

Transitions in Earth's Convective Regimes: Causes, Styles, and Consequences

By

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Thesis

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INTRODUCTION

While the theory of plate tectonics is often attributed to Alfred Wegener, known as the father of the concept, it in fact emerged through centuries of observations, hypotheses, empirical evidence, and scientific debate. The modern theory, formally proposed in 1933 by Arthur Holmes (Holmes, 1933), and mathematically formalized in the 1960s, holds that the motion of surface plates is the observable manifestation of solid-state convection within Earth’s mantle. In one of the first such formulations, Turcotte and Oxburgh (1967) modeled Earth’s mantle as a high-Rayleigh-number fluid convecting to cool, or, in William Hopkins’ words from 1839, to achieve the “refrigeration of the globe.”(Hopkins, 1839)

However, modern terminology surrounding plate tectonics has drifted from this whole-Earth perspective. Phrases such as “slab pull,” “ridge push,” and “passive mantle upwelling” suggest a surface-driven system, where plates move independently and the mantle simply responds. This language implies a passive mantle – a system in which convection would cease if plate subduction did. Yet the high Rayleigh number of Earth’s mantle (10^6 – 10^8) far exceeds the threshold for convective instability ($Ra_{\text{crit}} \sim 10^3$), indicating that mantle convection is not only active but inevitable. This inference likely holds even for planets like Venus or Mars, where plate tectonics may be absent, but mantle convection likely persists.

I linger on these linguistic choices because they reflect deeper assumptions about Earth’s interior dynamics, assumptions my research seeks to reassess. The analytical framework introduced by Turcotte and Oxburgh implies that plate velocities and surface

heat loss should increase with mantle temperature. Yet these predictions are inconsistent with geologic inferences of plate speeds and mantle thermal evolution. To reconcile this discrepancy, some have proposed a negative feedback mechanism in which higher mantle temperatures lead to increased melt production, causing lithospheric dehydration, strengthening, and ultimately a slowdown in surface motion and heat flux. However, this view places disproportionate control on the surface, suggesting that lithospheric strength can regulate the entire mantle convective system. I argue that this inversion arises from surface-centric terminology, rather than from a geodynamically grounded understanding of mantle behavior.

An essential part of the the development of plate tectonics was the introduction of the concept of the asthenosphere, introduced by Joseph Barrell in 1914. He described it as “a thick earth-shell marked by a capacity to yield readily to long-enduring strains,” which “must exercise a fundamental control in terrestrial mechanics.” With the later discovery of radioactivity as a source of internal heating, the asthenosphere became central to early models of mantle convection and, ultimately, plate tectonics.

The body of work I present in this dissertation builds on that legacy. I focus on the viscosity structure of Earth’s mantle, particularly the asthenosphere, and explore how its evolution governs transitions in tectonic regime. While analytical models have long connected surface observables like plate velocity and heat flow to the mantle’s thermal state, these models are often tuned to the present-day Earth. Yet, early Earth likely operated under very different conditions, involving alternate convection regimes: active-lid (as today), sluggish-lid, and stagnant-lid. Building on the analytical framework of Crowley and O’Connell (2012), I develop a new theory that characterizes these regimes and reveals a multi-solution space for a given set of mantle parameters. This leads to the derivation of new scaling laws, the introduction of a dimensionless number, the O’Connell number—that characterizes plate resistance, and the identification of bifurcations that delineate the boundaries of multi-stability in the system. Removing the assumption of steady-state,

I perform a linear stability analysis to assess the dynamical viability of each convective mode. This work represents the first chapter of my dissertation and has been published in *AGU Solid Earth* (Al Asad et al., 2023), with co-authors H. C. P. Lau, J. W. Crowley, and A. Lenardic.

I also investigate the relationship between mantle convection dynamics and magnetic field generation throughout Earth’s history. I further develop the theory of Crowley and O’Connell (2012) to allow for self-consistent and natural transitions in mantle convective regime (Al Asad and Lau, 2024). I used this modified theory to address a longstanding challenge in Earth evolution: sustaining the long-lived magnetic field, especially before the nucleation of the inner core. Taking an Earth-system approach by coupling my theoretical mantle framework with a core model, my findings suggest that while the Archean Earth exhibited mobile plates, similar to present day, the forces driving plate motion were fundamentally different. These differences have consequential impacts on surface and subsurface processes of the past, and our interpretation of the geologic record. The heat distribution and stress patterns within Earth, governed by different convective regimes, directly affect heat flow at the core-mantle boundary, which drives core convection and sustains Earth’s magnetic field. My research has shown that the persistence of Earth’s magnetic field is consistent with a transition from a sluggish-lid to an active-lid tectonic regime. This insight underscores the importance of considering planetary systems holistically: even if surface expression, like plate velocities, appear similar across different regimes, their subsurface dynamics can have profound implications on planetary evolution. This work has been published in *Science Advances* (Al Asad and Lau, 2024) with co-author H. C. P. Lau and represent the second chapter of my dissertation.

This work also emphasizes the critical role of the asthenosphere in governing the tectonic regime throughout Earth’s history, and paves the way for connections to be identified to the evolution and distribution of factors that determine its viscosity, namely

water and melt. Despite its significance, our understanding of the evolution of water distribution within Earth’s mantle is still in its infancy, where only general bulk budget considerations have been explored. To address this gap, I have developed a model that captures the evolution of water within the mantle and its influence on viscosity both directly (via viscous weakening) and indirectly (via, e.g., melt generation and the dehydration of residual mantle). This fully coupled model integrates insights from various experimental studies to construct a more nuanced picture of how water dynamics influence convective processes. Through this work, I identify a feedback loop whereby deep melting dehydrates the asthenosphere, increasing viscosity and accelerating plate motion, which enhances regassing via subduction. Shallower melting reverses this trend, allowing water retention in the upper mantle, lowering viscosity, and triggering transitions between tectonic regimes. These dynamics yield transient behavior in plate speeds and cooling rates that may leave imprints in the rock record. This is the third chapter of my dissertation and is being prepared for publication in *Geochemistry, Geophysics, Geosystems*. with co-authors H. C. P. Lau and G. Hirth.

Taken together, the chapters of this dissertation aim to move beyond the surface-driven narrative of plate tectonics and re-center mantle convection, and especially the evolving role of the asthenosphere, as the engine of planetary dynamics.

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CHAPTER 1

Modes of Mantle Convection, their Stability, and What controls their Existence

1.1 Introduction

Idealized analytical models of mantle convection have provided much insight on the forces that drive mantle convection, tectonic plates, and heat loss through Earth's surface. Perhaps the most influential result of these models stems from the seminal study of Turcotte and Oxburgh (1967), who derived the classic Rayleigh-Nusselt number relationship (i.e., $\text{Nu} \propto \text{Ra}^\beta$) in the infinite Prandtl number limit for an isoviscous rheological model, establishing the familiar value $\beta = 1/3$ which remains deeply influential throughout studies of mantle convection. Although the assumption of an isoviscous mantle is not considered realistic for Earth's mantle, as the temperature-, compositions-, pressure-, and strain rate- dependence of viscosity results in significant depth and lateral variations, the predicted plate velocities from these scaling models for present-day conditions agree to first order with observations.

The addition of viscosity variations to numerical and experimental fluid dynamics studies of mantle convection (e.g. Jaupart and Parsons, 1985; Davaille and Jaupart, 1993, 1994; Buffett et al., 1994; Conrad and Hager, 1999) revealed the existence of other modes of convection. For example, in the case of strongly temperature-dependent viscosity, which results in large viscosity variations within the domain, convective modes that exhibit small-scale convection below a rigid lid or stagnant-lid convection can emerge. Solomatov (1995) derived scaling laws for three convective regimes that result from distinct rheological models. If the rheology is weak temperature-dependent viscosity, he showed that the mantle convects in what is commonly referred to as an active-lid mode, where the whole mantle convects. With intermediate temperature dependence, a transitional convection mode arises (though the specific planform of this convective regime was not described). And finally, with strong temperature dependence, the so-called stagnant lid mode develops. In addition, developments of mantle convection models which describe the plates using plastic, rather than viscous, rheology, find that the choice of yield stress for the plate determines the mode of the convective regime: low yield stress values result in the active-lid regime, intermediate values in an episodic regime, and high values in a stagnant-lid regime (e.g. Fowler, 1993; Moresi and Solomatov, 1998).

These efforts typically result in analytical descriptions of key parameters in the convection models (such as plate velocities, boundary layer thickness, and Nusselt number), and regime-space diagrams that describe a unique convective regime based on two or more model parameters, such as the internal, surface Rayleigh number, or the viscosity contrast (Solomatov, 1995; Lourenço et al., 2020), and in which the boundaries of the different regimes are well-defined. The possibility of multiple mantle convection modes existing for the same rheological model and thermal state was introduced in Sleep (2000) (see Figures 5–9 therein) with an added possibility that multiple modes could exist where more than one of which may be stable.

The idea of bistability was further strengthened by the analytical thermal boundary

layer model of Crowley and O’Connell (2012) and numerical results of Weller and Lenardic (2012). They demonstrated that, for a single mantle-plate convective system described by the same rheological model and thermal state, multiple convective regimes can coexist. Crowley and O’Connell (2012) identified three, non-stagnant-lid mantle convection modes: one mode that corresponds to active-lid convection, and two sluggish-lid modes in which the plate is involved in the convection but at varying degrees. We will return to more detailed discussion of Crowley and O’Connell (2012), but it is worth mentioning that one of sluggish lid modes, as we will demonstrate, reflects a balance between the buoyancy of the plate and its own dissipation and is similar to how Solomatov (1995) described the “transitional” regime in his study. However, our approach and resulting scaling law we derive here is different. The second sluggish-lid mode, to our knowledge, has not been previously described.

In addition to understanding the general fluid dynamical behavior that such analysis provides, or possible convective modes of other planetary bodies, the interpretation of past Earth mantle temperatures seems to necessitates a different, stable mode of convection in the past. For example, $\beta = 1/3$ found for the active-lid regime results in what has been termed the “Mantle Thermal Catastrophe” (Davies, 1980; Korenaga, 2006). This relatively high, positive β value indicates that when the mantle convects at higher Ra values (i.e., more vigorously), a greater degree of heat loss occurs at the surface, and when integrated backwards in time, results in runaway mantle temperatures.

More complicated forms of mantle convection often result in reduced β values and provide a pathway for avoiding such a catastrophe. For example, Christensen (1985) found that numerical models of mantle convection with temperature-dependent viscosity resulted in non-fixed β values that depend on both the surface and internal Rayleigh numbers, and that β , for today’s mantle, lies between 0 and 0.1, although computational limitations meant that his models could not reach Ra values appropriate for the Archaen. Conrad and Hager (1999) similarly suggested that by accounting for the dissipation in a

relatively strong plate and an otherwise isoviscous mantle, β values fall to approximately zero. In their approach, dissipation within the plate is the main resistive force against plate motion and thus controls plate velocity and surface heat flux. Korenaga (2003) added to their model the effect of depth- and temperature-dependent viscosity, as well as chemical buoyancy on the plate thickness and found that β is not constant and can even become negative for high values of mantle potential temperature, indicating an inverse relationship between the vigor of convection and surface heat flux. When they use the variable β value to reconstruct past mantle temperatures, they are able to avoid high Archean temperatures. However, they assume that plate tectonics was active in its current form for the past 4 Ga, and do not account for changes in the tectonic regime of the mantle-lithosphere system.

Whereas the aforementioned studies focus on reducing β in the Nu–Ra relationship, others explore alternatives for solving the thermal catastrophe. For example, the study by Grigné et al. (2005) focuses on how reducing the constant of proportionality between the two quantities, due to changes in the wavelength of the convective cell at different stages of the Wilson cycle, can also avoid the thermal catastrophe. In addition, water cycling and its effect on mantle viscosity (Seales et al., 2022) has also been proposed to limit the increase in viscosity due to mantle cooling, thus reducing Ra rather than β .

In this work we extend the analytical boundary layer model of Crowley and O’Connell (2012), in which the mantle system includes a strong lithosphere and multiple internal layers of variable viscosity. Although it could be viewed as an effort to combine both approaches of more complicated mantle viscosity structure like that in Christensen (1985), while including dissipative forces within the plate, like that in Conrad and Hager (1999), there is an important distinction between it and previous models. Crowley and O’Connell (2012) theory (hereafter, COC12) allows for the plate and mantle to move at different velocities (U_P and U_M , respectively), and thus permits the self-consistent solution of several modes of convection that depart from the classic view, where $\beta \sim 1/3$. This

is the “active-lid” mode, likely the dominant mode in operation today, where the plate moves at approximately the same velocity as the mantle (i.e., $U_P \approx U_M$). Additionally, they find several “sluggish-lid” modes, in which the plate moves much slower than the mantle (i.e., $U_P \ll U_M$) that also satisfy the energetics of the mantle-lithosphere system. These modes were also found to develop in numerical simulations of mantle convection (Höink and Lenardic, 2010; Lenardic and Crowley, 2012). Moreover, Lenardic and Crowley (2012) not only found these modes to develop, but across the same range of parameters expected by COC12. The direction of the mantle convection community towards the idea of intermittent convective modes (e.g. Silver and Behn, 2008; O’Neill, 2020) merits detailed study of these modes to establish scaling laws that describe their behaviour, as well as to determine their stability.

Our extension involves tackling these two important undertakings. First, we closely examine the local lithospheric and global energy balances to derive scaling laws for all the solutions to the governing equations. We show that the classical $Nu \propto Ra^{1/3}$ scaling is recovered in the active-lid solution and present new scaling laws for the sluggish-lid regimes. In those regimes, Ra alone is unable to fully characterize the behavior of the mantle and we introduce a new non-dimensional number, the O’Connell number (Oc). Together, Ra and Oc are able to describe the dynamics of the system. Second, we determine the stability of the different regimes and the characteristic decay timescale towards or away from each specific regime. Such timescales are conceptually useful as an indication of the likelihood of any given state. We find that the active-lid and the lower sluggish-lid branches are stable, whereas the intermediate branch of sluggish-lid regime is not.

The paper proceeds as follows: in Section 1.2.1 we describe the model introduced in COC12, summarizing key equations and results from that study. Within their framework, we derive new scaling laws for plate velocity and heat flux (Section 1.2.2). We follow this with Section 1.2.3 where we depart from the main assumption of steady-state in COC12’s original presentation of their model and perform a stability analysis of the multiple regimes

of convection and explore themes surrounding our results. We conclude this study by discussing the key implications of this extended theory for Earth and beyond (Section 1.3). This includes mapping out the regime phase diagram where we discuss the boundaries between the existence and disappearance of mantle convection modes (Section 1.3.1) and implications for Earth’s thermal history and how it changes with different styles of convection (Section 1.3.2).

1.2 Extending Crowley and O’Connell’s Theory

1.2.1 Recapitulation of Crowley and O’Connell’s theory and their main results

The original analytical model presented in Crowley and O’Connell (2012) combines two energy balance equations: one for the whole system (mantle and lithosphere) and one for the lithosphere alone. Together these are solved to produce the plate and mantle’s velocity profile, the associated viscous stresses, and lateral pressure gradient profiles. The important features of their model are the inclusion of plate bending dissipation, a layered viscosity profile, the contribution of the pressure gradient in driving flow, and, most importantly, the removal of the assumption that the mantle and plate flow at the same velocity. The result of these features is the existence of one to multiple solutions for the mantle’s velocity profiles at any given thermodynamic and mechanical state. Each solution corresponds to distinct styles of mantle convection. Here, we recapitulate the energy equations for the whole mantle and lithosphere as introduced in Crowley and O’Connell (2012). For a comprehensive description and discussion of the model, we refer the reader to their original paper. In Table 1.1, we tabulate all the symbols and our adopted values.

Fig. 1.1 shows the geometry of the model setup, with labels to relevant material parameters and length-scales. (Note that the figure depicts three layers, but the model allows for any number of distinct layers of different material properties.) The global energy

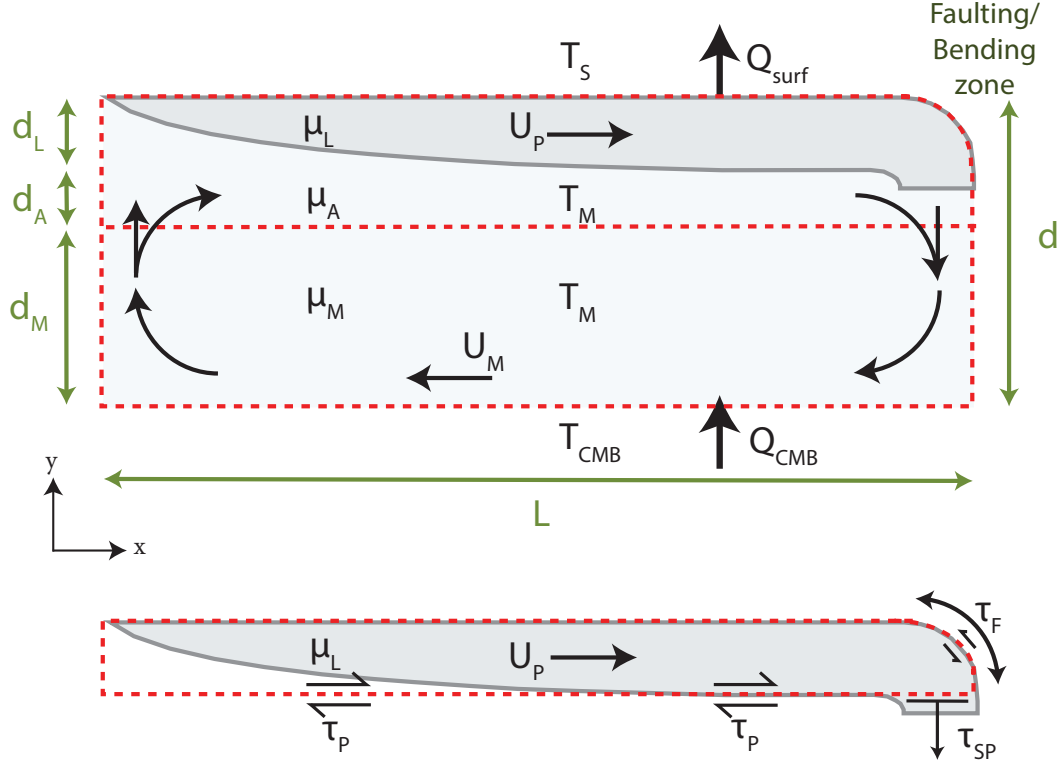


Figure 1.1: **Schematic Diagram of Model Geometry:** Top box represents the global domain, where any number of mantle layers may be included (we have shown three: the lithosphere, the asthenosphere, and the lower mantle), where parameters associated with each subscripted L, A, and M, respectively. The bottom box represents the lithospheric domain. Here, d_X , μ_X and U_X are the thicknesses, viscosities, and horizontal (in the x direction), velocities of layers X . T and Q are the temperature and heat flow, respectively, where “S” and “CMB” subscripts denote the surface and core-mantle boundary, respectively. The x – and y –directions are also shown, where x is the direction perpendicular to the spreading ridge axis and y is the vertical direction from the CMB to the surface. See Table 1.1 for definitions and adopted values. Reproduced from Crowley and O’Connell (2012) with permission (Figure 1)

Table 1.1: Table of Variables and Parameters

Symbol	Description	Value and unit adopted (if applicable)
α	Coefficient of thermal expansion	10^{-5} K^{-1}
g	Acceleration due to gravity	9.8 m s^{-2}
d	Thickness of domain	3000 km (Fig. 1.1)
C_p	Heat capacity	$1200 \text{ J kg}^{-1} \text{ K}^{-1}$
d_L	Thickness of the lithosphere	Eq. 1.4 (Fig. 1.1)
c_0	Constant in Eq. 1.15	2.32^a
κ	Thermal diffusivity	10^{-6} ms^{-2}
L	Plate width	6000 km
k_c	Thermal conductivity	$3.2 \text{ Wm}^{-1}\text{K}^{-1}$
μ_0	Reference mantle viscosity	10^{21} Pa s
T_0	Reference temperature for the viscosity	1300 K
E	Activation energy for the viscosity	200 kJ mol^{-1}
R_g	Ideal gas constant	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
ρ	Density of mantle	3500 kg m^{-3}
$H(t)$	Internal Heat ^b	^c W kg^{-1}

^b The time dependent form of $H(t) = \sum_i H_{i0} \exp(-t/\lambda_i)$

^c Fuentes et al. (2019)

balance equation for the whole mantle system is

$$\frac{\alpha g d}{C_p} \langle Q \rangle = d_L \tau_Y U_P + \Phi_M, \quad (1.1)$$

where the left-hand side of the equation represents the rate of change of whole mantle potential energy and scales linearly with the depth-averaged advected heat flow $\langle Q \rangle$. Here, α , g , d , and C_p are the thermal expansivity, the acceleration due to gravity, the depth of the mantle, and the specific heat capacity, respectively. The terms on the right-hand side represent the rate of dissipation in the mantle due to the bending of the lithosphere and the viscous dissipation within the mantle, respectively, the latter of which is a function of the viscosity and velocity profiles of the mantle. In these terms, d_L is the thickness of the lithosphere, τ_Y is the effective yield stress and U_P is the plate velocity. It should be noted that no assumptions are required for the plate rheology in this model. Regardless of the real plate rheology or the true mechanics of plate bending, there is always an

amount of work or energy that is required for plate deformation. If this energy is given by some model dependent Φ_L , then one can always calculate an effective yield stress that is consistent with the energetics as $\tau_Y = \Phi_L/(d_L U_P)$. The lithospheric energy balance is

$$\frac{\alpha g d}{C_p} \langle Q \rangle_L = (d_L L P_{x_P} + L \tau_P + d_L \tau_R) U_P, \quad (1.2)$$

where the term on the left-hand side represents the rate of change of lithospheric potential energy and $\langle Q \rangle_L$ is the depth-averaged heat flow through the lithosphere. The right-hand side terms represent the sources and sinks. The first is the pressure gradient at the base of the lithosphere which drives the plate away from the ridge, where L and P_{x_P} are the horizontal length of the plate in the direction of plate flow (Fig. 1.1) and horizontal pressure gradient at the base of the plate, respectively. The second term is the traction at the base of the plate, τ_P , which is positive if the plate is moving faster than the mantle below (i.e., a sink), or negative if it is moving slower (i.e., a source). The last term is the net resistive stress given by $\tau_R = \tau_Y - \tau_{SP}$, the difference between the effective yield stress of the plate and the stress due to slab pull.

Multiple plate velocities may satisfy Eqs. 1.1 and 1.2 and these represent multiple mantle convective regimes in which the dominant energy balances vary. Some of the permitted solutions include convective regimes that are layered, where the subducted material does not reach the base of the mantle but returns to the surface from mid-mantle depths. While we do not focus on such solutions here, we note that this class of layered convection regimes could represent several historically important planforms of convection. For example, we observe solutions in which small scale convection occurs in the top layer, reminiscent of the so-called Richter rolls, hypothesized to develop in highly temperature-dependent viscosity fluids (e.g. Richter, 1973; Buffett et al., 1994). In other cases, the two layers are more comparable in size and could be considered as a mode consistent with early arguments for independently convecting largescale mantle reservoirs, as indicated by geochemical observations of mantle derived basalt (e.g., Jacobsen and Wasserburg, 1979;

Hofmann, 1997). Because it is widely accepted that significant mass exchange occurs between the upper and lower mantle (e.g., Morgan and Shearer, 1993; Van der Hilst et al., 1997), we will discuss only the whole-mantle class of solutions.

For the set of parameters that cover a wide range of possible thermodynamical and mechanical states of the mantle, there exists up to three whole-mantle convection modes. These solutions are displayed in Fig. 1.2a, where we show values of U_P that satisfy Eqs. 1.1 and 1.2 as a function of Ra . For all calculations in this study, the mantle is assumed to be isoviscous, but that its temperature-dependent and is described by the Arrhenius equation

$$\mu_M(T) = \mu_0 e^{\frac{E}{Rg} \left(\frac{1}{T} - \frac{1}{T_0} \right)} \quad (1.3)$$

where μ_0 is the reference viscosity at the reference temperature T_0 . We will discuss effects of this choice over other more realistic choices in Section 1.3.3, but for now choose the simplest setting for demonstrative purposes. The resulting solutions lie along distinct branches and correspond to: the classic active-lid regime in which the plate and mantle beneath are strongly coupled, moving at approximately equal velocities (i.e., the uppermost branch marked by red circles; Fig. 1.2a), and two sluggish-lid regimes where the mantle moves at velocities that exceed the plate above. In Fig. 1.2a these are the middle branch (green circles) and lowermost branch (blue circles). Typical velocity profiles of active and sluggish-lid solutions are shown in Fig. 1.2b for a single Ra value (see vertical line in panel a).

The original COC12 model is able to capture the behaviour of the present-day mantle as well as possible past behaviours, but COC12 note that it has certain simplifications which could limit its use. One of these is because they describe a steady-state system, the analysis of stability may not be addressed. Without knowledge of their stability, COC12 sought to explore their existence by comparing these solutions to the numerical model of Höink and Lenardic (2010). They also noted through inspection of their solutions that

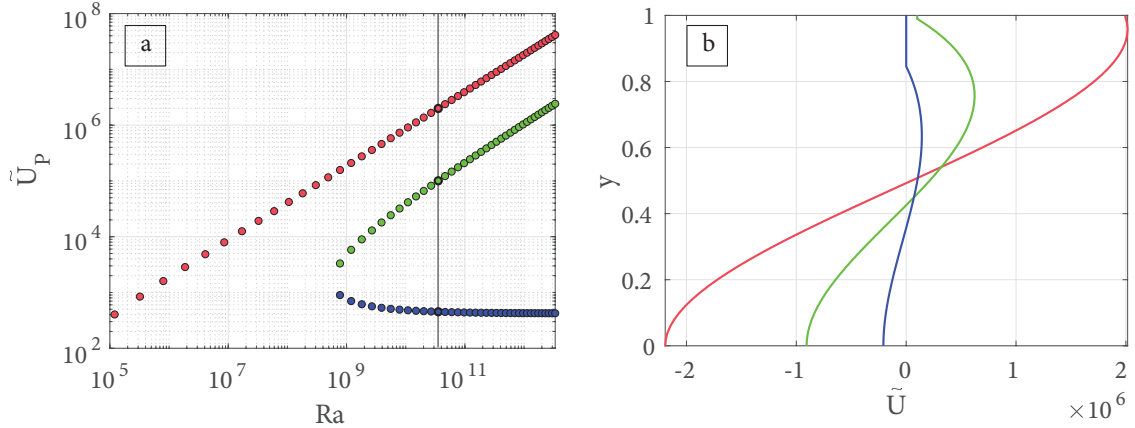


Figure 1.2: **Solution space of Crowley & O’Connell’s (2012) analytical model:** Plate velocity solutions of the coupled mantle (Eq. 1.1) lithosphere (Eq. 1.2) energy balance from COC12. (a) Plate velocity solutions as a function of Ra . (b) mantle horizontal velocity profiles as a function of depth for the three solutions at $Ra \approx 10^{10}$ (shown by the vertical line in (a)). The fastest plate velocity (red) corresponds to the active-lid solution, while the intermediate and slow velocities represent two distinct sluggish-lid regimes where the mantle velocity exceeds the plate velocity.

they correspond to certain force balances in the global (Eq. 1.1) and local (Eq. 1.2) energy equations, and began the process of deriving scaling laws for the plate velocity, which we will complete. Finally, they noted that a necessary condition for the existence of multiple solutions is that the plate hinders, rather than drives, the motion of the mantle beneath it, i.e., that τ_R is negative. While this condition must be met, it is alone not sufficient to fully determine where multiple solutions exist. We address this in Section 1.2.2.

In the rest of this study, we expand on the work accomplished by COC12 by directly addressing each of their highlighted areas of potential development, namely by (1) deriving the noted scaling laws, (2), investigating the bifurcations and (3), conducting the stability analysis. As will be seen, these extensions reveal a richness in these solutions that have important implications for mantle dynamics.

1.2.2 Scaling laws and force balances

Through the derivation of scaling laws, we show that the different solutions for the plate velocity that arise in COC12’s theory correspond to conditions in which different

terms in the (global and lithospheric) energy balances (Eqs 1.1 and 1.2), dominate. To begin, we use the half-space cooling model for the plate thickness as in Turcotte and Schubert (2002), where

$$d_L = c_0 \left(\frac{\kappa L}{U_p} \right)^{1/2}, \quad (1.4)$$

where κ is the thermal diffusivity and c_0 is a constant of proportionality that depends on the geometry of the system (typically $\mathcal{O}(1)$). We also use their expression for the surface convective heat flux, q_{conv} , in their thermal boundary layer model,

$$q_{\text{conv}} = 2k_c \Delta T \left(\frac{U_p}{\pi \kappa L} \right)^{1/2}, \quad (1.5)$$

where k_c is the thermal conductivity and ΔT is the temperature difference across the plate.

We make a few simplifying assumptions to enable derivation of compact laws from highly non-linear equations. First, we require that the plate is weak enough to allow for the plate to participate in the convection (i.e., we do not characterize the so-called stagnant-lid regime (Solomatov and Moresi, 2000)). Mathematically, the weak plate condition requires that the plate dissipation term is much smaller than the total rate of change of potential energy in the mantle, or, by Eq. 1.1,

$$\frac{\alpha g d}{C_p} \langle Q \rangle \gg d_L \tau_Y U_P, \quad (1.6)$$

such that

$$\frac{C_p}{\alpha g d} \frac{\tau_Y d_L U_P}{\langle Q \rangle} \ll 1. \quad (1.7)$$

Substituting Eqs. 1.4 and 1.5, this results in a new stress non-dimensionalization relevant for a convecting mantle with a strong lid, given by $\alpha \rho g d \Delta T$, which may be considered a

measure of the strength of the plate against the stresses induced by the convecting mantle beneath. We can therefore write the above condition in Eq. 1.7 as

$$\tau_Y^* \ll \frac{1}{c_0 \sqrt{\pi}} \approx \frac{1}{4}, \quad (1.8)$$

where $\tau_Y^* = \tau_Y / (\alpha \rho g d \Delta T)$ and d_L is provided by Eq. 1.4. This requirement nullifies the plate dissipation term and simplifies Eq. 1.1 to a balance between buoyancy and viscous dissipation in the mantle, or

$$\frac{\alpha g d}{C_p} \langle Q \rangle \approx \Phi_M. \quad (1.9)$$

The total viscous dissipation in the mantle varies linearly with the mantle viscosity, μ_M , and with the characteristic mantle velocity, U_M , squared. Specifically,

$$\Phi_M \approx 4c_1 \mu_M U_M^2, \quad (1.10)$$

where c_1 is a constant of $\mathcal{O}(1)$ that depends on the geometry and viscosity structure of the mantle (see also Eq. A15 in COC12). Combining Eqs. 1.9 and 1.10, and once more making use of Eqs. 1.4 and 1.5 yields the scaling relationship

$$\tilde{U}_M^2 \approx \frac{1}{4c_1} \sqrt{\frac{L}{\pi d}} \text{Ra} \tilde{U}_P^{1/2} \quad (1.11)$$

where the “ $\sim$ ” over-script represents the non-dimensional velocity using a factor of (κ/d) , and we have defined Ra as

$$\text{Ra} = \frac{\alpha \rho g \Delta T d^3}{\kappa \mu_M}. \quad (1.12)$$

Given these relations, we can derive the scaling laws for each convective mode.

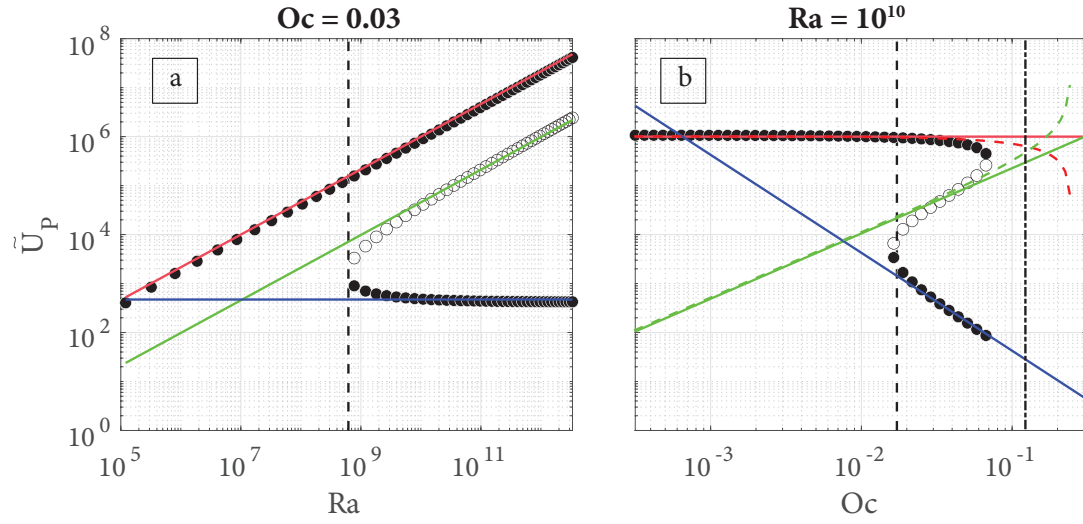


Figure 1.3: **Comparison of scaling laws against analytical results and their stability:** (a) results of the scaling laws (solid lines) as a function of Ra compared to the solutions of the full analytical model (circles) for $Oc = 0.03$, corresponding to variable τ_Y all of $\mathcal{O}(100)$. (b) similar to (a) but as a function of Oc and constant $Ra = 10^{10}$. The dashed vertical lines in (a) and (b) are the first bifurcation Rayleigh (Ra_{B1}) and O'Connell (Oc_{B1}) numbers given by Eqs 1.30 and 1.31, respectively. The vertical dash-dot line in (b) is the second bifurcation O'Connell number (Oc_{B2}) given by Eq. 1.36. In both panels, the red lines represent the active-lid scaling law, the green lines the traction-driven sluggish-lid, and blue lines, the P.E.-driven sluggish-lid solutions.

Active-lid Scaling

If the characteristic mantle and plate velocities are assumed to be approximately equal, which, once more, is the key characteristic of the active-lid mode, then by setting $\tilde{U}_M \approx \tilde{U}_P$ in Eq. 1.11 results in

$$\tilde{U}_M \approx \tilde{U}_P \approx \left(\frac{1}{4c_1} \right)^{2/3} \left(\frac{L}{\pi d} \right)^{1/3} \text{Ra}^{2/3} \quad (1.13)$$

and with this,

$$\begin{aligned} \text{Nu} &\equiv \frac{q_{\text{conv}}}{q_{\text{cond}}} \\ &= \left(\frac{2}{c_1} \right)^{1/3} \left(\frac{1}{\pi} \right)^{2/3} \left(\frac{L}{d} \right)^{-1/3} \text{Ra}^{1/3}. \end{aligned} \quad (1.14)$$

This scaling solution represents the classical active-lid solution, $U_P \propto \text{Ra}^{2/3}$ and $\text{Nu} \propto \text{Ra}^{1/3}$, in which the mantle and plate velocities are assumed to be equal (Turcotte and Oxburgh, 1967).

Sluggish-lid Scalings

By the nature of the sluggish-lid solutions, we cannot make the assumption that $\tilde{U}_M \approx \tilde{U}_P$, and therefore require another equation to derive the scaling laws. Recall that a key distinction between COC12's approach and that of previous studies that included plate dissipation (e.g., Conrad and Hager, 1999) is that COC12's approach allows the plate and mantle velocity to be different, thus permitting sluggish-lid solutions characterized by slow moving plates overlying a fast moving mantle.

The condition for the existence of the sluggish-lid solutions, as discussed in COC12, is that the net resistive stress $\tau_R = \tau_Y - \tau_{SP}$ must be positive, or in other words, that the net resistive stress hinders rather than drives the plate motion. Turning to Eqs. 1.5 and

1.4 and rearranging Eq. 1.2 yields

$$0 = \sqrt{\frac{L}{\pi d}} - \frac{LP_{x_p}\tilde{U}_P^{1/2}}{\alpha\rho g\Delta Td} - \frac{1}{c_0}\sqrt{\frac{L}{d}}\frac{\tau_P\tilde{U}_P}{\alpha\rho g\Delta Td} - \frac{\tau_R\tilde{U}_P^{1/2}}{\alpha\rho g\Delta Td}. \quad (1.15)$$

As this equation will be referred to repeatedly in the following derivations and discussion, we summarize the constituent terms: The first term is the rate of change of potential energy of the lithosphere and it is always positive (i.e., it works to drive the plate motion), this term is known in literature as ridge-push or gravitational sliding (Turcotte and Schubert, 2002). The last term is the work done by the bending and pull of the plate to cumulatively resist the plate motion, which is always negative. The second and third terms are, respectively, the work done by the pressure gradient and the traction at the base of the plate to either drive or resist the plate motion, depending on their sign. If the mantle below the plate is moving faster than the plate, as in the sluggish-lid solutions we are considering, then the traction and the pressure gradient at the base of the plate are negative and help drive the plate.

We will now make the following first order approximation for the pressure and basal traction terms. If $U_M \gg U_P$ at a depth l beneath the plate, then the pressure and traction terms may be approximated as

$$P_{x_p} \approx -c_2 \frac{\mu_M U_M}{l^2}, \quad (1.16)$$

$$\tau_P \approx -c_3 \frac{\mu_M U_M}{l}, \quad (1.17)$$

with both c_2 and c_3 being $\mathcal{O}(1)$. Substituting Eqs 1.16, 1.17, and 1.11 into Eq. 1.15, and using the approximation that for a simple flow geometry $l \approx d/2$, leads to

$$0 = \sqrt{\frac{L}{\pi d}} + c'_1 \text{Ra}^{-1/2} \tilde{U}_P^{5/4} + c'_2 \text{Ra}^{-1/2} \tilde{U}_P^{3/4} - \text{Oc} \tilde{U}_P^{1/2}. \quad (1.18)$$

Here we have introduced a new non-dimensional number, the O'Connell number (Oc),

where

$$\text{Oc} \equiv \frac{\tau_R}{\alpha \rho g d \Delta T}. \quad (1.19)$$

Oc represents the ratio between the net plate dissipation to the rate of change of potential energy of the lithosphere. Alternatively, it can be thought of as the net plate stress normalized by the induced stress due to the buoyancy of the mantle. The constants c'_1 and c'_2 are defined as

$$c'_1 = \frac{c_3}{\pi^{1/4} c_0 \sqrt{c_1}} \left(\frac{L}{d} \right)^{3/4} \quad (1.20)$$

$$c'_2 = \frac{2c_2}{\pi^{1/4} \sqrt{c_1}} \left(\frac{L}{d} \right)^{5/4}. \quad (1.21)$$

If c_1 , c_2 , and c_3 are all $\mathcal{O}(1)$, then for $L/d \approx \mathcal{O}(1)$, c'_1 and c'_2 are also $\mathcal{O}(1)$. If we further assume that the non-dimensional plate velocity is much greater than unity (i.e., $\tilde{U}_P \gg 1$), which dimensionally requires $U_P \gg 10^{-3}$ cm/year (a reasonable assumption for a non stagnant-lid solution), while noting that both the pressure term ($\propto U_P^{3/4}$) and traction term ($\propto U_P^{5/4}$) scale with $Ra^{-1/2}$, then we can simplify the expression further by ignoring the pressure term because it is smaller than the traction term ($U_P^{5/4} \gg U_P^{3/4}$). In this case the lithospheric energy balance simplifies to

$$0 \approx \sqrt{\frac{L}{\pi d}} + c'_1 Ra^{-1/2} \tilde{U}_P^{5/4} - \text{Oc} \tilde{U}_P^{1/2}. \quad (1.22)$$

This expression states a balance between the rate of change of potential energy and basal traction as the driving terms against the plate's net resistance. With this balance established, we are now able to derive the two scaling laws for the sluggish-lid solutions based on the dominant driving force.

Upper Branch: Traction-driven sluggish-lid. If the traction term dominates over the rate of change of potential energy as the driving mechanism, then Eq. 1.22 simplifies

to

$$0 \approx c'_1 \text{Ra}^{-1/2} \tilde{U}_P^{5/4} - \text{Oc} \tilde{U}_P^{1/2} \quad (1.23)$$

which results in the following scaling for plate velocity and Nusselt number

$$\tilde{U}_P \approx c_1'^{-4/3} \text{Ra}^{2/3} \text{Oc}^{4/3}, \quad (1.24)$$

$$\text{Nu} = 2 \left(\frac{d}{\pi L} \right)^{1/2} c_1'^{-2/3} \text{Ra}^{1/3} \text{Oc}^{2/3}, \quad (1.25)$$

respectively. In this convective regime $\tilde{U}_P$ and Nu cannot be expressed in terms of Ra alone but requires our newly introduced dimensionless number Oc to account for the effect of plate dissipation in the local lithospheric energy balance. The plate dissipation can be a dominant term in the local lithospheric energy balance while at the same time a small one in the global energy balance. The plate velocity still scales with $\text{Ra}^{2/3}$, but the dependence on $\text{Oc}^{4/3}$ with $\text{Oc} \ll 1/4$ (from Eq. 1.8 for any $0 \leq \tau_{\text{SP}} < \tau_Y$) means that this velocity is much smaller than the active-lid one. In Fig. 1.2a this Ra number scaling is evident with the middle branch (or upper sluggish-lid branch, green circles), which is parallel to the uppermost active lid branch (Oc is fixed in this plot).

Lower branch: Potential Energy (P.E.)-driven sluggish-lid. If instead the rate of change of potential energy dominates over the basal traction term as the driving mechanism, the lithospheric energy balance then becomes

$$0 \approx \sqrt{\frac{L}{\pi d}} - \text{Oc} \tilde{U}_P^{1/2} \quad (1.26)$$

which leads to the following plate velocity and Nusselt number scaling laws

$$\tilde{U}_P \approx \frac{L}{\pi d} \text{Oc}^{-2}, \quad (1.27)$$

$$\text{Nu} = 2 \left(\frac{1}{\pi} \right)^{1/4} \text{Oc}^{-1}, \quad (1.28)$$

respectively. In this case, neither quantities depend on Ra , but instead only on Oc .

Bifurcations: predicting when single or multiple solutions exist

In Fig. 1.2a, there is a critical bifurcation Rayleigh number, Ra_{B1} , above which multiple convective regimes can exist. This number is of importance as it may help determine whether a planet's thermo-mechanical state allows for sluggish-lid solutions or whether only the classical active-lid solution is permissible. In this section we derive expressions for the two bifurcation points, Ra_{B1} and Ra_{B2} (see Fig. 1.3b), in turn, based on the previous analysis. Far away from and above Ra_{B1} we expect that the stated force balances for the sluggish-lid solutions to hold, whereas close to it, this is not necessarily the case. Across this marginal region, since both solutions merge, we expect instead that both basal traction and the rate of change of potential energy contribute equally to drive plate motion. Therefore, we can find the bifurcation Rayleigh number by setting the rate of change of potential energy and traction terms to be equal, i.e., setting the first two terms on the right-hand side of Eq. 1.22 equal such that

$$c'_1 Ra_{B1}^{-1/2} \tilde{U}_P^{5/4} \approx \sqrt{\frac{L}{\pi d}}. \quad (1.29)$$

By solving for $\tilde{U}_P$ and substituting this back into the simplified lithospheric balance (Eq. 1.22), we arrive at

$$Ra_{B1} \approx c_1'^2 \left(\frac{L}{\pi d} \right)^{3/2} \left(\frac{2}{Oc} \right)^5. \quad (1.30)$$

By inverting this expression, we can additionally calculate the bifurcation O'Connell number, Oc_{B1} ,

$$Oc_{B1} = 2 \left(\frac{L}{\pi d} \right)^{3/2} c_1'^{2/5} Ra^{-1/5}. \quad (1.31)$$

These expressions imply that sluggish-lid solutions are only permissible in a certain subset of Ra – Oc values.

The second bifurcation point, which appears in Fig. 1.3b, bounds the other side of the multi-solution space across the Oc scale. For large enough values of Oc (but still $< 1/c_0\sqrt{\pi}$), only the P. E. branch of the sluggish-lid solutions satisfies the energy equations. In order to derive this second bifurcation point (Oc_{B2}), we return to the global energy equation (Eq. 1.1) and acknowledge that for these intermediate values of Oc , we cannot use the weak plate approximation (Eq. 1.7). The global energy balance for this scenario is given by

$$\sqrt{\frac{L}{\pi d}} \tilde{U}_P^{1/2} \approx c_0 \sqrt{\frac{L}{d}} \tau_Y^* \tilde{U}_P^{1/2} + 4c_1 Ra^{-1} \tilde{U}_M^2 \quad (1.32)$$

where we have once again used Eqs. 1.5, 1.4, and 1.10. This results in a modified relationship between the plate and mantle velocities

$$\tilde{U}_M^2 \approx \frac{1}{4c_1} \sqrt{\frac{L}{\pi d}} Ra \tilde{U}_P^{1/2} (1 - c_0 \sqrt{\pi} \tau_Y^*) \quad (1.33)$$

For these values of Oc , we can say that $\tau_Y^* \gg \tau_{SP}^*$, and therefore $\tau_Y^* \approx Oc$. As a result, the expression for the active-lid plate velocity becomes

$$\tilde{U}_P \approx \left(\frac{1}{4c_1} \right)^{2/3} \left(\frac{L}{\pi d} \right)^{1/3} Ra^{2/3} (1 - Oc c_0 \sqrt{\pi})^{2/3} \quad (1.34)$$

and the traction-driven branch of the sluggish-lid

$$\tilde{U}_P \approx c_1'^{-4/3} Ra^{2/3} Oc^{4/3} (1 - Oc c_0 \sqrt{\pi})^{-2/3}. \quad (1.35)$$

These equations (shown in dashed red and green lines in Fig 1.3b, respectively) are similar to what has been already derived in the scaling laws for the top two branches (Eqs. 1.13, 1.24), but are multiplied by a factor of $(1 - Oc c_0 \sqrt{\pi})^{\pm 2/3}$. As can be readily seen, this is

only significantly different from 1 when $\text{Oc} \rightarrow 1/(c_0\sqrt{\pi})$, i.e when the plate is sufficiently strong.

The second bifurcation can be approximated as the Oc number (Oc_{B2}) at which the two velocities from Eqs. 1.35 and 1.34 are equal. That value is given by

$$\text{Oc}_{B2} = \frac{c'_1}{2\sqrt{c_1}} \left(\frac{L}{\pi d} \right)^{1/4} \left[1 + \frac{c'_1 c_0}{2} \sqrt{\frac{\pi}{c_1}} \left(\frac{L}{\pi d} \right)^{1/4} \right]^{-1} \quad (1.36)$$

which is shown in Fig. 1.3b (vertical dash-dot line). The second bifurcation is constant and not a function of Ra , which means that the non-dimensional strength of the plate is the sole control on whether convection in the mantle below can cause the plate to participate in convection.

Scaling Law Summary

Altogether, the scaling laws stated by Eqs. 1.13, 1.24, and 1.27 show that at least three regimes are viable for mantle convection. The first, the active-lid solution, depends only on the global energy balance equation and agrees with the classic scaling found in many other studies (Turcotte and Oxburgh, 1967). For this solution, the plate and mantle characteristic velocities scale with the Rayleigh number, a measure of the vigor of convection against thermo-viscous dissipation. The second solution, a sluggish-lid one, depends both on the Rayleigh number and the O'Connell number, defined in Eq. 1.19, which measures the net resistance of the plate against thermal convection. The plate in this solution is driven by traction at its base due to the underlying convecting mantle and resisted by the strength of the plate against bending and faulting. The third solution, also a sluggish-lid one, depends inversely on the O'Connell number, and represents a solution in which the plate is driven by its local rate of change of potential energy (or ridge push/gravitational sliding) and resisted by its strength.

The results of the scaling analysis are shown in Fig. 1.3 compared to the results of the

analytical model of COC12. In the left panel we fix the Oc number and in the right panel we fix the Ra number. As can be seen, the scaling laws capture the full solutions to the global and lithospheric energy balances well and provide a useful framework in which to understand the multi-state mantle convective system. We also identify the critical Ra–Oc relationships at which we expect bifurcations from single to multi solution states to occur. In Section 1.3 we further elaborate on the physical implications of these results.

1.2.3 Non steady-state equations and regime stability

As presented, the results of the analytical model and the scaling laws derived provide no way of inferring which regime the mantle is more likely to exhibit when multiple regimes (solutions) exist. The missing piece is that the analysis so far assumes steady state and, as such, the absence of time derivatives means no information can be gained on the stability of the solutions or how the mantle evolves through the solution space.

In this section we change our approach from considering a steady-state system to that of a non steady-state one. While the actual change to specific terms is subtle, it opens several avenues for further analysis. The stability of steady-state (or fixed point) solutions, x_{fp} , to dynamical systems of the form $\dot{x} = f(x)$ (where any over-dotted variable denotes its time derivative) can be determined by considering the sign of df/dx (e.g., Strogatz, 2015). We will explore two aspects of stability in this context. First, we explore the sign of $df/dx|_{x=x_{\text{fp}}}$ which dictates whether small perturbations from x_{fp} grow or decay, where positive and negative signs represent local instability and stability, respectively. Second, we compare the relative magnitudes of $df/dx|_{x=x_{\text{fp}}}$ between different fixed points, which indicates the the rate of decay or growth of a given perturbation and thus provides a sense of the degree of (local) stability or instability of a fixed point.

In typical treatments of mantle convection, including that of COC12’s, the dynamical nature of the system is removed from the momentum equation since the mantle has a very high Prandtl number (i.e., inertial forces are negligible), and from the energy equation

by assuming steady state. If one considers mantle evolution over long timescales, the latter assumption is invalid. Upon removing the steady-state assumption in the energy equation, we seek to address the thermodynamic stability of each solution at any given Ra-Oc state. (Note that the kinematic stability of the solutions can only be assessed by use of full numerical implementations of the governing equations and is not the focus of this study.) In COC12, the rate of change of potential energy, Φ_P , is expressed as

$$\Phi_P = \int_V \Delta \rho g v dV, \quad (1.37)$$

where $\Delta \rho$ is the density variation and v the vertical velocity component of mantle flow. If one assumes $\Delta \rho$ is driven by only thermal variations (represented by δT), then

$$\Phi_P = \frac{\alpha g d}{C_p} \left[\frac{1}{d} \int_0^d \left(\int \int \rho C_p \delta T v dx dz \right) dy \right]. \quad (1.38)$$

Here, x is, as before, the dimension in line with plate flow, y is the vertical dimension of the mantle domain, and z is the third dimension along the ridge axis. The quantity within the square brackets is the depth averaged advected heat flow $\langle Q \rangle_{\text{adv}}$ (see Equations 15–18 in COC12). In steady-state, this is simply

$$\langle Q \rangle_{\text{adv}}^{\text{ss}} = \left(q_b + \frac{1}{2} H d \right) L = \left(q_s - \frac{1}{2} H d \right) L, \quad (1.39)$$

where q_b is basal heat flux at the CMB and H is the heat production due to radiogenic material per unit volume (see Equation 19 in COC12). This description lends itself to a straightforward removal of the steady-state assumption within the mechanical energy equation via inclusion of the rate of change of mantle cooling in the advected heat flow. For the non steady-state system, the energy that is used to cool or heat the system is a

source/sink term in the expression for the advected heat flow such that

$$\begin{aligned}\langle Q \rangle_{\text{adv}}^{\text{nss}} &= \left(q_b + \frac{1}{2} H d - \frac{1}{2} \rho C_p \dot{T} d \right) L, \\ &= \langle Q \rangle_{\text{adv}}^{\text{ss}} - \frac{1}{2} \rho C_p d L \dot{T}.\end{aligned}\tag{1.40}$$

Superscripts “nss” and “ss” refer to non steady-state and steady-state contributions, respectively, where $\langle Q \rangle_{\text{adv}}^{\text{ss}}$ is given by Eq. 1.39. Rewriting the global energy balance (i.e., Eq. 1.1) using Eq. 1.40 yields

$$\dot{T} = \left(\frac{2}{\alpha g \rho L d^2} \right) \left[\frac{\alpha g d}{C_p} \langle Q \rangle_{\text{adv}}^{\text{ss}} - (\tau_Y d_L U_P + \Phi_M) \right].\tag{1.41}$$

In order to determine the stability of the steady-state solutions, the rate of change of Eq. 1.41 with temperature is required, which may be expressed as

$$\frac{d\dot{T}}{dT} = \left(\frac{2}{\alpha g \rho L d^2} \right) \frac{d}{dT} \left[\frac{\alpha g d}{C_p} \langle Q \rangle_{\text{adv}}^{\text{ss}} - (\tau_Y d_L U_P + \Phi_M) \right].\tag{1.42}$$

The expression within the brackets is the global energy balance assuming steady-state (Eq. 1.1), rearranged so all terms are on the same side. As such, numerical solutions to this equation are found by locating U_P values at which zero-crossings occur. We label this version of Eq. 1.1 “ $\mathcal{G}(U_P, T)$ ”. Thus, its temperature derivative is

$$\frac{d\mathcal{G}}{dT} = \left(\frac{d\mathcal{G}}{dU_P} \right) \left(\frac{dU_P}{dT} \right),\tag{1.43}$$

where the first derivative on the right-hand side is recovered by determining the slope of $\mathcal{G}$ with U_P used to find the zero-crossings, as described. The second right-hand side derivative can be found from the scaling laws derived in Section 1.2.2. For the derivation of the temperature derivatives for each solution see Appendix A, where we find that for all solutions $dU_P/dT > 0$. (In our examples of an isoviscous mantle, derivatives recovered from the scaling laws are of sufficient accuracy. However, we expect that numerical

derivatives must be taken if the mantle is characterized by varying viscosity.) Therefore, the stability of the solutions, as governed by the sign of $d\dot{T}/dT$, is dictated by the first of the derivatives on the right-hand side of Eq. 1.43.

The results of this analysis are shown in Fig. 1.3 where open and closed circles represent unstable and stable solutions, respectively. As shown, the active-lid and the P. E.-driven sluggish-lid branches are stable, whereas the traction-driven sluggish-lid branch is unstable. The stability states of all branches do not change during their existence. Given this analysis, one could interpret that today's active-lid mantle convection regime, once entered, did not depart from this mode. We warn against such interpretation, however, because using linear stability analysis does not provide information for non-small perturbations from a fixed point (Menck et al., 2013). Therefore it is still possible that episodic types of transitions could occur (e.g., Silver and Behn, 2008; Lenardic, 2018).

The magnitudes of the slopes in the linear stability analysis, which are equivalent to the eigenvalues of the linearized system (e.g. Strogatz, 2015), provide additional insights into the behaviour of the system locally. The dimension of $d\dot{T}/dT$ is Time^{-1} , and its inverse can be interpreted as the local timescale of decay towards (for stable) or away from (for unstable) fixed points. A large magnitude of $(d\dot{T}/dT)^{-1}$ may result in a slow approach to a stable fixed point or a slow transition away from an unstable point. While we do recover numerical values of $d\dot{T}/dT$, we do not wish to over interpret their values in this parameterized model and linearized analysis. The timescales recovered lie between $\mathcal{O}(10 \text{ My})$ to $\mathcal{O}(10^4 \text{ My})$, which is entirely reasonable in a system with such high viscosity. While the interpretation of these timescales has the aforementioned limitations, their relative magnitudes still provide insight. This is best conceptualized by considering each fixed point as a potential well, if stable, or hill, if unstable, and the magnitude of $(d\dot{T}/dT)^{-1}$ as the local slope in the vicinity of the fixed point. Fig. 1.4b schematically depicts this image for several Ra cases, while Fig. 1.4a displays the solutions as a function of their absolute decay timescale (the inverse of the $d\dot{T}/dT$ slope). Moving from high to

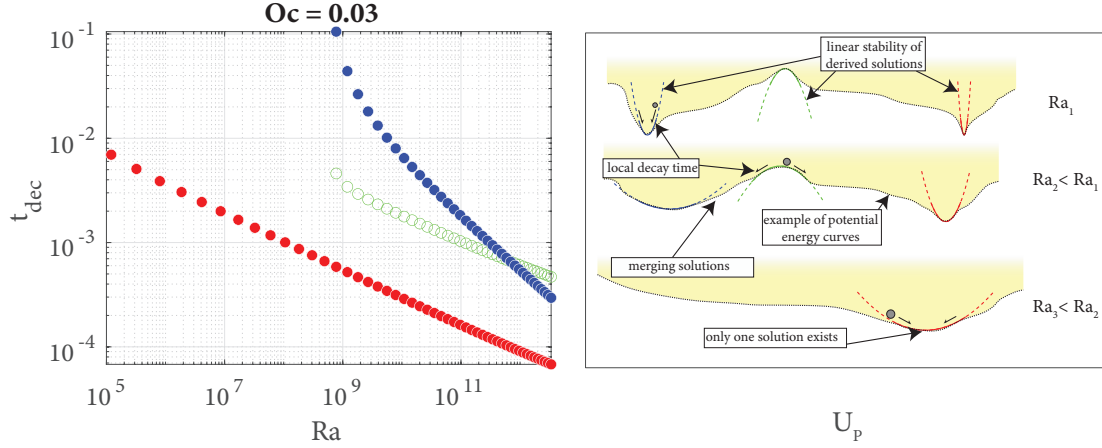


Figure 1.4: **Linear Stability Analysis:** (a) Decay times, t_{dec} , of solutions in Fig. 1.3a, where $t_{\text{dec}} = [d\dot{T}/dT]^{-1}$. As in Fig. 1.3a, closed and open circles represent stable and unstable solutions (or negative and positive t_{dec}), respectively, and colors of branches represent the active-lid (red), traction-driven (green) and potential energy-driven (blue) regimes. (b) Schematic diagram depicting solutions in (a) as potential hills or wells and their relative local slopes. Their colors are associated with colors in (a) and each row represents a different Ra number state.

low Ra (i.e., from top to bottom or right to left in Fig. 1.4, panels b and a, respectively), first we observe that all solutions are clustered around relatively small decay times, or steep local slopes. This is to be expected, since at higher Ra convection is more vigorous, which would be characterized by shorter timescales. As we move towards lower Ra (second row in Fig. 1.4b), each slope becomes gentler: the degree of local stability or instability lessens, and continues to do so, since their decay times continue to follow their upward trajectories (panel a). Finally, we reach the bifurcation point, more easily identified in Fig. 1.4a, where only the active-lid solution remains.

This linear stability analysis has provided some context for how the nature of each solution changes with Ra , but only for small perturbations from the fixed points. Further away, the trajectory between these fixed points, shown schematically in 1D as topography between the colored hills/wells (dotted black lines), is not known through this analysis. The full phase-space diagram that can more accurately depict the behaviour of the system can only be obtained by numerically solving the full Navier-Stokes equation.

1.3 Discussion

1.3.1 The Ra–Oc space of convection regimes

In the derived scaling laws for different convective regimes, we have been able to express the non-dimensional plate velocities, characteristic mantle velocities, and the Nusselt number in terms of two non-dimensional groups: the Rayleigh number, which is the familiar fluid dynamics number used to describe convection, and the O’Connell number, a new non-dimensional quantity that describes the strength of the plate against convection. We have also derived the conditions for the existence of multiple solutions. This enables construction of a regime phase diagram that synthesizes all the convective styles, shown in Fig. 1.5.

The likely parameter range for Earth’s mantle (shown as the yellow shaded region in Fig. 1.5) places it on the boundary between the multi-solution space and the active-lid-only space. Depending on the evolutionary path Earth took within the Ra–Oc space, it might have fallen in the multi-solution space where all convective styles are energetically permissible. For example, if the plate’s (dimensional) yield strength (τ_Y) linearly depends on temperature, then we expect Oc to remain constant (Eq. 1.19) throughout Earth’s evolution. In such a scenario, the Ra–Oc path taken is vertically downward. Therefore, it likely existed in a multi-state regime. If, on the other hand, τ_Y remains constant, then Oc decreases with time and it is possible that the Earth moved directly from the lower branch sluggish-lid solution into an active-lid one, without passing through the multi-solution space. The evolution of τ_Y with mantle temperature is thus a crucial element in the evolutionary path of Earth and deserves detailed investigation that are outside the scope of this study.

In the multi solution space, the likelihood of a specific convective regime to occur is a function of that solution’s stability and rate of decay away from or towards it. For example, even though the upper branch sluggish-lid solution is found to not be stable, its

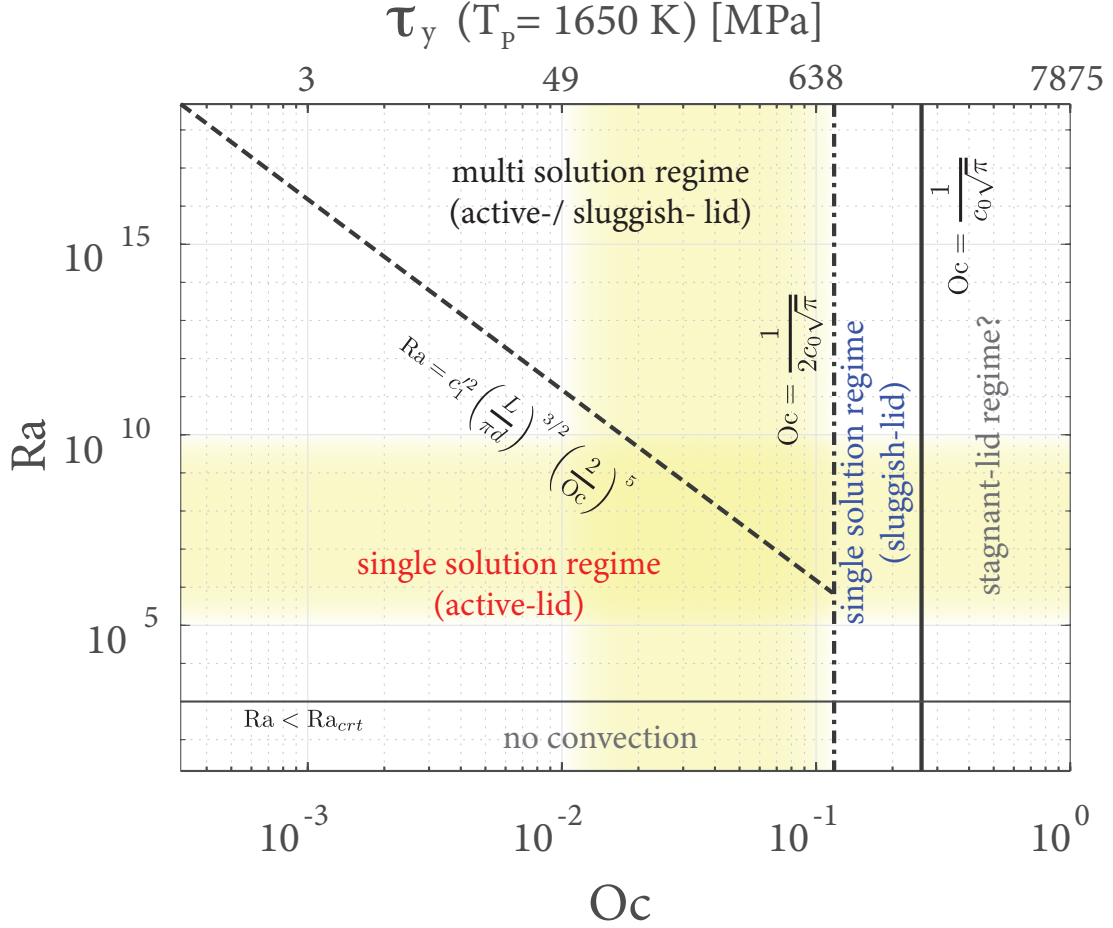


Figure 1.5: **Regime phase diagram for the possible convective style in the mantle in the Ra–Oc space:** The dashed line is the boundary between multiple states and active-lid only, given by Eq. 1.30, the dash-dotted line is the boundary between the multiple states and the lower branch sluggish-lid given by Eq. 1.36. The vertical solid line defines the weak-lid approximation, and we hypothesize that this represents the stagnant-lid boundary. Finally, the solid horizontal line is the critical $Ra = \mathcal{O}(10^3)$, below which the mantle does not convect. The region highlighted in yellow represents the range of Ra and Oc for Earth. Oc values correspond to plate strength in the order of tens to hundreds of MPa, as shown in the top axis

decay time close to the bifurcation point is $\mathcal{O}(1)$ Ga, allowing Earth to spend a significant time in that regime prior to moving towards one of the stable regimes. Additionally, the mantle need not exhibit a single regime at any one time and all three convective regimes could coexist, which is commonly observed in numerical (Jaupart and Parsons, 1985) and laboratory (Karani and Huber, 2017) convection experiments, and has been used in mantle thermal evolution models (Höink et al., 2013; Jellinek and Jackson, 2015). It is also worth noting that the numerical models of (Höink and Lenardic, 2010; Höink et al., 2011) found that the imposed aspect ratio of the convection in their simulations affects the type of flow that develops in the mantle, with smaller convection cells favoring sluggish-lid convection whereas longer convection cells favor the active-lid mode. While we have not derived an analytical expression for the stability of the different regimes to enable us to examine the effect of plate size, we can see that the different modes depend on the aspect ratio of the convective cell to different powers, hence their response to the aspect ratio is different.

1.3.2 Implications for Earth’s Thermal History

Scaling relationships of the type we derived in this study are most commonly used to estimate the thermal history and evolution of planetary bodies. As mentioned in section 1.1, using the active-lid scaling law for the evolution of the mantle results in unrealistic predictions of the mantle’s thermal state and has spurred many discussions and studies to improve the model. As a consequence, plate velocities, in this active-lid mode, become unrealistically high. Sluggish-lid solutions predict much lower plate velocities for a hotter mantle than the active-lid solution, and therefore are associated with lower convective heat loss at the surface. For example, for a mantle $\sim 200^\circ\text{C}$ hotter on average than today’s, and a plate with ~ 200 MPa effective yield strength, the active-lid scaling given by Eq. 1.13 predicts a plate velocity ~ 83 m/year (extremely high value), whereas the lower branch sluggish-lid (Eq. 1.24) predicts ~ 0.4 cm/year, much closer to the values observed today.

It may be tempting to use these laws to integrate the present-day mantle temperature backwards in time to assess the thermal history implied by these laws, but we refrain from doing that here for several reasons. First, it was shown by Lenardic and Crowley (2012) that the system experiences hysteresis, i.e. the history of the planet’s state matters in determining the convective mode. They found that numerical simulations agreed with the analytical model of COC12 in that where the COC12 model predicts multiple solutions to exist, numerical simulations give rise to different convective modes based on the direction of time integration. Which exact mode cannot be determined from the parameters at a single time alone. While the stability of the solutions can help constrain the evolutionary trajectory, Fig. 1.4 shows that the decay times towards and away from the steady-state solutions can be very long, and the planet may spend a significant fraction of time away from these states. Second, the rheology of the plate and the asthenosphere are important properties in evolution models; whereas we vary Ra and Oc independently in this study, they are both functions of the mantle and lithospheric temperature, composition, and rheology. Without better understanding how these parameters simultaneously evolve, and the details of their effect on the results, we refrain from using the scaling laws alone for evolution models.

1.3.3 Uses and Limitations of the Scaling Laws

We noted in our recapitulation of COC12 that one of the strengths of their model is its ability to accommodate for multiple viscosity layers. Yet here we have assumed and derived scaling laws for an isoviscous mantle. We made this assumption in order to simplify the model enough to be able to derive scaling laws and yet provide an insightful conceptual framework within which to work. Here, we briefly discuss the effect of an asthenosphere on the results. The terms that would be modified by the addition of a low viscosity asthenosphere are the pressure gradient (Eq. 1.16) and the traction (Eq. 1.17) at the base of the plate. The constants associated with these terms may be different

from unity, and the approximation for the small effect of the pressure gradient on driving the flow may not be true. This will affect the upper branch of the sluggish-lid scaling (Eq. 1.24) as well as the first bifurcation point (Eq. 1.30). The lower branch, which is driven by the plate itself and is not affected by the underlying mantle is not expected to change. Finally, the constant in the viscous dissipation term (Eq. 1.10) in the global energy balance will also change, but considering that this layer is small compared to the thickness of the lower mantle, it should not change the active-lid scaling significantly, but will affect the second bifurcation point.

Similarly, we focused on the whole-mantle convective regimes in our analysis because that is the mode in operation at present. To fully explore all possible regimes, one ought to consider the layered convective regimes. However, the geometry of the convection in these cases is not simple and many of approximations made to derive the scaling laws do not hold. Specifically, viscous shearing is likely to increase, and whether traction and the pressure gradient are negative or positive, which determines whether they drive or hinder the plate's motion, is not clear. Such complexity in the geometry and direction of the flow renders it difficult to make appropriate approximations to derive the scaling laws for these convective regimes. It is important to note that these limitations concern the derived scaling laws, they are irrelevant when solving the global and lithospheric energy balances numerically as in COC12. However, we emphasize the insight gained from developing these scaling laws.

1.4 Conclusion

The analytical boundary layer mantle convection model of Crowley and O'Connell (2012) yields, for a given thermo-mechanical condition, multiple modes of convection and their associated steady-state velocity structure for both the lithosphere and mantle system. We have focused on the three whole-mantle modes of convection: one active-lid and two sluggish-lid regimes and have extended the original analysis in several ways.

First, for each of the regimes we derived scaling laws that predict plate velocity as a function of the Rayleigh number and a new, non-dimensional number, the O’Connell number, that balances a plate’s resistive force over the underlying mantle buoyancy. The active-lid scaling recovers the classic result of Turcotte and Oxburgh (1967) where $Nu \sim Ra^{1/3}$, while the sluggish-lid regimes require both Ra and Oc to fully characterize their plate velocity scalings. We also identified two bifurcations and derive their Ra - and Oc -number bifurcation points, beyond which represents regions where either only active-lid solutions or the potential energy-driven sluggish-lid solutions exist. Second, we have removed the assumption of steady state which opens up several pathways to investigate the richness in behavior of these solutions. We performed a linear stability analysis to show that the active-lid and potential energy-driven sluggish-lid solutions are stable, while the traction-driven sluggish-lid solution is unstable and we examined their relative stabilities. Finally, through synthesis of the scaling laws and bifurcation points, we map the regime phase diagram of mantle convection, highlighting different Ra - Oc regions in which different convective may or may not exist. This kind of conceptualization is useful in the context of other planetary bodies.

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Appendix A

Appendix for Chapter 1: Expressions for temperature derivatives

Here we state the expressions for dU_P/dT from the scaling laws required to determine a given solution's stability (see Eq. 1.41). We first note that the mantle's viscosity varies with temperature as stated in Eq. 1.3. With the definitions of Ra and Oc (Eqs 1.12, 1.19, respectively), their temperature derivatives are

$$\frac{d \text{Ra}}{dT} = \frac{\text{Ra}}{T} \left(1 + \frac{E}{R_g T} \right) \quad (\text{A.1})$$

$$\frac{d \text{Oc}}{dT} = -\frac{\text{Oc}}{T}, \quad (\text{A.2})$$

With these relations, the temperature derivatives for U_P for each of the modes of convection are

$$\frac{dU_P}{dT} = \begin{cases} \frac{2}{3} \frac{U_P}{T} \left(1 + \frac{E}{R_g T} \right) & \text{for active-lid,} \\ \frac{2}{3} \frac{U_P}{T} \left(\frac{E}{R_g T} - 1 \right) & \text{for upper branch sluggish-lid,} \\ 2 \frac{U_P}{T} & \text{for lower branch sluggish-lid.} \end{cases} \quad (\text{A.3})$$

where the derivatives are taken for Eqs. 1.13, 1.24, and 1.27, respectively.

CHAPTER 2

Coupled Fates of Earth’s Mantle and Core: Early Sluggish-Lid Tectonics and a Long-lived Geodynamo

2.1 Introduction

The modern style of tectonics on Earth, plate tectonics, is characterized by large, rigid plates, separated by weak zones with high deformation rates: subduction zones, mid-ocean ridges, and transform faults. Plates progressively cool and become dense. Once unstable, they subduct into the mantle, creating space for new mantle material to erupt to the surface, actively participating in mantle convection which ultimately cools our planet. They move relative to one another at a rate of $\sim 1\text{--}10$ cm/yr. The Earth is unique in this surface expression of mantle convection in the solar system; we do not observe plate tectonics on other silicate planetary bodies (but these can instead exhibit other forms of tectonics, see, e.g. Stern et al. (2023)). Why Earth exhibits plate tectonics and when it initiated is the focus of many studies across Earth and planetary sciences. A recently invigorated idea is that Earth did not always operate in the plate tectonics regime,

but transitioned into it from a different tectonic mode (Höink et al., 2013; Jellinek and Jackson, 2015; Lenardic, 2018; Fuentes et al., 2019; Stern et al., 2023). However, there is disagreement on what exactly preceded plate tectonics, and when the transition (or transitions) occurred (Brown et al., 2020; Bédard, 2018; Cawood et al., 2018; Gerya, 2014; Stern, 2005).

The paleomagnetic record suggests the continual presence of relative and moderate plate motion as early as 3.25 gyr (Swanson-Hysell, 2021; Brenner et al., 2020, 2022) (Fig. 2.1a), indicative of some form of surface mobility. Whether this mobility is driven by the negative buoyancy of the plates themselves (i.e., slab pull, as is characteristic of modern plate tectonics and what we term “active-lid” tectonics), or by the motion of the mantle below (i.e., “sluggish-lid” tectonics) (Korenaga, 2006; Foley, 2018; Lenardic, 2018), is a fundamental question in our understanding of planetary evolution. The history of cooling, degassing, atmospheric and oceanic evolution, magnetic field generation, as well as crustal generation and orogenesis are all shaped by the forces that drive plate motion (Stern et al., 2023). As such, this calls for rigorous and systematic exploration into the cause of transitions between tectonics regimes (Cathles et al., 2023).

Mantle cooling drives core cooling, which, along with the solidification of the inner core, drives convection in the liquid core. This convection produces Earth’s magnetic field, through the geodynamo (Buffett et al., 1996). It is well established that a substantial global magnetic field has existed through most of Earth history (Tarduno et al., 2020; Bono et al., 2019) (Fig. 2.1b), and yet meeting this basic observation of our planet comes with difficulty. Today, both thermal and chemical convection act to power the geodynamo. The former convective source arises from the outward flow of heat to the mantle and the release of latent heat due to solidification of the inner core, while the latter convective source arises from the expulsion of light elements during the solidification of the inner core. Prior to inner core nucleation (ICN), the vigor of convection is lowered without sources of chemical buoyancy, substantially diminishing the power available to drive dynamo action.

This is exacerbated, in two ways by the high thermal conductivity of the liquid core alloy (k_c): (1) Such values increase the estimates of the adiabatic core-mantle boundary (CMB) heat flow, and the associated entropy sink, which must be overcome to sustain convection and therefore the dynamo action (Pozzo et al., 2012; Gomi et al., 2013); and (2) they yield young ages of the inner core spanning ≤ 500 My to ≤ 1.5 Gy (Gomi et al., 2013; Davies, 2015; Labrosse, 2015). A young inner core implies that over a substantial portion of Earth history, Earth’s long-lived dynamo must have been driven by thermal convection alone. While there is currently little agreement on the value of k_c , ranging from ~ 20 W/m/K (Hsieh et al., 2020; Konôpková et al., 2016) to ~ 160 W/m/K (De Koker et al., 2012), recent efforts to reconcile this range have converged to values between 50–100 W/m/K (Pozzo et al., 2022). The upper range of these values have been considered prohibitively high for geodynamo generation purely by thermal convection (e.g. Driscoll and Davies (2023)). In light of this, there have been proposals of more exotic mechanisms that may have powered a dynamo prior to ICN, which we address in the Discussion section, but they have been found insufficient to drive dynamo generation (Landeau et al., 2022). Therefore, the presence of the magnetic field requires that the core cooling rate is high enough to overcome conduction and sustain convection in the core, i.e. the net entropy in the core ($E_{\text{net}} \geq 0$).

Because thermal convection in the fully liquid core is mainly driven by cooling to the mantle above, the evolution of one cannot be considered without the other. The mantle must accommodate the heat input from the core via vigorous convection and redistribution of the heat, ultimately cooling to space. In contrast to active-lid tectonics, a stagnant-lid regime – whereby the mantle convects completely decoupled below an immobile lithosphere – limits mantle cooling. This leads to the equilibration of core and mantle temperatures causing convection in the core to cease (Breuer et al., 2010). Therefore, the paleomagnetic evidence for a substantial global magnetic field through most of Earth’s history (Tarduno et al., 2020; Bono et al., 2019) (Fig. 2.1b) is often linked

to existence of plate tectonics across the same period of time (Swanson-Hysell, 2021). This conflation, however, presents yet another problem: If Earth’s modern plate tectonic regime was dominant in the Archean (4.0–2.5 ga), the predicted mantle temperature is be much higher than indicated by the geologic record (shown in Fig. 2.1c. The plates are also predicted to move and subduct at unrealistically high rates compared to the geologic record, Fig. 2.1b. This results in a rapidly cooling mantle which would drive the geodynamo for only a brief period of time before core convection inevitably ceases.

In this study, we explore the coupled evolution of the mantle’s mobile-lid tectonic regime (we do not consider the stagnant-lid phase of Earth’s evolution if it existed) and key properties of the core (the development of an inner core and magnetic field), dictated by the flow of heat between the two domains. We ground-truth our analysis in the context of geologic observations as guiding constraints. We base our mantle evolution on a semi-analytical model that, without prescribing *a priori*, self-consistently predicts sluggish-lid and active-lid modes of convection (Crowley and O’Connell, 2012; Al Asad et al., 2023). We extend this theory by allowing transitions between the tectonic modes to naturally emerge and coupling it to a core evolution model (adapted from Labrosse (2015)). We explore a wide parameter space and show that despite the large uncertainty in the environment of the Archean Earth, most parameter combinations result in sluggish-lid tectonics during the Archean, with plate speeds similar to (or slightly elevated from) present-day but are instead partially decoupled from the mantle by the presence of a low viscosity layer. Furthermore, we show that this style of convection is capable of modulating the CMB heat flow in such a way that drives sufficiently vigorous thermal convection to power the core dynamo all the way up to ICN, even when using the upper limit of the core thermal conductivity values (≥ 100 W/m/K) derived from experimental and *ab initio* studies (Pozzo et al., 2022), which have been considered prohibitively high to sustain a thermal dynamo (Driscoll and Davies, 2023).

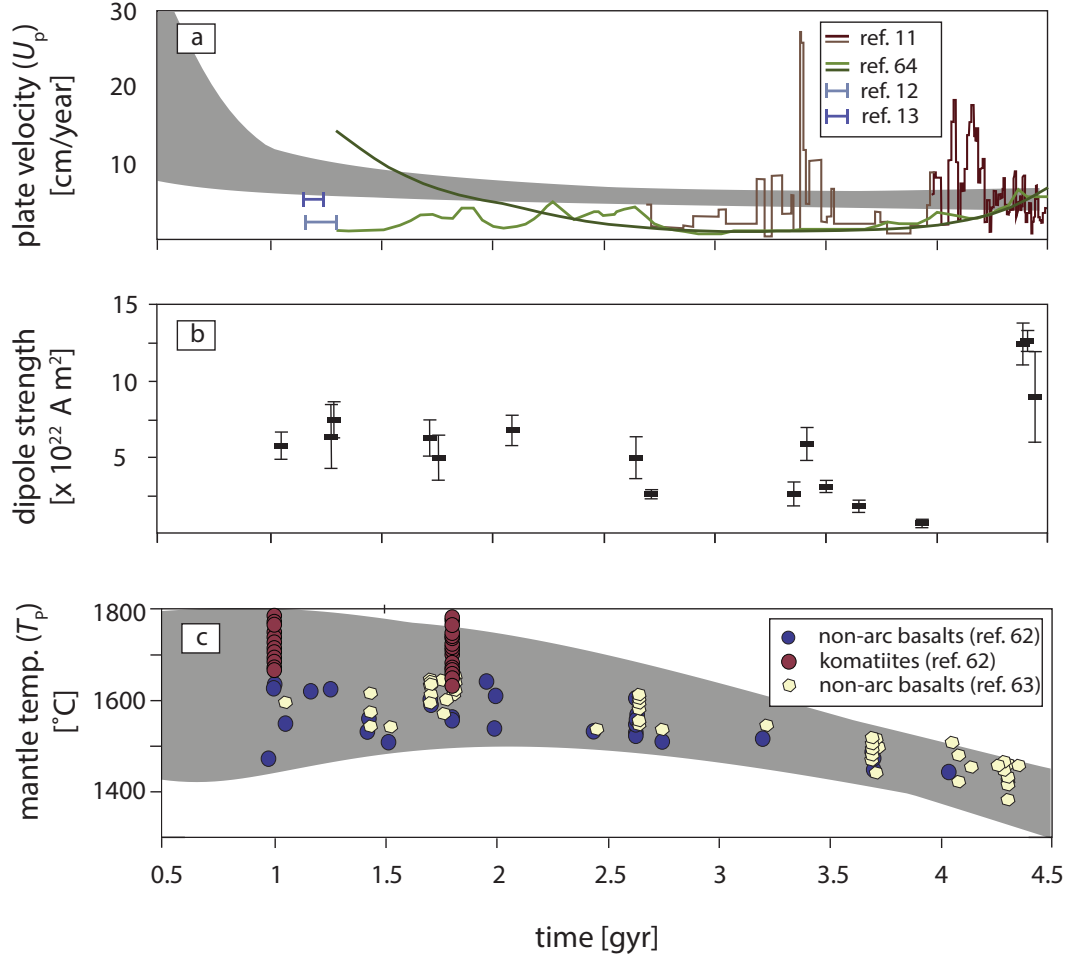


Figure 2.1: **Compilation of geologic Earth evolution observations:** (a) Reconstructed plate velocities, U_p (pink lines: Swanson-Hysell (2021); blue lines: Brenner et al. (2020, 2022); green lines: O'Neill et al. (2022)). (b) Compilation of magnetic dipole strength Bono et al. (2019). (c) Mantle potential temperature, T_p , inferred from non-arc basalts and komatiites Herzberg et al. (2010); Ganne and Feng (2017). In panels (a) and (c), the shaded regions represents the envelope of successful histories with $k_c = 100 \text{ W/m/K}$, the same as fully successful simulations in Fig. 2.2.

2.2 Model Summary

We solve our coupled system many times to extensively explore the uncertain parameters in Earth’s initial conditions and structure (resulting in a suite of $> 10^6$ simulations, Table 2.4), most important of these are the mantle’s viscosity structure, and core thermal conductivity. We consider 4 layers of the mantle: the lithosphere, asthenosphere, bulk mantle, and lowermost mantle. We hold the lithospheric viscosity constant throughout the model run at 3×10^{23} Pa s. Although this assumption is an oversimplification of the lithospheric evolution, it is sufficient in this present study, and we return to it in the Discussion section below. The viscosities of the rest of the mantle layers are temperature-dependent: the asthenospheric viscosity is a constant fraction of the bulk mantle viscosity (where the ratio of bulk mantle to asthenospheric viscosity, r_μ , is varied from 1 (with no asthenosphere) to 1000 (very weak asthenosphere)). The lowermost mantle viscosity is also temperature-dependent, calculated based on the lower thermal boundary layer temperature. We vary the reference viscosity of the lowermost mantle independently from that of the bulk mantle, and depending on our choice of reference lowermost mantle viscosity and the temperature evolution, the lowermost mantle may be more or less viscous than the bulk mantle. We also consider the case of an isoviscous mantle where the lowermost mantle viscosity is equal to the mantle viscosity. Although we do not consider the pressure-dependency of viscosity explicitly, or the existence of a compositional layer in the lower mantle, setting the lowermost mantle reference viscosity as a free parameter allows us to investigate the effect of some viscosity variations with depth on the evolution. For details, the reader is referred to the Method section. Each simulation spans 4.0 billion years, starting 500 myr after Earth formation, i.e. after core-mantle differentiation and mantle solidification (we do not consider the evolution of a basal magma ocean, which may be important in the geodynamo evolution, see Labrosse et al. (2007)).

We use the ratio, r_U , between the average mantle velocity below the lithosphere and lithosphere (or plate) velocity, to define the mantle convection mode. If $r_U \sim 1$, we classify

the mantle as operating in the active-lid regime, whereas $r_U \geq 10$ signifies the sluggish-lid regime. Since these modes are defined by a velocity ratio, sluggish-lid tectonics may still result in substantial absolute plate velocities. We consider a simulation as successful if it ends with realistic present-day, $t = t_f$, features of: (I) mantle potential temperature ($T_P(t = t_f) = 1375 \pm 75^\circ\text{C}$); (II) active-lid convection; and (III) the inner core radius ($r_{\text{ic}}(t = t_f) = 1220 \pm 240 \text{ km}$). Finally, (IV), that the geodynamo is in existence over the entire simulation (i.e., $E_{\text{net}} > 0$ for all simulation time).

2.3 Results

We begin by examining the behavior of three different classes of evolution scenarios: active-lid only (simulations that operate in the active-lid mode throughout the model duration, shown in red envelopes in Fig. 2.2), sluggish-lid only (blue envelopes in Fig. 2.2), and transitioning regimes, which start in the sluggish-lid then gradually move towards the active-lid mode (a subset of which is shown in green envelopes in Fig. 2.2). These do not necessarily reach $r_U \sim 1$, but have $r_U < 10$. The solid colored curves in each envelope represent individual demonstrative examples of each behavior. In the active-lid evolution scenarios, the mantle cools quickly (as shown in red envelopes Fig. 2.2a) due to high initial plate velocities (Fig. 2.2b). Depending on the initial temperature of the core and the viscosity of the lowermost mantle, the inner core may either grow to be too large (in the case of independently varying lowermost mantle viscosity), or not grow at all (in the isoviscous mantle case)(Fig. 2.2c). In turn, this causes the CMB heat flow (Fig. 2.2e) and the net entropy production (Fig. 2.2d) to often fall below the required value to maintain dynamo action. This behavior is characteristic of models in which the initial mantle temperature and viscosity ratio are low. The models that always operate in the sluggish-lid mode (shown in blue envelopes), on the other hand, heat up initially, then modestly cool, resulting in present-day mantle temperatures that are too hot (Fig. 2.2a), due to low plate velocities (Fig. 2.2b). Not many (0.5%) simulations fall in this category

(most simulations that begin in sluggish-lid tectonics cool to an extent that a transition to active-lid tectonics occurs), as it requires very high r_μ , high initial mantle and core temperatures, and high mantle thermal conductivity. The fact that core temperature and lower mantle structure play a role in maintaining the sluggish-lid regime indicates that high heat input from the core (Fig. 2.2e) keeps the mantle hot enough to remain in the sluggish-lid mode. Although we do not explicitly model the stagnant-lid mode, the evolution of the temperature in the stagnant-lid would likely resemble that of the extreme cases of the sluggish-lid modes. Only 8% of these scenarios are able to reproduce acceptable final inner core radius. However, those that do meet this criterion are also able to sustain the geodynamo. Finally, the models that start in the sluggish-lid mode then transition into the active-lid ones, the subset of which that meets criteria I-IV is shown in the green envelopes in Fig. 2.2, have moderate initial plate velocities and therefore cause only moderate temperature cooling. It is worth noting that of the cases that transition from the sluggish-lid towards the active-one and which satisfy the inner core radius criterion, 95% of them are able to sustain a long-lived geodynamo. They require the existence of an asthenosphere and are the only models that are able to produce reasonable present-day mantle temperature, inner-core radius, tectonic mode, while maintaining a dynamo throughout the model time. Examples for the velocity profiles at the beginning and end of the model duration for each of these scenarios is shown in Fig. 2.2f, g, and h for the active-lid, sluggish-lid, and transitioning case, respectively. For the interested reader, we show the evolution of the velocity ratio, the Urey ratio, the lowermost mantle viscosity and lower boundary layer thickness, as well as CMB temperature evolution in Appendix B.1.

As noted, the presence of an asthenosphere is the most important parameter that governs the tectonic regime during this period, where if present, an asthenosphere strongly correlates with sluggish-lid tectonics, which is in accord with numerical studies (Höink and Lenardic, 2010). Fig. 2.3a illustrates this behavior where we plot the median r_ν

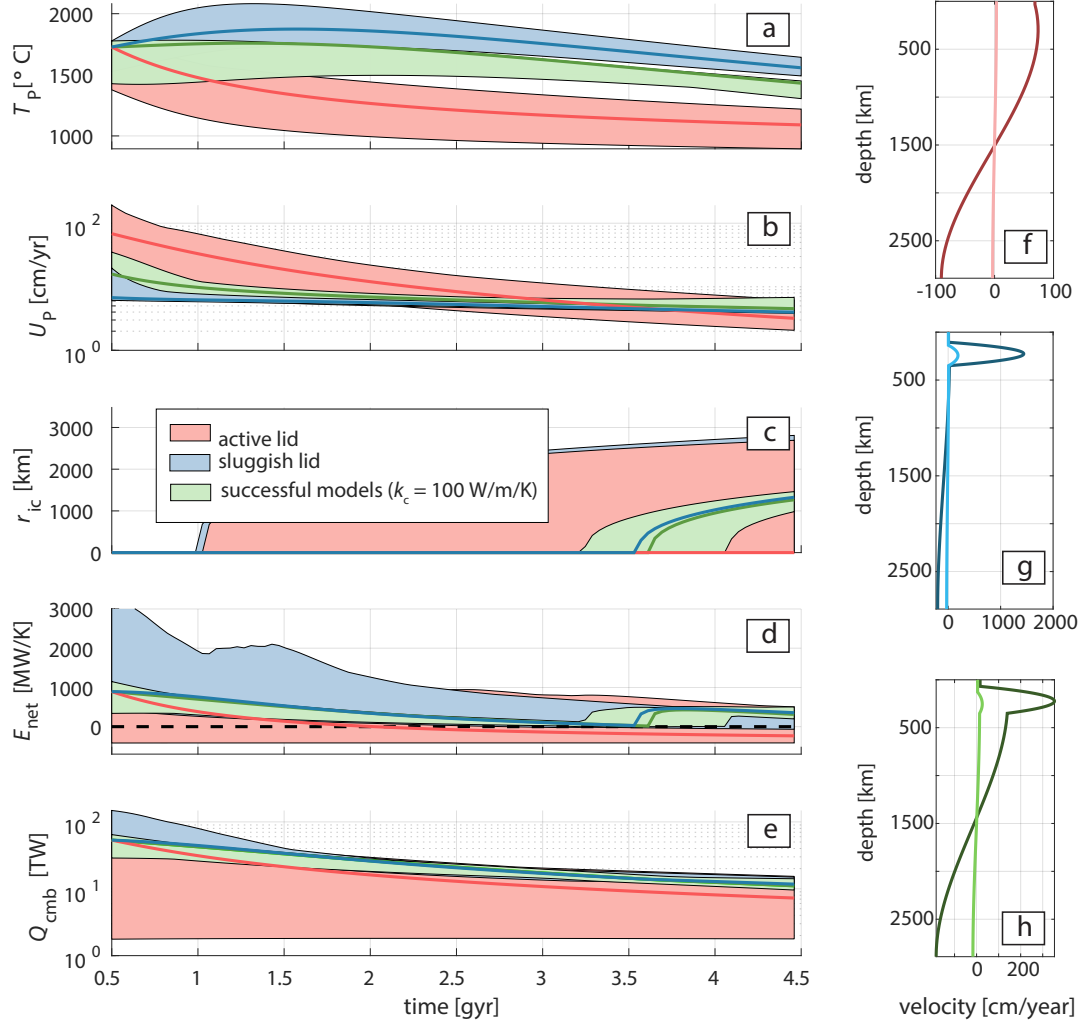


Figure 2.2: **Evolution of key quantities in simulations (where $k_c = 100$ W/m/K):** Panels (a)-(e) show the mantle potential temperature (T_p), plate velocity (U_p), inner core radius (r_{ic}), net entropy production (E_Φ), and CMB heat flow (Q_{cmb}), respectively. In all panels, red shaded envelopes span simulations that exhibit active-lid tectonics ($r_u \sim 1$ at all times), blue shaded envelopes span simulations that are always in the sluggish-lid mode ($r_u > 10$ at all times), and the green shaded envelopes span those that satisfy all criteria (I)-(IV), which transition from the sluggish- to the active-lid mode. The solid curves of each color show individual examples of the three behaviors shown with shading. They have identical mantle and core structures and initial conditions, except with different mantle-to-asthenosphere viscosity ratio (r_μ), where in the red curve $r_\mu = 1$, the blue curve $r_\mu = 1000$, and the green curve $r_\mu = 100$. Panels (f)-(h) show the horizontal velocity profiles (from the surface to the CMB), that correspond to the examples highlighted in (a)-(e) in the specified color. The dark colors in each of these panels is the velocity profile at the beginning of the simulation, while the light color is at the end. Note the different horizontal scale in each of the panels.

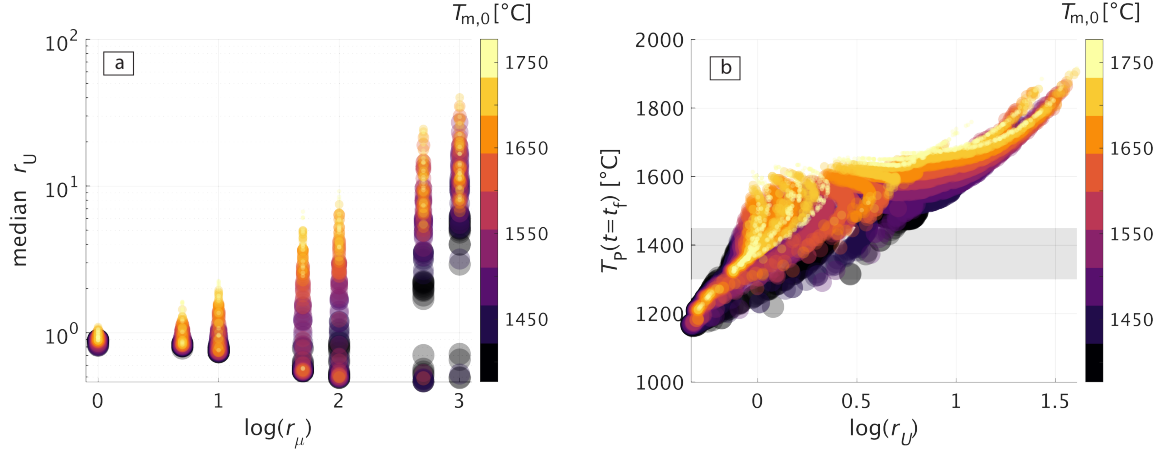


Figure 2.3: The degree of sluggishness and its effect on mantle observations: We use the median mantle-to-plate velocity ratio during each simulation, r_U , and asthenospheric-to-bulk mantle viscosity ratio, r_μ , as proxies for the degree of sluggishness. (a) Median r_U values (over each simulation) against r_μ for all simulations, where color-scale and inverse of size indicates initial mantle temperature, $T_P(t=0)$. (b) Final mantle temperature, $T_P(t=t_f)$, and its dependence on the median r_U in the simulation time. The gray shaded region in shows the acceptable range for the final mantle temperature (criterion I).

(r_U again representing the velocity ratio of the average mantle to the plate) over every simulation for all r_μ tested. When r_μ increases, the degree of sluggishness does too. The spread of results indicates that other factors, like mantle temperature (color-scale of symbols), also contribute to tectonic mode. Other parameters that we fix in our model (e.g., asthenospheric thickness, convective cell size, plate viscosity) can also influence convective mode, we test their effect in the Sensitivity Test section (Appendix B.2) in the Supplemental Material. Furthermore, in order to meet present-day mantle temperature constraints (criterion I), the Archean tectonic regime must have been sluggish-lid. This is shown in the plot of $T_P(t=t_f)$ against r_U , where the dominant trend is controlled by r_U (Fig. 2.3b).

Having established that r_μ dictates mantle convective mode and temperature evolution, we consider the effect of the mantle's tectonic mode on the core. Core temperature evolution (and thus r_{ic}) is governed mainly by the lowermost mantle viscosity, $\mu_{0,bl}$ (and thus the lower boundary layer thickness, Eq. 2.9), as depicted in Fig. 2.4a, which shows the

present-day inner core radius decreasing with increasing lower mantle reference viscosity. However, the degree of tectonic sluggishness (r_U) also plays an important role in dictating the core evolution, extending to dynamo generation. If we select evolution scenarios in which all parameters except the initial mantle temperature and asthenospheric viscosity ratio (r_μ) are fixed, it is clear that r_μ plays an important role in the entropy budget (Fig. 2.4b), and can change whether a certain evolution scenario can (i.e., scenarios with positive entropy, high viscosity ratio) or cannot (i.e., scenarios with negative entropy, low viscosity ratio) sustain the geodynamo prior to ICN. We focus here on the time just prior to ICN because during this period, there is the least entropy production due to the decrease in cooling entropy and the unavailability of inner-core solidification related sources (Eq. 2.17). In other words, this is the most difficult period of time to sustain the geodynamo.

Our results indicate that it is possible to sustain a thermally-driven geodynamo if the dominant tectonic regime for early Earth is sluggish-lid, even for high thermal conductivity values (> 100 W/m/K). Fig. 2.4c shows histograms of the successful simulations against core thermal conductivity. Evolution scenarios that are able to maintain a thermally driven dynamo for a given k_c^* are also able to maintain it for all $k_c \leq k_c^*$. This cumulative nature is illustrated by the continuous histogram (solid black line, Fig. 2.4c). The pink histogram represents the differences between the populations within the cumulative distribution. It is important to stress that increasing k_c will always make sustaining the geodynamo more difficult, but Fig. 2.4c demonstrates that a non-negligible portion of our simulations can do so even with $k_c \geq 100$ W/m/K ($\sim 9\%$ of the differential distribution).

Collectively, fully successful simulations are characterized by initial condition ranges of 1450–1750°C and 4400–5400°C mantle and CMB temperatures, respectively, with the latter depending on core thermal conductivity; asthenospheric-to-bulk mantle viscosity ratios of 50–1000; core thermal conductivity values < 120 W/m/K; and inner core age ranges from 0.5–1.5 Gy, depending on k_c . Meeting the criteria also places some constraints

on model parameters, shown in Appendix B.3.

While our success criteria focus on present day constraints, our successful simulations are also in accord with geological inferences of plate velocity and mantle potential temperature, discussed earlier, and shown by overlaying successful simulations (where $k_c = 100$ W/m/K) against such geological data (Fig. 2.1a and c). Thus, an early mantle characterized by sluggish-lid tectonics is not only required to meet key present-day characteristics, but is consistent with geological core and mantle constraints in Earth’s past.

2.4 Discussion

Our results indicate that maintaining a thermally-driven dynamo prior to inner core-nucleation with realistic mantle and core temperature evolution requires that Earth operates in the sluggish-lid mode early in its history, which in turn requires the existence of an asthenosphere. In our simulations, the possibility of a sluggish-lid tectonic regime arises because we assume that the dissipation in the plate is larger than the stresses caused by slab pull (Eq. 2.25, and Al Asad et al. (2023)). However, the relative strength of these terms may also evolve with time (Korenaga, 2006; Foley, 2018), and if slab pull proceeds to a point at which it exceeds plate dissipation stresses, there may be more abrupt transitions than our models predict. We illustrate a simple example of this in Appendix B.4. In addition, as we show in Appendix B.2, other factors, such as the size of the convection cell and the effective plate strength, also play an important role in determining the convective regime, which may alter the successful parameter space (e.g. the range of successful viscosity ratios), but not the conclusions of this study. Nevertheless, the observed and likely trend of these factors in the Archean still favor the idea of early sluggish-lid tectonics. For example, we (Appendix B.2), and others (Höink et al., 2011) find that smaller plates promote sluggish-lid tectonics. Geologic observations of Archean plate margin-length (Abbott and Menke, 1990), as well as geodynamic modelling (Mallard et al., 2016), indicate that plates were likely smaller during this time. Finally, as well as

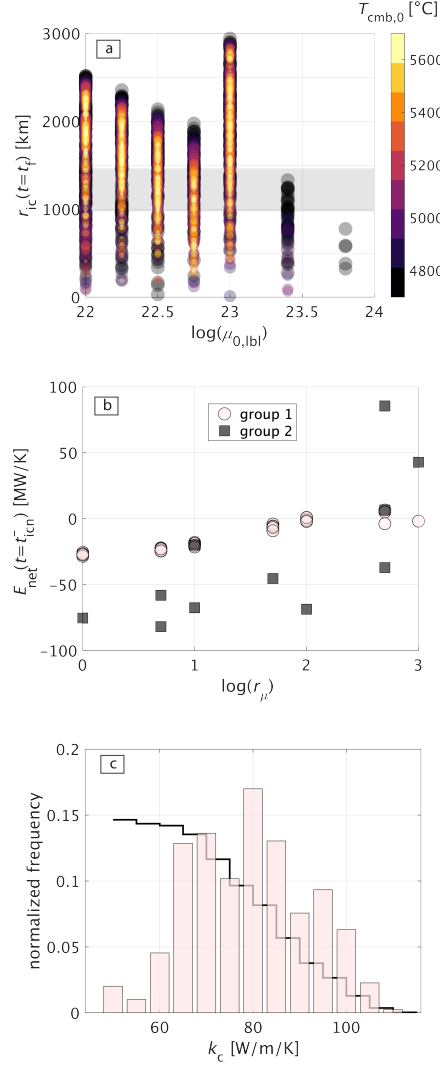


Figure 2.4: **Meeting core success criteria:**(a) Present-day inner core radius, $r_{ic}(t = t_f)$, as a function of lower boundary mantle viscosity. Color-scale shows initial core temperature ($T_{cmb,0}$). The departure from the trend for $\log(\mu_{0,lbl}) = 23$ is because this value refers to an isoviscous lower mantle (i.e. $\mu_{lbl}(T) = \mu_m(T)$) rather than using the lower mantle temperature to compute the lower mantle viscosity. (b) Net entropy production (E_Φ) just prior to ICN with r_μ for simulations that satisfy the inner core radius condition (criterion III) for two sets of parameters: group (1) with the parameters $\mu_{0,lm} = 10^{22.5}$ Pa s, $T_{cmb,0} = 5600^\circ\text{C}$, $k_c = 100$ W/m/K. group (2) with an isoviscous lower bulk mantle, $T_{cmb,0} = 5000^\circ\text{C}$, $k_c = 70$ W/m/K. Both groups have $C_{P,c} = 750$ J/kg/K, and MG core model Masters and Gubbins (2003) Table 2.4). (d) Histograms of fully successful simulations (meeting criteria I-IV) against core thermal conductivity, k_c . Solid black line histogram demonstrates cumulative effect of sustaining the dynamo, where for any given set of model parameters and a given k_c^* , all criteria are met for any $k_c \leq k_c^*$. The solid pink histogram removes this cumulative effect.

large plate dissipation, synonymous to early sluggish-lid tectonics is the existence of an early asthenosphere. While we do not explore specific mechanisms to produce this crucial weak layer, our work shows the importance of further exploring asthenospheric evolution (e.g., its expansion or reduction in thickness, Appendix B.4), its source (e.g., partial melt, water content), and its dynamics (Cathles et al., 2023). This is an important direction for future studies of tectonic evolution.

We note that our treatment of the mantle is quite simplified, as we do not include processes that likely matter in the tectonic evolution, especially for the early Earth. For example, we do not include the evolution of the lithospheric viscosity with temperature, which results smaller plate dissipation. However, the weakening of the lithosphere also result in lower slab pull stresses due to slab break off (Foley, 2018), possibly countering the effect of decreased dissipation. Partial melting and crustal production also likely play an important role in upper mantle dynamics. Crustal production can increase slab buoyancy, thickness (Korenaga, 2006), as well as thermal insulation (Lenardic et al., 2011) due to the partitioning of radiogenic elements in the melt (Mareschal et al., 2017). Partial melting can also alter the rheology of the residual mantle (Hirth and Kohlstedt, 1996). Nevertheless, our model self-consistently captures transitions between multiple modes of convection within the explored parameter space and is thus sufficiently complete to address our hypotheses.

2.4.1 Mechanisms for Generating an Early Dynamo

The lack of agreement between experimental and *ab initio* studies on k_c (e.g., Gomi et al. (2013); Basu et al. (2020)) has spurred new approaches to reconcile the range of estimates. For example, by measuring k_c directly (Hsieh et al., 2020; Konôpková et al., 2016), by measuring iron-light-element alloys rather than pure iron (Zhang et al., 2021), or by accounting for missing processes in numerical calculations (Pozzo et al., 2022). The convergence towards high k_c values (50–100 W/m/K) have motivated the search for

alternative mechanisms to power the geodynamo prior to ICN, such as silicate convection in a basal magma ocean (Davies et al., 2015), chemical interaction between the core and the mantle (Badro et al., 2016; O’Rourke et al., 2017; Mittal et al., 2020), tidally and/or precessionally driven convection (Malkus, 1968). While these processes may indeed have contributed to the core entropy budget, they seem unable to power the geodynamo alone (Landeau et al., 2022). Nevertheless, each mechanism carries a distinct prediction of the nature of the dynamo produced. In particular, some of these mechanisms may not predict a dipole-dominated geodynamo. In contrast, our proposal of a thermally-driven dynamo inside the liquid core is expected to produce a dipole-dominated geodynamo (and thus similar to the geometry of the present-day field). Such a prediction may be used to test the viability of our proposed mechanism as more paleomagnetic data is gathered and, within that field, our finds have important implications on the longevity of the so-called geocentric axial dipole (“GAD”) hypothesis.

We make several assumptions regarding the core structure in our model that are important to address. First, we assume the core to be isentropic (Eq. 2.3) and convection occurs in the entire liquid core. We do not include the evolution of a stably-stratified layer (Lister and Buffett, 1998; Buffett and Seagle, 2010) (whether of thermal or chemical origins). While there is strong evidence for its existence today (Buffett, 2014), a recent study that explored the evolution of both a thermal and chemical stratification layer found that long-lived magnetic field generation is mostly inconsistent with a long-lived stably stratified layer (Nakagawa et al., 2023). However, some solutions are permissible if this layer is considerably thin (<75 km), which would allow for the isentropic assumption made in this study. Another important assumption in our model is the that we do not consider ohmic dissipation in the core. These terms represent entropy sinks in the core and when considered, increase the difficulty of maintaining a thermal geodynamo. However, the value of this dissipation term is highly uncertain, even for today’s magnetic field, because of the difficulty of estimating the magnetic power spectrum in the core, and the

magnitude of the toroidal magnetic field (Roberts et al., 2003). By that token, we also do not consider other mechanisms that may help with geodynamo generation, such as chemical interactions between the core and the mantle (O’Rourke et al., 2017), which depends on the core cooling rate and may provide substantial entropy in the early Earth.

2.4.2 Other Indications of Tectonic Transitions

The onset of plate tectonics and transitions in tectonic regime cannot, of course, be considered without evaluation of geologic data. And while this is not the topic of this paper, we highlight a few observations that are consistent with the idea of tectonic transitions (for a review of this topic, see, e.g., Brown et al. (2020); Windley et al. (2021); Hawkesworth et al. (2020)). Much geological evidence suggests transitions in plate-mantle dynamics occurring towards the end of the Archean from a bifurcation in metamorphic style ~ 2 ga (Brown et al., 2020). Though the interpretation of this trend is contentious (Draper and Bone, 1981; Stern, 2005) and we do not claim this to be evidence for a sluggish- to active-lid transition, such a transition would no doubt lead to global changes in metamorphic style. While plate velocities do not substantially vary over time in our successful models, the underlying mantle velocities do, thus resulting in large changes in the stresses (both vertical and horizontal) imparted by the mantle on the plate. Similarly, the seeming lack of Archaen-aged blueschist rocks (indicative of modern-day subduction style) has been used to argue for a change in tectonic style (Cawood et al., 2018; Hawkesworth et al., 2020). Since the sluggish-lid mode is associated with higher mantle temperatures and lower lithospheric stresses (though does not preclude subduction), such geological data may be interpreted in the context of mantle transitions. Moreover, the nature of Earth’s metamorphic transition has been suggested to be gradual (Holder et al., 2019), which could be consistent with our smooth r_U evolution (e.g., green curves in Fig. 2.2). Put together, we propose that the tectonic regime of the Earth evolved smoothly from the sluggish-lid mode to the present-day active-lid one, as schematically depicted in Fig. 2.5.

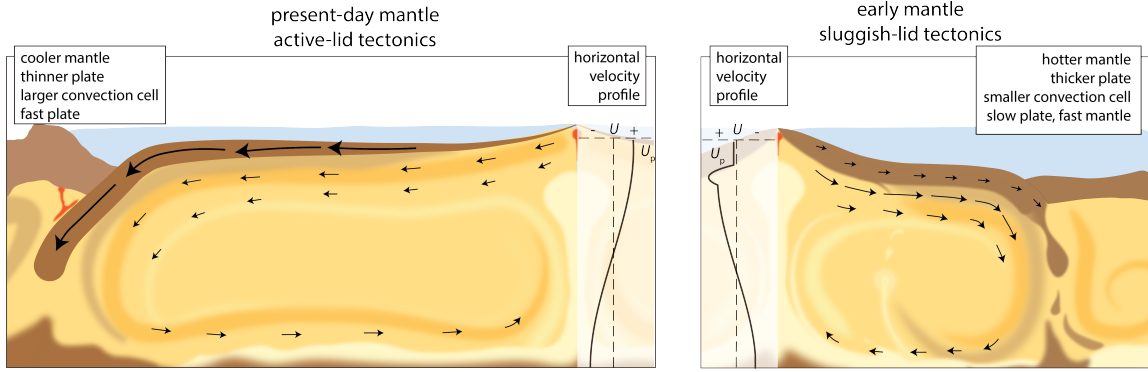


Figure 2.5: **Schematic diagram depicting mantle convective modes:** Features of each mode are listed in the insets, where, if sluggish-lid dominated early in Earth history, as our simulations suggest, the early Earth would have been characterized by thicker plates due to slow plate motion (exaggerated here) and fast mantle flow that drives plate motion (see schematic horizontal velocity profile). In contrast, the later, active-lid mode would feature thinner plates fully coupled to the mantle. For illustrative purposes, arrow size and density indicates relative magnitude of flow velocity.

Our simulations connect the deepest part of Earth to the surface expression of mantle convection and have revealed critical interdependencies between large-scale systematics of core, mantle and plate dynamics, able to address some basic Earth science questions.

2.5 Materials and Methods

Our goal is to evolve the energy balances of the core and the (plate-)mantle system over Earth history. We begin with stating the core energy and entropy balances and how they are calculated, followed by a similar section for the mantle system, considering both the mechanical and thermal energy balances. We then describe how each system is coupled and evolved and the details of our parameter sweep to produce our suite of simulations.

2.5.1 Core Equations

In the following sections, we describe our approach to calculate the core’s thermal (Part (i)) and entropy (Part (ii)) budgets. We treat the core as spherically symmetric, i.e.,

parameters and variables vary only with radius. When appropriate, bulk averages of the entire core are assumed. Our philosophy in the core analysis is to capture the important physics that govern whether a geodynamo can or cannot exist. Note that, where necessary, we distinguish core and mantle quantities with the subscripts “c” and “m”, but otherwise omit them for notational brevity.

(i) Core Energy Balance

The evolution of the core’s average temperature can be described using the simple energy balance that states that the secular cooling of the core is equal to the difference between the heat sources and sinks. While the energetics of the core is a topic covered in detail in many comprehensive papers (Buffett et al., 1996; Gubbins et al., 2003; Nimmo, 2015), we summarize the relevant equations and assumptions here. The dominant energy sources that we consider in the core are latent heat and compositional (gravitational) energy due to inner core freezing; while the sink is heat lost to the mantle through core-mantle boundary (CMB) heat flow. This balance is

$$Q_{s,c} = Q_g + Q_L - Q_{cmb}, \quad (2.1)$$

where $Q_{s,c}$, Q_g , Q_L , and Q_{cmb} are the terms that represent secular cooling, the gravitational energy, the latent heat release, and the heat lost through the CMB. The secular cooling of the core is given by

$$Q_{s,c} = - \int_{V_c} \rho_a(r) C_{P,c} \frac{\partial T_{a,c}}{\partial t} dV, \quad (2.2)$$

where t is time, T_a is the adiabatic temperature of the core that corresponds to ρ_a , the adiabatic density profile, and $C_{P,c}$ is the heat capacity of the core, dV is the volume element, and V_c is the volume of the core. We use the fourth order polynomial fit from Labrosse (2015) to the adiabatic density profile derived from PREM (Dziewonski and

Anderson, 1981), and its associated adiabatic temperature profile, given by

$$T_{a,c}(r, t) = T_{\text{cmb}}(t) \left(\frac{\rho_a(r)}{\rho_{\text{cmb}}} \right)^\gamma = T_{\text{cmb}}(t) \left(\frac{\rho_0}{\rho_{\text{cmb}}} \right)^\gamma \left(1 - \frac{r^2}{L_\rho^2} - A_\rho \frac{r^4}{L_\rho^4} \right)^\gamma, \quad (2.3)$$

where, instead of using the inner-core boundary (ICB) as the reference point, as is done in (Labrosse, 2015), we use the CMB. ρ_0 and ρ_{cmb} are, respectively, the adiabatic density of the liquid core at the center and at the CMB, γ is the Grüneisen parameter, L_ρ and A_ρ are the polynomial fit parameters. Performing the integral in Eq. 2.2, and only keeping the terms up to order 4, results in

$$Q_{s,c} = -\frac{4}{3}\pi r_c^3 C_{P,c} \rho_0 \frac{\partial T_{\text{cmb}}}{\partial t}. \quad (2.4)$$

The gravitational energy term which arises due to the redistribution of the core material when light elements are released at the ICB is

$$Q_g = \int_{V_c} \rho_a \mu^* \frac{d\xi}{dt} dV, \quad (2.5)$$

where μ^* is the chemical potential difference across the ICB due to compositional differences and ξ is the concentration of light elements in the outer core, assuming that no light elements are incorporated into the inner core and that they are evenly distributed in the outer core. The exact expression for μ^* is in Labrosse (2015), but it is worth noting here that it is a function of β , the coefficient of chemical expansion, which relates to the density jump across the CMB due to compositional differences alone, $\Delta_\xi \rho$ (excluding the difference due to the phase transition between the liquid and solid states), and the concentration of light elements in the outer core, ξ ($\beta = (\Delta_\xi \rho)/(\rho_0 \xi)$). The final expression for the gravitational heat is given by

$$Q_g = 8\pi^2 \xi_0 G \rho_0^2 \beta \frac{r_{\text{ic}}^2 L_\rho^2}{f_c(r_c/L_\rho, 0)} \left[f_\chi \left(\frac{r_c}{L_\rho} \right) - f_\chi \left(\frac{r_{\text{ic}}}{L_\rho} \right) \right] \frac{dr_{\text{ic}}}{dt}, \quad (2.6)$$

where ξ_0 is the initial concentration of light elements in the core, G is the gravitational constant, f_c and f_χ are geometric functions that arise from integration over the core's volume defined in Labrosse (2015) for notational convenience (see the Appendix therein). Q_L is the energy release associated with the freezing of the inner core, given by

$$Q_L = 4\pi r_{ic}^2 \rho_a(r_{ic}) T_{L,c}(r_{ic}) \Delta S \frac{dr_{ic}}{dt}, \quad (2.7)$$

where ΔS is the entropy of crystallization per unit mass of solidified (inner) core, and $T_L(r_{ic})$ is the liquidus temperature of core material at the inner core boundary given by (Labrosse, 2015):

$$T_{L,c}(r) = T_{L,0} + K_0 \left(\frac{\partial T_{L,c}}{\partial P} \right)_\xi \frac{r_{ic}^2}{L_\rho} + \left(\frac{\partial T_{L,c}}{\partial \xi} \right)_P \frac{\xi_0 r_{ic}^3}{L_\rho^3 f_c(r_c/L_\rho, 0)}, \quad (2.8)$$

where $T_{L,0}$ is the liquidus at the central pressure of the core. The partial derivatives represent the rate of change of the liquidus with pressure and light element concentration while holding the other variable constant, respectively as they appear in the equation. Finally, the heat flow across the CMB, Q_{cmb} , is given by Fourier's law:

$$Q_{cmb} = 4\pi r_c^2 \frac{k_{lm} \Delta T_{lm}}{d_{lm}}, \quad (2.9)$$

where k_{lm} is the thermal conductivity of the lowermost mantle, ΔT_{lm} is the temperature difference across the CMB (i.e., $\Delta T_{lm} = T_{cmb} - T_{lm}$), and d_{lm} is the thickness of the lower thermal boundary layer. The expressions for ΔT_{lm} and d_{lm} are in the mantle section below.

The growth of the inner core (r_{ic}) is determined by the intersection of the core adiabat and the liquidus given by Eqs 2.3 and 2.8 above, and the rate of the growth, dr_{ic}/dt , can be expressed as $(dr_{ic}/dT_{cmb})(dT_{cmb}/dt)$, the first term is evaluated numerically, and the second is the only unknown variable left which we solve for.

(ii) Core Entropy Balance

Since we are not modeling heat transport within the core and are mainly concerned with the average evolution of the core, the thermal conductivity of the core does not enter the temperature evolution equation above. However, in order to evaluate the ability of any resulting core evolution to sustain a dynamo, an important consideration is the net generation of entropy, the main sink of which is heat conduction. The entropy may be considered as a proxy for whether a global magnetic field is generated. The analogous entropy balance to the energy balance (Eq. 2.1) is

$$E_{s,c} + E_L + E_g = E_k + E_\Phi, \quad (2.10)$$

where the left-hand side represents the sources of entropy. Subscripts are associated with those shown in the energy balance (Eq. 2.1). The right-hand side comprises the entropy sinks due to heat conduction (E_k) and magnetic and viscous diffusion (E_Φ). The entropy sources can be derived from the equivalent heat flow terms multiplied by a thermodynamic efficiency term (Buffett et al., 1996; Nimmo, 2015). This efficiency term is the difference of the inverse between the production temperature and the destruction temperature associated with that source. For example, secular cooling is produced throughout the core volume and depends on the integral of the core adiabat (T_{eff}) (see the Appendix in Labrosse (2015)), and is lost at the CMB at a temperature of T_{cmb} . Therefore, the entropy associated with cooling is defined as

$$E_{s,c} = \left(\frac{1}{T_{\text{eff}}} - \frac{1}{T_{\text{cmb}}} \right) Q_{s,c}. \quad (2.11)$$

Similarly, the entropy production term associated with the latent heat release at the ICB is

$$E_L = \left(\frac{1}{T_{\text{icb}}} - \frac{1}{T_{\text{cmb}}} \right) Q_L, \quad (2.12)$$

where T_{icb} is the temperature at the ICB, determined using the adiabatic temperature profile in Eq. 2.3. Because we assume no light element flux from the core to the mantle, the entropy production term associated with gravitation buoyancy has higher efficiency (Buffett et al., 1996) and is given by

$$E_g = \frac{1}{T_{\text{cmb}}} Q_g. \quad (2.13)$$

Moving onto the right-hand side terms, we first consider E_k , which represents a major entropy sink. E_k is

$$E_k = \int_{V_c} k \left(\frac{\Delta T_{\text{a,c}}}{T_{\text{a,c}}} \right)^2 dV, \quad (2.14)$$

where $T_{\text{a,c}}$ is once more the adiabatic temperature of the core, given by Eq. 2.3. If we take the core conductivity to be constant, i.e., $k(r) = k_c$, and perform the integral over the volume of the core, we arrive at

$$E_k = 16\pi\gamma^2 k_c L_\rho \left[f_k \left(\frac{r_c}{L_\rho} \right) - f_k \left(\frac{r_{\text{ic}}}{L_\rho} \right) \right], \quad (2.15)$$

where f_k is another geometric integral. Finally, to evaluate the ability of convection in the core to sustain a dynamo, the entropy sources must exceed the sinks, as a minimum requirement. Without making assumptions about the dynamic of the core convection, this means that

$$E_k + E_\Phi > E_g + E_L + E_{\text{c,s}}, \quad (2.16)$$

or

$$E_\Phi = E_g + E_L + E_{\text{c,s}} - E_k > 0. \quad (2.17)$$

This is considered a minimum requirement because we have not included the entropy sink due to magnetic and viscous dissipation in the entropy budget. The magnetic dissipation is a function of the strength of the magnetic field and the wavelength of convection and while we may introduce errors due to its omission, this omission is commonly applied

to avoid making assumptions about the dynamics within the core driving the dynamo (Buffett et al., 1996). Similarly, viscous dissipation omission is common, due to the low viscosity of liquid iron at core conditions. With Eqs 2.11–2.13, 2.15 and 2.17, we can evaluate, at any given time in our evolution, whether the core produces a magnetic field or not. The relevant core parameters used are listed in Table 2.1.

Table 2.1: Core Parameters

Parameter	Notation	Value	Reference
Outer core radius	r_{cmb}	3485 km	Dziewonski and Anderson (1981)
Gravitational Constant	G	$6.67 \times 10^{-11} \text{ N/m}^2/\text{kg}^2$	
Adiabatic Density at the center	ρ_0	12451 kg/m ³	Dziewonski and Anderson (1981); Labrosse (2015)
Core density at the CMB	ρ_c	9906 kg/m ³	Dziewonski and Anderson (1981) derived from Labrosse (2015)
Grüneisen parameter	γ	1.5	Vočadlo et al. (2003)
2 nd order coefficient for adiabatic density profile	L_ρ	$8039 \times 10^3 \text{ km}$	Labrosse (2015)
4 th order coefficient for adiabatic density profile	A_ρ	0.484	Labrosse (2015)
Core bulk modulus at the center	K_0	1403 GPa	Dziewonski and Anderson (1981); Labrosse (2015)
Core mean to CMB temperature factor	η_c	1.13	derived from Labrosse (2015) ¹
Entropy of crystallization	ΔS	$0.3 \times 10^6 \text{ J/kg}$	Hirose et al. (2013); Labrosse (2015)
Rate of change of liquidus with pressure	$\left(\frac{\partial T_{\text{Lc}}}{\partial P}\right)_\xi$	$-21 \times 10^3 \text{ K}$	Alfe et al. (1999); Labrosse (2015)
Rate of change of liquidus with light element concentration	$\left(\frac{\partial T_{\text{Lc}}}{\partial \xi}\right)_P$	9 GPa/K	Alfè et al. (2007); Labrosse (2015)

2.5.2 The Plate-Mantle System

The mechanical and thermal evolution of the plate and mantle are governed by conservation of momentum and conservation of energy, respectively. In analytical mantle convection models, since bulk average properties are considered, the momentum equation is often written in the form of conservation of mechanical energy. The two conservation equations (total and mechanical energy) are coupled through the plate velocity. The plate velocity is obtained by solving the mechanical energy equation and governs the heat loss through the surface in the total energy equation. In the following two sections, we outline the models we use to calculate the plate velocity (and the entire velocity profile of the mantle) from the mechanical energy equation. We then explain how the plate velocity is used to evolve the mantle temperature.

(i) Mechanical Energy

Here we outline the forms of the mechanical energy equation used in our model, based on the formulation of Crowley & O’Connell (hereafter, COC12 (Crowley and O’Connell, 2012)) with two important improvements. First, we remove the assumption of steady state convection from the mechanical energy equation. Although this assumption is often made in analytical models of mantle convection, it is strictly inconsistent with thermal evolution models, which are based on an imbalance between supplied and lost energies. In addition, embedded within this assumption is the removal of the hysteresis (or the path dependence) of the mechanical energy equation, which has been shown to be important in numerical models (Lenardic and Crowley, 2012). Conceptually, path dependence means that the convective state of the mantle does not only depend on the thermal state at an instance in time, but also on its prior evolution of previous states. This change, while practically straightforward to implement, represents a philosophical departure from many previous applications. Second, we introduce a lowermost mantle layer that represents the thermal boundary layer between the core and the mantle, that has its own mechanical properties (specifically, reference viscosity). This change is simple but is important for coupling the mantle with the core.

Following COC12, it can be shown that the conservation of mechanical energy equation in 2D can be written as

$$\Phi_P = - \int_S (u_i P \delta_{ij} - u_i \tau_{ij}) dS_j + \Phi_L + \Phi_M, \quad (2.18)$$

which states the rate of change of potential energy in the mantle (Φ_P) is balanced by the work done by viscous forces in the mantle (Φ_M), the work done to bend or shear the lithosphere (Φ_L), and the work done of the surface forces of the system (represented by the terms within the surface integral). Here, u_i is the velocity vector, P is the pressure, δ_{ij} is the Kronecker delta function and τ_{ij} is the deviatoric stress. The subscripts i, j

follow the Einstein convention.

We model the mechanical energy of the mantle using 2D Cartesian geometry (see Figure 1 in Al Asad et al. (2023)), and we represent all the quantities per unit length along the z -direction (along strike of the ridge/subduction boundaries). The rate of change of potential energy in the system, is proportional to the depth-averaged vertically advected heat flow, $\langle Q \rangle_{\text{adv}}$, and for high Rayleigh number systems, such as the mantle, advection dominates such that the advected heat flow is approximately equal to the total heat flow $\langle Q \rangle$. In steady state systems, $\langle Q \rangle_{\text{ss}}$ is

$$\langle Q \rangle_{\text{ss}} = (q_{\text{cmb}} + \frac{1}{2}Hd)L = (q_{\text{surf}} - \frac{1}{2}Hd)L, \quad (2.19)$$

where q_{cmb} and q_{surf} are respectively the CMB and surface heat fluxes, and d and L are the depth and length of the system. H is the radiogenic heating per unit volume in the mantle (Q_{rad} in Eq. 2.28 divided by the mantle volume V_{m}). In the non-steady state system we aim to consider, Eq. 2.19 no longer holds because the change of internal energy of the mantle can act to change the total advected heat flow. When the mantle is heating/cooling, a part of the energy is not advected but is used to heat/cool the system. We must depart from the original balance stated in COC12 and modify the equation above to

$$\langle Q \rangle_{\text{nss}} = \left(q_{\text{cmb}} + \frac{1}{2}Hd - \frac{1}{2}\rho_{\text{m}}C_{\text{P,m}}\frac{dT_{\text{m}}}{dt} \right) L, \quad (2.20)$$

to account for the non-steady-state (nss) balance of the advected heat flow. Here, T_{m} is the average mantle temperature, and ρ_{m} and $C_{\text{P,m}}$ are the average mantle density and heat capacity, respectively. The geometry of these expressions are Cartesian (as formulated in COC12) such that the areas of the surface and the CMB are assumed to be equivalent. This overestimates the CMB heat flow and is inconsistent with the core model presented above. In order to account for this, we instead use the version of this equation consistent with spherical geometry provided in Appendix B.5.

Performing the integral in Eq. 2.18, we consider the lithospheric system $\langle Q \rangle_L$ alone, and the whole mantle system, $\langle Q \rangle_{\text{nss}}$, separately, yields

$$\frac{\alpha g d_l}{C_{P,m}} \langle Q \rangle_L = L P_{x_p} d_l U_p + L \tau_p U_p + \tau_R d_l U_p, \quad (2.21)$$

$$\frac{\alpha g d}{C_{P,m}} \langle Q \rangle_{\text{nss}} = \tau_F d_l U_p + \tau_Y d_l U_p + \Phi_M. \quad (2.22)$$

Here, α is the coefficient of thermal expansion, g is the gravitational acceleration, d_l is the thickness of the lithosphere at the subduction zone, for which we use the half-space cooling model

$$d_l = c_0 \left(\frac{\kappa L}{U_p} \right)^{1/2}, \quad (2.23)$$

where c_0 is a constant, κ is the thermal diffusivity, and U_p is the plate velocity. $\langle Q \rangle_L$ is the depth-averaged advected heat flow through the lithosphere and is equal to half the surface heat flow (Q_{surf}), given by

$$Q_{\text{surf}} = 2k_{\text{um}} \Delta T \left(\frac{U_p L}{\pi \kappa} \right)^{1/2}, \quad (2.24)$$

where k_{um} is the thermal conductivity of the upper mantle, $\Delta T = T_u - T_s$ is the temperature difference across the lithosphere (i.e., the difference between the upper mantle, T_u , and surface, T_s , temperatures, see below). Whether for the lithosphere alone or the entire mantle, Eqs 2.21 and 2.22 state that the rate of change of potential energy in the system is balanced by the work done on the interior and boundary of the system.

For the lithospheric energy balance (Eq. 2.21), P_{x_p} is the lateral pressure gradient evaluated at the base of the lithosphere, τ_p is the viscous stress at the base of the lithosphere, and τ_R is the net resistive stress of the lithosphere. It represents the difference between the stresses that hinder and drive lithospheric flow and is given by

$$\tau_R = \tau_Y + \tau_F - \tau_{\text{SP}}, \quad (2.25)$$

where τ_Y is the effective yield stress of the lithosphere, τ_F is the faulting stress, and τ_{SP} is the stress due to slab pull.

The work done on the surface of the lithosphere is concentrated at the lower boundary due to the lateral pressure gradient (first term on the right-hand side of Eq. 2.21) and the traction at the base of the lithosphere (second term on the right-hand side of Eq. 2.21) due to the flow of the mantle beneath the plate, both of which are functions of the mantle's flow pattern and viscosity structure. The vertical boundary at the subduction zone also contributes work in the form of faulting and slab pull. Within the volume of the lithosphere, work is done to bend the lithosphere (where our expressions make no assumption on the rheological model). The individual contributions of τ_Y , τ_F and τ_{SP} do not matter in the lithospheric energy balance, but rather the sum of these contributions, which has a considerable role in determining the mode of convection in the mantle. If slab pull is larger than the sum of bending and faulting stresses, τ_R is negative, it leads to a plate-mantle coupling state in active-lid mode. In this case, the plate moves due to its own negative buoyancy. However, when τ_R is positive, active and multiple sluggish-lid solutions are possible that represent different dominant balances in the energy equations (Al Asad et al., 2023; Crowley and O'Connell, 2012). While these stresses evolve with the evolution of the mantle, we set each of them as constant model parameters (values given in Table 2.2) following the justification in COC12, and for the lack of analytical models that adequately describe their behavior with time. While the mechanical strengths are constant, the dissipation associated with these terms varies with lithospheric thickness and plate velocity. It is worth noting that the rate of change of potential energy in the lithosphere is equivalent to the work commonly referred to as ridge push (Hager and O'Connell, 1978).

For the mantle energy balance (Eq. 2.22), the whole mantle's change in potential energy is balanced by the work done on the boundaries and within the mantle. The only surface contribution is the faulting stress due to both boundaries being stress-free.

Table 2.2: Mantle Parameters. All values, except η_m are taken from Crowley and O’Connell (2012). η_m is from Stevenson et al. (1983)

Parameter	Notation	Value
Upper mantle thermal conductivity	k_{um}	3.2 W/m/K
Thermal expansion coefficient	α	$2.5 \times 10^{-5} /K$
Average Density	ρ_m	$3.5 \times 10^3 \text{ kg/m}^3$
Gravitational acceleration	g	10
Thermal diffusivity	κ	$7.6 \times 10^{-7} \text{ m}^2/\text{s}$
Viscosity activation energy	E	200 kJ/mol
Universal gas constant	R_g	8.31 J/mol/K
Viscosity reference temperature for bulk mantle	$T_{m,0}$	1300 K
Mantle reference viscosity	$\mu_{0,m}$	10^{23} Pa s
Critical Rayleigh number	Ra_{crit}	2000
Mantle thickness	d	$2891 \times 10^3 \text{ km}$
Mantle average to potential temperature factor	η_m	1.3
Convection cell length	L	$6000 \times 10^3 \text{ km}$
Mantle heat capacity	$C_{P,m}$	1200 J/kg/K
Lithospheric yield strength	τ_Y	300 MPa
Lithospheric faulting strength	τ_F	0
Slab pull stress	τ_{SP}	200 MPa
Half-space cooling model constant	c_0	2.32
Surface temperature	T_s	273 K
Base of the asthenosphere	b_a	350 km

Within the volume of the mantle, both deformation of the lithosphere (second term on the right-hand side of Eq. 2.22), and viscous deformation due to mantle flow contribute work on the mantle (third term on right-hand side of Eq. 2.22). Viscous dissipation is a function of the viscosity and flow structure of the mantle and has contributions due to lateral shearing and corner flow. While the expressions above make no assumption of such structure, in our model, the mantle is subdivided into the four major regions: the lithosphere, asthenosphere, bulk mantle, and lower mantle. The thickness and mechanical properties of these regions depend on their thermal state and therefore evolve with time.

We follow the approximations and method of solution outlined by COC12 (see their Appendix) to solve the mantle and lithospheric energy equations for the plate velocity. We present the equations consistent with our model, due to the addition of the lower thermal boundary layer, in Appendix. B.7.

(ii) Thermal evolution

Upon determining the plate velocity, U_p , the surface heat flow is obtained (Eq. 2.24) and with this, we are able to evolve the mantle temperature. The secular cooling of the mantle, similarly to Eq. 2.1 for the core, is determined by the difference between the energy sources and sinks in the mantle and is given by

$$Q_{s,m} = Q_{\text{rad}}(t) + Q_{\text{cmb}} - Q_{\text{surf}}, \quad (2.26)$$

where $Q_{s,m}$, Q_{rad} , are the energy sources in the mantle and represent the mantle's secular heat and radiogenic heating, respectively, given by

$$Q_{s,m} = -V_m \rho_m C_{P,m} \frac{dT_m}{dt}, \quad (2.27)$$

and

$$Q_{\text{rad}}(t) = V_m \rho_m \sum_i C_i H_i e^{t \ln(2)/\tau_i}, \quad (2.28)$$

where V_m is the mantle volume and ρ_m is the average mantle density. Radiogenic heat is produced in the mantle by many radioactive elements, assumed to be uniformly distributed within the mantle (Listed in Table 2.3). The total heat produced is the sum of the individual elements i with concentration C_i , heat production (per unit mass) H_i , and half-life τ_i . Substituting Eq. 2.27 into Eq. 2.26 give the temperature evolution of the mantle.

Table 2.3: Mantle Radioactive Elements. Concentration are from Palme and O'Neill (2003) assuming chondritic Earth composition. Heat production and half-life values are from Dye (2012) via Jaupart et al. (2015)

Element	Concentration [ppm]	Heat Production [W/kg]	Half Life [years]
U ²³⁸	0.9927×0.0218	95.1×10^{-6}	4.46×10^9
U ²³⁵	0.0072×0.0218	568.4×10^{-6}	7.04×10^8
K ⁴⁰	260	29.1×10^{-6}	1.26×10^9
Th ²³²	0.0834	26.3×10^{-6}	1.4×10^{10}

As noted, the CMB heat flux depends on the difference across the thermal boundary layer (with thickness d_{lm}), as well as the lowermost mantle viscosity, thermal conductivity and thermal boundary layer thickness (see Eq. 2.9). The latter can be determined from consideration of local instability when the local Rayleigh number, Ra_{lm} , exceeds the critical Rayleigh number for convection, Ra_{crit} . The lower mantle Rayleigh number is defined by

$$\text{Ra}_{\text{lm}} = \frac{\alpha \rho_{\text{m}} g \Delta T_{\text{lm}} d_{\text{lm}}^3}{\mu_{\text{lm}} \kappa}, \quad (2.29)$$

where α is the thermal expansion coefficient, κ is the thermal diffusivity and μ_{lm} is the lower mantle viscosity. This viscosity is governed by the Arrhenius relationship

$$\mu_{\text{R}}(T) = \mu_{\text{R},0} e^{\frac{E}{R_{\text{g}}} \left(\frac{1}{T_{\text{R}}} - \frac{1}{T_{\text{R},0}} \right)}, \quad (2.30)$$

where μ_{R} is the viscosity of any given region, R. In this specific case, $\text{R} = \text{lm}$, the lower mantle. Here, $\mu_{0,\text{R}}$ is the reference viscosity defined at $T = T_{0,\text{R}}$, T_{R} is the temperature of region R ($= T_{\text{lm}}$), E is the activation energy and R_{g} is the ideal gas constant. The temperature of the lower mantle is determined by extrapolating down a linear adiabat, given by

$$T_{\text{a,m}}(y) = 2T_{\text{P}} (\eta_{\text{m}} - 1) \frac{(y - d_1)}{(d - d_1)} + T_{\text{P}}, \quad (2.31)$$

where y is the depth from the surface. This adiabatic profile is set such that the mantle potential (T_{P}) and average (T_{m}) mantle temperatures have the relationship $T_{\text{m}} = \eta_{\text{m}} T_{\text{P}}$. The mantle parameters we use are listed in Table 2.2.

2.5.3 Coupling the Core and Mantle Evolutions

The core and mantle systems are coupled via the expressions for heat flow at the CMB (Eq. 2.9), which appears in both the core energy balance (Eq. 2.1) and the mantle energy balance (Eq. 2.26) and depends on the temperature difference between the core and the mantle and thus both of their evolutions. Beginning with the initial conditions

($t = t_0$) of the mantle temperature ($T_{m,0}$) and the core temperature at the CMB ($T_{cmb,0}$), we determine the initial mantle viscosity structure (Eq. 2.30), the thickness of the lower thermal boundary layer (Eq. 2.29) and Q_{cmb} (Eq. 2.9). We then solve for the plate velocity numerically using the method described in the Supplemental Text and COC12. Finally, we use the plate velocity solution to calculate the surface heat flow (Eq. 2.24) and can evaluate the rate of change of mantle (dT_m/dt) and core at the CMB (dT_{cmb}/dt) temperatures using Equations 2.26 and 2.1, respectively. With these two derivatives, the temperatures of both the mantle and core are advanced to the next time step using a first order Euler method, with a time step of 40 My. We tested several time steps and since the evolution paths are smooth over our 4.0 billion year histories, our simple time stepping scheme is efficient and accurate. We begin the evolution at time $t_0 = 500$ My to use initial conditions consistent with a differentiated Earth and solidified mantle.

Across different simulations, we vary the initial temperature ($T_{m,0}$), the initial core temperature at the CMB ($T_{cmb,0}$), the asthenospheric ($\mu_{0,a}$) and lower mantle ($\mu_{0,lm}$) reference viscosities. The range of values tested for each of these parameters is listed in Table 2.4, which result in $\sim 10^6$ thermal histories. For each of these histories, we then calculate the entropy budget (Eq. 2.17) using different values of core thermal conductivity (k_c), also listed in Table 2.4, resulting in $> 40 \times 10^6$ entropy histories. With these histories and tracked variables such as the plate and mantle velocities, the mantle temperature, the inner core radius and the core entropy production, we are able to present results described in the Main Text.

Table 2.4: Parameters in Simulations. Values superscripted with “L” in left column are taken from Labrosse (2015), PREM is from Dziewonski and Anderson (1981), and MG is from Masters and Gubbins (2003).

Parameter	Symbol	Values		Number of Increments
Mantle initial potential temperature	$T_{m,0}$	1650–2050 K		9
CMB initial temperature	$T_{\text{cmb},0}$	4700–5700 K		11
Ratio of bulk mantle to asthenospheric viscosities	r_μ	1—1000		7
Lower mantle reference viscosity	$\mu_{0,\text{lm}}$	$10^{22} - 10^{25}$ Pa s		10
Core thermal conductivity	k_c	50–120 W/m/K		15
Lower mantle thermal conductivity	k_{lm}	4, 10 W/m/K		2
Core heat capacity	$C_{P,c}$	715, 750, 840 J/kg/K		3
Core model:		PREM	MG	2
compositional density jump across ICB	$\Delta\rho_\xi$	380 kg/m ³	580 kg/m ³	
present-day light element concentration ^L	ξ_{pd}	4%	5.6%	
present-day center melting temperature ^L	$T_{L0,\text{pd}}$	5700 K	5500 K	

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Appendix B

Additional Material for Chapter 2

B.1 The Evolution of Other Mantle and Core Quantities

In this section, we show the evolution of other important quantities in our model results that we do not include in the Main Text in Fig. 2 and which may be of interest to some readers. Fig. B.1 is complementary to Fig. 2 in the Main Text and is organized the same way: the red envelope represents the active-lid evolution scenarios, characterized by a velocity ratio $r_U \sim 1$ (Fig. B.1a), the blue envelopes are the sluggish-lid scenarios, with $r_U > 10$ (Fig. B.1a), and the transitioning scenarios, where the mantle starts spend time in the sluggish-lid regime (Fig. B.1a). The solid curves are the same as the ones shown in Fig. 2 in the main text. The quantities shown in this figure are: the velocity ratio (r_U), the Urey ratio, the lowermost mantle absolute viscosity (μ_{lm}), the lower thermal boundary layer thickness (d_{lm}) and the CMB temperature evolution (T_{CMB}).

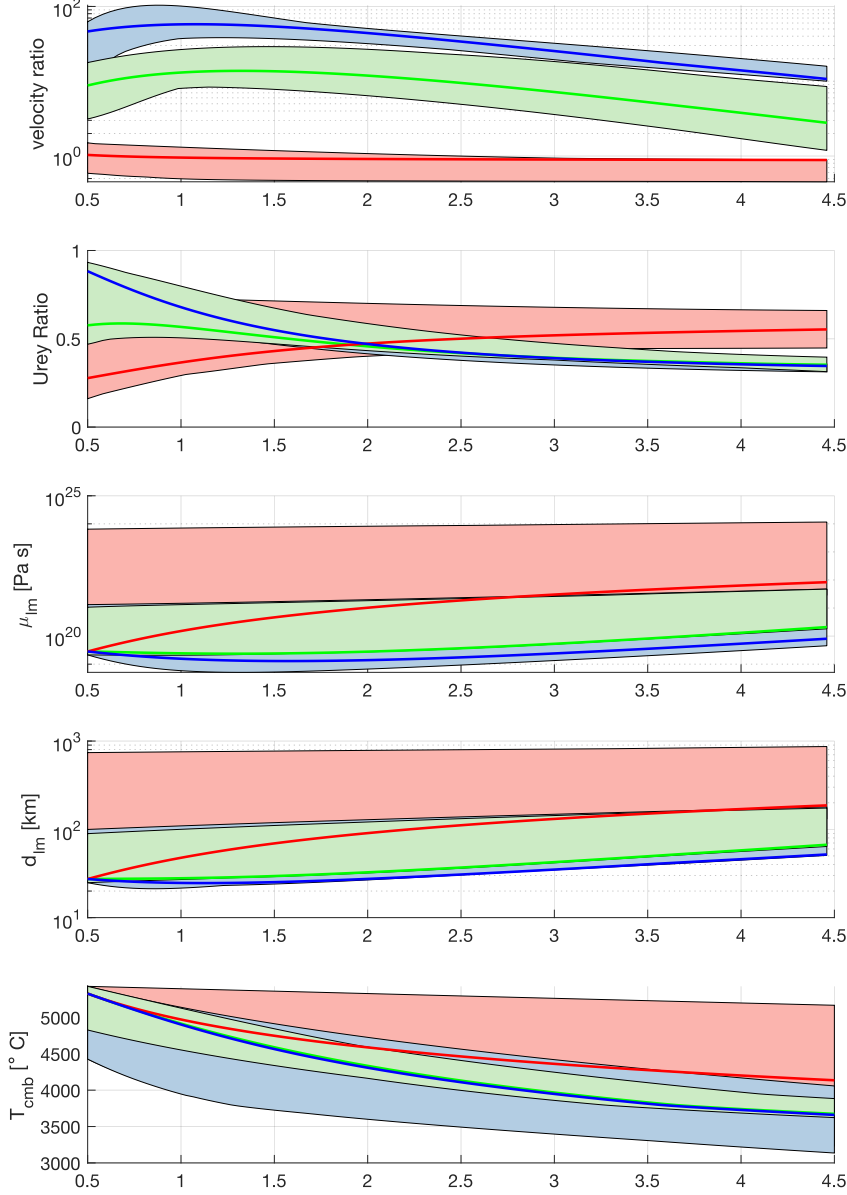


Figure B.1: **Evolution of key quantities in simulations (where $k_c = 100$ W/m/K):** Panels (a)-(e) show the velocity ratio (r_U), the Urey ratio, the lowermost mantle absolute viscosity (μ_{lm}), the lower thermal boundary layer thickness (d_{lm}) and the CMB temperature evolution (T_{CMB}), respectively. In all panels, red shaded envelopes span simulations that exhibit active-lid tectonics ($r_U \sim 1$ at all times), blue shaded envelopes span simulations that are always in the sluggish-lid mode ($r_U > 10$ at all times), and the green shaded envelopes span those that satisfy all criteria (I)-(IV), which transition from the sluggish- to the active-lid mode. The solid curves of each color show individual examples of the three behaviors shown with shading. They have identical mantle and core structures and initial conditions, except with different mantle-to-asthenosphere viscosity ratio (r_μ), where in the red curve $r_\mu = 1$, the blue curve $r_\mu = 1000$, and the green curve $r_\mu = 100$.

B.2 Convection Mode Sensitivity Tests

We augment the calculations in the Main Text by varying parameters assumed to be fixed (i.e., do not evolve) in the main results: the effective strength of the plate (τ_{yf} , Fig. B.2a), the cell aspect ratio (L/d , Fig. B.2b), the base depth of the low viscosity region (b_a Fig. B.2c), and, for completion, the viscosity ratio (r_μ Fig. B.2d). The fixed values used in the main results are found in Table S2. We use the median r_U value over the entire simulation as a proxy for convective mode. As can be seen, some of these parameters, especially τ_{yf} and L/d also control the convective mode.

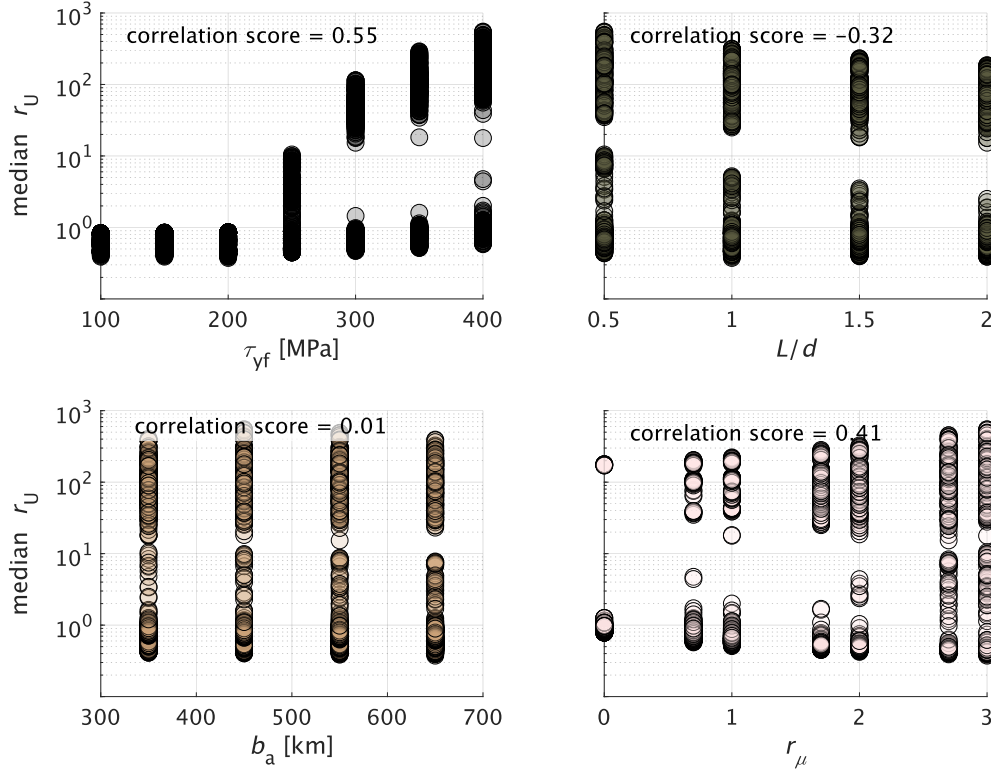


Figure B.2: **Parameters that control mantle convection regime:** plots of median velocity ratio r_U as a function of (a) effective yield strength of the plate τ_{yf} , (b) cell aspect ratio L/d , (c) the base of the asthenosphere b_a , and (d) viscosity ratio r_μ . the correlation score between each quantities on the x- and y- axes is included in the figure.

B.3 Permitted Parameter Values to Meet Success Criteria

We have varied 8 parameters in our simulations, shown in Table 2.4. Here we show the distribution of parameters that meet certain success criteria. In Fig. B.3, we show the distribution of the parameters we tested (blue), those that meet criterion (I) (orange), criterion (III) (yellow), and all success criteria (purple).

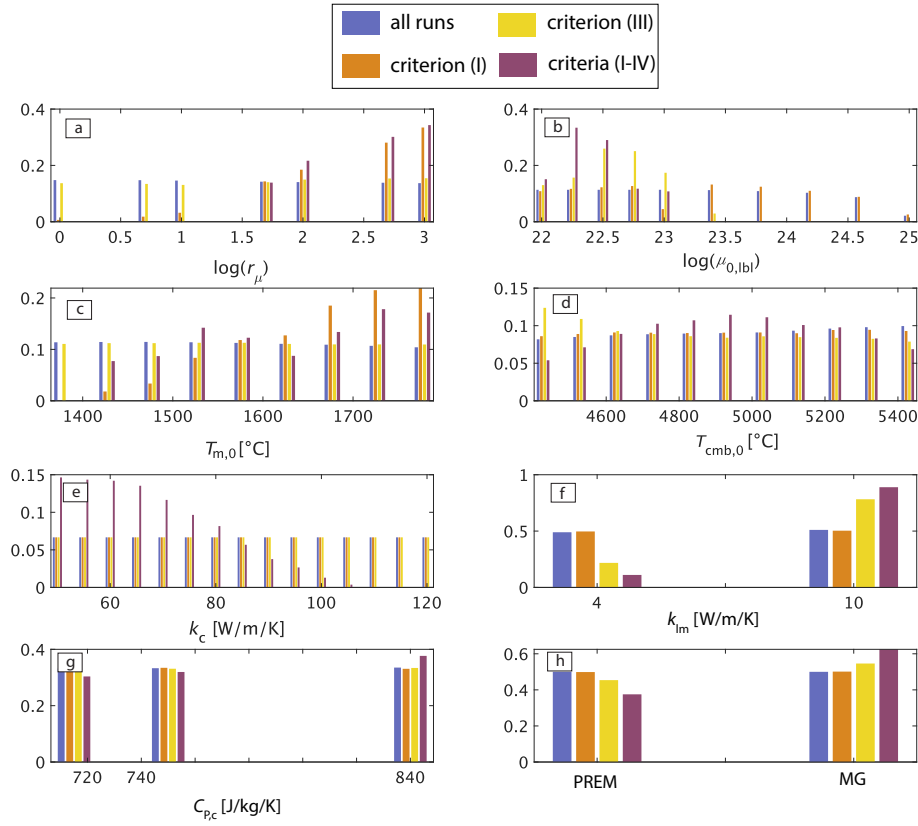


Figure B.3: **Distribution of parameters that meet success criteria** (a) viscosity ratio (r_μ), (b) lower mantle reference viscosity ($\mu_{0,ibl}$), (c) initial mantle ($T_{m,0}$) and (d) core ($T_{cmb,0}$) temperatures, (e) core thermal conductivity (k_c), (f) lower mantle thermal conductivity (k_{lm}), (g) core heat capacity ($C_{p,c}$), and (h) core model, PREM Dziewonski and Anderson (1981), and MG Masters and Gubbins (2003).

B.4 Time Evolving parameters

In this section, we first show that the early asthenosphere plays an important role in Earth evolution and determining the tectonic regime. We test two evolution scenarios for asthenospheric thickness evolution: (1) the depth to the base of the asthenosphere decreases linearly with time from 350–0 km, blue curves in Fig. B.4. (2) the depth to the base of the asthenosphere increases linearly with time from 0–350 km, red curves in Fig. B.4, against a reference case with constant asthenospheric base depth of 350 km (black curves in Fig. B.4). The decreasing (blue) and constant (black) cases, which have an asthenosphere present at the start of the simulation, evolve very similarly (except for the velocity ratio), and both lead to reasonable evolution scenarios. If the asthenosphere is not present at the beginning of the simulation, as is the case with the red curve, then the mantle cools too fast. Increasing and decreasing the viscosity ratio with time has the same effect.

Second, we evolve the effective plate strength parameter τ_{yf} linearly with time from 350–200 MPa, while holding slab pull constant at $\tau_{sp} = 250$ MPa. This changes the force balance on the plate such that plate dissipation exceeds slab pull first, allowing for a sluggish-lid solution, then slab pull exceeding plate dissipation, promoting active-lid convection. This is shown in Fig. B.5 in blue curves compared to the reference case in black. This test shows the importance of understanding the lithospheric strength evolution with time.

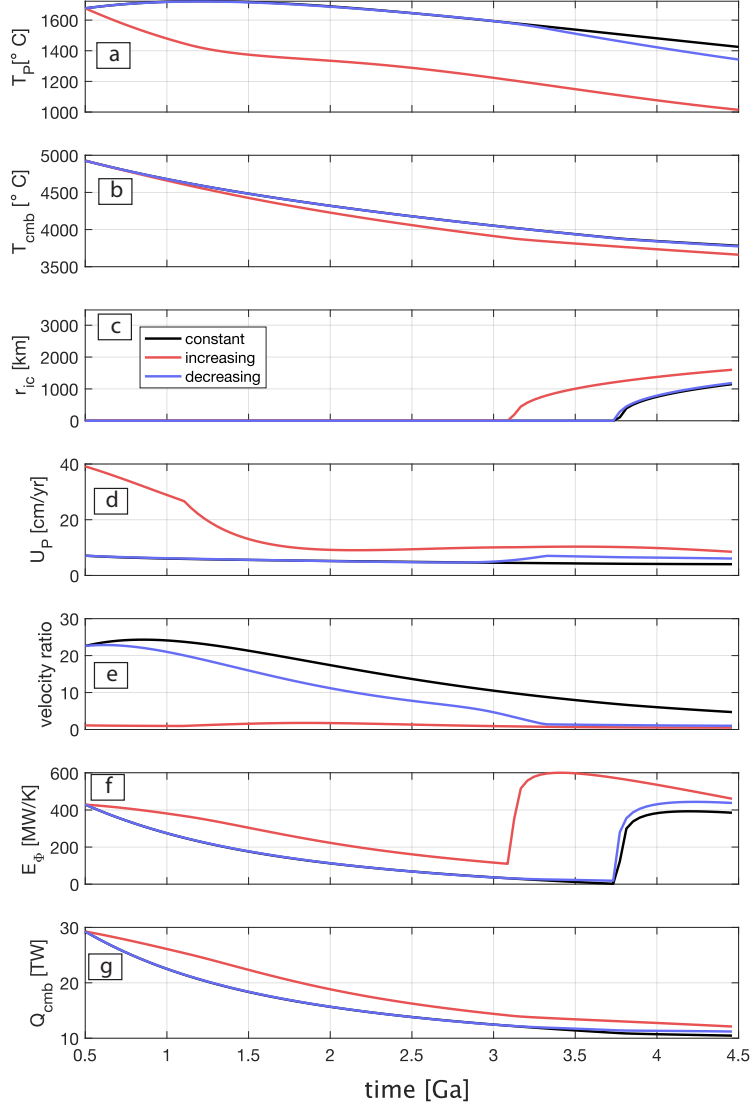


Figure B.4: **Simulations with evolving asthenospheric thicknesses:** Black – reference case in black where the depth to the asthenosphere base is constant (350 km) throughout the simulation time. Red – asthenospheric depth increases linearly from 0 (no asthenosphere) to 350 km at the end. Blue – asthenospheric depth decreases linearly from 350 km to 0. All simulations have an asthenospheric viscosity ratio $r_\mu = 500$. from (a) - (g) the panels show the mantle potential temperature, CMB temperature, inner core radius, plate velocity, mantle-to-plate velocity ratio, net entropy production and CMB heat flow, respectively.

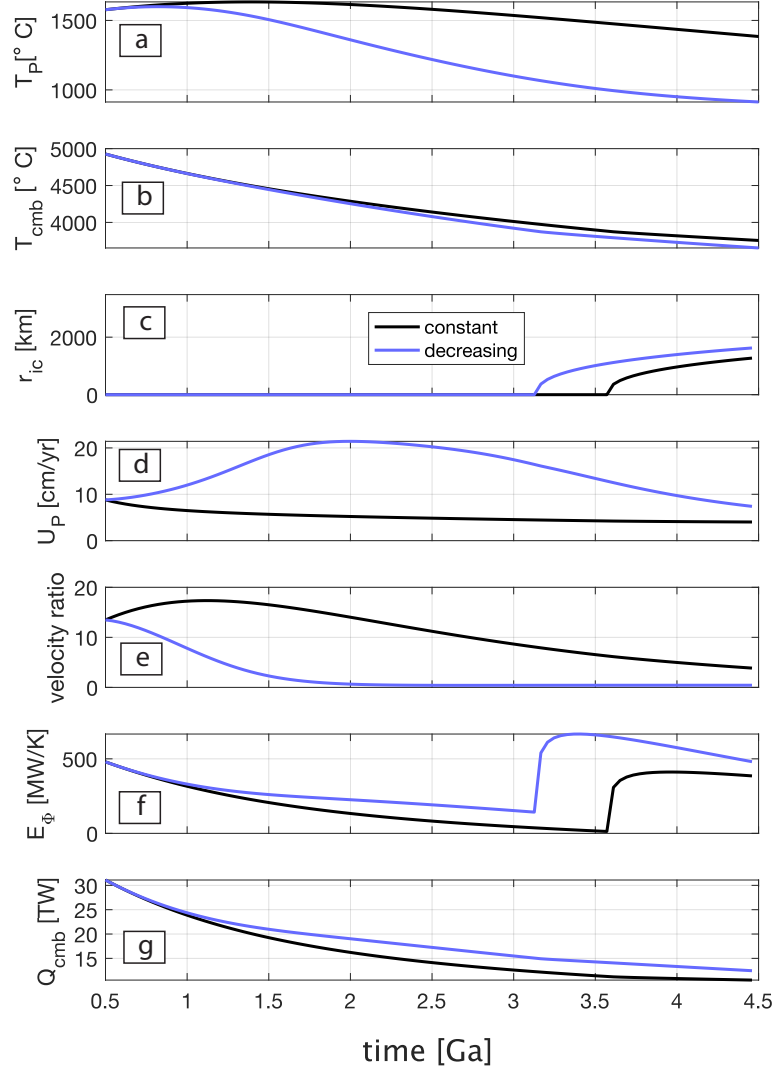


Figure B.5: **Simulations with evolving effective plate strength:** Black – reference case in black where τ_{yf} is constant (350 MPa) throughout the simulation time. Blue – τ_{yf} decreases linearly from 350–200 MPa. Both simulations have an asthenospheric viscosity ratio $r_\mu = 500$ and slab pull $\tau_{sp} = 250$ MPa. from (a) - (g) the panels show the mantle potential temperature, CMB temperature, inner core radius, plate velocity, mantle-to-plate velocity ratio, net entropy production and CMB heat flow, respectively.

B.5 Derivation of the Heat Flow Through the Mantle in Polar Coordinates

Although the mantle model requires that we consider Cartesian geometry, to solve the velocity equation analytically (see next section), some quantities may be derived for the more realistic 2D polar mantle geometry. The most important of these quantities is arguably the depth-averaged advected heat flow that enters in the calculation of the rate of change of potential energy in the whole mantle equation. This is especially true because we are coupling the mantle model to a core model, and the heat flow at the CMB in each of them must be consistent.

The heat flux advected in the mantle from the CMB in 2D polar geometry is given by

$$\frac{dq}{dr} + \frac{2q}{r} = H + \rho_m C_{P,m} \dot{T}, \quad (\text{B.1})$$

with the boundary condition $q(r_c) = q_{\text{cmb}}$ where q is the heat flux, H is the radiogenic heat production per unit volume (Q_{rad}/V_m), and the over-dot represents the time derivative. The solution to the equation is

$$q(r) = \frac{1}{3} \left(H + \rho_m C_{P,m} \dot{T} \right) r + \left(\frac{r_c}{r} \right)^2 \left[q_{\text{cmb}} - \frac{1}{3} \left(H + \rho_m C_{P,m} \dot{T} \right) r_c \right] \quad (\text{B.2})$$

The heat flow per unit length along the axis ridge is derived from Eq. B.2 by multiplying by the arclength ($L \times (r/r_p)$) where r_p is the radius of the planet. To take the depth average, we integrate the heat flow equation from r_c to r_p normalized by the mantle

thickness $(r_p - r_c)$. This yields

$$\langle Q \rangle_{\text{nss}} = \frac{L}{r_p - r_c} \left\{ \frac{1}{9r_p} \left(H + \rho_m C_{P,m} \dot{T} \right) (r_p^3 - r_c^3) + \frac{r_c^2}{r_p} \left[q_{\text{cmb}} - \frac{r_c}{3} \left(H + \rho_m C_{P,m} \dot{T} \right) \right] \ln \left(\frac{r_p}{r_c} \right) \right\}. \quad (\text{B.3})$$

This expression is used instead of the expression stated in the Methods Section (Eq. 2.20).

B.6 Mantle Model Geometry

We use a 4-layer 2D Cartesian mantle model which has 4 distinct layers each with different mechanical properties: the lithosphere, asthenosphere, bulk mantle, and lowermost mantle (each labeled with subscripts “l”, “a”, “m”, and “lm”). The total thickness of the mantle, d , is subdivided into the individual layers: the lithosphere (d_l) is determined using Eq. 2.23, the lowermost mantle (d_{lm}) using Eq. 2.29, the boundary between the mantle and asthenosphere is always at a depth of 350 km, such that the thickness of the bulk mantle is $d_m = d - 350 - d_{\text{lm}}$ and the thickness of the asthenosphere is $d_a = 350 - d_l$. Each layer has a distinct viscosity: μ_l , μ_a , μ_m and μ_{lm} , respectively. In our calculations μ_l is constant in time (Table 2.2). The asthenospheric and mantle reference viscosities are fixed in time with a chosen r_μ such that $\mu_{0,a} = \mu_{0,m}/r_\mu$. We test several values of r_μ , (see Table 1). The mantle and lower mantle viscosities are governed by the temperature dependence in Eq. 2.30 (and thus evolve over the histories). Each have their own associated reference viscosities and are calculated using different temperatures: the mantle at the average mantle temperature, T_m , and the lowermost mantle at the average temperature of the CMB and lower mantle $T_{\text{lbl}} = (T_{\text{lm}} + T_{\text{cmb}})/2$, where T_{lm} is calculated by evaluating Eq. 2.31 at the base of the mantle.

B.7 Calculating the Plate Velocity in a 4-layer Model

To calculate the lateral velocity of the mantle in our 4-layer model, as depicted in Fig. B.1, we solve the momentum equation in the interior of the convection cell (away from regions of vertical motion), as outlined in COC12. In this region, we can write the (lateral) component of the momentum equation as

$$0 = -P_x(y) + \mu(y) \frac{\partial^2}{\partial y^2} U(y), \quad (\text{B.4})$$

and assume the pressure gradient varies linearly with depth using

$$\frac{\partial P}{\partial x} = Ay + b, \quad (\text{B.5})$$

where A and b are model parameters that depend on the flow profile which we solve for. The momentum equation is then

$$\mu(y) \frac{\partial^2}{\partial y^2} U(y) = Ay + b. \quad (\text{B.6})$$

The appropriate boundary conditions for our model are

$$U(d) = U_p, \quad (\text{B.7})$$

and

$$\tau_{lm} = \mu_{lm} \frac{\partial U}{\partial y} \bigg|_{y=0} = 0, \quad (\text{B.8})$$

and we require that both the horizontal velocity and stress are continuous between the layers. Within a given layer n , with constant viscosity μ_n , the momentum equation has the solution of the general form

$$U_n(y) = U_{n+1} + \frac{y - y_{n+1}}{6\mu_n} [6\tau_{n+1} + (y - y_{n+1})(3b + Ay + 2Ay_{n+1})], \quad (\text{B.9})$$

where quantities with the subscript “ $n + 1$ ” denote quantities evaluated at the base of the layer above, and as such, y_{n+1} is height of the boundary between layer n and the layer above. The velocity, U_{n0} , and stress τ_{n0} at the base of each layer can be calculated from equation B.9 with y_{n0} the height at the base of layer, with

$$U_{n0}(y = y_{n0}) = U_{n+1} + \frac{y_{n0} - y_{n+1}}{6\mu_n} [6\tau_{n+1} + (y_{n0} - y_{n+1})(3b + Ay_{n0} + 2Ay_{n+1})], \quad (\text{B.10})$$

$$\tau_{n0}(y = y_{n0}) = \tau_{n+1} - \frac{1}{2}(y_{n+1} - y_{n0}) [2b + A(y_{n0} + y_{n+1})]. \quad (\text{B.11})$$

We note that the evaluation of U_n and τ_n is not restricted to the boundaries between the layer and adjacent layers, but may be determined continuously within each layer. We now present the streamlined equations for each layer, with all left-hand side quantities evaluated at the base of their associated layers. These are

$$U_a = U_p - \frac{d_a}{6\mu_a} \{6\tau_p - d_a [3b + A(3d_m + 3d_{lm} + 2d_a)]\}, \quad (\text{B.12})$$

$$\tau_a = \tau_p - \frac{1}{2}d_a [2b + A(2d_m + 2d_{lm} + d_a)], \quad (\text{B.13})$$

for the velocity and stress at the base of the asthenosphere,

$$U_m = U_a - \frac{d_m}{6\mu_m} \{6\tau_a - d_m [3b + A(3d_{lm} + 2d_m)]\}, \quad (\text{B.14})$$

$$\tau_m = \tau_a - \frac{1}{2}d_m [2b + A(2d_{lm} + d_m)], \quad (\text{B.15})$$

at the base of the mantle, and

$$U_{lm} = U_m - \frac{d_{lm}}{(6\mu_{lm})} [6\tau_m - d_{lm}(3b + 2Ad_{lm})], \quad (\text{B.16})$$

$$0 = \tau_m - \frac{1}{2}d_{lm}(2b + Ad_{lm}), \quad (\text{B.17})$$

at the base of the base of the lowermost mantle (i.e., at the CMB). Conservation of mass requires that the integrated lateral velocity over the entire mantle is zero, yielding

$$U_p(d_l + d_a) + U_a d_m + U_m d_{lm} - \frac{d_a^2}{2\mu_a}\tau_p - \frac{d_{lm}^2}{2\mu_{lm}}\tau_m - \frac{d_m^2}{2\mu_m}\tau_a + \\ \left(\frac{d_a^4}{8\mu_a} + \frac{d_a^3 d_{lm}}{6\mu_a} + \frac{d_a^3 d_m}{6\mu_a} + \frac{d_{lm}^4}{8\mu_{lm}} + \frac{d_{lm} d_m^3}{6\mu_m} + \frac{d_m^4}{8\mu_m} \right) A + \\ \left(\frac{d_a^3}{\mu_a} + \frac{d_m^3}{\mu_m} + \frac{d_{lm}^3}{\mu_{lm}} \right) \frac{b}{6} = 0. \quad (\text{B.18})$$

Finally, to close the system, we re-arrange the lithospheric mechanical energy equation to

$$\tau_p = \frac{\alpha g d_l}{C_{P,m}} \frac{\langle Q \rangle_L}{LU_p} - d_l (d_{lm} + d_m + d_a) A - d_l b - \frac{d_l}{L} \tau_R \quad (\text{B.19})$$

The set of Eqs B.12–B.19 can be solved for the unknown variables U_a , U_m, U_{lm} , τ_p , τ_a , τ_m , A , and b in terms of U_p . These are solved for numerically within the whole mantle mechanical energy equation. As noted by Fuentes et al. (2019), the dissipation equation in COC12 requires a minor correction and is

$$\Phi_n = \frac{L(y_{n+1} - y_n)}{2\mu_n} \left\{ \tau_n^2 + \frac{\tau_{n+1}(y_n - y_{n+1})}{3} [3b + A(y_n + 2y_{n+1})] + \right. \\ \left. \frac{(y_0 - y_1)^2}{60} [20b^2 + 5Ab(3y_0 + 5y_1)] + \frac{(y_0 - y_1)^2}{60} A^2 (3y_0^2 + 9y_0 y_1 + 8y_1^2) \right\}. \quad (\text{B.20})$$

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CHAPTER 3

The Distribution of Water in the Mantle and its Effect on Tectonic Transitions

3.1 Introduction

With the discovery that water in the structure of olivine significantly weakens its rheology (Karato, 1986; Hirth and Kohlstedt, 1995), hypotheses on the role of water in mantle convection (Schubert, 1997; Dixon et al., 2004) and plate tectonics (Mian and Tozer, 1990) emerged. A number of studies have sought to quantify the role of water in the long term evolution of planetary mantles (Sandu et al., 2011; Crowley et al., 2011; Chotalia et al., 2020; Seales and Lenardic, 2020). For Earth, these efforts are limited by significant uncertainties in the present-day mantle water content, with estimates ranging from 2 to 10 ocean mass equivalents (e.g. Dong et al., 2021; Ni et al., 2017, and references therein). Further, while the effect of water on olivine flow laws is relatively well established (Karato et al., 1986; Hirth and Kohlstedt, 1995; Korenaga and Karato, 2008), the rheological impact of water on other dominant mantle phases such as wadsleyite, ringwoodite, and bridgmanite remains poorly constrained (Karato, 2010; Karato et al., 2025; Muir and Brodholt, 2018).

By and large, evolution models use scaling relationships to evaluate the cooling history of a planet. Such models rely on averaged mantle properties through a Rayleigh number-dependent relationship for surface heat flux:

$$\text{Nu} = \frac{q_{\text{surf}}}{(k\Delta T/d)} = \left(\frac{\text{Ra}}{\text{Ra}_{\text{crt}}} \right)^{\beta}, \quad (3.1)$$

where Nu is the Nusselt number, a non-dimensional heat flux defined as the ratio of the convective heat flux (q_{surf}) normalized by the conductive heat flux through the entire domain in the absence of convection $k\Delta T/d$, where k is the thermal conductivity, ΔT is the temperature difference between the interior temperature and the surface, and d is thickness of the mantle. Ra is the mantle Rayleigh number, which measures the vigor of convection and given by Eq. 3.2 below. Ra_{crt} is the critical Rayleigh number at which the onset of convection occurs, with $\text{Ra}_{\text{crt}} \sim \mathcal{O}(10^3)$. β is the important scaling parameter that describes the relationship between convection and cooling, and its value depends on the strength of the surface plates (or rather the relative contribution of dissipation in the plate compared to dissipation within the mantle) (Christensen, 1985; Korenaga, 2003). The thermal Rayleigh number is defined in terms of interior properties of the mantle as

$$\text{Ra} = \frac{\alpha \rho g \Delta T d^3}{\mu \kappa}, \quad (3.2)$$

where α is the thermal expansion coefficient, ρ is the density, μ is the viscosity and κ is the thermal diffusivity. In the simplest case, one can assume that all these properties are constant with time and evaluate the surface heat flux using Eq. 3.1 above, and thus the mantle cooling history. However, the strong dependence of viscosity on temperature does not permit this simplification, and instead it is common to use temperature-dependent viscosity of the form (or a variation of it)

$$\mu(T) = \mu_0 \exp\left(\frac{E}{R_g T}\right), \quad (3.3)$$

where E is the activation energy, R_g is the universal gas constant, and μ_0 is a reference viscosity. The role of water in the aforementioned studies enters into mantle evolution through its effect on viscosity with the following modification

$$\mu(T) = \mu_0 f(\chi) \exp\left(\frac{E}{R_g T}\right), \quad (3.4)$$

where $f(\chi)$ characterizes water weakening, a function of χ , the mantle water content. Water cycling between surface and mantle occurs via melting/outgassing and subduction/regassing. Although these fluxes are difficult to constrain through time, they can be bounded by topographic data (Parai and Mukhopadhyay, 2012; Korenaga et al., 2017; Chotalia et al., 2023) and geochemical tracers such as noble gases (Parai and Mukhopadhyay, 2018). Due to these minimal constraints on long-term water exchange, paired with the use of scaling laws, lack of studies on the effect of water on the rheology of many mantle minerals, most previous studies consider the bulk mantle water content and use known olivine rheologies to infer planetary cooling.

However, it has been long recognized that the viscosity *structure* of the mantle, rather than just its representative average viscosity, plays an important role in mantle convection and the tectonic style. The so-called “Frank-Kamenetskii” or “ θ ” parameter (in early literature labeled p , Solomatov, 1995) quantifies temperature-dependent viscosity contrasts and, alongside Ra , governs tectonic regime behavior. This approach is similar to the “strong plate” hypothesis, where cold and strong plates result in a surface dissipation that slows cooling and modifies scaling laws (e.g., $\beta < 0.3$) to fit petrological constraints (Korenaga, 2003). Additionally, low-viscosity zones (LVZs) can dramatically alter mantle convection planforms (Richards et al., 2001; Lenardic et al., 2006; Weller and Lenardic, 2012; Crowley and O’Connell, 2012). Models lacking LVZs may misrepresent mantle

dynamics and cooling paths (Al Asad and Lau, 2024).

Present-day geodetic estimates of mantle viscosity support the existence of an LVZ below the lithosphere (Mitrovica and Forte, 2004; Lau et al., 2016). Proposed mechanisms for LVZ formation and its relationship to the seismic low velocity zone include partial melting, increased temperature (Hua et al., 2023), and water weakening (see, e.g, Karato, 2012). These factors all influence upper mantle rheology, particularly that of olivine, as shown in deformation experiments. In addition, grain-size evolution models (e.g., Dannberg et al., 2017) demonstrate that including such processes increases lateral viscosity heterogeneity, affecting slab and plume dynamics.

In this study, we investigate how mantle viscosity structure and convective regime evolve in response to the distribution of water and melt. We extend prior work by resolving the vertical distribution of water in the mantle over time, rather than assuming an average bulk value. To do this, we develop a self-consistent convection model that incorporates water cycling (melting, regassing) and its effects on rheology.

Unlike classical approaches, our model does not assume a scaling-law-based cooling history (i.e., we do not prescribe β). Instead, we dynamically couple the thermo-mechanical evolution of the plate and mantle system to water redistribution, allowing us to test the role of water in sustaining mobile-lid tectonics. We describe our modeling framework and tested parameter space in Section 3.2, and analyze water’s influence on convection and feedback mechanisms in Section 3.3.

3.2 Model Description

There are three fully coupled components in the model presented here: (1) the thermo-mechanical evolution of the mantle, (2) the thermal evolution of the core, and (3) the evolution of water within the mantle. The details of the first two components are described in Al Asad and Lau (2024) and we point the reader to that study for a comprehensive

description of components (1) and (2). For the purposes of this study, the core model serves simply as a lower boundary condition for the mantle. For brevity, every time we discuss use of the thermo-mechanical mantle convection model, we will refer to this as the AL24-MC model.

In the following, we describe how AL24-MC has been modified to include the effects of melt and water. The thermal evolution of the model is captured via the mantle potential temperature, T_p , according to the thermal energy balance which includes: radiogenic heating (Q_{rad} assumed to be evenly distributed throughout the mantle), conductive heat flux from the core (Q_{cmb}), convective heat flux to the surface (Q_{surf}), and the new addition in this model, latent and sensible cooling of melt (Q_{melt}). The rate of change of the mantle’s internal temperature can then be expressed as:

$$\rho C_p \eta_m \frac{dT_p}{dt} = Q_{\text{rad}} + Q_{\text{cmb}} - Q_{\text{surf}} - Q_{\text{melt}} \quad (3.5)$$

where ρ is the average mantle density, C_p is the volumetric heat capacity, and η_m is scalar that relates the potential temperature to the average mantle temperature. The various heat flow terms, with the exception of the melt-related heat term, are as described in AL24-MC. We describe the calculation of Q_{melt} in Section 3.2.1, but note here that instead of using a prescribed intrusive/extrusive volcanism ratio, the amount of extruded volcanism is dynamically determined based on the concept of the melt focusing distance (Katz, 2008; Keller et al., 2017).

We divide the mantle into five layers that represent important mechanical and mineralogical boundaries (lithosphere, upper mantle/asthenosphere, mantle transition zone, lower mantle and lowermost mantle). In the following, the mantle layers correspond to the following rheology-defining minerals, captured below by variable $\mathcal{C}$: olivine for the asthenosphere, wadsleyite for the mantle transition zone, and a bridgmanite for the lower and lowermost mantle. The thickness of each of these layers is determined dynamically

at each time step in our model. For the lithospheric thickness, we use the half-space cooling model, that is, the lithosphere is defined thermally. For the transition depth between olivine and wadsleyite, we use the relationship in Katsura et al. (2004), and keep the ringwoodite-bridgmanite transition constant at 660 km, making it not temperature-dependent (see, e.g., Ishii et al., 2018). Finally, the thickness of the lowermost mantle, or the CMB thermal boundary layer, is calculated based on the critical Rayleigh number for the onset of convection at those conditions.

For the lithosphere, we will adopt a rheology-agnostic way of calculating dissipation due to the bending of the subducting plate (Crowley and O’Connell, 2012). The viscosities of the rest of the layers are calculated as the vertical-average based on a 1-D profile of pressure P , temperature T , water content χ and retained melt fraction ϕ (all of which vary with depth, z), according to

$$\mu(\mathcal{C}, T, P, \chi, \phi) = A(\mathcal{C}) f_{\text{H}_2\text{O}}^{-r(\mathcal{C})}(\chi) \exp\left(\frac{E^*(\mathcal{C}) + PV^*(\mathcal{C})}{R_g T}\right) \exp(-\alpha\phi) \quad (3.6)$$

where μ is the viscosity, and for the given composition $\mathcal{C}$, A is its viscosity prefactor, E^* is its activation energy, and V^* is its activation volume. Hereafter, we will assume implicit dependency on $\mathcal{C}$ to avoid notational clutter, unless required for clarity. We will, however, make explicit dependence on all evolution (i.e., time-dependent) variables. R_g the universal gas constant, α is the melt-dependence exponent (Hirth and Kohlstedt, 2003), ϕ is the melt fraction retained in the mantle, $f_{\text{H}_2\text{O}}(\chi)$ is the water fugacity and its superscript $r(\mathcal{C})$ is the water fugacity exponent.

The form of the water-dependent rheology function is taken from Mei and Kohlstedt (2000). However, as noted by Li et al. (2008), for geodynamic models, one tracks the water content (χ) rather than its fugacity, which is difficult to determine. Therefore, we use the relationship in Li et al. (2008) for the conversion between water content and fugacity

$$\ln f_{\text{H}_2\text{O}} = c_0 + c_1 \ln C_{\text{OH}} + c_2 \ln^2 C_{\text{OH}} + c_3 \ln^3 C_{\text{OH}}, \quad (3.7)$$

where C_{OH} is the molar hydroxyl concentration (i.e., H/10⁶ Si) of the rheologically-important (and water-dependent) mineral, calculated such that a water content of $\chi = 1$ ppm H₂O by weight in the mantle is equivalent to $C_{\text{OH}} = 6.3 \text{ H}/10^6 \text{ Si}$ for olivine (Hirth and Kohlstedt, 1996). The constants c_n are listed in Li et al. (2008).

A Note On the Rheology of Mantle Mineral Assemblages

As we will show in the results below, the relative viscosity of the different layers is the main control on tectonic styles in our models. Therefore, it is worth discussing the rheological laws that we use in the model and how they were chosen.

Olivine flow laws have been extensively studied in lab experiments and it has been found to deform by both diffusion and dislocation creep mechanisms at upper mantle conditions. Because olivine is both the volumetrically dominant and weaker component of the upper mantle, its deformation behavior dominates the upper mantle rheology. In the diffusion regime, grain size strongly affects viscosity through a power-law relationship, and given the broad temperature range in our model, not accounting for grain-size evolution may be a notable simplification. However, tracking the evolution of the grain size requires a Lagrangian approach to a mantle parcel (Dannberg et al., 2017; Okamoto and Hiraga, 2024) that is not possible in our model set-up. Dislocation creep, on the other hand, is not dependent on grain size but is characterized by a non-linear rheology law. In our formulation, we model only the diffusion creep mechanism for the entire mantle, which is another notable simplification. However, it was noted by Hirth and Kohlstedt (2003) that the use of a linear rheology law (i.e., one with no stress dependence) simulates the non-linear dislocation creep rheology if one uses $E_{\text{eff}}^* = E_{\text{exp}}^*/2$. As for the water effect, experiments on olivine found that the presence of water causes considerable weakening (Karato et al., 1986) with values of r ranging from 0.7 to 1.2. To summarize, we use a

Newtonian viscosity model for olivine (no stress-dependence), with reduced E^* value and high degree of weakening by water, shown in Table 3.1.

For the flow laws within the mantle transition zone (which we assume to be made of wadsleyite given the limited experimental results for bridgmanite), we use the values reported in Dannberg et al. (2017), based on the work of Shimojuku et al. (2004, 2009). Shimojuku et al. (2004, 2009) are both Si self-diffusion, rather than deformation, experiments. However, experimental studies of the effect of water on the rheology of wadsleyite have not been as systematically conducted to the same extent as those on olivine. Instead, we use the results of Si self-diffusion experiments of Druzhbin et al. (2022) which finds an r value of 0.8 ± 0.1 . However, we note that self-diffusion measurements do not agree with the results of deformation experiments of olivine (Fei et al., 2013), which may indicate missing physical diffusion processes in Si self-diffusion experiments. Nevertheless, due to the lack of deformation experiments that consistently investigate the effect of water on the rheology of wadsleyite, we use the results of Druzhbin et al. (2022).

The rheology of the lower mantle is yet harder to investigate experimentally, with the dominant deformation mechanism(s) still actively debated, with diffusion creep, dislocation creep (Karato et al., 2025), and pure climb (Cordier et al., 2023) being the main candidates. With little agreement, our assumption of a diffusion-creep-dominant deformation mechanism is consistent also with the lack of seismic anisotropy (Long and Silver, 2009). As for the effect of water on the rheology of bridgmanite (the main lower mantle mineral), the *ab initio* calculations of Muir and Brodholt (2018) indicate no water dependence ($r = 0$). However, it was noted that while bridgmanite is the volumetrically dominant mineral in the lower mantle, it is likely stronger than ferropericlase, which can lead to stress-localization in the lower mantle and result in a deformation behavior that is different from pure bridgmanite rheology (Karato et al., 2025).

Finally, we scale the viscosity parameters so that they results in reasonable present-day estimates of mantle rheology using the geodetically-constrained 1-D profile of Lau et al.

(2016). This requires an assumption about the present-day mantle water content (in the transition zone and upper mantle), and given the sensitivity of the rheology to the water content, we note that this choice will affect the behavior of the model. We assume that the present-day mantle water content in the upper mantle is 100 wt ppm (Ni et al., 2017, and references therein), and in the transition zone 10000 ppm (Pearson et al., 2014). The rheology parameters used in our model are listed in Table 3.1.

Table 3.1: Rheological parameters for major mantle phases used in this study

Parameter	Olivine	Wadsleyite	Bridgmanite
Water fugacity exponent r	1.0–1.2	0.8 ± 0.1	0
Activation energy E^* (kJ mol ⁻¹)	375	231	299
Activation volume V^* (cm ³ mol ⁻¹)	2.24	4.48	2.91
Prefactor A (Pa ⁻¹ s ⁻¹)	2.24×10^{-8}	4.48×10^{-12}	2.91×10^{-13}
Melt parameter α	30	30	30

The Water Cycle

We track the water content in the mantle as a function of depth but not in the hydrosphere (i.e., in Earth’s oceans and atmosphere). While the volume of water and the extent of land exposure on the surface are important constraints on the degassing and regassing of the mantle (e.g., Parai and Mukhopadhyay, 2012; Korenaga et al., 2017; Chotalia et al., 2023), it is not the present focus of this study but an extension to be made in future work.

In general, water is believed to cycle through the mantle by the following processes (also illustrated schematically in Fig. 3.1):

1. Hydrous melting at mid-ocean ridges (MORs), mantle plumes and subduction zones preferentially partitions water into the melt phase, leaving the residual mantle dry. Some residual melt is retained in the mantle which can affect its viscosity.
2. Oceanic crust and lithosphere are hydrated as they travel from MORs to subduction zones through hydrothermal alteration and deep faulting. Eventually, they subduct

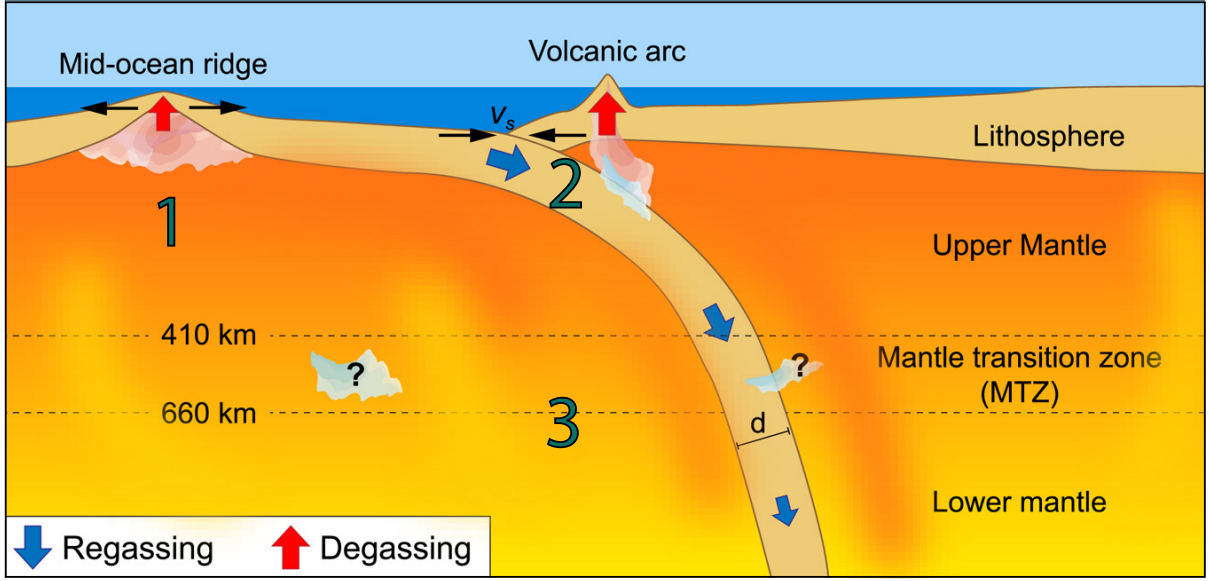


Figure 3.1: **Deep mantle water cycle and processes that redistribute water:** Processes labeled with numbers correspond to the ones listed in the text: (1) melting and outgassing at the MOR, (2) hydration of the lithosphere and regassing of the water into the mantle, and, (3) convective mixing within the mantle. The figure depicts the top ~ 1000 km of the mantle. Modified from Wang et al. (2025)

water-carrying minerals in the slab. At certain pressure-temperature conditions, such minerals destabilize, releasing their water content causing arc volcanism and outgassing at shallow depths (few hundred km), or hydration of the surrounding mantle at deeper depths.

3. Water within the mantle is redistributed as a consequence of mantle convection.

In the next sections, we detail how each of these processes are represented in our model.

3.2.1 Hydrated Melting at the MOR and Formation of the Oceanic Crust

Hydrated Melting

Melting at the MOR is due to the decompression of upwelling mantle. The water content of the solid mantle can reduce the solidus causing melt to start deeper. In this study, we apply the hydrated melt model of Katz et al. (2003) because it captures the

essential characteristics of hydrous melting relevant for our geodynamic considerations, which we will briefly summarize below (but we point the reader to the original study for further detail).

The main elements of their model are that the melt fraction F is a function of pressure, temperature, water content and composition (specifically, the relative amount of clino- and orthopyroxene). Water depresses the solidus and liquidus according to a power law, increasing the melt fraction and depth of melting compared to the equivalent anhydrous scenario. The upwelling mantle goes through adiabatic decompression melting based on the difference between the geotherm and the water-modified solidus-liquidus. Once melting occurs, water partitions into the melt phase using a constant partitioning coefficient and a pressure-dependent water saturation curve.

As the mantle upwells, we assume that the rate of melt production is equal to the rate of melt extraction, i.e the melting process is steady-state (Jha et al., 1994; Braun et al., 2000) in each upwelling column (or distance x away from the ridge axis). We use Darcy's flow law for melt migration, where

$$\frac{K\Delta\rho g}{\eta} = \int \dot{M} dz, \quad (3.8)$$

along with the Kozeny-Carmen relationship between (small) melt fraction, permeability K , and retained melt fraction ϕ

$$K = K_0 \bar{\phi}^n, \quad (3.9)$$

to calculate the column-averaged retained melt fraction $\bar{\phi}$. In the above equations $\Delta\rho$ is the difference between solid and melt densities, g is the gravitational acceleration, η is the melt viscosity and $\dot{M}$ is the melt production rate. For simplicity, we assume that melt density and viscosity are constant, while in reality, both of these quantities are functions

of melt composition and temperature. We do not track melt composition in our model.

Eq. 3.8 states that the Darcy velocity, which represents the rate of melt migration (left-hand side of the equation), is balanced by the depth-averaged melt production rate. In Eq. 3.9, K_0 is a constant which depends on the grain size and the geometry of the pore space, and n is a power law exponent, usually taken to be 2 or 3 (Bear, 2013). The melt production rate, $\dot{M}$ is given by

$$\dot{M}(x, z) = V(x, z) \frac{dF}{dz}. \quad (3.10)$$

Here, $V(x, z)$ is the vertical velocity field, which is determined based on the 1-D horizontal velocity profile (output by AL24-MC), the assumed upwelling region, and a local conservation of mass criteria, which we describe in Appendix C.1. As previously mentioned, $F(z)$ is the degree of partial melting, calculated here based on the hydrous melting model of Katz et al. (2003) and simplified decompression-related melt productivity from McKenzie (1984) (specifically Appendix A in the reference). For the solidus curve beyond 8 GPa (the pressure limit of Katz et al., 2003), we use the results of Pierru et al. (2022) for a pyrolitic composition and modify the curve slightly to smoothly merge it with the shallow mantle curve presented in Katz et al. (2003).

Due to the model set-up of AL24-MC, upwelling occurs only at one end of the rectangular cell, i.e. at the MOR. The upwelling mantle in that region represents both what is commonly referred to as “passively” upwelling mantle due to plate divergence, as well as active upwelling from mantle convection. The term “passive upwelling” is normally used as a description of plate-driven tectonics, or kinematically when $|U| \approx |V|$, where $|U|$ is the magnitude of the plate velocity and $|V|$ is the magnitude of the upwelling velocity. Although we do not prefer to use this term, we use it to note the relationship between our model geometry and some plate-driven convection models. Under certain thermo-mechanical conditions (e.g., under hot mantle and high CMB heat flux conditions,

and in the presence of a weak asthenosphere) active upwelling can lead to $|V| > |U|$. Therefore, melting in our model can be thought of as the combined amount due to passive upwelling as well as the active mantle.

Melt and Water Extraction

When melting in the mantle occurs, water in the hydrated mantle partitions into the melt phase, and when melt is extracted through volcanism at the MOR, the residual mantle is dehydrated compared to the source mantle. However, not all melt produced in the upwelling region is extracted at the MOR. The horizontal distance over which melt laterally migrates to the MOR is known as the melt-focusing distance, and is the subject of a number of multiphase flow studies (e.g. Katz, 2008; Keller et al., 2017), typically performed within the framework of active-lid tectonics (and therefore passive upwelling scenarios). Understanding how melt is focused in the sluggish-lid regime, or generally in the presence of a fast-flow region below the plate, is also relevant for the model we are developing here. Without better understanding of lateral melt migration under the conditions of interest here (i.e., for sluggish-lid tectonics), we turn to the empirical relationship in Katz (2008) which relates the compaction length, δ_c , to the melt-focusing distance, x_{mfd} using $x_{\text{mfd}} \text{ (m)} = 6000 \text{ m} + 2.7 \times \delta_c$. The compaction length is given by (McKenzie, 1984; Katz, 2008)

$$\delta_c = \sqrt{K\zeta/\eta} \approx \left(\frac{\zeta_R \phi_0}{\bar{\phi}} \right)^{1/2} \sqrt{K\mu/\eta}, \quad (3.11)$$

where ζ is the matrix bulk viscosity, ζ_R is the bulk-to-shear viscosity ratio of the matrix at the reference porosity ϕ_0 with $\zeta = \zeta_R \mu \frac{\phi_0}{\bar{\phi}}$. Once more, η is the melt viscosity and μ is the matrix (solid) viscosity, and we note that our symbols for melt and solid viscosity are flipped compared to the equations of McKenzie (1984).

The mass of melt, M^{melt} , that is extracted from the mantle (per unit length of ridge

segment) in a given time interval Δt is then given by the mean melt fraction multiplied by the mass of mantle that upwells through the melt extraction zone

$$M^{\text{melt}} = (\bar{F} - \bar{\phi}) \rho_{\text{m}} \Delta t \int_{z_{\text{start}}}^{z_{\text{end}}} \int_0^{x_{\text{mfd}}} V(x, z) dx dz \quad (3.12)$$

where $\bar{F}$ is the mean melt fraction, given by

$$\bar{F} = \frac{1}{z_{\text{start}} - z_{\text{end}}} \int_{z_{\text{start}}}^{z_{\text{end}}} F(z) dz, \quad (3.13)$$

and $\bar{\phi}$ is the depth-averaged retained melt fraction calculated using Eqs 3.8–3.10. The difference $(\bar{F} - \bar{\phi})$ represents the fraction of melt extracted from the mantle melt zone. The depths z_{start} and z_{end} denote where melt in the mantle starts and ends, respectively, and are determined from the difference between the geotherm and solidus. The total mass of water lost from the mantle is given by

$$M^{\text{water}} = M^{\text{melt}} \bar{\chi}^{\text{melt}}, \quad (3.14)$$

where $\bar{\chi}^{\text{melt}}$ is the mean water content of the melt. By conservation of mass, the thickness of the crust d_{c} produced by this melt is

$$d_{\text{c}} = \frac{M^{\text{melt}}}{\rho_{\text{c}} \Delta t u_{\text{p}}}, \quad (3.15)$$

where ρ_{c} is the density of the crust and u_{p} is the plate velocity, output by AL24-MC. If the produced crust is thicker than the eclogite transition pressure limit, then we assume that the eclogitic crust and its water content are returned to the mantle and mixed into the mantle transition zone, where eclogite becomes neutrally buoyant.

3.2.2 Lithospheric Hydration and Mantle Regassing

We adopt a simplified model for the hydration of the crust and lithosphere as they travel from the MOR to the subduction zone and, upon subduction, we track the water released from the subducting slabs into the mantle. This simplified treatment does not capture many of the elements that influence the dynamics of subducting slabs, mantle wedge, and flux volcanism. Rather, we endeavor to capture some first-order effects of the thermo-mechanical convection state of the mantle on the deep water cycle. We consider each of these broad processes (lithospheric hydration and mantle regassing) below.

Lithospheric Hydration

As a simplifying measure, we assume that 50% of the melt water content is lost to the hydrosphere (Anderson et al., 2025), and the rest is retained in the crust. As the plate continues to travel from the MOR to the subduction zone, it is hydrated through interactions with the hydrosphere via three main mechanisms: (1) hydrothermal alteration, (2) deep thermal fracturing (Zhang et al., 2023), and (3) the deposition of hydrated sediments with water-filled pore spaces. We only include the first two mechanisms in our model because we assume pore fluids are squeezed out at shallow depths (a similar assumption was made by Van Keken et al., 2011) and the sediments are either accreted onto the overlying plate or destabilized at shallow depths in the mantle, and therefore do not contribute to the regassing of the mantle.

Hydrothermal alteration can cause significant hydration of the shallow oceanic crust near the ridge system due to the interaction of seawater and hot basalt, which, through metasomatism, precipitates hydrous silicate minerals (Kleine et al., 2020). The process is generally limited to the upper crust because increases in temperature and pressure within the deeper parts of the crust result in dehydration reactions. In reality, analysis of ocean drilling cores and theoretical models of hydrothermal alterations show that the extent of the alterations as well as the resulting minerals depend on ocean chemistry, the

composition of the crust, and temperature. Thus, our model, as presented here, does not represent these processes in detail. However, using observations of hydrothermal systems, we can parameterize the hydration state of the upper crust due to hydrothermal alteration reactions. We use a parametrization that captures the dependence on porosity/permeability of the upper oceanic crust, which controls water/rock ratio and thus plays an important role in the relevant reactions, and the time the crust spends interacting with the water. The permeability of the ridge system is found to be inversely proportional to the spreading rate (Jellinek et al., 2020), leading to the following parameterization

$$\chi_p^{\text{ht}}(z) = \chi_{\text{surf}}^{\text{ht}}(u_p) \exp\left(-\frac{z}{z_c^{\text{ht}}}\right), \quad (3.16)$$

where χ_p^{ht} is the water content in the crust due to hydrothermal alteration, and z_c^{ht} is the penetration depth for hydrothermal alteration. In this relationship, the exponentials capture the effect of depth-dependent decrease in permeability. Finally, $\chi_{\text{surf}}^{\text{ht}}$ is the surface hydration rate, which is a function of plate velocity because fracture density (or topographic roughness) has been found to inversely correlate with plate velocity (Jellinek et al., 2020), i.e. $\chi_{\text{surf}}^{\text{ht}} = \chi_{\text{surf}}(u_p/u_{\text{ref}})^{-1}$, where χ_{surf} is constant and u_{ref} is some reference velocity to be fit. Similarly, hydration due to deep thermal fracturing – which involves hydration reactions that may occur away from the hydrothermal system and may reach high pressures and temperatures – is represented by

$$\chi_p^{\text{frac}}(z) = \chi_{\text{surf}}^{\text{frac}}(u_p) \exp\left(-\frac{z}{z_c^{\text{frac}}}\right). \quad (3.17)$$

As with fracture density, we assume the fault density is also inversely proportional to the plate velocity based on the relationship between transform fault spacing and spreading rate (e.g. Sandwell, 1986; Ren et al., 2022).

Therefore, the total amount of water in the subducted slab is then given by

$$W_p = 0.5\bar{\chi}^{\text{melt}}d_c + \int_a^0 [\chi_p^{\text{ht}}(z) + \chi_p^{\text{frac}}(z) + \chi_s(z)] dz, \quad (3.18)$$

where $a = d_l + d_c$ is the total thickness of the slab (mantle lithosphere + crust) and χ_s is the mantle water content calculated using the hydrous melting model as explained in the previous section.

Mantle Regassing

The water contained within the oceanic slab as it reaches the subduction zone is recycled into the interior through subduction and downwellings. We ignore the water contained within sediment pore space because it is usually extracted from the slab at shallow depths and efficiently outgassed to the hydrosphere (Ni et al., 2017). Our parameterization for water flux out of the subducted slab and into the mantle depends on five factors: (1) the initial slab water concentration upon reaching the subduction zone (using calculations described in the previous section); (2) the mantle potential temperature T_m^p , where higher T_m^p results in more slab dehydration; (3) the decrease of water flux with depth due to the reduction of water remaining within the slab; (4) the time the slab spends at any specific depth, which is inversely proportional to the subduction/downwelling velocity v_{sub} ; and finally, (5) the partitioning coefficient of water between the slab minerals (basalt/olivine) and the surrounding mantle, $D_{\text{OH}}^{\text{“slab”}-\text{mineral}}$.

To capture these effects, the water flux into the surrounding mantle from the subducted slab is parameterized using

$$\frac{dW}{dz} = \tau_{\text{deh}} D_{\text{OH}}(z) \left(\frac{1}{v_{\text{sub}}} \right) W(z), \quad (3.19)$$

where dW/dz is the water output from the slab as a function of depth, $W(z)$ is the water content in the subducted slab, such that $W(z = 0) = W_p$, given by Eq. 3.18 above. The speed of slab dehydration is characterized by the reaction timescale τ_{deh} . Eq. 3.19 broadly

captures the characteristics that have been found by in-depth studies of slab dehydration (e.g. Van Keken et al., 2011).

3.2.3 Mantle Mixing

The final process that can change the water distribution in the mantle is convection-related mixing. We calculate the mantle overturn time using the average horizontal velocity magnitude of the mantle, $\langle u(z) \rangle$, and the vertical velocity at the upwelling region, v_{upwell} , which are each calculated at every time step using the AL24-MC model. With this, the overturn time τ^{ot} is

$$\tau^{\text{ot}} = 2L / \langle u \rangle + 2d / v_{\text{upwell}}, \quad (3.20)$$

where L is the width of the convection cell and d is the thickness of the mantle. We redistribute the water over the mantle based on the ratio of the time step in our evolution Δt and τ^{ot} according to

$$\chi(z)^{\text{mixed}} = \begin{cases} \chi(z)^{\text{unmixed}} [1 - \Delta t / (a\tau^{\text{ot}})] + \chi^{\text{avg}} [\Delta t / (a\tau^{\text{ot}})] & \Delta t < a\tau^{\text{ot}} \\ \chi^{\text{avg}} & \Delta t \geq a\tau^{\text{ot}}. \end{cases} \quad (3.21)$$

The use of the factor a when comparing the overturn timescale to the time step in our model is based on numerical studies on the relationship between convection and mixing, which find the value of $a \sim 40$ (Olson et al., 1984; Christensen, 1985). All the constant parameters used in this study are listed in Table 3.2.

Table 3.2: Model Constants and Parameters Used in This Study

Parameter	Description	Value / Expression
<i>General Physical Constants</i>		
ρ	Average mantle density	4400 kg m ⁻³
ρ_L	Lithospheric density	3300 kg m ⁻³
ρ_c	Density of crust	2900 kg m ⁻³
g	Gravitational acceleration	9.81 m s ⁻²
C_p	Heat capacity of mantle	1200 J kg ⁻¹ K ⁻¹
R_g	Universal gas constant	8.314 J mol ⁻¹ K ⁻¹
T_s	Surface temperature	273 K
η_m	Scaling factor for $T_{\text{pot}} \rightarrow T_{\text{avg}}$	~ 1.3
L	Convection cell width	5796 km
<i>Rheological & Flow Parameters</i>		
ϕ_0	Reference porosity	0.05
K_0	Permeability constant	10 ⁻⁶ m ²
n	Permeability exponent	2
η_{melt}	Melt viscosity	1 Pa s
ζ_R	Bulk-to-shear viscosity ratio	2.5
$\Delta\rho$	Solid-melt density difference	500 kg m ⁻³
<i>Mixing & Melting Parameters</i>		
a	Mixing efficiency factor	40
F	Degree of melting	Output of melt model
dF/dz	Melt production rate	$\sim -\partial F/\partial z$
T_{solidus}	Mantle solidus	Pierru et al. (2022)
T_{liquidus}	Mantle liquidus	Katz et al. (2003)
<i>Regassing & Hydration Parameters</i>		
χ_{surf}	Surface hydration scaling	5%
C_{OH}	Molar OH concentration from ppm	1/6.3 mol kg ⁻¹ ppm ⁻¹
u_{ref}	Reference plate velocity	5 cm year ⁻¹
z_c^{ht}	Hydrothermal alteration depth exponent	1500 m
z_c^{frac}	Thermal cracking depth exponent	10000 m
τ_{deh}	Dehydration timescale	5×10^{-12} s ⁻¹
<i>Hydroxyl Partitioning Coefficient</i>		
$D_{\text{OH}}^{\text{ol}}$	basalt-olivine	$10^{-9\gamma}$, where $\gamma = z/z_b^{\text{wds}\dagger}$
$D_{\text{OH}}^{\text{wd}}$	basalt-wadsleyite	0.5
$D_{\text{OH}}^{\text{br}}$	basalt-bridgmanite	0.005

[†] where z_b^{wds} is the depth of olivine-wadsleyite transition

3.2.4 Parameter Exploration

The model developed in this study involves a number of non-linearly coupled processes, and therefore the behavior of the system is difficult to predict. As noted, the main purpose of this study is to understand the effect of *water distribution* within the mantle on the dynamics of mantle convection. In particular, how this effect manifests as sluggish or active lid tectonics, and transitions in between. To this end, we run many iterations of the model varying four different initial conditions: mantle potential temperature (T_m^0), CMB temperature (T_{cmb}^0), total mass of water in the mantle (M_w^0), and the distribution of water in the mantle (dist^0), as well as one model parameter: the effective plate strength (τ_{yf}).

For the initial distribution of water within the mantle, we test three distribution scenarios: constant, exponentially increasing, and exponentially decreasing with radius. These scenarios are based on studies of magma ocean solidification (Elkins-Tanton, 2012; Boukaré et al., 2018) where we assume that the partitioning behavior of water-melt is similar to that of iron-melt. As such, the three scenarios indicate three differing hypotheses: Constant water fraction corresponds to early mantle convection during magma ocean solidification; an exponential increase of water fraction with depth corresponds to a late overturn scenario; and an exponential decrease of water fraction with depth corresponds to post-magma ocean solidification but pre-overturn distribution (Boukaré et al., 2018). Of course, the assumption of similarity between the distributions of iron and water neglects the volatile nature of water. It is possible that as water is partitioned into the liquid phase during magma ocean solidification, the interaction between the liquid phase and the atmosphere could lead to degassing of much of the water content within the liquid melt into the atmosphere, leaving a very dry mantle behind. However, the formation of an early proto-crust and a thick atmosphere on the surface may limit the degassing process and maintain a correlation between the distributions of iron and water (Bower et al., 2022). The behavior of water during accretion and magma ocean solidification is beyond the scope of this study, and we test these different scenarios as possible end

members for the distribution of water. We use Latin Hypercube Sampling (LHS McKay et al., 1979) to generate 2000 model realizations spanning the parameter ranges listed in Table 3.3. Compared to grid search, LHS achieves more efficient and uniform coverage of the multidimensional parameter space with fewer samples, and unlike standard Monte Carlo methods, LHS ensures that each parameter range is sampled evenly, reducing clustering and improving representation.

Table 3.3: Parameters explored in the model ensemble.

Parameter	Symbol	Units	Explored Range / Values
Initial mantle potential temperature	T_p^0	°C	1450 – 1750
Initial CMB temperature	T_{cmb}^0	°C	4227 – 5127
Effective plate yield stress	τ_{yf}	MPa	220 – 300
Initial total mantle water content	M_w^0	ocean mass (OM)	0.1 – 2
Initial water distribution	dist^0	–	Exponential decrease
			Uniform
			Exponential Increase

3.3 Results

3.3.1 Trends in Convective Evolution

To investigate the role of water redistribution in driving tectonic transitions, we begin by identifying dominant evolutionary trends across our model ensemble. Following AL24-MC, we use the mantle-to-plate velocity ratio $r_u = \langle |u(z)| \rangle / u_p$ as a diagnostic of tectonic style, where values closer to unity indicate more active-lid behavior. We analyze how this ratio evolves over time across the ensemble.

As a means to identify broad characteristics across our 2000 evolutions, we use dynamic time warping (Sakoe and Chiba, 2003), a clustering method designed to group together distinct characteristics present in the time evolution of our simulations. First, we cluster the results based on whether or not the time-evolution of the velocity ratio is smooth or exhibits abrupt/transient changes, as is observed in some evolution scenarios. To account for variability in the timing of these abrupt changes, we perform dynamic time warping

on each normalized time series and cluster them based on Euclidean distance. Iterative clustering is applied to the largest groups to isolate additional trends. In cases with ambiguous membership, we manually assign models to clusters based on proximity in feature space.

This results in three principal evolutionary patterns (Fig. 3.2): (a) persistent active-lid behavior, (b) secular transition from sluggish- to active-lid, and (c) transient, abrupt changes in convective regime. Each panel in Fig. 3.2 summarizes a cluster. The first column shows normalized velocity ratio trajectories; the second shows their ensemble mean and standard deviation. Subsequent columns display distributions of average velocity ratio $\langle r_u \rangle$, initial mantle potential temperature T_p^0 , total mantle water content M_w^0 , effective plate strength τ_{yf} , and initial water distribution (dist^0).

There are two primary model parameters that determine the behavior of the tectonic regime: the effective plate strength (τ_{yf} and the initial distribution of water (dist^0). To a lesser extent, the initial mantle temperature T_p^0 and total water content in the mantle M_w^0 also appear to influence the behavior.

Models in cluster (a), persistent active-lid, are characterized by cold, dry upper mantles or relatively weak plates. This cluster appears most commonly when water is initially concentrated in the lower mantle due to an exponential decrease with radius distribution and low total water content. With no mechanism to weaken sub-plate viscosity, these conditions favor early and persistent active-lid regime, reflected by consistently low velocity ratios. These cases ($\sim 30\%$ of the ensemble) are consistent with unchanging tectonic behavior and, as argued in Al Asad & Lau (2024), may not reflect Earth-like evolution.

Cluster (b), models with a secular transition from sluggish- to active-lid tectonics, contains scenarios where moderately strong plates and modestly hydrated upper mantles support a gradual transition toward active-lid tectonics. Some models in this group exhibit a gradual increase in sluggishness before settling into a steady decline in r_u . These secular

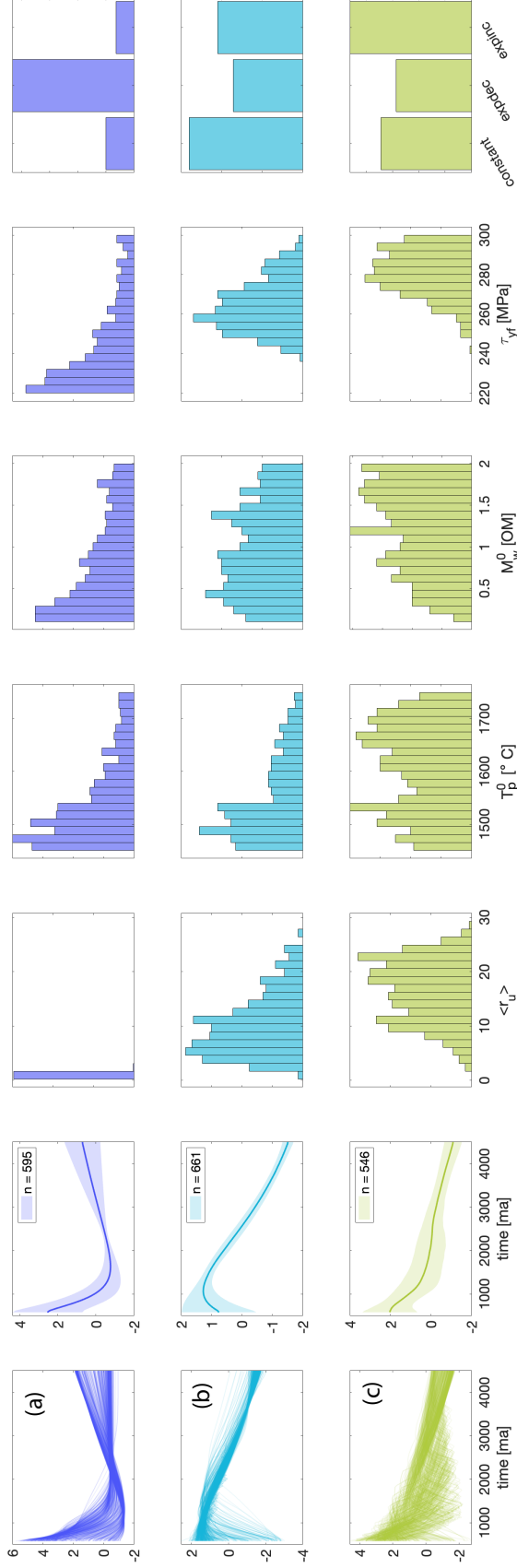


Figure 3.2: Trends in Tectonic Evolution: Clusters of velocity ratio evolution trends in our model results: Top row (a) represents the cluster with little change in the velocity ratio over time, as indicated by the low average velocity ratio histogram in the third column and its narrow spread. Middle row (b) represents the cluster showing a gradual decrease in the velocity ratio with time (although some period of increase may be present). Bottom row (c) is the cluster which exhibits abrupt changes to the velocity ratio with time. For each of these clusters, we show the normalized velocity ratio ensemble in the first column, the mean and standard deviation in the second column, the histogram of average velocity ratio throughout each member model evolution in the third column (r_u), the histogram of the initial mantle potential temperature T_p^0 in the fourth column, the initial total water content of the mantle M_w^0 in the fifth column, the effective plate strength τ_{yf} in the sixth column, and the initial water distribution $dist^0$ in the seventh column.

transitions reflect cooling-driven shifts in viscosity structure.

Cluster (c) comprises models with abrupt peaks and troughs in velocity ratio, indicative of transient excursions toward different tectonic regimes. These events do not correspond to permanent regime transitions but rather to failed or incomplete ones. For example, a sharp drop in r_u may signal a transient move toward active-lid convection, which is ultimately suppressed by internal feedbacks.

Though transient, these events can last hundreds of millions to billions of years and significantly influence the planet’s thermal evolution. In many cases, the system eventually converges towards an active-lid state. The typical trajectory involves: an early increase in r_u (increasing sluggishness), followed by a sharp slope change towards lower r_u (approaching active-lid), which may stabilize or reverse temporarily before resuming a secular decline. These dynamic behaviors are associated with intermediate to high plate strengths and illustrate the sensitivity of tectonic evolution to mantle viscosity structure. In the next section, we analyze the causes of these transient shifts and identify the parameters that control their onset and duration.

3.3.2 Transient Events in Convection Regime

The subset of models exhibiting transient behavior in the velocity ratio are predominantly those with high effective plate yield strength, as shown in Fig. 3.2c, column 6. Notably, the timing of these transient events – particularly the local minima (hereafter, “troughs”) – is positively correlated with τ_{yf} , with higher values leading to later occurrences (Fig. 3.3c). By contrast, neither the initial total mantle water content nor its radial distribution (Figs. 3.3b, d) appears to correlate strongly with event timing. However, as we demonstrate below, the time evolution of asthenospheric water content does show a clear association. Additionally, there is a weak negative correlation between the initial mantle temperature and event timing, such that hotter initial conditions lead to earlier transients.

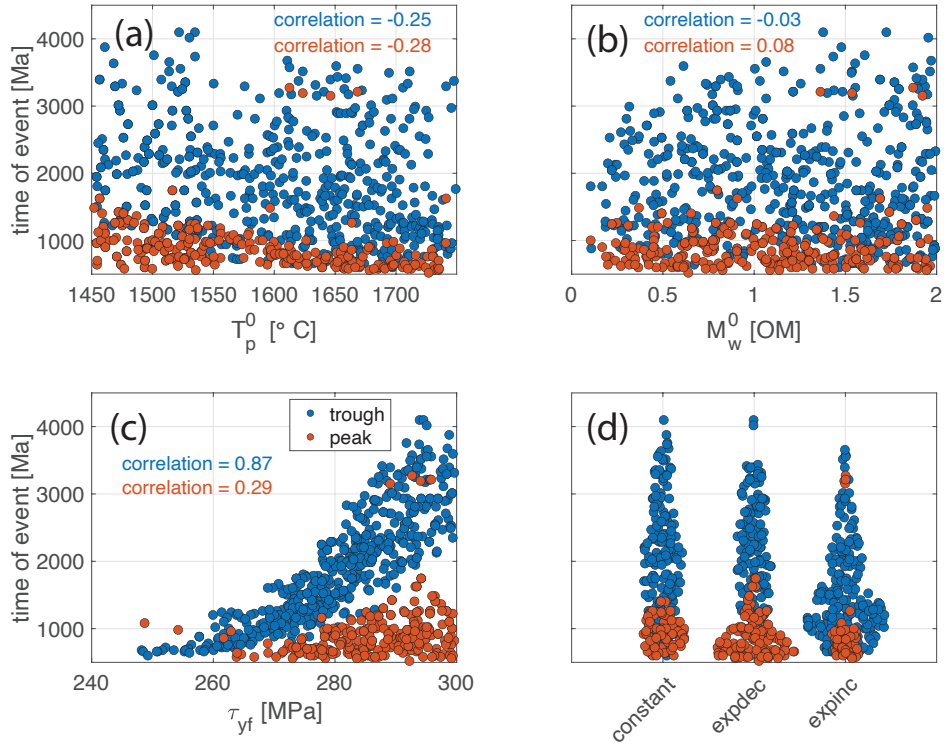


Figure 3.3: **Timing of transient events as a function of model parameters:** (a) Initial mantle potential temperature, (b) initial mantle total water content, (c) effective plate strength, and (d) initial water distribution. The blue markers represent the timing of the troughs, which are ubiquitous in this subset (cluster c in Fig. 3.2), and the orange markers represent the timing of the peak, which are present in some of the evolution models within cluster c.

Troughs correspond to a reversal in tectonic evolution, specifically marking a sudden shift away from active-lid behavior back towards a sluggish-lid regime. In models with strong plates and initially hot mantles, the plate–mantle coupling is initially weak due to low asthenospheric viscosity. As the asthenosphere cools and dehydrates, its viscosity increases, which strengthens the coupling and results in more efficient surface mobilization. This trend then suddenly reverses, and the system kicks back to the sluggish-lid regime, an unexpected behavior that we investigate further.

In Fig. 3.4, we show the time evolution of several key quantities across this model ensemble. Panel (a) displays the velocity ratio time series with annotated peaks (orange markers) and troughs (blue markers). Panel (b) shows that the asthenospheric viscosity tracks these variations, with low viscosities corresponding to high velocity ratios (i.e., decoupled, sluggish-lid behavior) and increasing viscosity leading to lower ratios and stronger coupling. The mantle temperature evolves smoothly over time (Fig. 3.4c), which rules it out as the direct driver of the transients. The vertically-averaged asthenospheric water content (Fig. 3.4d), however, exhibits variability that mirrors the velocity ratio evolution.

Moreover, transient behavior is closely linked to changes in melt dynamics. Specifically, troughs tend to occur when the depth of first melting shifts to shallower levels (Fig. 3.4e). This change modifies the efficiency of water removal and retention in the mantle, which we elaborate on in the next section. Fig. 3.4f shows that the local minima are associated with plate velocity maxima, and because plate velocity controls the rate of water regassing into the mantle (Eq. 3.18), it provides a plausible feedback mechanism between the surface and the subsurface.

3.3.3 Water-Melt-Viscosity Feedback

Further clustering of these transient evolution scenarios (via k -means applied to normalized velocity ratio time series) reveals a subset of models with high upper mantle

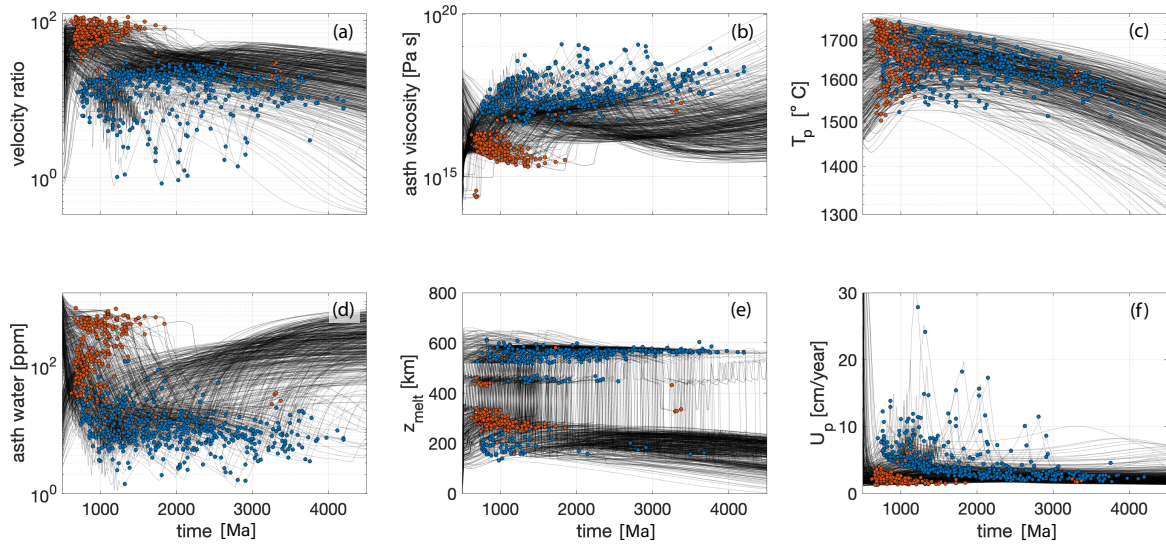


Figure 3.4: **Time evolution of model ensemble with transient events**, with timing of peaks (orange markers) and troughs (blue markers) in the velocity ratio marked. (a) Velocity ratio as a function of time. These curves were used to cluster the data and pick the peak and trough locations. (b) Average asthenospheric water content evolution with the peak and trough timings determined from (a) overlain. (c) Temperature evolution, (d) plate velocity evolution, (e) depth of melt initiation, and (f) mantle vertical velocity at the edge of the convection cell.

water content (i.e., exponentially decreasing with depth), high total mantle water content, moderately strong plates, and elevated initial mantle temperatures. These models typically begin in a sluggish-lid regime and experience early transients. Notably, they lack an early peak in the velocity ratio, instead evolving quickly towards an active-lid state before abruptly reverting to sluggish-lid behavior.

This pattern can be explained through the feedback mechanism illustrated in Fig. 3.5. In a representative model, high upper mantle water content (Fig. 3.5b, g) and elevated mantle temperatures (Fig. 3.5d) lead to deep melting near or within the transition zone (Fig. 3.5a), and rapid water loss from the mantle. As convection cycles material through the melt zone, the upper mantle dries significantly, increasing its viscosity (Fig. 3.5c) and strengthening plate–mantle coupling (Fig. 3.5f). This enhanced coupling accelerates plate motion (Fig. 3.5e), increasing regassing rates that enables rehydration of the upper mantle (Fig. 3.5b, g). Rehydration lowers asthenospheric viscosity and thus weakens coupling,

effectively buffering the system against a full transition to active-lid convection. Similar feedbacks have been proposed by Seales et al. (2022) in the context of whole-mantle water budgets.

However, the abruptness of observed transients suggests that this continuous feedback alone cannot explain the behavior. Instead, we argue that melt dynamics play a critical role. As the upper mantle dries, the intersection of the mantle adiabat with the water-dependent solidus moves to shallower depths. Because the solidus depression tracks the heterogeneous water distribution, the resulting shift in melting depth is abrupt. Initially, when melting occurs deeper in the mantle, any water that is introduced to the upper mantle is likely lost. Once melting becomes restricted to shallower depths, rehydration becomes more efficient, lowering viscosity and forcing the system back toward sluggish-lid convection. Given the exponential dependence of olivine viscosity on water content (Hirth and Kohlstedt, 2003), even modest changes in hydration can drive significant rheological effects.

For the sub-cluster comprised of colder models, melting starts shallower and regassing and mixing gradually increases upper mantle water content. As the mantle warms by radiogenic heating due to the inefficient cooling of the sluggish-lid or hydrates over time, the melting front shifts downward, marking the location of the velocity ratio peaks. The deep melting enhances water loss and initiating a similar feedback loop to that described previously, i.e., an increase in asthenospheric viscosity that strongly couples to the plate, driving increased plate motion and a trend towards the active-lid regime. This progression explains the velocity ratio peaks (i.e., jerks towards sluggish-lid ratios) followed by troughs (reversion to active-lid) observed in some models. Two key characteristics contribute to the abruptness of melt depth changes in these scenarios. The first is that the distribution of water in the upper mantle does not smoothly vary, with much of the water regassed into the mantle at depths less than 200 km lost to arc-style volcanism. Further, as mentioned, we also assume that when the eclogitic crust founders, it stagnates in the mantle transition

zone, and can increase the water content of the transition zone. The second is that the mantle adiabat does not smoothly vary, with discontinuities arising from phase transitions. Together, these characteristics promote sudden shifts in melt depth and, consequently, mantle viscosity structure.

3.3.4 Effect on Mantle Temperature Evolution

Although reproducing Earth’s thermal history is not the primary aim of this study, we briefly examine a subset of our models that end with a mantle potential temperature between 1300 and 1400 °C, consistent with present-day estimates (Fig. 3.6). Approximately 10% of the models fall within this range. As shown in Fig. 3.6a, this final temperature can be reached from a wide range of initial conditions, with total mantle cooling over Earth history varying from modest (~ 150 °C) to substantial (> 300 °C).

Most of these evolutions have smoothly varying temperature and velocity ratio evolutions (Fig. 3.6c), with modest (< 5 cm/year) plate velocities throughout the time of the model (Fig. 3.6b), while a small number of cases exhibit the transient behaviors described in the earlier section. During excursions towards the active-lid regime, the rate of mantle cooling increases, and then returns to a slower cooling rate at the end of the excursion. There is no consistent timing for the transient events.

The most robust predictor of reaching present-day mantle temperatures is the effective plate yield strength, with values clustering around ~ 250 MPa (Fig. 3.6e). There is also a modest preference for models with higher initial water content (Fig. 3.6d), but no clear dependence on the initial radial water distribution (Fig. 3.6f).

The inclusion of time-dependent water evolution and redistribution broadens the range of acceptable initial conditions that yield realistic present-day mantle temperatures. Unlike Al Asad and Lau (2024), who found that high initial temperatures and viscosity ratios were required, we find that a wide range of initial thermal states can converge on plausible outcomes. Given the variability in geochemical estimates of past mantle temperatures

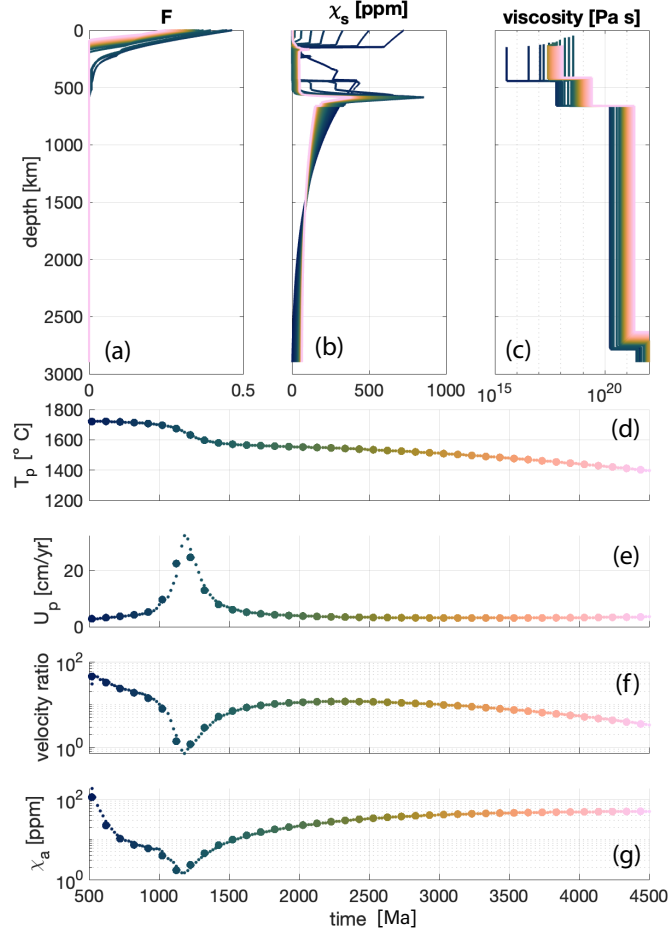


Figure 3.5: **Example of an evolution scenario with transient behavior.** Panels (a)-(c) display 1-D vertical profiles of: degree of partial melt (F), water content (χ_s) in ppm, and mantle viscosity (μ) in Pa s, respectively. Panels (d)-(g) display the time evolution of: mantle potential temperature T_p , plate velocity, velocity ratio, and water content in the asthenosphere, respectively. The colors of the depth-profiles in (a)-(c) correspond to the time they represent, associated with the colored circles in (d)-(g).

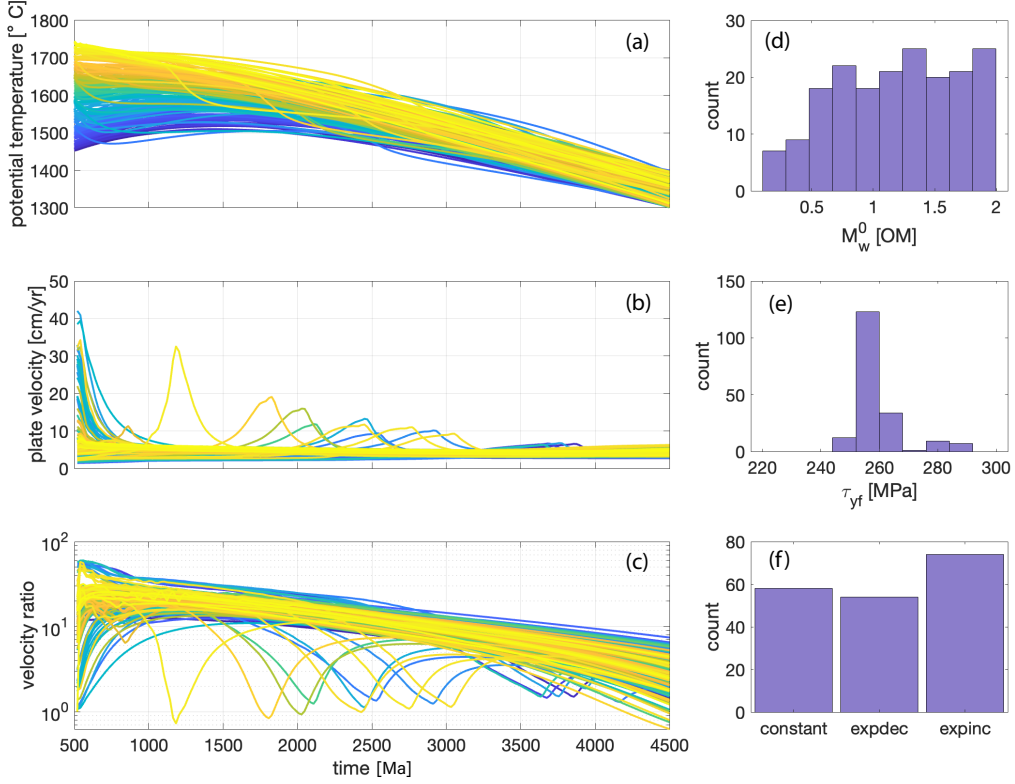


Figure 3.6: **Evolution scenarios which end in mantle potential temperatures between 1300 and 1400 °C.** (a) Mantle potential temperature evolution, (b) plate velocity evolution, (c) velocity ratio evolution. (d) histogram of total initial water content, (e) histogram of effective plate strength, (f) histogram of initial water distribution. Models are color coded by the initial mantle potential temperature.

(e.g. Seales et al., 2022; Keller and Schoene, 2018; Condie et al., 2016), we do not prefer any single thermal trajectory, but instead emphasize that by introducing the modeling mantle water evolution, the realm of possible evolution paths is substantially expanded.

3.4 Discussion

3.4.1 Model Limitations

The results presented here are sensitive to the vertical distribution of water in the mantle, which influences the solidus temperature and thus the depth of melting. We use the pyrolitic solidus from Pierru et al. (2022), which is smooth. However, the mantle

adiabat includes discrete phase transitions, so the crossing point between adiabat and solidus may vary non-smoothly, even when the water profile is smooth. Moreover, we use the peridotite-based melt parameterization of Katz et al. (2003), extrapolated to transition zone and lower mantle conditions. This introduces uncertainty, as the water-dependent melting behavior of transition zone minerals may differ from peridotite.

Additionally, in many of our simulations, water accumulates preferentially in the upper mantle. This contrasts with some observational evidence suggesting that the transition zone may be more hydrated than the upper mantle (Pearson et al., 2014; Ni et al., 2017). This discrepancy likely arises from limitations in the regassing function, including uncertainties in water partitioning, slab dehydration depths, and our neglect of slab stagnation (Wang et al., 2025), all of which are factors that influence water delivery to different mantle layers. However, we note that the work of Van Keken et al. (2011) suggests that most of the slab water content is released at shallow depths, and therefore hydration of the mantle transition zone may be related to other mechanisms, such as the water filter hypothesis (Bercovici and Karato, 2003), or eclogitic enrichment of the transition zone by early crustal foundering, as we find in our model. These assumptions introduce uncertainty into the exact water distribution but do not undermine the broader feedbacks we investigate between water, viscosity, and melting.

A major structural limitation of the model is its one-dimensional geometry. Processes such as mid-ocean ridge melting and subduction hydration occur at spatially distinct regions of the mantle, but our model laterally averages mantle properties. As a result, dehydration and regassing are treated as occurring simultaneously and instantaneously, an unrealistic assumption that can lead to artificial feedbacks. This limitation highlights that our model is not a full representation of mantle convection dynamics, but rather a demonstration that laterally averaged water cycling can drive complex temporal behavior.

Finally, our model assumes a globally oceanic lithosphere (i.e., no continental lithosphere) with convective heat loss at the surface. In reality, the emergence, growth, and

movement of continents modify the global heat budget and influence mantle convection patterns (Jellinek et al., 2020). Continental insulation, cell size variations, and the supercontinental cycle can all impact tectonic regimes. While we do not capture these effects, our goal is not to reconstruct Earth’s tectonic history, but to highlight a plausible feedback mechanism that has been largely underexplored: the coupling between mantle water evolution, melt depth, and viscosity structure.

3.4.2 Implications for Tectonic Transition in Terrestrial Planets

While we have focused our interpretation of the results on Earth, the inferred feedback mechanism likely operates in any terrestrial planet with a broken surface (plates), and material exchange between the surface and the interior via volcanism, delamination and foundering. If liquid water is stable on the surface of a planet, then it may hydrate the crust and lithosphere over time, allowing the crust and lithosphere to carry water into the interior of the planet. The efficiency of water delivery into the subsurface depends on the rate of resurfacing, and in some cases, water may be lost via volcanism faster than it can be replenished via “subduction”.

If there is no surface-to-interior material exchange, and water is not introduced into the upper mantle, the upper mantle might become too stiff. This introduces an additional rheological lithosphere (e.g. Braun et al., 2000), which, as suggested by Korenaga (2003), increases dissipation required to bend and break the lithosphere. The result is a lithosphere that is too stiff for subduction, forming a stagnant-lid. Additionally, extreme viscosity variations and phase changes between the upper and lower mantle result in layered convection (Buffett et al., 1994), inhibiting the redistribution of water within the mantle and affecting the viscosity structure evolution.

These scenarios are plausible for Venus, where liquid water is not stable on the surface and may not be recycled into the interior of the planet. Some geological interpretations of surface topography suggest that Venus may have operated in a mobile-lid regime (Smrekar

et al., 2018; Capitanio et al., 2024). It is possible that this behavior ceased with the drying of the upper mantle (Mian and Tozer, 1990). Testing this hypothesis for Venus within our model formulation requires minor additional developments, namely the inclusion of a rheological lithosphere and true plastic rheology, and will be the focus of future studies.

3.5 Summary and Conclusion

In this study, we investigate how the redistribution of water within the mantle affects the structure of viscosity, the evolution of tectonic regimes, and the thermal history of a terrestrial planet. We develop a 1.5-D thermo-mechanical evolution model that self-consistently couples mantle convection, plate dynamics, water cycling (via melting and regassing), and their feedbacks on mantle rheology. Unlike classical models that rely on parameterized scaling laws, we explicitly track the vertical distribution of water and its evolving impact on viscosity and melting depth, enabling us to probe transient events and feedback mechanisms not captured by bulk-viscosity approaches. We extend the AL24-MC mantle convection model by incorporating a thermochemical water evolution module that captures melting, extraction, regassing, and convective redistribution of water in a layered mantle. Using a suite of 2000 simulations, we vary initial thermal and volatile conditions, along with plate yield stress, to examine the interplay between water cycling and tectonic transitions.

Our model ensemble reveals three broad classes of tectonic evolution: (1) fully active-lid convection, (2) smooth, temperature-controlled transitions toward more active lid behavior, and (3) scenarios that exhibit transient, non-monotonic behavior in the mantle-to-plate velocity ratio, which represent failed transitions. In particular, peaks in the velocity ratio represent the start of an abrupt trend towards full plate-mantle coupling, while troughs indicate reversals away from the active-lid regime. These trends result from internal feedback mechanisms rather than boundary condition changes.

We identify a key feedback involving water content, melt depth, and asthenospheric viscosity. As the upper mantle dehydrates due to deep melting and inefficient regassing, its viscosity increases, enhancing plate–mantle coupling and accelerating plate motion. This in turn increases water delivery into the mantle via subduction. When melting depths become shallower, water can be retained in the upper mantle, reducing viscosity and reversing the trend. These abrupt shifts in melt depth lead to transient reversals in tectonic regime that significantly modulate the rate of mantle cooling.

Finally, we emphasize that our model is not designed to reproduce the specific tectonic history of Earth. Instead, it demonstrates that first-order feedbacks between water distribution, viscosity, and melt structure can lead to complex, emergent behavior even in simplified geometries. Bulk-averaged models may miss these subtleties. Understanding the stability, timing, and climatic implications of such transitions requires further study, particularly in more spatially resolved geometries that can capture the lateral separation of melting and regassing zones.

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Appendix C

Additional Information for Chapter 3

C.1 Mantle Velocity field

We calculate the 2-D velocity field of the mantle, shown in Fig. C.1, from the 1-D horizontal profile and the prescribed vertical flow structure (see Crowley and O’Connell, 2012). The horizontal velocity profile with depth is calculated according to the energy balance equations described in Crowley and O’Connell (2012) and the vertical flow is assumed to be confined to a region of width equal to the lower mantle and lower thermal boundary layer thicknesses ($d_{\text{lm}} + d_{\text{m}}$) of the cell ends. At the depth where the horizontal velocity vanishes (z_{r} , where “r” represents the depth of the return flow), the vertical velocity decreases linearly from a maximum value $v_{\text{m}}^{\text{max}}$, calculated again from the energy and mass conservation equations, to zero at $x = d_{\text{lm}} + d_{\text{m}}$. This flow geometry is imposed in our convection model. Within the upwelling region, the horizontal velocity vector decreases from its value at the center of the cell to zero at each depth. And finally, the maximum vertical velocity at each depth away from z_{r} is calculated based on a depth-wise mass balance according to

$$v_{\text{m}}^{\text{max}} = v_{\text{m}}^{\text{max}}(z_{\text{r}}) \pm u(z_{\text{r}})\Delta z(d_{\text{lm}} + d_{\text{m}}) \quad (\text{C.1})$$

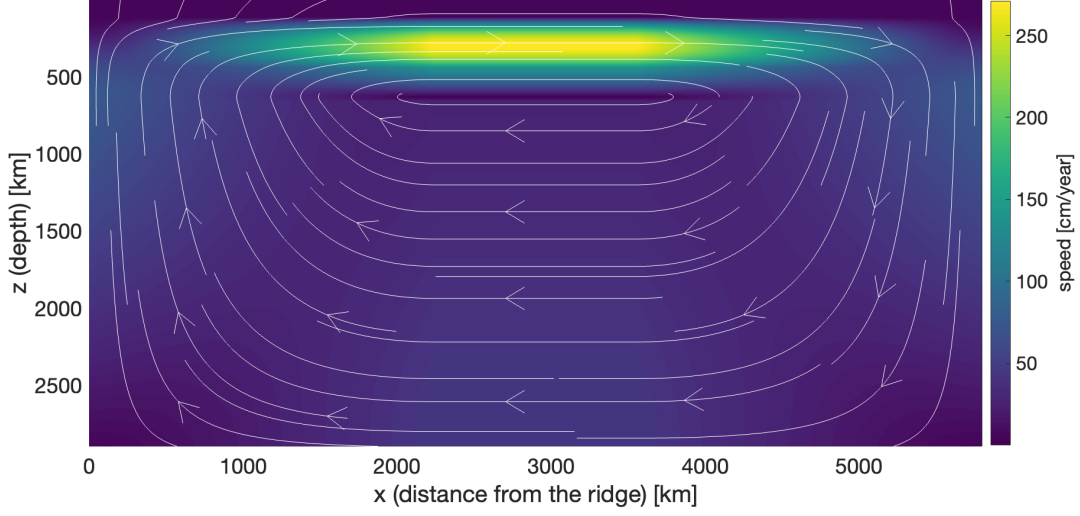


Figure C.1: **2-D velocity flow field:** Derived 2D velocity flow field based on the 1-D profile, the assumed upwelling region width, and local mass conservation. The colors represent the magnitude of the flow field in cm x/year, while the arrow the pattern.

C.2 Mass Conservation During Melting and Crustal Production

To conserve the volume of the model during melting and crustal production, we must adjust the size (depth) of the mantle domain based on the amount of melt extracted from the mantle to form the crust. The compression of the mantle that undergoes partial melting is proportional to the amount of partial melting F . Thus, if the original depth interval vector is Δz such that the depth vector at point i is defined as $z_k = \sum_i^k \Delta z_i$, then we can define the compressed depth vector as $z_k^{\text{new}} = d_c + \sum_i^k \Delta z_i(1 - F_i)$, where d_c is the thickness of the crust produced by extracting that melt.

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CONCLUSION

This dissertation builds on and extends fundamental theories of mantle convection and plate tectonics, with a focus on the evolution of viscosity structure, convective regimes, and deep volatile cycling. By revisiting and expanding analytical frameworks, I have shown that Earth’s tectonic behavior may be governed by feedbacks between water distribution, melt dynamics, and asthenospheric viscosity, feedbacks that can produce abrupt regime transitions and modulate the thermal evolution of the mantle. These insights open several avenues for future inquiry into the coupled dynamics of the solid Earth and its surface and atmospheric systems.

My early work revisited and expanded on the analytical framework of and O’Connell (2012), showing that the convective behavior of a planet can be non-unique for a given thermo-mechanical state. I derived new scaling laws, introduced the O’Connell number to quantify plate resistance, and mapped out a bifurcation space in which multiple stable and unstable solutions may coexist. I then extended this theory to account for time-dependent behavior, performing a stability analysis to identify which regimes are dynamically favored and how perturbations might trigger tectonic transitions.

Building on this foundation, I developed a coupled model of the mantle and core to investigate how these tectonic regimes affect planetary thermal and magnetic evolution. I showed that a transition from a sluggish-lid to an active-lid regime is consistent with the sustained generation of Earth’s magnetic field in the absence of an inner core. This result links convective regime transitions with one of the most enduring features of Earth’s

history, its magnetic field, and demonstrates how subtle changes in mantle dynamics can cascade into profound surface and planetary-scale consequences. I found that the presence of a long-lived asthenosphere is essential in the operation of sluggish-lid convection.

In the latter part of this work, I turned my attention to the evolution of water within the mantle as a possible origin for the asthenosphere and its feedback on tectonic behavior. I developed a semi-parameterized model that tracks water cycling via melt production, regassing, and convective redistribution. I showed that water content influences viscosity both directly, by weakening the mantle, and indirectly, by modifying melt depth and dehydration efficiency. This water–melt–viscosity feedback leads to transient shifts in convective regime that modulate mantle cooling and tectonic behavior, offering a new mechanism for episodic geologic evolution. These findings pave the way for future research aimed at understanding mantle dynamics as a feedback system, not a fixed-state process.

I see this dissertation as laying a foundation for a broader research that seeks to unify mantle dynamics, volatile cycling, and tectonic evolution under a common theoretical framework. My previous work on water–melt–viscosity feedbacks has opened up new questions about the onset and sustainability of plate tectonics, which I will continue to pursue. I am particularly interested in how the redistribution of water throughout Earth’s mantle may have influenced early crustal growth and tectonic stability, and how those changes are encoded in the geologic and petrologic record. These questions invite tighter integration between geodynamic modeling and geochemical constraints.

I also intend to extend the models I have developed to explore the timing and nature of abrupt tectonic events in Earth’s history, such as the Great Oxidation Event. My findings suggest that convective transitions, triggered by water cycling, could lead to large-scale changes in volcanic outgassing and crustal exposure, with direct implications for atmospheric evolution. Connecting these feedbacks to events in the geologic record will be key to understanding how the Earth’s surface environment and interior co-evolve.

Finally, while this work has focused primarily on Earth, the frameworks I've built are designed to be general. They can be adapted to investigate the divergent tectonic histories of other rocky planets, especially Venus. I am particularly motivated by the question of whether differences in deep water cycling can explain the absence of plate tectonics on Venus and its broader implications for planetary habitability. The ability to model both active- and stagnant-lid regimes in a unified way, while accounting for the evolution of water, is central to that effort.

This dissertation has shown that transitions in mantle dynamics, far from being minor fluctuations, may fundamentally shape the evolution of a planet. I look forward to continuing this line of inquiry, building on the conceptual tools developed here to ask deeper questions about the physics of planetary interiors and their connections to the surfaces and we observe.