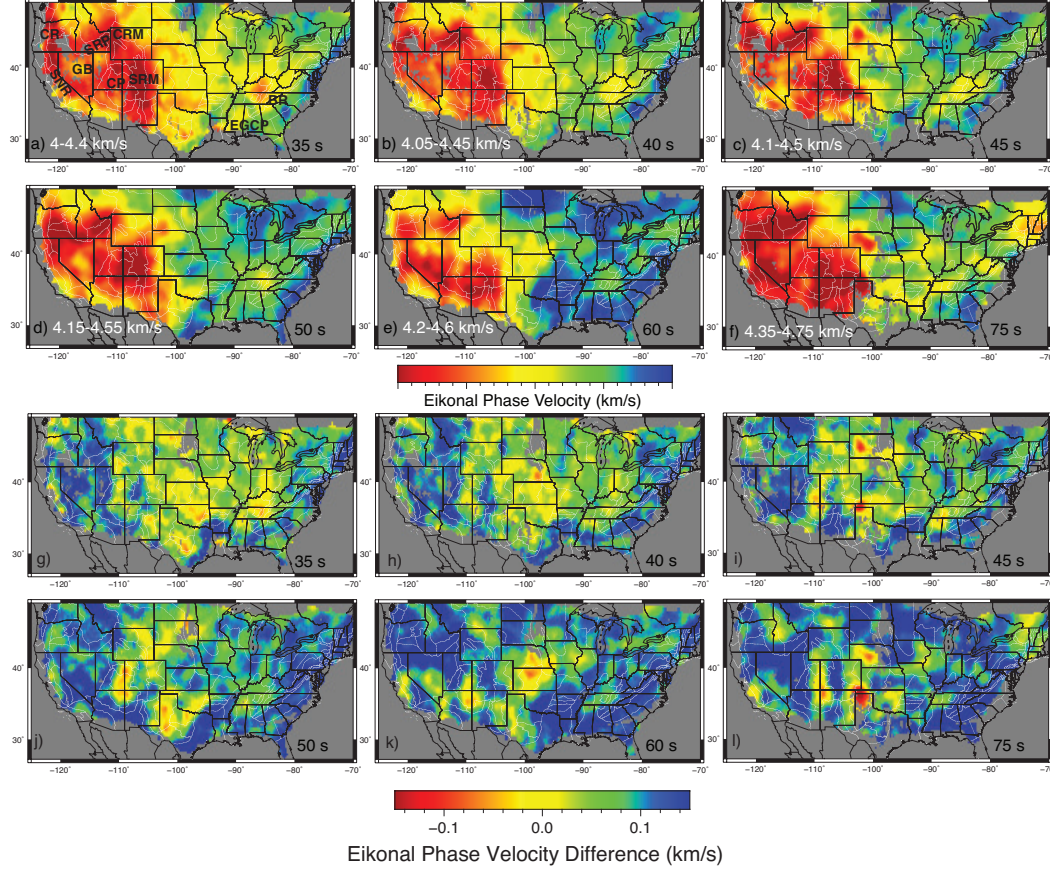


Figure 4.3: (a-f) New Love wave phase velocity maps, generated with data that meet our quality control approach. The period corresponding to each map is noted below at the bottom-right corner of each subplot, and the phase velocity range of the colorbar is shown on the bottom left. Pixels shown in gray had measurements from five or fewer events after quality control. Overlain on each map as white lines are physiographic provinces (after Fenneman (1928)). CR is Cascade Range; SNR is Sierra Nevada Range; BR is Blue Ridge Province; SRP is Snake River Plain; GB is Great Basin; CP is Colorado Plateau; SRM is Southern Rocky Mountains; EGCP is Eastern Gulf Coast Plain; CRM is Central Rocky Mountains. (g-l) Differences between our measured Love wave phase velocity maps and the phase velocity maps predicted using the isotropic shear velocity model SR16 (Shen and Ritzwoller, 2016); predicted phase velocities are subtracted from measured phase velocities.

Mountains. These low velocities extend northward along the Central Rocky Mountains and are lowest at the eastern tip of the Snake River Plain. In a slight difference from the maps of Foster *et al.* (2014), we also observe low velocities along the Cascades Range. Moderately low velocities are present along the Sierra Nevada Range and the eastern edge of the Great Basin, producing a ‘ring’ around the Basin and Range. In

the eastern half of the U.S., the lowest phase velocity anomalies are located at the boundary between Kentucky and Tennessee. Many of the aforementioned zones of relatively low velocity are co-located with anomalously thick crust (Shen and Ritzwoller, 2016). At these short periods, the highest phase velocity values are located near and also along the Eastern Gulf Coastal Plain and also at the border of North Dakota and Minnesota, potentially related to the Superior Craton.

At longer periods ( $T \geq 50$  s), the locations of the lowest velocity anomalies are shifted slightly relative to shorter periods. A prominent anomaly exists beneath the Colorado Plateau rather than the Southern Rocky Mountains, and at 60 and 75 s, one of the lowest-velocity anomalies parallels the border between California and Nevada, slightly east of the Sierra Nevada Range. As at shorter periods, we find a low-velocity anomaly along the Snake River Plain. In the eastern U.S., a zone of slightly low velocities is northwest of the Blue Ridge Province in the period range 45–60 s. At 75 s, we observe distinct low-velocity anomalies that correspond to the locations of the New England Upwelling (Levin *et al.*, 2018) and the Reelfoot Rift, neither of which appear at shorter periods.

We quantify uncertainty in our maps by calculating, at each pixel, the standard deviation of all the measurements retained after quality control (Fig. S4). At all periods, the average uncertainty across all pixels is less than 0.2 km/s. We also conduct a test to verify that the systematic phase velocity differences resulting from the interference corrections are not from random error. At pixels where the phase velocity decreases by more than 0.05 km/s after quality control, we generate  $10^4$  random realizations from the uncorrected distribution, each containing  $N$  samples, where  $N$  is the number of measurements remaining after the correction. We determine the median phase velocity from each realization and calculate the percentage of realizations for which the median phase velocity is higher than the corrected phase velocity (Fig. S5). For the majority of pixels at all periods, more than 90% of the random

realizations meet this requirement, suggesting that the changes in phase velocity due to data selection reflect reduction of the bias due to overtone interference rather than random variability introduced by changing the distribution from which the median is calculated.

## 4.7 Discussion

### 4.7.1 Effects of Our Quality-Control Approach and Implications for Overtone Interference Globally

Even within the U.S., there are spatial variations in the degree to which a Love wave data set is biased by overtone interference and the effect of our quality control scheme on the features in phase-velocity maps (Fig. 2e,f,g). This is due to a station’s location relative to the geographic distribution of earthquake sources as well as lateral variations in seismic velocity. In this section we conduct a simple experiment to explore the range in  $\Delta T$  values and the extent of Love wave overtone interference at different locations on the Earth.

We divide the Earth’s surface into pixels and assume that every pixel records all earthquakes in the Global CMT catalog (Ekström *et al.*, 2012) that occurred during 2014 and 2015 and had  $M_w \geq 6$ . For every source-pixel pair, we calculate  $\Delta T$  by integrating the FM and first-overtone group velocity maps (Section 4.4.2) along the great-circle path. Each pixel is then characterized by a distribution of  $\Delta T$  values, each representing the degree of overtone interference, with smaller  $\Delta T$  indicating stronger interference. We summarize this distribution with a single value: the percentage of measurements at each pixel with  $\Delta T$  value greater than  $1.2\times$  the period of interest, corresponding to our preferred quality control threshold for data selection.

Maps of this quantity (Fig. 4) show that the likelihood of measuring Love waves that are minimally affected by overtone interference depends strongly on where the

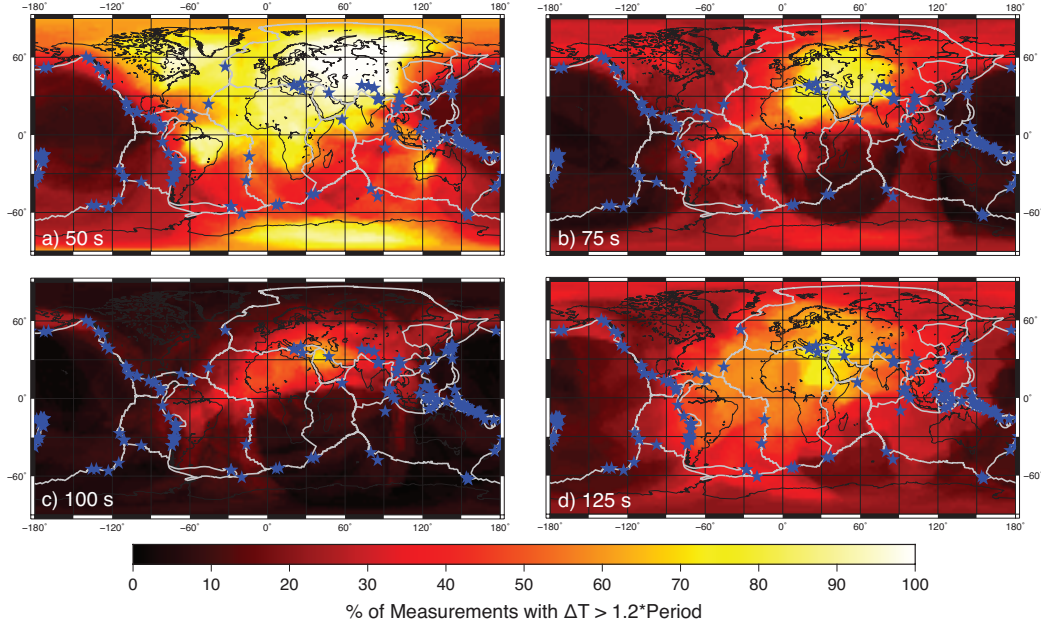


Figure 4.4: (a-d) Maps showing the percentage of measurements that would satisfy our quality control criteria, assuming every pixel on the Earth’s surface records every earthquake from the Global CMT catalog (Ekström *et al.*, 2012) with  $M_w \geq 6$  through 2014 and 2015. Locations of earthquakes used in this test are plotted as blue stars. Plate boundaries from Bird (2003) are overlain as gray lines.

measurements are made. Locations where interference-free measurements are likely correspond to source-receiver paths where a considerable fraction of the propagation length is through continental lithosphere, where the FM and overtone group velocities are reasonably well separated (Fig. 1a). Our analysis shows that, at all periods, these locations tend to be in Europe, northern Africa, and western Asia. Locations where overtone interference affects a high percentage of Love wave measurements include the Pacific Ocean, the southern half of the Indian Ocean, the western U.S., and the eastern half of Australia. In addition to these geographical variations, there is a tendency for more high-quality measurements to be possible at the shortest and longest periods analyzed ( $T \leq 50$  s and  $T \geq 125$  s) because of the larger group-velocity separation between the FM and overtones. For example, a measurement on the east coast of the U.S. has a 75% chance of passing our quality control criterion at 50 s, but a 30% chance of doing so at 100 s.

This experiment explains the regional variations in overtone interference observed with our USArray measurements. Fig. 2e,f,g shows a larger difference in the average phase velocities estimated with and without quality control at pixels in the western U.S. than in the eastern U.S. This is consistent with the results in Fig. 4a, which shows that locations in the western U.S. are more likely to record contaminated measurements. Thus, the application of our quality control criterion will more strongly impact average phase velocity measurements in the western U.S.

#### 4.7.2 Implications for Radial Anisotropy

Our Love wave phase velocity maps offer a new opportunity to study radial anisotropy in the North American lithosphere. Although a full inversion for depth-dependent shear velocity and radial anisotropy (e.g. Gao and Lekić, 2018) is beyond the scope of this study, here we examine how well a high-resolution model of isotropic shear velocity in the U.S. can explain our observations and where, if anywhere, radial anisotropy is required.

We use the shear velocity model of Shen and Ritzwoller (2016) (hereafter referred to as SR16) to test the null hypothesis that radial anisotropy is not required in the lithosphere beneath USArray. SR16 was developed from joint inversion of Rayleigh wave phase velocity and ellipticity and P-to-s receiver functions and thus constrains  $V_{SV}$ . We sample depth-dependent profiles from SR16 on an evenly spaced grid, prescribe that  $V_{SH} = V_{SV}$  and  $V_{PH} = V_{PV}$ , and use MINEOS to predict Love wave phase velocity, with details as described by Babikoff and Dalton (2019). We test end-member scenarios for depth-dependent attenuation and find that realistic attenuation variations have a negligible impact on the predicted Love wave phase-velocity maps and our conclusions about radial anisotropy.

Fig. 3g-3l plots the difference between our Love wave phase velocity maps (Fig. 3a-3f) and the predictions from SR16. In the following discussion we assume that

difference values not equal to zero indicate failure of our assumption of isotropy,  $V_{SH} = V_{SV}$ . The alternative option— that the lithosphere is isotropic but SR16 inadequately captures the seismic structure— is unlikely, since numerous other studies have produced models of  $V_{SV}$  that agree well with SR16 (Zhou *et al.*, 2022; Netto *et al.*, 2019), and Babikoff and Dalton (2019) found good agreement between their observed Rayleigh wave phase velocity maps and predictions of SR16. Nonetheless, a future 3-D inversion for radial anisotropy will carefully consider such a scenario.

We find that even after we have suppressed the bias to higher phase velocities due to overtone interference, at many locations our Love wave phase velocities are higher than the predictions. At short periods, these higher-than-predicted phase velocities are mostly distributed in the western and southeastern U.S. and closely follow the distribution of thin crust in the SR16 model. At long periods, higher-than-predicted phase velocities are found in most of the U.S.

In the western U.S., Moschetti *et al.* (2010) imaged crustal radial anisotropy using Love wave measurements from ambient noise at short periods ( $T \leq 32$  s) and Rayleigh wave measurements in the period band  $6 \text{ s} \leq T \leq 100 \text{ s}$ . They found widespread positive radial anisotropy ( $V_{SH} > V_{SV}$ ) in the middle-lower crust, and showed that this radial anisotropy was confined to provinces that underwent extension in the Cenozoic. This is consistent with our observations, particularly at 40 s, 45 s, and 50 s. At these periods, we find higher-than predicted phase velocities in the Basin and Range Province and north of the Snake River Plain, coincident with regional extension (Moschetti *et al.*, 2010). On the other hand, regions not traditionally associated with extension, such as the Colorado Plateau, the Sierra Nevada Range, the Snake River Plain, and the Columbia Plateau in Oregon, do not show higher-than-predicted velocities and positive radial anisotropy in our images. At longer periods ( $T > 50$  s), there is a reorganization in the patterns of phase velocity differences, and we instead observe higher-than-predicted phase velocities along the Snake River Plain

and west of the Colorado Plateau, potentially suggestive of a change in the patterns of radial anisotropy at subcrustal depths.

We observe different behavior in the central and eastern U.S. Here, at periods  $\leq 45$  s, Love wave phase velocities agree much more closely with predictions from the isotropic model, particularly at longitudes east of  $-105^\circ$ . More broadly, this hints that radial anisotropy may not be ubiquitous in continental crust, as has been suggested (Dalton and Gaherty, 2013). In the eastern U.S., we observe regions of higher-than-predicted phase velocities to the southeast of the Blue Ridge Province, in the Appalachian Highlands. At longer periods, we observe differences closer to zero in the central U.S. and in the vicinity of the New England Upwelling, as well as higher-than-predicted velocities along the Blue Ridge Province.

We conclude by emphasizing that the differences in observed and predicted phase velocity (Fig. 3g-3l) are similar in magnitude to the differences between Love wave phase velocity maps that are and are not corrected for overtone interference (Fig. 2e-2g). This underscores the importance of correcting for overtone-interference effects in order to accurately resolve radial anisotropy.

## 4.8 Conclusion

We present an approach to eliminate overtone interference in regional Love wave phase velocity measurements, and we apply it to measurements made at EarthScope USArray stations. By isolating measurements with a large group arrival time separation between the fundamental mode and the first overtone, we show that we are able to eliminate three sources of contamination due to overtone interference: a systematic bias to anomalously high velocity, elevated scatter in phase velocity measurements, and oscillation as a function of epicentral distance. Although the focus of this study is the U.S., we also quantify how 3-D velocity heterogeneity and the global distribution of earthquake sources produce geographic variability around the globe in the

likelihood that a Love wave measurement is contaminated by overtone interference.

Using the quality-controlled USArray data set, we present Love wave phase velocity maps spanning the conterminous U.S. in the period range 35–75 seconds. We observe correlations with known physiographic features. By comparing our maps to predictions of an isotropic shear velocity model, we show that radial anisotropy is required in the upper mantle beneath much of the U.S. and in the crust of the western U.S. However, predictions from the isotropic model can explain the Love wave phase velocities at shorter periods throughout the central U.S.

Our approach eliminates the first-order source of noise and systematic bias in Love wave phase velocities. It will also reduce contamination in Love wave amplitudes, and thereby suggests a path forward to measure secondary Love wave observables including amplification and attenuation.

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## 4.10 Supplemental Material

### 4.10.1 Introduction

This supporting information contains five supplementary figures, a description of the format of the phase velocity data set that is presented with this study, and a description of the processing steps used to calculate phase velocities. Text S1 describes the specific pre-processing steps used to convert travel-time measurements into Eikonal phase velocities. Figure S1 presents a comparison of four radially anisotropic models of the upper mantle beneath the U.S. Figure S2 shows, for all periods in this study, the distance-dependence of Love wave measurement error when different subsets of the full data set are isolated based on  $\Delta T$  values. Figure S3 compares our phase velocity images to those from the study of Ekström (2017). Figure S4 visualizes the uncertainties at all pixels of our phase velocity maps for all periods analyzed. Figure S5 presents results of a statistical test to evaluate the robustness of the overtone interference correction.

A zipped file containing our data set of phase velocity maps at the six periods used in this study accompanies this document.

#### 4.10.2 Text S1

Our measured travel times from a single earthquake are pre-processed in several ways, broadly following the approach of Babikoff and Dalton (2019). Firstly, they are averaged in cells with longitude and latitude dimensions of  $0.5^\circ \times 0.5^\circ$ . They are then interpolated onto a gridded surface under tension, with cell size  $0.25^\circ \times 0.25^\circ$ . The surfaces are smoothed with a Gaussian filter of width 400 km. Raw measurements and pixels on this surface are only retained in the final calculation if stations exist in three non-overlapping evenly spaced azimuthal bins around the pixel or station location.

We generate phase velocities with Eikonal tomography Lin *et al.* (2009). To calculate the gradient of travel time, we use a finite-difference method in spherical coordinates. This gradient is taken separately for each event, and an individual phase-velocity map for each event is solved for from the inverse of  $\sqrt{|\nabla\tau|^2}$ . As an outlier-removal step, individual phase-velocity measurements that differ by more than 1 km/s from the value predicted at that location from the GDM52 dispersion model Ekström (2011) are removed. To construct a final map at any period from all the individual event-based phase velocity maps, we take the median of all the phase-velocity measurements at any pixel. After this, pixels at the edge of the study area are removed. The map is then smoothed using a smoothing length equal to half the wavelength at the period of interest. Individual pixels in the map are only retained if 5 or more measurements contribute.

#### 4.10.3 Dataset S1

The accompanying .zip file contains six files, each of which contains a phase velocity map for a different period considered in this study. The file naming convention is *Love\_phvel\_period*, where the variable *period* spans the values 35, 40, 45, 50, 60, or 75 s. Each file contains three columns. The first column is longitude, the second is latitude,

and the third column is phase velocity at the pixel defined by the corresponding latitude/longitude pair.

Figure S1

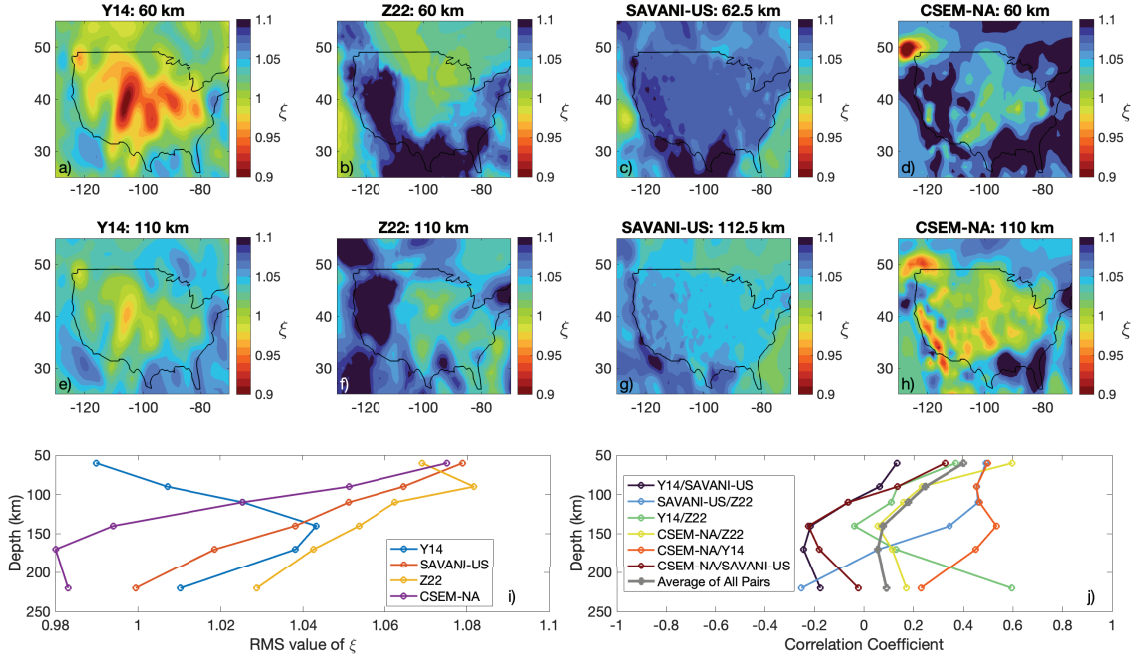


Figure 4.5: **Figure S1:** Map view plots of four different models of radial anisotropy ( $\xi = (V_{SH}/V_{SV})^2$ ) at depths of 60 km (a-d) and 110 km (e-h). (i): The average (RMS) strength of radial anisotropy within the conterminous U.S is shown as a function of depth for all models. (j) pair-wise correlation coefficient for all model pairs and the average of all pairs for structure within the conterminous U.S. Calculations (i) and (j) only used structure within the black polygon. The models used are Y14 Yuan *et al.* (2014), Z22 Zhu *et al.* (2017), SAVANI-US Porritt *et al.* (2021), and CSEM-NA Fichtner *et al.* (2018).



Figure S2

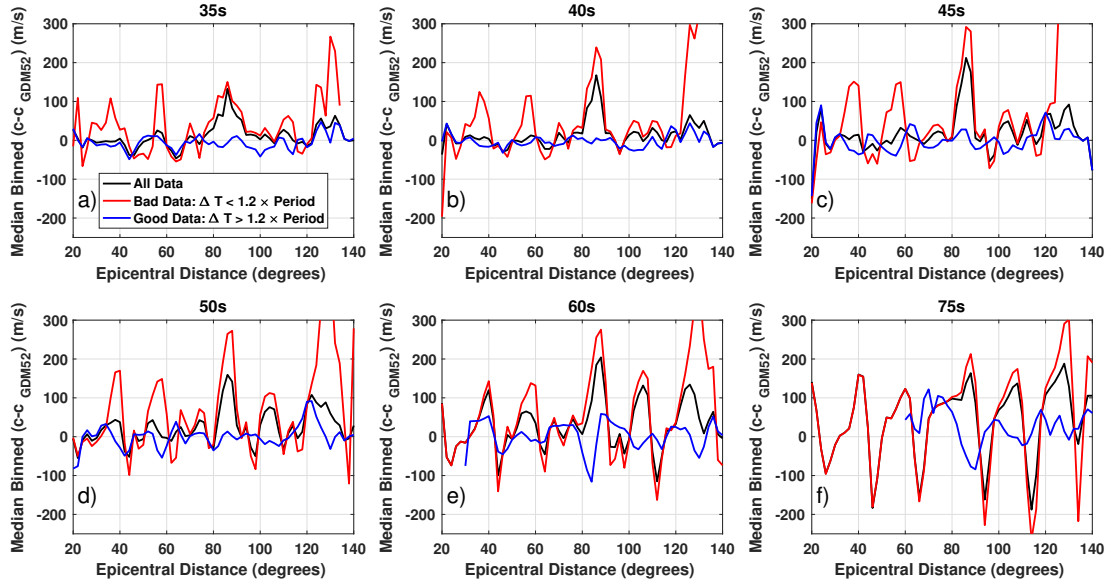


Figure 4.6: **Figure S2:** (a-f): Distance dependence of error in phase velocity measurements for different periods, shown in the title. Blue line: Only using data that pass our quality control criteria. Red line: Only data that fail our quality control criteria. Black line: All data.

**Figure S3**

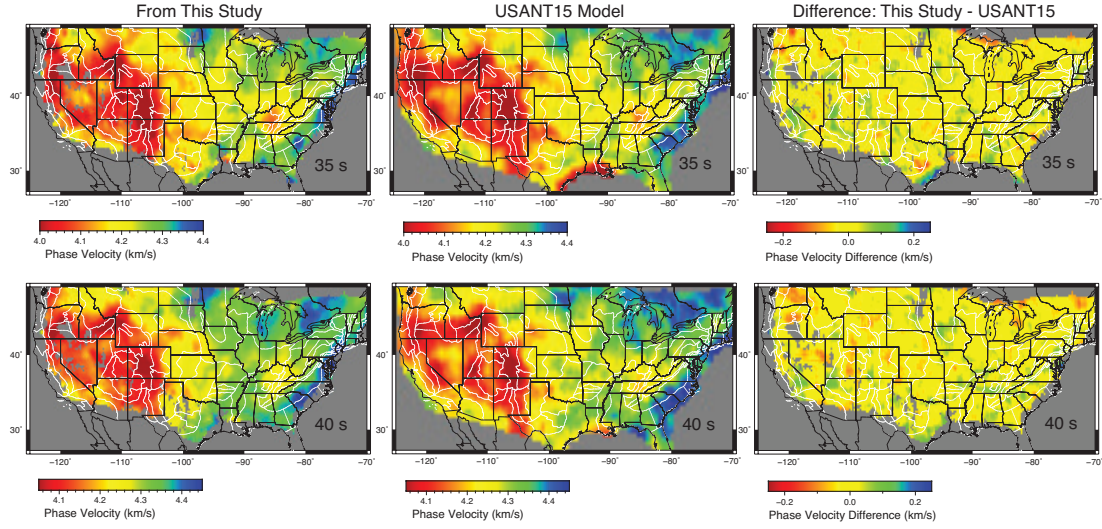


Figure 4.7: **Figure S3:** Comparison of earthquake-derived phase velocity maps generated from this study with ambient-noise-derived maps from the USANT15 model Ekström (2017). Left column: map view plots of our models. Center: map view plots of the USANT15 model. Right: Difference between both models, with the USANT15 maps subtracted from our maps. Top row: 35 s. Bottom row: 40 s.

Figure S4

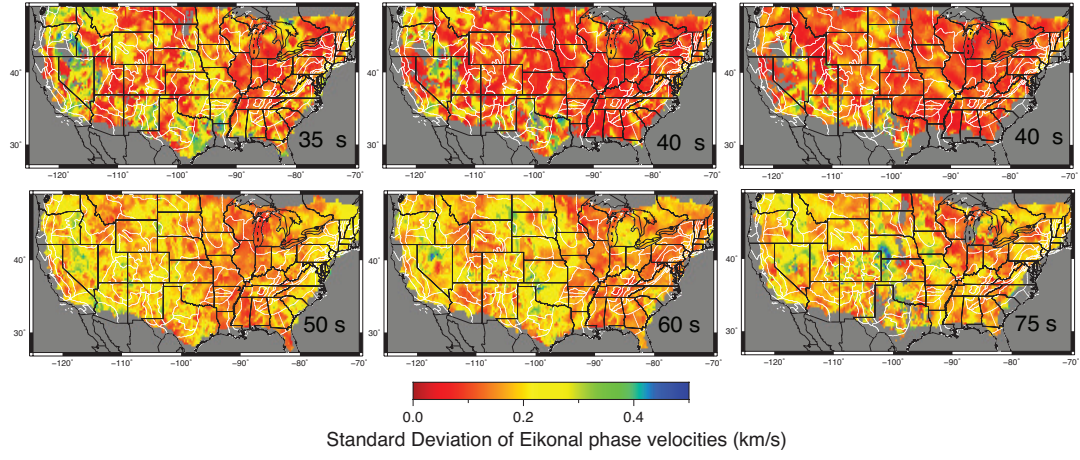


Figure 4.8: **Figure S4:** Standard deviation of phase velocity measurements at every pixel after quality control, visualized in map view for all periods in this study. The period corresponding to each map is shown in the bottom-right corner of each map.

Figure S5

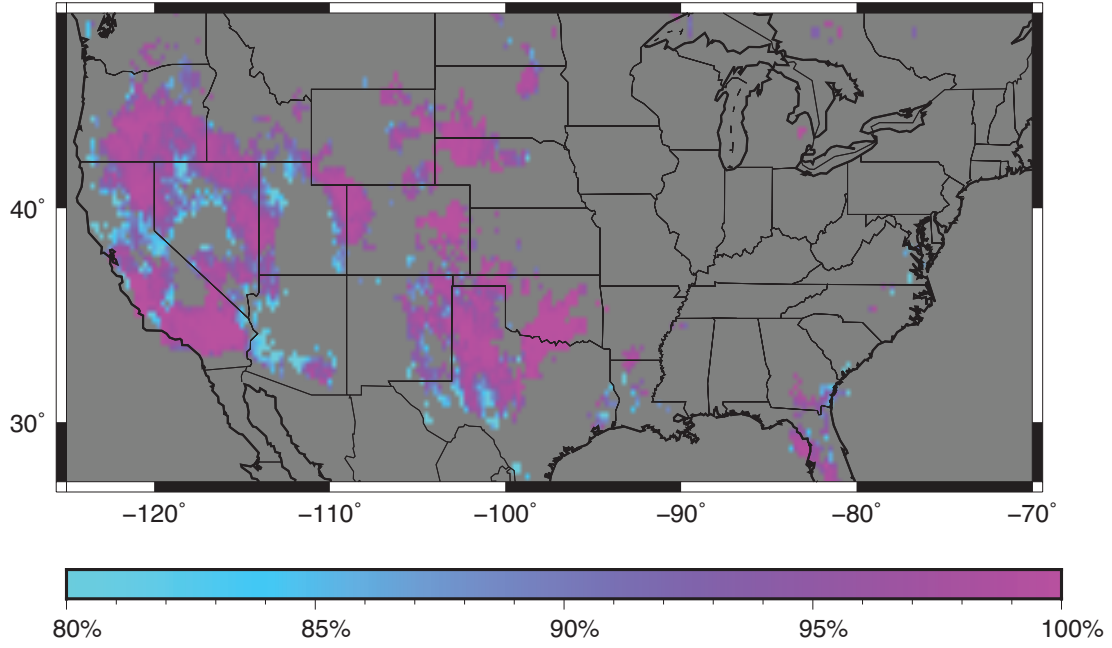


Figure 4.9: **Figure S5:** Results of a bootstrapping analysis for measurements at 50 s to explore the robustness of the phase velocity change after correction for overtone interference. We first identify the 2807 pixels in which the median phase velocity after quality control decreases by at least 0.05 km/s relative to the uncorrected value (i.e., before quality control). In each pixel, we generate  $10^4$  random realizations of size  $N$  from the original uncorrected distribution, where  $N$  is the size of (i.e., number of events in) the corrected distribution, and determine the median phase velocity from each realization. Plotted are the percentage of these realizations for which the median phase velocity of the random realization is greater than the corrected median phase velocity.

# **Chapter 5** | Radial Anisotropy in the Uppermost Mantle Beneath the Continental U.S from a Joint In- version of Rayleigh and Love wave Phase Ve- locities

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There do not yet exist high-resolution images of the radially anisotropic character of the crust and uppermost mantle beneath the continental U.S. These images can provide direct constraints on the distribution of finite strain within the mantle lithosphere, helping to validate or eliminate certain geodynamic hypotheses for the formation and destruction of continental lithosphere, the lateral distribution of crustal deformation, and the location of asthenospheric flow.

In this study, we synthesize several recent Rayleigh and Love wave phase velocity images of the continental U.S. to solve for the best-fitting model of radial anisotropy, horizontally polarized shear wavespeed ( $V_{sh}$ ) and vertically polarized shear wavespeed ( $V_{sv}$ ) in the crust and uppermost mantle of the continental U.S. The resulting model, which provides an excellent fit to observations, shows that the crust is isotropic in much of the central and eastern U.S. Where crustal radial anisotropy is required, regions of radial anisotropy ( $\xi = \frac{V_{sh}}{V_{sv}}^2$ ) greater and less than unity are well correlated with regions of extension and compression respectively. In the model, a sharp change in the patterns of radial anisotropy occurs at approximately 75 km. In the upper mantle, we observe regions of  $\xi \geq 1$  in the asthenosphere beneath the Snake River Plain, consistent with lateral flow from a mantle upwelling. We find that  $\xi \leq 1$  beneath the Colorado Plateau, consistent with lithospheric downwelling. We show that accounting for radial anisotropy when calculating isotropic velocities can have a substantial impact on applications such as thermochemical inferences. The isotropic part of our model contains higher isotropic velocities in the eastern U.S. than other recent studies, including Shen and Ritzwoller (2016).

We also perform a suite of synthetic tests to explore the robustness of our model, highlighting (1) the requirement of radial anisotropy to fit the data, (2) the need to allow variations in both horizontally and vertically polarized shear wave velocity to avoid artifacts, and (3) the similarity of the results that are obtained with different sets of Rayleigh wave measurements from earthquakes.

## 5.1 Introduction

Images of radial anisotropy are powerful tools to study Earth structure. At global scales, studies such as Becker *et al.* (2008) have used models of radial anisotropy in combination with geodynamic simulations to constrain the rheology of the upper mantle. In relatively well understood tectonic settings such as ocean basins, radial anisotropy has been an effective tool to understand lithosphere-asthenosphere dynamics. Eddy *et al.* (2022) was able to use radial anisotropy to infer fossil deformation within the Pacific plate and shear in the asthenosphere beneath. Kendall *et al.* (2022) showed that variations in plate speed resulted in changes in asthenospheric strain rates and first-order variations in the strength of radial anisotropy. The extent and even existence of radial anisotropy at deeper depths, such as the transition zone and lower mantle, is more debated. In the transition zone, images of radial anisotropy have also been used to infer a reduction in water content relative to what was previously suspected (Chang and Ferreira, 2019). In the lower mantle, studies have linked regions of radial anisotropy  $\xi \leq 1$  to deformation from subducting slabs.

Our understanding of the radial anisotropy in the crust is at a surprisingly nascent stage. Models of seismic wave propagation often assume an isotropic crust, but recent studies of crustal radial anisotropy using measurements of ambient noise in the western U.S. (Moschetti *et al.*, 2010) and northwestern Canada (Dalton and Gaherty, 2013) showed evidence for substantial radial anisotropy, suggesting that radial anisotropy might be a widespread feature of the continental crust. Studies of the radial anisotropy in the Tibetan Plateau have also found evidence for crustal radial anisotropy (Xie *et al.*, 2013). More widespread studies of the existence (or lack thereof) of crustal radial anisotropy are therefore essential at crustal scales, since a confirmation of ubiquitous crustal radial anisotropy would impact a wide range of seismic studies of the mantle and necessitate more nuanced calculations for predic-

tions of wavespeed as a function of depth. At much more local scales, studies of radial anisotropy at calderas, primarily from ambient noise, have presented links between magmatic activity and radial anisotropy via the formation of mid-crustal sill complexes (Jiang *et al.*, 2018, 2023; Chambers *et al.*, 2021; Jaxybulatov *et al.*, 2014).

In the upper mantle, there is reasonably good agreement between different studies on the long-wavelength variations in radial anisotropy beneath oceans (Nettles and Dziewoński, 2008; Ekström and Dziewonski, 1998; Eddy *et al.*, 2022). However, less is known about radial anisotropy in the continental upper mantle, especially at short wavelengths. At large scales, studies have found an increase in radial anisotropy at depths of below  $\approx 200 - 250\text{km}$ , and attributed this increase in radial anisotropy to the transition between the lithosphere and asthenosphere and the development of asthenospheric shear and anisotropy due to mantle flow (Gung *et al.*, 2003). At more regional scales, Yoshizawa and Kennett (2015) found an increase in radial anisotropy that they associated with the lithosphere-asthenosphere transition, and also associated increases in radial anisotropy within the continental lithosphere to the presence of a mid-lithospheric discontinuity, though the widespread nature of MLD detections from receiver functions (Krueger *et al.*, 2021) is sometimes cited as a reason as to why radial anisotropy could not be the global origin of the MLD (Selway *et al.*, 2015). Much of our understanding of radial anisotropy in continental settings comes from studies beneath the North American continent. Using full-waveform imaging, Yuan *et al.* (2011) found generally positive ( $\xi \geq 1$ ) radial anisotropy in the lithosphere beneath North America and more heterogeneous anisotropy beneath the inferred LAB. In a higher-resolution model update, Yuan *et al.* (2014) linked anomalies in radial anisotropy to continental sutures and suggest the development of anisotropy due to fossil accretion processes. Studies have also found layers of different radial anisotropy in the cratonic lithosphere of North America (Zhou *et al.*, 2022; Marone *et al.*, 2007).

While imaging radial anisotropy with SH and SV teleseismic body wave arrivals



(Porritt *et al.*, 2014) sometimes shows little variation in radial anisotropy with depth, joint inversions of body wave arrivals with multiple datasets have instead found long-wavelength variations in radial anisotropy, with a tendency to isotropic structure in the deeper lithosphere ( $\geq 175$  km). Overall, few studies exist that model radial anisotropy at high resolution; prior to the early 2010s, studies of radial anisotropy beneath North America were limited in resolution due to limited data coverage, but the advent of USArray (Meltzer *et al.*, 1999) has theoretically enabled new possibilities in high-resolution radially anisotropic imaging.

However, in practice, inversions for radial anisotropy are fraught with pitfalls. Numerous works have documented challenges with these inversions, including trade-offs between radial anisotropy and vertical layering (multiple isotropic layers) (Bodin *et al.*, 2015), the difficulty of accounting for crustal structure (Chang *et al.*, 2014), and tradeoffs between compressional wave velocity and radial anisotropy (Gao and Lekić, 2018). The latter challenge is partially rooted in the differing depth sensitivities of the Rayleigh and Love wave sensitivity kernels (Fig. 1), which impact the retrieval of  $V_{sv}$  and  $V_{sh}$  structure differently. A challenge at the level of data collection has been the difficulty of retrieving accurate Love wave phase velocity measurements. Due to interference from overtones, these measurements are systematically biased to higher values when measured at regional scales and over short path lengths. As a result, Love wave phase velocity maps from earthquakes had not been developed for the continental U.S. until Hariharan and Dalton (2022) developed and applied an approach to eliminate Love wave overtone interference. Consequently, the only images of radial anisotropy in the mantle beneath the U.S. had been from full-waveform inversion (Yuan *et al.*, 2014; Zhu *et al.*, 2017; Fichtner *et al.*, 2018) or regional inversions embedded in global-scale models (Porritt *et al.*, 2021; Nettles and Dziewoński, 2008). The maps from Hariharan and Dalton (2022) have presented new high-resolution observations at longer periods, rendering the time ripe for the development of a radially

anisotropic model that extends to lithospheric and asthenospheric depths.

## 5.2 Data and Methods

### 5.2.1 Inversion Approach

We jointly invert Rayleigh and Love wave phase-velocity maps, which are primarily sensitive to  $V_{SV}$  and  $V_{SH}$ , respectively, for isotropic and anisotropic structure beneath the continental U.S. We use the SR16 tomographic model (Shen and Ritzwoller, 2016) as the starting and reference model. It was determined from Rayleigh wave phase velocity and ellipticity as well as P-s receiver functions and therefore reliably resolves  $V_{SV}$  and crustal thickness. It is well-known that without prior information or additional data constraints such as from converted body waves, inversions (such as the one described in this section) that use only surface wave data cannot adequately resolve a sharp discontinuity such as the crust-mantle boundary (Lebedev *et al.*, 2013). By using the SR16 model—which contains a laterally varying moho discontinuity—as a starting model and only solving for perturbations to the velocity structure and not the moho directly, we address this issue and avoid potential trade-offs between a poorly-resolved Moho and the velocity structure above and beneath it. By starting with a model that is presumably very close to the true model, we also expect to obtain relatively accurate results.

We separately solve for anisotropic and isotropic shear velocity perturbations to the reference model, following the approach of Kustowski *et al.* (2008), which allows for easier control on separate isotropic and anisotropic smoothness variations. We define the anisotropic velocity perturbation as  $\frac{\delta as}{as} = \frac{\delta V_{sh}}{V_{sh}} - \frac{\delta V_{sv}}{V_{sv}}$  and the isotropic shear velocity perturbation as  $\frac{\delta V_s}{V_s} = \frac{1}{2}(\frac{\delta V_{sh}}{V_{sh}} + \frac{\delta V_{sv}}{V_{sv}})$ . These perturbations can be linked to the observed Rayleigh or Love wave phase velocity perturbations as:

$$\int_0^R ((K_{SH}(\omega, r) + K_{SV}(\omega, r)) \frac{\delta V_s}{V_s}(r) + \frac{1}{2}(K_{SH}(\omega, r) - K_{SV}(\omega, r)) \frac{\delta a_s}{a_s}(r)) dr = \frac{\delta c}{c}(\omega) \quad (5.1)$$

where  $K_{SH}$  is the frequency- ( $\omega$ ) and radius- ( $r$ ) dependent sensitivity kernel linking perturbations in Rayleigh or Love wave phase velocities  $\frac{\delta c}{c}(\omega)$  at a given frequency to perturbations in  $V_{sh}$ , and  $K_{SV}$  is the sensitivity kernel linking perturbations in Rayleigh or Love wave phase velocities to perturbations in  $V_{sv}$  (Fig.1).

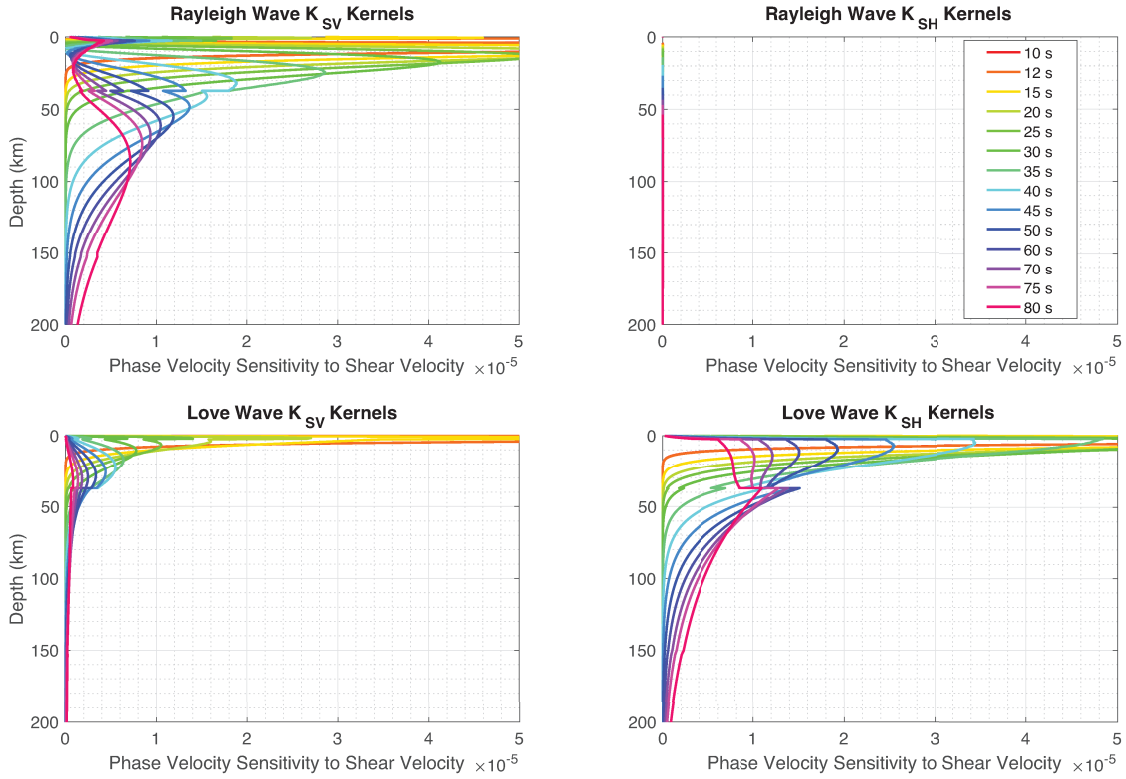


Figure 5.1: Example of Rayleigh and Love wave phase velocity sensitivity kernels to vertically and horizontally polarized shear wave velocity, calculated for the SR16 model at the longitude/latitude co-ordinates -112,38.25.

The starting model, SR16, has isotropic velocity, which we achieve by assigning  $V_{sh} = V_{sv}$ . We compute laterally variable sensitivity kernels for each pixel using the SR16 model at that pixel. This decision improves the accuracy of the inversion because inaccuracies in the starting model can have a particularly dramatic effect on Love wave sensitivity kernels and strongly influence inversion results. (Rychert and

Harmon, 2017).

We use the formulation in equation 1 to jointly invert Rayleigh and Love wave phase velocities in a 1-D inversion at every pixel, for isotropic and anisotropic velocity structure as a function of depth by framing equation 1 as an inverse problem  $\mathbf{G}\mathbf{m} = \mathbf{d}$  with the least-squares solution:

$$\mathbf{m} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{d} \quad (5.2)$$

where  $\mathbf{m}$  is a model vector composed of variations in isotropic and anisotropic velocity perturbations in depth,  $\mathbf{d}$  is a data vector composed of measured Rayleigh and Love wave phase velocity perturbations with respect to predictions from the starting model, and  $\mathbf{G}$  is a matrix (Figure 2) composed of Rayleigh and Love wave sensitivities to isotropic and anisotropic velocity variations. After solving for  $\frac{\delta as}{as}$  and  $\frac{\delta V_s}{V_s}$ , we convert back to  $V_{sh}$ ,  $V_{sv}$  and radial anisotropy  $\xi = (\frac{V_{sh}}{V_{sv}})^2$  to facilitate model interpretation. To stabilize the inversion and obtain physically reasonable values, we smooth the model vertically by minimizing the first derivative of the isotropic and anisotropic velocity perturbations using a roughness matrix  $\mathbf{W}$ . In practice, this matrix is concatenated vertically with the  $\mathbf{G}$  matrix consisting of the sensitivity kernels (Fig.1). The extent to which we damp towards a smoother model is governed by a smoothing parameter  $\lambda$  which multiplies the  $\mathbf{W}$  matrix and is chosen via an L-curve analysis presented in section 3 (Hansen, 1992).

In practice, we parameterize our model laterally on an evenly spaced grid with  $0.5^\circ$  latitude and longitude spacing and vertically in layers of 5 km thickness. Deeper than 150 km, where the SR16 model is not defined, the starting model is fixed to radially anisotropic PREM (Dziewonski and Anderson, 1981). We quantify the performance

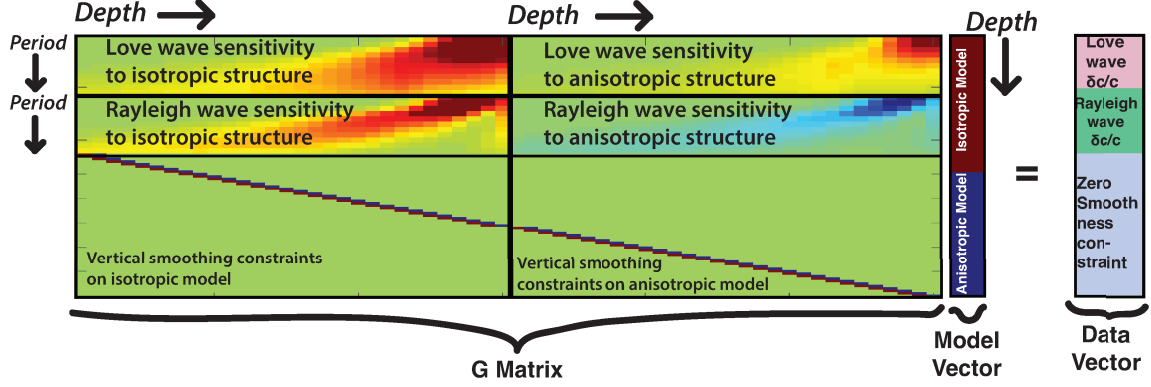


Figure 5.2: Schematic illustration of our implementation of the joint inversion of Rayleigh and Love wave phase velocities for isotropic and radially anisotropic velocity variations.

of the inversion and the fit to the data using the variance reduction  $\mathbf{V}_R$ , defined as:

$$\mathbf{V}_R = 100 \times \left( 1 - \frac{\sum_{i=1}^N \left( \frac{\delta c}{c}_i(obs) - \frac{\delta c}{c}_i(pred) \right)^2}{\sum_{i=1}^N \left( \frac{\delta c}{c}_i(obs) \right)^2} \right) \quad (5.3)$$

where  $N$  corresponds to the number of  $\frac{\delta c}{c}$  measurements (each of which corresponds to either a different period or a different wave —Rayleigh or Love—) used in the joint inversion.

### 5.2.2 Data

The data used in this inversion are pixel-based Rayleigh and Love wave phase velocity maps measured by four previous studies from earthquake and ambient noise observations. The Love wave phase velocities were measured by Hariharan and Dalton (2022) on earthquakes and Ekström (2017) on ambient noise. The measurements of Ekström (2017) were derived from analyses of the spectra of noise cross-correlations based on the approach of Aki (1957). The earthquake-derived phase velocity maps were calculated using Eikonal tomography (Lin *et al.*, 2009) with a quality control approach designed to eliminate overtone interference (Hariharan and Dalton, 2022). Jointly, these measurements span the period band 5-75 s, and they overlap at periods

between 30-40 s. Hariharan and Dalton (2022) found good agreement between the phase velocity maps from the two independent measurement techniques at these periods, suggesting that they were compatible and could be used together in inversions. We also use Rayleigh wave phase velocities measured by Babikoff and Dalton (2019) on earthquakes and by Ekström (2017) on ambient noise. Babikoff and Dalton (2019) found exceptionally high correlation between their phase velocity maps and the corresponding ambient-noise maps at overlapping periods, suggesting that the two sets of measurements are compatible. The Rayleigh wave earthquake-derived measurements were made using Helmholtz tomography (Lin and Ritzwoller, 2011). The Love wave measurements we use are defined at periods of 12, 15, 20, 25, 30, 35, 40, 45, 50, 60, and 75 s. The Rayleigh wave measurements we use are defined at periods of 10, 12, 15, 20, 25, 30, 35, 40, 50, 60, and 80 s.

In the inversion, we fit perturbations to the measured phase velocities calculated with respect to predictions from the starting model. At every pixel in our model we use the normal-mode code MINEOS (Masters *et al.*, 2011) to predict frequency-dependent phase velocities  $c_{MINEOS}(\omega)$  from the SR16 model at that pixel and calculate the phase velocity perturbation  $\frac{\delta c}{c}(\omega) = \frac{c_{measured}(\omega) - c_{MINEOS}(\omega)}{c_{MINEOS}(\omega)}$ , for both Rayleigh and Love waves. Figure 3 shows plots of  $\frac{\delta c}{c}(\omega)$ . Consistent with the results of Hariharan and Dalton (2022), we find that  $\frac{\delta c}{c}$  is generally positive for Love waves and increases with increasing period.

For Rayleigh waves,  $\frac{\delta c}{c}$  is generally close to zero, but shows a subtle increase to values greater than zero at longer periods (Figure 3). We note the appearance of a slight apparent jump in  $\frac{\delta c}{c}$  for the Rayleigh waves at periods transitioning from ambient noise measurements to measurements on earthquakes. To test whether the Rayleigh wave  $\frac{\delta c}{c}$  at longer periods is robust, we also explore using  $\frac{\delta c}{c}$  measurements calculated using the Rayleigh wave phase velocity maps of Jin and Gaherty (2015) (Figure 3b) instead of those of Babikoff and Dalton (2019). Jin and Gaherty (2015) showed that

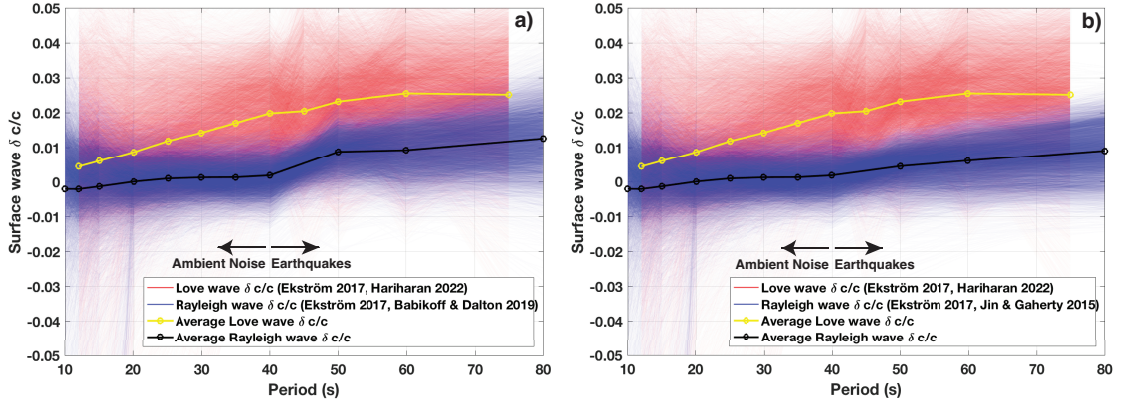


Figure 5.3: Measured  $\delta c/c$ , calculated with respect to the phase velocity predictions of SR16. Red lines correspond to Love waves and blue lines correspond to Rayleigh waves. Each line corresponds to a different pixel. The Love wave phase velocity maps used are from Ekström (2017) and Hariharan and Dalton (2022), and the Rayleigh wave phase velocity maps used are from Ekström (2017), Babikoff and Dalton (2019) (a), and Jin and Gaherty (2015) (b). Each panel uses a different set of Rayleigh wave measurements at periods greater than 40 s.

their maps also had strong correlation between the ambient-noise measurements of Ekström (2017).

Figure 4 explicitly compares the average  $\frac{\delta c}{c}$  for the two sets of Rayleigh-wave earthquake phase velocity maps, and shows that for both Rayleigh wave data sets an increase in  $\frac{\delta c}{c}$  from 40 s to 50 s is present, although this is to a slightly greater extent for the maps of Babikoff and Dalton (2019). This suggests that the increase in Rayleigh wave  $\frac{\delta c}{c}$  at longer periods is a robust feature of the data.

### 5.2.3 Tests with Synthetic Data

To demonstrate the inversion for radially anisotropic structure, we present a suite of synthetic tests aimed at recovering Gaussian perturbations to  $V_{sh}$ ,  $V_{sv}$ , and radial anisotropy. We explore the recovery of radially anisotropic structure and the recovery of purely isotropic structure, paying attention to the amplitude of the recovered perturbations, and the trade-offs between the recovered isotropic and anisotropic heterogeneity.

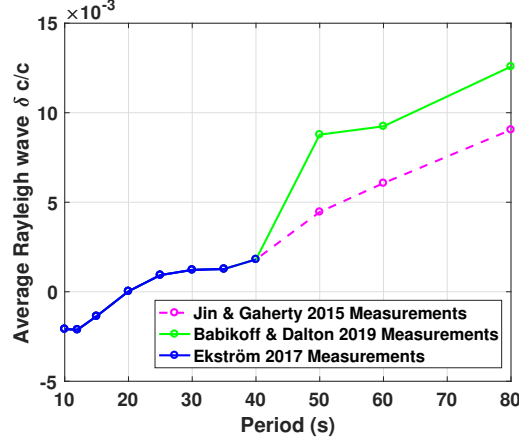


Figure 5.4: Comparison of average Rayleigh wave  $\frac{\delta c}{c}$  at periods greater than 40 s for the earthquake-derived phase velocity measurements of Babikoff and Dalton (2019) and Jin and Gaherty (2015), with respect to predictions from the SR16 model. The results from Ekström (2017) at shorter periods are also shown for context.

For all of these tests, the best-fit model from the inversions is able to predict data that fits the synthetic ‘observations’ very well (Fig. 5a,d,g,j,m). The tests show that:

- The inversion is able to recover an isotropic model perturbation (due to both  $V_{sh}$  and  $V_{sv}$ ) adequately, with minimal contamination in radial anisotropy (Fig.5 a-c).
- When starting from an isotropic model and trying to recover radial anisotropy due to both  $V_{sh}$  and  $V_{sv}$ , the model is able to place the peak of the signal in the correct depth range. However, the amplitude is slightly diminished and spread out over a slightly larger large depth range. (Fig.5 d-f).
- When radial anisotropy is due to  $V_{sv}$  as opposed to  $V_{sh}$ , the model recovers the amplitude of the anomaly more accurately and with better localization in depth. (Fig.5 g-l).
- When the true model is isotropic and if starting with a radially anisotropic model that is due to incorrect  $V_{sh}$  and  $V_{sv}$ , the inversion is able to reduce the amplitude of radial anisotropy accordingly. However, the inability to recover



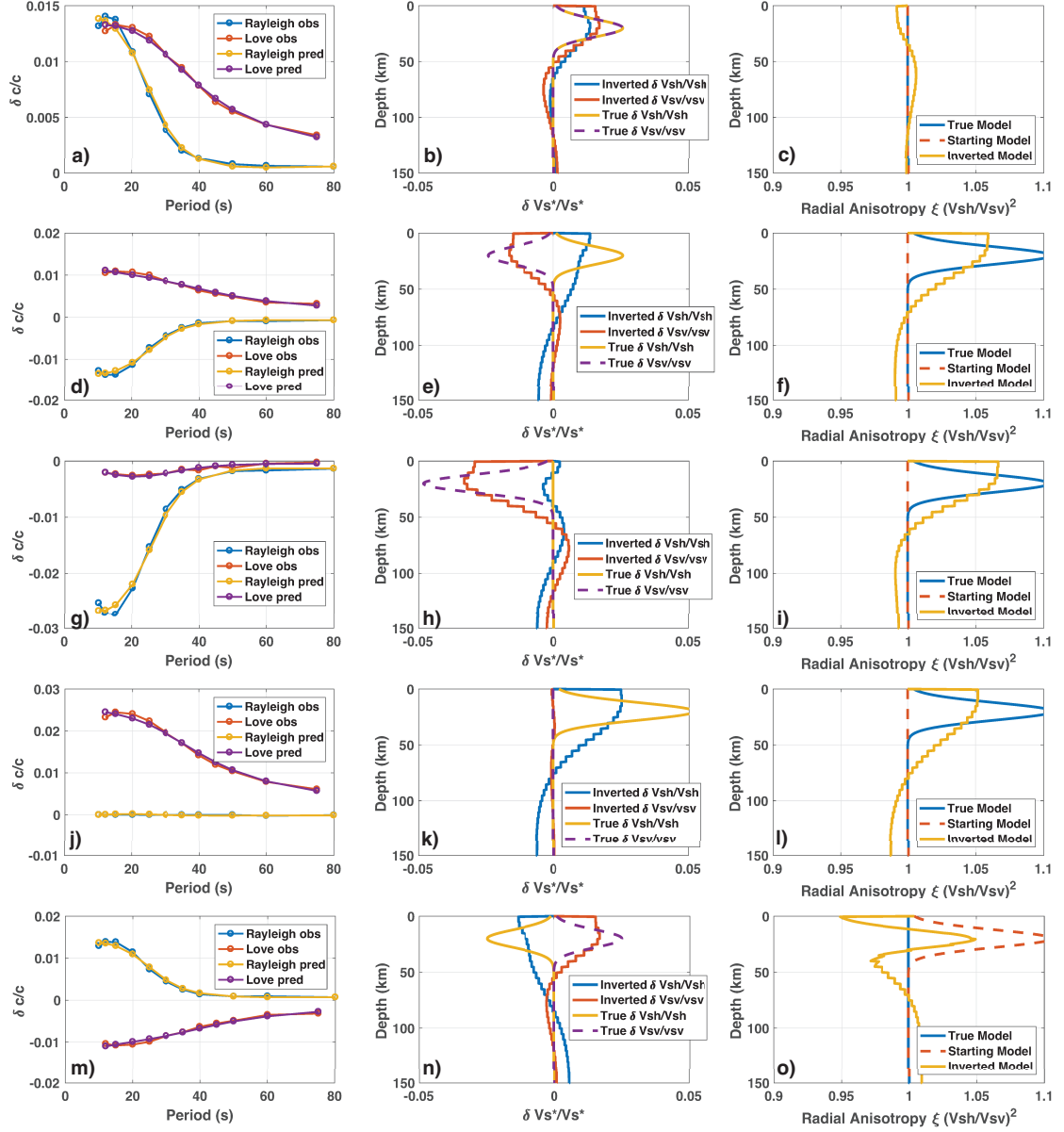


Figure 5.5: Tests with synthetic data exploring the performance of the joint inversion. Left column: Plots of best-fit and observed Rayleigh and Love wave  $\frac{\delta c}{c}$  expressed as a fraction. Middle column: Plots of true and recovered  $\frac{\delta V_{sh}}{V_{sh}}$  and  $\frac{\delta V_{sv}}{V_{sv}}$ . Right column: Plots of true and recovered radial anisotropy  $\xi$ . Each row corresponds to a test exploring a different use case of the inversion. (a-c): A test exploring the ability to recover a purely isotropic perturbation to the starting model. (d-f): A test exploring the ability to recover a radially anisotropic model caused by perturbations in both  $V_{sh}$  and  $V_{sv}$ . (g-i): A test exploring the ability to recover a radially anisotropic model caused by only a perturbation in  $V_{sv}$ . (j-l): A test exploring the ability to recover a radially anisotropic model caused by only a perturbation in  $V_{sh}$ . (m-o): A test exploring the ability to recover an isotropic model when the starting model is radially anisotropic due to both incorrect  $V_{sv}$  and  $V_{sh}$ .

both the same amplitude of both  $V_{sv}$  and  $V_{sh}$  leads to smaller-wavelength oscillations in radial anisotropy.

A recurring theme that is apparent upon inspection of these synthetic tests is the fact that the inversion does not recover  $V_{sh}$  and  $V_{sv}$  equally; the  $V_{sv}$  component of the result is consistently more accurate. This is likely due to the different shape of the phase velocity kernels for Love and Rayleigh waves; the Love wave kernels do not exhibit the same local maximum in depth that the Rayleigh wave kernels do. This may lead to more non-uniqueness in the solution and a poorer ability to localize anomalies in depth.

Overall, our synthetic tests show that the inversion is able to recover the broad location and shape of radially anisotropic anomalies to first order and fit the data well. However, the amplitude of radial anisotropy recovered is consistently lower than in the true model, and this fact must be kept in mind when interpreting models. In addition, while the broad conclusions of the synthetic tests discussed previously are likely generalizable with increasing depth due to the gradually varying shapes of the kernels, the amplitude of the recovered anomalies likely varies based on the depth of the target anomalies. To probe the depth-dependent amplitude degradation in recovered isotropic and anisotropic images, we calculate model resolution matrices (Menke, 2015). When the joint inversion is reframed as  $\mathbf{m} = (\mathbf{G}^{-1}\mathbf{G} + \lambda\mathbf{W}^{-1}\mathbf{W})\mathbf{G}^T\mathbf{d}$ , the model resolution matrix can be defined as

$$\mathbf{R} = (\mathbf{G}^{-1}\mathbf{G} + \lambda\mathbf{W}^{-1}\mathbf{W})\mathbf{G}^T\mathbf{G} \quad (5.4)$$

Visualizing this matrix (Fig. 6) illustrates the recovery of amplitude as a function of depth; perfect recovery would result in resolution matrices equal to the identity matrix  $\mathbf{R} = \mathbf{I}$ .

Taking the resolution matrices we calculate (Fig. 6) at face value, the inversion re-

sults in weakening recovery of amplitudes with increasing depth. At shallow (crustal) depths, we recover approximately 75% of the input amplitude, but this diminishes to  $\sim 30\%$  at deeper depths. At the deepest depths, the resolution matrices suggest smearing of anomalies at length scales of around 20 km. The smearing is slightly greater for the anisotropic model than for the isotropic model. While the low values of amplitude recovery reported here might seem like a cause for concern, it is important to keep in mind that they reflect the inversion’s ability to recover the amplitude of an unrealistically sharp input perturbation of a specific wavelength of 15 km.

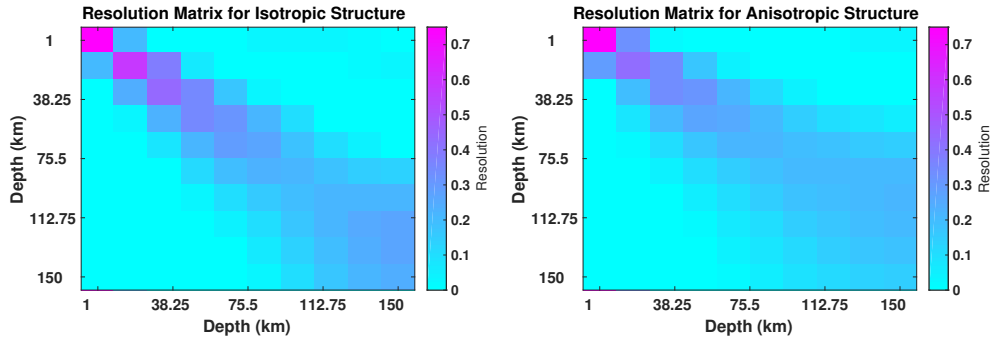


Figure 5.6: Left: Model resolution matrix for the isotropic part of the model. Right: Model resolution matrix for the anisotropic part of the model.

### 5.3 Results

In this section, we report on the performance of the joint inversion outlined in the previous section when it is applied to real data. We also describe the main features of the resulting model of shear velocity and radial anisotropy.

Our preferred model discussed in this section, SR16-ANI, corresponds to a smoothness parameter of  $\lambda = 0.3$ , arrived at through an L-curve analysis (Figure 7) where we run the joint inversion across the entire geographic region of interest but down-sample the grid laterally for computational expediency, solving for a 3-D model with the same vertical resolution as our final model but slightly worse horizontal resolution. We repeat this inversion for a range of smoothing parameters spanning four

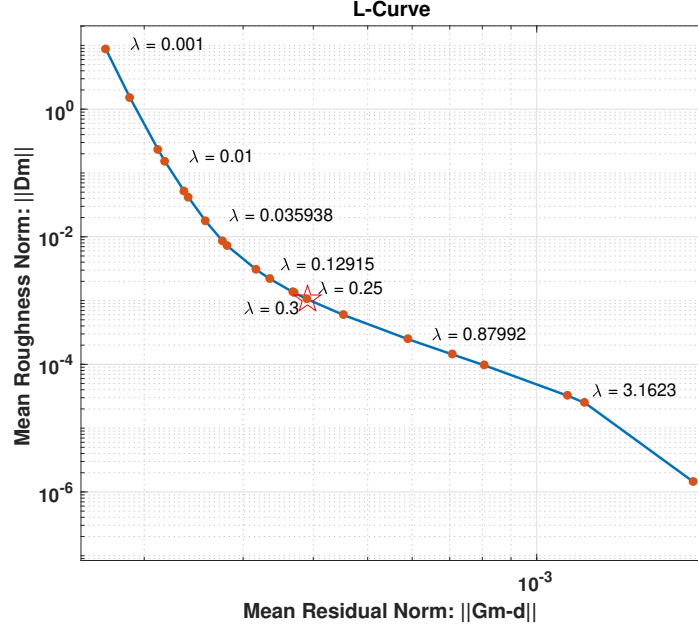


Figure 5.7: L-curve for the joint inversion, illustrating the logic for the choice of the preferred smoothing parameter (shown as a star). Plotted are the norm of the data error and the model roughness for different values of the smoothing parameter  $\lambda$ , annotated on the plot.

orders of magnitude. The L-curve has a characteristic shape that allows us to pick the smoothing parameter corresponding to the ‘elbow’. For simplicity, we damp the isotropic and anisotropic parts of the model by the same smoothing parameter  $\lambda$ . This effectively results in stronger damping to the anisotropic part of the model than the isotropic part of the model due to the weaker anisotropic sensitivities in the  $\mathbf{G}$  matrix.

In Figure 8, we illustrate an example of the inversion as applied to real data by examining data from a single pixel (located at latitude=38.25, longitude=-112). At this pixel, we observe slightly positive Love wave  $\delta c/c$  at shorter periods (Fig. 8a) that increases with increasing period. To explain this, the SR16-ANI result at this location contains faster  $V_{sh}$  than the reference model, especially at the greater depths that are sampled by the longer-period Love waves. Concurrently, the Rayleigh wave  $\delta c/c$  is slightly less than zero at shorter periods and close to zero at longer periods,

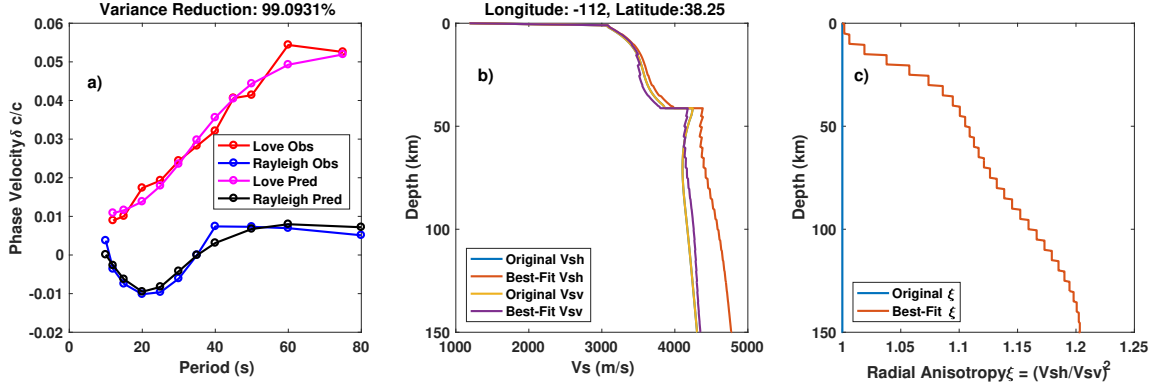


Figure 5.8: Example of the joint inversion as applied to real data at a single pixel (longitude/latitude coordinates are -112,38.25) in Utah, illustrating (a) fit to the observations, (b) the changes to  $V_{sh}$  and  $V_{sv}$  solved for in the inversion, and (c) the resulting model of radial anisotropy.

suggesting that the SR16 starting model provides a reasonably good fit to the Rayleigh wave observations. To explain this, the inversion produces  $V_{sv}$  that is slightly slower than the reference model at shallower depths, with even less perturbation at greater depths (Fig. 8b). The increase in  $V_{sh}$  at greater depths results in increasing radial anisotropy with depth (Fig. 8c). Overall, the inversion manages to fit both observed  $\frac{\delta c}{c}$  curves very well and achieves a high variance reduction for this pixel.

We find very high levels of variance reduction when the inversion is applied to the entire data set (Fig. 9), with a median variance reduction of 94% (Fig. 9a). When the variance reduction is decomposed into the individual contributions from Love and Rayleigh waves, we find that SR16-ANI is able to fit the Love wave data slightly better than the Rayleigh wave data on average (Fig. 9b). When this variance reduction is further decomposed into the period-dependent contributions for both Rayleigh and Love waves, we find that there is generally little variation in variance reduction as a function of period. The notable exception is the Rayleigh wave measurements at a period of 40 s, for which there is a sharp minimum in variance reduction and the observations are not fit as well as the other measurements. This minimum corresponds to the ‘jump’ in Rayleigh wave  $\frac{\delta c}{c}$  (Fig. 3a) that occurs between periods of 40 and 50

s, at the transition from the ambient noise to the earthquake phase velocity measurements. Since  $\frac{\delta c}{c}$  curves predicted by realistic seismic models tend to be smooth as a function of period, the sharp jump in  $\frac{\delta c}{c}$  values that exists in the measured data is not able to be well-fit in the inversion; the inversion likely systematically overpredicts average  $\frac{\delta c}{c}$  at 40 s in order to smoothly transition from the measurements at 35 s to 50 s. Nonetheless, in general SR16-ANI is able to fit the data very well.

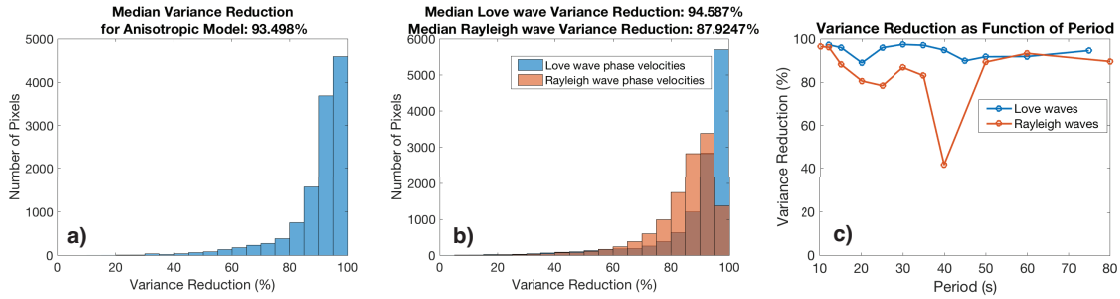


Figure 5.9: Examination of data fit for the joint inversion. (a) Histogram of variance reduction values at individual pixels. (b) Histogram of variance reduction at individual pixels, shown separately for Love and Rayleigh wave data. (c) Variance reduction for Love and Rayleigh wave data as a function of period.

We now present and describe depth slices of SR16-ANI beneath the continental U.S. (Figure 10). At mid-crustal depths (Fig. 10a), radial anisotropy  $\xi > 1$  generally occurs in the western U.S. Specifically, the Great Basin in the western U.S. is defined by a region  $\xi > 1$ . North of this region, the Snake River Plain is isotropic ( $\xi \approx 1$ ). Just north of the Snake River Plain, in the Rocky Mountain Basin & Range and the Omineca Extended Belt, we once again see a region of  $\xi > 1$ . In the southwest, the Sierra Nevada range is the sole region of  $\xi < 1$ . The Cascades Range bounds the western edge of the region with  $\xi > 1$  in the western U.S. and is relatively isotropic. The easternmost edge of the model also shows regions where  $\xi < 1$ . Examining deeper crustal structure at a depth of 40 km (Fig. 10b), the region of  $\xi > 1$  becomes relatively patchier, expanding slightly to regions eastward of the Great Basin but avoiding the Colorado Plateau and the Snake River Plain of the Great Basin. We also begin to see regions of subtle  $\xi > 1$  ringing the Colorado Plateau.

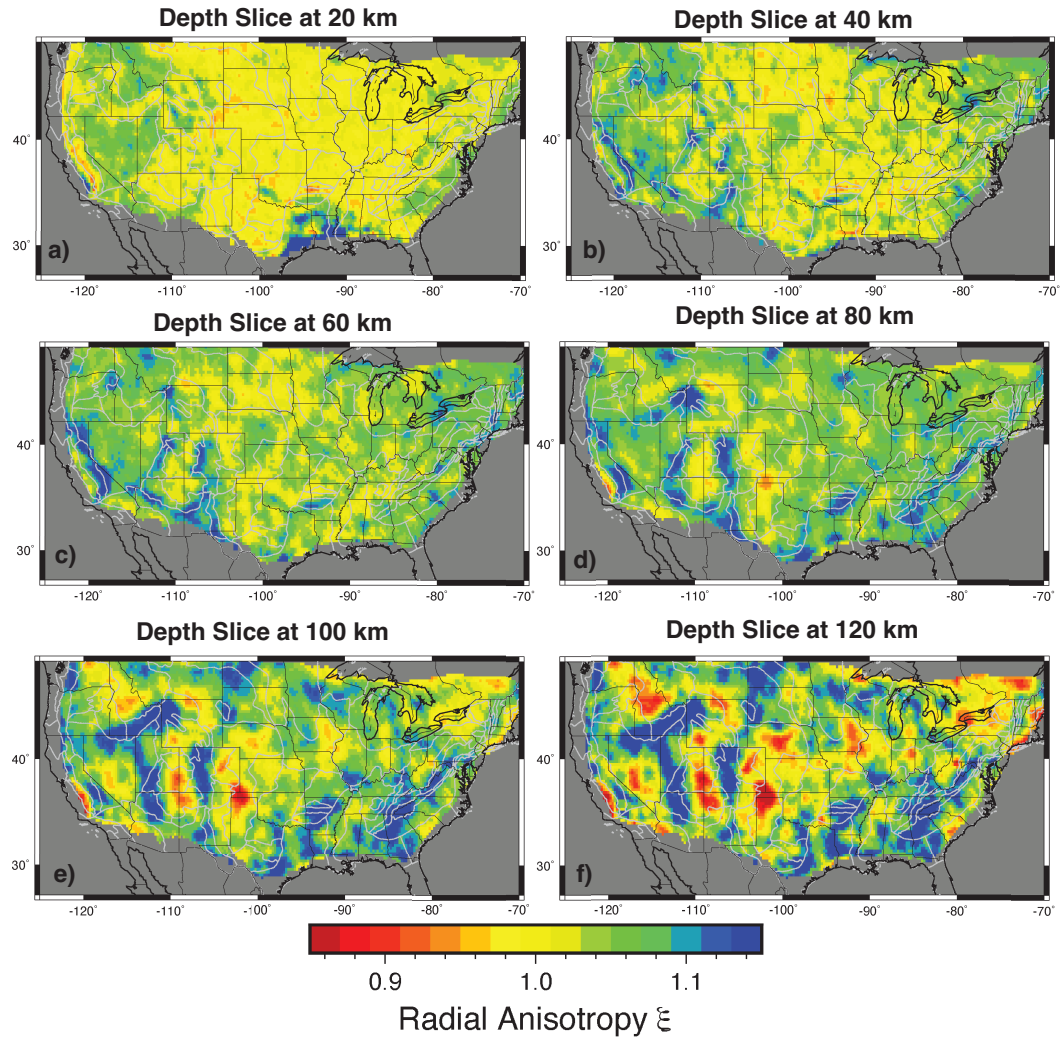


Figure 5.10: Images of radial anisotropy from our preferred model, at depths of (a) 20 km, (b) 40 km, (c) 60 km, (d) 80 km, (e) 100 km, and (f) 120 km. Overlain on each map as white lines are physiographic provinces (after Fenneman (1928)).

At deeper depths, and moving into the mantle, (Fig. 10 c,d) the patterns of radial anisotropy change dramatically. The regions with  $\xi > 1$  become more widespread, spanning most of the continental U.S., with maxima along the Appalachians, beneath the Snake River Plain, and encircling the Colorado Plateau. At even greater depths in the western U.S., (Fig. 10 e,f), there continues to be strong radial anisotropy beneath the Snake River Plain and east and west of the Colorado Plateau, as well as along the Appalachian Front. Intriguingly, the region beneath the Colorado Plateau itself shows substantial  $\xi < 1$ .

We also consider the variation with depth in our velocity and radial anisotropy images by calculating the RMS average of structure across our study region as a function of depth (Fig. 11a,b). Consistent with the visualization in Figure 10, radial anisotropy is weakest at crustal depths, and increases with depth, finding a maximum at around 85 km (Fig. 11b). The decrease in the strength of anisotropy at greater depths reflects the development of regions where there is substantial  $\xi \leq 1$ , despite there also being more regions where  $\xi \geq 1$  at deeper depths. To study the vertical coherence of radial anisotropy, we present a radial correlation function (Fig. 11c). This illustrates that the most dramatic re-organization in the geographic patterns of radial anisotropy occurs at a depth of 75 km. Anisotropic structure beneath this depth is generally well-correlated, whereas structure at shallower depths is not as well-correlated.

We also examine variations in shear velocity structure as a function of depth (Fig. 11a). On average, across the entire study region, the inverted  $V_{sh}$  is consistently higher than the inverted  $V_{sv}$ , particularly at depths beneath the Moho. A striking feature of the average profiles is that at depths greater than approximately 80 km, the inverted  $V_{sv}$  diverges from the SR16  $V_{sv}$ , with noticeably higher values. This increase is also visible in the Voigt averaged  $V_s$  and  $V_{sh}$  RMS. The  $V_{sv}$  increase is consistent with the Rayleigh wave  $\frac{\delta c}{c} \geq 0$  values observed at longer periods in our data set (Fig.



3).

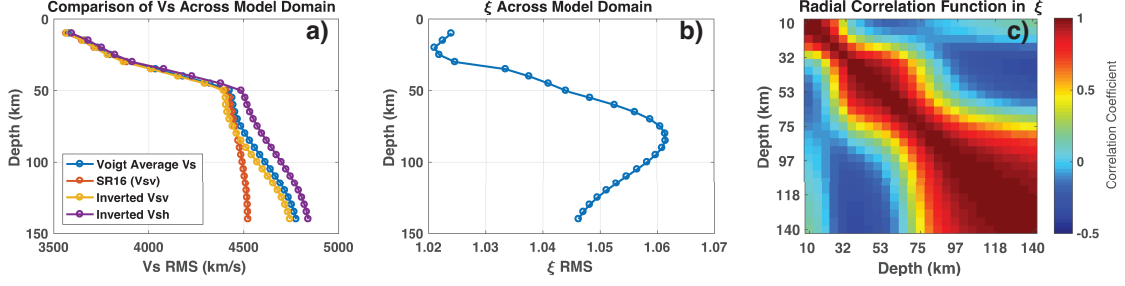


Figure 5.11: Comparison of depth-dependent RMS variations in shear velocity and radial anisotropy. (a) Comparison of depth-dependent Voigt average,  $V_{sh}$ , and  $V_{sv}$  from our model and  $V_{sv}$  from the SR16 model, averaged across the continental U.S. (b) Comparison of depth-dependent RMS in radial anisotropy. (c) Radial correlation function of radial anisotropy in depth.

To further probe the origin of the radial anisotropy variations shown in Figs. 10 and 11, we present images of the perturbations in  $V_{sh}$  and  $V_{sv}$  relative to the reference model (Fig. 12). For comparison, we also show images of the  $V_{sv}$  in the reference model and the Voigt averaged  $V_s$  that is obtained from our new model. These images show that in general, the  $dV_{sh}/V_{sh}$  perturbations solved for in our model are consistently larger than the corresponding  $dV_{sv}/V_{sv}$  perturbations. This is not surprising, since the reference model consists of  $V_{sv}$ . At crustal depths, the patterns in perturbations to  $V_{sh}$  are very similar to the maps of radial anisotropy; we find the largest perturbations in the western and easternmost U.S. At greater depths, we find  $V_{sh}$  faster than the reference model throughout the U.S., with exceptions at 60 km along the Rocky Mountain Front, and at greater depths, beneath the Colorado Plateau, north of the Snake River Plain, and southwest of the Great Basin.  $V_{sv}$  perturbations are relatively weaker. At crustal depths, there is very little perturbation to  $V_{sv}$ ; this only begins to noticeably change at 60 km, where there is a slight tendency toward decreased  $V_{sv}$  in the eastern U.S., contributing to  $\xi \geq 1$  in the eastern U.S. at that depth (Figure 10). At even greater depths, we find a tendency for our model to solve for large positive perturbations to  $V_{sv}$  in the central and eastern U.S., particularly

east of -100 degrees longitude.

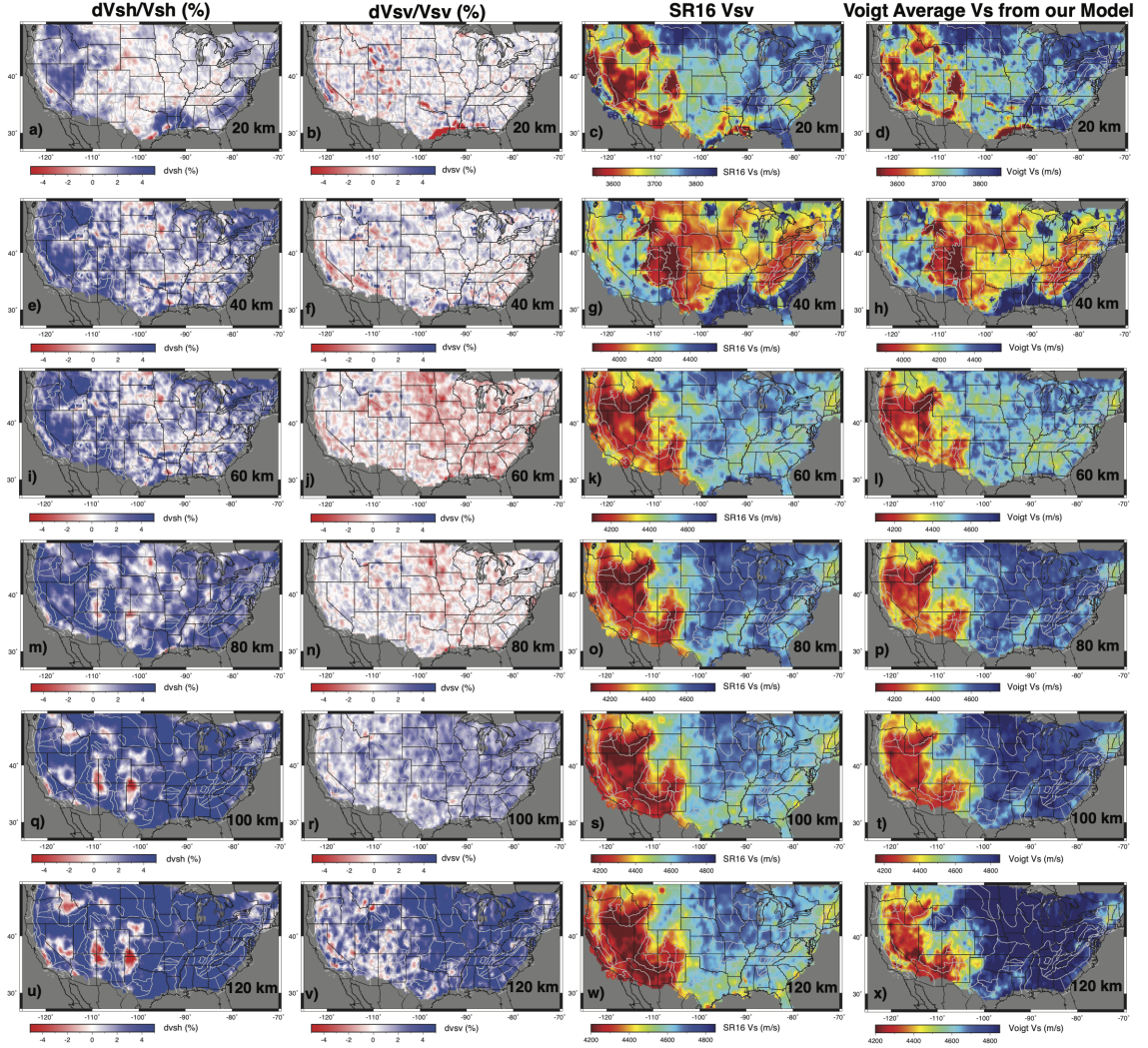


Figure 5.12: Depth slices of  $\delta V_{sv}$  and  $\delta V_{sh}$  from our model,  $V_{SV}$  from the SR16 model, and the Voigt-averaged  $V_s$  from our model at depths of (a-d) 20 km, (e-h) 40 km, (i-l) 60 km, (m-p) 80 km, (q-t) 100 km, and (u-x) 120 km. Overlain on each map as white lines are physiographic provinces (after Fenneman (1928)).

The patterns in velocities to first order agree well between our final model and the SR16 model- an unsurprising result, given the SR16 model is the starting model. However, the general tendency toward radial anisotropy  $\xi > 1$  generally manifests in the form of higher Voigt averaged shear velocities than in the starting model  $V_{sv}$ . In addition, the increase in both  $V_{sh}$  and  $V_{sv}$  in the central and eastern U.S. in the upper mantle results in substantial differences between the Voigt averaged  $V_s$  and the SR16

$V_s$  -often approaching 0.2 km/s in these regions. On the other, the strong radial anisotropy in the western U.S. also results in a strong increase in Voigt averaged  $V_s$  relative to the SR16  $V_s$  in regions such as the upper mantle beneath the Snake River Plain.

## 5.4 Discussion

### 5.4.1 Implications of Decisions Made in the Inversion Process

In this section, we present several synthetic tests to assess the robustness of certain features of the model presented in the previous section.

#### Is Radial Anisotropy Necessary to Fit the Data?

To test the null hypothesis of whether anisotropy is required in the final model, we perform a test where we repeat the inversion described in the Results section. However, this time, the anisotropic part of the inversion is heavily damped and effectively forced to zero. We explore how well the data is fit in this case and how the purely isotropic model adapts to fit the data.

Figure 13 reports on the variance reduction for both inversions. The variance reduction for the isotropic inversion is approximately 25% lower than the variance reduction for the inversion that allows anisotropy (Fig. 13a). To understand whether this means that an isotropic model could still result in an improvement in fit to both Love and Rayleigh wave data sets, we analyze the variance reduction for the Rayleigh and Love waves separately (Fig. 13b,c). We find that the isotropic inversion is able to obtain a very high variance reduction for the Love waves, but the Rayleigh waves suffer dramatically, with many of the pixels recording negative variance reductions, implying that the Rayleigh wave  $\frac{\delta c}{c}$  values are better fit by the starting model than the inverted model.

The reason that the Rayleigh waves are so dramatically impacted is rooted in the fact that the data error (i.e.,  $\frac{\delta c}{c}$ , the ability of the starting model to fit the observations) is much larger for Love waves. By virtue of minimizing the least-squares error, the inversion fundamentally prefers to fit the Love wave data. Consequently, the high values of variance reduction are most noticeable for the Love wave data,

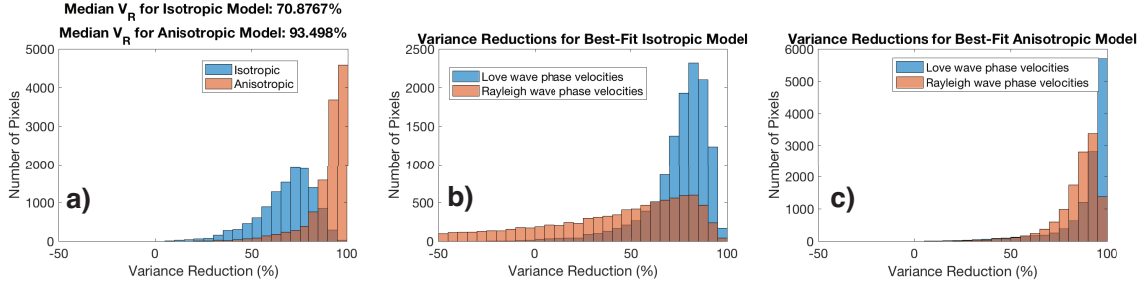


Figure 5.13: Comparison of the data fit provided by the anisotropic joint inversion shown in Section 3 with an isotropic joint inversion, in which the anisotropic component of the model is heavily damped to zero. (a): Comparison of variance reduction in each pixel for both inversions. (b): Comparison of Love and Rayleigh wave variance reduction for the isotropic inversion. (c): Comparison of Love and Rayleigh wave variance reduction for the anisotropic inversion.

and the final ‘isotropic’ model most resembles a  $V_{sh}$  model. However, this turns out to not be reconcilable with the Rayleigh wave data, which consequently suffers and shows extremely low, and often negative variance reduction. The pixels that the anisotropic inversion determines requires a large isotropic increase in shear velocities (and maybe also a small anisotropic component) are closely linked to the subset that have a positive variance reduction for Rayleigh waves in the isotropic inversion. This analysis serves as a powerful demonstration of the Rayleigh-Love discrepancy for structure at crustal and upper-mantle depths. Furthermore, by explicitly solving for radial anisotropy in the crust, we eliminate the issue of crustal contributions to signals used to infer radial anisotropy in the mantle (Panning *et al.*, 2010; Lekić *et al.*, 2010). Overall, the substantial difference in average variance reduction between both inversions suggests that the radial anisotropy we observe likely reflects real Earth structure.

### Effect of Allowing Perturbations to $V_{sv}$

The starting model that we use, SR16 (Shen and Ritzwoller, 2016), is a well-constrained model of  $V_{sv}$ , incorporating constraints from Rayleigh wave phase and group velocities, Rayleigh wave ellipticity, and receiver functions (Shen and Ritz-

woller, 2016). The robustness of the  $V_{sv}$  captured by this model is highlighted by the fact that our Rayleigh wave  $\frac{\delta c}{c}$  values are lower in magnitude than the corresponding Love wave measurements (Fig. 3); in other words, the Rayleigh wave phase velocities predicted by the starting model are quite similar to the observed values. The question naturally arises: Is it necessary to adjust  $V_{sv}$  at all, or could the Love and Rayleigh wave data sets be fit just as well by an inversion that solves only for  $V_{sh}$ ? We explore what the effect is of setting the  $V_{sv}$  in our final model to the  $V_{sv}$  in the SR16 model by conducting an additional inversion where we only solve for  $V_{sh}$  perturbations and only fit the Love wave phase velocities. We use the same smoothness parameter as in the inversion described in the Results section.

In Figure 14, we present the variance reduction from this inversion, as well as RMS plots of radial anisotropy and  $V_{sh}$ . Both inversions achieve nearly identical and high variance reductions for the Love wave data (Fig. 14a). However, the inversion that uses only Love wave data and only solves for  $V_{sh}$  achieves a relatively larger  $V_{sh}$  at deeper depths, and consequently predicts much higher radial anisotropy at depths greater than  $\sim 80$  km than in the joint inversion. This is a consequence of using the starting SR16  $V_{sv}$ , which our Rayleigh wave data suggest is artificially low (Fig. 3).

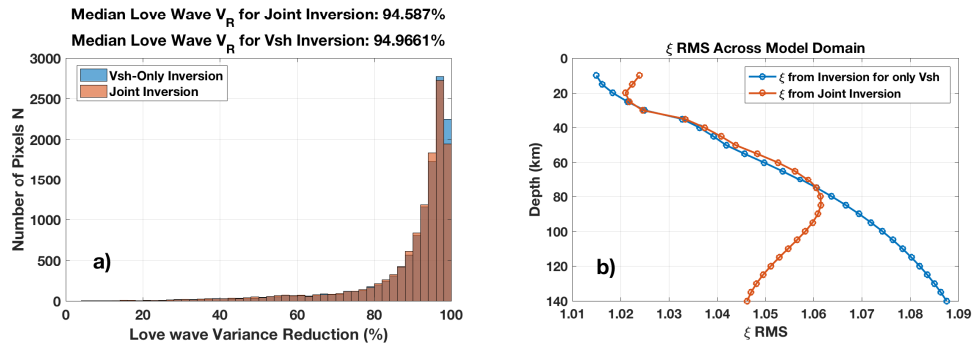


Figure 5.14: Comparison of a joint inversion of Rayleigh and Love waves for isotropic and anisotropic  $V_s$  with an inversion for just  $V_{sh}$  using only Love wave data. (a): Comparison of variance reduction for both inversions. (b): Comparison of depth-averaged radial anisotropy for both inversions.

This exercise highlights the necessity of accounting for an increase in isotropic  $V_s$

at deeper depths in order to accurately calculate radial anisotropy. If the high  $V_s$  that our joint inversion suggests was truly only in  $V_{sv}$  and consequently only reflected in the Rayleigh wave data, then the values of radial anisotropy that both inversions solved for would be much more similar. In other words, the generally positive perturbations in  $V_{sh}$  that the Love wave measurements impose on the inversion reflect two separate contributions: an anisotropic component (with  $V_{sh} \geq V_{sv}$ ) and an isotropic component (i.e.,  $V_{sv}$  in the SR16 starting model is biased toward low values).

### Choice of Rayleigh Wave Data Set

The higher  $V_{sv}$  velocities that we obtain at depths greater than 70 km relative to the SR16 starting model (Fig. 11) are a striking feature of our model. To ensure that this feature is not an artifact due to error or systematic bias in the Rayleigh wave measurements, we perform a test where we repeat the joint inversion of Rayleigh and Love wave phase velocities that is described in the Results section. In this test, we replace the earthquake-derived phase velocities from Babikoff and Dalton (2019) that are used at periods greater than 40 s with phase velocities from the maps of Jin and Gaherty (2015) that are measured at the same periods (Fig. 3c). We otherwise use the same settings for the inversion as shown previously, including the same amount of vertical smoothing. When comparing the two resulting models, we find that we are able to obtain very similar levels of data fit (variance reduction) when using either of the Rayleigh wave data sets (Fig. 15a), with a slightly higher variance reduction on average achieved when the inversion using the Jin and Gaherty (2015) data set. This facilitates a more detailed comparison of heterogeneity in the models.

We find that increased  $V_{sv}$  relative to the SR16 model at depths greater than 70 km is a feature of models obtained using either of the Rayleigh wave data sets (Fig. 15b), although it is subtly weaker for the Jin and Gaherty (2015) data set, consistent with the slightly lower average  $\frac{\delta c}{c}$  values in this data set (Fig. 3). The

average distribution of  $V_{sh}$  as a function of depth is similar for both models (Fig. 15c), which is to be expected given that the Love wave measurements, which are the same in both inversions, are the primary control on  $V_{sh}$  heterogeneity. Due to the slightly lower  $V_{sv}$  for the inversion using the Jin and Gaherty (2015) data set, we observe slightly higher radial anisotropy on average for the corresponding model (Fig. 15d). To ensure that our RMS comparisons are not obliterating important local differences between models, we examine correlation as a function of depth between models. At all depths, agreement between model parameters is good, with correlation coefficients consistently greater than 0.93.

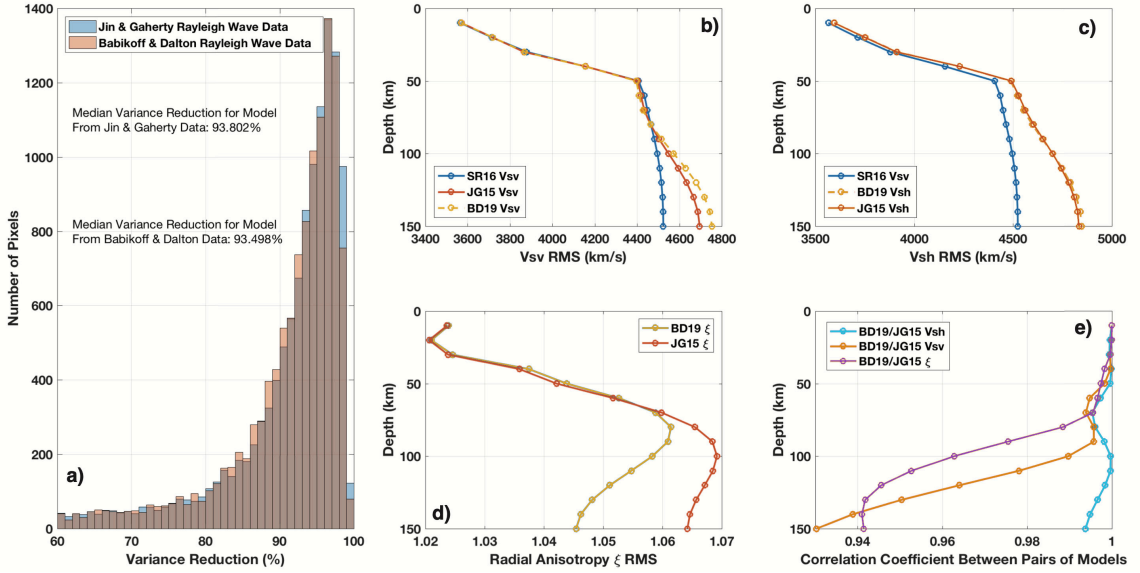


Figure 5.15: Comparison of the two different velocity models that are obtained when the Rayleigh wave datasets of either Babikoff and Dalton (2019) or Jin and Gaherty (2015) are used. (a): histogram of variance reduction for all individual pixels for both models. (b) compares  $V_{sv}$ , (c) compares  $V_{sh}$ , and (d) compares radial anisotropy  $\xi$ , by taking the RMS average of all quantities across the continental U.S. (e): Correlation coefficient between depth slices of both models.

This comparison illustrates that models obtained using either of the Rayleigh wave earthquake data sets are quite similar. While this may not be surprising given that both sets of Rayleigh wave phase velocities are measured using Helmholtz tomography (Lin and Ritzwoller, 2011) with phase delays measured using the ASWMS toolkit



(Jin and Gaherty, 2015), the actual construction of event-specific phase velocity maps differs noticeably between the two studies. Furthermore, both Jin and Gaherty (2015) and Babikoff and Dalton (2019) compared their phase velocity maps to other studies, collectively including the maps of Ekström (2017), Foster *et al.* (2014), and Porter *et al.* (2016), finding good agreement between other Rayleigh wave phase velocity maps measured with other methods. Taken jointly, the similarity between models suggests that the choice of Rayleigh wave data set is not the cause of the observed increase in  $V_{sv}$  at deeper depths relative to the starting model.

#### 5.4.2 Implications of the Preferred Model

In this section, I interpret our preferred isotropic and anisotropic model and discuss the implications in the context of regional tectonics and seismic observations.

#### Variations in Crustal Radial Anisotropy

As discussed in Section 3, the deviations from isotropy that we observe in the crust are most noticeable in the western U.S. and easternmost U.S. In a subset of the western U.S., Moschetti *et al.* (2010) used ambient noise Love wave measurements spanning periods up to 30 s to study radial anisotropy at crustal and sub-crustal depths. While their study only considered a subset of the western U.S. and only focused on crustal depths, we note very good agreement between the radial anisotropy in their model and ours (Figure 16), which are derived from independent data.

Physiographic provinces in the western U.S. line up well with our model of crustal radial anisotropy. We find that the regions of the strongest radial anisotropy correspond to the Great Basin, the Northern Basin and Range, and the Omineca Extended Province, all regions that have experienced extension. The regions of  $\xi \geq 1$  likely reflect finite strains related to Cenozoic extension (Moschetti *et al.*, 2010) the fact that we observe radial anisotropy throughout the crust in these regions suggests that the

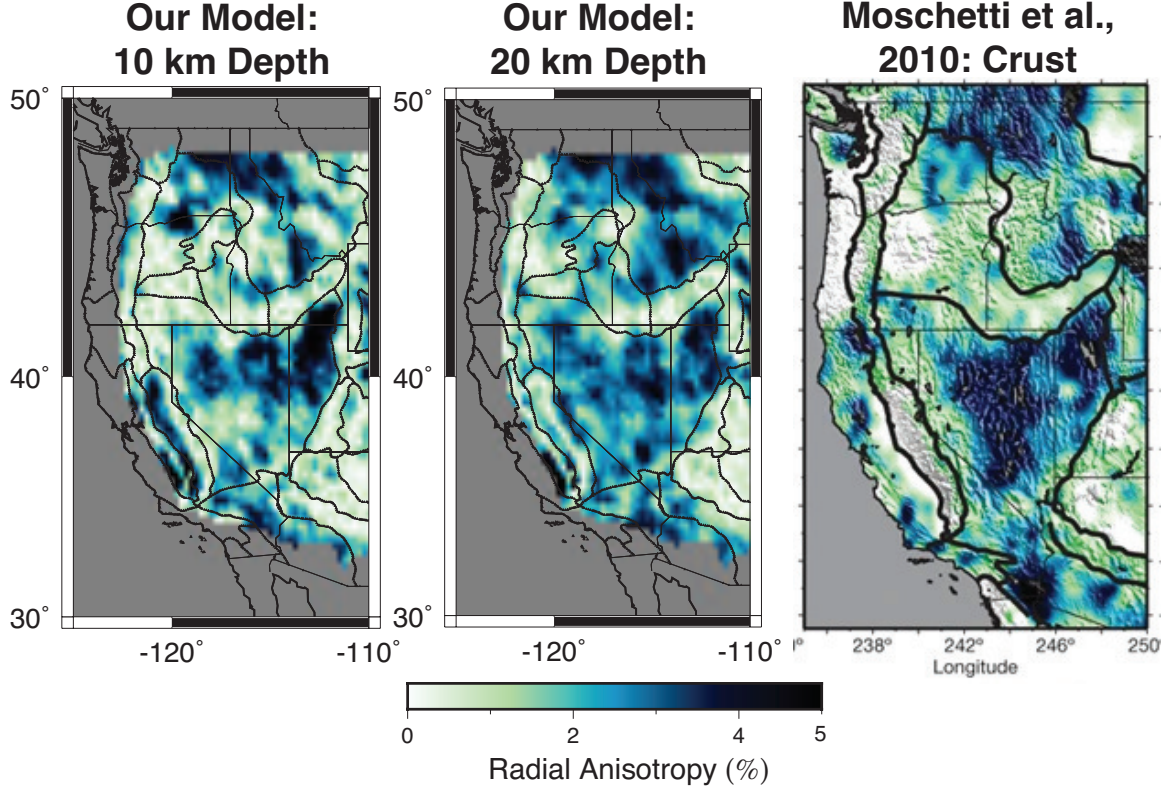


Figure 5.16: Comparison of radial anisotropy in our model at crustal depths in the western U.S. with the model of Moschetti *et al.* (2010). The rightmost panel is directly taken from Moschetti *et al.* (2010). To facilitate comparison, our model is plotted with the same color scale and bounds on latitude and longitude. Note that Moschetti *et al.* (2010) define radial anisotropy differently, as  $\xi = \frac{2|V_{sh}-V_{sv}|}{(V_{sh}+V_{sv})}$  and so we recalculate our radial anisotropy values to be consistent with their definition. Note that the absolute value in their definition means that regions where we observe  $(\frac{V_{sh}}{V_{sv}})^2 \leq 1$  such as the Sierra Nevada and Cascades Ranges may appear to have the same behavior as regions with  $\xi \geq 1$ .

extension impacted the entire crust. More generally, links between radial anisotropy and crustal extension are not limited to the western U.S.; Feng and Ritzwoller (2019) also observe radial anisotropy in Alaska in regions identified as having undergone mid-Cretaceous extension.

In the western U.S., radial anisotropy is found not only in regions of extension but also in regions of compression; the Sierra Nevada range, which exhibits strong negative radial anisotropy ( $\xi \leq 1$ ) at upper-crustal depths, provides the strongest example of  $\xi \leq 1$  in the crust. This likely reflects the development of vertical or subvertical foliations associated with orogenesis, which has been observed in the field (Edelman, 1991). The Cascades Range may also provide a weaker example of this, as the depth slices at 10 and 20 km show some evidence for  $\xi \leq 1$ , although this is at the edge of our image.

Some of the radial anisotropy in the eastern U.S. (Fig. 10) can also be linked to episodes of extensional deformation. The zone of positive radial anisotropy ( $\xi \leq 1$ ) in West Virginia directly overlaps the Rome trough, a graben system associated with Cambrian extension (Hickman and Harris, 2015). Other correlations are less clear. For instance, the zone of radial anisotropy on the eastern margin of the model underlies the Central Piedmont Shear Zone, but the mechanism linking this deformation to radial anisotropy  $\xi \geq 1$  needs to be further understood. Unlike in the upper mantle, where olivine is dominant, radial anisotropy in the crust may also be due to the alignments of minerals including micas, plagioclase, and amphiboles (Lloyd *et al.*, 2009; Bernard and Behr, 2017). This complicates interpretation, although one possibility could be the alignment of plagioclase parallel to the shear plane of the Piedmont Shear Zone which would result in positive radial anisotropy, since its fast axis is perpendicular to the plane of shear (Brownlee *et al.*, 2017). Moving northeast, the zone of positive radial anisotropy at the eastern edge of the model domain in New England overlaps with the Acadian altiplano, and may be associated with ductile flow linked

to the collapse of the plateau  $\approx 300$  million years ago (Hillenbrand *et al.*, 2021).

Overall, the fact that radial anisotropy within the crust across the majority of our study region –and particularly in the central U.S.– is negligible must be noted. This validates the commonly made but less-commonly supported assumption of crustal isotropy when modeling wave propagation and also when solving for models of shear velocity and anisotropy in the mantle without accounting for the crustal contribution explicitly.

### Variations in Upper Mantle Radial Anisotropy

The upper mantle in the continental U.S. is generally characterized by zones of positive radial anisotropy that are larger in amplitude than in the crust (Fig. 10). These zones also tend to be more continuous with depth than in the crust (Figure 11c), though the extent to which this is an artifact of lower vertical resolution in the upper mantle than in the crust remains to be determined (Fig. 1; Fig. 6).

The radially anisotropic model is highly heterogeneous, and so I only discuss a few salient features of it here. One of the most intriguing features is the Colorado Plateau, which is characterized by isotropic velocity at depths  $\leq 80$  km, despite being surrounded by zones of positive anisotropy. However, beneath the lithosphere–asthenosphere boundary it exhibits strong radial anisotropy  $\xi \leq 1$  that persists to the lower edge of the model domain (Fig. 17). This is consistent with downwelling of the Colorado Plateau lithosphere in the form of a lithospheric drip (Levander *et al.*, 2011; Reid *et al.*, 2012); the finite strain this drip undergoes as it downwells (Chen, 2021) results in vertically oriented fast axes. To the west and east of the downwelling drip, edge-driven convection (Van Wijk *et al.*, 2010) could presumably result in zones of alternating horizontally and vertically aligned fast axes, which manifest as the observed  $\xi \geq 1$  and  $\xi \leq 1$  that are adjacent to the Colorado Plateau and each represent one convection cell. Directly east and west of the Colorado Plateau, surface

volcanism (younger than 26 Ma) has been observed, and this correlates with crustal radial anisotropy  $\xi \leq 1$ . This is consistent with a narrative where melt ponds beneath the LAB and vertically percolates through the crust and lithosphere, resulting in  $\xi \leq 1$ . However, it is more difficult to explain why the region beneath where surface volcanism is observed shows  $\xi \geq 1$ , and more generally, whether the wavelength of convection that might be suggested by our images of radial anisotropy is consistent with the locations of observed volcanism. East of a longitude of -100 degrees, radial anisotropy is generally weaker than in the west, but also shows a consistent increase in the amplitude of the deviations from unity at depths below approximately 100 km (e.g. Figure 18). East of -100 degrees, structure beneath 100 km is likely still within cratonic lithosphere. The depth of the transition in radial anisotropy from a region of fairly weak anisotropy to a region of stronger heterogeneous anisotropy may be consistent with the observations of the depths of two distinct lithospheric layers of anisotropy (Yuan *et al.*, 2014).

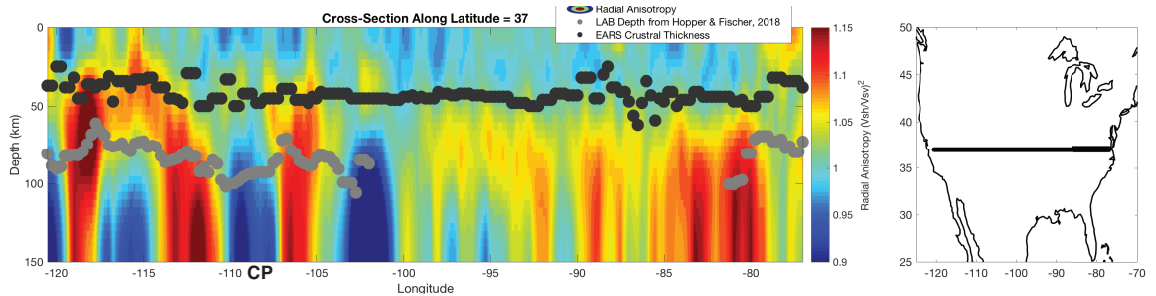


Figure 5.17: Cross-section of our model of radial anisotropy along a latitude of 37 degrees. Overlain are S-p receiver function observations of negative velocity gradients corresponding to the lithosphere-asthenosphere boundary (Hopper and Fischer, 2018), and crustal thicknesses from Croswell and Owens (2005).

Studies using SKS splitting measurements and Rayleigh wave phase velocities have noted strong upper mantle anisotropy at sub-crustal depths in this region (Wagner and Long, 2013), with one study suggesting horizontal flow at the base of the plate (Walker *et al.*, 2004) due to spreading of a mantle upwelling in Nevada. On the other hand, our radial anisotropy measurements at upper mantle depths do not show obvi-

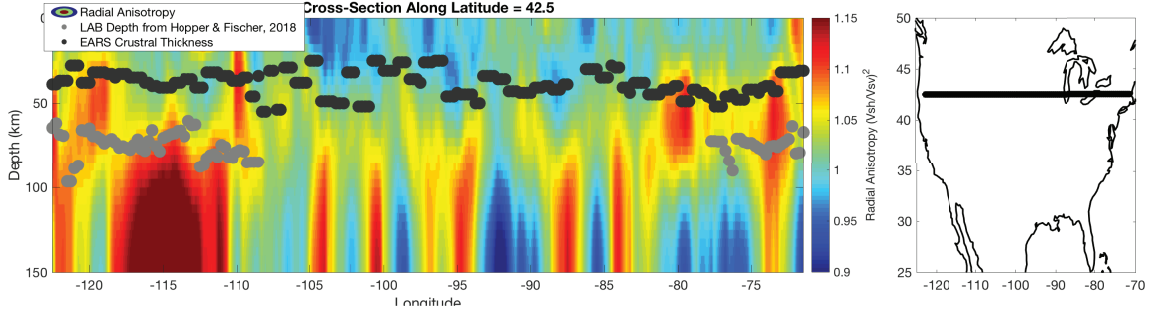


Figure 5.18: Cross-section of our model of radial anisotropy along a latitude of 42.5 degrees. Overlain are S-p receiver function observations of negative velocity gradients corresponding to the lithosphere-asthenosphere boundary (Hopper and Fischer, 2018), and crustal thicknesses from Croswell and Owens (2005).

ous correlations with the delay time amplitude in shear-wave splitting measurements across the U.S, such as those in the global database of Wüstefeld *et al.* (2009), making correlations with inferred hypotheses for shear-wave splitting observations tenuous or suggesting that the signal in the SWS measurements could be dominated by deeper structure than is imaged here. Further eastward, we also do not yet have a geodynamic explanation for the strong radial anisotropy in the upper mantle that parallels the Appalachians.

### Implications for Shear Velocities

Perhaps the strongest changes to isotropic shear velocity that our joint inversion suggests are in the eastern U.S., where the Voigt averaged shear velocity we solve for can reach  $\approx 0.2$  km/s faster than the reference  $V_{sv}$  model. This is a substantial difference ( $\sim 5\%$ ) and is essential to take into account when inferring compositional and thermal properties from shear velocities. At this stage, it is unclear as to why this quantity is underestimated in the SR16 model, but the tests in this section and the behavior of the raw data suggests that it is not an artifact. A comparison of the RMS  $V_s$  with our model and other models of the continental U.S (Figure 19), after interpolating other models onto the same longitude/latitude locations of our model,

suggests that the increase in  $V_{sv}$  in our model at depths until  $\approx 90$  km is within the range suggested by other models, but exceeds other models' predictions at deeper depths. Additional careful testing is important to ascertain the robustness of the increased  $V_s$  at deeper depths.

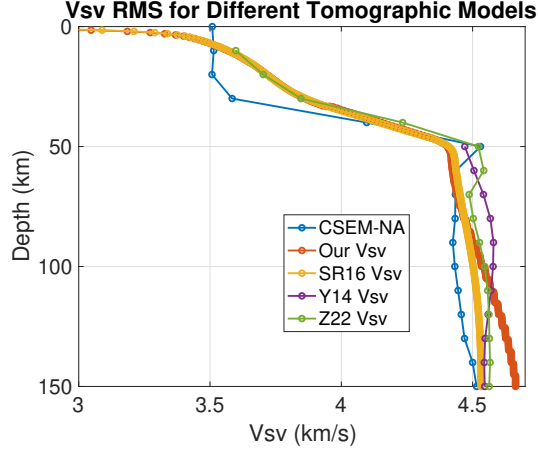


Figure 5.19: Comparison between the  $V_{sv}$  RMS as a function of depth in our model and other radially anisotropic models evaluated at the same locations. Models used include Z22 (Zhu *et al.*, 2017), Y14 (Yuan *et al.*, 2014), CSEM-NA (Fichtner *et al.*, 2018), and the SR16 starting model (Shen and Ritzwoller, 2016).

It is important to account for radial anisotropy when deciding on the right value of  $V_s$  to use for interpretation, particularly in situation where the radial anisotropy deviates substantially from unity. To illustrate this, I compare the Voigt averaged  $V_s$  with the  $V_{sv}$  for all pixels in our model (Fig. 20). At the largest values of radial anisotropy that we measure, the difference between the Voigt averaged value and the  $V_{sv}$  value can reach and exceed 0.2 km/s, which is a substantial amount with the potential to significantly perturb conclusions about thermochemical structure (Havlin *et al.*, 2021).

#### 5.4.3 Future Refinements to the Model

Several refinements can be made to the inversion algorithm to improve the quality of the model and take advantage of the linearized framework in which it is cast. Firstly,

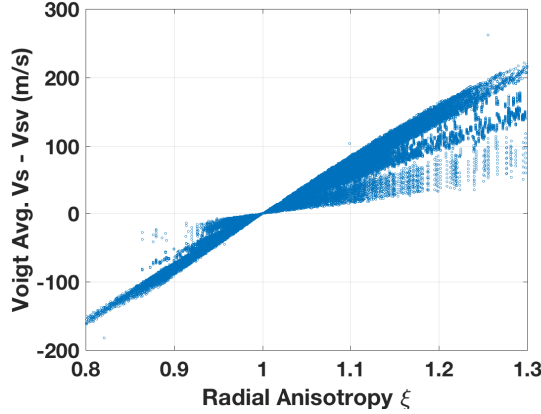


Figure 5.20: Plot of the difference between measured voigt average and  $V_{sv}$  at individual pixels in our model, as a function of radial anisotropy at that pixel.

A benefit of jointly inverting for isotropic and anisotropic structure is the ability to apply different levels of smoothing to each of these components of the model. At present, we have not taken advantage of this ability, but a data-driven approach to arrive at optimal smoothing parameters may lead to a more accurate model in the future.

Thirdly, while vertical smoothing is necessary to condition the inverse problem, it may have a detrimental impact on the inversion’s ability to accurately recover the amplitude of anomalies. The resolution matrices shown in Figure 5 suggest that amplitudes are poorly recovered; recalculating these resolution matrices with reduced smoothing showed slightly higher amplitudes, suggesting shifting the smoothing parameter to the left region of the L-curve ‘elbow’ may be beneficial.

In terms of data, the synthetic tests shown in section 2.3 have highlighted that the inversion encounters more difficulties recovering the amplitudes of  $V_{sh}$  heterogeneity than for  $V_{sv}$ . It is possible that given more information in the form of measurements at more periods, the inversion might perform better in terms of accurately solving for  $V_{sh}$  heterogeneity. Naturally, the inclusion of other, complementary datatypes, such as amplification measurements, ellipticity measurements, group velocity measurements, and particularly converted wave measurements, would likely also improve



the performance of the inversion, and more importantly, eliminate the necessity of relying on the SR16 model as a starting model.

## 5.5 Conclusion

We present a radially anisotropic model of the continental United States, SR16-ANI. SR16-ANI is solved from an inversion of Love and Rayleigh wave phase velocities from earthquake and ambient noise measurements and represents a modification to the isotropic SR16 model (Shen and Ritzwoller, 2016). Our preferred model exhibits a mostly isotropic crust, with notable deviations in the western and easternmost U.S. Radial anisotropy is more widespread in the uppermost mantle and can be associated with geodynamic processes including asthenospheric shear and lithospheric downwelling. Our model represents the culmination of two decades of high-resolution phase velocity images obtained using USArray, and provides new constraints that can feed into geodynamic models.

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## Chapter 6 | Some Future Directions for Studies of Overtone Interference

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## 6.1 Abstract

In this chapter, I develop some future directions that emerge as natural extensions of the work presented in this thesis. I first survey work that further probes the phenomenon of overtone interference by investigating how it impacts other types of surface-wave observations and how it might manifest on other planetary bodies. Following this, I present a tool supporting these studies by allowing for easy calculation of overtone and fundamental mode excitation.

## 6.2 Overtone Interference in other Seismic Observables

The work presented so far in this thesis has catalogued the impact of overtone interference on minor-arc measurements of Rayleigh and Love wave phase, amplitude, and phase velocity. However, we do not yet understand the extent and impact of overtone contamination in other surface-wave observables. Here, I present initial results that analyze overtone interference in a range of Rayleigh and Love wave observables, including group velocities, attenuation, and amplification. These results can provide the reader with guidance as to how overtone interference might manifest in their measurements, what the magnitude of the resulting errors are, and what some approaches to avoid these errors could be. More generally, the synthetic tests I present here offer examples to guide in the assessment of errors in other seismic observables.

### 6.2.1 Overtone Interference in Group Velocity Measurements

Measurements of group velocity provide useful information to infer the shear velocity structure of the Earth’s interior. Moreover, since group velocity sensitivity kernels have a different shape when compared to phase velocity sensitivity kernels and generally have sensitivity to shallower Earth structure, they offer complementary information about elastic structure to that provided by phase velocity measurements.

The most recent studies probing overtone interference for Rayleigh and Love waves (e.g. Hariharan *et al.*, 2022, 2020; Foster *et al.*, 2014) have focused on contamination in phase arrival times, as opposed to group arrival times. The study of Ma and Masters (2014) showed that path-integrated Love and Rayleigh wave group velocity measurements did experience contamination from overtones, but that study did not investigate this contamination further. It is likely that understanding how overtone interference impacts group velocities will enable seismologists to both make more accurate measurements and have increased confidence in these measurements.

Here, I pose several main questions that need to be answered in order to understand the phenomenon of overtone interference in group velocities.

1. How does the relative magnitude of group velocity contamination compare to that of phase velocities?
2. What controls the amplitude and wavelength of group arrival time interference patterns?
3. How does group arrival time interference vary with epicentral distance?
4. How does group arrival time interference vary with the period, group and phase velocity, and relative excitation of the interfering modes?

To address these questions, I present a series of synthetic experiments. First, using the normal-mode code MINEOS, I generate two sets of synthetic seismograms; one containing select overtones and the fundamental mode and one containing just the fundamental mode. Comparisons of measurements between both these seismograms allow us to isolate the contribution to measurements from overtone interference. I make group velocity measurements on synthetic seismograms using the definition of group arrival time *sensu stricto*; I calculate the waveform envelope using the Hilbert transform and then record the time corresponding to the peak of the envelope as the

group arrival time. Figure 6.1 shows an example of the measurement technique, as applied to transverse-component synthetic seismograms containing the fundamental mode and the first overtone.

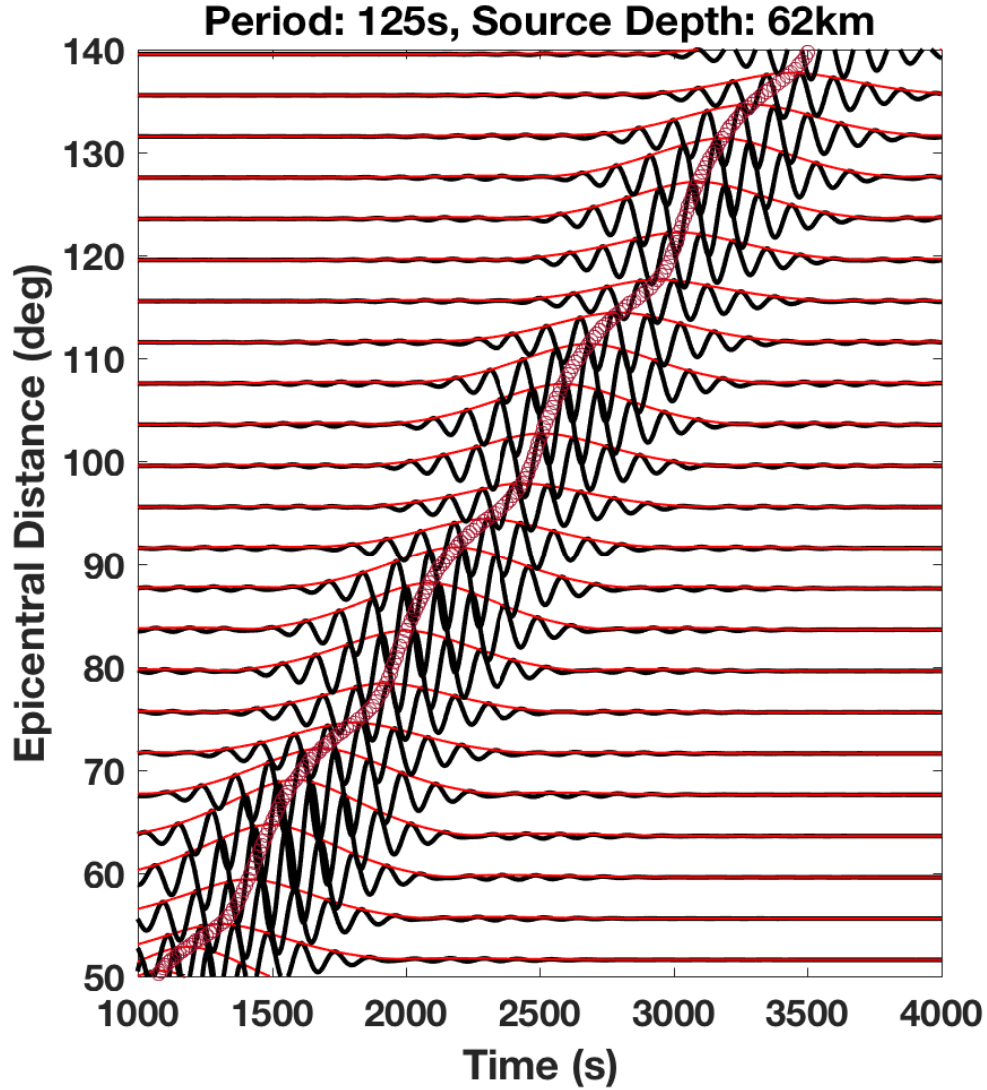


Figure 6.1: Group arrival time measurements for transverse-component seismograms bandpass-filtered at a period of 125 s. The seismograms contain only the fundamental mode and the first overtone. Plotted as red dots are the group arrival time measurements. Overlain as red lines are the calculated envelopes from which group arrival time measurements are made.

Figure 6.2 presents the resulting measurements, plotting the difference between these group arrival measurements and the group arrival times that are measured

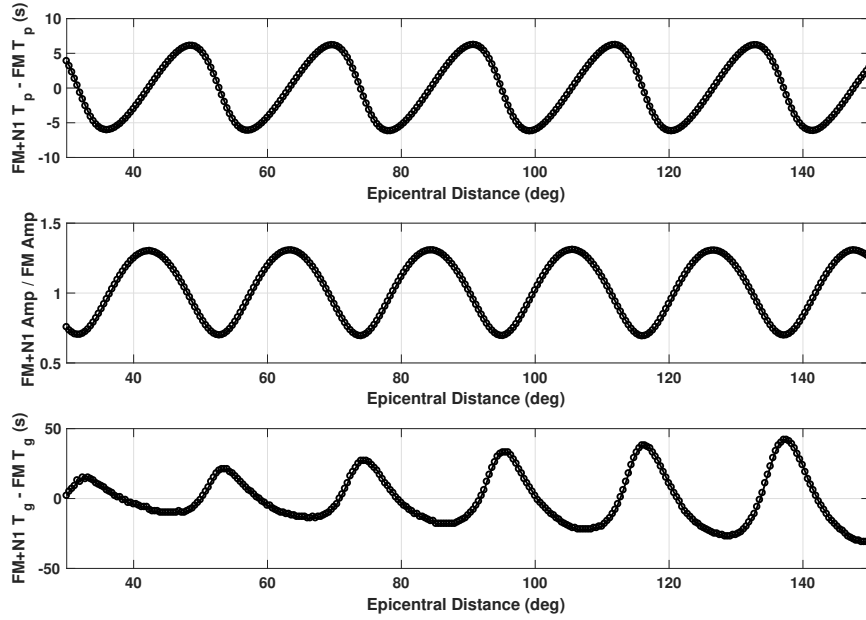


Figure 6.2: Comparison of (top) phase time interference pattern, (middle) amplitude interference pattern, and (bottom) group time interference patterns, for a single earthquake. Measurements are made at stations along a great-circle path.

on seismograms containing only the fundamental mode. A comparison between the group arrival time and the phase and amplitude interference patterns highlights several interesting features. Firstly, while the strength (amplitude) of the phase and amplitude patterns are constant as a function of epicentral distance, the same is not true for the group arrival time interference; the strength of group arrival time interference increases linearly as a function of epicentral distance for this example. The group arrival time interference also has the same wavelength as the phase and amplitude interference pattern. Even at short epicentral distances, the magnitude of the group arrival time interference is consistently much larger than the corresponding contamination in phase velocities. Finally, it is crucial to note that the group arrival time interference pattern is asymmetric, much like the corresponding interference pattern in phase arrival times. Therefore, measurements of group velocities that use an inter-station method will experience some form of systematic bias.

To further understand the origin of the variation of interference with distance, I perform tests where I vary the group velocity, period, and phase velocity of the interfering waves. For wave propagation in realistic settings, these three variables are inextricably linked and vary as a function of period; it is challenging to isolate all but one of these variables using simulations on mode synthetics. Instead, I simplify the problem by approximating the propagating surface wave as a gaussian wavepacket. In this case, the displacement  $u(x, t)$  of a mode as a function of space and time can be expressed as follows:

$$u(x, t) = A_0 e^{-i(\omega t - kx)} e^{-i \frac{(t - t_g)^2}{2\sigma^2}} \quad (6.1)$$

Here, the first exponential represents the ‘carrier’;  $A_0$  is the excitation of the wavepacket,  $\omega$  is the angular frequency, and  $k$  is the wavenumber, defined as  $k = \omega/c$  where  $c$  is the phase velocity. The second exponential represents the ‘envelope’, where  $t_g$  is the group time (defined as  $t_g = x/v_g$  where  $v_g$  is the group velocity) and  $\sigma$  is the pulse width.

I simulate the condition of overtone interference by generating two wavefields; one comprised of a sum of two gaussian wavepackets and one containing just one gaussian wavepacket. I measure group arrival times using the same method as was applied to mode synthetics previously. First, as a benchmark of this approach for simulating surface-wave observables, I replicate the interference pattern shown in the bottom panel of figure 6.2. To best match this case, both pulses have a period of 125 s. The fundamental-mode pulse has an excitation of 1, and the overtone pulse has an excitation of 0.49. The fundamental-mode pulse has a group velocity of 4.42 and phase velocity of 4.72. The overtone pulse has a group velocity of 4.61 and a phase velocity of 6.31. The pulse width  $\sigma$  is set equal to the period of the

waveform. In figure 6.3, I plot the group time interference pattern generated using gaussian wavepackets. There is strong qualitative agreement with the interference pattern shown in the previous figure; in particular, we replicate the wavelength of the interference pattern, the location of peaks and troughs, and the trend of interference increasing with epicentral distance. This shows that the gaussian wavepackets are a reliable tool to investigate group velocity overtone interference.

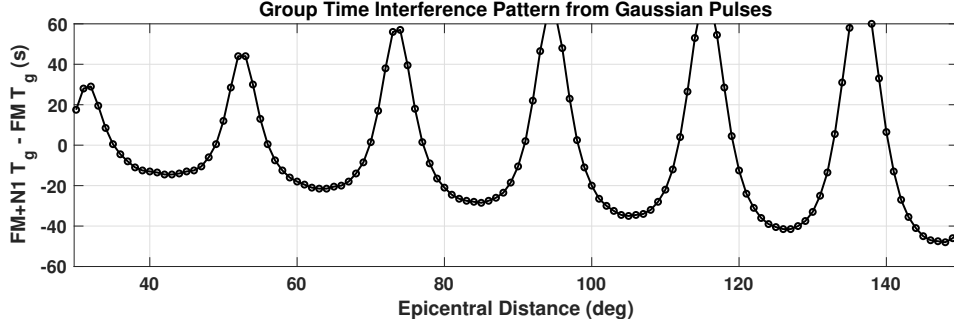


Figure 6.3: Interference pattern in group arrival times, as in the bottom row of the previous figure. The measurements are made on gaussian wavepackets.

Having shown that interference can be simulated in this manner, I now systematically vary, (while holding all other variables constant), the period, group velocity, and phase velocity of the interfering pulses. Doing so allows us to more better understand the mechanisms and origin of group arrival time interference. The results are presented in figure 6.4.

When only the wavepackets' group velocity is varied, the wavelength of the interference pattern does not change. However, when the period and phase velocity of the wavepackets are varied, the wavelength shows a systematic response. This is consistent with the interference pattern wavelength being defined as  $\lambda = \frac{T}{1/c_0 - 1/c_1}$  (Hariharan *et al.*, 2022). The results in Figure 6.4 also show that only group velocity and period variations impact the strength of interference. Finally, the left and center panels show that for some periods and group velocities, the interference does not just increase as a function of epicentral distance, but at a critical distance, begins

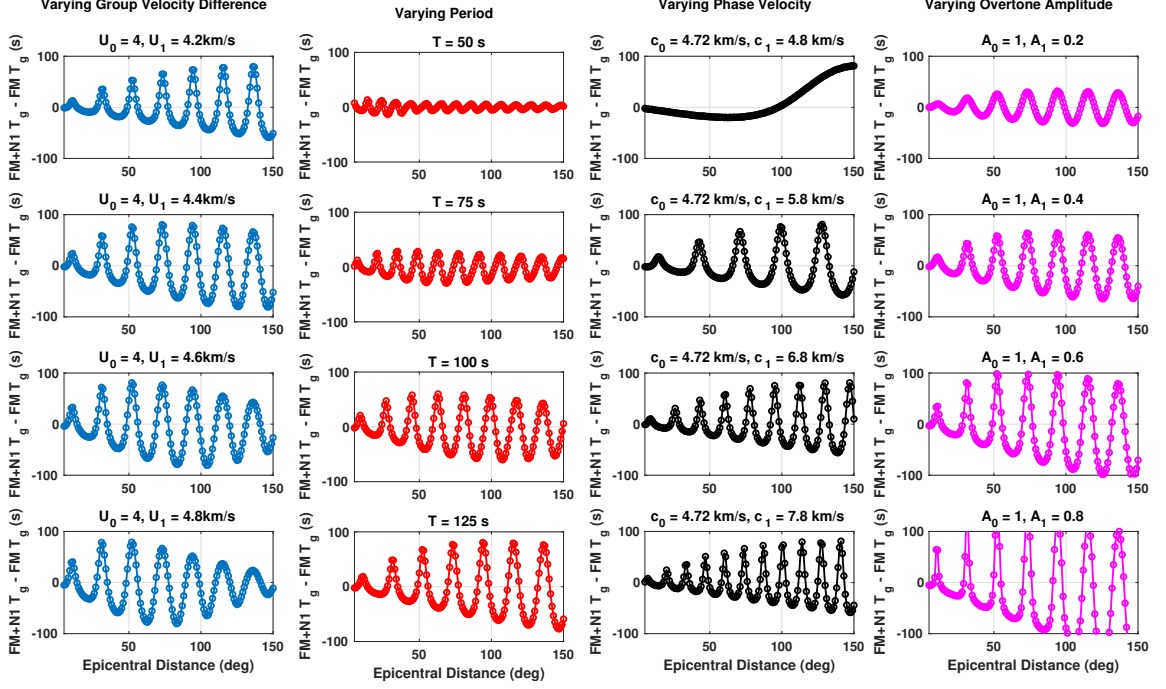


Figure 6.4: Tests to understand the controls on group time overtone interference. Each column shows the impact of varying one variable while holding all others constant.

to decrease. The distance at which this decrease begins to take place (1) decreases with increasing group velocity separation, (2) increases with increasing period and (3) decreases with increasing overtone amplitude.

By plotting an example of the envelopes formed by the interfering waveforms as a function of distance (Figure 6.5), we can see that the distance at which interference begins to decrease most likely corresponds to the point at which the envelope formed by the two waveforms forms two separate maxima. It is therefore a function of variables that impact how well separated in time both the fundamental-mode and overtone waveforms are; the periods of the wave, the relative amplitudes of both, and also the difference between their group velocities.

We can now summarize the mechanism for group velocity overtone interference. At the shortest epicentral distances, the two waveforms have not had much time to separate, and so the group time of the composite, multi-modal envelope is not well



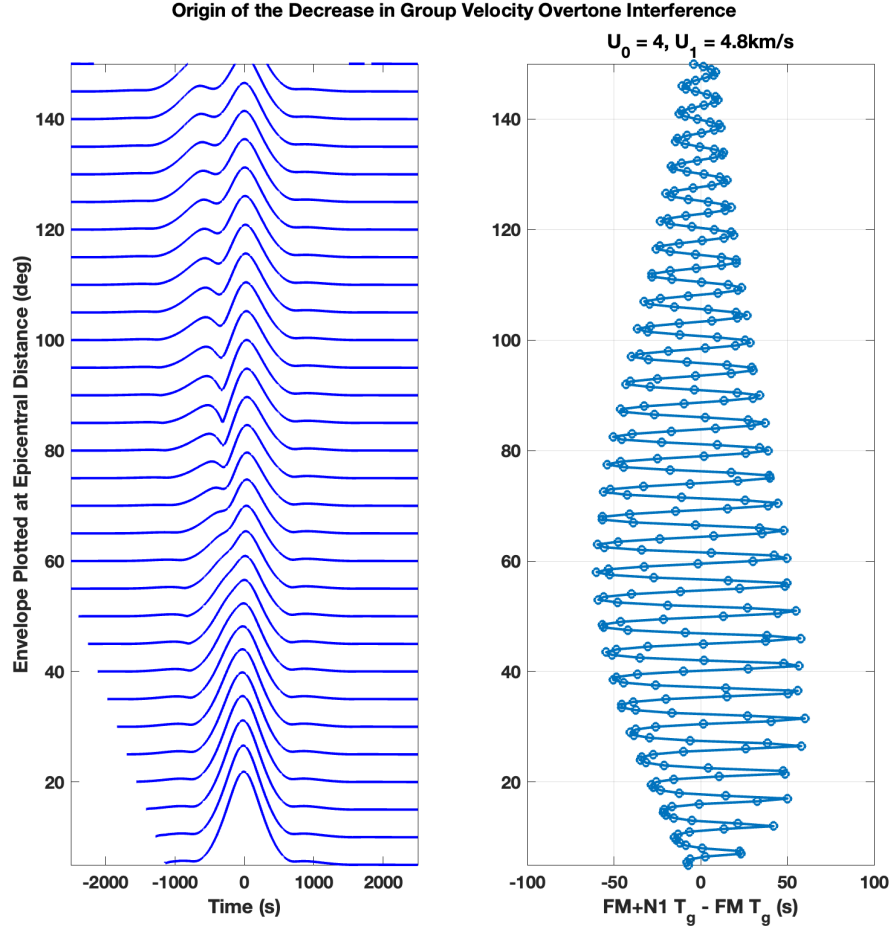


Figure 6.5: Left: Record section showing envelopes of the two interfering waveforms. The envelopes are time-shifted by subtracting the fundamental-mode only group arrival time from the time axis. Right: Group time interference pattern, rotated so that the y-axis is shared with the left panel. Note that the decrease in interference on the right panel coincides with the appearance of a secondary maximum in the waveform envelope in the left panel.

perturbed. As epicentral distance increases, the group time of the composite envelope varies more dramatically, due to the greater group time separation between the two interfering modes. Finally, at a certain epicentral distance (and more importantly, a sufficient group time separation between the two modes), a secondary peak manifests in the composite envelope. More energy is partitioned into this peak with increasing distance and consequently the interference in the group time of the main peak, which

is what is directly measured, begins to decrease with increasing epicentral distances. This peak forms more quickly (that is, with decreasing group time separation) if the overtone has a stronger excitation.

Overall, these results show that theoretically, the distance and period/excitation dependent behavior of group velocity interference may mean it is in fact more complex a phenomenon than interference in phase and amplitude. However, teleseismic measurements of group velocity take place under a narrow range of excitation/group time separation conditions, particularly for Love waves. To understand how the behavior we have described above may manifest in more typical datasets, we perform a more realistic synthetic test. We generate a dataset of 200 randomly chosen earthquakes with source depths between 15-50 km from the GCMT catalog (Ekström *et al.*, 2012). For each of these earthquakes, we make group time measurements on stations spaced along a great-circle path. The measurements are made on synthetics containing both all modes and just the fundamental mode. I convert every group time measurement into a path-integrated group velocity measurement by using  $v_g = X/t_g$ . I also generate group time interference patterns for every earthquake by differencing the group time measured on synthetics containing all modes by the group time measured on synthetics containing only the fundamental mode. The results are plotted in figure 6.6 for measurements made at a single period.

The measurements on different sources are fairly consistent. At short distances, the interference in group velocities (Figure 6.6, top row) is perturbed between 0.15-0.1 km/s away from the true value. The strength of this contamination decreases approximately linearly with increasing epicentral distance. The traveltime interference patterns on representative earthquake sources are also all fairly consistent (Figure 6.6, middle row). They show that interference does increase with increasing epicentral distance until approx. 100 degrees, when the strength of the traveltime interference patterns start to decrease. The increase of traveltime interference with distance is

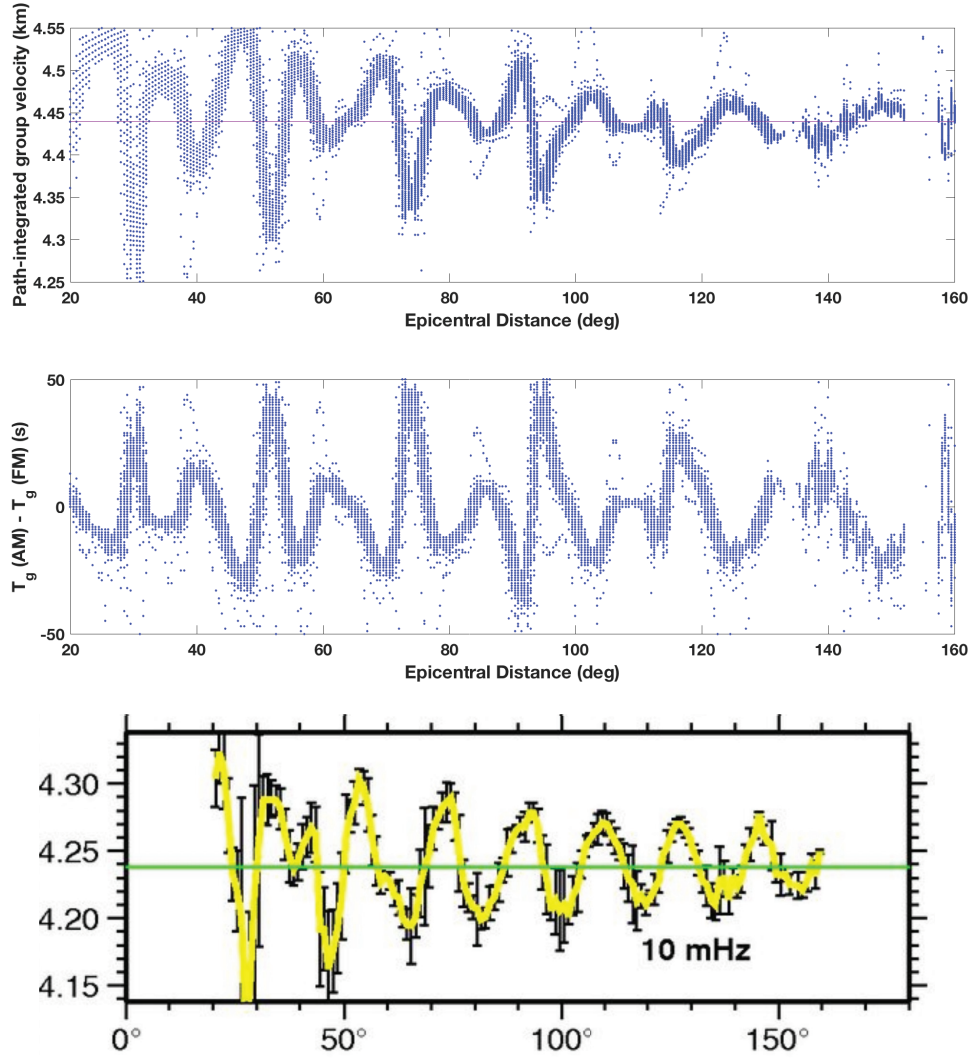


Figure 6.6: Top: Love wave group velocity measurements for 200 earthquakes from the GCMT catalog. Overlain as a solid red line is the expected group velocity predicted by MINEOS. Middle row: Group time interference patterns for the same earthquakes. Bottom row: Comparison with a similar test from the study of Ma et al., 2014. All measurements shown are at a period of 100 s.

relatively subtle (on the magnitude of 20 seconds), and does not noticeably impact the fact that averaging a static error over distances that increase from 10-100 degrees naturally causes the contamination in group velocities to increase dramatically over these large distance. As a benchmark of our group velocity measurements, I also show (Figure 6.6, bottom row) the result from (Ma et al., 2014). The magnitude of the interference shown in this result is approximately consistent in strength with

our results, though the absolute values are different, likely because the synthetics the measurements are made on in that study were computed for a different 1-D Earth model.

In summary, group velocity overtone interference, while similar to phase and amplitude interference in many respects, also shows key differences, particularly in regards to its dependence on epicentral distance. Future work to understand this phenomenon further should explore the impact of group velocity interference on Rayleigh wave measurements, particularly in the distance range prone to major-arc overtone interference. For both Love and Rayleigh waves, future work should also investigate the magnitude of the contamination for array-based measurements, and in particular, to what extent the systematic contamination for Love waves can cause a bias in inversions that use inter-station methods. Furthermore, future work should explore the impact of overtone interference on more common measurement approaches such as the matched-filter approach implemented in the codes of Herrmann (2013).

### 6.2.2 Overtone Interference in Global Attenuation Measurements

Measurements of Rayleigh wave attenuation provide critical information to infer the anelastic structure of the Earth’s interior. The different responses of anelasticity and the elastic shear moduli enable joint inferences of melt and thermal/chemical variations in the Earth’s interior; consequently, measurements of attenuation have been a high priority for seismologists over the last decades.

Attenuation measurements can be derived from accurate measurements of surface wave amplitudes (Dalton and Ekström, 2006). The results of Hariharan *et al.* (2020) and Hariharan *et al.* (2022) showed that the amplitude measurements may experience contamination from overtone interference. However, the magnitude of the error that this contamination introduces into global attenuation measurements is unknown. In this section, I use synthetic data to quantify the extent to which overtone interference can impact global estimates of Rayleigh wave attenuation.

The synthetic data is generated using mode summation as implemented in the MINEOS code, for the 1D oceanic model ATL2a (James *et al.*, 2014). The distribution of sources is 200 earthquakes at depths between 15-50 km, randomly drawn from the GCMT catalog. This synthetic dataset earthquakes was first developed and described in Hariharan *et al.* (2022). At every station, I measure Rayleigh and Love wave amplitudes using fourier analysis. I then calculate the ratio of the amplitude measured on synthetics containing all modes to the amplitude measured on synthetics containing only the fundamental mode; this was referred to in previous sections as the amplitude interference pattern. I now propagate this amplitude interference pattern into estimates of errors in attenuation measurements.

First, we can link attenuation to the measured surface wave amplitudes at a source and receiver as follows:

$$A_r = A_s e^{-\frac{\pi X}{UT} Q^{-1}} \quad (6.2)$$

Where  $A_1$  is the amplitude measured at the receiver,  $A_s$  is the amplitude at the source,  $X$  is the path length between the source and the receiver,  $U$  is the group velocity,  $T$  is the period, and  $Q^{-1}$  is the inverse quality factor, or attenuation. Rearranging, we get:

$$Q^{-1} = \frac{-UT \ln(A_r/A_s)}{\pi X} \quad (6.3)$$

The previous equation is valid when there are no errors in the amplitude measured at the receiver. In the presence of overtone interference, we can represent the error in the amplitude measurements by saying that  $A_{measured} = f A_r$ . Using mode synthetics, we can measure the error  $f$  by saying that  $f = A_{AM}/A_{FM}$ , where  $A_{AM}$  is the amplitude measured on synthetics containing all modes and  $A_{FM}$  is the amplitude measured on synthetics containing only the fundamental mode.

We can distinguish between the *true* and the *measured* attenuation as follows:

$$Q_{true}^{-1} = \frac{-UT \ln(A_r/A_s)}{\pi X} \quad (6.4)$$

$$Q_{measured}^{-1} = \frac{-UT \ln(f A_r/A_s)}{\pi X} \quad (6.5)$$

Differencing the above equations, we get:

$$Q_{true}^{-1} - Q_{measured}^{-1} = \frac{-UT}{\pi X} \left( \ln \frac{1}{f} \right) \quad (6.6)$$

By using this formulation, we do not need to measure the amplitude at the earthquake source  $A_s$ , and can more precisely isolate how the error from overtone interference  $f = A_{AM}/A_{FM}$  maps into attenuation estimates. The above equation also tells us qualitatively about other behavior we can expect when considering errors in attenuation estimates from overtone interference.

1. Since period is in the numerator, we can expect a constant error from overtone interference to have a larger impact at longer periods.
2. Since path length  $X$  is in the denominator, a constant error from overtone interference has a weaker impact at longer epicentral distances.

I apply this analysis to amplitude measurements on the synthetic dataset. Figure 6.7 summarizes the results; the quantity plotted on the left is the quantity  $f = A_{AM}/A_{FM}$ ; the amplitude interference pattern, which shows the perturbation in surface wave amplitude measurements due to overtone interference. The quantity being plotted in the center is  $Q_{true}^{-1} - Q_{measured}^{-1}$ , but normalized by the true attenuation predicted at the period of interest using the MINEOS codes, such that the quantity being plotted is  $Q_{error}^{-1} = 100 \times (Q_{true}^{-1} - Q_{measured}^{-1})/Q_{true}^{-1}$ . The quantity being plotted on the right is the average of the absolute value of  $Q_{error}^{-1}$ , but calculated in a moving window.

Next, we repeat this analysis for Love waves.

For Rayleigh waves, the analysis above shows that errors in attenuation measurements are minimal at epicentral distances between 60-110 degrees. However, at distances between 120-150 degrees, when major-arc overtone interference begins to dominate, estimates of attenuation can exhibit errors of 20% or more. In this distance range, a typical measurement may experience 10% error at 100 s. This increases with increasing period; at 175 s, a typical measurement may experience up to 15% error. This analysis also highlights that the impact of minor-arc overtone interference is also quite pronounced. Despite the contamination in amplitudes rarely exceeding 1.1 at the distances where minor-arc overtone interference takes place, the low value of path length at these distances mean that even low amounts of contamination can have a substantial effect on the inferred error in attenuation estimates. Therefore, the typical measurement at an epicentral distance of 50 degrees exhibits 5% error at 50 s at 20 % error at 175 s. Overall, while there is a distance range where measure-

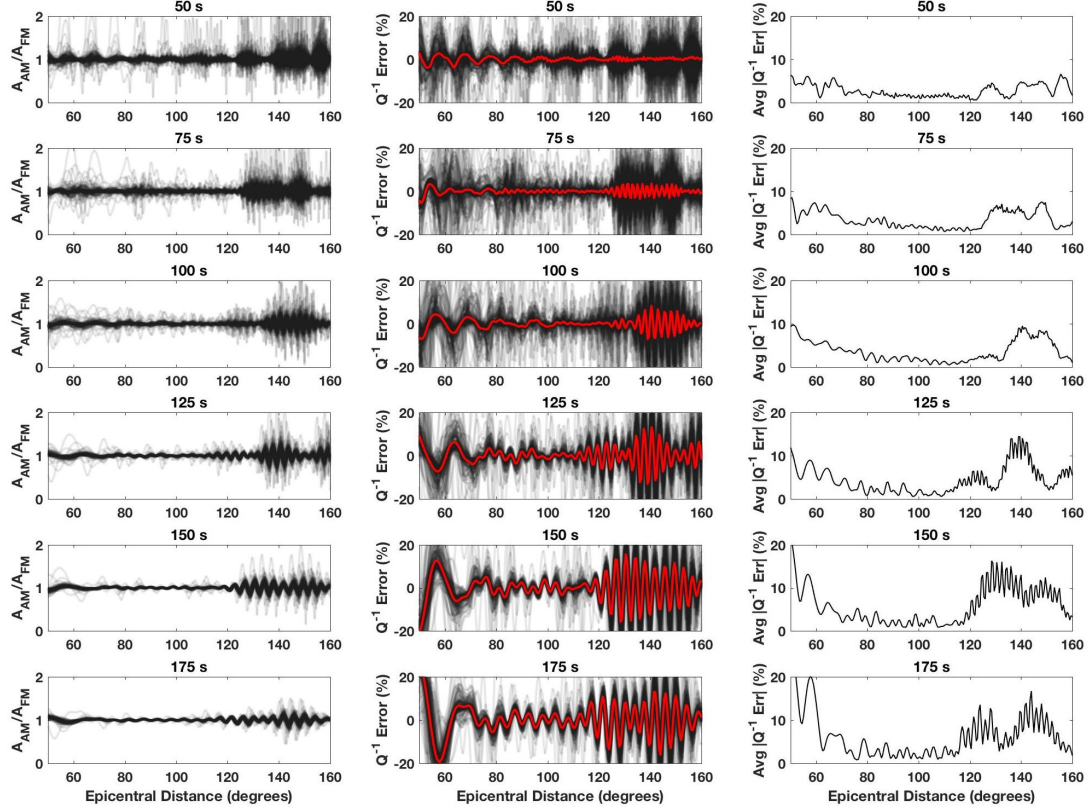


Figure 6.7: Error in global Rayleigh wave attenuation estimates. Left: Amplitude interference patterns for a set of 200 randomly selected events from the Global CMT catalog with source depth between 15 and 50 km. Center: Percentage difference between true and measured attenuation. The quantity is plotted separately for all 200 earthquakes, and the red line is the average across all events. Right: Average absolute difference between true and measured attenuation. All quantities are plotted as a function of epicentral distance.

ments are relatively accurate, our analysis suggests that attenuation measurements can experience 5-20% error with increasing period in the distance ranges prone to both minor-arc and major-arc overtone interference.

For Love waves (Figure 6.8), the situation is much more dire. Superposed atop an oscillatory variation, measurements consistently show errors in attenuation estimates exceeding 50%, decreasing slightly with increasing epicentral distance. Love wave overtone interference is more pronounced at shorter periods due to the relative closeness of the fundamental mode and 1st overtone group velocity curves at these periods. This stronger interference at shorter periods trades offs with the general



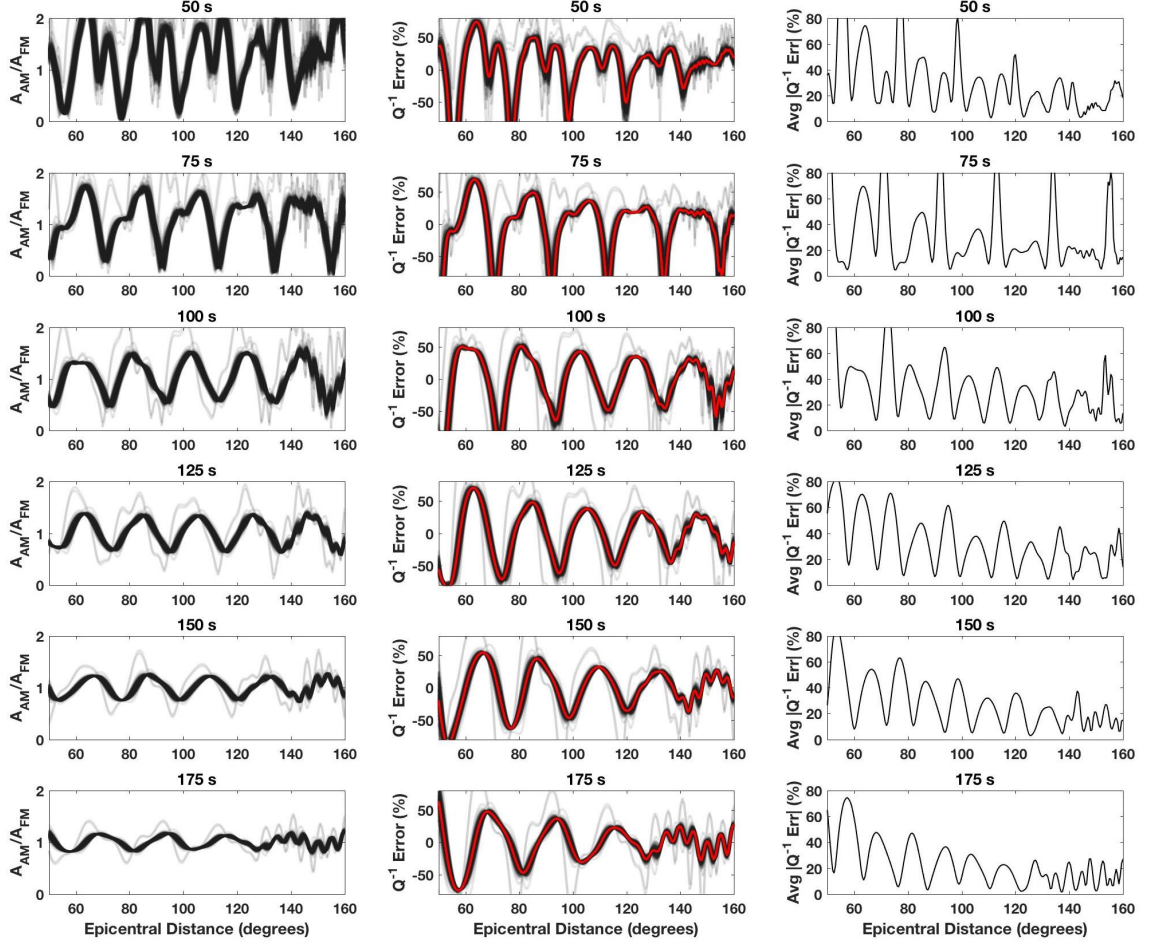


Figure 6.8: As in previous figure, but for Love waves instead of Rayleigh waves.

increase in attenuation error with increasing periods (due to  $T$  in the numerator of equation 6.5), resulting in a subtle decrease in the strength of contamination with increasing period. The overall massive amplitudes of attenuation errors suggest that without careful quality control, it is not suitable to make attenuation measurements using Love waves.

I close this section by noting that the results here are only relevant to global-scale estimates of Rayleigh wave attenuation made using a ray-theoretical framework. For instance, in order to understand how overtone interference impacts array-based Rayleigh wave attenuation such as that calculated by Bao *et al.* (2016) at regional scales, synthetic studies that measure attenuation using the same approach will be

necessary.

### 6.2.3 Overtone Interference in Surface-Wave Amplification Measurements

Measurements of the amplification of surface waves provide unique constraints on the elastic structure of the Earth’s interior. The sensitivity kernels of amplification measurements complement those of both phase and group velocity. Recent work has shown that these measurements provide robust constraints on sediment and crustal thickness. However, we do not yet precisely understand the impact that overtone interference has on amplification measurements. In this section, I present tests that explore the extent to which overtone interference impacts regional estimates of surface wave amplification.

For this analysis, I make amplitude measurements on synthetic seismograms including the fundamental mode and all overtones recorded at a dense array with receivers placed at the locations of USArray stations. Every earthquake from the GCMT catalog that occurred in 2016 at source depths between 15 and 50 km and with  $M_w \geq 6$  (Figure 6.9) is recorded at all these receivers. Amplitude (and phase) measurements are made using the ASWMS toolkit (Jin and Gaherty, 2015). The amplitude measurements are corrected for the effects of attenuation, source radiation, and geometric spreading by dividing the measured amplitudes (measured on seismograms including all modes) by the amplitude measured on a corresponding synthetic seismogram which only includes the fundamental mode.

Amplification analyses are conducted using the approach of Eddy and Ekström (2020): for every event  $k$  and every pair of stations  $(i, j)$  I make event-specific logarithmic amplification measurements  $d_{ij}^k = \log(A_i/A_j)$ , where the indices  $i$  and  $j$  correspond to a pair of stations within 2 degrees of each other. These measurements are averaged over all events to generate a single relative amplification measurement at each station pair  $(i, j)$ . These measurements are then inverted in a least-squares

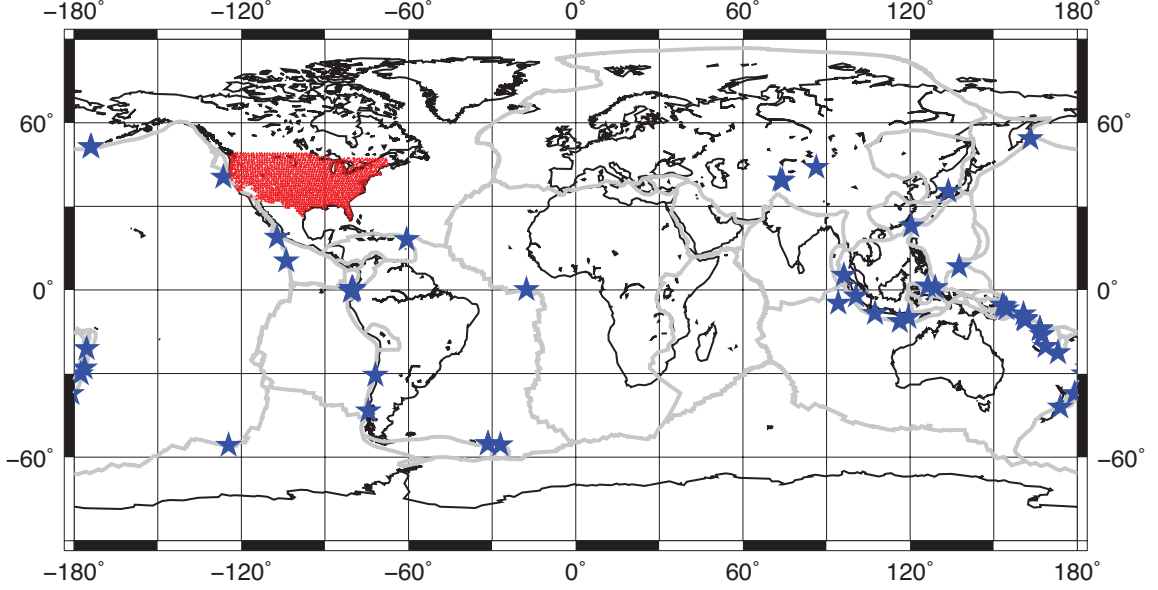


Figure 6.9: Map showing the location of all sources (blue) and the array geometry (red) used in the synthetic tests described in this subsection. Plate boundaries (Bird, 2003) are overlain as gray lines.  $N_{eq} = 52$ ,  $N_{stations} = 1681$ .

framework for maps of relative amplification at each station. By dividing the amplitudes at two nearby stations, this amplitude-ratio approach isolates the contributions to amplitude due solely to local earth structure at each station. In an Earth with no lateral heterogeneity, as is the case in our normal-mode synthetic seismograms, we would expect relative amplification to be equal to 1 at all stations.

I first explore the impact of overtone interference on the individual  $d_{ijk}$  amplification measurements, by plotting the logarithmic  $d_{ijk}$  values as a function of epicentral distance (Figure 6.10, left column). Deviations from 0 reflect contamination from overtone interference. The variation in  $d_{ijk}$  values as a function of epicentral distance highlights the impact of both major and minor-arc overtone interference; at short epicentral distances ( $X < 50$ ), as well as at long epicentral distances ( $X > 120$ ), measurements show striking deviations from 0; these correspond to the impact of minor-arc and major-arc overtone interference respectively. A striking feature of the interference observed in the  $d_{ij}^k$  observations is a strong decrease as a function of increasing period, by around a factor of 4 moving from 50 s to 125 s. Such a pronounced

decrease in the strength of interference is not observed in phase, amplitude, or phase velocity interference (Hariharan *et al.*, 2022, 2020). Consequently, the spread of the station-pair averaged  $d_{ij}$  values decreases dramatically from short to long periods (Figure 6.10, left column).

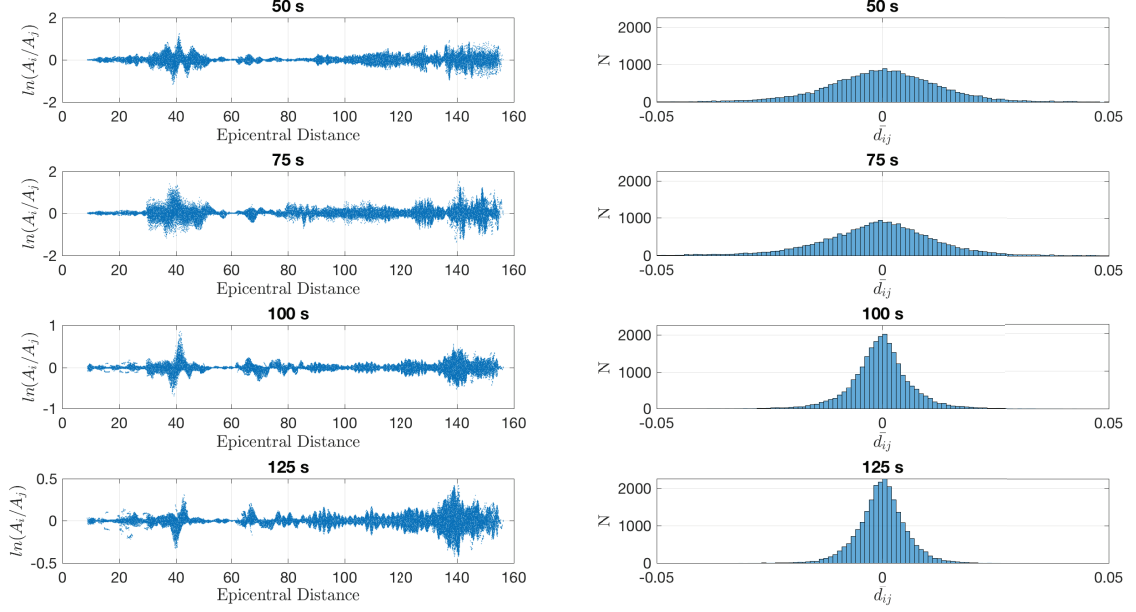


Figure 6.10: Left column: Distribution of individual  $d_{ijk}$  measurements as a function of epicentral distance to the station pair midpoint. Note the change in the y-axis range as a function of period. Right column: Distribution of the station-pair averaged measurements. Each row corresponds to a different period.

When the resulting datasets are inverted for maps of relative site amplification (Figure 6.11), we find that, consistent with the trend of increasing contamination in the measured  $\bar{d}_{ij}$  values at shorter periods, the maps of amplification show more structure at shorter periods. Interestingly, such structure manifests in somewhat streak-like patterns, similar to those observed in Hariharan *et al.* (2020).

Why do both the maps and the individual measurements exhibit such a pronounced decrease in contamination with increasing period? I hypothesize that the cause may be the increase in interference wavelength with increasing period. The wavelength of the amplitude interference pattern is given by  $\lambda = \frac{T}{1/c_0 - 1/c_1}$  (Hariharan

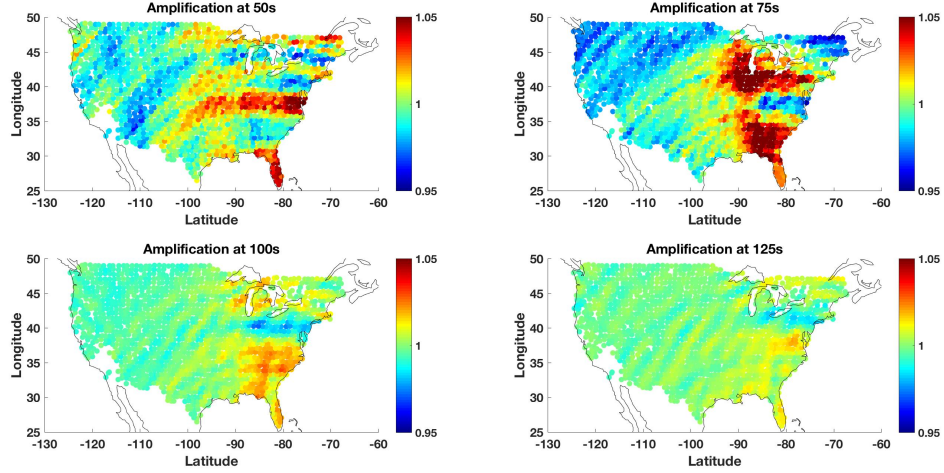


Figure 6.11: Maps of site amplification inverted for using the synthetic dataset shown in Figure 6.9. Each panel corresponds to a different period.

*et al.*, 2022). Thus, at shorter periods, we expect more rapid variations in amplitude measurements. Since in the amplitude ratio method, we take the ratios of amplitude measurements as the datum, then a shorter interference wavelength may lead to more pronounced errors in these measurements, since we limit the range around each station to a radius of 2 degrees. Of course, another factor may be period-dependent variations in relative excitation; Hariharan *et al.* (2022) showed that the minima in fundamental-mode excitation typically increases with increasing period, meaning that the depths sampled here may be more likely to occur closer to these minima, resulting in relatively more overtone excitation. The maps shown also give us a sense of how much contamination we can expect in inverted maps; around  $\pm 0.05$  variations in site amplification for datasets of a similar size and spanning a similar range in depth.

### 6.3 Overtone Interference on Other Planetary Bodies

Seismic imaging is a powerful tool to characterize the physical properties and structure of planetary interiors. To enable these investigations, deployments of seismometers on Venus, Mars, and the Moon have already occurred, and missions are underway

to deploy seismometers on icy moons such as Titan. In these missions, surface-wave measurements are typically accounted for as planned observables and consistently prove their utility; on Mars, surface wave measurements have been used to infer crustal thickness (Kim *et al.*, 2022), locate events (Panning *et al.*, 2023), measure radial anisotropy (Beghein *et al.*, 2022), and even infer lateral heterogeneity when multi-orbit measurements are available (Li *et al.*, 2023). On icy moons such as Europa, studies have shown that the shape of group-velocity dispersion curves can provide constraints on the thickness of the body’s ice shell, even in the absence of a precisely known source (Panning *et al.*, 2006), and that model space search approaches can be used to accurately solve for measurements of ice shell thickness when measurements of fundamental-mode Rayleigh wave group velocity are available (Maguire *et al.*, 2021). Measurements of surface wave dispersion on Titan, projected to be obtained as part of the Dragonfly mission, will constrain the extent to which methan clathrates are present at the top of the ice shell (Marusiak *et al.*, 2022a).

These extra-terrestrial surface wave measurements, like their terrestrial counterparts, are presumably vulnerable to overtone interference. However, the extent to which we should anticipate overtone interference in these measurements has not been considered. To explore this question, I calculate and plot the group velocities of the fundamental mode and overtones in the period range of 25-150 s for Rayleigh and Love waves observed at several of the planetary bodies with current or planned future seismometer deployments (Fig. 6.12); the Earth’s moon, Mars, and Titan, the largest moon of Saturn. For these calculations, I use the planetary models of Garcia *et al.* (2011), Khan *et al.* (2018), and Marusiak *et al.* (2022b).

A visual examination of the group velocities for these bodies shows that for both Rayleigh and Love waves observed on the Moon, Mars, and Titan, the fundamental mode surface wave typically propagates at a group velocity that is relatively distinct from that of the higher modes. There is a slight exception at the shortest periods

examined ( $T \approx 25s$ ) examined for Mars and perhaps also Titan, where the fundamental mode and overtones have similar group velocities. This is consistent with studies on other icy moons; for example, Maguire *et al.* (2021) simulated wave propagation on Europa and observed overtone interference from at least the first three branches ( $n=1-3$ ) in their short-period fundamental-mode measurements ( $T \approx 25s$ ).

Overall, this simple calculation suggests that overtone interference may not be as much of an issue for measurements on other planetary settings as it is for measurements on Earth, but this is an issue that merits further study; factors that remain to be understood include the expected relative excitation for overtones and the fundamental mode given reasonable source mechanisms and the range of expected lateral variations in frequency-dependent group velocity heterogeneity.

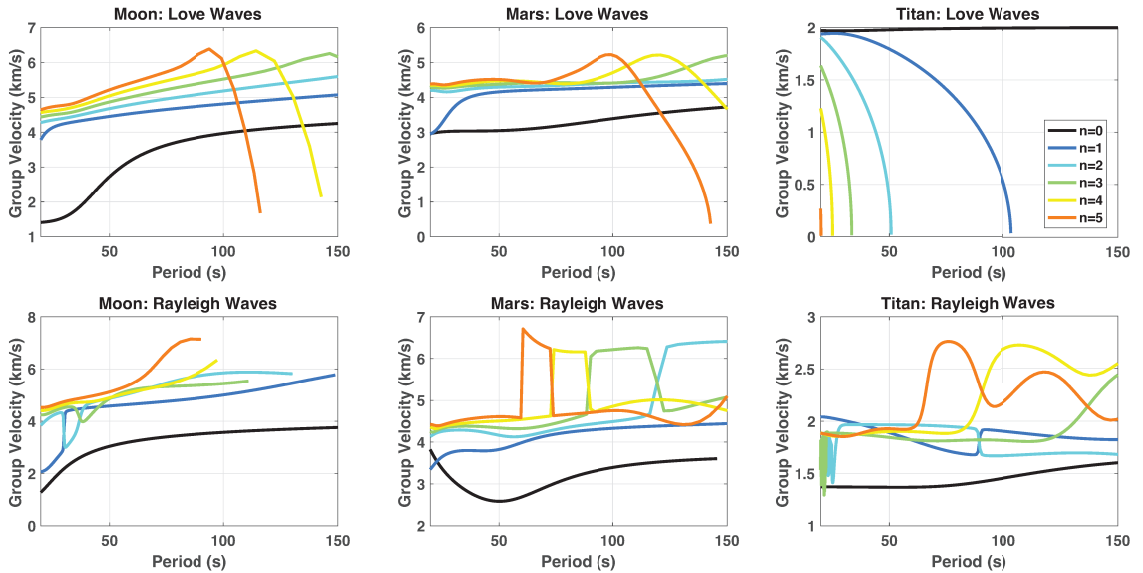


Figure 6.12: Rayleigh and Love wave group velocities for the fundamental mode and first five overtones, for a selection of extra-terrestrial bodies with past or future planned seismometer deployments.

## 6.4 Tools for Calculating Mode-Dependent Surface-Wave Excitation

### 6.4.1 Summary

The work in this thesis and in particular, chapter 2, highlights the importance of accurate calculation of the excitation of different surface-wave modes for obtaining accurate measurements. The open-source ‘ExciteSurf’ software package allows users to rapidly calculate the frequency-dependent radiation pattern as well as source excitation (amplitude) and phase for the Rayleigh and Love wave fundamental mode and overtones generated from an earthquake. By incorporating pre-calculated eigenfunctions, the majority of the computational effort is eliminated and the calculations can be completed extremely rapidly. Options are provided to calculate excitation for homogenous continental and oceanic Earth models, as well as 1D models for the Moon, Mars, and Titan.

### 6.4.2 Statement of need

Measurements of the frequency-dependent phase and amplitude of surface waves generated from earthquakes are essential to improve our understanding of Earth structure and earthquake source processes. Successful modeling and use of these measurements requires accurately accounting for the contribution to these measurements from the earthquake of interest, which has a significant impact on the character of surface wave measurements. For instance, studies using surface-wave observables for imaging of earth structure sometimes implement quality-control steps to eliminate surface wave measurements near nodes in the radiation pattern (Godfrey *et al.*, 2019).

However, calculating the fundamental-mode and overtone radiation pattern is a non-trivial problem and easy-to-use codes to enable this calculation are non-existent, particularly for applications at the scale of a large dataset of multiple earthquakes and



for surface wave overtones. The IRIS-supported surface-wave radiation pattern tool (<http://ds.iris.edu/ds/products/surface-wave-radiation-patterns/tools/surface-wave-radiation-patterns/>) can only be run for a single earthquake at a time, and only for a limited range of depths and frequencies. Traditional, in-house workflows to implement this calculation require solving the surface-wave eigenproblem, which is a computationally difficult task. More generally, the involved nature of the surface-wave eigenproblem has precluded the availability of easy-to-use codes to calculate quantities derived from these eigenfunctions, including surface-wave excitation, receiver polarization, and mode coupling.

‘ExciteSurf’ is a package with versions for both MATLAB and Python users for the calculation of surface-wave radiation pattern and rapid computation of surface-wave source phase and amplitude. ‘ExciteSurf’ works within a normal-mode framework, and uses a database of pre-calculated eigenfunctions calculated using MINEOS Masters *et al.* (2011) to circumvent the majority of the computational effort in a typical workflow to calculate surface-wave excitation. The code is designed with ease-of-use as a first priority, and only requires a single input file with the source depth and moment tensor specified for earthquakes of interest in order to run. ‘ExciteSW’ was designed to be used by both researchers and students. The ease of computing surface wave radiation patterns will enable more researchers to conduct nuanced analyses of surface wave measurements by accounting for the contribution from earthquake source parameters. This can allow for accurate comparisons of surface wave measurements and predictions. Furthermore, estimates of overtone excitation can allow scientists to make estimates of overtone contamination in surface-wave measurements due to both minor and major-arc overtone interference, and use these estimates as a means to improve the quality of measured datasets of fundamental-mode dispersion (Hariharan *et al.*, 2022) by eliminating the impact of overtone interference (Foster *et al.*, 2014; Hariharan *et al.*, 2020).

### 6.4.3 Implementation

‘ExciteSurf’ implements equation 11.34 in (Dahlen and Tromp, 1998) as follows:

$$\begin{aligned}
 R = & \omega[(M_{rr}\dot{U}_s + (M_{\theta\theta} + M_{\phi\phi})r_s^{-1}(U_s - \frac{1}{2}kV_s))e^{i\pi/4} \\
 & + (-1)^q(\dot{V}_s - r_s^{-1}V_s + kr_s^{-1}U_s)(M_{r\phi}\sin\Psi + M_{r\theta}\cos\Psi)e^{-i\pi/4} \\
 & - kr_s^{-1}V_s[M_{\theta\phi}\sin(2\Psi) + \frac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\cos(2\Psi)]e^{i\pi/4}]
 \end{aligned} \tag{6.7}$$

$$\begin{aligned}
 L = & \omega((-1)^q(\dot{W}_s - r_s^{-1}W_s)(M_{r\theta}\sin(\Psi) - M_{r\phi}\cos(\Psi))e^{-i\pi/4} - \\
 & kr_s^{-1}W_s[\frac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\sin(2\Psi) - M_{\theta\phi}\cos(2\Psi)]e^{i\pi/4})
 \end{aligned} \tag{6.8}$$

where  $\omega$  is the angular frequency,  $k$  is the wavenumber,  $M_n$  represents the components of the moment tensor,  $r_s$  is the source radius,  $\Psi$  is the azimuth measured counterclockwise from south,  $q$  is the wavegroup index, and  $U_s$ ,  $V_s$ ,  $W_s$ ,  $\dot{U}_s$ ,  $\dot{V}_s$ , and  $\dot{W}_s$  are the normalized displacement eigenfunctions and their derivatives, respectively, evaluated at the source depth.

The source amplitude  $A_R$  and  $A_L$  for Rayleigh and Love waves respectively is the absolute value of  $R$  and  $L$ , and the source phase  $\Phi_R$  and  $\Phi_L$  for Rayleigh and Love waves is obtained by taking the arctangent of the ratio of the real and imaginary components of  $R$  and  $L$ .

To run the code, the user must edit the parameters in a ‘setup\_parameter’ script to assign whether they are interested in Rayleigh or Love waves, which frequencies(periods) they are interested in, which overtones they are interested in, which wavegroup(major or minor-arc waves) and which velocity model they will use for this calculation. The user must also provide an input file where each row corresponds to an earthquake of interest and the columns contain source depth and moment tensor for every earthquake. The user can then run the ‘Calc\_Excitation\_Ratios\_MAIN’ script, which begins the calculation process. The code will loop over every earthquake, frequency, and mode that the user is interested in, extracting the value of

the displacement eigenfunction at the most relevant source depth and frequency using equations 6.7 and 6.8 to calculate source amplitude and phase. The code loads the eigenfunctions relevant to the earth/planetary model. The output files generated are absolute excitation for every earthquake of interest, and for every overtone. A separate output file is generated for each period of interest. Finally, the library also comes with easy tools to visualize eigenfunctions and phase/group velocities for different Earth and Planetary models of radially varying seismic properties. At present, the code library is in development to incorporate a version in python and a version with interactive, web-based functionality.

#### **6.4.4 Access**

The ‘ExciteSurf’ software package can be freely downloaded from:

[https://github.com/AnantHariharan1996/Calculate\\_Excitation\\_Ratios](https://github.com/AnantHariharan1996/Calculate_Excitation_Ratios)

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